

Title: Anyonic information theory and quantum foundations

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Abstract: In this talk, I present the latest works on anyonic information theory and how it is linked to aspects of quantum foundations. First, the theory of 2+1 D non-abelian anyons will be introduced. The newly discovered notion of anyonic creation operators will be presented, as well as their use as local elements of reality within the Deutsch-Hayden interpretation of quantum mechanics. Lastly, I will show strange properties of anyonic entanglement that appear due to the lack of a tensor product structure, such as the different spectra of marginals in bipartite systems. This property makes the Von Neumann entropy a bad entanglement measure. I will explain the challenges of defining entanglement measures for anyonic systems and current approaches.

Zoom link: <https://pitp.zoom.us/j/99863263804?pwd=MUhkYTBzcUlwTmJ0Z3F4aFo3Rkt6QT09>

Anyonic quantum information and quantum foundations

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Motivation

- ▶ Fermionic information perspective¹. Quantum foundations should study fundamental theories. No-signalling vs. local-tomography.
- ▶ I would like to use constructor theory to classify all information theories.
- ▶ System composition is essential in that effort. Local ontic/generalised states for quantum mechanics². **Does the construction hold for constrained systems?**
- ▶ For fermions, local ontic states are the creation operators³.

¹Tibau Vidal et al. 2021; D'Ariano et al. 2014.

²Deutsch and Hayden 2000; Brassard and Raymond-Robichaud 2017.

³Tibau Vidal, Vedral, and Marletto 2022.

Why anyons?

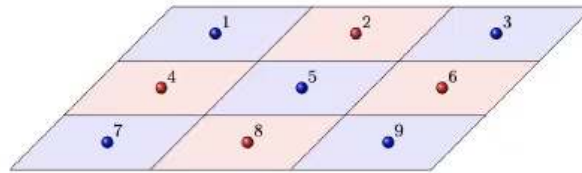
- ▶ There are no tensor product structures or creation operators. How is composition described?
- ▶ Fundamental theory of nature used in condensed matter physics. Proposed to be used for topological quantum computation.
- ▶ Constrained quantum systems. Connections with quantum gravity?
- ▶ Anyons are described diagrammatically⁴. Not a symmetric monoidal category, though.

Objective 1: Understand the notion of locality in anyons without a tensor product-like structure.

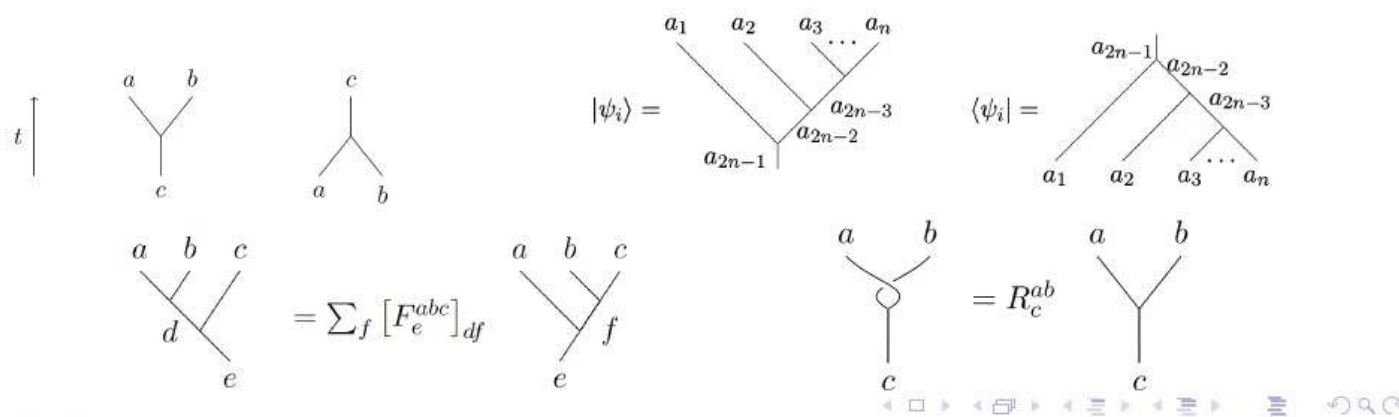
Objective 2: Study the information properties and structures.

⁴Bonderson, Shtengel, and Slingerland 2008; Coecke, Fritz, and Spekkens 2016. 

Anyons

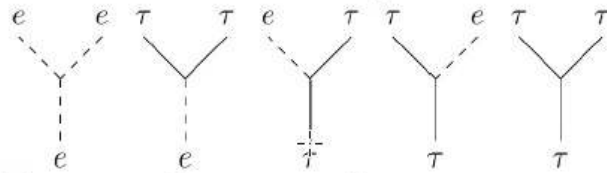


Each anyon theory has its particle types with fusion rules and braiding factors. $a = \{e, \tau\}$ $\tau \times \tau = e + \tau$ $a \times e = a$



Anyons as constrained systems

Fibonacci matrix representation for the 2 anyon system.



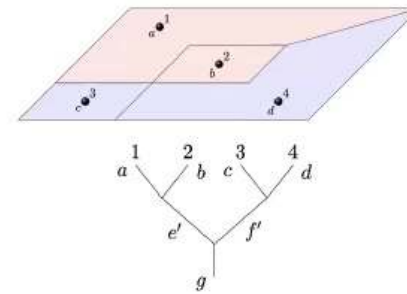
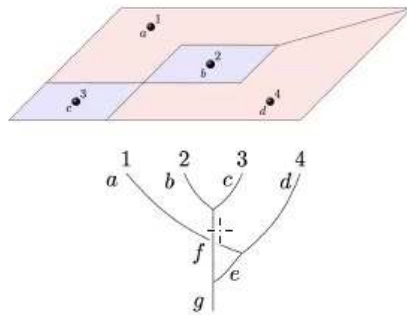
$$\rho = \begin{pmatrix} a_{00} & a_{10} & & & \\ a_{01} & a_{11} & & & \\ & & b_{00} & b_{10} & b_{20} \\ & & b_{01} & b_{11} & b_{21} \\ & & b_{02} & b_{12} & b_{22} \end{pmatrix}$$

You can interpret it as a constrained 3 qubit system.

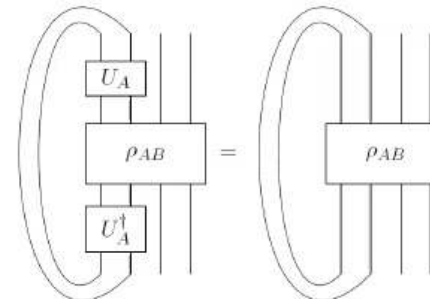
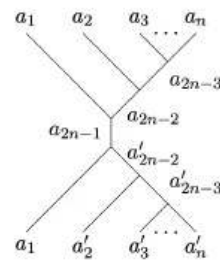
$$\hat{C} = \begin{pmatrix} 0 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 0 & & & & \\ & & & & 1 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & 0 \end{pmatrix} \quad |\psi\rangle \in \mathcal{H}_{phys} \Leftrightarrow \begin{cases} \hat{C}|\psi\rangle = 0 \\ [|\psi\rangle\langle\psi|, \hat{Z}_A] = 0 \end{cases}$$

We do not have a tensor product structure between the left and the right anyons!

Anyon subsystems



$$\begin{array}{c} 1 \\ | \\ \boxed{U} \\ | \\ 1 \end{array}
 \begin{array}{c} 2 \\ \dots \\ \dots \\ 2 \end{array}
 \begin{array}{c} n \\ \dots \\ \dots \\ n \end{array}
 = \sum_{\substack{a_1 \dots a_{2n-1} \\ a'_2 \dots a'_{2n-2}}} U^{a_2 \dots a_{2n-1}}_{a'_2 \dots a'_{2n-2}}$$



We have local unitaries and observables (also partial tracing).
Thus, we have subsystems⁵

⁵Chiribella 2018.

Anyonic creation operators

We require $U_B^\dagger \tau_A U_B = \tau_A$. Not observables

$$\begin{aligned}
 \tau_A^{(0)} &= \begin{array}{c} e \quad e \\ \diagdown \quad \diagup \\ e \\ \diagup \quad \diagdown \\ \tau \quad e \end{array} + \begin{array}{c} e \quad \tau \\ \diagdown \quad \diagup \\ \tau \\ \diagup \quad \diagdown \\ \tau \quad \tau \end{array} & \tau_A^{(1)} &= \begin{array}{c} e \quad e \\ \diagdown \quad \diagup \\ e \\ \diagup \quad \diagdown \\ \tau \quad e \end{array} + \begin{array}{c} e \quad \tau \\ \diagdown \quad \diagup \\ \tau \\ \diagup \quad \diagdown \\ \tau \quad \tau \end{array} \\
 \tau_B^{(0)} &= \begin{array}{c} e \quad e \\ \diagdown \quad \diagup \\ e_i \\ \diagup \quad \diagdown \\ e \quad \tau \end{array} + R_e^{\tau\tau\tau} \begin{array}{c} \tau \quad e \\ \diagdown \quad \diagup \\ e_i \\ \diagup \quad \diagdown \\ \tau \quad \tau \end{array} & \tau_B^{(1)} &= \begin{array}{c} e \quad e \\ \diagdown \quad \diagup \\ e_i \\ \diagup \quad \diagdown \\ e \quad \tau \end{array} + R_\tau^{\tau\tau\tau} \begin{array}{c} \tau \quad e \\ \diagdown \quad \diagup \\ \tau \\ \diagup \quad \diagdown \\ \tau \quad \tau \end{array}
 \end{aligned}$$

We combine the annihilating terms to minimize the number of operators needed to generate the local algebra of observables. We need **two annihilation operators** per lattice site.

Local elements of reality

In the Deutsch-Hayden interpretation of QM, we can **use annihilation operators as the local elements of reality**.

You can have a local realistic account of anyons. Reinterpreting the Heisenberg picture.

$$\begin{aligned}
 |\psi_0\rangle \text{ is fixed.} \quad \vec{\hat{q}}(t) &= \left(\tau_A^{(0)}(t), \tau_A^{(1)}(t), \tau_B^{(0)}(t), \tau_B^{(1)}(t) \right) = \\
 &= U^\dagger \left(\tau_A^{(0)}(0), \tau_A^{(1)}(0), \tau_B^{(0)}(0), \tau_B^{(1)}(0) \right) U
 \end{aligned}$$

$$\text{Tr} \left(\hat{O}_{AB} U |\psi_0\rangle\langle\psi_0| U^\dagger \right) = \sum_j o_j \text{Tr} \left(p_j \left(\vec{\hat{q}}(t), \vec{\hat{q}}^\dagger(t) \right) |\psi_0\rangle\langle\psi_0| \right)$$

Creation operators are part of the ontology. **This violates Leibnitz principle!**

Asymmetric marginals

Information theory perspective. Usually the Von Neumann entropy is used to quantify entanglement in **pure states**.

$S(\rho_A) = S(\rho_B)$ because we have a Schmidt decomposition

$$|\psi\rangle_{AB} = \sum_j \sqrt{p_j} |\psi_j\rangle_A \otimes |\varphi_j\rangle_B.$$

The lack of tensor product allows us to find:

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \tau \quad e \\ \diagdown \quad \diagup \\ \text{---} \\ \tau \end{array} + \begin{array}{c} \tau \quad \tau \\ \diagdown \quad \diagup \\ \text{---} \\ \tau \end{array} \right)$$

$$\rho_A = \begin{array}{c} | \\ \tau \end{array}$$

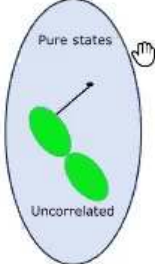
$$\rho_B = \frac{1}{2} \left(\begin{array}{c} | \\ \tau \end{array} + \begin{array}{c} | \\ e \end{array} \right)$$

Thus, $S(\rho_B) = 1$ and $S(\rho_A) = 0$

Entanglement quantification

We can find all pure uncorrelated states. States that satisfy:

$$\text{Tr}(\hat{O}_A \cdot \hat{O}_B \rho) = \text{Tr}(\hat{O}_A \rho_A) \cdot \text{Tr}(\hat{O}_B \rho_B).$$

$$|\psi\rangle_{AB} = \alpha \begin{array}{c} \tau \quad e \\ \diagdown \quad \diagup \\ \tau \end{array} + \beta \begin{array}{c} \tau \quad \tau \\ \diagdown \quad \diagup \\ \tau \end{array} \quad \text{or} \quad |\psi\rangle_{AB} = \alpha \begin{array}{c} e \quad \tau \\ \diagdown \quad \diagup \\ \tau \end{array} + \beta \begin{array}{c} \tau \quad \tau \\ \diagdown \quad \diagup \\ \tau \end{array}$$


Local operations $U_A \cdot U_B$ is the largest group that leaves the uncorrelated states set invariant. Similar structure to alignable states transformations. Mappings are state-dependent?

Relative entropy, teleportation protocol is possible?, entanglement distillation,...

Conclusions

- ▶ Anyons provide a different perspective on subsystem and entanglement structures of constrained systems.
- ▶ We have found anyonic annihilation operators, that can act as local elements of reality.
- ▶ Tensor product decompositions are not necessary to define subsystems and entanglement.
- ▶ Pure anyonic states have asymmetric marginal spectra. The Von Neumann entropy is a bad entanglement measure.
- ▶ Local operations do not cover the uncorrelated state set. We need state-dependent maps, similar to alignable states in QRF's. Relevant to entanglement-coherence invariants.

Possible future directions

- ▶ Study free operations using process theory. Not describable by unitaries. Link to anyonic entanglement measures.
- ▶ Connect to alignable states in perspectival quantum reference frames and subsystem relativity.
- ▶ Study the relation of local ontic states with process theories, explore the consequences of breaking Leibnitz principle.
- ▶ Link process theories with constructor theory regarding subsystem composition. Classify all information theories.
- ▶ Revisit reconstruction efforts where local tomography is emphasised, how to recover constrained quantum theories?.
- ▶ Express and analyse the BMV experiment in diagrammatic form. Explore consequences for quantum gravity.