Title: Anyonic information theory and quantum foundations

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Abstract: In this talk, I present the latest works on anyonic information theory and how it is linked to aspects of quantum foundations. First, the theory of 2+1 D non-abelian anyons will be introduced. The newly discovered notion of anyonic creation operators will be presented, as well as their use as local elements of reality within the Deutsch-Hayden interpretation of quantum mechanics. Lastly, I will show strange properties of anyonic entanglement that appear due to the lack of a tensor product structure, such as the different spectra of marginals in bipartite systems. This property makes the Von Neumann entropy a bad entanglement measure. I will explain the challenges of defining entanglement measures for anyonic systems and current approaches.

Zoom link: https://pitp.zoom.us/j/99863263804?pwd=MUhkYTBzcUlwTmJ0Z3F4aFo3Rkt6QT09

Anyonic quantum information and quantum foundations

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Motivation

- Fermionic information perspective[1]. Quantum foundations should study fundamental theories. No-signalling vs. local-tomography.
- I would like to use constructor theory to classify all information theories.
- System composition is essential in that effort. Local ontic/generalised states for quantum mechanics[2]. **Does the construction hold for constrained systems?**
- For fermions, local ontic states are the creation operators[3].

[1] Tibau Vidal et al. 2021; D'Ariano et al. 2014.
[2] Deutsch and Hayden 2000; Brassard and Raymond-Robichaud 2017.
[3] Tibau Vidal, Vedral, and Marletto 2022.

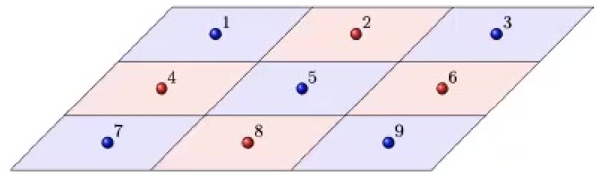
Why anyons?

- There are no tensor product structures or creation operators. How is composition described?
- Fundamental theory of nature used in condensed matter physics. Proposed to be used for topological quantum computation.
- Constrained quantum systems. Connections with quantum gravity?
- Anyons are described diagrammatically[4]. Not a symmetric monoidal category, though.

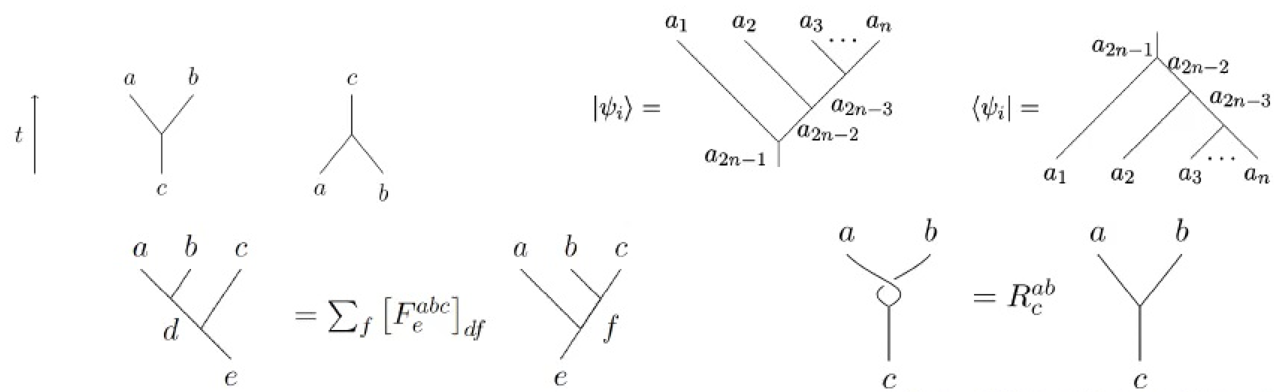
Objective 1: Understand the notion of locality in anyons without a tensor product-like structure.
Objective 2: Study the information properties and structures.

[4] Bonderson, Shtengel, and Slingerland 2008; Coecke, Fritz, and Spekkens 2016.

Anyons

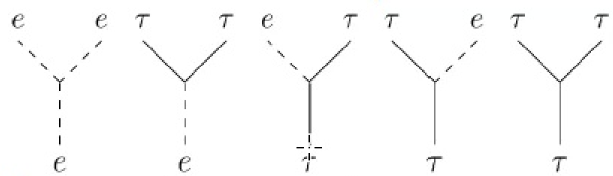


Each anyon theory has its particle types with fusion rules and braiding factors. $a = \{e, \tau\}$ $\qquad \tau \times \tau = e + \tau$ $\qquad a \times e = a$



Anyons as constrained systems

Fibonacci matrix representation for the 2 anyon system.



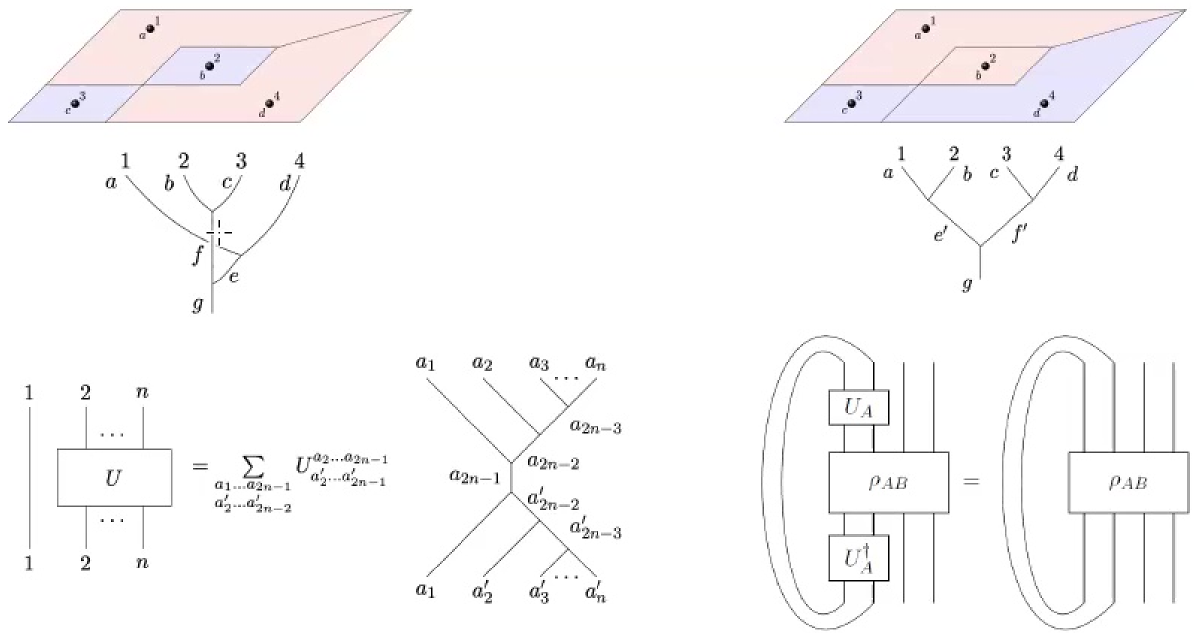
$$\rho = \begin{pmatrix} a_{00} & a_{10} & & & \\ a_{01} & a_{11} & & & \\ & & b_{00} & b_{10} & b_{20} \\ & & b_{01} & b_{11} & b_{21} \\ & & b_{02} & b_{12} & b_{22} \end{pmatrix}$$

You can interpret it as a constrained 3 qubit system.

$$\hat{C} = \begin{pmatrix} 0 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 0 & & & & \\ & & & & 1 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & 0 \end{pmatrix} \quad |\psi\rangle \in \mathcal{H}_{phys} \quad \Leftrightarrow \quad \begin{matrix} \hat{C}\,|\psi\rangle = 0 \\ [|\psi\rangle\langle\psi|\,,\hat{Z}_A] = 0 \end{matrix}$$

We do not have a tensor product structure between the left and the right anyons!

Anyon subsystems

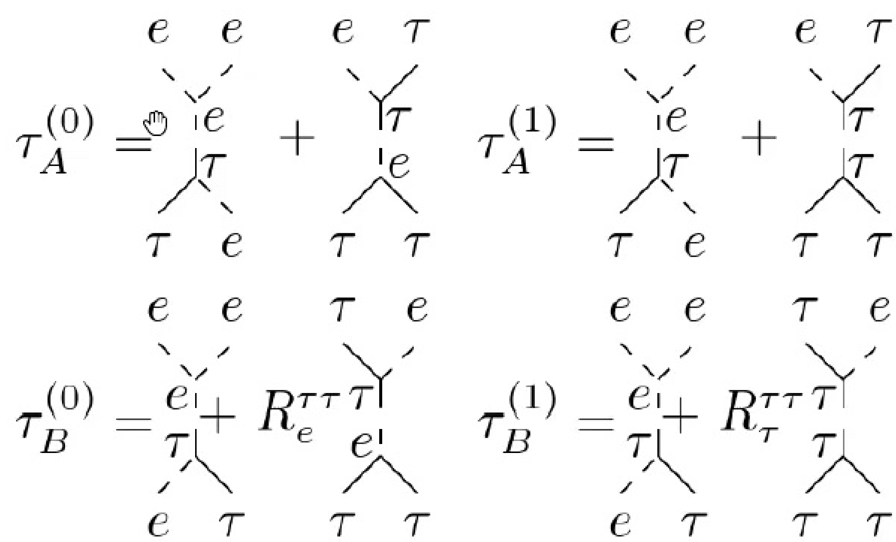


We have local unitaries and observables (also partial tracing).
Thus, we have subsystems[5]

[5] Chiribella 2018.

Anyonic creation operators

We require $U_B^\dagger \tau_A U_B = \tau_A$. Not observables



We combine the annihilating terms to minimize the number of operators needed to generate the local algebra of observables.

We need **two annihilation operators** per lattice site.

Local elements of reality

In the Deustch-Hayden interpretation of QM, we can **use annihilation operators as the local elements of reality**.

You can have a local realistic account of anyons. Reinterpreting the Heisenberg picture.

$|\psi_0\rangle$ is fixed.

$$\vec{\hat{q}}(t) = \left(\tau_A^{(0)}(t), \tau_A^{(1)}(t), \tau_B^{(0)}(t), \tau_B^{(1)}(t)\right) =$$
$$= U^\dagger \left(\tau_A^{(0)}(0), \tau_A^{(1)}(0), \tau_B^{(0)}(0), \tau_B^{(1)}(0)\right) U$$

$$\mathrm{Tr}\left(\hat{O}_{AB} U \left|\psi_0\rangle\langle\psi_0\right| U^\dagger\right) = \sum_j o_j \, \mathrm{Tr}\left(p_j\left(\vec{\hat{q}}(t), \vec{\hat{q}}^\dagger(t)\right) \left|\psi_0\rangle\langle\psi_0\right|\right)$$

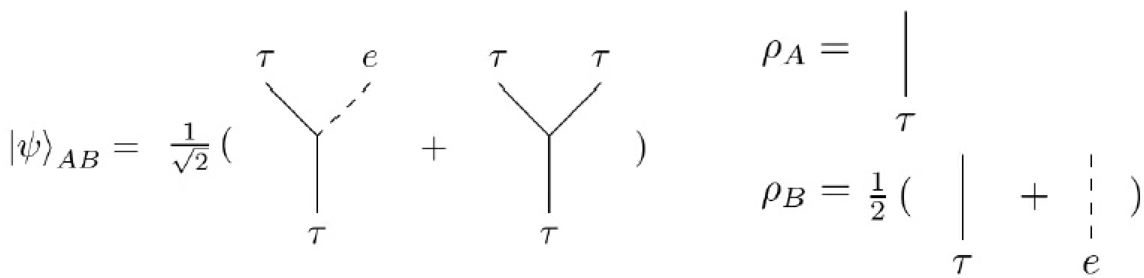
Creation operators are part of the ontology. **This violates Leibnitz principle!**

Asymmetric marginals

Information theory perspective. Usually the Von Neumann entropy is used to quantify entanglement in **pure states**.

$S(\rho_A) = S(\rho_B)$ because we have a Schmidt decomposition $|\psi\rangle_{AB} = \sum_j \sqrt{p_j}\, |\psi_j\rangle_A \otimes |\varphi_j\rangle_B$.

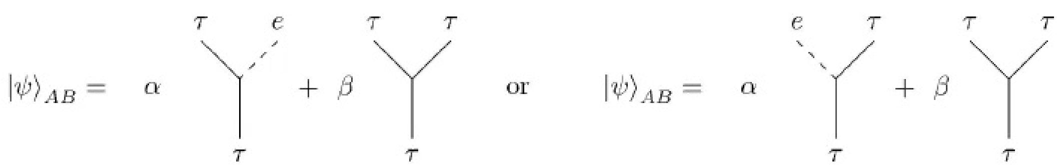
The lack of tensor product allows us to find:

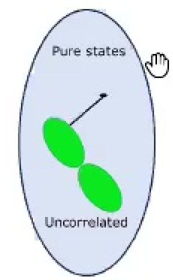


Thus, $S(\rho_B) = 1$ and $S(\rho_A) = 0$

Entanglement quantification

We can find all pure uncorrelated states. States that satisfy:
$\mathrm{Tr}\left(\hat{O}_A \cdot \hat{O}_B \rho\right) = \mathrm{Tr}\left(\hat{O}_A \rho_A\right) \cdot \mathrm{Tr}\left(\hat{O}_B \rho_B\right)$.





Local operations $U_A \cdot U_B$ is the largest group that leaves the uncorrelated states set invariant. Similar structure to alignable states transformations. Mappings are state-dependent?

Relative entropy, teleportation protocol is possible?, entanglement distillation,...

Lenovo

Conclusions

- Anyons provide a different perspective on subsystem and entanglement structures of constrained systems.
- We have found anyonic annihilation operators, that can act as local elements of reality.
- Tensor product decompositions are not necessary to define subsystems and entanglement.
- Pure anyonic states have asymmetric marginal spectra. The Von Neumann entropy is a bad entanglement measure.
- Local operations do not cover the uncorrelated state set. We need state-dependent maps, similar to alignable states in QRF's. Relevant to entanglement-coherence invariants.

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Possible future directions

- Study free operations using process theory. Not describable by unitaries. Link to anyonic entanglement measures.
- Connect to alignable states in perspectival quantum reference frames and subsystem relativity.
- Study the relation of local ontic states with process theories, explore the consequences of breaking Leibnitz principle.
- Link process theories with constructor theory regarding subsystem composition. Classify all information theories.
- Revisit reconstruction efforts where local tomography is emphasised, how to recover constrained quantum theories?.
- Express and analyse the BMV experiment in diagrammatic form. Explore consequences for quantum gravity.