Title: Towards the identification of Quantum Theory: Operational Approach

Speakers: Sutapa Saha

Series: Quantum Foundations

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Abstract: In spite of its immense importance in the present-day information technology, the foundational aspects of quantum theory (QT) remain still elusive. In particular, there is no such set of physically motivated axioms which can answer why Hilbert space formalism is the only natural choice to describe the microscopic world. Hence, to shed light on the unique formalism of QT, two different operational frameworks will be described in the primitive of various convex operational theories. The first one refers to a kinematical symmetry principle which would be proposed from the perspective of single copy state discrimination and it would be shown that this symmetry holds for both classical and QT - two successful descriptions of the physical world. On the other hand, studying a wide range of convex operational theories, namely the General Probabilistic Theories (GPTs) with polygonal state spaces, we observe the absence of such symmetry. Thus, the principle deserves its own importance to mark a sharp distinction between the physical and unphysical theories. Thereafter, a distributed computing scenario will be introduced for which the other convex theories except the QT turn out to be equivalent to the classical one even though the theories possess more exotic state and effect spaces. We have coined this particular operational framework as 'Distributed computation with limited communication' (DCLC). Furthermore, it will be shown that the distributed computational strength of quantum communication will be justified in terms of a stronger version of this task, namely the 'Delayed choice distributed computation with limited communication' (DC2LC). The proposed task thus provides a new approach to operationally single out quantum theory in the theory-space and hence promises a novel perspective towards the axiomatic derivation of Hilbert space quantum mechanics.

References:
Phys. Rev. A (Rapid)100, 060101 (2019)
Ann. Phys.(Berlin)2020,532, 2000334 (2020)
arXiv:2012.05781 [quant-ph](2020)

Zoom link: https://pitp.zoom.us/j/92924188227?pwd=ODJYQXVoaUtzZmZIdFlmcUNIV3Rmdz09

Towards the identification of quantum theory : operational approach

Sutapa Saha

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January 19, 2023

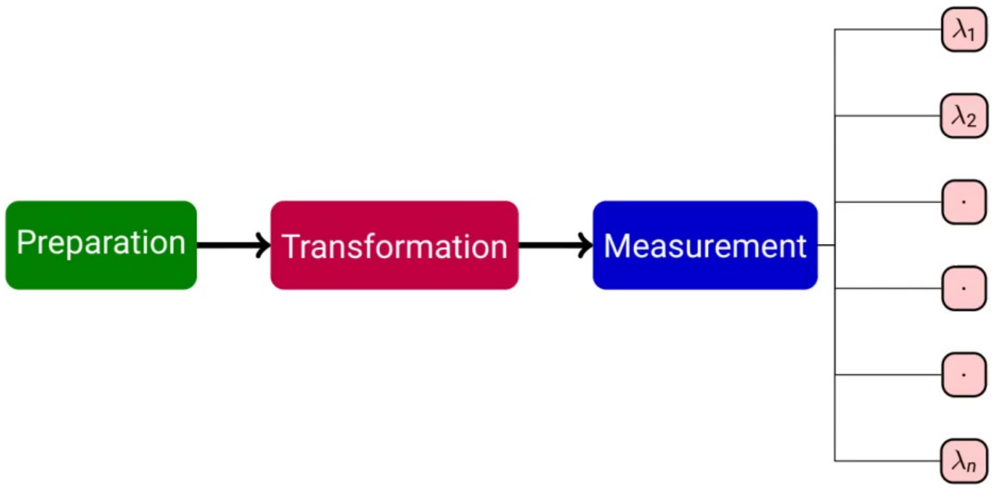
Plan of Talk :

1. Motivation
2. Introduction to Generalized Probabilistic Theories (GPT)
3. How to Study ?
4. GPT in Polygon State Space
5. First Work : Information Symmetry and Polygons
6. Second Work : A task of Distributed Computation : QT and beyond
7. Discussion
8. Other works

Why GPT ?

- Foundational motivation :
  - What geometrical or mathematical structure do you need to represent nature ?
  - How to axiomatize QT ?
  - Does the non-classical phenomena only belong to QT ?
- Operational motivation :
  - How do you certify quantum resources authenticated in a device independent manner ?

Operational theory



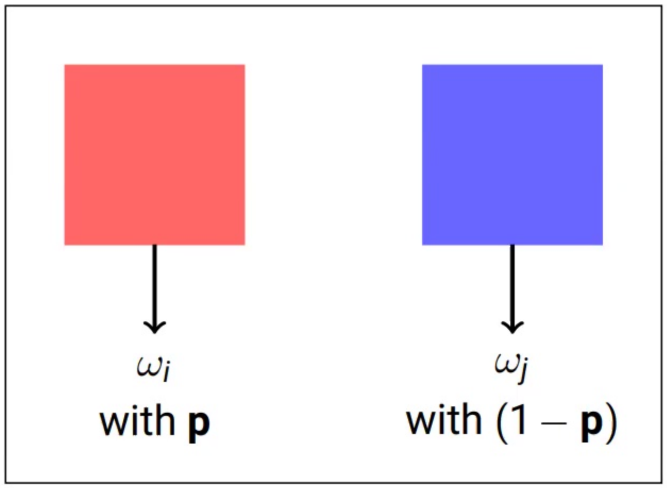
$$\text{State} \longrightarrow \begin{pmatrix} p(a|M_1) \\ p(b|M_1) \\ \vdots \\ --\,-\,-- \\ p(a|M_2) \\ p(b|M_2) \\ \vdots \\ --\,-\,-- \\ \vdots \end{pmatrix} \in \mathbb{R}^n, \; M_1, M_2 \ldots \in \mathcal{F}^1$$

$$\Downarrow$$

3 non coplaner
measurements for
2-level quantum systems

[1] L.Hardy, arXiv: quant-ph/0101012 (2001); J. Barrett, Phys. Rev. A **75**, 032304 (2007).

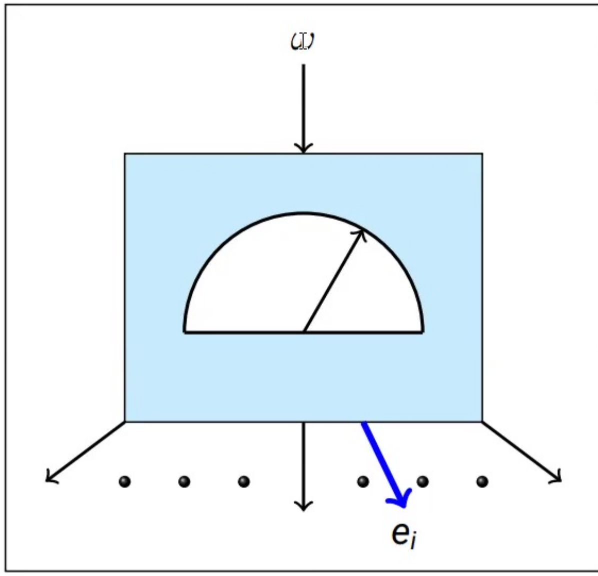
Any physical system $\Rightarrow$ a real vector space $\mathcal{A}$



state of the ensemble
$\mathbf{p}\omega_i + (1-\mathbf{p})\omega_j$
$\omega_i, \omega_j \in \Omega_{\mathcal{A}}$

$\Omega_{\mathcal{A}}$ is convex

Mathematical Description of GPT : Observables

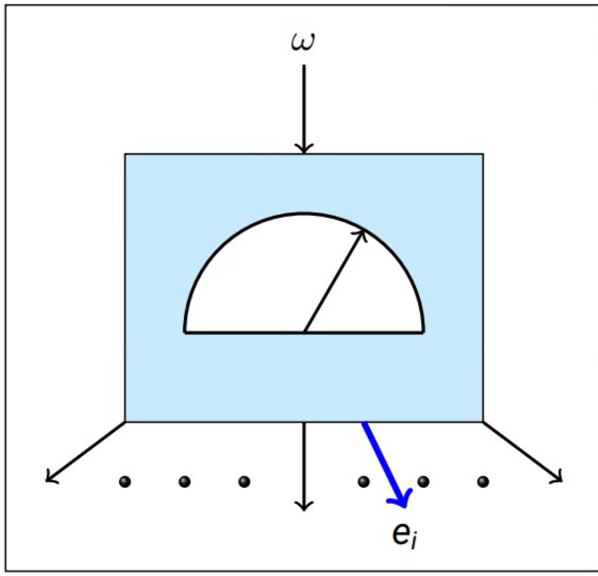


$$\mathbf{p}_i = e_i(\omega) \in [0,1]$$

$$\mathcal{E}_{\mathcal{A}} \equiv \{e_i\} : \Omega_{\mathcal{A}} \mapsto [0,1]$$

Mathematical Description of GPT : Observables

$$\sum_i \mathbf{p}_i = \sum_i e_i(\omega) = u(\omega)$$



$$\mathbf{p}_i = e_i(\omega) \in [0,1]$$

$$\mathcal{E}_{\mathcal{A}} \equiv \{e_i\} : \Omega_{\mathcal{A}} \mapsto [0,1]$$

$$u \in \mathcal{E}_{\mathcal{A}} : u(\omega) = 1, \quad \forall \omega \in \Omega_{\mathcal{A}}$$

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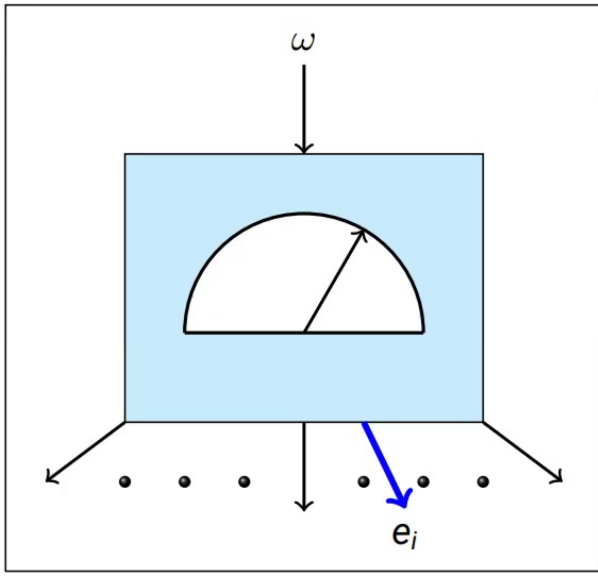


$$\mathbf{p}_i = e_i(\omega) \in [0,1]$$

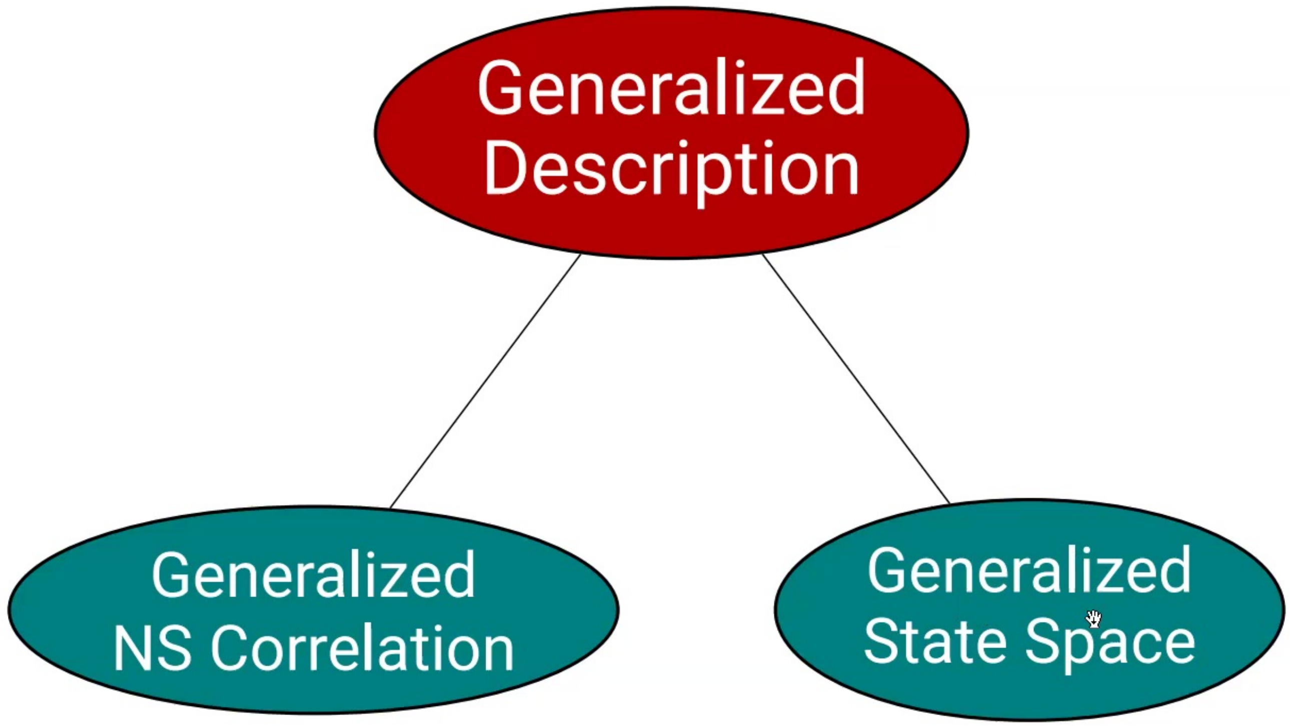
$$\mathcal{E}_{\mathcal{A}} \equiv \{e_i\} : \Omega_{\mathcal{A}} \mapsto [0,1]$$

$u \in \mathcal{E}_{\mathcal{A}} : u(\omega) = 1,$
$\forall \omega \in \Omega_{\mathcal{A}}$

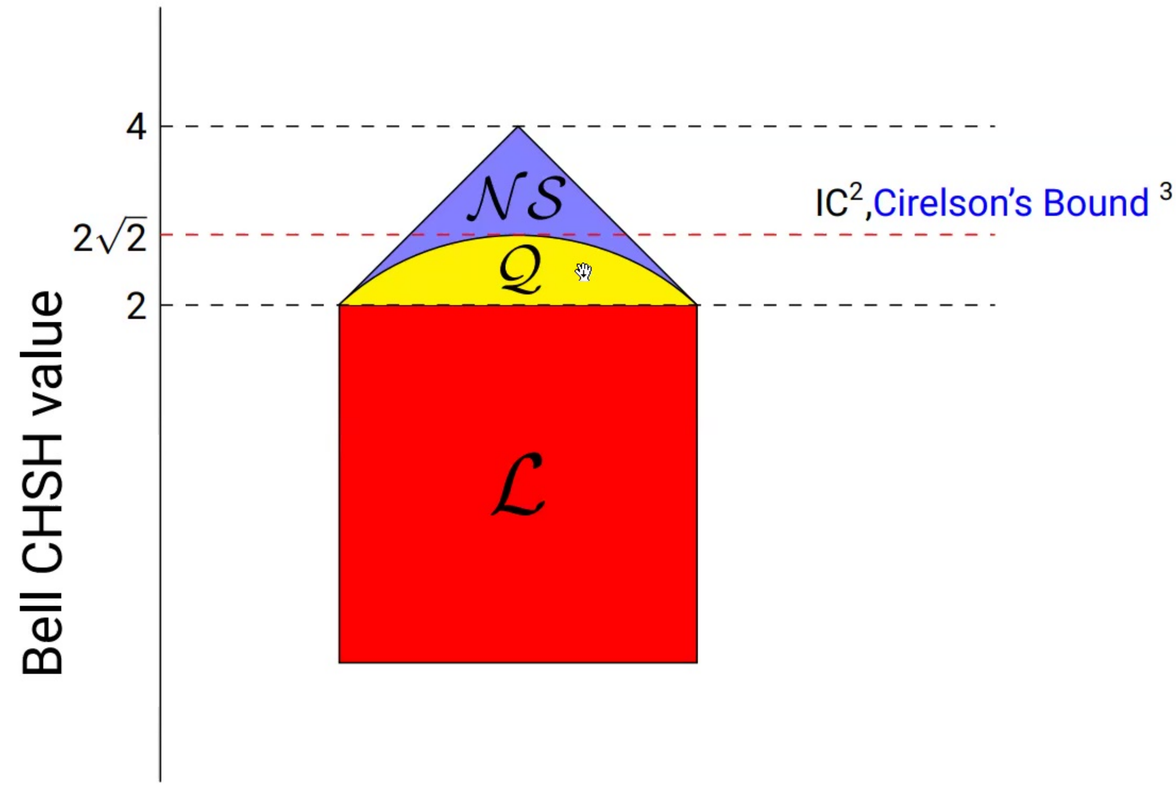
Functional
or, effects

$$e_k[p\omega_i + (1-p)\omega_j] = pe_k(\omega_i) + (1-p)e_k(\omega_j),$$
$$\forall e_k \in \mathcal{E}_{\mathcal{A}} \;\&\; \forall \omega_i, \omega_j \in \Omega_{\mathcal{A}}$$

Two different approaches

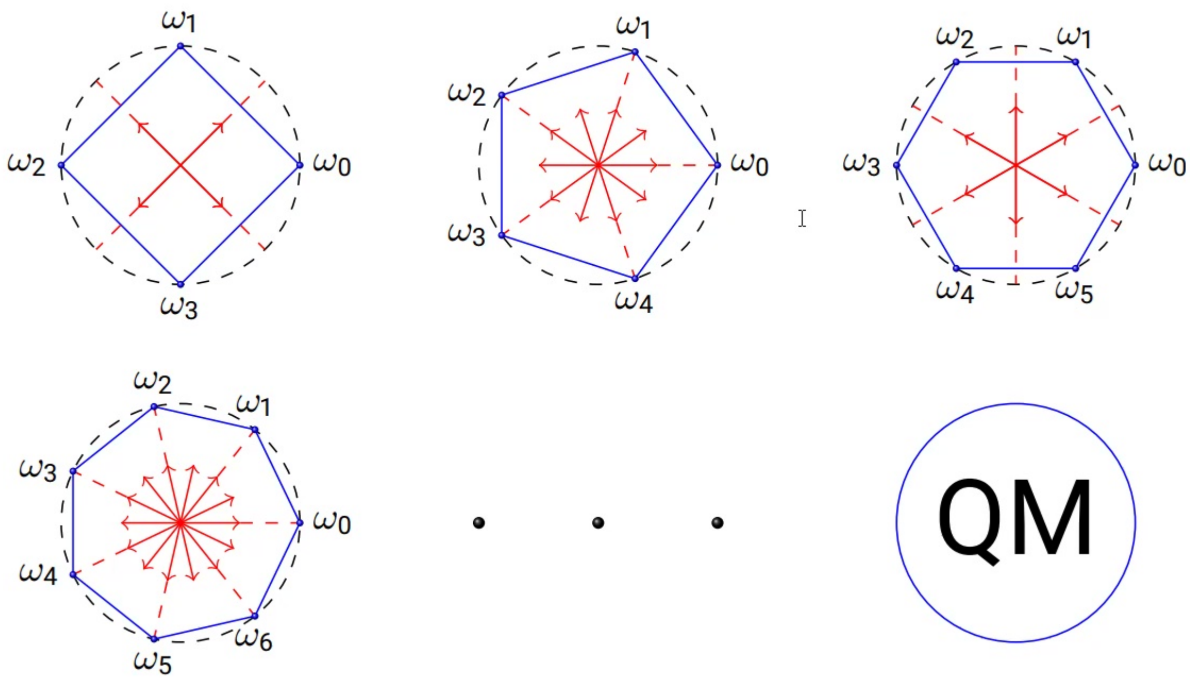


Generalized NS Correlation



[2] M. Pawlowski, *et. al.*, Nature, **461**, 1101 (2009)
[3] B. S. Cirelson, Letters in Math. Phys. **4**, (2), 93 (1980)

Generalized State Space : Why Polygons ?



- M. P. Muller and C. Ududec, Phys. Rev. Lett., **108**, 130401 (2012)
- P. Janotta, *et. al.*, New J. Phys., **13**, 06324 (2011)
- P. Janotta and H. Hinrichsen, J. Phys. A: Math. Theo. **47**, 32 (2014)
- M. Winczewski, *et. al.*, arXiv:1810.02222 (2018)

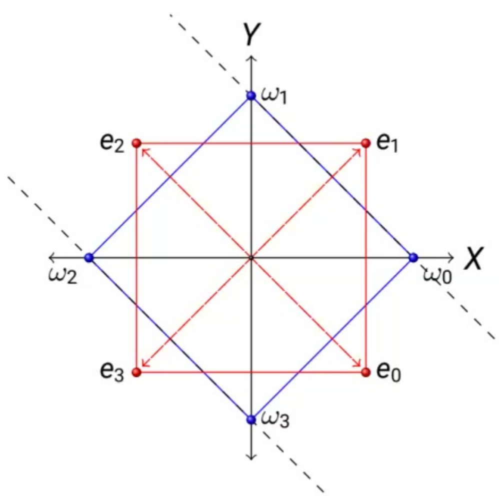
Even-gon

State Space $\rightarrow \ \Omega_n \equiv \{\omega_i\}$

$$\omega_i := \begin{pmatrix} r_n \cos \frac{2\pi i}{n} \\ r_n \sin \frac{2\pi i}{n} \\ 1 \end{pmatrix}$$

where, $i \in \{0, \cdots, (n-1)\}$ &

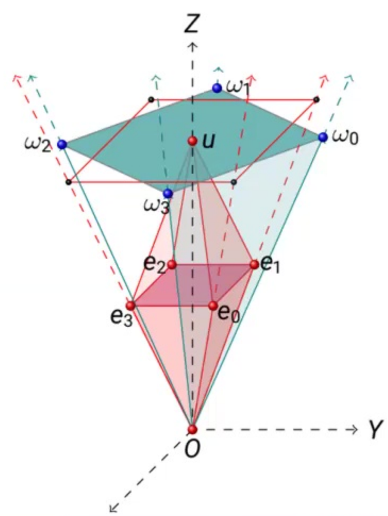
$$r_n = \sqrt{\sec \frac{\pi}{n}}$$



Effect Space $\rightarrow \ \mathcal{E}_n \equiv \{e_i, u, \Theta\}$

$$e_i := \frac{1}{2}\begin{pmatrix} r_n \cos \frac{(2i-1)\pi}{n} \\ r_n \sin \frac{(2i-1)\pi}{n} \\ 1 \end{pmatrix};$$

$$u := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \Theta := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$



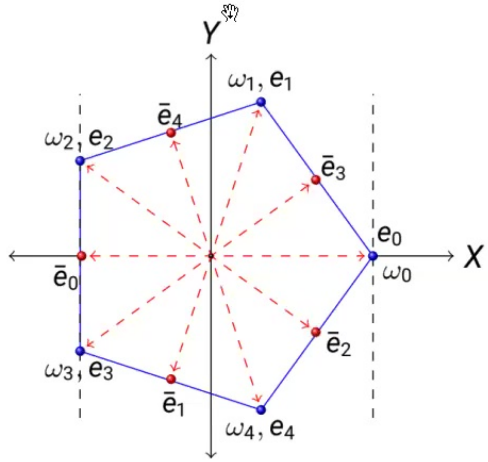
Odd-gon

State Space $\rightarrow \Omega_n \equiv \{\omega_i\}$

$$\omega_i := \begin{pmatrix} r_n \cos \frac{2\pi i}{n} \\ r_n \sin \frac{2\pi i}{n} \\ 1 \end{pmatrix}$$

where, $i \in \{0, \cdots, (n-1)\}$ &

$$r_n = \sqrt{\sec \frac{\pi}{n}}$$



Effect Space $\rightarrow \mathcal{E}_n \equiv \{e_i, u, \Theta\}$

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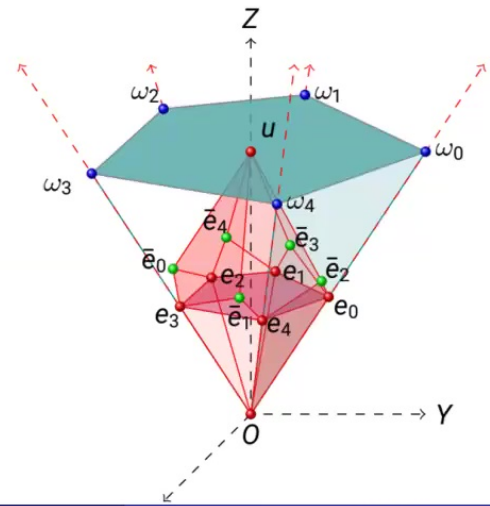
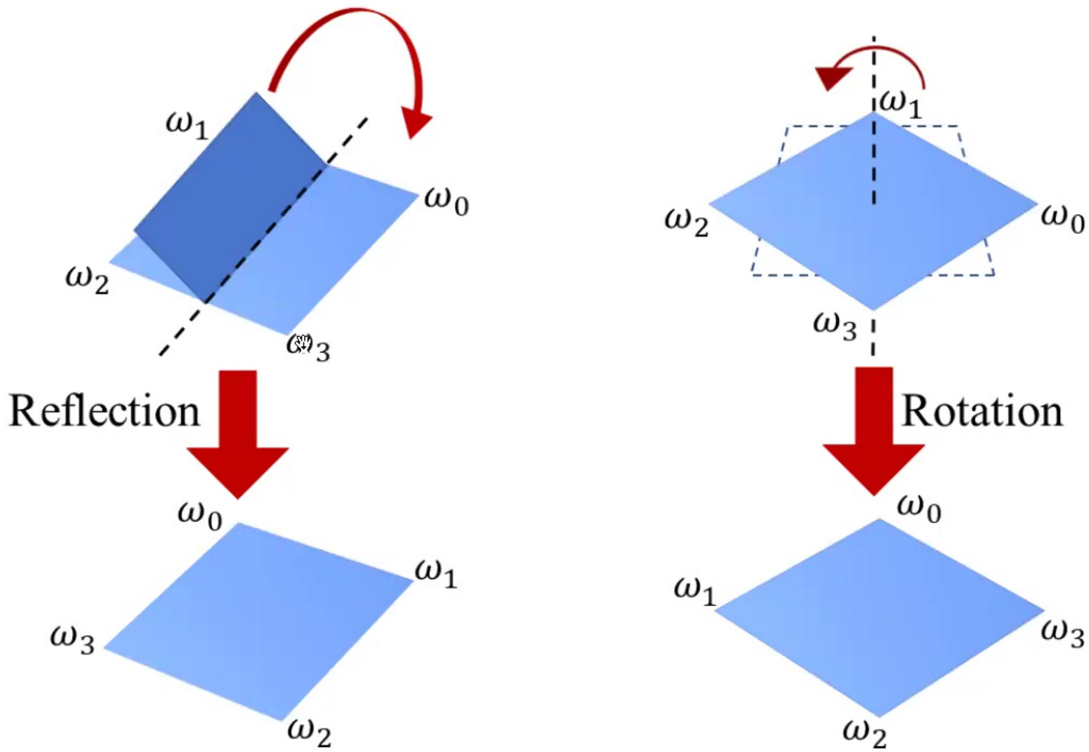
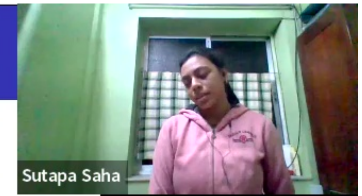


Transformations :

$$\mathbb{T}_n \equiv \{\mathcal{T}_k^p \,|\, k \in \{0, \cdots, (n-1)\};\; p \in \{+,-\}\}$$



Bipartite Composition : State & Effect Space :



State Space

$$\boxed{\Omega_n^A \otimes_{min} \Omega_n^B \subseteq \Omega_n^{AB} \subseteq \Omega_n^A \otimes_{max} \Omega_n^B}$$

Effect Space

$$\boxed{\mathcal{E}_n^A \otimes_{min} \mathcal{E}_n^B \subseteq \mathcal{E}_n^{AB} \subseteq \mathcal{E}_n^A \otimes_{max} \mathcal{E}_n^B}$$

$$\Omega_n^A \otimes_{max} \Omega_n^B \equiv \text{upperbound for } \Omega_n^{AB}$$

$$= 0 \leq \mathrm{Tr}[(\mathcal{E}_n^A \otimes_{min} \mathcal{E}_n^B)^T \Omega_n^{AB}] \leq 1$$

$$\mathcal{E}_n^A \otimes_{max} \mathcal{E}_n^B \equiv \text{upperbound for } \mathcal{E}_n^{AB}$$

$$= 0 \leq \mathrm{Tr}[\mathcal{E}_n^{AB^T} (\Omega_n^A \otimes_{min} \Omega_n^B)] \leq 1$$

Bipartite Compositions : Transformation & Models :

Transformation

$$\mathbb{T}_{n\otimes 2} := \mathbb{T}^{AB} \equiv \{\mathcal{S}, \mathcal{T}_k^p \otimes \mathcal{T}_l^q | \, k,l \in \{0,\cdots,(n-1)\};\ p,q \in \{+,-\}\}$$
$$\mathcal{S}(\omega^A \otimes \omega^B) = \omega^B \otimes \omega^A;\ \forall\, \omega^A \in \Omega^A \ \&\ \omega^B \in \Omega^B$$

Models

- **Type-I** $\downarrow$ $\{\Omega^{product} \bigcup \Omega^{entangled}, \mathcal{E}^{product}\}$ $\Downarrow$ PR Model[4]
- **Type-II** $\downarrow$ $\{\Omega^{product}, \mathcal{E}^{product} \bigcup \mathcal{E}^{entangled}\}$ $\Downarrow$ HS Model[5]
- **Type-III** $\downarrow$ Intermediate $\Downarrow$ Hybrid & Frozen Model[7]

[4] S. Popescu & D. Rohrlich, Found. Phys., **24**, 379-385 (1994)
[5] M. Dall'Arno *et.al.*, Phys. Rev. Lett, **119**, 020401 (2017)

First Work

Information Symmetry and Polygons[6]

[6] Phys. Rev. A (Rapid Comm.) **100**, 06010 (2019)

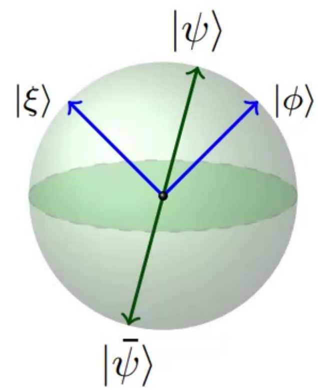
State Discrimination

Classical Theory

$\Downarrow$

Perfect Discrimination

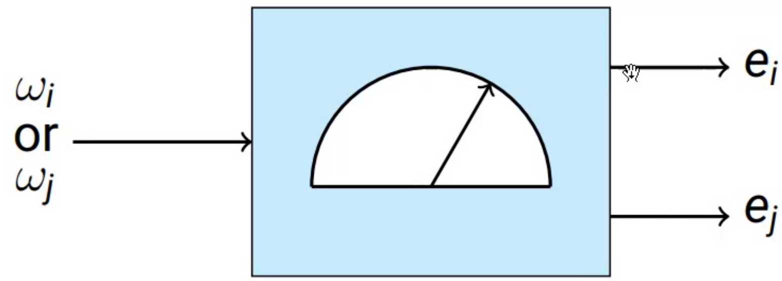
Quantum Theory



$\langle\psi|\bar{\psi}\rangle = 0 \Rightarrow$ **Perfect Discrimination**

$\langle\phi|\xi\rangle \neq 0 \Rightarrow$ **Probabilistic Discrimination**

Minimum Error State Discrimination



$$\mathbb{M}^\star \equiv \{e_i, e_j \mid e_i + e_j = u\}$$

$$p_E|_{min} = \frac{1}{2}\left[p_{ij} + p_{ji}\right] = \frac{1}{2}\left[p(e_i|\omega_j) + p(e_j|\omega_i)\right]$$

Generalized Information Symmetry (GIS)

Principle

Each pair of states having identical minimal type of subjective ignorance can be optimally discriminated with symmetric error measurement.

$$\omega_i = p\omega_m + (1-p)\omega_n$$
$$\omega_j = q\omega_k + (1-q)\omega_l$$
$$p = q$$
$$\Downarrow$$

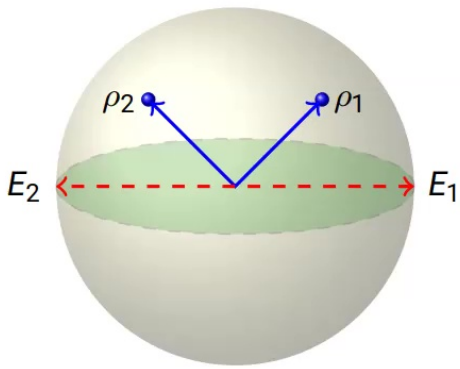
identical minimun
subjective ignorance

$$\boxed{\textbf{GIS} \Rightarrow \; p_{ij} = p_{ji}}$$

Generalized Information Symmetry (GIS)

Principle

Each pair of states having identical minimal type of subjective ignorance can be optimally discriminated with symmetric error measurement.



$$\rho_1 = \frac{1}{2}(\mathbb{I} + \vec{r}_1.\vec{\sigma})$$

$$\rho_2 = \frac{1}{2}(\mathbb{I} + \vec{r}_2.\vec{\sigma})$$

identical minimun subjective ignorance

$$\Downarrow$$

$$\boxed{|\vec{r}_1| = |\vec{r}_2|}$$

MESD measurement [7] $\Rightarrow$ $(\vec{r}_1 - \vec{r}_2).\vec{\sigma}$

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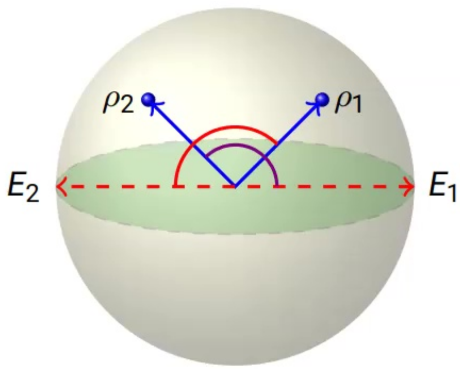

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identical minimun subjective ignorance

$$\Downarrow$$

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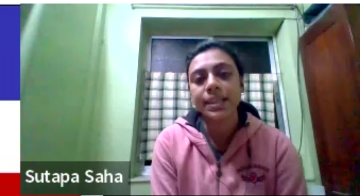
MESD measurement [7] $\Rightarrow$ $(\vec{r}_1 - \vec{r}_2).\vec{\sigma}$

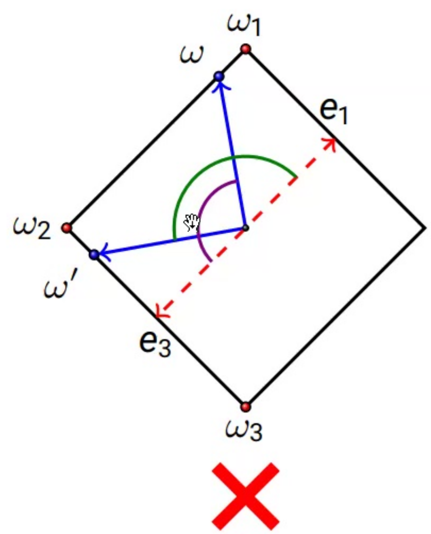
[7] C. W. Helstrom, J. Stat. Phys., **1**, 239 (1969)

Generalized Information Symmetry (GIS)



Principle

Each pair of states having identical minimal type of subjective ignorance can be optimally discriminated with symmetric error measurement.



$$\omega = p\omega_1 + (1-p)\omega_2$$
$$\omega' = p\omega_2 + (1-p)\omega_3$$

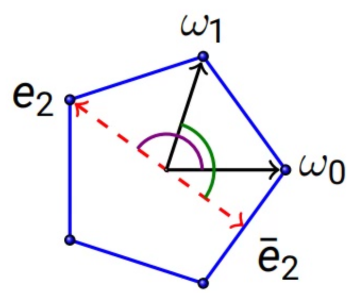
A special case : Information Symmetry (IS)

What about the satisfaction of GIS for the rest of polygons ?

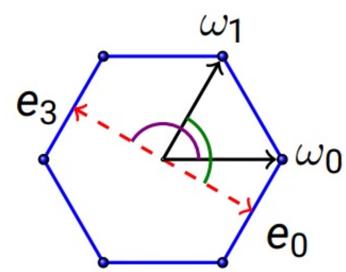
$$\Downarrow$$

No subjective ignorance

IS Principle : Each pair of pure states can be optimally discriminated with symmetric error measurement.

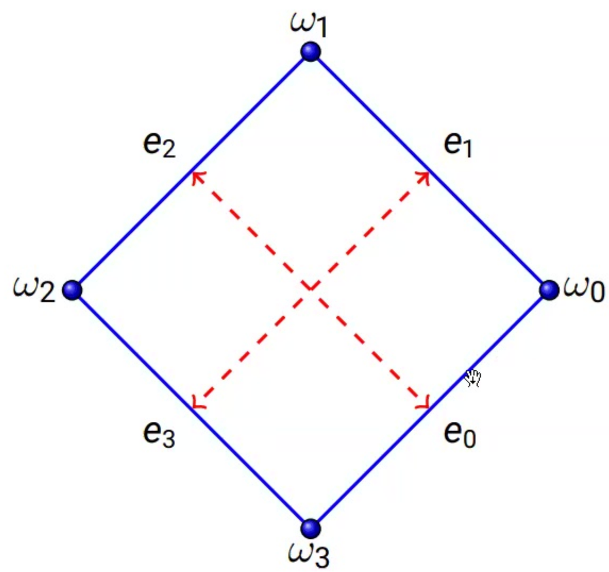


$p(e_2|\omega_0) \neq p(\bar{e}_2|\omega_1)$



$p(e_0|\omega_1) \neq p(\bar{e}_3|\omega_0)$

IS and Squit



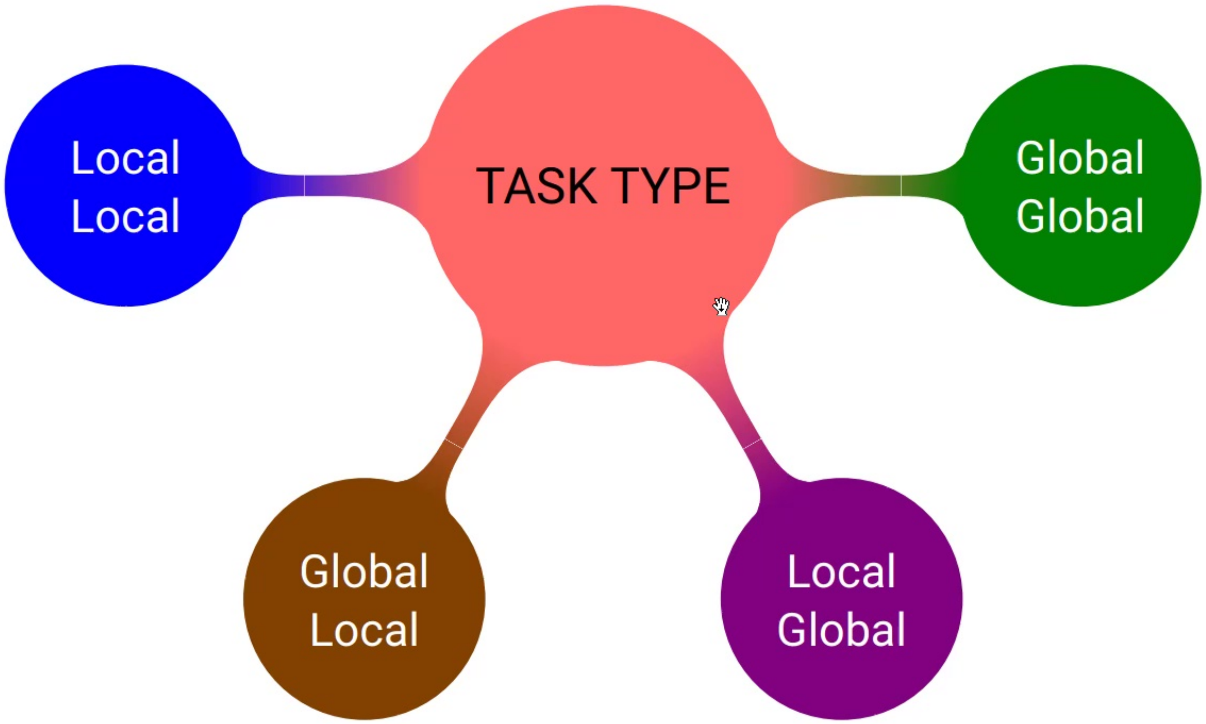
Each pair is perfectly distinguishable

Second Work

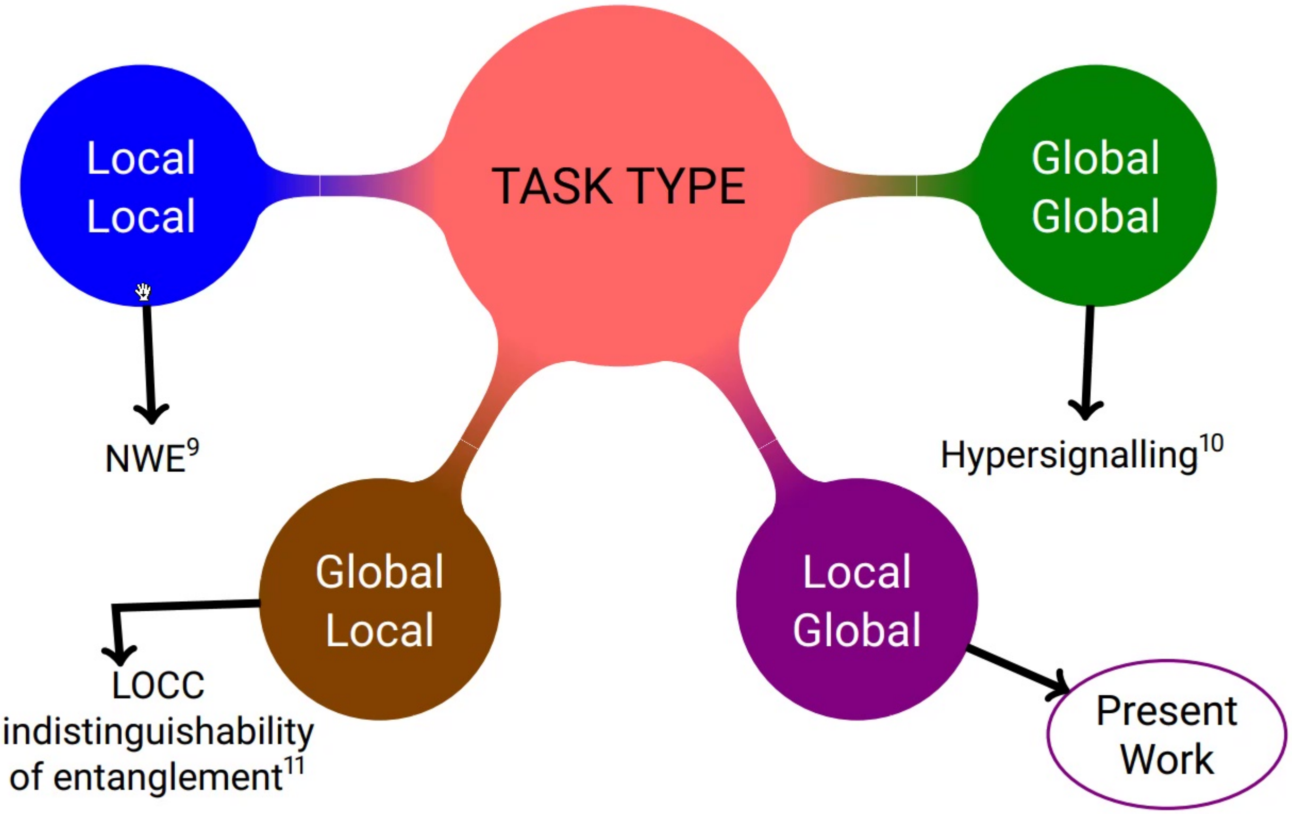
A task of Distributed Computation : QT and beyond [8]

[8] arXiv: 2012.05781[quant-ph] & Ann. Phys. (Berlin) 2000334 (2020)

Different Information Processing Tasks :



Different Information Processing Tasks :

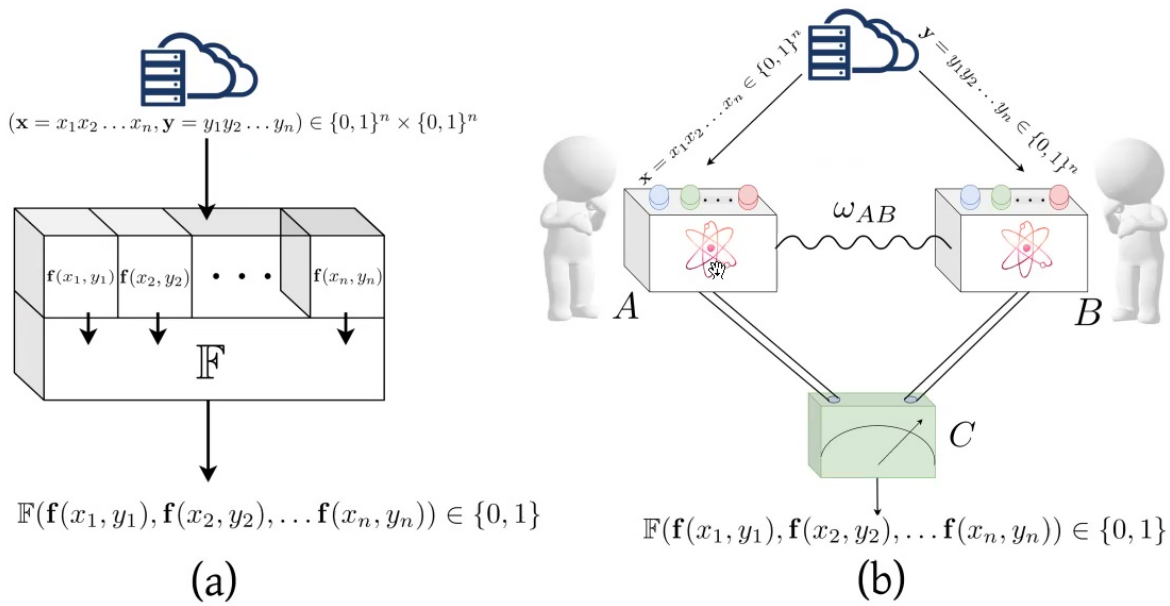


[9] C. H. Bennett, *et. al.*, Phys. Rev. A, **59**, 1070 (1999); S. S. Bhattacharya, *et. al.*, Phys. Rev. Research **2**, 012068(R) (2020)

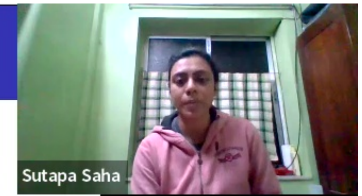
[10] M. Dall'Arno, *et. al.*, Phys. Rev. Lett. **119**, 020401 (2017)

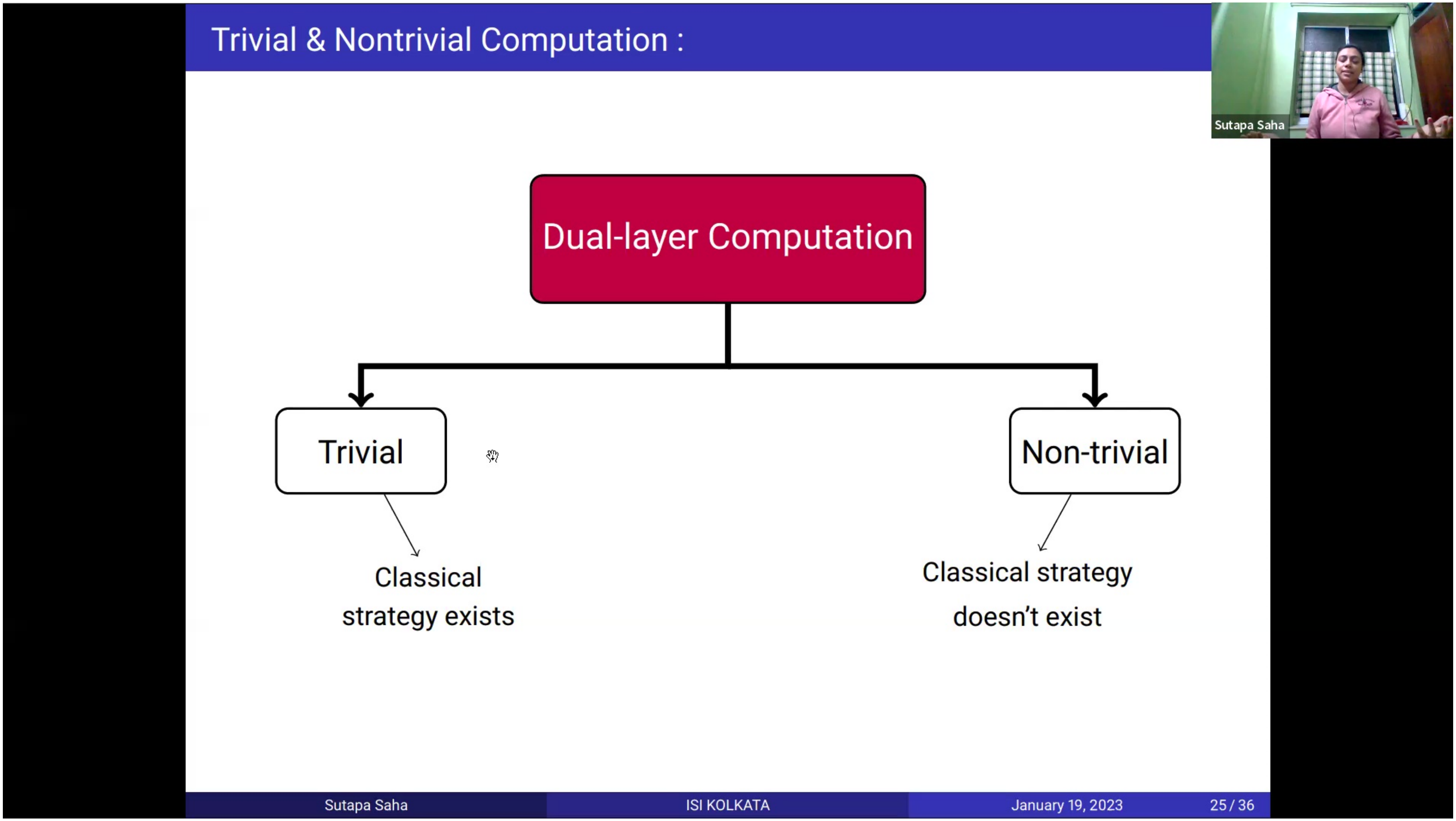
[11] J. Walgate, *et. al.* , Phys. Rev. Lett. **85**, 4972 (2000); M. Banik, *et. al.*, Phys. Rev. Lett. **126**, 210505 (2021)

Distributed Computing Scenario :



Total no. of dual layer computation $(\mathbb{F}, \mathbf{f})$ for DCLC(n) = $2^{2^2} \times 2^{2^n}$




Trivial & Nontrivial Computation :
Dual-layer Computation
Trivial
Non-trivial
Classical strategy exists
Classical strategy doesn't exist
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January 19, 2023
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Sutapa Saha

Propositions :

Proposition 1 :

1. At least one of $\mathbb{F}$ or $\mathbf{f}$ $\rightarrow$ a constant function.
2. At least one of $\mathbb{F}$ or $\mathbf{f}$ $\rightarrow$ a single bit function.
3. $\mathbb{F}$ $\rightarrow$ a symmetric function on inputs & either $\mathbf{f}(a_1, a_2) = \mathbb{F}(a_1, a_2)$ or $\mathbf{f}(a_1, a_2) = \mathbb{F}(\bar{a}_1, \bar{a}_2)$.

$\downarrow$

Total no. of trivial computations in DCLC(2) $= 176$

Corollary : Satisfaction of any condition of Proposition 1 $\implies$ $(\mathbb{F}, \mathbf{f}) \in$ DCLC(n)is trivial ($n \geq 2$).

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Corollary : Satisfaction of any condition of Proposition 1 $\implies$ $(\mathbb{F}, \mathbf{f}) \in$ DCLC(n)is trivial ($n \geq 2$).

Proposition 2 :

Perfect accomplishment of any nontrivial $(\mathbb{F}, \mathbf{f}) \in$ DCLC(2) $\implies$ Presence of entanglement in theory.[a]

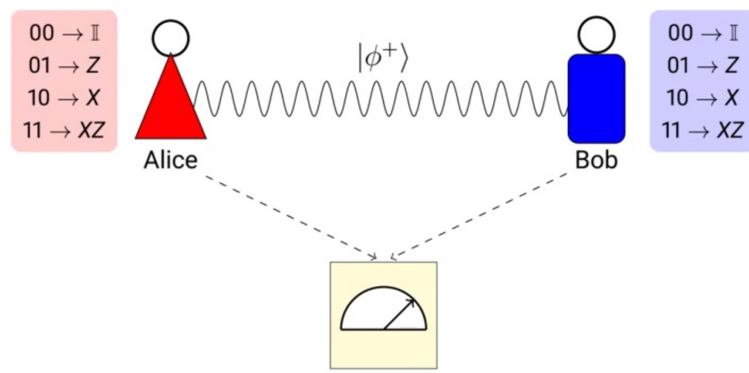
[a] G. P. Barker, Lin. Alg. Appl., **39**, 263 (1981);
G. Auburn, *et. al.*, arXiv:1911.09663 [math.FA] (2019)

Main Results :

Theorem 1 :

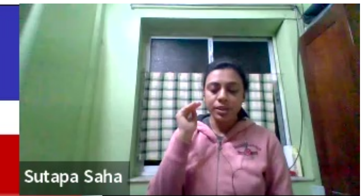
f is a balanced function $\Longleftrightarrow$ perfect accomplishment of a nontrivial $(\mathbb{F}, \mathbf{f}) \in$ DCLC(2) in QT .

A special case : $(\mathbb{F} \equiv \vee, \mathbf{f} \equiv \oplus)$



$$\mathbb{M} \equiv \{|\phi^+\rangle\langle\phi^+|, \mathbb{I} - |\phi^+\rangle\langle\phi^+|\}$$

Main Results :



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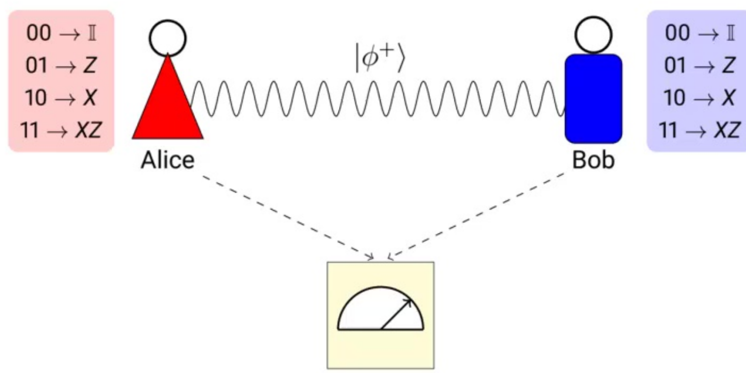


$$\mathbb{M} \equiv \{|\phi^+\rangle\langle\phi^+|, \mathbb{I} - |\phi^+\rangle\langle\phi^+|\}$$

Theorem 2 :

No nontrivial accomplishment with any extreme bipartite polygon model.

The generalisation : DCLC(n)

1. Satisfaction of any condition of Proposition 1 $\implies$ $(\mathbb{F}, \mathbf{f}) \in \text{DCLC}(n)$ is trivial $(n \geq 2)$.
2. Balanced $\mathbf{f}$ $\implies$ quantum accomplishment of nontrivial $(\mathbb{F}, \mathbf{f}) \in \text{DCLC}(n)$ where $n > 2$.

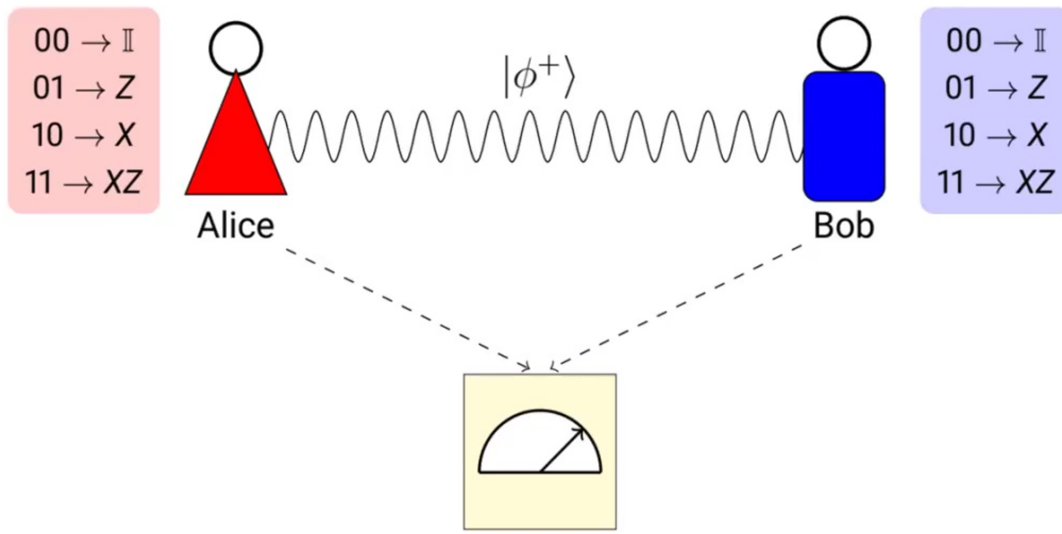
   Required resource,

   1. for even $n \rightarrow \frac{n}{2}$ e-pairs.
   2. for odd $n \rightarrow \frac{n}{2}$ e-pairs & 1 product qubit.
3. No success in rest of the theories.

The Delayed Choice version DC^{2}LC :

f $\rightarrow$ known & $\mathbb{F} \rightarrow$ declared later $\implies$ DC^{2}LC.

1. Quantum protocol for DC^2LC(2) (**f** $\rightarrow$ balanced) :

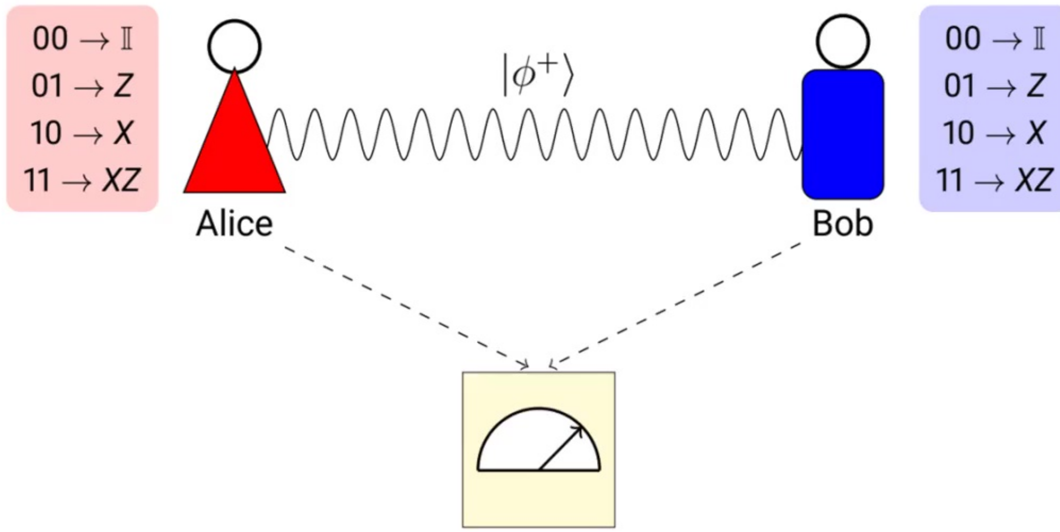


$$\mathbb{M}_{\mathcal{O}} \equiv \{ |\phi^+\rangle\langle\phi^+| , |\phi^-\rangle\langle\phi^-| , |\psi^+\rangle\langle\psi^+| , |\psi^-\rangle\langle\psi^-| \}$$

The Delayed Choice version DC^{2}LC :

f $\rightarrow$ known & $\mathbb{F} \rightarrow$ declared later $\implies$ DC^{2}LC.

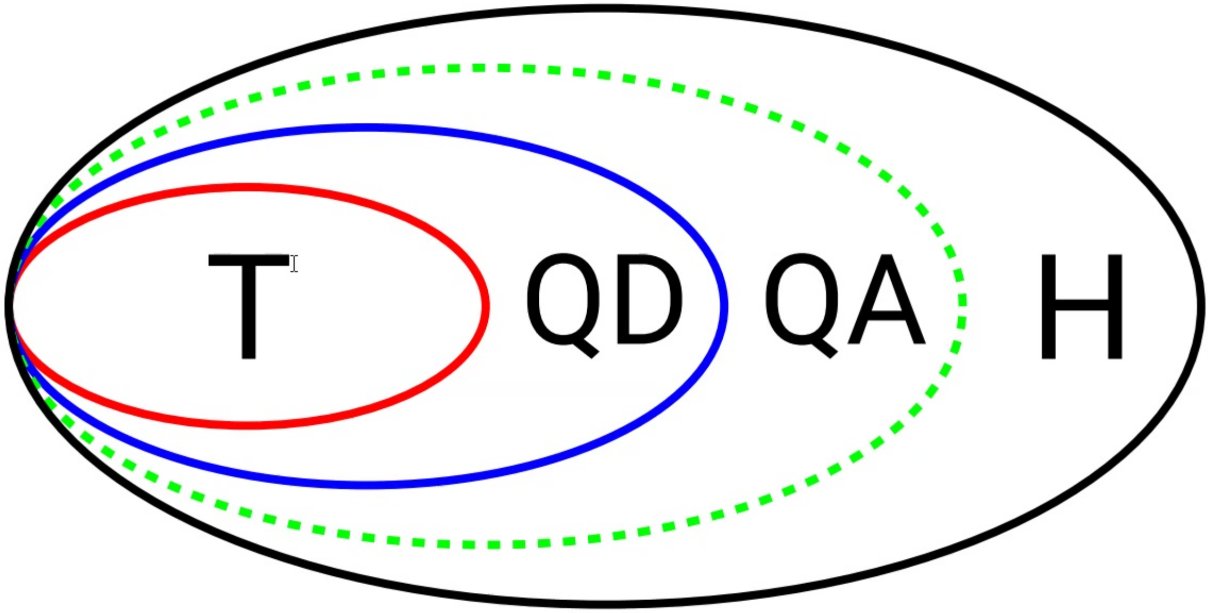
1. Quantum protocol for DC^2LC(2) (**f** $\rightarrow$ balanced) :



$$\mathbb{M}_{\mathcal{O}} \equiv \{|\phi^+\rangle\langle\phi^+|, |\phi^-\rangle\langle\phi^-|, |\psi^+\rangle\langle\psi^+|, |\psi^-\rangle\langle\psi^-|\}$$

2. No success in other theories.

The Range of DCLC :



T = Trivial; QD = Quantum Deterministic; QA = Quantum Advantage; H = Hard/ No Quantum Advantage

Discussion

Information Symmetry and its Generalisation

1. Generally *symmetries* involves dynamical structure in physics. IS / GIS are the counter-examples: A kinematical symmetry to rule out the unphysicallity.
2. A party-independent simple formalism. Doesn't involve any state-update rule.

Discussion

A task of Distributed Computation : QT and beyond

1. DCLC – An initiative to conclude the structure of bipartite quantum systems operationally.
2. Exploiting all the elements of the composite structure (i.e., preparations, transformations and measurements), QT can dominate over a large class of theories even with more exotic space-like [a] as well as time-like correlations [b].
3. A special case of DCLC(n): $\mathbf{f} \equiv \oplus$ & $\mathbb{F} \equiv \vee$ mimics quantum fingerprinting [c].
4. An open direction $\rightarrow$ the complete characterization of DCLC(n).

[a] S. Popescu and D. Rohrlich, *Found. of Phys.* **24**, 379 (1994)
[b] M. Dall'Arno *et. al. Phys. Rev. Lett.* **119**, 020401 (2017)
[c] H. Buhrman *et.al.*, Phys. Rev. Lett, **87**, 167902 (2001)

Other questions I have tried to answer

- **Is there other nonlocality beyond QT?** T. Muruganandan, S. G. Naik, T. Guha, M. Banik and **S. Saha**, arXiv:2205.05415 (2022).
- **What if events placed without causal structure?** G. Chiribella, M. Banik, S. S. Bhattacharya, T. Guha, M. Alimuddin, A. Roy, **S. Saha**, S. Agarwal and G. Kar, New J. Phys. **23**, 033039 (2021).
- **How difficult is to discriminate quantum states?** S. Sen, E. P. Lobo, S. G. Naik, R. K. Patra, T. Gupta, S. Ghosh, **S. Saha**, M. Alimuddin, T. Guha, S. S. Bhattacharya and M. Banik, Phys. Rev. A **105**, 032407 (2022)

Other questions I have tried to answer

- **How to activate indistinguishability from distinguishable set?**
  S. Ghosh, T. Gupta, A. Ardra, A. Das Bhowmik, **S. Saha**, T. Guha and A. Mukherjee, Phys. Rev. A (Letter) **106** L010202 (2022)
  T. Gupta, S. Ghosh, A. Ardra, A. Das Bhowmik, **S. Saha**, T. Guha, R. Rahaman and A. Mukherjee, arXiv:2202.03127 (2022)
- **How to share nonlocality sequentially? S. Saha**, D. Das, S. Sasmal, D. Sarkar, K. Mukherjee, A. Roy and S. S. Bhattacharya, Quantum Inf Process (2019) 18: 42.
- **How to construct nonlocal operator? S. Saha**, S. S. Bhattacharya, A. Roy, A. Mukherjee and R. Rahaman,arXiv:1802:00214 (2018).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Thank you!!