Title: On quantum corrections to the celestial operator product in gravity

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Abstract: The question of whether the holomorphic collinear singularities of graviton amplitudes define a consistent chiral algebra has garnered much recent attention. I will discuss a version of this question for infinitesimal perturbations around the self-dual sector of 4d Einstein gravity. The singularities of tree amplitudes in such perturbations do form a consistent chiral algebra. However, at loop level new poles are generated, the simplest of which are described the 1-loop effective graviton vertex. These quantum corrections violate associativity of the operator product. I will argue that this failure can be traced to an anomaly in the twistor uplift of self-dual gravity. Associativity can be restored by coupling to an unusual 4th-order gravitational axion, which cancels the anomaly by a Green-Schwarz mechanism. Alternatively, the anomaly vanishes in certain theories of self-dual gravity coupled to matter, including in self-dual supergravity.

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On quantum corrections to the celestial operator product in gravity

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Quantum fields and strings seminar, January 2022

Overview

Recently there's been significant progress in the understanding of quantum self-dual Yang-Mills theory, achieved by exploiting twistor methods. [Costello, 21; Costello, Paquette, 22; Costello, Paquette, Sharma, 22; . . .]

This is closely related to developments in the celestial holography program. [Guevara *et al.*, 21; Strominger, 21; Ball *et al.*, 22; . . .]

This talk concerns the application of similar twistor methods in the context of self-dual gravity. Based on arXiv:2211.06417 [RB, 22] and arXiv:2208.12701 [RB, Sharma, Skinner, 22].

Self-dual Yang-Mills

Let A be a connection on a principal G-bundle P over a 4d Riemannian manifold $\mathcal{M}$. (For G a complex semi-simple Lie group with Lie algebra $\mathfrak{g}$.)

The self-dual Yang-Mills (SDYM) equations read

$$F(A) = *F(A)\,.$$

Introducing a Lagrange multiplier field $B \in \Omega^2_-(\mathcal{M};\mathfrak{g}^\vee)$ can define SDYM as a perturbative quantum field theory using the action

$$S_{\text{SDYM}}[B,A] = \int_{\mathcal{M}} \langle B, F(A)\rangle\,.$$

The SDYM equations are integrable.

Their solutions admit the action of an infinite dimensional hidden symmetry algebra. [Ward, 77; Chau *et al.*, 83; Chakravarty, Mason, Newman, 88]

Closely related to the chiral algebra of positive-helicity asymptotic symmetries arising from soft theorems in YM. [Guevara *et al.*, 21; Strominger, 21] Denote this by $\mathcal{V}_{\mathrm{SDYM}}$.

As a Lie alegbra

$$\mathcal{V}_{\mathrm{SDYM}} = \mathfrak{g}[v^{\dot{1}}, v^{\dot{2}}, z, z^{-1}] \,.$$

As a chiral algebra is generated by $j^{\mathsf{a}}[m,n](z)$ for $\mathsf{a} = 1, \ldots, \dim \mathfrak{g}$ and $m, n \in \mathbb{Z}_{\geq 0}$. OPEs are

$$j^{\mathsf{a}}[p,q](z) j^{\mathsf{b}}[r,s](0) \sim \frac{1}{z} f^{\mathsf{ab}}{}_{\mathsf{c}}\, j^{\mathsf{c}}[p+r, q+s](0) \,.$$

The OPE describes the singularity in a YM tree amplitude as two external positive helicity gluons become collinear.

Also describes the collinear singularities of 1-loop amplitudes (and hence all amplitudes) in SDYM. [Ball *et al.*, 21]

However, recently further argued by Costello and Paquette that $\mathcal{V}_{\mathrm{SDYM}} \oplus_{\mathrm{ad}} \tilde{\mathcal{V}}_{\mathrm{SDYM}}$ describes the collinear limits of tree form factors in SDYM. [Costello, Paquette, 22]

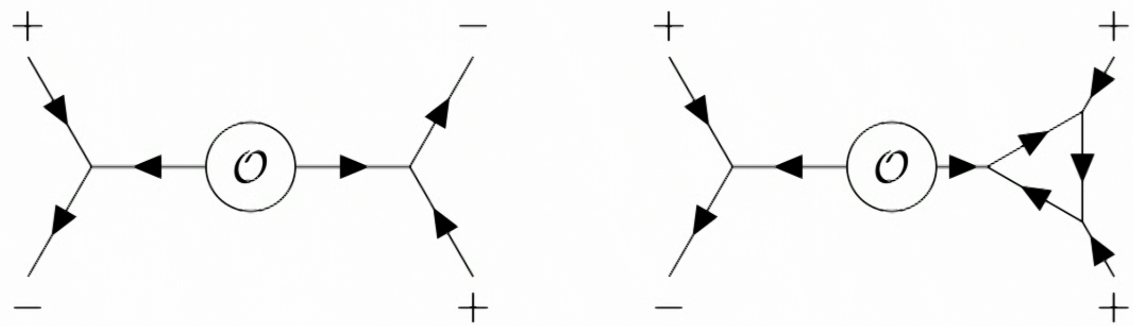
Here $\tilde{\mathcal{V}}_{\mathrm{SDYM}}$ denotes the adjoint of $\mathcal{V}_{\mathrm{SDYM}}$. It is generated by $\tilde{\jmath}^{\mathsf{a}}[m,n](z)$ for $\mathsf{a} = 1, \ldots \dim \mathfrak{g}$ and $m, n \in \mathbb{Z}_{\geq 0}$, and has the following non-trivial OPEs

$$j^{\mathsf{a}}[p,q](z)\tilde{\jmath}^{\mathsf{b}}[r,s](0) \sim \frac{1}{z} f^{\mathsf{ab}}{}_{\mathsf{c}}\,\tilde{\jmath}^{\mathsf{c}}[p+r, q+s](0)\,.$$

Form factors are amplitudes in the presence of a local operator. A particularly interesting local operator is

$$\mathcal{O} = \frac{1}{2}\kappa^{-1}(B, B)\,,$$

for which corresponding tree and 1-loop form factors are:



The collinear limits of form factors in SDYM are modified at 1-loop. They do not define a consistent chiral algebra. [Costello, Paquette, 22]

To get a consistent quantum deformed chiral algebra the all-plus amplitudes must vanish. This can be achieved, e.g., by coupling to appropriate fermionic matter, or something more exotic. [Costello, 21; Costello, Paquette, 22]

There is then a correspondence:

$$\begin{array}{ccc} \text{local operators} & \leftrightarrow & \text{conformal blocks} \\ \text{form factors} & \leftrightarrow & \text{chiral algebra correlators} \end{array}$$

Self-dual Einstein gravity

In this talk we'll see that many of these same ideas apply in the context of self-dual Einstein gravity.

Why consider this?

- 4d Einstein gravity is interesting but challenging.
- Self-dual Einstein gravity is less interesting but less challenging: it's classically integrable, 1-loop exact and finite.
- It retains some important features of full Einstein gravity: it's 4-dimensional and has propagating degrees of freedom.
- It can be deformed to full Einstein gravity, and the simplicity of the self-dual sector can be leveraged to understand this deformation.

Self-dual Einstein gravity

Let g be a Riemannian metric on the 4d manifold $\mathcal{M}$. The self-dual vacuum Einstein equations (without cosmological constant) state that

$$C = *C\,, \qquad \mathrm{Ric} = 0\,,$$

where C denotes the Weyl tensor and Ric the Ricci tensor. Introducing vierbeins

$$g = e^{\dot{\alpha}\alpha} \odot e_{\dot{\alpha}\alpha}\,,$$

the self-dual vacuum Einstein equations can be written as

$$\frac{1}{2}\mathrm{d}\big(e^{\dot{\alpha}\alpha} \wedge e_{\dot{\alpha}}{}^{\beta}\big) = 0\,.$$

These equations are slightly easier to understand from the perspective of the action: the self-dual Palatini action is

$$S_{\mathrm{GR}}[\Gamma, e] = \frac{1}{2}\int_{\mathcal{M}} e^{\dot{\alpha}\alpha} \wedge e_{\dot{\alpha}}{}^{\beta} \wedge \left(\mathrm{d}\Gamma_{\alpha\beta} + \kappa^2 \Gamma_{\alpha}{}^{\gamma} \wedge \Gamma_{\gamma\beta}\right).$$

It differs from the tetradic Palatini action by a topological Nieh-Yan term.

In the weak coupling limit $\kappa^2 \to 0$ we obtain an action for the self-dual vacuum Einstein equations

$$S_{\mathrm{SDGR}}[\Gamma, e] = \frac{1}{2}\int_{\mathcal{M}} e^{\dot{\alpha}\alpha} \wedge e_{\dot{\alpha}}{}^{\beta} \wedge \mathrm{d}\Gamma_{\alpha\beta}\,.$$

Working perturbatively around flat space $e^{\dot{\alpha}\alpha} = \mathrm{d}x^{\dot{\alpha}\alpha} + \delta e^{\dot{\alpha}\alpha}$ we can use this to define self-dual Einstein gravity (SDGR) as a perturbative quantum field theory.

Celestial chiral algebra

The self-dual vacuum Einstein equations are integrable.

Self-dual Einstein metrics are acted upon by an infinite dimensional hidden symmetry group. [Penrose, 76; Park, 90; Dunajski, Mason, 00] Recently has been identified as the chiral algebra of positive-helicity asymptotic symmetries arising from soft theorems in GR. [Guevara *et al.*, 21; Strominger, 21]

Also describes the collinear singularities of 1-loop amplitudes (and hence all amplitudes) in SDGR. [Ball *et al.*, 21]

As Lie algebra it's the loop algebra of $\mathrm{Ham}(\mathbb{C}^2)$, itself the Lie algebra of Hamiltonian vector fields on $\mathbb{C}^2$ equipped with its standard symplectic structure.

Write $\mathcal{V}_{\mathrm{SDGR}} = L(\mathrm{Ham}(\mathbb{C}^2))$ for this infinite dimensional symmetry algebra. As a chiral algebra generated by $w[m,n](z)$ for $m,n \in \mathbb{Z}_{\geq 0}$. OPEs are

$$w[p,q](z)w[r,s](0) \sim \frac{1}{z}(ps - qr)w[p+r-1, q+s-1](0)\,.$$

Some notable features:

- $w[0,0]$ is central.
- $w[1,0]$, $w[0,1]$ correspond to supertranslations.
- $w[2,0]$, $w[1,1]$, $w[0,2]$ correspond to superrotations. They generate an $\mathfrak{sl}_2(\mathbb{C})$ current algebra at level 0.

Connection to amplitudes is clearer if we organise these into 'hard' generating functions.

Interpret z as an inhomogeneous coordinate on $\mathbb{CP}^1$ with $\lambda = (z, 1)$. Then let

$$w(\tilde{\lambda}, \lambda) = \sum_{m,n \in \mathbb{Z}_{\geq 0}} \frac{(\tilde{\lambda}^{\dot{1}})^m (\tilde{\lambda}^{\dot{2}})^n}{m!n!} w[m,n](z) ,$$

in terms of which

$$w(\tilde{\lambda}_1, \lambda_1) w(\tilde{\lambda}_2, \lambda_2) \sim -\frac{[12]}{\langle 12 \rangle} w(\tilde{\lambda}_1 + \tilde{\lambda}_2, \lambda_2) .$$

Here $\langle 12 \rangle = \sqrt{2}(z_1 - z_2)$, $[12] = -\sqrt{2}\epsilon_{\dot{\alpha}\dot{\beta}} \tilde{\lambda}_1^{\dot{\alpha}} \tilde{\lambda}_2^{\dot{\beta}}$.

Forming null momenta $p_i^{\dot{\alpha}\alpha} = \tilde{\lambda}_i^{\dot{\alpha}} \lambda_i^{\alpha}$, we recognise the tree graviton splitting amplitude

$$\mathrm{Split}_{-}^{\mathrm{tree}}(1^+, 2^+) = -\frac{[12]}{\langle 12 \rangle} .$$

There are no local operators in gravitational theories. Next best thing is an infinitesimal deformation:

$$S_{\mathrm{SDGR}} \mapsto S_{\mathrm{SDGR}} + \epsilon \int_{\mathbb{R}^4} \mathcal{O} \,.$$

BRST variation of $\mathcal{O}$ must be de Rham exact $\delta\mathcal{O} = \mathrm{d}\mathcal{O}'$. By ascent corresponds to a local operator of positive ghost number.

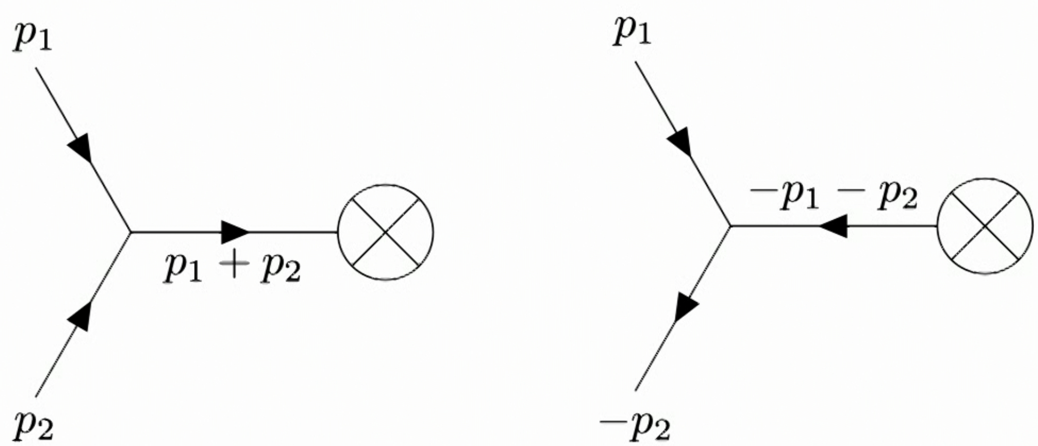
A particularly interesting example is the deformation to full GR

$$\mathcal{O} = \frac{1}{2} e^{\dot\alpha\alpha} \wedge e_{\dot\alpha}{}^{\beta} \wedge \Gamma_{\alpha\gamma} \wedge \Gamma^{\gamma}{}_{\beta} \,.$$

Then can consider amplitudes in the presence of the infinitesimal deformation.

The only non-vanishing tree amplitude in SDGR is the 3-point vertex.

Hence, the collinear singularities of tree amplitudes in the presence of an infinitesimal deformation are universal, i.e., they do not depend on $\mathcal{O}$.



Crossed dot represents a tree amplitude in the deformation by $\mathcal{O}$.

The holomorphic collinear limits of tree graviton amplitudes in infinitesimal deformations of SDGR are described by $\mathcal{V}_{\mathrm{SDGR}} \oplus_{\mathrm{ad}} \tilde{\mathcal{V}}_{\mathrm{SDGR}}$.

Here $\tilde{\mathcal{V}}_{\mathrm{SDGR}}$ is the adjoint, generated by $\tilde{w}[m,n](z)$ for $m,n \in \mathbb{Z}_{\geq 0}$ and with non-vanishing OPEs

$$w[p,q](z)\tilde{w}[r,s](0) \sim \frac{1}{z}(ps - qr)\tilde{w}[p+r-1, q+s-1](0)\,.$$

This encodes the tree splitting amplitude

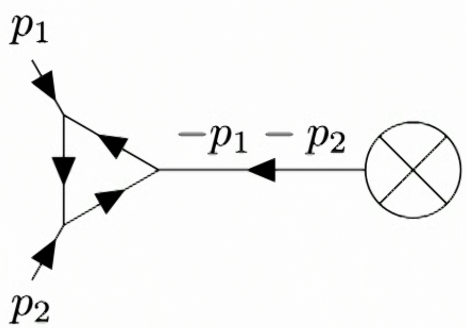
$$\mathrm{Split}_+(1^+, 2^-) = -\frac{[12]}{\langle 12 \rangle}\,.$$

There are no 1-loop splitting amplitudes in gravity, at least in the true collinear limit $p_2 \xrightarrow{\parallel} p_1$. [Bern *et al.*, 98]

However the chiral algebra encodes the singularities in holomorphic collinear limits $\lambda_2 \xrightarrow{\parallel} \lambda_1$, i.e., at fixed $\tilde{\lambda}_1, \tilde{\lambda}_2$. These get new contributions in generic loop amplitudes. [Brandhuber *et al.*, 07; Dunbar *et al.*, 10]

The 1-loop all-plus amplitudes present in SDGR do not acquire new collinear singularities, however the 1-loop amplitudes in generic infinitesimal deformations do see these new contributions.

The leading singularity at 1-loop is generated by the following diagram:



Loop integral is described by the effective graviton vertex

$$\mathcal{M}_3^{1-\mathrm{loop}}(1^+, 2^+, 3^+) = -\frac{\mathrm{i}}{180(4\pi)^2}\frac{[12]^2[23]^2[31]^2}{P_{12}^2}\,.$$

$(P_{ij} = p_i + p_j,\ P_{ij}^2 = \langle ij\rangle[ji].)$

Including propagator gives a 1-loop graviton 'holomorphic splitting amplitude'

$$\mathrm{Split}_+^{1-\mathrm{loop}}(1^+,2^+) = \frac{\mathrm{i}\mathcal{M}_3^{1-\mathrm{loop}}(1^+,2^+,P_{12}^+)}{P_{12}^2} = \frac{1}{180(4\pi)^2}\frac{[12]^4}{\langle 12\rangle^2}\,.$$

This vanishes in the true limit $p_2 \xrightarrow{\parallel} p_1$.

Introduces a new term in the operator product

$$w(\tilde\lambda_1,\lambda_1)w(\tilde\lambda_2,\lambda_2) \sim \mathrm{Split}_+^{1-\mathrm{loop}}(1^+,2^+)\tilde w(\tilde\lambda_1+\tilde\lambda_2,(\lambda_1+\lambda_2)/2)$$
$$\sim \frac{1}{180(4\pi)^2}\frac{[12]^4}{\langle 12\rangle^2}\tilde w(\tilde\lambda_1+\tilde\lambda_2,(\lambda_1+\lambda_2)/2)\,.$$

Decomposing into soft modes gives

$$w[p,q](z)w[r,s](0) \sim \frac{2}{5\pi^2}\frac{R_4(p,q,r,s)}{(4!)^2}\frac{1}{z^2}\tilde{w}[p+r-4,q+s-4]\left(\frac{z}{2}\right),$$

where

$$R_\ell(p,q,r,s) = \sum_{k=0}^{\ell}(-)^k\binom{\ell}{k}[p]_{\ell-k}[q]_k[r]_k[s]_{\ell-k}\,.$$

Remark

$R_\ell(p,q,r,s)$ intertwines $\mathfrak{sl}_2(\mathbb{C})$ representations

$$(\mathbf{p}+\mathbf{q}+\mathbf{1})\otimes(\mathbf{r}+\mathbf{s}+\mathbf{1})\to(\mathbf{p}+\mathbf{q}+\mathbf{r}+\mathbf{s}+\mathbf{1}-\mathbf{2}\ell)\,.$$

In fact intertwines representations of $\mathfrak{sl}_2(\mathbb{C})\ltimes H_3(\mathbb{C})$.

Certainly not the only 1-loop correction - however in general they are tightly constrained by symmetry. The first deformed w, w OPEs are

$$w[3,0](z)w[0,3](0) \sim \frac{\beta_{4,4}}{z} w[0,0]\tilde{w}[0,0](0)\,,$$

$$w[4,0](z)w[0,3](0) \sim \frac{\beta^{2,1}_{5,4}}{z} w[1,0]\tilde{w}[0,0](0) + \frac{\beta^{1,2}_{5,4}}{z} w[0,0]\tilde{w}[1,0](0)\,,$$

$$w[4,0](z)w[0,4](0) \sim \frac{\alpha}{z^2}\tilde{w}[0,0]\left(\frac{z}{2}\right) + \frac{1}{z}\Big(\beta^{3,1}_{5,5} w[1,1]\tilde{w}[0,0]$$

$$+ \beta^{2,2}_{5,5}(:w[1,0]\tilde{w}[0,1]: + :w[0,1]\tilde{w}[1,0]:) + \beta^{1,3}_{5,5} w[0,0]\tilde{w}[1,1]\Big)(0)\,.$$

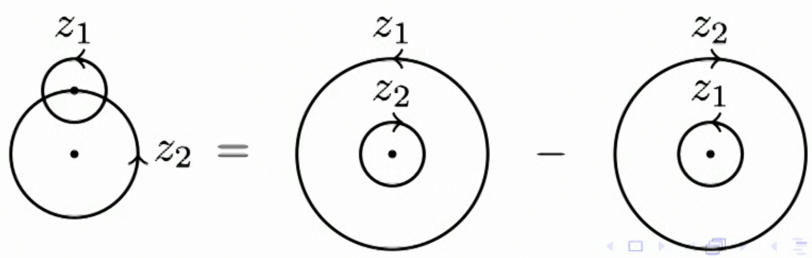
Have seen that $\alpha = 2/5\pi^2$.

In general r.h.s. involves double poles accompanied by $\tilde{w}$ and simple poles accompanied by $:w\tilde{w}:$, $\partial_z \tilde{w}$.

Easy to see that this does not define a consistent chiral algebra. Associativity of the operator product necessitates

$$\oint_{|z_2|=2} \mathrm{d}z_2 \left(\oint_{|z_{12}|=1} \mathrm{d}z_{12}\, \mathcal{O}_1(z_1)\mathcal{O}_2(z_2) \right) \mathcal{O}_3(0)$$
$$= \oint_{|z_1|=2} \mathrm{d}z_1\, \mathcal{O}_1(z_1) \left(\oint_{|z_2|=1} \mathrm{d}z_2\, \mathcal{O}_2(z_2)\mathcal{O}_3(0) \right)$$
$$- \oint_{|z_2|=2} \mathrm{d}z_2\, \mathcal{O}_2(z_2) \left(\oint_{|z_1|=1} \mathrm{d}z_1\, \mathcal{O}_1(z_1)\mathcal{O}_3(0) \right)$$

for any triplet of operators $\{\mathcal{O}_i(z)\}_{i=1}^{3}$. Here we're using the following equivalence of contours.



Choosing

$$\mathcal{O}_1(z) = w[3,0](z)\,, \quad \mathcal{O}_2(z) = zw[0,3](z)\,, \quad \mathcal{O}_3(z) = w[2,2](z)\,.$$

We find that l.h.s. gives

$$\frac{3}{2}\alpha\tilde{w}[0,0](0)\,,$$

whereas r.h.s. vanishes. However, we know that $\alpha = 2/5\pi^2 \neq 0$.

Remark

Tempting to throw away $\tilde{w}[0,0]$, but repeating calculation with

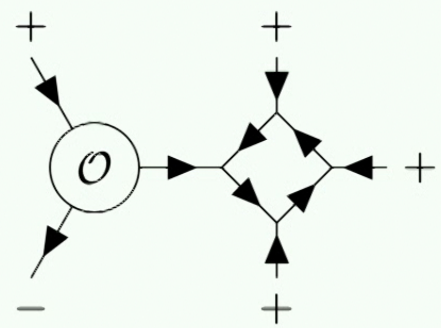
$$\mathcal{O}_1(z) = w[4,0](z)\,, \quad \mathcal{O}_2(z) = zw[0,3](z)\,, \quad \mathcal{O}_1(z) = w[2,2](z)$$

gives a discrepancy $6\alpha\tilde{w}[1,0]$.

This failure of associativity signals that the collinear singularities of amplitudes in infinitesimal deformations are not universal. The 1-loop all-plus amplitudes are the source of this non-universal behaviour.

Example

In the infinitesimal deformation towards full Einstein gravity the 1-loop 5-point mostly-plus amplitudes acquires a non-universal collinear singularity.



Twistors

Penrose's non-linear graviton provides an identification between:

- 4-dimensional manifolds $\mathcal{M}$ with a self-dual vacuum Einstein metric.
- 3-dimensional complex manifolds $\mathcal{PT}$ admitting a holomorphic fibration over $\mathbb{CP}^1$ and an $\mathcal{O}(2)$-valued symplectic form on the fibres. (Together with some further qualifiers.)

$\mathcal{PT}$ is the twistor space of $\mathcal{M}$. [Penrose, 76]

Points $x \in \mathcal{M}$ correspond to rational curves $\mathcal{L}_x \subset \mathcal{PT}$. In Euclidean signature this provides a non-holomorphic fibration $\mathcal{PT} \to \mathcal{M}$ with fibre over x given by $\mathcal{L}_x$.

Example

The twistor space of $\mathbb{R}^4$, denoted $\mathbb{PT}$, is the total space of the vector bundle

$$\mathcal{O}(1) \oplus \mathcal{O}(1) \to \mathbb{CP}^1 \,.$$

Using coordinates $v^{\dot\alpha}$ for $\dot\alpha = \dot 1, \dot 2$ on fibres and z on base the symplectic form is $\mathrm{d}v^{\dot 2} \wedge \mathrm{d}v^{\dot 1}$.

In perturbation theory can construct $\mathcal{PT}$ by deforming the flat twistor space $\mathbb{PT}$. Almost complex structure deformations are encoded in a Beltrami differential

$$\bar\partial \mapsto \bar\nabla = \bar\partial + \mathcal{L}_V$$

for $V \in \Omega^{0,1}(T^{1,0}_{\mathbb{PT}})$. The almost complex structure deformation determined by $\bar\nabla$ is integrable when the Nijenhuis tensor vanishes

$$N = \bar\nabla^2 = \bar\partial V + \frac{1}{2}[V, V] = 0 \,.$$

Almost complex structure deformations of this type preserve the 2-form $\mathrm{d}v^{\dot{2}} \wedge \mathrm{d}v^{\dot{1}}$ if V is Hamiltonian in the sense

$$V = \{h,\ \} = \epsilon^{\dot{\beta}\dot{\alpha}} \partial_{v^{\dot{\alpha}}} h \, \partial_{v^{\dot{\beta}}}$$

for $h \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(2))$. For a Hamiltonian deformation the Nijenhuis tensor is itself Hamiltonian, $N = \{T,\ \}$, where

$$T = \bar{\partial} h + \frac{1}{2}\{h, h\} \in \Omega^{0,2}(\mathbb{PT}, \mathcal{O}(2))\,.$$

A natural twistor action is then (holomorphic) Poisson-BF theory [Mason, Wolf, 09]

$$S_{\mathrm{PBF}}[g, h] = \frac{1}{2\pi \mathrm{i}} \int_{\mathbb{PT}} g \wedge T$$

for $g \in \Omega^{3,1}(\mathbb{PT}, \mathcal{O}(-2))$. Classically equivalent to our perturbative action for SDGR on spacetime. [Sharma, 21; RB *et al.*, 22]

The chiral algebra arises in multiple ways in the twistorial theory [Costello, Paquette, 22]. Most relevant for us, it arises as the universal holomorphic surface defect supported on a twistor line $\mathcal{L}_x$.

Defect couples to Poisson-BF theory via

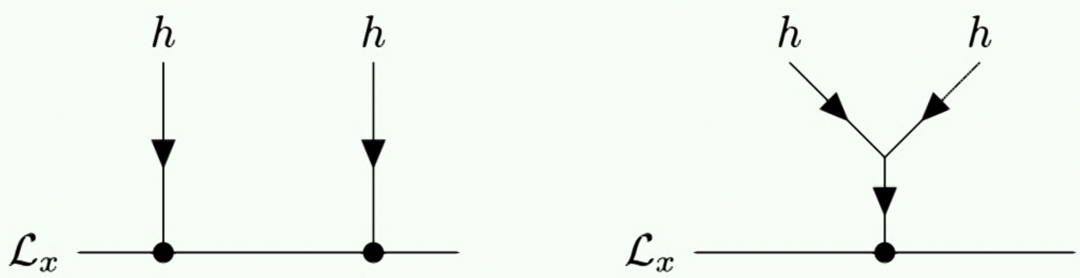
$$\sum_{m,n\in\mathbb{Z}_{\geq 0}} \frac{1}{m!n!}\int_{\mathcal{L}_x}\frac{\mathrm{d}z}{2\pi\mathrm{i}}\left(w[m,n](z)\partial^m_{v^{\dot{1}}}\partial^n_{v^{\dot{2}}}h+\tilde{w}[m,n](z)\partial^m_{v^{\dot{1}}}\partial^n_{v^{\dot{2}}}g\right)$$

for operators $w[m,n](z), \tilde{w}[m,n](z)$ living on the $\mathcal{L}_x$.

OPEs between the operators on the defect are determined by BRST invariance. At tree level we recover $\mathcal{V}_{\mathrm{SDGR}} \oplus_{\mathrm{ad}} \tilde{\mathcal{V}}_{\mathrm{SDGR}}$.

Example

Linearised BRST variation of the following diagrams cancel,

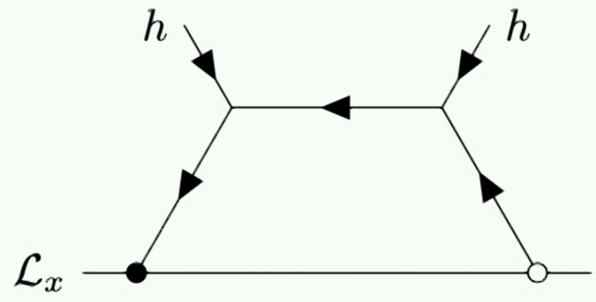


necessitating

$$w[p,q](z)w[r,s](0) \sim \frac{1}{z}(ps - qr)w[p+r-1, q+s-1](0)\,.$$

Example

At 1-loop there are other diagrams which can contribute.



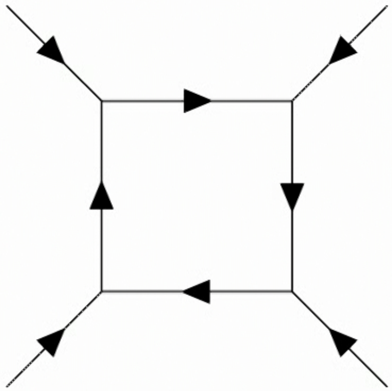
The above necessitates a correction

$$w[4,0](z)w[0,4](0) \sim \frac{2}{5\pi^2}\frac{1}{z^2}\tilde{w}[0,0](0) + \mathcal{O}\left(\frac{1}{z}\right).$$

By symmetry arguments this can be leveraged to get precisely the double poles introduced by the 1-loop effective vertex.

But we know this does not define a consistent chiral algebra. What is going wrong?

Twistor uplift of SDGR suffers from an anomaly which can be attributed to the failure of the following diagram to be BRST invariant. [RB, Sharma, Skinner, 22]



Can be identified with the 4-point 1-loop all-plus amplitudes on spacetime, where it represents a global anomaly in integrability.

Restoring associativity

To obtain a consistent chiral algebra we must cancel the twistorial anomaly, or equivalently eliminate the non-vanishing 1-loop all-plus amplitudes in SDGR.

There are multiple ways of doing this:

- Couple to scalars, fermions, gauge bosons and gravitinos so that a count of the degrees of freedom weighted by Grassmann parity gives 0. This occurs in self-dual SUGRA and chiral higher-spin gravity. In these cases $\alpha = 0$.
- Alternatively couple to an exotic 4^{th}-order scalar on spacetime, cancelling the twistorial anomaly by a Green-Schwarz mechanism. α remains non-vanishing.

On twistor space couple to a field $\eta \in \Omega^{2,1}(\mathbb{PT})$ obeying $\bar\partial\eta = 0$. Action is

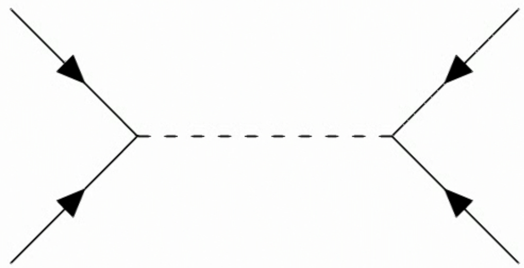
$$S_\eta[\eta; h] = \frac{1}{4\pi \mathrm{i}} \int_{\mathbb{PT}} \left(\bar\partial^{-1}\eta \wedge \bar\nabla \eta + \mu\, \epsilon^{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\, \eta \wedge \partial_{v^{\dot\delta}}\partial_{v^{\dot\alpha}} h \wedge \partial_{v^{\dot\gamma}}\partial_{v^{\dot\beta}} \partial h \right).$$

η descends to a scalar field ρ on spacetime, and the above action becomes

$$S_\rho[\rho; g] = \int_{\mathbb{R}^4} \left(\mathrm{vol}_g \, \frac{1}{2}(\triangle_g \rho)^2 + \frac{\mu}{\sqrt{2}}\, \rho\, R^\mu{}_\nu \wedge R^\nu{}_\mu \right).$$

Here $R^\mu{}_\nu \wedge R^\nu{}_\mu$ is the Pontryagin class, revealing ρ to be a 4th-order gravitational axion.

The twistorial anomaly, or equivalently the 1-loop all-plus amplitudes, are cancelled by tree level axion exchange



if the coupling constant μ is tuned so that

$$\mu^2 = \frac{1}{5!}\left(\frac{\mathrm{i}}{2\pi}\right)^2 .$$

This relies on the following trace identity for the fundamental of $\mathfrak{sl}_2(\mathbb{C})$

$$\mathrm{tr}(X^4) = \frac{1}{2}\mathrm{tr}(X^2)^2 .$$

In particular, our twistorial arguments suggest that SDGR coupled to this 4^{th}-order gravitational axion admits a quantum chiral algebra governing the holomorphic collinear limits of amplitudes in its infinitesimal deformations.

This is a kind of quantum group which plays a role analogous to the Yangian for the principal chiral model.

If so, the previously identified associativity failure should no longer be present. This is indeed the case precisely if

$$\alpha = \frac{2}{5\pi^2} \, ,$$

the same value obtained from the effective vertex, and from the direction calculation on twistor space.

Conclusions

- Asked whether the holomorphic collinear singularities of amplitudes in infinitesimal deformations of SDGR define a consistent chiral algebra.
- True at tree level, but at 1-loop collinear behaviour is modified.
- Do not obtain a consistent chiral algebra - associativity of the operator product is violated.
- Failure can be traced to the presence of 1-loop all-plus amplitudes, which introduce non-universal behaviour.
- From the twistor perspective, chiral algebra describes the universal holomorphic surface defect supported on a twistor line. Associativity failure can be attributed to an anomaly.
- Cancelling the anomaly cures previously identified failure.

Future work

- Can we better understand these quantum groups? Simplest example probably arises for self-dual $\mathcal{N} = 1$ SUGRA.
- In case of SDYM there is a correspondence:

$$\begin{array}{ccc} \text{local operators} & \leftrightarrow & \text{conformal blocks} \\ \text{form factors} & \leftrightarrow & \text{chiral algebra correlators} \end{array}$$

  Is there an analogous statement in gravity?
- Poisson-Chern-Simons theory on twistor space conjecturally describes the $\mathcal{N} = 2$ string on spacetime. What is its chiral algebra?
- Recently a remarkable new holographic duality has been obtained using twistor methods. [Costello, Paquette, Sharma, 22] Is there a SDGR counterpart?

Thank you for listening.