Title: Computational Approaches to Many-Electron Problems

Speakers: Xiao-Bo Li

Series: Machine Learning Initiative

Date: January 24, 2023 - 10:30 AM

URL: https://pirsa.org/23010107

Abstract: In this talk, I will present two recent works on electronic lattice models, both of which utilize novel numerical algorithms to achieve a deeper understanding of the many-electron problem. Competing and intertwined orders including inhomogeneous patterns of spin and charge are observed in many correlated electron materials, such as high-temperature superconductors. In arXiv:2202.11741, we introduce a new development of constrained-path auxiliary-field quantum Monte Carlo (AFMQC) method and study the interplay between thermal and quantum fluctuations in the two-dimensional Hubbard model. We identify a finite-temperature phase transition below which charge ordering sets in. Quantum gas microscopy has developed into a powerful tool to explore strongly correlated quantum systems. However, discerning phases with off-diagonal long range order requires the ability to extract these correlations from site-resolved measurements. In the second work arXiv:2209.10565, we study the one-dimensional extended Hubbard model using the variational uniform matrix product states algorithm. We show that a multi-scale complexity measure can pinpoint the transition to and from the bond ordered wave phase with an off-diagonal order parameter, sandwiched between diagonal charge and spin density wave phases, using only diagonal descriptors.

Zoom link: https://pitp.zoom.us/j/97325001971?pwd=QXNmU2I0L3BxQVErMEZWSDF2bGZ0QT09

Computational Approaches to Many-Electron Problems

Bo Xiao [1]

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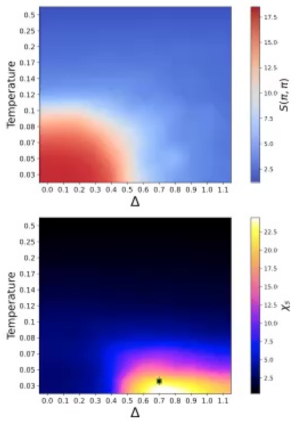
Center for Computational Quantum Physics

Research Summary

Simulating Fermion-Boson Interactions

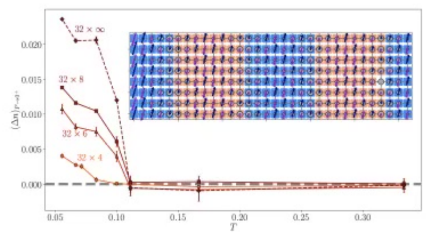
- Long-range electron-phonon interactions.
- Charge density wave, superconductivity, disorder.



Xiao *et al.* Phys. Rev. B **99**, 205145 (2019); Hébert, **Xiao**, *et al.* Phys. Rev. B **99**, 075018 (2019); **Xiao** *et al.* Phys. Rev. B **103**, L060501

Simulating Fermionic Systems

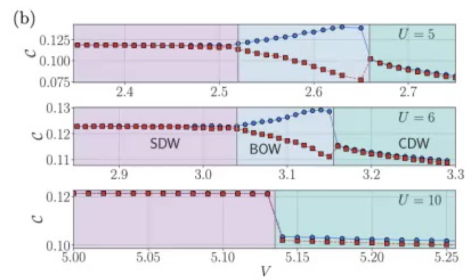
- Doped two-dimensional Hubbard model: stripe order, superconductivity etc.
- Handshakes between high-temperature DQMC results and ground-state DRMG results. AMO physics.
- Algorithm development: control the sign problem etc.



Xiao *et al.* preprint arXiv:2202.11741 (*to appear on Phys. Rev. X); Tarat,* **Xiao**, *et al. Phys. Rev. B* **105**, *045107 (2022),* **Xiao** *et al. (in preparation).*

Unsupervised Detection of Off-Diagonal Long-Range Order

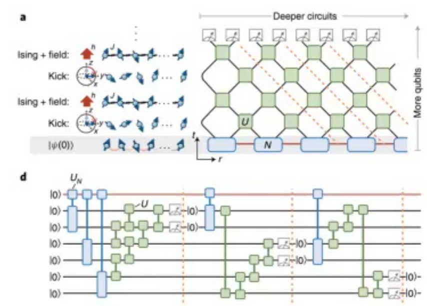
- Obtain the ground-state wavefunction in the thermodynamic limit.
- Emulate projective and diagonal measurements.
- Off-Diagonal long-range order can be detected from real-space snapshots using complexity measurements.



Xiao *et al.* preprint arXiv:2209.10565

Quantum Computation Inspired Algorithms

- Inspired by experiments done in a trapped-ion quantum processor. Chertkov *et al.* Nat. Phys. **18**, 1074-1079 (2022).
- Simulate real-time evolution using MPS.

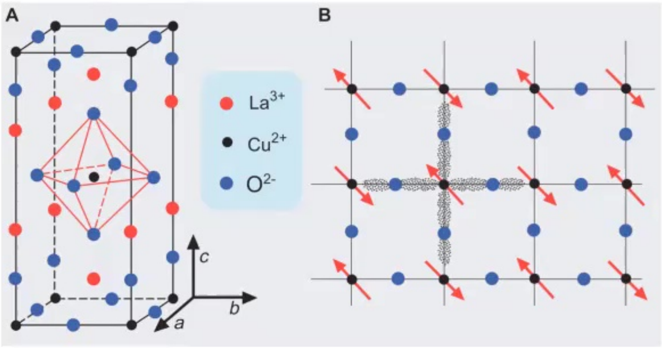


Xiao, Tindall, Stoudenmire (on-going).

Temperature Dependence of Spin and Charge Orders in the Doped Two-Dimensional Hubbard Model

Xiao, He, Georges, Zhang arXiv:2202.11741 (to appear on Phys. Rev. X)

Stripe Orders in High-Temperature Superconductors (HTSCs)



- Crystal structure of $\mathrm{La_2CuO_4}$
- The "parent compound" of the $\mathrm{La_{2-x}Sr_xCuO_4}$ family of HTSCs.
- Weak electronic couplings in the interplane direction.

J. Orenstein and A. Millis, Science **288**, 5465 (2000)
E. Dagotto, Science **309**, 257 (2005)
P. A. Lee *et al.*, Rev. Mod. Phys. **78**, 17 (2006)
M. Imada *et al.*, Rev. Mod. Phys. **70**, 1039 (1998)
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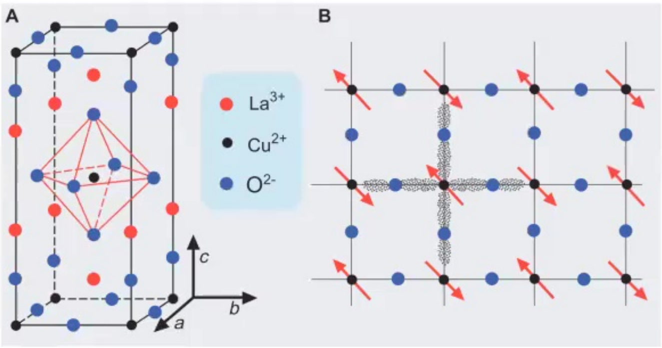

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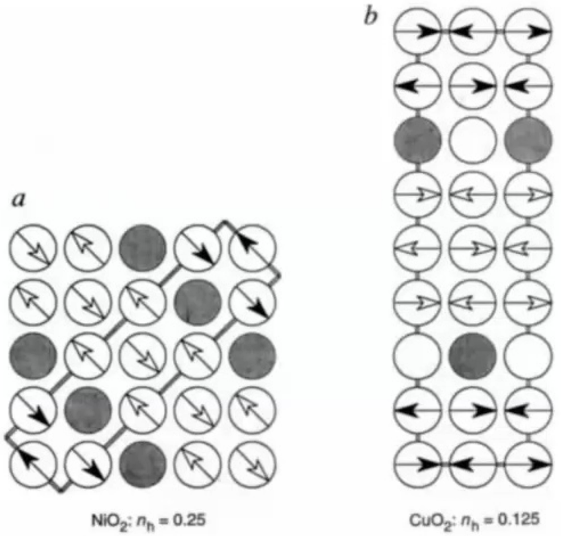
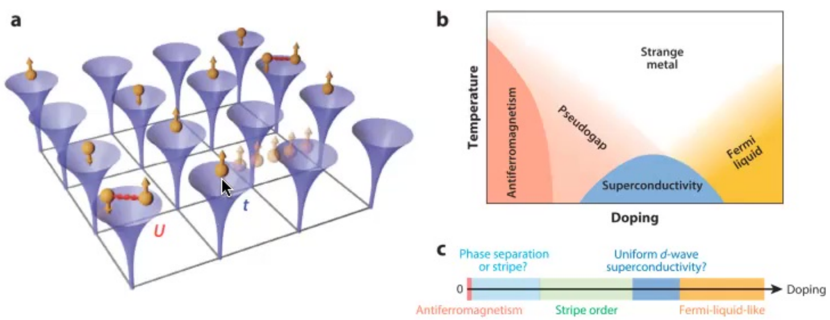


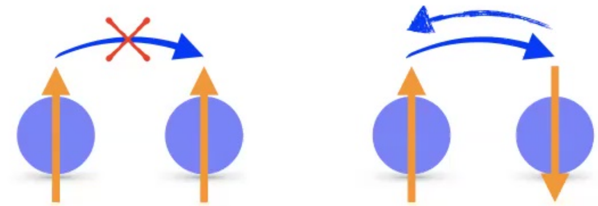
- (a) Hole-doped $\mathrm{La_2NiO_4}$.
- (b) $\mathrm{La_{1.6-x}Nd_{0.4}Sr_xCuO_4}$ with $x = 0.12$.
- Static stripe & anomalous suppression of superconductivity.

J. M. Tranquada *et al.* Nature **375**, 561-563 (1995)
J. M. Tranquada *et al.* Phys. Rev. Lett. **78**, 338 (1997)
M. Fujita *et al.* Phys. Rev. Lett. **88**, 167008 (2002)

The single-band Hubbard model in two dimensions (2D)

$$\hat{H} = -t \sum_{\langle \mathbf{i},\mathbf{j}\rangle,\sigma} \left(c^{\dagger}_{\mathbf{i}\sigma} c_{\mathbf{j}\sigma} + c^{\dagger}_{\mathbf{j}\sigma} c_{\mathbf{i}\sigma}\right) + U \sum_{\mathbf{i}} \left(n_{\mathbf{i}\uparrow} - \frac{1}{2}\right)\left(n_{\mathbf{i}\downarrow} - \frac{1}{2}\right) - \mu \sum_{\mathbf{i},\sigma} n_{\mathbf{i}\sigma}$$



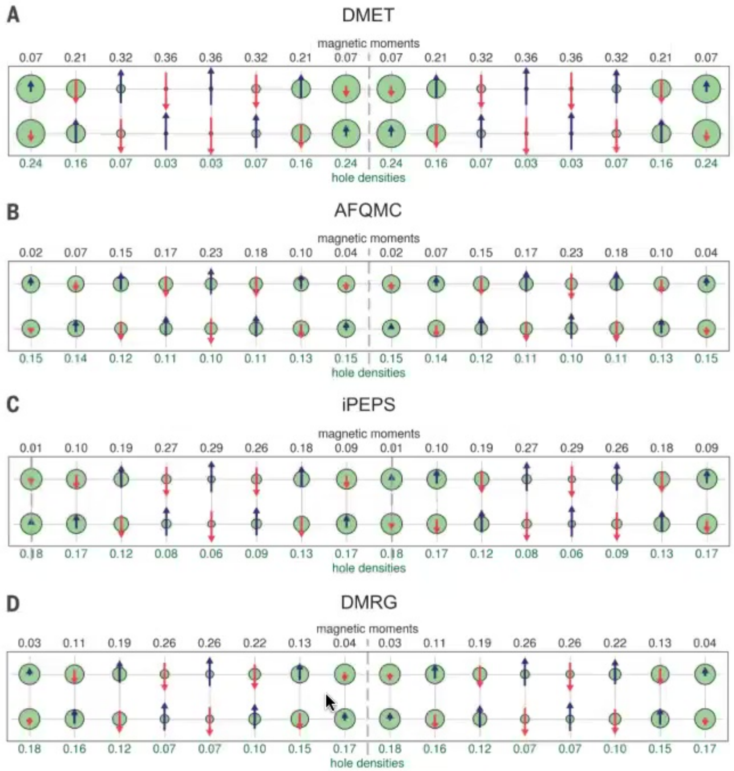


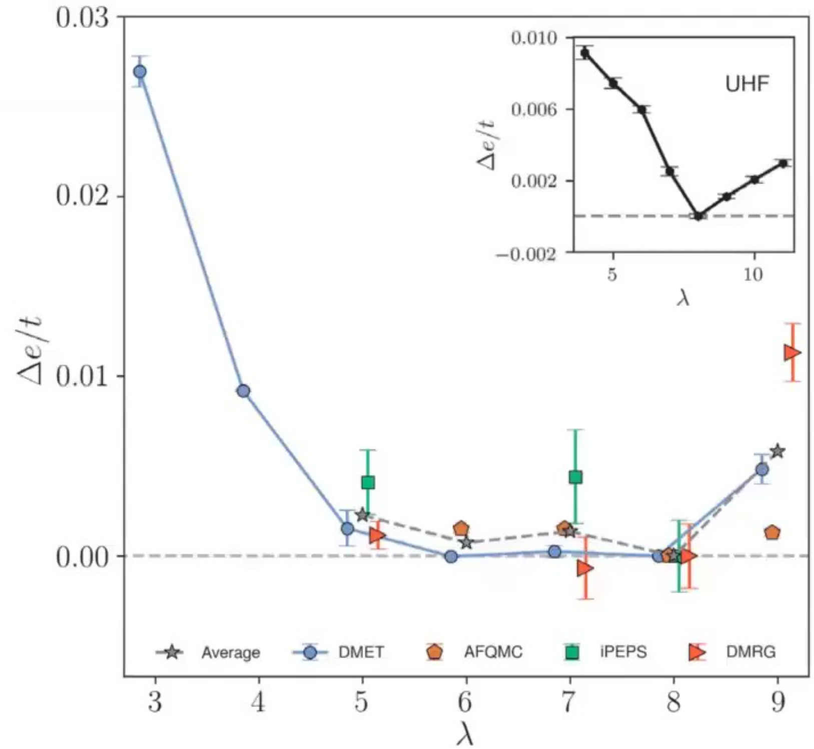
$\Delta E^2 = 0$ $\qquad$ $\Delta E^2 \propto -t^2/U = -J$

- **Hole doping:** $\delta = 1 - \rho$ where $\rho = N_e/N$. Use chemical potential μ to tune the particle number in a grand canonical ensemble.
- **Strong-coupling limit:** the Coulomb interaction $U/t = 6, 8$ is comparable to the bandwidth $W/t = 8$. $\epsilon(\mathbf{k}) = -2t(\cos k_x + \cos k_y)$.

M. Qin *et al.* Annual Review of Condensed Matter Physics **13**, 275-302 (2022), Arovas *et al.* Annual Review of Condensed Matter Physics **13** 239-274 (2022) etc.

Stripe Order in the Doped Hubbard Model

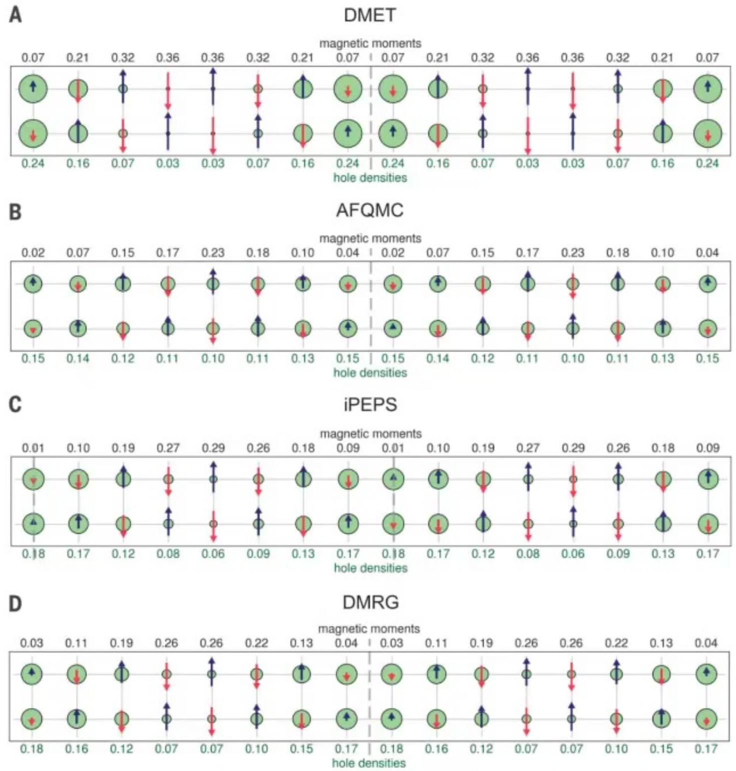




Filled stripe: $\lambda_s = 2/\delta$ (spin) and $\lambda_c = 1/\delta$ (charge). $U/t = 8$ and $\delta = 1/8$. B. Zheng *et al.*, Science **358**, 1155 (2017) M. Qin *et al.* Phys. Rev. X **10**, 031058 (2020)

Relative energies of stripe states at $T = 0$. Multi-messengers. Different from real materials (half-filled stripes).

Stripe Order in the Doped Hubbard Model



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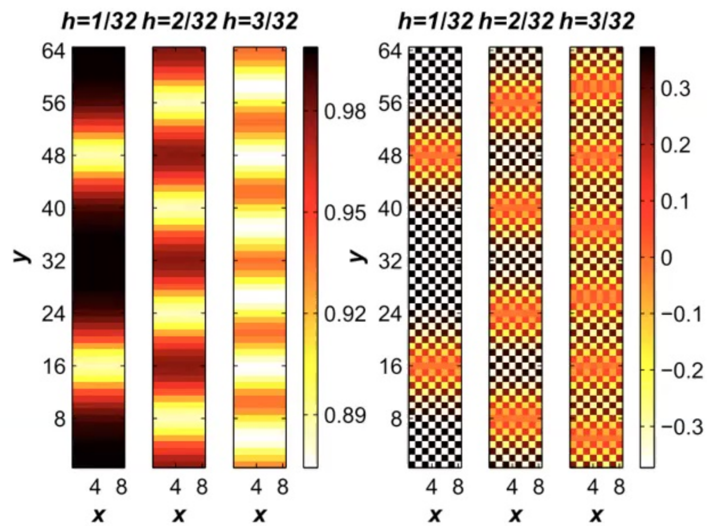
- Stripe order in mean-field approaches.
  J. Zannen *et al.*, Phys. Rev. B **40**, 7391(R) (1989)
  D. Poiblanc *et al.*, Phys. Rev. B **39**, 9749 (1989)
  H. J. Schulz, Phys. Rev. Lett. **64**, 1455 (1990)

J. Xu *et al.* Journal of Physics: Condensed Matter **23**, 50 (2011)



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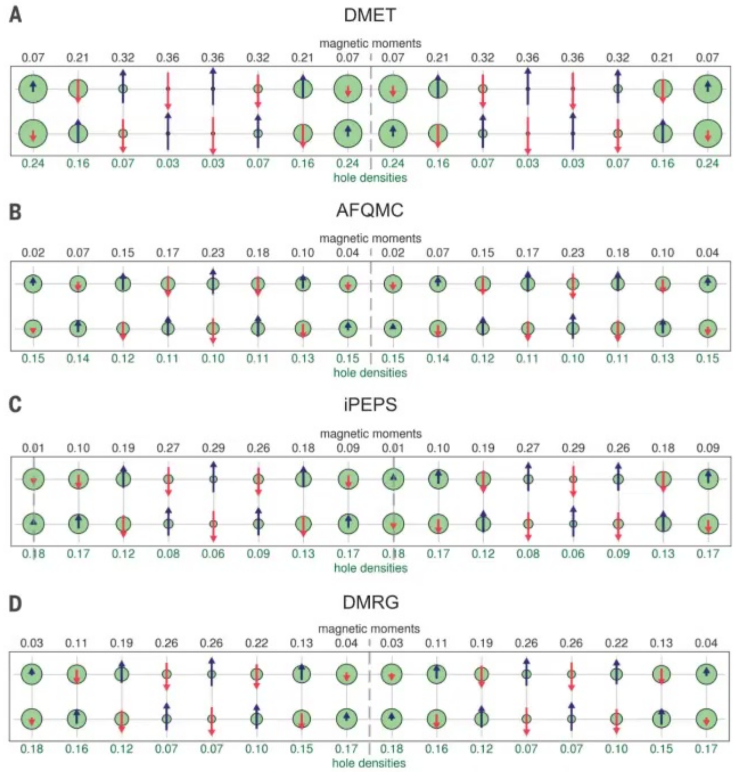
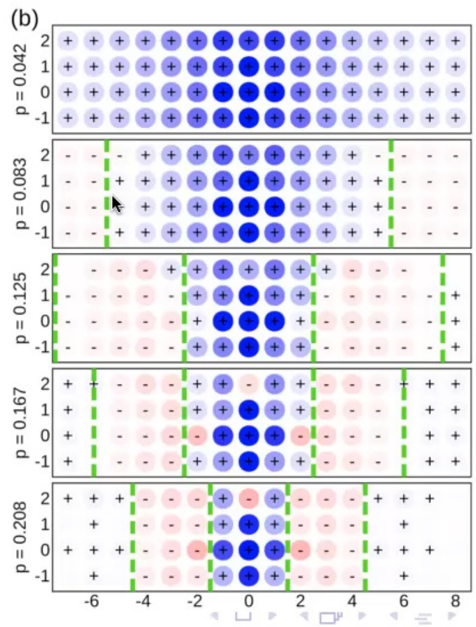


- Stripe order in determinant quantum Monte Carlo (DQMC) approaches. Sign problem and therefore finite/high temperature.
  E. Huang *et al.* Science **358** 1161-1164 (2017),

  E. Huang *et al.* npj Quantum Materials **3** 1-6 (2018) etc.



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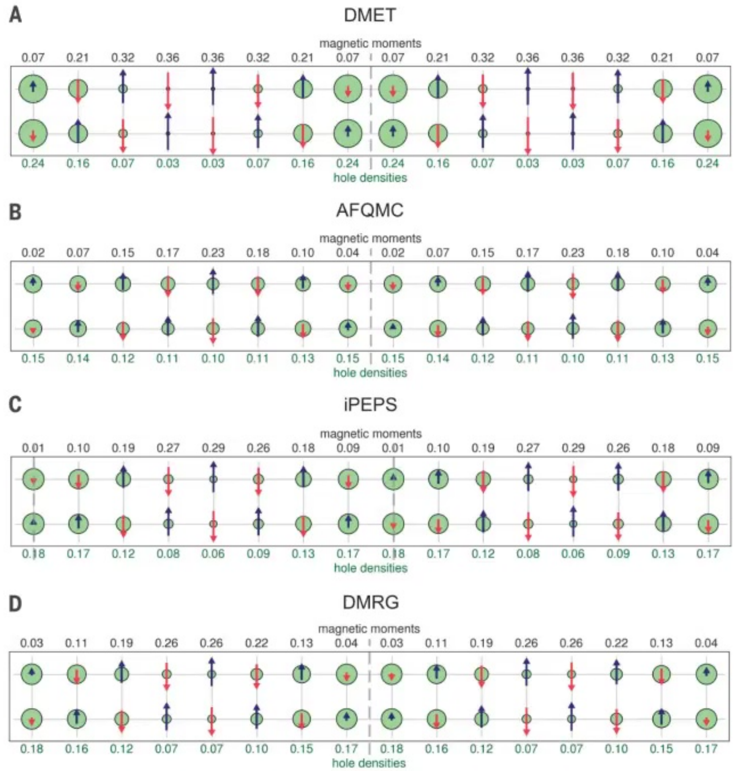
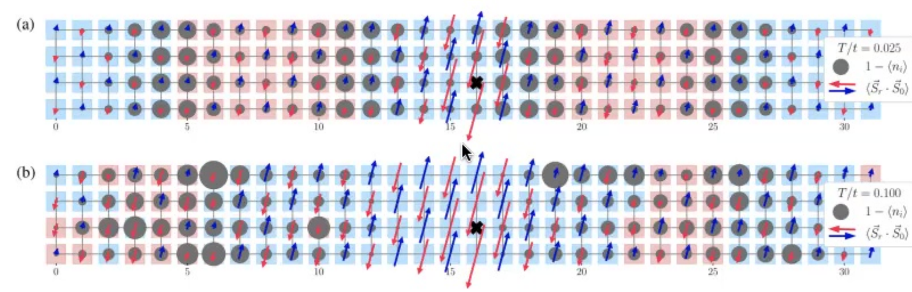


Filled stripe: $\lambda_s = 2/\delta$ (spin) and $\lambda_c = 1/\delta$ (charge). $U/t = 8$ and $\delta = 1/8$. B. Zheng *et al.*, Science **358**, 1155 (2017) M. Qin *et al.* Phys. Rev. X **10**, 031058 (2020)

- Stripe order in tensor networks (TN) based approaches. Entanglement and therefore finite/narrow cylinder width e.g. $W = 4$ for fermionic cases.

  $T = 0$: H. Jiang *et al.* Science **365** 1424-1428 (2019), Y. Jiang *et al.* Phys. Rev. Research **2**, 033073 (2020) etc.

  Finite T: A. Wietek *et al.* Phys. Rev. X **11**, 031007 (2021)



$U/t = 10$, $\delta = 1/16$ on a 32×4 cylinder.

$t - J$ model and its variants at $T = 0$: S.R. White, and D.J. Scalapino Phys. Rev. Lett. **80**, 1272 (1998), Phys. Rev. Lett. **81** 3227, (1998), Phys. Rev. Lett. **91**, 136403 (2003), S. Jiang *et al.* PNAS **118**, 44 (2021) etc.

Finite-Temperature Auxiliary-Field Quantum Monte Carlo (AFQMC)

- The expectation value of any physical observable at $T > 0K$,

$$\langle \hat{O} \rangle \equiv \frac{Tr\left(\hat{O}e^{-\beta \hat{H}}\right)}{Tr(e^{-\beta \hat{H}})}.$$

- Map a $2D$ quantum problem into a $(2+1)D$ classical problem (trace over fermionic degrees of freedom).

$$\mathcal{Z} \equiv Tr\left(e^{-\beta \hat{H}}\right) = Tr\underbrace{\left[e^{-\Delta\tau \hat{H}} \dots e^{-\Delta\tau \hat{H}} e^{-\Delta\tau \hat{H}}\right]}_{L}$$

$$= \int dXP(X) det\left[I + B(\mathbf{x}_L) \dots B(\mathbf{x}_2)B(\mathbf{x}_1)\right],$$

where $e^{-\Delta\tau H} \approx e^{-\Delta\tau K/2} e^{-\Delta V} e^{-\Delta\tau K/2}$.

R. Blankenbecler *et al.*, Phys. Rev. D **24**, 2278 (1981), White *et al.* Phys. Rev. B **40**, 506 (1989) etc.

- *Hubbard-Stratonovich transformation:* e.g. introducing a discrete auxiliary field

$$e^{-\Delta\tau U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right)} = \frac{1}{2} e^{-\frac{1}{4}\Delta\tau U} \sum_{x=\pm 1} e^{\alpha x \left(n_\uparrow - n_\downarrow\right)}, U > 0$$

J. Hirsch Phys. Rev. B **28**, 4059(R) (1983)

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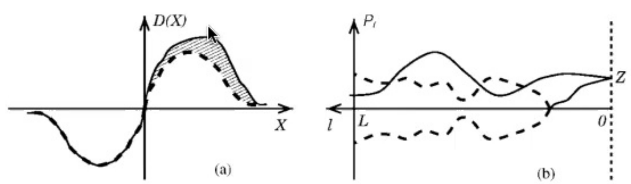


S. Zhang, Phys. Rev. Lett. **83**, 2777 (1999)

- $\langle n \rangle = 1$: no sign problem due to the particle-hole symmetry.

Loh *et al.* Phys. Rev. B **41**, 9301 (1990)

- $\langle n \rangle \neq 1$: sign problem exists.

Constrained Path and Trial Density Matrix

Finite-temperature auxiliary-field quantum Monte Carlo (AFQMC)

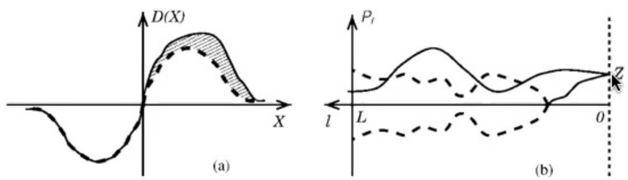
- The expectation value of any physical observable at $T > 0K$,

$$\langle \hat{O} \rangle \equiv \frac{Tr\left(\hat{O}e^{-\beta\hat{H}}\right)}{Tr(e^{-\beta\hat{H}})}.$$

- Generating all possible auxiliary-field paths time slice by time slice

$$\mathcal{P}_\ell\left(\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_\ell\}, \mathcal{B}\right) > 0,$$

$$\mathcal{P}_\ell^T = \det\left[I + \left(\prod_{m=1}^{L-\ell} B_T\right) B(\mathbf{x}_\ell)\ldots B(\mathbf{x}_1)\right] > 0.$$



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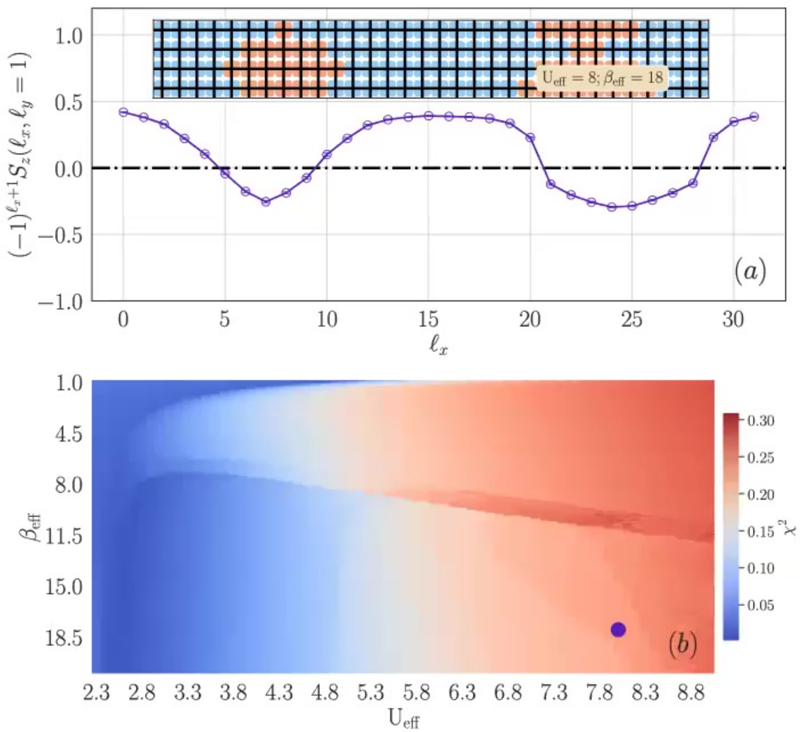
Self-Consistent Constraint: AFQMC & Unrestricted Hartree-Fock

An independent-particle (IP) Hamiltonian Xiao, He, Georges, and Zhang arXiv:2202.11741 (to appear on Phys. Rev. X)

$$H_{IP} = -t \sum_{\langle \mathbf{i},\mathbf{j}\rangle,\sigma} \left(c_{\mathbf{i}\sigma}^{\dagger} c_{\mathbf{j}\sigma} + c_{\mathbf{j}\sigma}^{\dagger} c_{\mathbf{i}\sigma}\right) + U_{\text{eff}} \sum_{\mathbf{i},\sigma} \langle n_{\mathbf{i}\bar{\sigma}} \rangle n_{\mathbf{i}\sigma} - \mu_{\text{eff}} \sum_{\mathbf{i},\sigma} n_{\mathbf{i}\sigma} + \sum_{\mathbf{i},\sigma} v_{\mathbf{i}\sigma} n_{\mathbf{i}\sigma}$$



- $U/t = 8$, $\delta = 1/8$, and $\beta = 18$.
- Initializing the IP Hamiltonian @ $U_{\text{eff}} = 8$, and $\beta_{\text{eff}} = 18$.
- Edge spin pinning fields.

$$v_{i_x,\uparrow} = -v_{i_x,\downarrow} = (-1)^{i_y} v; \;\; i_x = 0$$

- Minimizing

$$\chi^2(U_{\text{eff}}, \beta_{\text{eff}}) = \frac{1}{N} \sum_{\mathbf{i},\sigma} \left(\langle n_{\mathbf{i}\sigma} \rangle_{\text{IP}} - \langle n_{\mathbf{i}\sigma} \rangle_{\text{QMC}}\right)^2$$

Self-Consistent Constraint: AFQMC & Unrestricted Hartree-Fock

An independent-particle (IP) Hamiltonian

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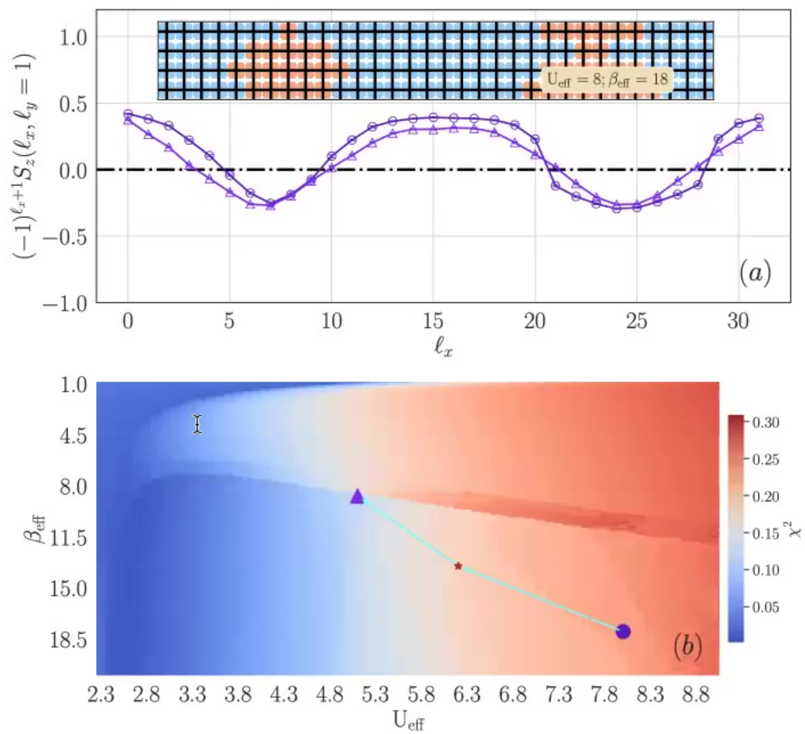

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Self-Consistent Constraint: AFQMC & Unrestricted Hartree-Fock

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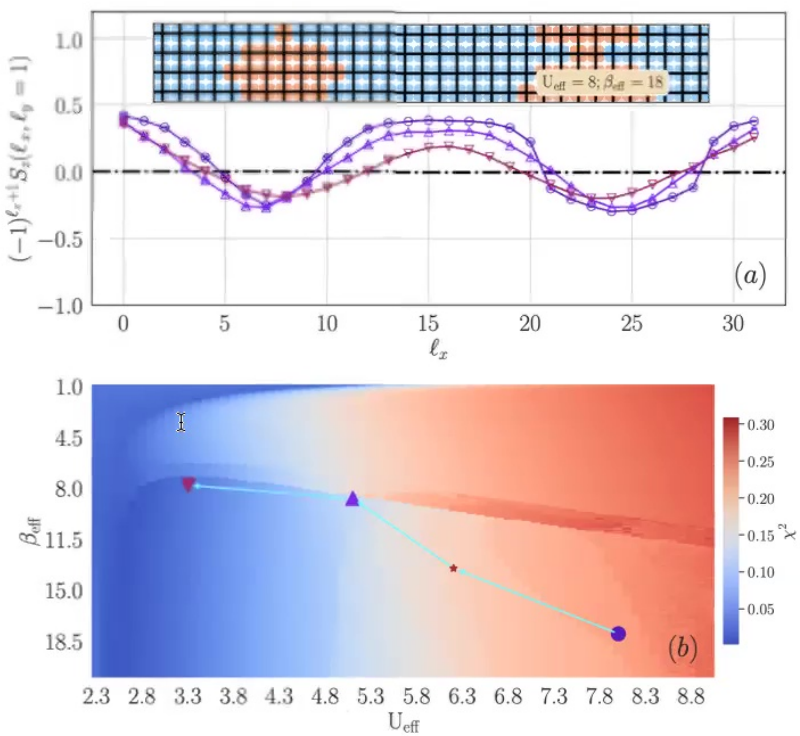

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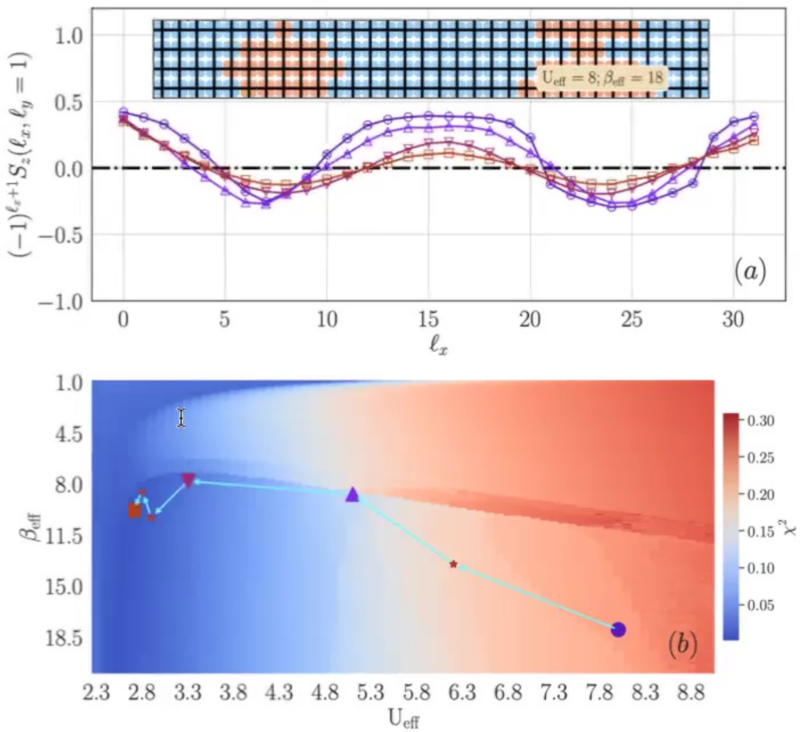

- $U/t = 8$, $\delta = 1/8$, and $\beta = 18$.
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- Minimizing

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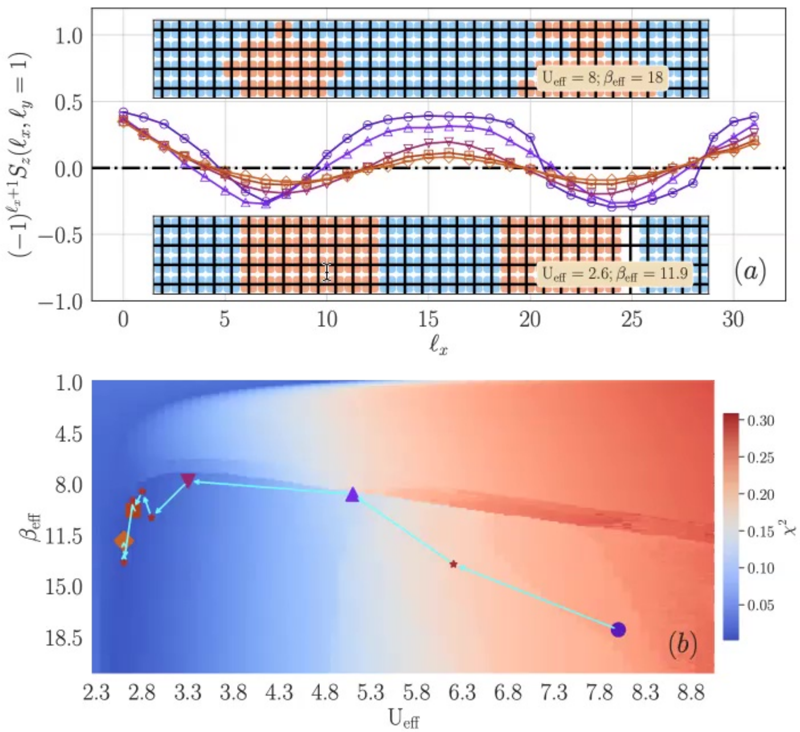
An independent-particle (IP) Hamiltonian

$$H_{\mathrm{IP}} = -t \sum_{\langle \mathbf{i},\mathbf{j}\rangle,\sigma} \left(c^{\dagger}_{\mathbf{i}\sigma} c_{\mathbf{j}\sigma} + c^{\dagger}_{\mathbf{j}\sigma} c_{\mathbf{i}\sigma} \right) + U_{\mathrm{eff}} \sum_{\mathbf{i},\sigma} \langle n_{\mathbf{i}\bar{\sigma}} \rangle n_{\mathbf{i}\sigma} - \mu_{\mathrm{eff}} \sum_{\mathbf{i},\sigma} n_{\mathbf{i}\sigma} + \sum_{\mathbf{i},\sigma} v_{\mathbf{i}\sigma} n_{\mathbf{i}\sigma}$$



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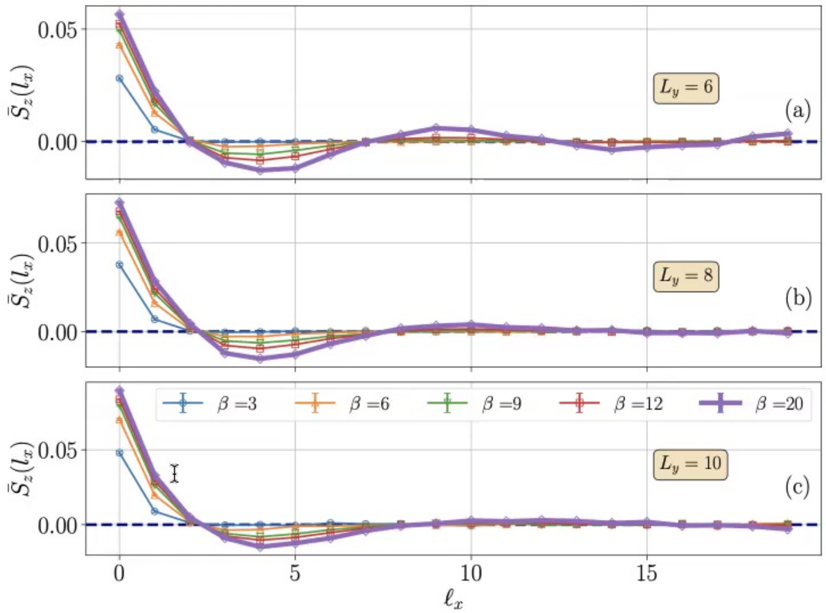
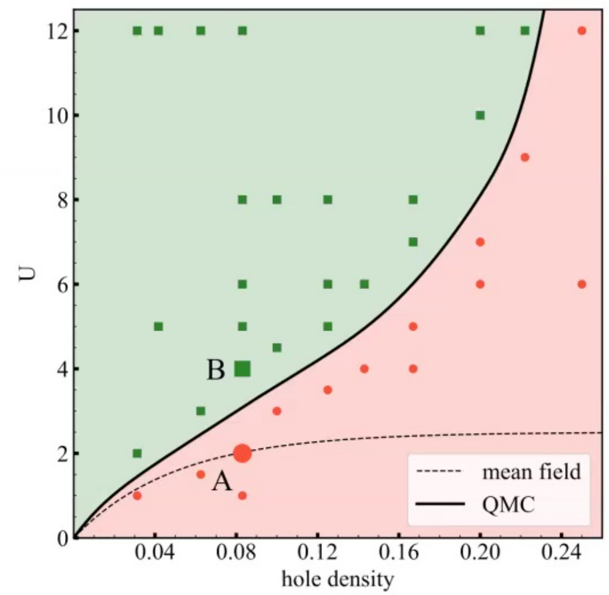
Short-Ranged Spin and Charge Orders



- $U/t = 6$ and $\delta = 1/5$.
- Rung-average

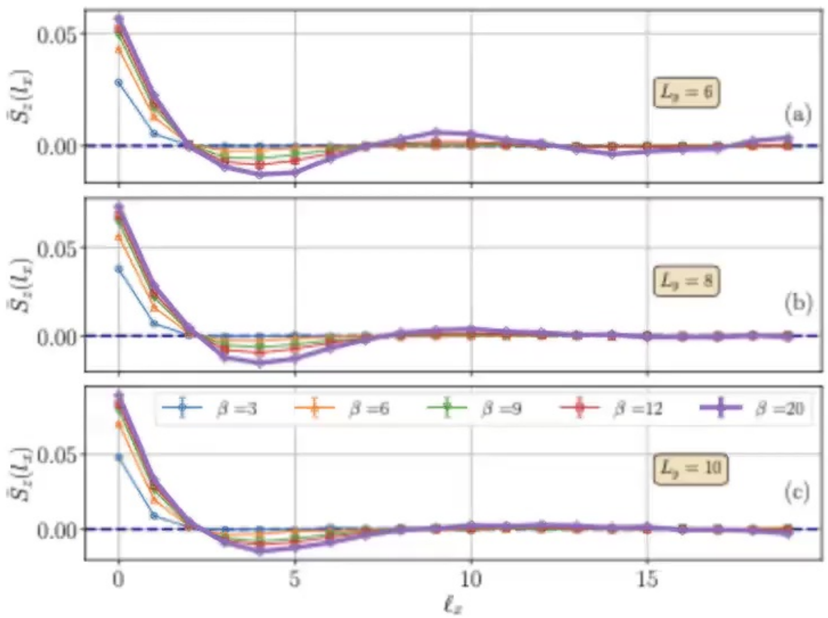
$$\bar{S}_z(\ell_x) = \frac{1}{L_y} \sum_{\ell_y=0}^{L_y-1} (-1)^{\ell_x+\ell_y} \langle S_z(\ell_x, \ell_y) \rangle$$

- Measure spin/charge densities using linear response by introducing symmetry-breaking spin pinning fields.
  F. Assaad *et al.* Phys. Rev. X **3**, 031010 (2013)
- Two-point functions e.g. spin and charge correlation functions. Fourier transformation. Commensurate AFM order to incommensurate spin density wave crossover in momentum space.



S. Sorella preprint arXiv:2101.07045, H. Xu Phys. Rev. Research **4**, 013239 (2022), F. Simkovic *et al.* preprint: arXiv:2209.09237 etc.

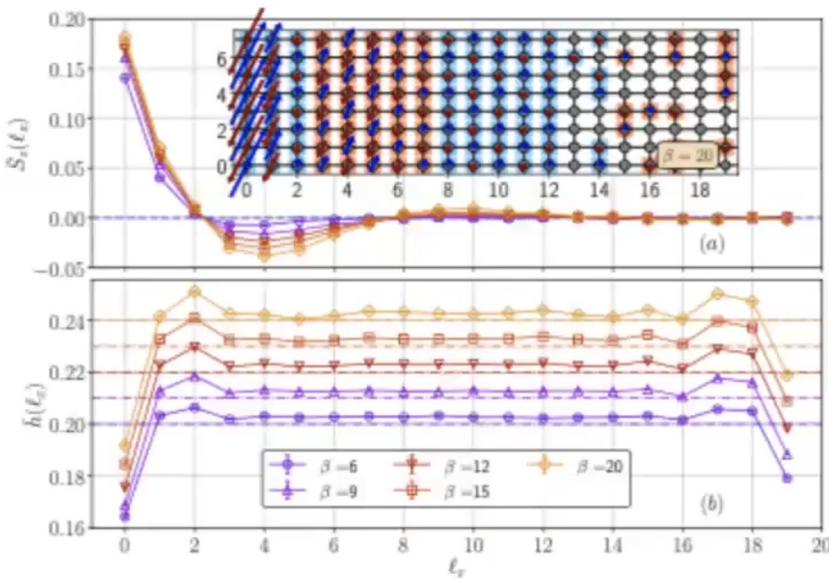
Short-Ranged Spin and Charge Orders



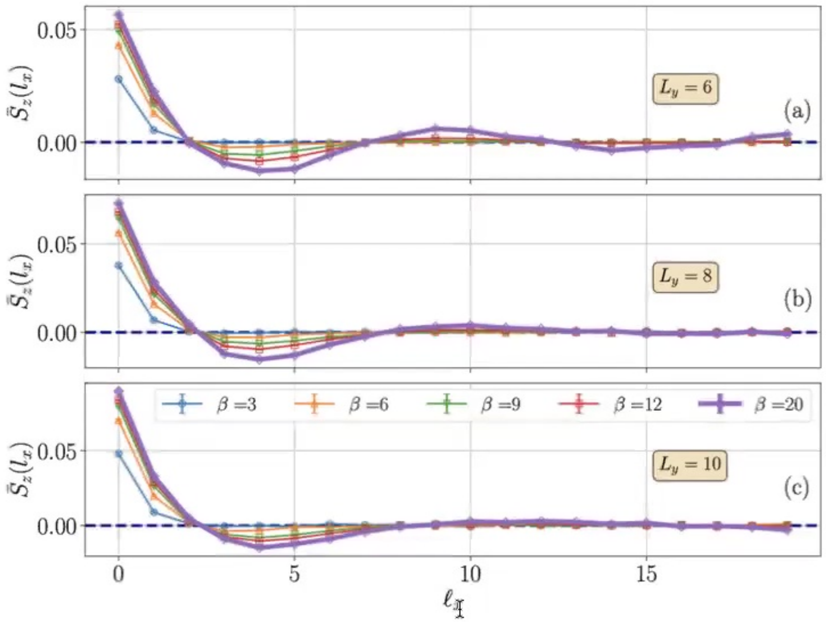
- $U/t = 6$ and $\delta = 1/5$.
- Rung-average

$$\bar{h}(\ell_x) = 1 - \frac{1}{L_y}\sum_{\ell_y=0}^{L_y-1}\langle n_{\ell,\uparrow} + n_{\ell,\downarrow}\rangle$$

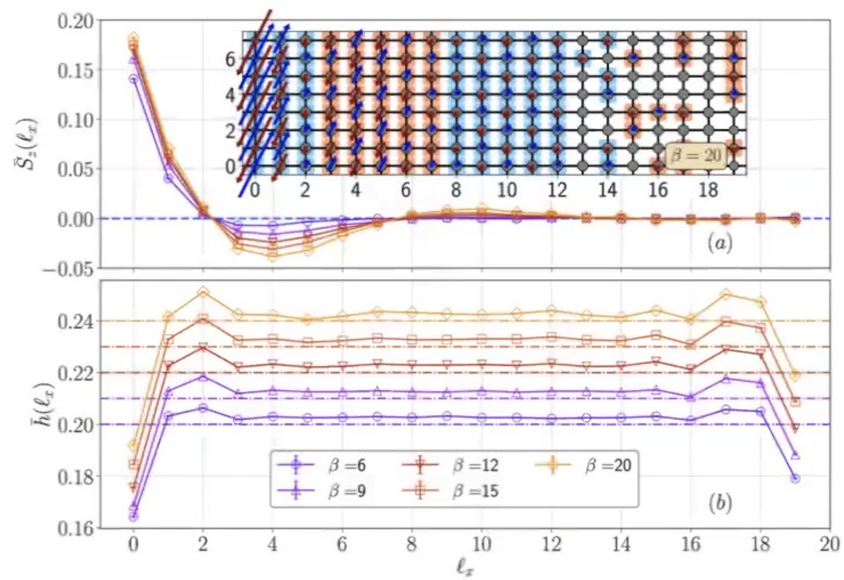
- Measure spin/charge densities using linear response by introducing symmetry-breaking spin pinning fields.
  F. Assaad *et al.* Phys. Rev. X **3**, 031010 (2013)



Short-Ranged Spin and Charge Orders



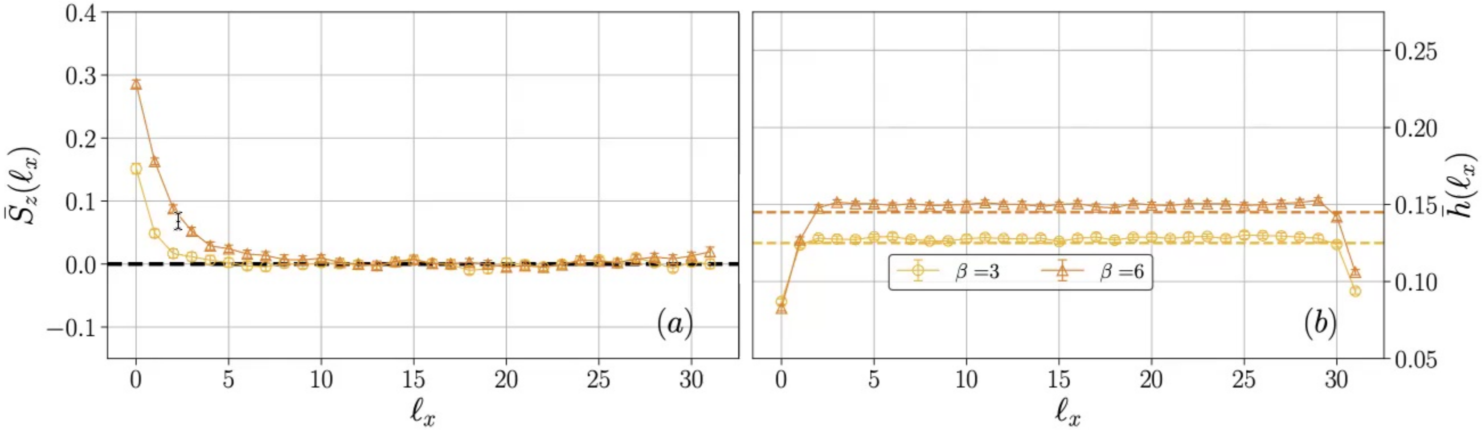
- Measure spin/charge densities using linear response by introducing symmetry-breaking spin pinning fields. F. Assaad *et al.* Phys. Rev. X **3**, 031010 (2013)



- $U/t = 6$ and $\delta = 1/5$.
- Rung-average

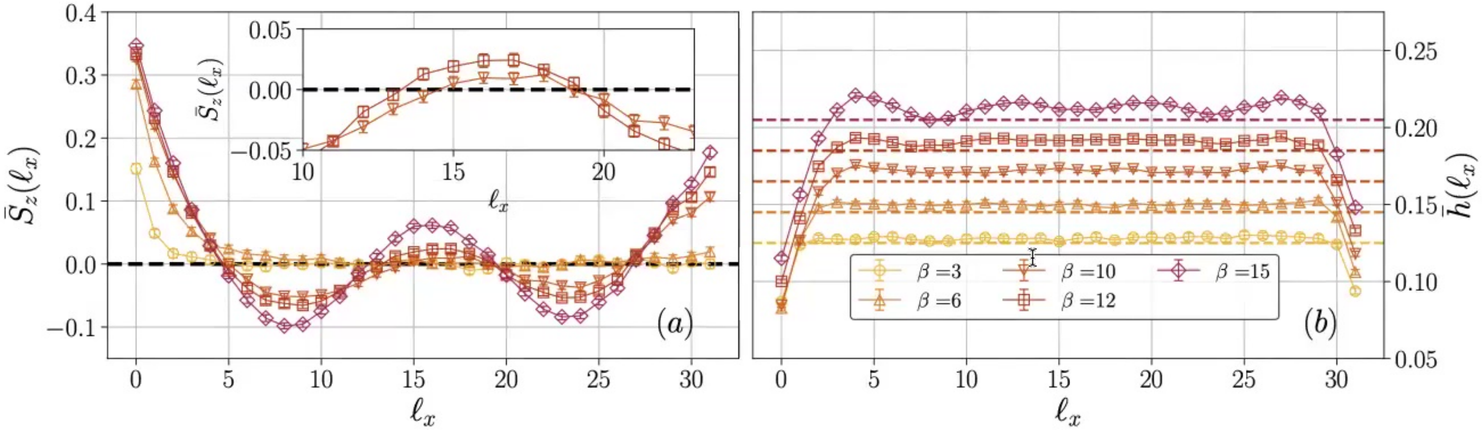
$$\bar{h}(\ell_x) = 1 - \frac{1}{L_y} \sum_{\ell_y=0}^{L_y-1} \langle n_{\ell,\uparrow} + n_{\ell,\downarrow} \rangle$$

Long-Range Spin and Charge Orders



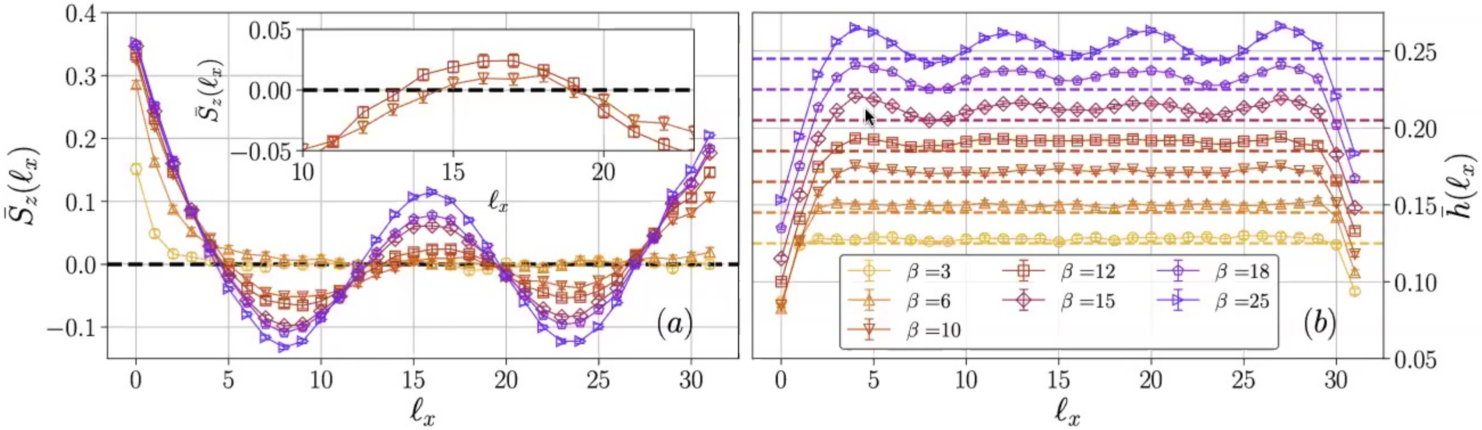
- $U/t = 8$ and $\delta = 1/8$.

Long-Range Spin and Charge Orders



- $U/t = 8$ and $\delta = 1/8$.

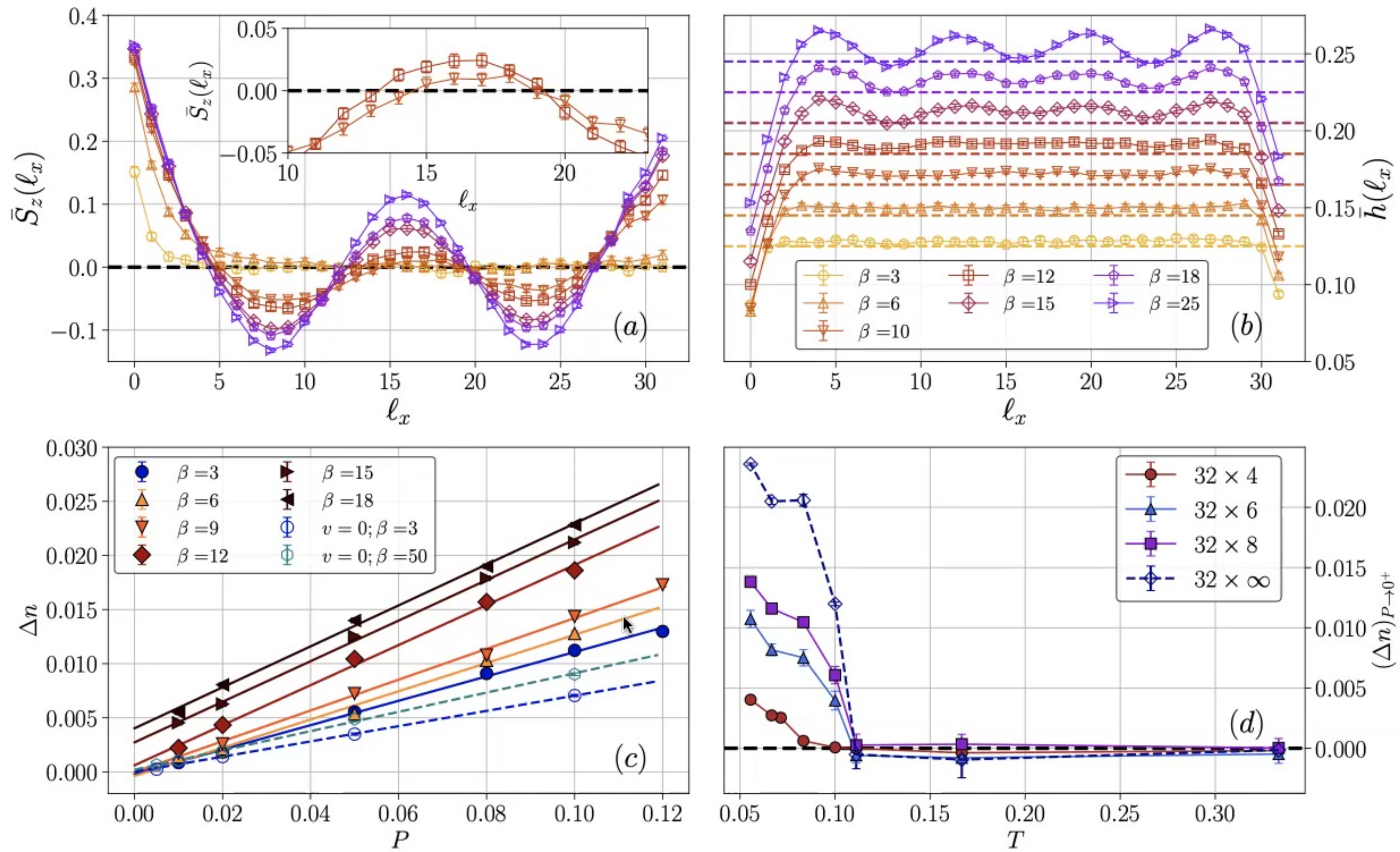
Long-Range Spin and Charge Orders



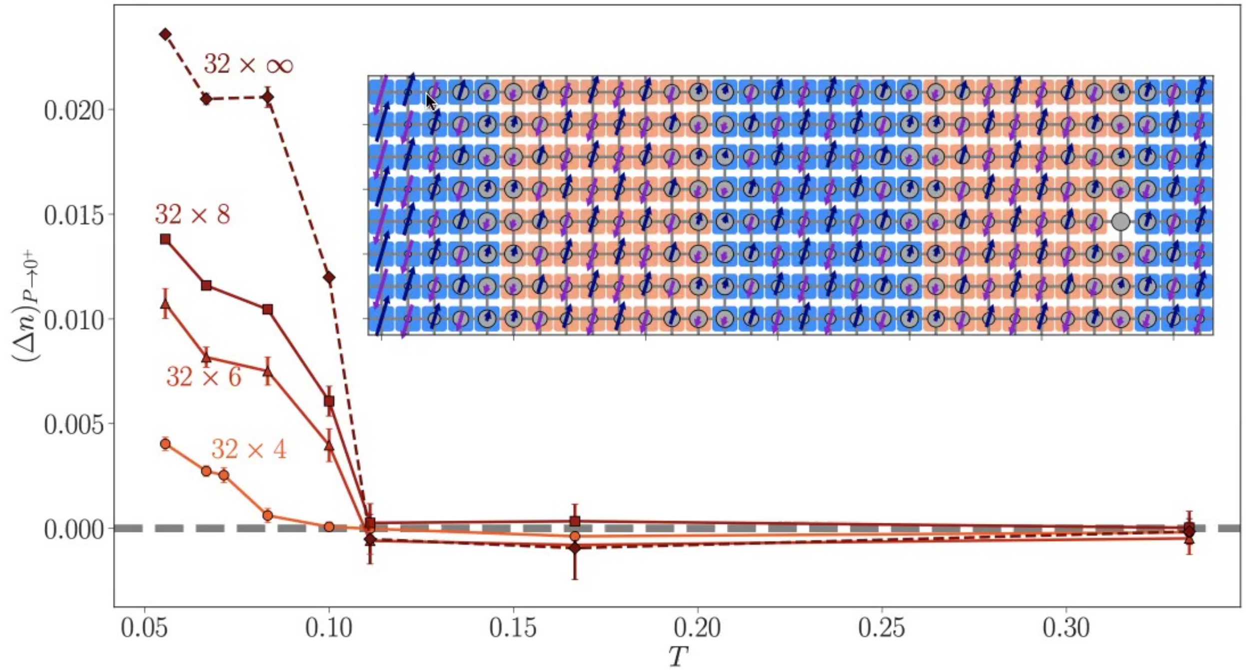
- $U/t = 8$ and $\delta = 1/8$. $\lambda_s = 2\lambda_c$.
- Charge perturbation $H_c = P \sum_{i_x, i_y, \sigma} \sin(\kappa i_x + \phi) n_{(i_x, i_y), \sigma}$ with $P \to 0$.

G. Ehlers *et al.*, Phys. Rev. B **95**, 125125, (2017).

Long-Range Spin and Charge Orders



Discussion and Conclusion

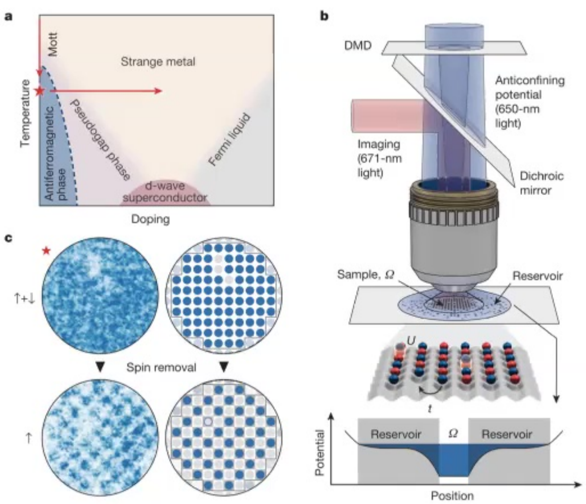


- Long-range stripe order at $\delta = 1/8$ ($U/t = 8$) and short-range spin and charge orders at $\delta = 1/5$ ($U/t = 6$).
- Charge order becomes long-range at finite T_c at $\delta = 1/8$.

Extracting Off-Diagonal Order from Diagonal Basis Measurements

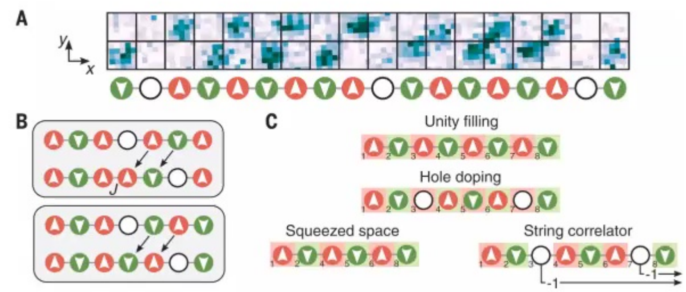
Xiao, Moreno, Fishman, Sels, Khatami, Scalettar arXiv:2209.10565

Quantum Simulations Using Ultracold Atoms in Optical Lattices



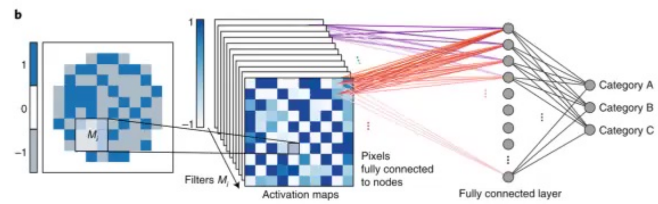
- Projective measurements of the spin-resolved local occupation $\hat{n}_{i,\sigma}$.
- Temperature: computed from the spin correlation function $C_d = \frac{1}{\mathcal{N}_{\mathbf{d}}} \frac{1}{S^2} \sum_{\mathbf{r},\mathbf{s}\in\Omega;\ \mathbf{d}=\mathbf{r}-\mathbf{s}} \langle S^z_{\mathbf{r}} S^z_{\mathbf{s}} \rangle - \langle S^z_{\mathbf{r}} \rangle \langle S^z_{\mathbf{s}} \rangle$.

A. Mazurenko *et al.*, Nature **545**, 462-466 (2017), I. Bloch *et al.* Nat. Phys. **8** 267-276 (2012), I. Bloch *et al.* Rev. Mod. Phys. **80**, 885 (2008) etc.



- Doped Hubbard chain: spin-charge separation.

T. Hilker *et al.*, Science **357**, 484-487 (2017)

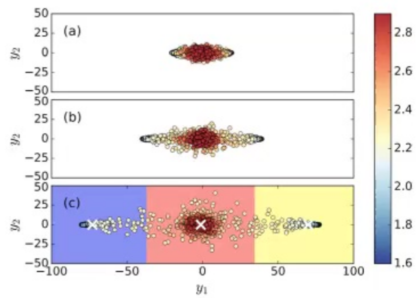


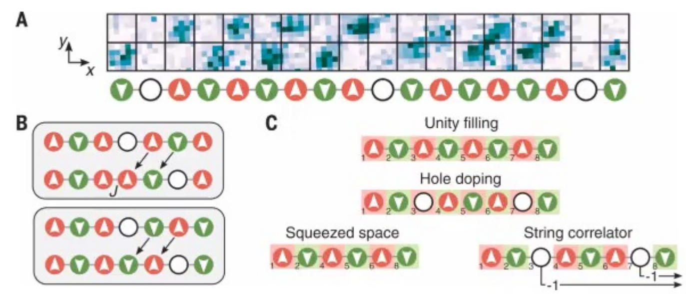
- Supervised learning using convolutional neural networks (CNN).

A. Bohrdt *et al.*, Nat. Phys. **15** 921-924 (2019), C. Miles *et al.* arXiv.2112.10789 etc.

Discovering Phase Transitions with Machine Learning

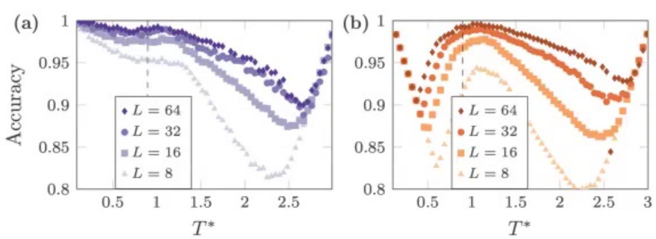
Synthetic data

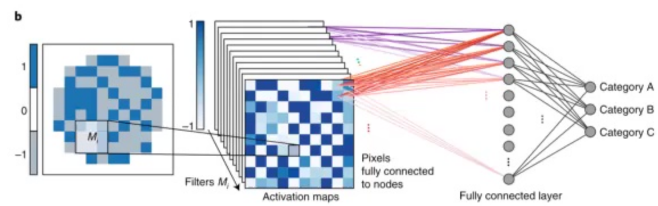




Learn the order parameter/critical T_c for the Ising model in 2D, L. Wang Phys. Rev. B **94**, 195105 (2016), W. Hu *et al.* Phys. Rev. E **95**, 062122 (2017), S. Wetzel Phys. Rev. E **96**, 022140 (2017), critical T_c/exponent J. Carrasquilla, and R. Melko Nat. Phys. **13**, 431-434 (2017) etc.

- Doped Hubbard chain: spin-charge separation.
  T. Hilker *et al.*, Science **357**, 484-487 (2017)



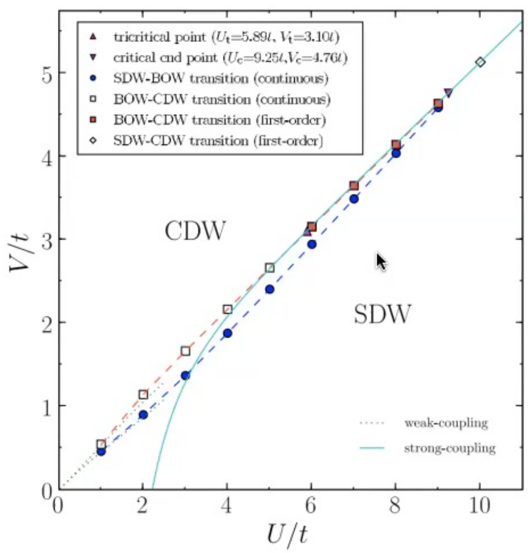


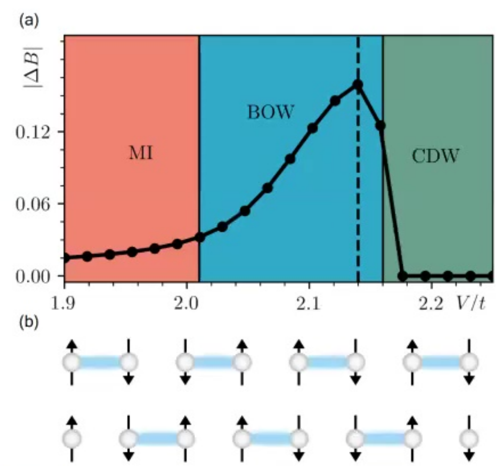
Learning by confusion for the XY model in 2D, M. Beach *et al.* Phys. Rev. E **97**, 045207 (2018). J. Rodriguez-Nieva *et al.* Nat. Phys. **15**, 790-795 (2019) etc.

- Supervised learning using convolutional neural networks (CNN).
  A. Bohrdt *et al.*, Nat. Phys. **15** 921-924 (2019), C. Miles *et al.* arXiv.2112.10789 etc.

Extended Fermi-Hubbard Model in One Dimension

$$\hat{H} = -t \sum_{\langle i,j \rangle} \left(\hat{c}^{\dagger}_{i,\sigma} \hat{c}_{j,\sigma} + \hat{c}^{\dagger}_{j,\sigma} \hat{c}_{i,\sigma} \right) + U \sum_{i} \hat{n}_{i,\uparrow} \hat{n}_{i\downarrow} + V \sum_{\langle i,j \rangle} \hat{n}_i \hat{n}_j$$





- $T = 0$ at half filling.
- 1st-order, 2nd-order, BKT-type quantum phase transitions, tri-critical points, topological properties.

- QMC based: P. Sengupta *et al.*, Phys. Rev. B **65**, 155113 (2002), A. Sandvik *et al.*, Phys. Rev. Lett. **92**, 236401 (2004) etc.
- Tensor networks based: E. Jeckelmann, Phys. Rev. Lett. **89**, 236401 (2002), S. Ejima, *et al.*, Phys. Rev. Lett. **99**, 216403 (2007), M. Dalmonte *et al.*, Phys. Rev. B **91**, 165136 (2015), S. Julià-Farré *et al.* Phys. Rev. Research **4**, L032005 (2021) etc.

Detecting the Quantum Phase Transition to the Off-Diagonal Bond-Order Wave

- Spin structure factor:

$$S_{spin}(q) = \frac{1}{N}\sum_{j,k} e^{iq(j-k)}\langle S_j^z S_k^z\rangle .$$

- Charge structure factor:

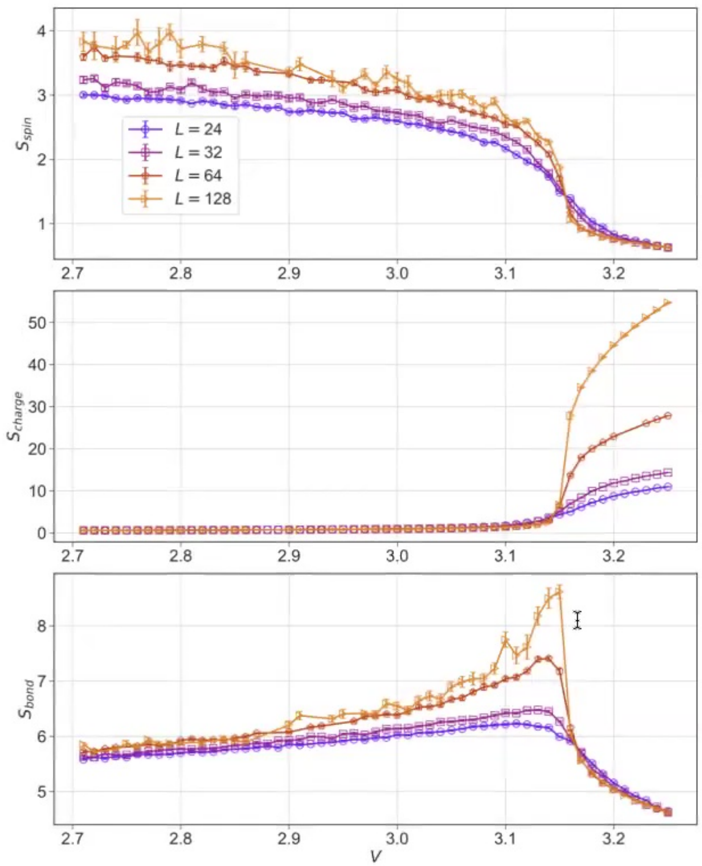
$$S_{charge}(q) = \frac{1}{N}\sum_{j,k} e^{iq(j-k)}\left(\langle n_j n_k\rangle - \langle n_j\rangle^2\right) .$$

- Bond-order wave structure factor

$$S_{bond}(q) = \frac{1}{N}\sum_{j,k} e^{iq(j-k)}\left(\langle k_j k_k\rangle - \langle k_j\rangle^2\right) ,$$

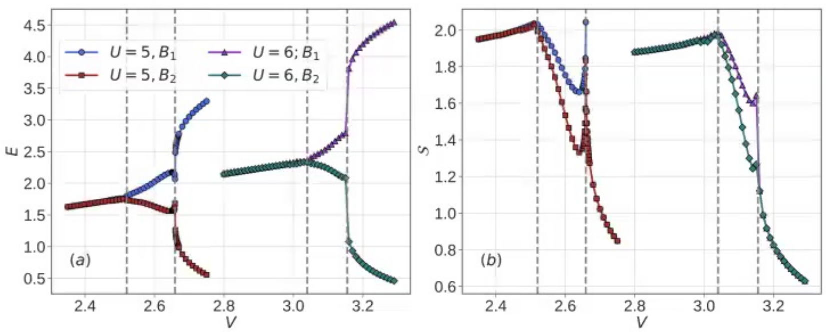
$$k_j = \sum_{\sigma=\uparrow,\downarrow}\left(c^\dagger_{j+1,\sigma}c_{j,\sigma} + H.c.\right) .$$

- $U/t = 6$. Data generated using world-line quantum Monte Carlo (WLQMC).
  J. Hirsch Phys. Rev. B **26**, 5033 (1982), F. Assaad *et al.* Computational Many-Particle Physics, 277-356 (2008) etc.



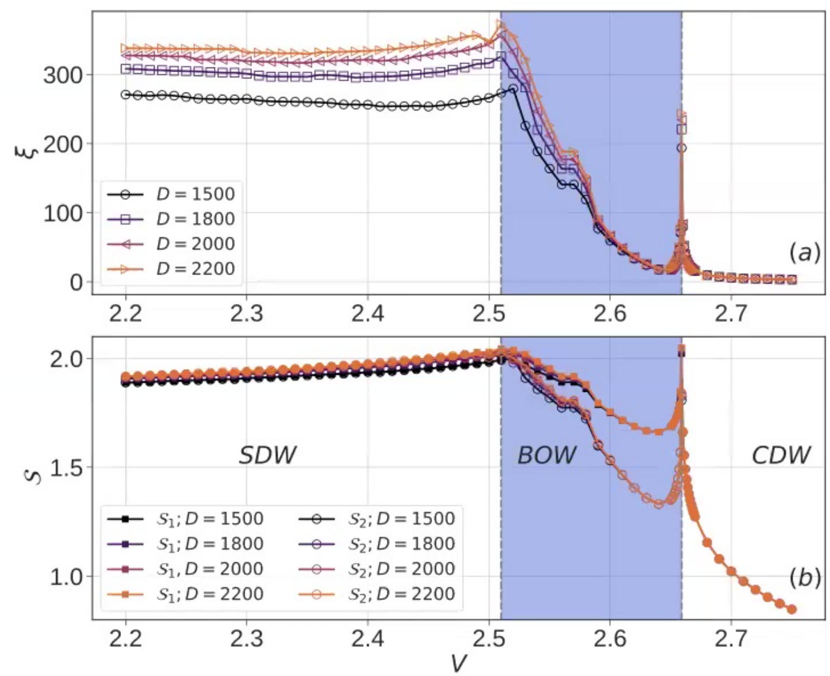
Reaching Thermodynamic Limit using Variational Uniform Matrix Product States

- Inspired by tangent space ideas, variational uniform matrix product states (VUMPS) is formulated in thermodynamic limit.
- Translational invariance removes soliton induced by the use of open boundary condition.
  V. Zauner-Stauber *et al.* Phys. Rev. B **97**, 045145 (2018), V. Zauner-Stauber *et al.* Phys. Rev. B **97**, 235155 (2018), L. Vanderstraeten *et al.* SciPost Phys. Lect. Notes, 7 (2019)



- Energy E_i and von Neumann entanglement entropy $\mathcal{S}_i$ associated with two bonds in a two-site unit cell.

$$\mathcal{S} = -Tr\rho_A \log \rho_A = -Tr\rho_B \log \rho_B.$$



- Bond dimension dependence of the correlation length

$$C(r) \sim e^{-r/\xi}, \xi = -\frac{1}{\log|\lambda_2/\lambda_1|}.$$

Learn the Snapshots

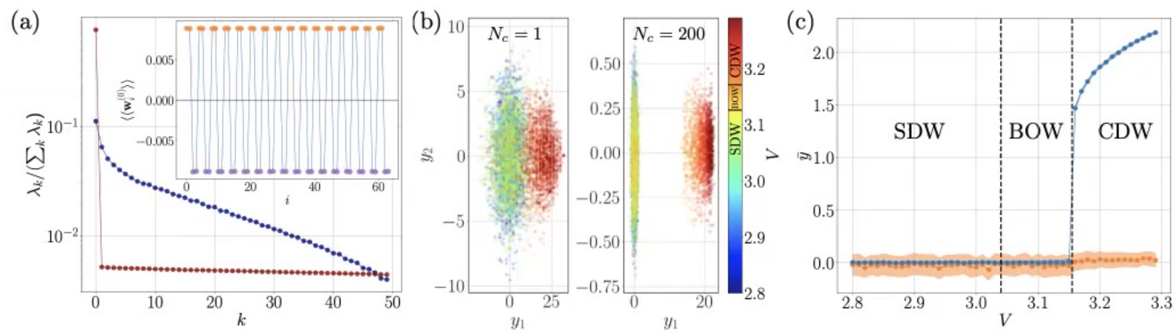
Principal component analysis (PCA)

Kernel PCA

Variational autoencoder/autoencoder

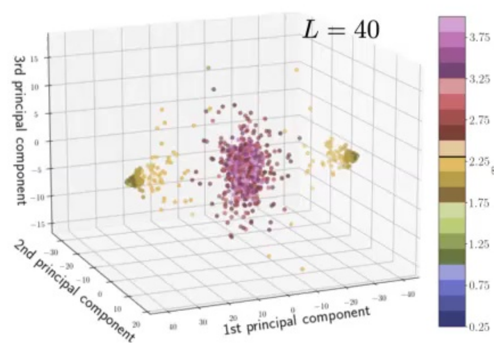
t-distributed stochastic neighbor embedding

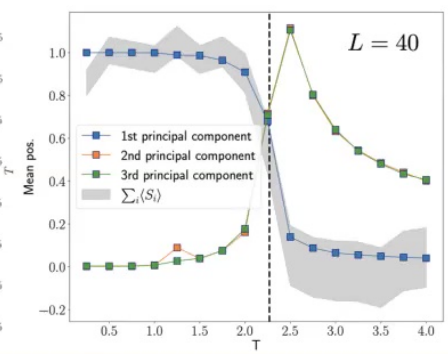
neural networks etc.



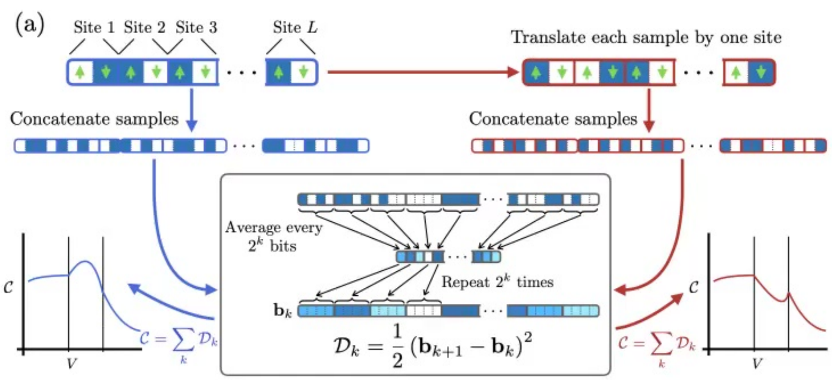
$U/t = 6$, $\Delta V = 0.1$, $N_s = 50000$ samples. Tracing the system down to a single site and sample from the resulting density matrix.

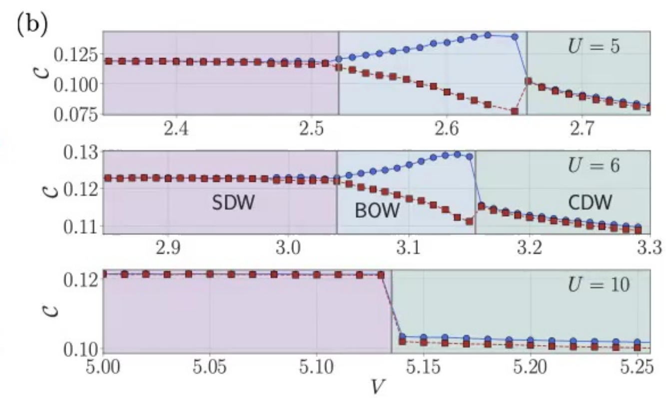
Comparing to 2nd-order transition e.g. the Ising model in 2D.





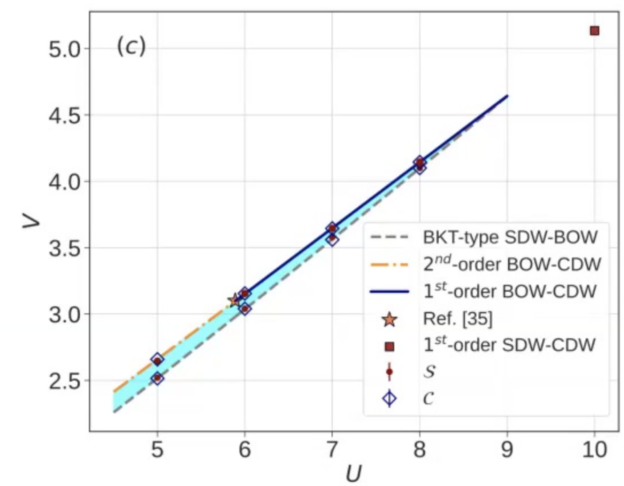
Multi-Scale Structural Complexity

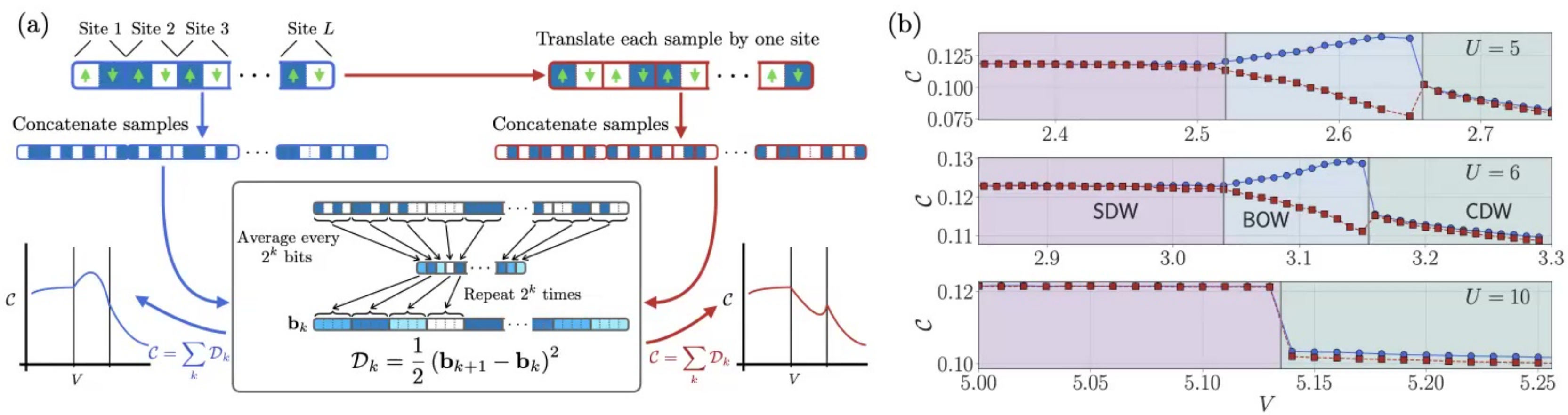




- Off-diagonal long-range order can be detected using the structural complexity analysis.
- Snapshots need to be generated from one of the degenerate ground state of the BOW phase.
- Open boundary condition in optical lattice experiments. Pinning fields.

A. Bagrov *et al.* PNAS **117**, 30241 (2020), O. Sotonikov *et al.* npj Quantum Information **8**, 41 (2022) etc.





- The off-diagonal long-range order can be detected using snapshots if these snapshots are generated from one of the degenerate ground states of the BOW phase.

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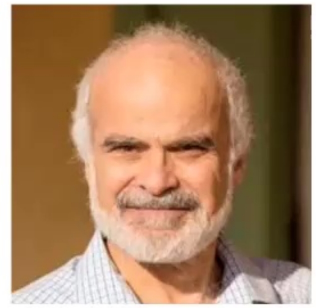

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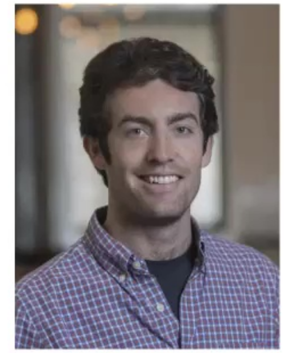

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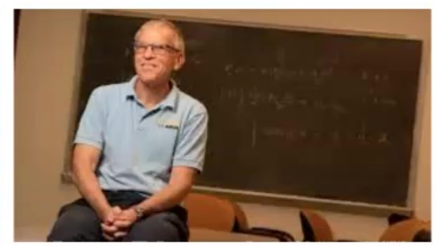

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Thank you for your attention