Title: Effective Field Theory and Symmetries

Speakers: John Donoghue

Collection: Symmetries Graduate School 2023

Date: January 31, 2023 - 11:00 AM

URL: https://pirsa.org/23010092

Abstract: These lectures will cover the concepts and techniques of effective field theory. I will try to introduce several of the useful techniques which do not usually get covered in the standard QFT courses and books. We will start with the effective field theory aspects of QED, and end with the treatment of general relativity as a quantum field theory using effective field theory techniques.

EFT & Symmetries - Preliminaries

John Donoghue (UMass Amherst)

Jan 31, 2023

at Perimeter Institute

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webpage: blogs.umass.edu/donoghue/

John Donoghue's web pages

WELCOME NEWS RESEARCH INTERESTS TEACHING PERSONAL LINKS DYNAMICS OF THE STANDARD MODEL

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WELCOME

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I am a theoretical physicist at the University of Massachusetts at

EFT Winter school at Perimeter Institute (Jan/Feb 2023):

I will post my notes for the school here

Dynamics of the Standard Model is now free in Open Access:

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Physics is exptl. science

EFT is humble QFT

- don't need complete
  - scales

Effective

- low energy approx
- useful

- don't need complete
  - scales

Effective
- low energy approx
- useful

EFT is tool

Bookkeeping

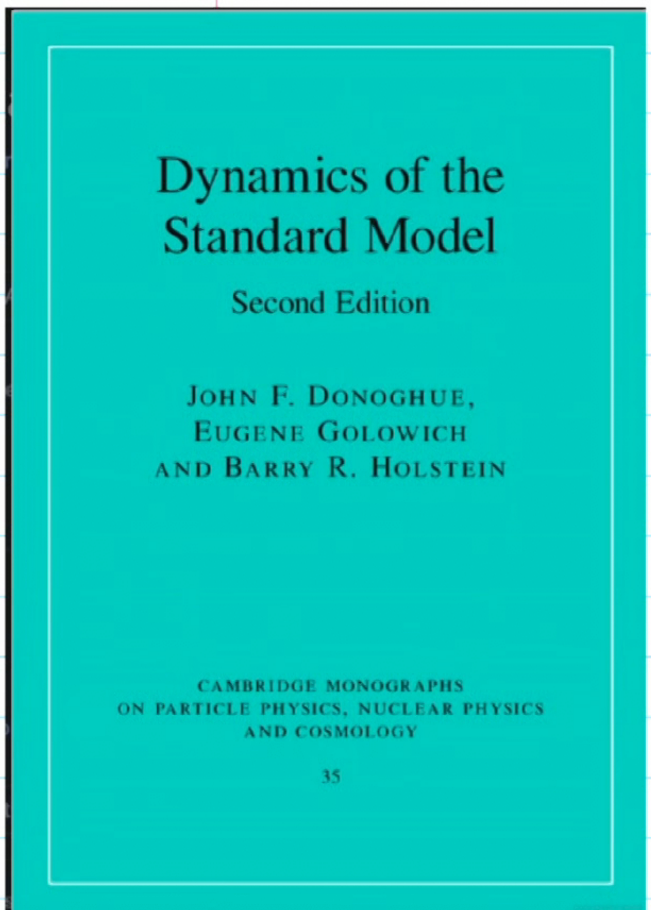
- notes
- handwriting
- $\hbar, c$ -

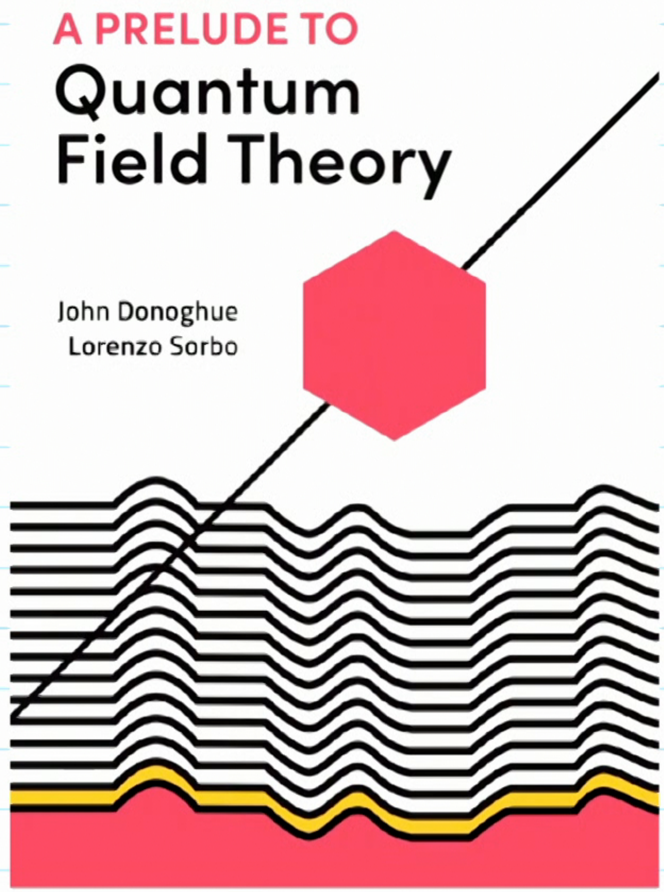
Bookkeeping

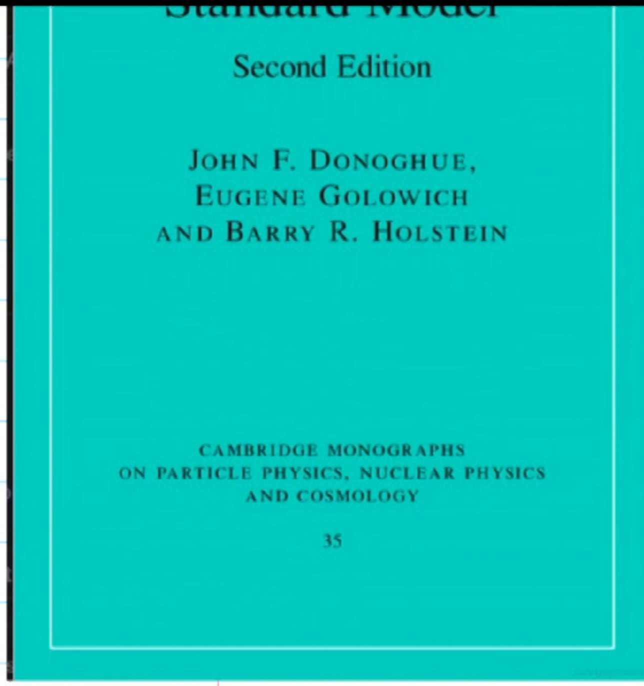
- notes
- handwriting
- $\hbar, c = 1$
- metric $(+ - - -)$ $\quad p_\mu p^\mu = E^2 - \vec{p}^{\,2} = m^2$

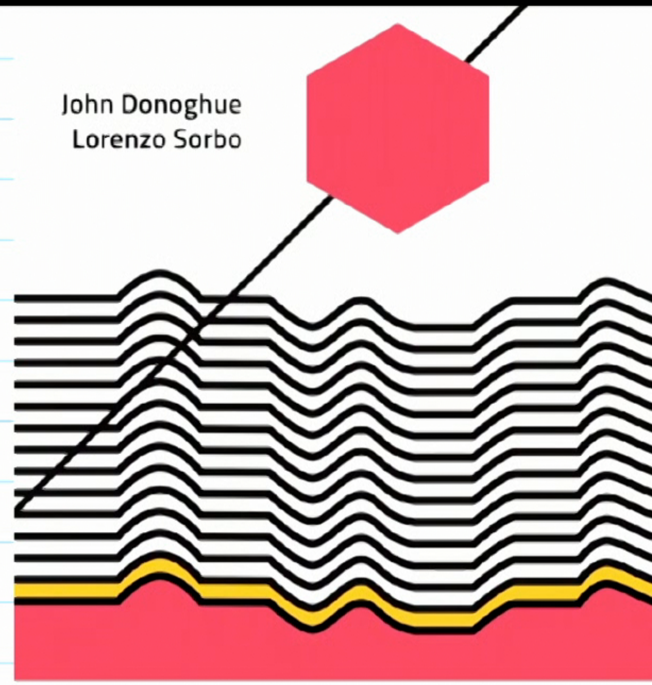
- Notes
- Handwriting
- $\hbar, c = 1$
- metric $(+ - - -)$ $\quad p_\mu p^\mu = E^2 - \vec{p}^{\,2} = m^2$

$$iD_F = \frac{i}{p^2 - m^2 + i\epsilon}$$

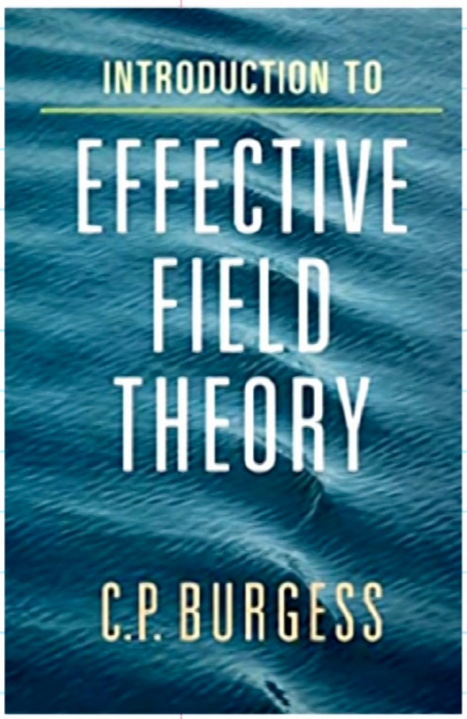

Dynamics of the
Standard Model
Second Edition
JOHN F. DONOGHUE,
EUGENE GOLOWICH
AND BARRY R. HOLSTEIN
CAMBRIDGE MONOGRAPHS
ON PARTICLE PHYSICS, NUCLEAR PHYSICS
AND COSMOLOGY
35

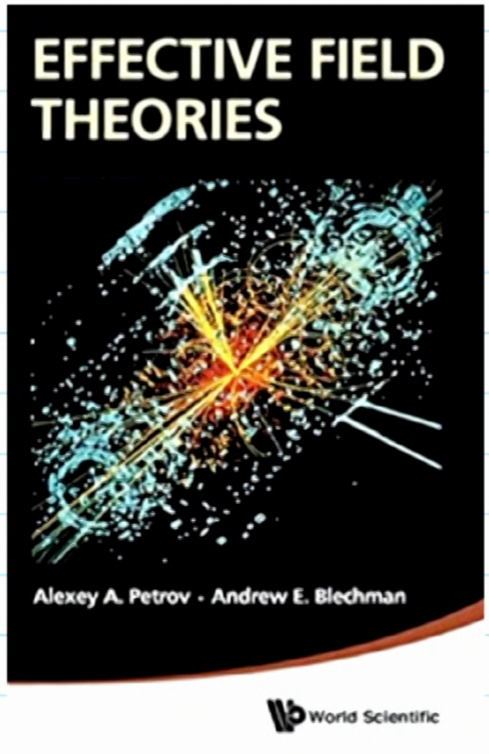

A PRELUDE TO
Quantum
Field Theory
John Donoghue
Lorenzo Sorbo





Dynamics of the Standard Model is now available for FREE on an open access agreement. You can find it by googling "SCOAP3 books" or by using the link on my web page https://blogs.umass.edu/donoghue/


INTRODUCTION TO
EFFECTIVE FIELD THEORY
C.P. BURGESS

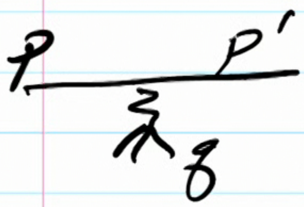

EFFECTIVE FIELD THEORIES
Alexey A. Petrov · Andrew E. Blechman
World Scientific


Ulf-G Meißner
and Akaki Rusetsky
EFFECTIVE
FIELD THEORIES

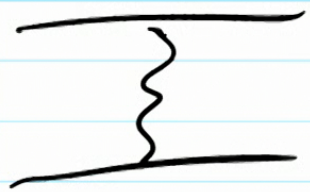
Effective Field Theory #1

QED

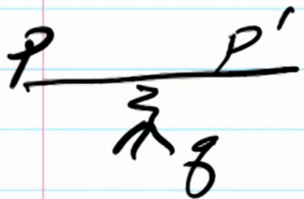
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi$$



$= -ie\gamma_\mu \qquad \propto -i(2p+q)_\mu$



$= -i\mathcal{M}$



$= -ie\gamma_\mu \qquad \propto -i(2p+q)_\mu$

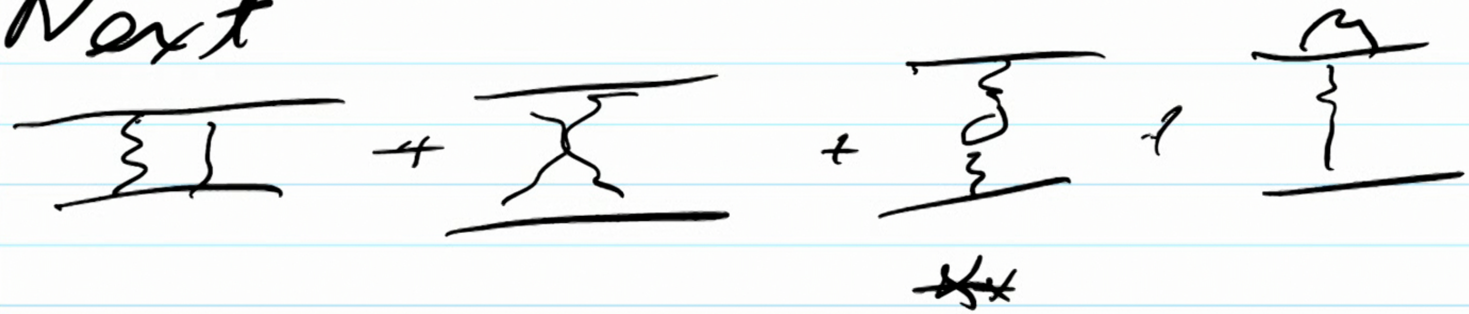
$= -i\mathcal{M} = (-ie)^2\, \bar{u}\gamma_\mu u \, \frac{-i g_{\mu\nu}}{q^2 - i\varepsilon} \, \bar{u}\gamma^\nu u$

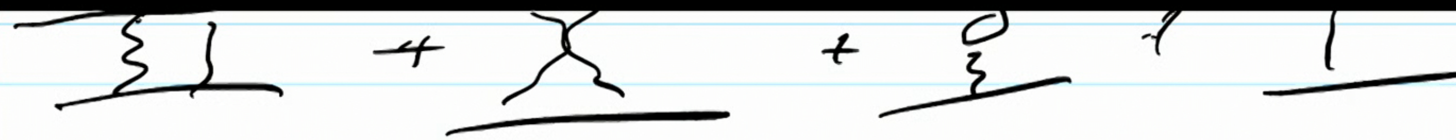
$$\mathcal{M} \xrightarrow[NR]{} \frac{e^2}{\vec{q}^{\,2} - i\epsilon}$$

$$V(r) = \int \frac{d^3q}{(2\pi)^3} e^{-iq\cdot r} V(q) \to \frac{\alpha^2}{4\pi r}$$

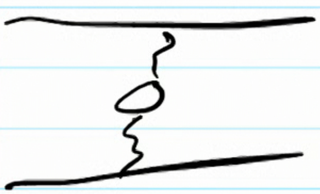
$$\frac{d\sigma}{d\Omega} = \frac{4m^2\alpha^2}{q^4}$$

Next

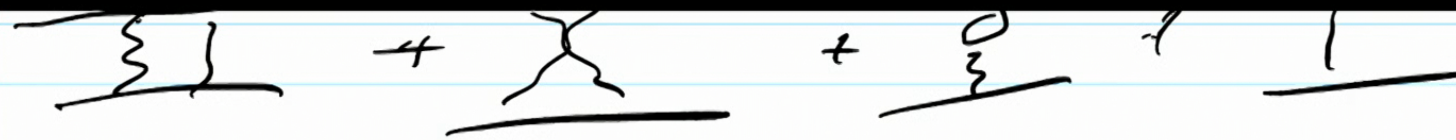




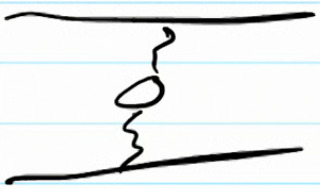
 $= i\,\Pi_{\mu\nu}(q^2) = (q_\mu q_\nu - g_{\mu\nu} q^2)\,\Pi(q^2)$

 $= \left[\quad\right] = \dfrac{-i g_{\mu\alpha}}{q^2}(q_\alpha q_\beta - g_{\alpha\beta} q^2)\,\Pi\left(\dfrac{-i g_{\beta\nu}}{q^2}\right)$

$\leftarrow \Pi(q^2)$



 $= i\Pi_{\mu\nu}(q^2) = (q_\mu q_\nu - g_{\mu\nu} q^2)\,\Pi(q^2)$

 $= \left[\quad\right] = \dfrac{-ig_{\mu\alpha}}{q^2}(q_\alpha q_\beta - g_{\alpha\beta} q^2)\,\Pi\left(\dfrac{-ig_{\beta\nu}}{q^2}\right)$

$\leftarrow \Pi(q^2)$

$= \mathcal{M}_0\left(-\Pi(q^2)\right)$

Now

$$\frac{d\sigma}{d\Omega} = \frac{4m^2}{q^4}\left[\alpha\left(1-\Pi(q^2)\right)\right]^2$$

$$\frac{d\sigma}{d\Omega} = \frac{4m^2}{q^4}\left[\alpha\left(1-\Pi(q^2)\right)\right]^2$$

Proceed?

— R_e measure

α

$$\frac{d\sigma}{d\Omega} = \frac{4m^2}{q^4}\left[\alpha_0\left(1-\Pi(q^2)\right)\right]^2$$

Proceed?

– Re measure

$$\alpha_0\left[1-\Pi(q^2\approx 0)\right] = \frac{1}{137}$$

Proceed?

— Re measure

$$\alpha_0 \left[1 - \Pi(q^2 \approx 0) \right] = \frac{1}{137}$$

$$\ldots \;=\; \frac{\quad}{137} = \alpha$$

To this order

$$\alpha_0\left[1 - \Pi(q^2)\right] = \alpha\left[1 - \underbrace{\left(\Pi(q^2) - \Pi(0)\right)}_{\Delta\Pi}\right]$$

$$\Delta \overline{\Pi}(q^2) = -\frac{\alpha}{15\pi}\frac{q^2}{m^2}$$

$\int$

$$-\frac{1}{g_c^2}\left[1-\Pi(q^2)\right] \longrightarrow \frac{\alpha}{4\pi r}-\frac{\alpha}{\cancel{\;}}\,\delta^3(x)$$

μ

$$\Delta\Pi(q^2) = -\frac{\alpha}{15\pi}\frac{q^2}{m^2}$$

Then

$$\frac{-i}{q^2}\left[1-\Delta\Pi(q^2)\right] \longrightarrow \frac{\alpha}{4\pi r} - \frac{\alpha}{15m^2}\delta^3(x)$$

Effective Lagrangian

$$\Delta \mathcal{L} = -\frac{e^2}{240\pi^2 m^2} F_{\mu\nu} \Box F^{\mu\nu}$$

$$\Delta \mathcal{L} = -\frac{e^2}{240\pi^2 m^2} F_{\mu\nu} \Box F^{\mu\nu}$$

Feynman rule

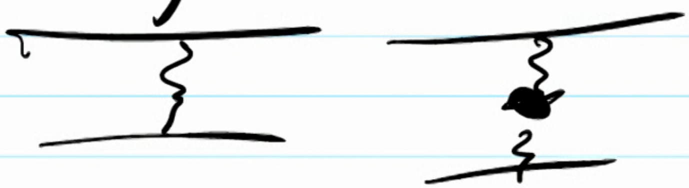
$$= -\frac{i\alpha}{15\pi m^2} q^2 \left(g_{\mu\nu} q^2 - q_\mu q_\nu\right)$$

Using

Feynman rule

$$= -\frac{i\alpha}{15\pi m^2}\, q^2 \left(g_{\mu\nu} q^2 - q_\mu q_\nu\right)$$

Using it



$$= -i\mathcal{M}$$

Dim. analysis

$$S = \int d^4x \, \mathcal{L}$$

$$\Delta x \, \Delta p \sim 1 \implies [x] = \frac{1}{E}, \quad \partial \sim [E]$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \implies [A] = E^1$$

$$[\phi] = E^1$$

$$\psi = E^{3/2}$$

$F\Gamma$

$$\psi = E^{3/2}$$

$$c\, F \Box F \sim E^6$$

$$\Rightarrow c = \frac{1}{m^2}$$

$$[\phi] = E^1$$

$$\psi = E^{3/2}$$

$$c\, F \Box F \sim E^6$$

$$\Longrightarrow c = \frac{1}{m^2}$$

"non-renorm."

$$i\Pi^{\alpha\beta}(q) = -(-ie_0)^2 \int \frac{d^4p}{(2\pi)^4}\ \mathrm{Tr}\left[\gamma^\alpha \frac{i}{\not{p} - m + i\epsilon}\gamma^\beta \frac{i}{\not{p} - \not{q} - m + i\epsilon}\right].$$

$$i\Pi^{\alpha\beta}(q) = -(-ie_0)^2 \int \frac{d^4p}{(2\pi)^4}\ \mathrm{Tr}\left[\gamma^\alpha \frac{i}{\not{p} - m + i\epsilon}\gamma^\beta \frac{i}{\not{p} - \not{q} - m + i\epsilon}\right].$$

$$\Pi(q) = \frac{e_0^2}{12\pi^2}\left[\frac{1}{\epsilon} + \ln(4\pi) - \gamma \right.$$
$$\left. - 6\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2 - q^2 x(1-x)}{\mu^2}\right) + \mathcal{O}(\epsilon)\right]$$
$$= \frac{e_0^2}{\ \ } \left\{ \frac{1}{\epsilon} + \ln(4\pi) - \gamma + \frac{5}{3} - \ln\frac{-q^2}{\mu^2} + \cdots \qquad (|q^2| \gg m^2), \right.$$

Calculate $\Pi(q^2)$

$$i\Pi^{\alpha\beta}(q) = -(-ie_0)^2 \int \frac{d^4p}{(2\pi)^4} \operatorname{Tr}\left[\gamma^\alpha \frac{i}{\not{p} - m + i\epsilon}\gamma^\beta \frac{i}{\not{p} - \not{q} - m + i\epsilon}\right].$$

$$i\Pi^{\alpha\beta}(q) = -(-ie_0)^2 \int \frac{d^4p}{(2\pi)^4}\, \mathrm{Tr}\left[\gamma^\alpha \frac{i}{\not p - m + i\epsilon}\gamma^\beta \frac{i}{\not p - \not q - m + i\epsilon}\right].$$

$$\begin{aligned}\Pi(q) &= \frac{e_0^2}{12\pi^2}\left[\frac{1}{\epsilon} + \ln(4\pi) - \gamma \right.\\ &\qquad \left. -6\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2 - q^2 x(1-x)}{\mu^2}\right) + \mathcal{O}(\epsilon)\right]\\ &= \frac{e_0^2}{12\pi^2}\begin{cases}\dfrac{1}{\epsilon} + \ln(4\pi) - \gamma + \dfrac{5}{3} - \ln\dfrac{-q^2}{\mu^2} + \cdots & (|q^2| \gg m^2),\\ \dfrac{1}{\epsilon} + \ln(4\pi) - \gamma - \ln\dfrac{m^2}{\mu^2} + \dfrac{q^2}{5m^2} + \cdots & (m^2 \gg |q^2|).\end{cases}\end{aligned}$$

$$
-6\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2-q^2x(1-x)}{\mu^2}\right)+\mathcal{O}(\epsilon)\Bigg]
$$

$$
=\frac{e_0^2}{12\pi^2}\begin{cases}\dfrac{1}{\epsilon}+\ln(4\pi)-\gamma+\dfrac{5}{3}-\ln\dfrac{-q^2}{\mu^2}+\cdots & (|q^2|\gg m^2),\\[2ex] \underbrace{\dfrac{1}{\epsilon}+\ln(4\pi)-\gamma-\ln\dfrac{m^2}{\mu^2}}_{\Pi(0)}+\dfrac{q^2}{5m^2}+\cdots & (m^2\gg|q^2|).\end{cases}
$$

$$
\begin{aligned}
\Pi(q) &= \frac{e_0^2}{12\pi^2}\left[\frac{1}{\epsilon} + \ln(4\pi) - \gamma \right. \\
&\qquad \left. - 6\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2 - q^2 x(1-x)}{\mu^2}\right) + \mathcal{O}(\epsilon)\right] \\
&= \frac{e_0^2}{12\pi^2}\begin{cases} \dfrac{1}{\epsilon} + \ln(4\pi) - \gamma + \dfrac{5}{3} - \ln\dfrac{-q^2}{\mu^2} + \cdots & (|q^2| \gg m^2), \\ \underbrace{\dfrac{1}{\epsilon} + \ln(4\pi) - \gamma - \ln\dfrac{m^2}{\mu^2}}_{\Pi(0)} + \dfrac{q^2}{5m^2} + \cdots & (m^2 \gg |q^2|). \end{cases}
\end{aligned}
$$

Dim. reg. $\qquad \epsilon = \dfrac{4-d}{2}$

$$\int \frac{d^4p}{(2\pi)^4} \to \mu^{4-d}\int \frac{d^dp}{(2\pi)^d}$$

$\Pi(0)$

Dim. reg. $\qquad \varepsilon = \frac{4-d}{2}$

$$\int \frac{d^4 p}{(2\pi)^4} \to \mu^{4-d} \int \frac{d^d p}{(2\pi)^d}$$

$$\Pi(0) \sim \ln m^2/\mu^2$$

$$\int \frac{d^4p}{(2\pi)^4} \to \mu^{4-d} \int \frac{d^dp}{(2\pi)^d}$$

$$\Pi(0) \sim \ln m^2/\mu^2$$

any particle

any particle
undiscovered!

Appelquist Carazzone thm.

Effects of heavy particle appear either

Effects of heavy particle appear either as a renorm. of a coupling in renorm. theory or is suppressed by powers of heavy mass

(

Effects of heavy particle appear either as a renorm. of a coupling in renorm. theory or is suppressed by powers of heavy mass (in a local $\mathcal{L}$)

as a renorm. of a coupling in renorm. theory
or is suppressed by powers of heavy mass
(in a local $\mathcal{L}$)

Locality – Uncertainty principle

(in a local $\mathcal{L}$)

Locality - Uncertainty principle

Caveat : Not true if symmetry is broken

$$\left.\begin{matrix} m_t \text{ large} \\ m_b \text{ small} \end{matrix}\right\} m_t^2 \text{ effect}$$

break SU(2)

First organizing principle
+ locality / decoupling

First organizing principle
+ locality / decoupling

Why do quantum calc. work?

$$\mathcal{M} = \sum_I \frac{\langle f|V|I\rangle\langle I|V|i\rangle}{E - E_I}$$

all I, not discovered yet!!

First organizing principle
+ locality / decoupling

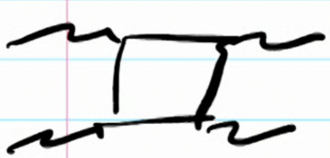
Why do quantum calc. work?

$$\mathcal{M} = \sum_I \frac{\langle f|V|I\rangle\langle I|V|i\rangle}{E - E_I}$$

↑ all I, not discovered yet!!

answer, U.P., A.C. decoupling

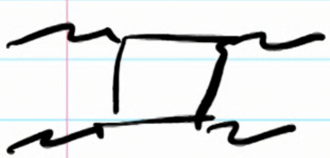
Next term



$$\mathcal{L} = \frac{e^4}{(4\pi)^2\, 90\, m^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \right]$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

Next term



$$\mathcal{L} = \frac{e^4}{(4\pi)^2\, 90\, m^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right]$$

$$\frac{1}{m^4} \qquad E^4 \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$\mathcal{L} = \frac{e^4}{(4\pi)^2\, 90\, m^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right]$$

$$\frac{1}{m^4} \qquad E^4 \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

Role of gauge symmetry:

not $A_\mu A^\nu A^\mu A_\nu$ ✗

$$\mathcal{L} = \frac{e^4}{(4\pi)^2\, 90\, m^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right]$$

$\frac{1}{m^4}$ E^4 $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$

Role of gauge symmetry:

not $A_\mu A^\nu A^\mu A_\nu$ X

must be $\frac{1}{m^4} F^4$

What if not massive?

$\Delta \Pi \sim + \dots \ln q^2/\mu^2$ $\qquad q^2 \ll m$

No local $\mathcal{L}$

Analytic $$\int \frac{d^4q}{(2\pi)^4}\, q^2 e^{iq\cdot x} = -\Box \int \frac{d^4q}{(2\pi)^4}\, e^{iq\cdot x}$$

$$= -\Box\, \delta^4(x)$$

No local $\mathcal{L}$

Analytic $\int \frac{d^4q}{(2\pi)^4}\, q^2 e^{iq\cdot x} = -\Box \int \frac{d^4q}{(2\pi)^4}\, e^{iq\cdot x}$

$= -\Box\, \delta^4(x)$

Nonanalytic $\int \frac{d^4q}{(2\pi)^4}\, \ln q^2\, e^{iq(x-y)} = L(x-y)$

Not local

Non analytic $\int \frac{d^4 q}{(2\pi)^4} \ln q^2 e^{iq(x-y)} = L(x-y)$

Not local

This would be

$$S_{eff} = \int d^4x\, d^4y\, F_{\mu\nu}(x)\, L(x-y)\, F^{\mu\nu}(y)$$

not local

This would be

$$S_{eff} = \int d^4x\, d^4y\, F_{\mu\nu}(x)\, L(x-y)\, F^{\mu\nu}(y)$$

Low energy effects are <u>not local</u>

Second organizing princ.

Low energy effects are <u>not local</u>

dynamical

Second organizing princ.

Second organizing princ.

Energy expansion

Second organizing princ.

Energy expansion

Taylor expansion $\frac{q^2}{m^2} + \frac{q^4}{m^4} + \cdots$

$\Rightarrow$

Second organizing princ.

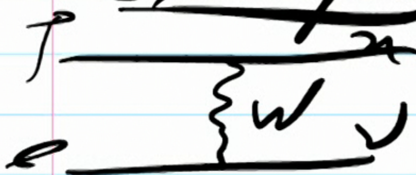
Energy expansion

Taylor expanding $\frac{q^2}{m^2} + \frac{q^4}{m^4} + \ldots$

$$\Rightarrow \mathcal{L} = F \frac{\Box}{m^2} F + F \frac{\Box^2}{m^4} F$$

$\times$

Example:



$$= \frac{g^2}{M_W^2} J_\mu J^\mu + \frac{g^2}{M_W^4} J \Box J$$

$$\frac{1}{q^2 - M_W^2} = -\frac{1}{M^2} - \frac{q^2}{M_W^2} + \cdots$$

Third organizing princ.

- matching or measuring

known theory

Third organizing princ.

– matching or measuring

known theory

unknown or difficult

Path Integral Language

1) Quantization

Z

1) Quantization

$$Z[J] = N \int [d\phi]\, e^{i\int d^4x\, [\mathcal{L}(\phi, \partial\phi) + J\phi]}$$

all field # at each point

"source"

all field $\pm$
at each point

source

2) <u>Free field</u> —

2) Free field $\mathcal{L} = \mathcal{L}_0 = \frac{1}{2}(\partial_\mu \varphi)^2 - m^2 \varphi^2$

$Z[J] = N e^{i}$

2) Free field $\mathcal{L} = \mathcal{L}_0 = \frac{1}{2}(\partial_\mu \varphi)^2 - m^2 \varphi^2$

$$Z[J] = N\, e^{-i \int d^4x\, d^4y\, J(x)\, i D_F(x-y) J(y)}$$

3) Quad

$$\mathcal{L} = \mathcal{L}_0 = \frac{1}{2}(\partial_\mu \phi)^2 - m^2\phi^2$$

$$Z[J] = N e^{-i\int d^4x\, d^4y\, J(x)\, i D_F(x-y) J(y)}$$

3) Quadratic - gaussian

$$\int [d\phi]\, e^{-i\int d^4x\, \phi \mathcal{D} \phi} = \#\left[\det \mathcal{D}\right]^{-\frac{1}{2}}$$

$$= \#\, e^{-\frac{i}{2}\int d^4x \langle x | \ln \mathcal{D} | x \rangle}$$