Title: Topological quantum matter and quantum computing

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Non-abelian anyons.

Braiding non-abelian anyons.

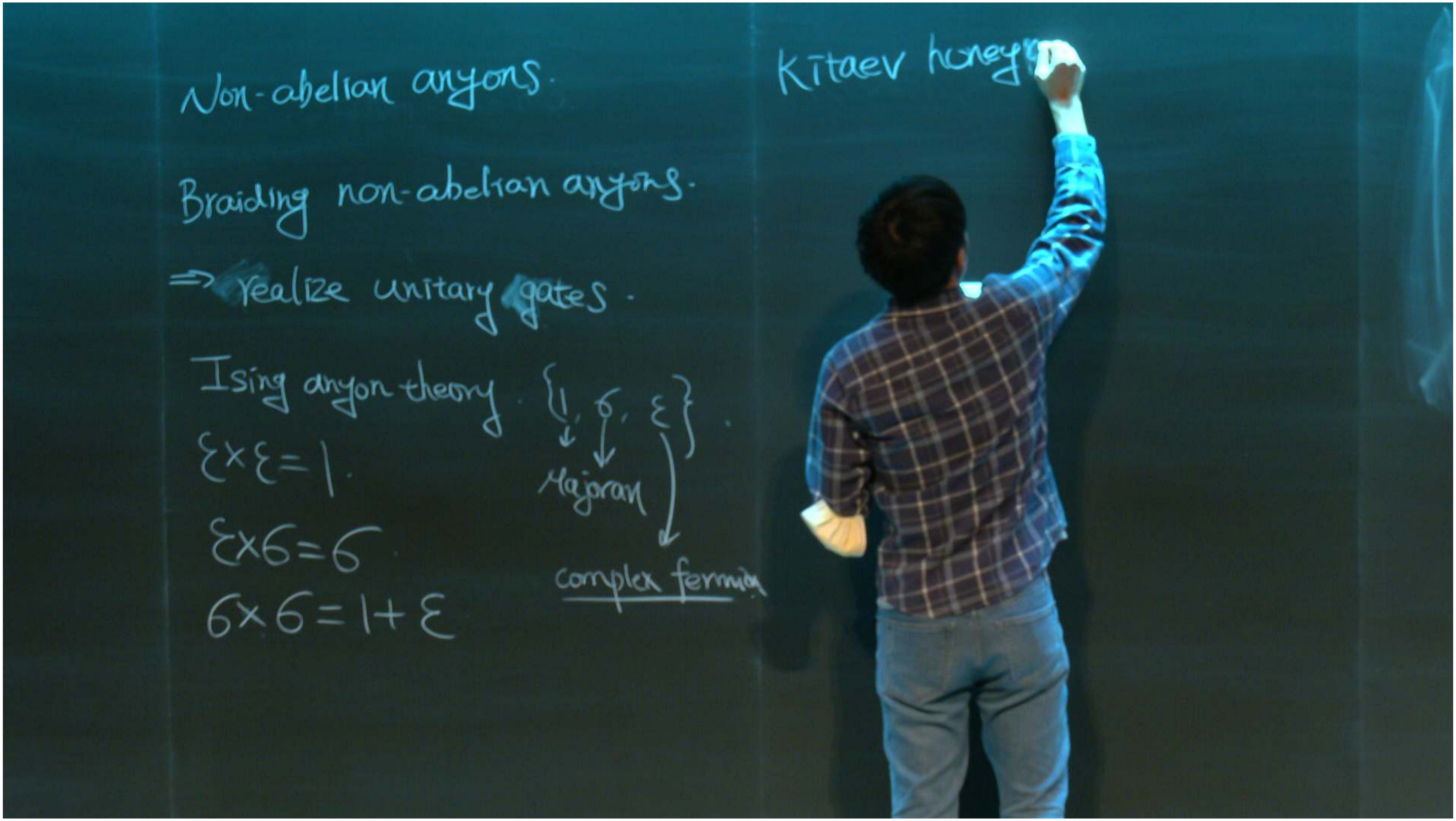
$\Rightarrow$ realize unitary gates.

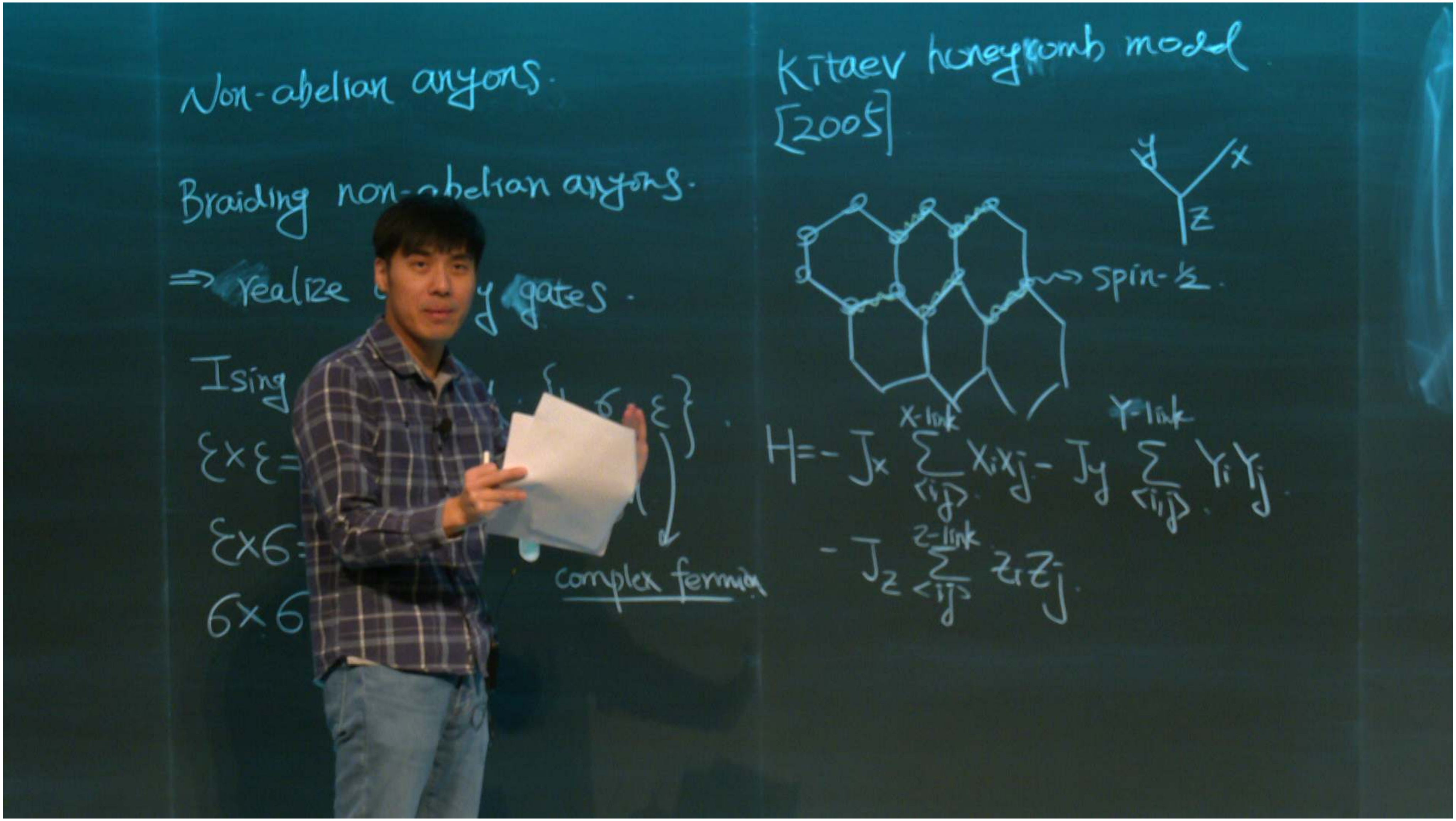
Ising anyon theory. $\{1, \sigma, \varepsilon\}$.

$\varepsilon \times \varepsilon = 1$.

Majoran

complex fermion


Non-abelian anyons.
Braiding non-abelian anyons.
⇒ realize unitary gates.
Ising anyon theory. {1, σ, ε}.
ε×ε=1.
ε×σ=σ.
σ×σ=1+ε
Majoran
complex fermion
Kitaev


Non-abelian anyons.
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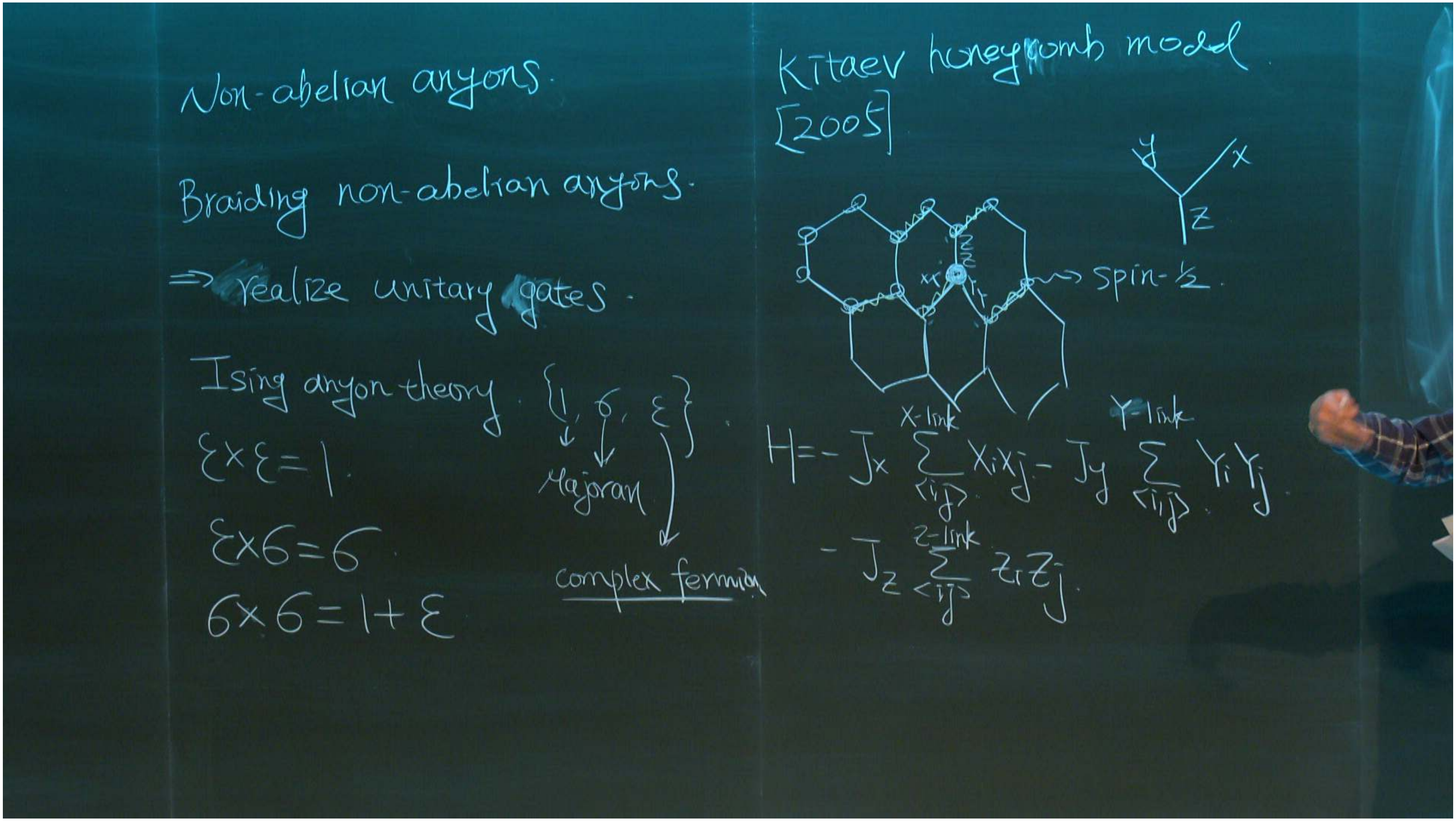
⇒ realize gates.
Ising
complex fermion
Kitaev honeycomb model
[2005]
spin-1/2.
X-link
Y-link
Z-link
H = - J_x ∑_{<ij>} X_i X_j - J_y ∑_{<ij>} Y_i Y_j
- J_z ∑_{<ij>} Z_i Z_j.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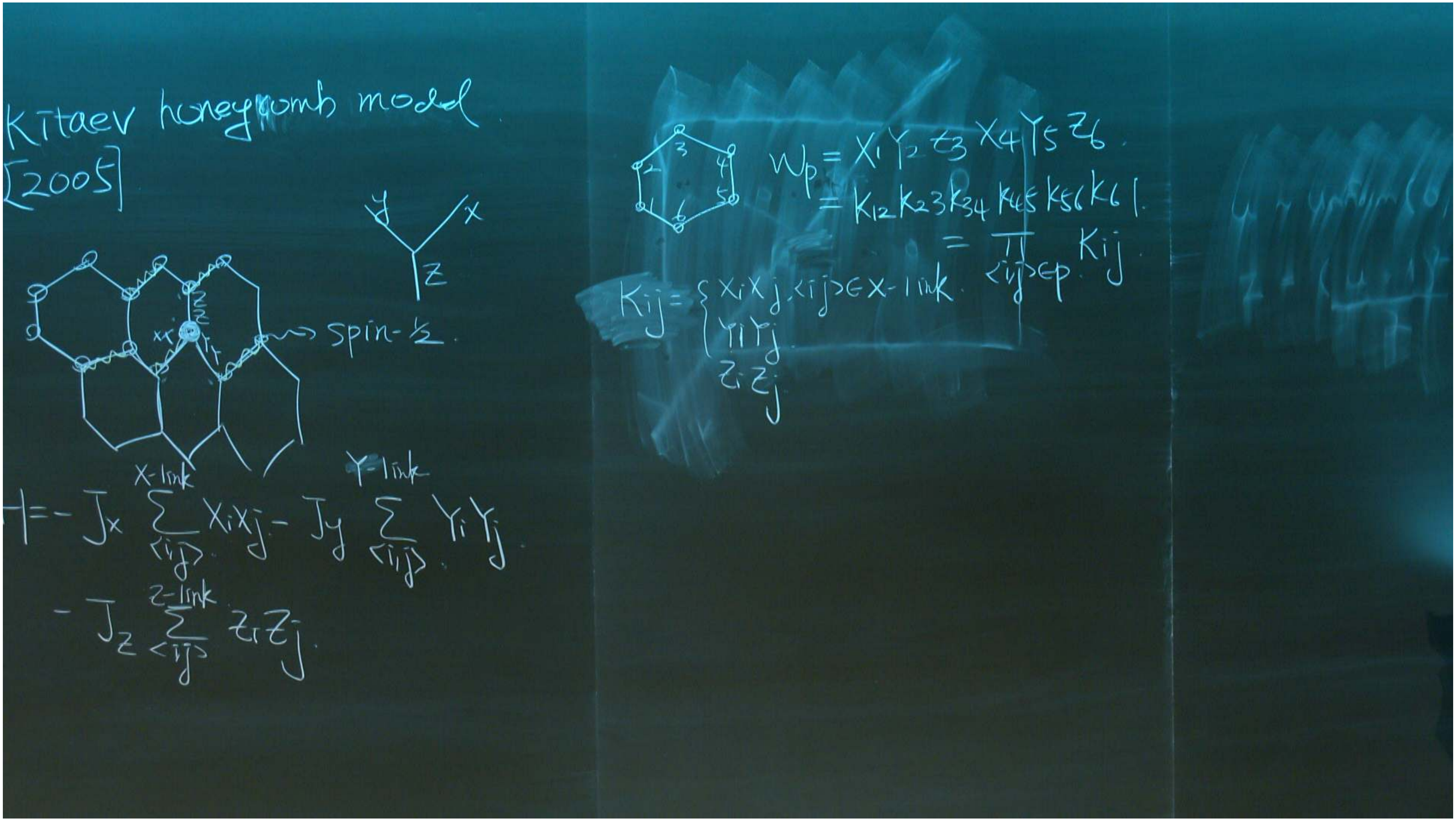
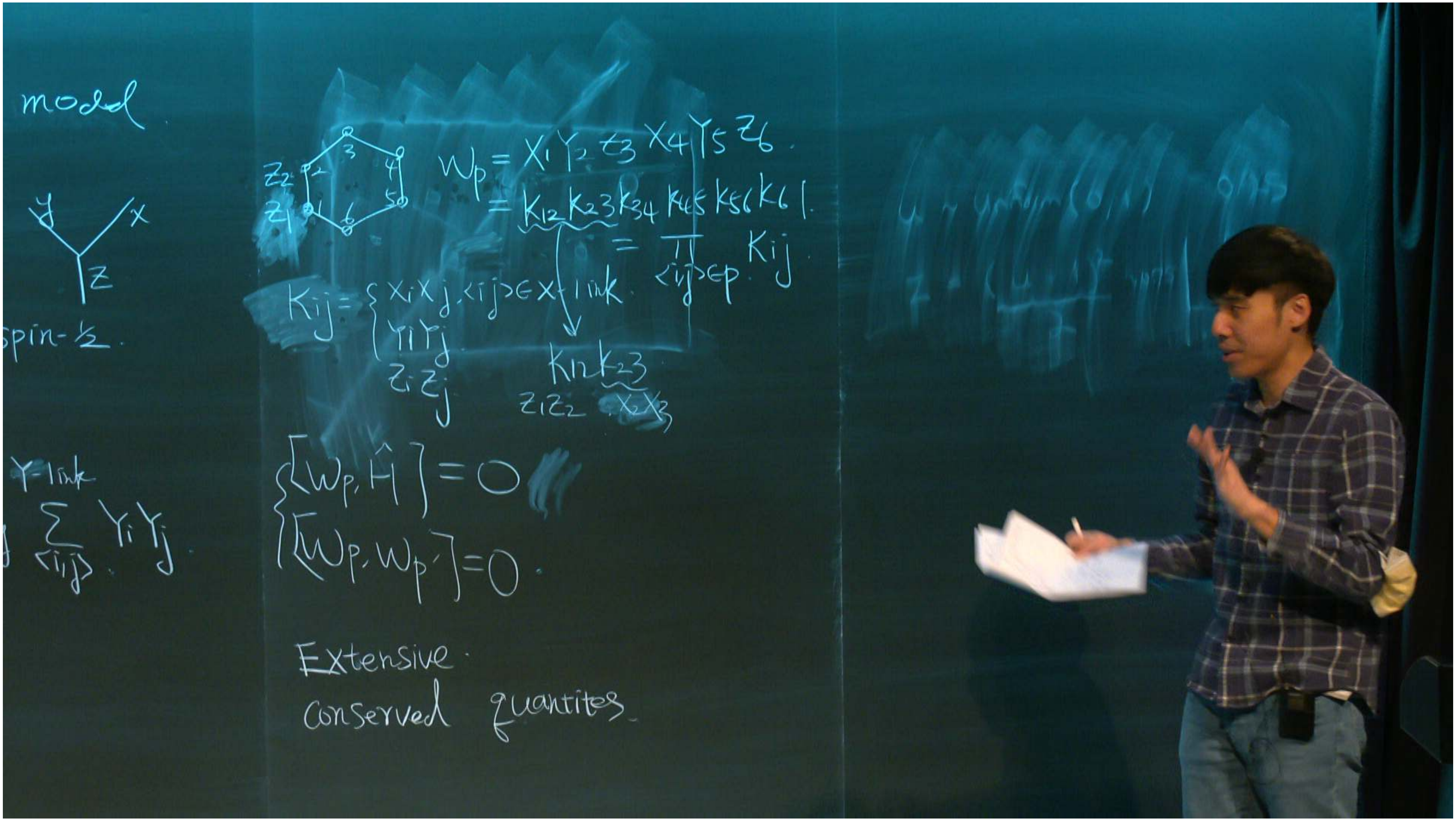
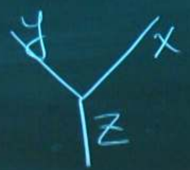
Kitaev honeycomb model.
[2005]
y
x
z
spin-½.
X-link
Y-link
H=-Jx Σ<i,j> XiXj - Jy Σ<i,j> YiYj
Z-link
- Jz Σ<i,j> ZiZj.


Kitaev honeycomb model
[2005]
y
x
z
spin-½
X-link
Y-link
Z-link
$H = -J_x \sum_{\langle ij \rangle} X_i X_j - J_y \sum_{\langle ij \rangle} Y_i Y_j - J_z \sum_{\langle ij \rangle} Z_i Z_j$
1 2 3 4 5 6
$W_p = X_1 Y_2 Z_3 X_4 Y_5 Z_6 = K_{12} K_{23} K_{34} K_{45} K_{56} K_{61} = \prod_{\langle ij \rangle \in p} K_{ij}$
$K_{ij} = \begin{cases} X_i X_j, & \langle ij \rangle \in X\text{-link} \\ Y_i Y_j \\ Z_i Z_j \end{cases}$


model
y x z
spin-1/2
Y-link
∑ ⟨ij⟩ YiYj
z2 z1 2 3 4 5 6 1
Wp = X1 Y2 Z3 X4 Y5 Z6
= K12 K23 K34 K45 K56 K61
= ∏ ⟨ij⟩∈p Kij
Kij = XiXj ⟨ij⟩∈X-link
YiYj
ZiZj
K12 K23
Z1Z2 X2X3
[Wp, H] = 0
[Wp, Wp'] = 0
Extensive
conserved quantites

model



spin-$\frac{1}{2}$.

Y-link

$\sum_{\langle i,j\rangle} Y_i Y_j$.

$$W_p = X_1 Y_2 Z_3 X_4 Y_5 Z_6 = K_{12} K_{23} K_{34} K_{45} K_{56} K_{61} = \prod_{\langle ij\rangle \in p} K_{ij}.$$

$$K_{ij} = \begin{cases} X_i X_j, & \langle ij\rangle \in x\text{-link}. \\ Y_i Y_j \\ Z_i Z_j \end{cases}$$

$$K_{12} K_{23} = Z_1 Z_2 \, X_2 X_3$$

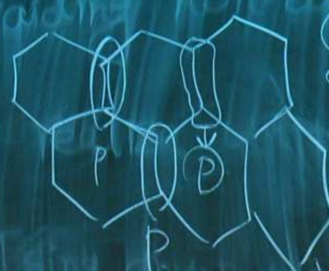
$$\begin{cases} [W_p, \hat{H}] = 0 \\ [W_p, W_{p'}] = 0. \end{cases}$$

Extensive conserved quantites.

Hilbert space splits into sectors labelled by $\{W_p = \pm 1\}$.

For a given $\{W_p\}$'s.

Size of subspace.

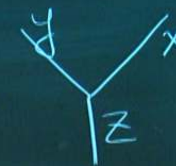


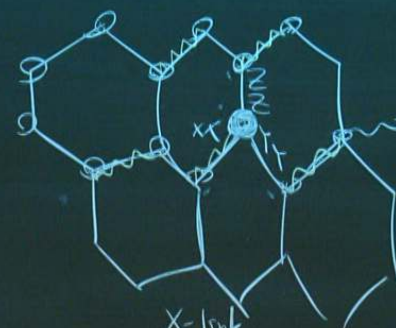
$\left\{ \begin{array}{l} n \; \# \text{ of lattice sites.} \\ \frac{n}{2} : \# \text{ of } W_p \end{array} \right.$

$$\text{Subspace} = \frac{2^n}{2^{n/2}} = 2^{\frac{n}{2}} = (\sqrt{2})^n.$$

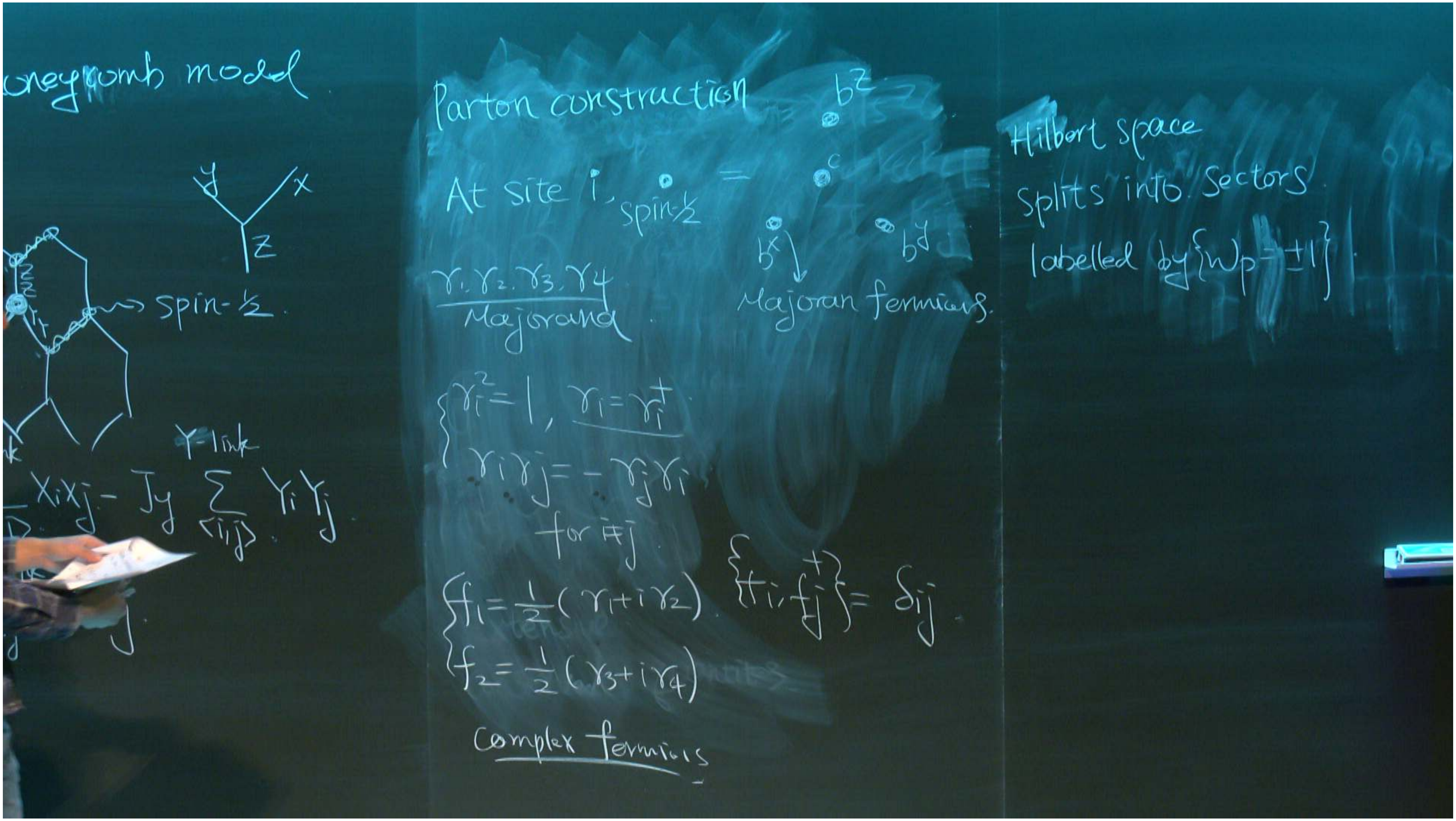
Kitaev honeycomb model

[2005]

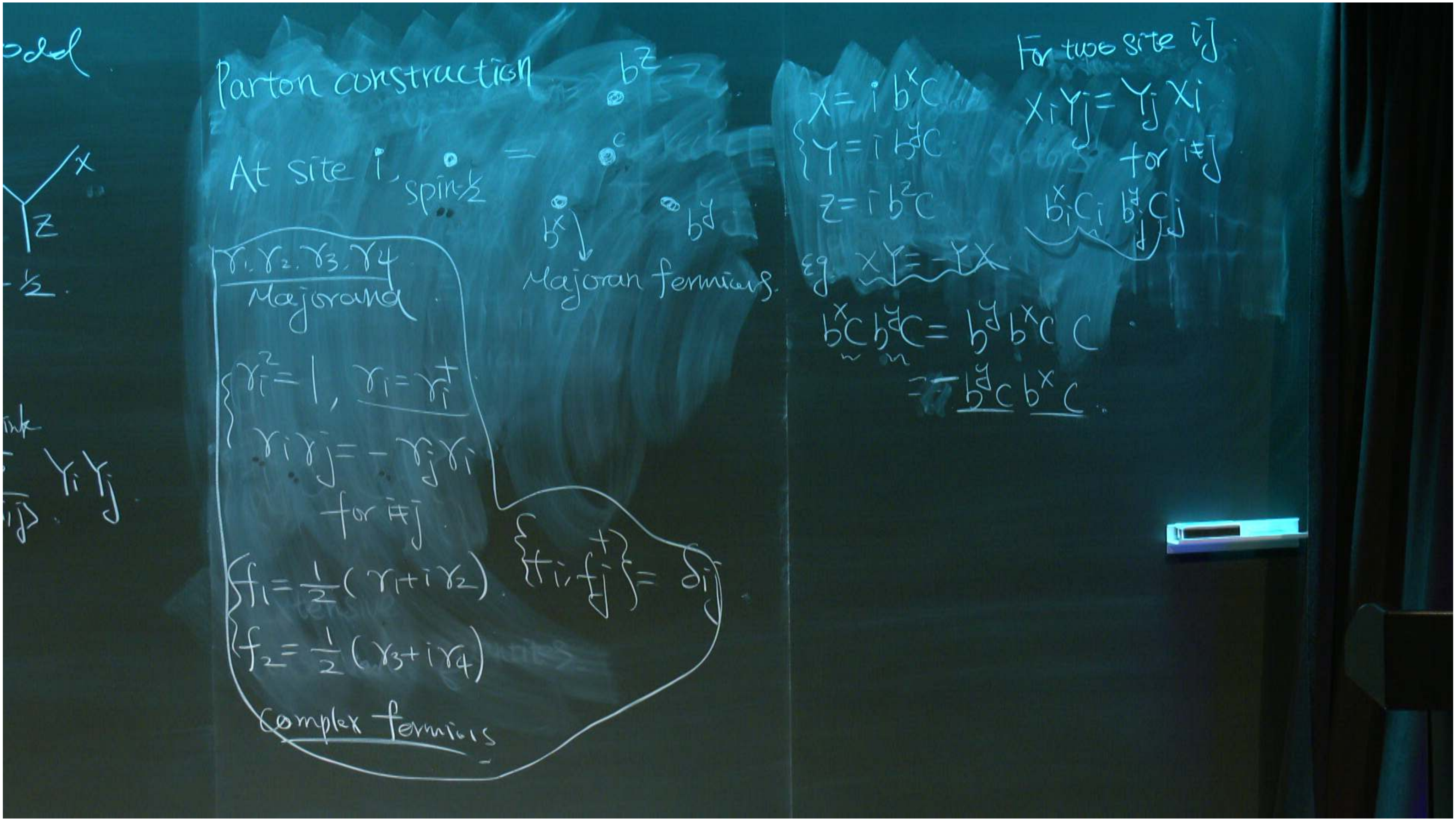


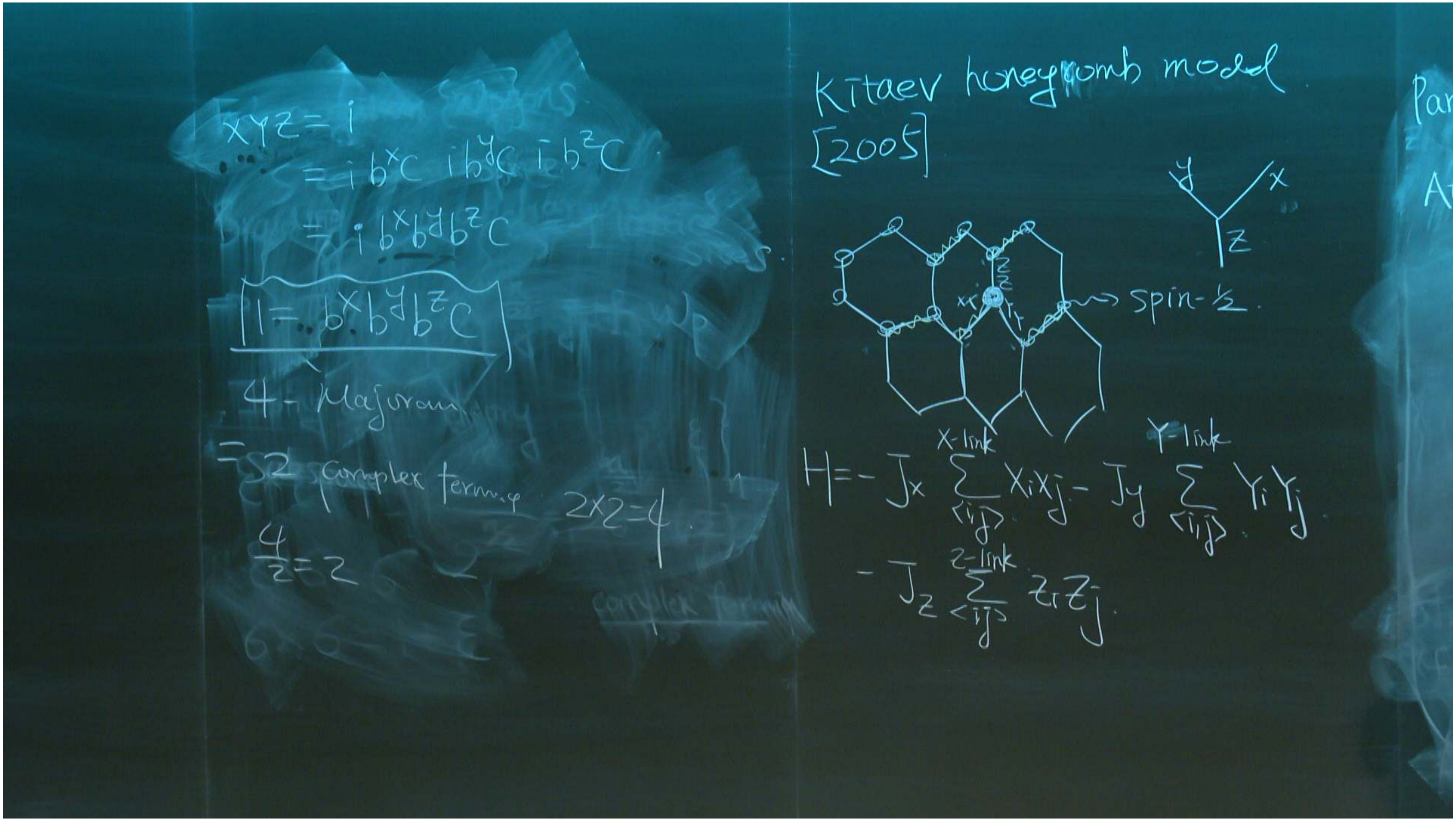


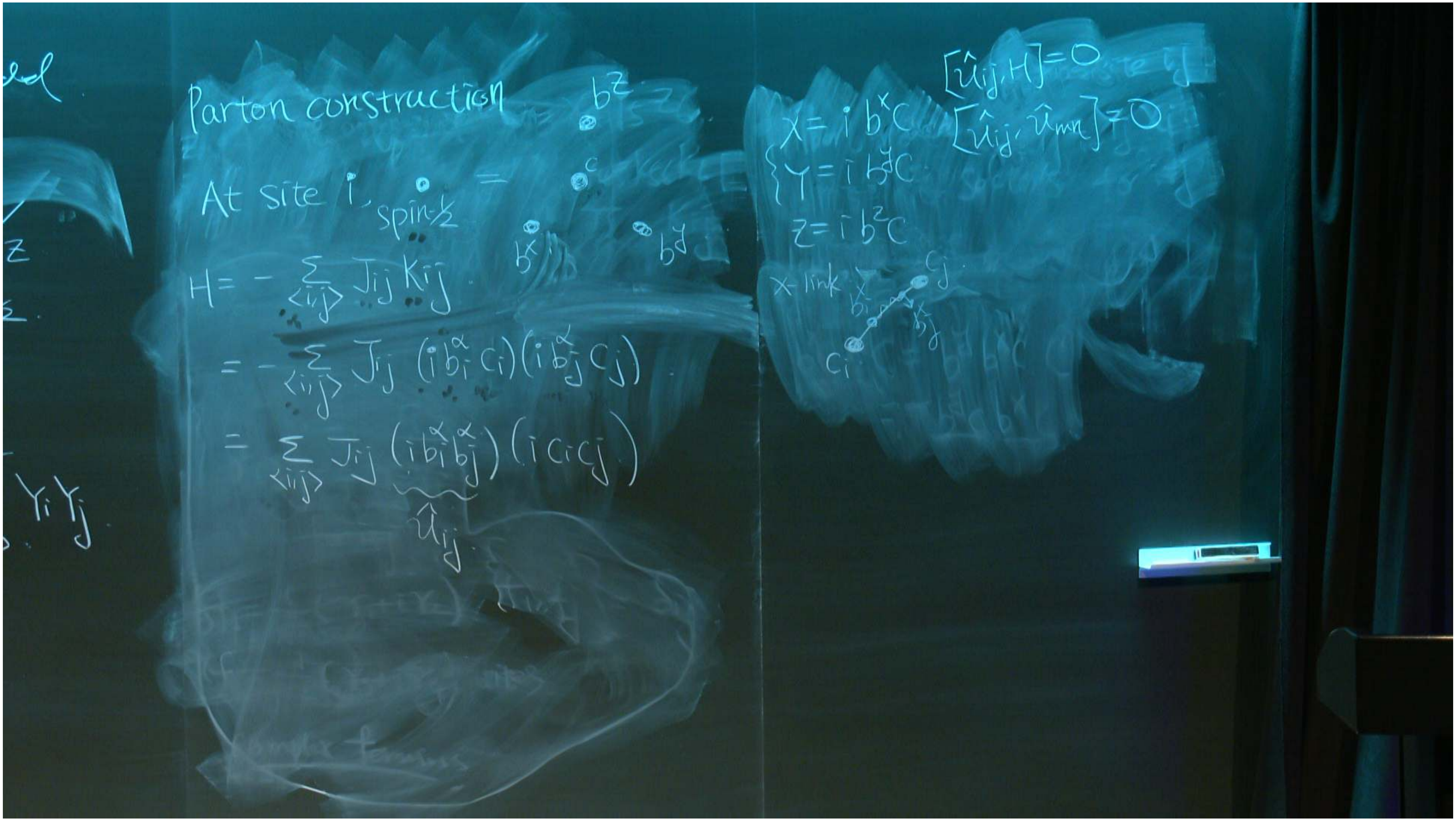
$$H = -J_x \overset{\text{X-link}}{\sum_{\langle ij \rangle}} X_i X_j - J_y \overset{\text{Y-link}}{\sum_{\langle ij \rangle}} Y_i Y_j$$
$$- J_z \overset{\text{Z-link}}{\sum_{\langle ij \rangle}} Z_i Z_j.$$



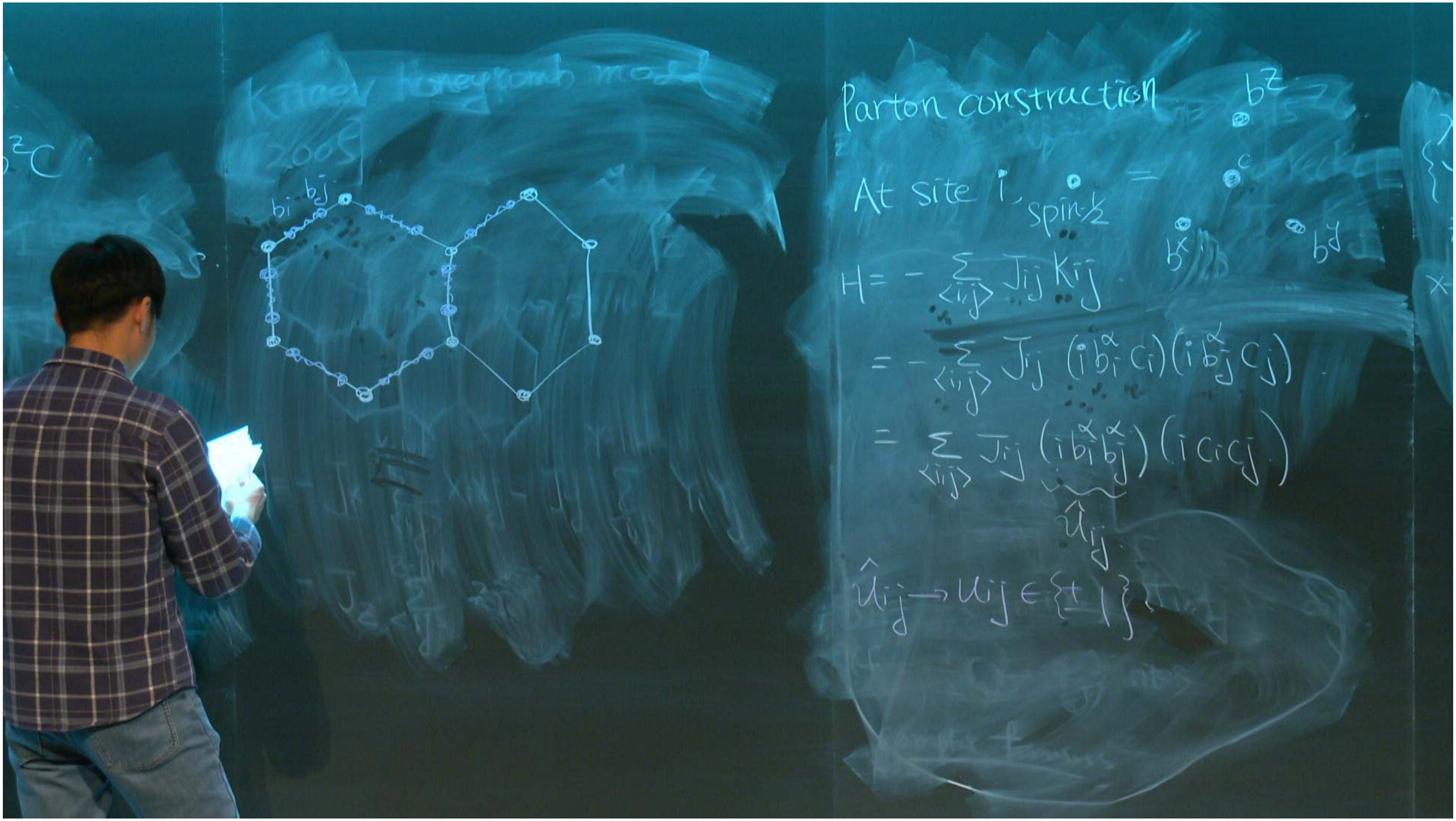
oneycomb model
y x z
spin-½
Y-link
$X_i X_j - J_y \sum_{\langle i,j \rangle} Y_i Y_j$
Parton construction
At site i, spin-½ = b^z c b^x b^y
Majoran fermions
$\gamma_1 . \gamma_2 . \gamma_3 . \gamma_4$
Majorana
$\gamma_i^2 = 1, \ \gamma_i = \gamma_i^\dagger$
$\gamma_i \gamma_j = -\gamma_j \gamma_i$ for $i \neq j$
$f_1 = \frac{1}{2}(\gamma_1 + i\gamma_2)$
$f_2 = \frac{1}{2}(\gamma_3 + i\gamma_4)$
$\{f_i, f_j^\dagger\} = \delta_{ij}$
Complex fermions
Hilbert space
Splits into Sectors
labelled by $\{W_p = \pm 1\}$

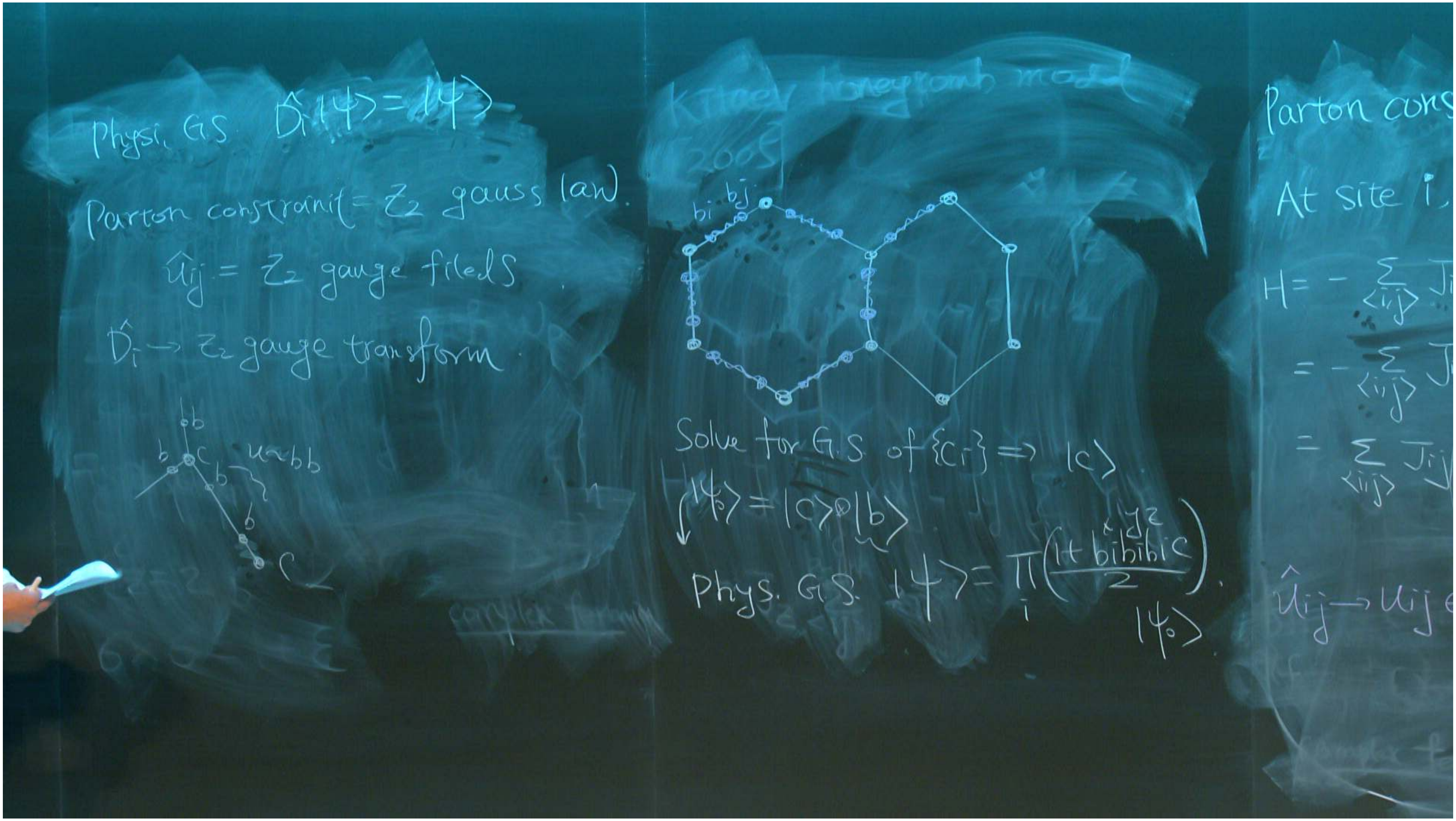

Parton construction
At site i, spin-1/2
b^z
b^x b^y
c
Majoran fermions
γ1 γ2 γ3 γ4
Majorana
γi^2 = 1, γi = γi^†
γi γj = − γj γi
for i≠j
f1 = 1/2 (γ1 + i γ2)
f2 = 1/2 (γ3 + i γ4)
{fi, fj^†} = δij
Complex fermions
X = i b^x c
Y = i b^y c
Z = i b^z c
Eg. XY = −YX
b^x c b^y c = b^y b^x c c
= − b^y c b^x c
For two site i j
Xi Yj = Yj Xi
for i≠j
b^x_i c_i b^y_j c_j

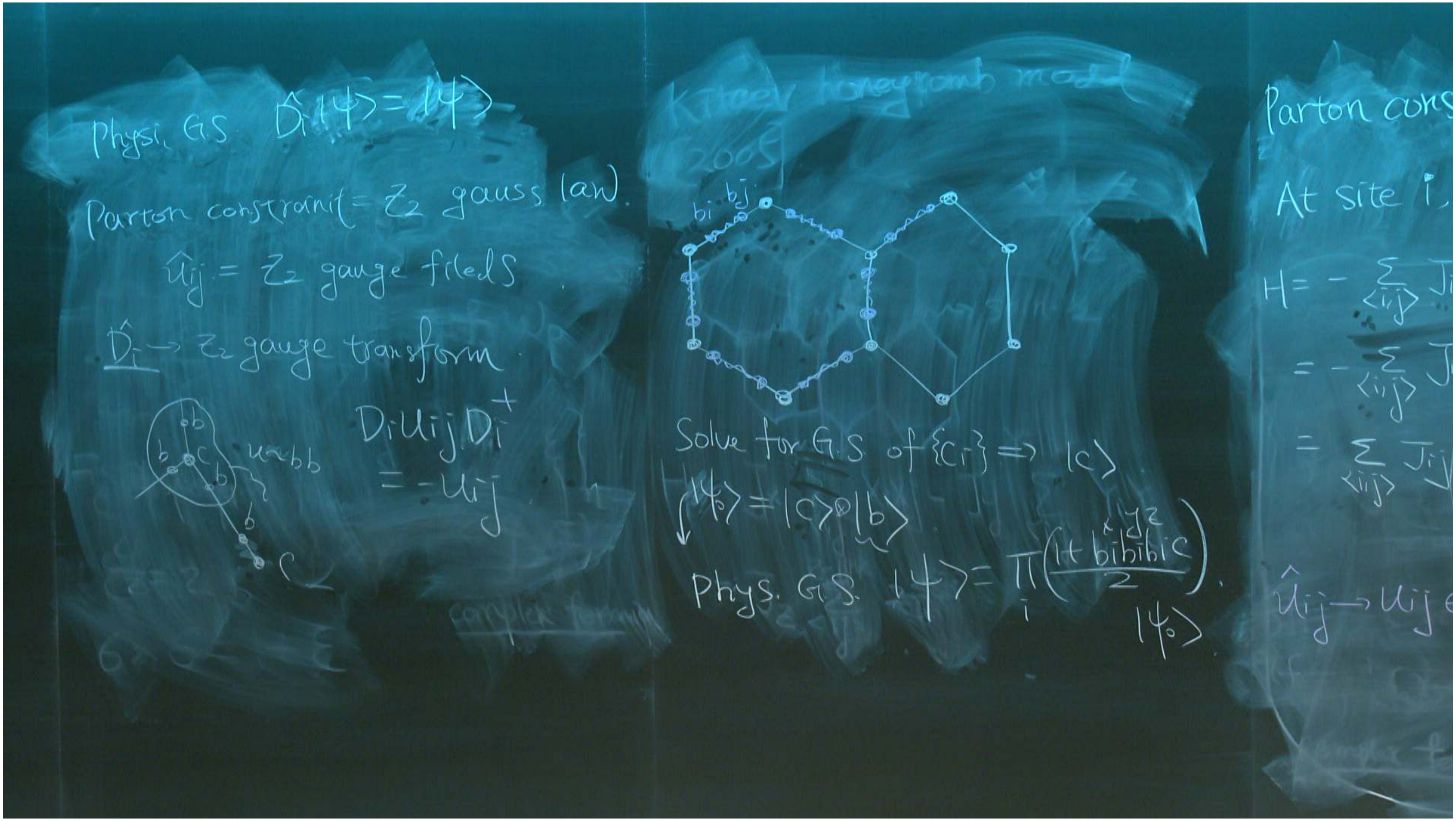

XYZ = i
= ib^x c ib^y c ib^z c
= i b^x b^y b^z c
1 = b^x b^y b^z c
4 Majorana
= 2 complex fermion
2×2 = 4
4/2 = 2
Kitaev honeycomb model
[2005]
y
x
z
spin-1/2
X-link
Y-link
H = - J_x Σ_⟨ij⟩ X_i X_j - J_y Σ_⟨ij⟩ Y_i Y_j
- J_z Σ_⟨ij⟩ Z_i Z_j
Z-link

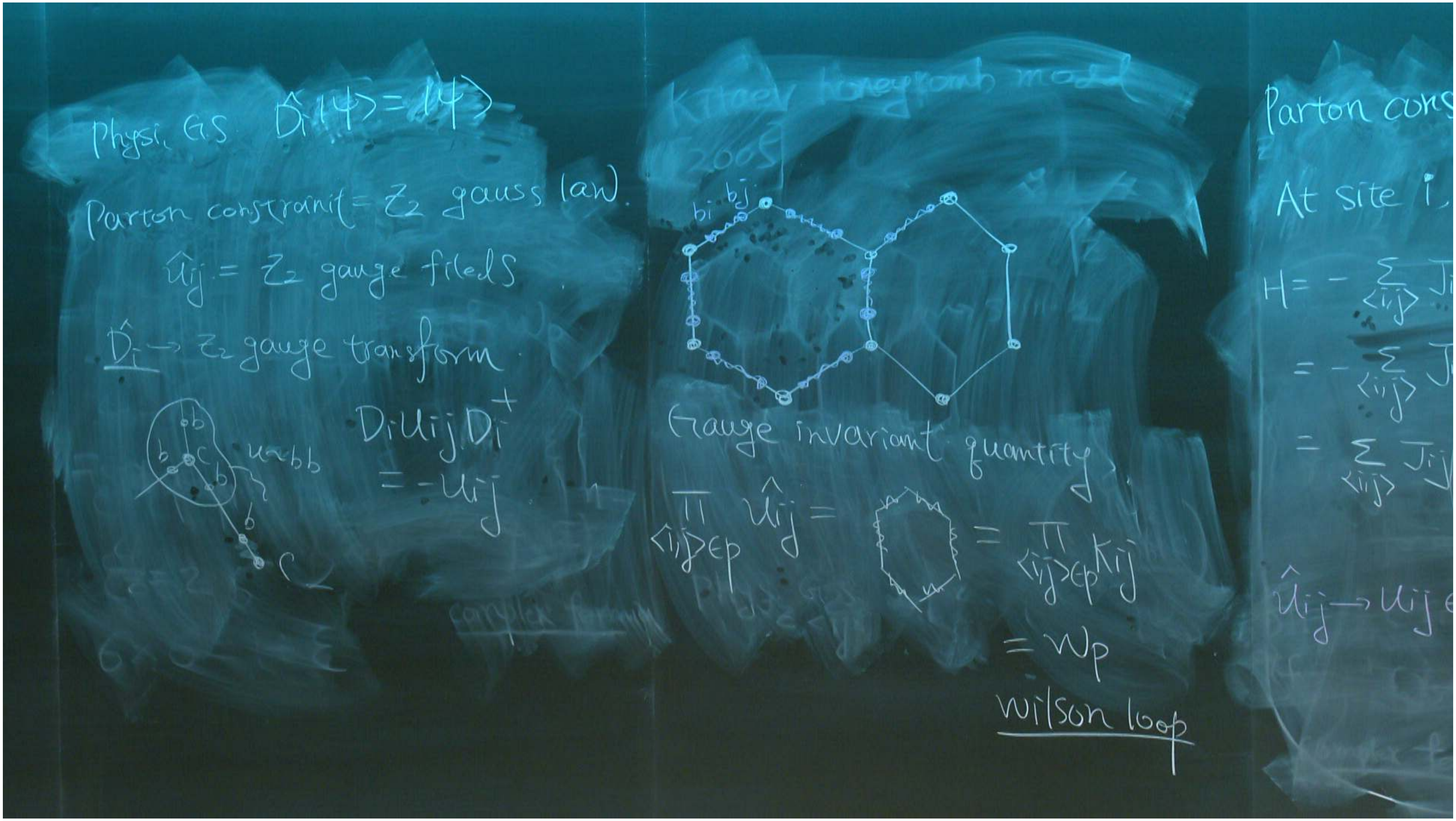


Parton construction
At site i, spin-1/2
b^z
c
b^x
b^y
$H = -\sum_{\langle ij \rangle} J_{ij} K_{ij}$
$= -\sum_{\langle ij \rangle} J_{ij} (i b_i^\alpha c_i)(i b_j^\alpha c_j)$
$= \sum_{\langle ij \rangle} J_{ij} \underbrace{(i b_i^\alpha b_j^\alpha)}_{\hat{u}_{ij}} (i c_i c_j)$
$X = i b^x c$
$Y = i b^y c$
$Z = i b^z c$
$[\hat{u}_{ij}, H] = 0$
$[\hat{u}_{ij}, \hat{u}_{mn}] = 0$
x-link
c_j
b_i^x
b_j^x
c_i
$Y_i Y_j$


Parton construction
At site i
spin-½
H = − Σ⟨ij⟩ Jij Kij
= − Σ⟨ij⟩ Jij (i bαi ci)(i bαj cj)
= Σ⟨ij⟩ Jij (i bαi bαj)(i ci cj)
ûij
ûij → uij ∈ {± 1}


Physi. G.S. $\hat{D}_i|\psi\rangle = |\psi\rangle$
Parton constraint = Z_2 gauss law.
$\hat{u}_{ij} = Z_2$ gauge fileds
$\hat{D}_i \to Z_2$ gauge transform.
Solve for G.S. of $\{c_i\} \Rightarrow |c\rangle$
$|\psi_0\rangle = |c\rangle \otimes |b\rangle$
Phys. G.S. $|\psi\rangle = \prod_i \left(\frac{1 + b_i^x b_i^y b_i^z c}{2}\right) |\psi_0\rangle$
Parton cons
At site i,
$H = -\sum_{\langle ij \rangle} J$
$= -\sum_{\langle ij \rangle} J$
$= \sum_{\langle ij \rangle} J_{ij}$
$\hat{u}_{ij} \to u_{ij}$


Physi. G.S. $\hat{D}_i|\psi\rangle = |\psi\rangle$
Parton constraint = Z_2 gauss law.
$\hat{u}_{ij} = Z_2$ gauge fileds
$\hat{D}_i \to Z_2$ gauge transform.
$D_i u_{ij} D_i^+ = -u_{ij}$
Kitaev honeycomb model 2005
Solve for G.S of $\{c_i\} \Rightarrow |c\rangle$
$|\psi_0\rangle = |c\rangle \otimes |b\rangle$
Phys. G.S. $|\psi\rangle = \prod_i \left(\frac{1 + b_i^x b_i^y b_i^z c}{2}\right) |\psi_0\rangle$
Parton cons
At site i,
$H = -\sum_{\langle ij \rangle} J$
$\hat{u}_{ij} \to u_{ij}$


Physi. G.S $\hat{D}_i|\psi\rangle = |\psi\rangle$
Parton constraint = Z_2 gauss law.
$\hat{u}_{ij}$ = Z_2 gauge filedS
$\hat{D}_i \rightarrow$ Z_2 gauge transform
$D_i u_{ij} D_i^{+} = -u_{ij}$
$u \sim bb$
Kitaev honeycomb model
2005
b_i b_j
Gauge invariant quantity
$\prod_{\langle ij\rangle \in p} \hat{u}_{ij} = \prod_{\langle ij\rangle \in p} K_{ij}$
$= W_p$
wilson loop
Parton cons
At site i,
$H = -\sum_{\langle ij\rangle} J$
$= -\sum_{\langle ij\rangle} J$
$= \sum_{\langle ij\rangle} J$
$\hat{u}_{ij} \rightarrow u_{ij}$

Parton construction

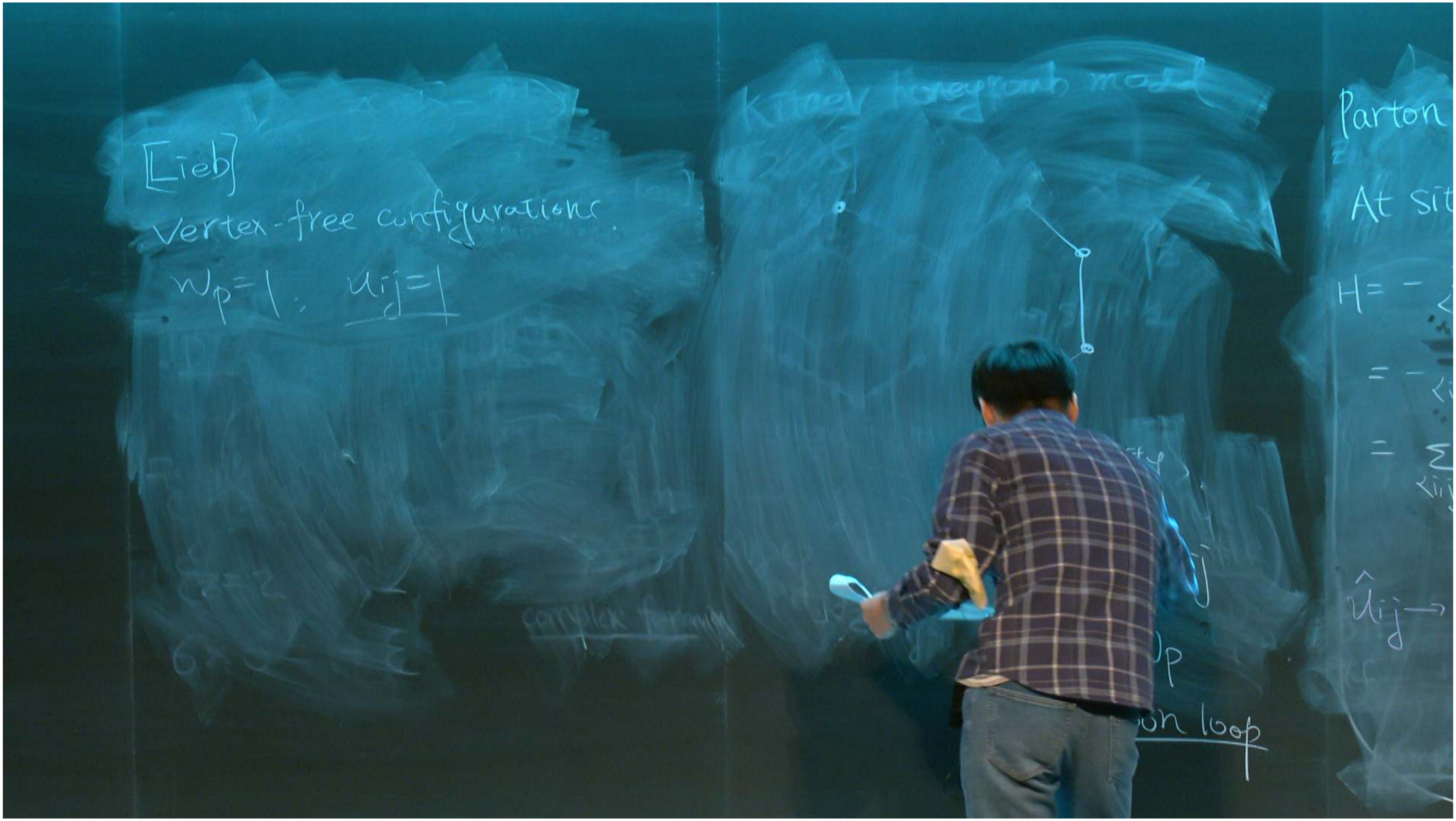
At site i, spin-$\frac{1}{2}$ $\bullet = $ (b^z, c, b^x, b^y)

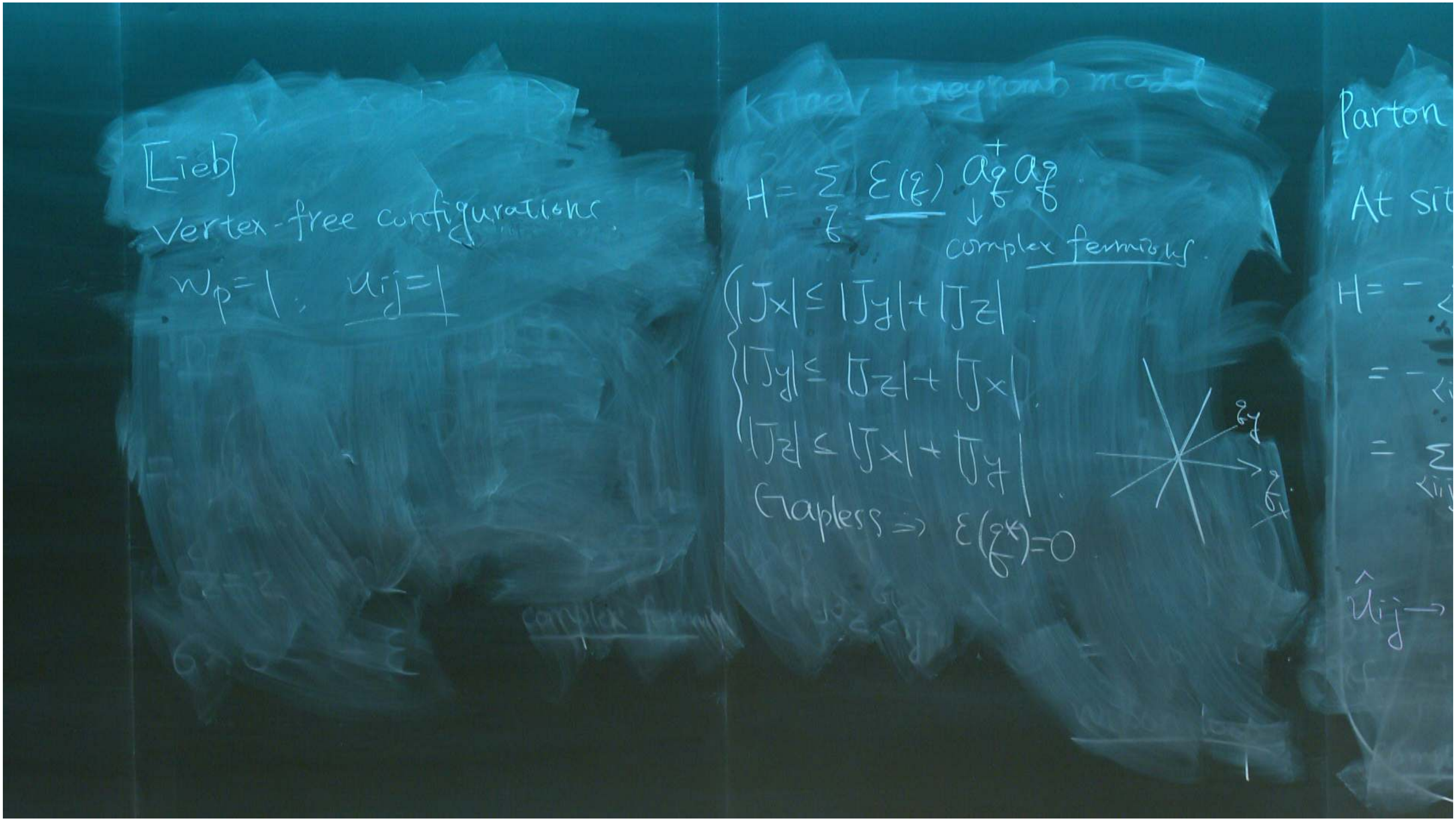
$$H = -\sum_{\langle ij\rangle} J_{ij} K_{ij}$$

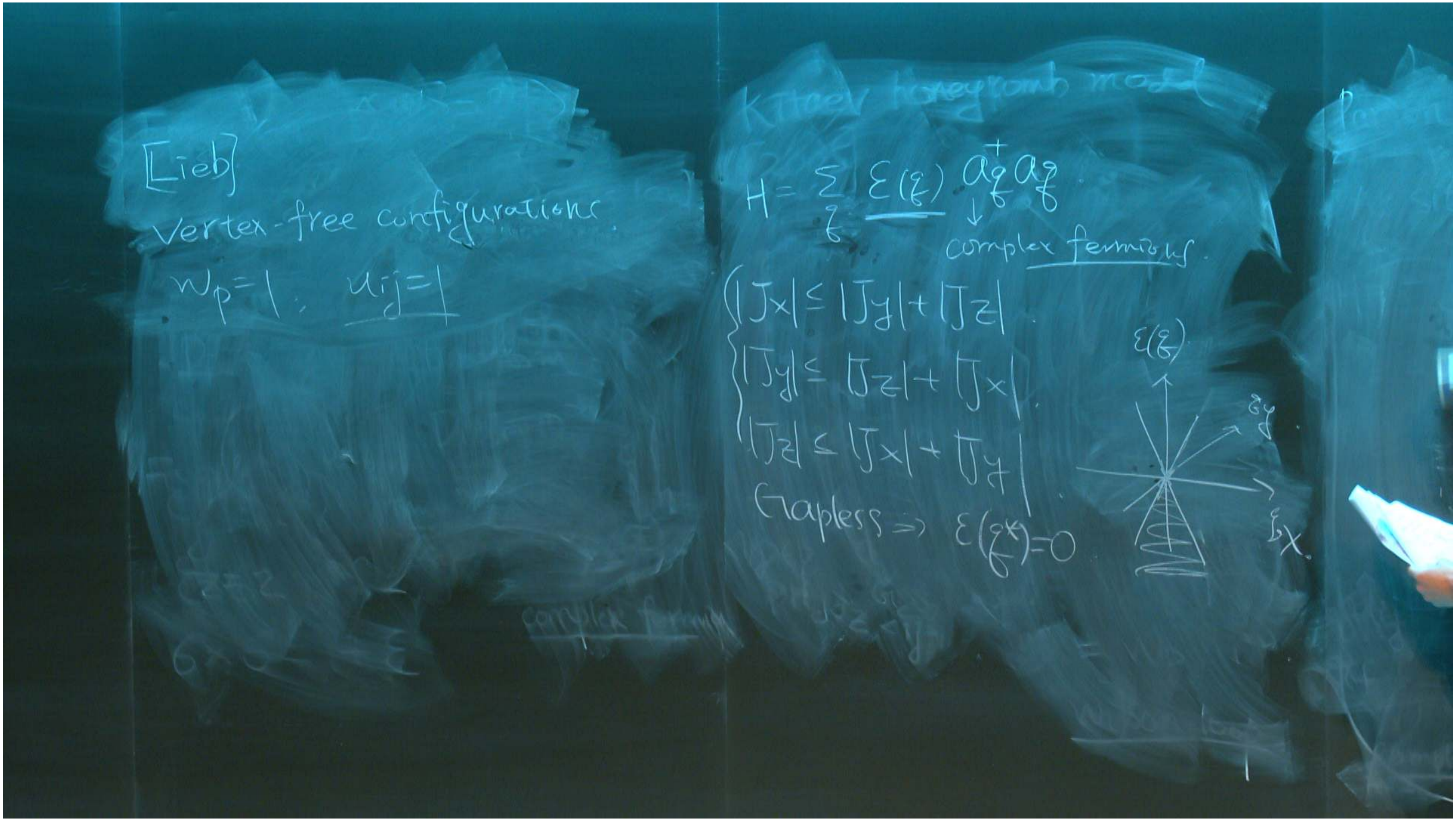
$$= -\sum_{\langle ij\rangle} J_{ij} \left(i b_i^\alpha c_i\right)\left(i b_j^\alpha c_j\right)$$

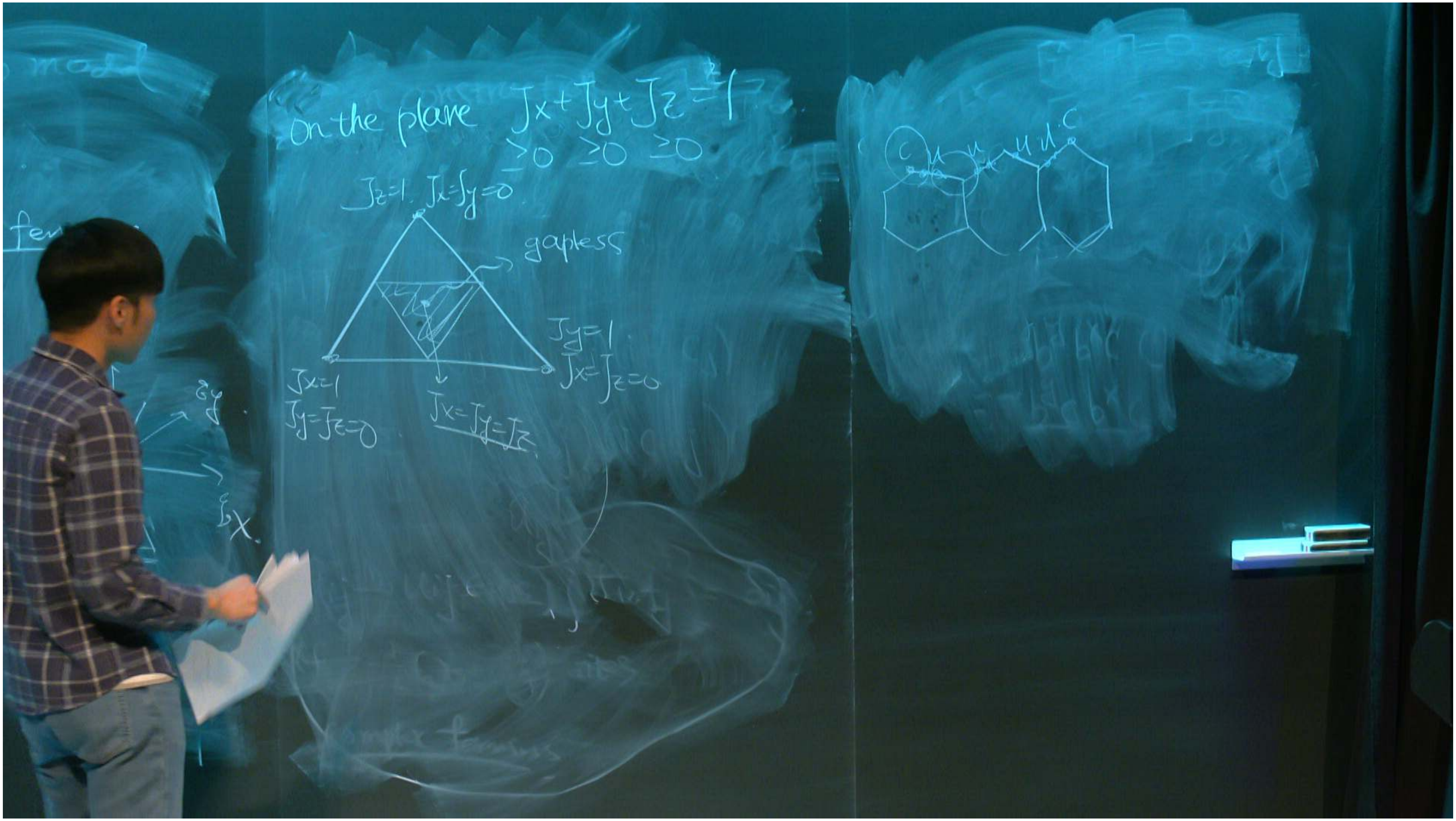
$$= \sum_{\langle ij\rangle} J_{ij} \underbrace{\left(i b_i^\alpha b_j^\alpha\right)}_{\hat{u}_{ij}} \left(i c_i c_j\right)$$

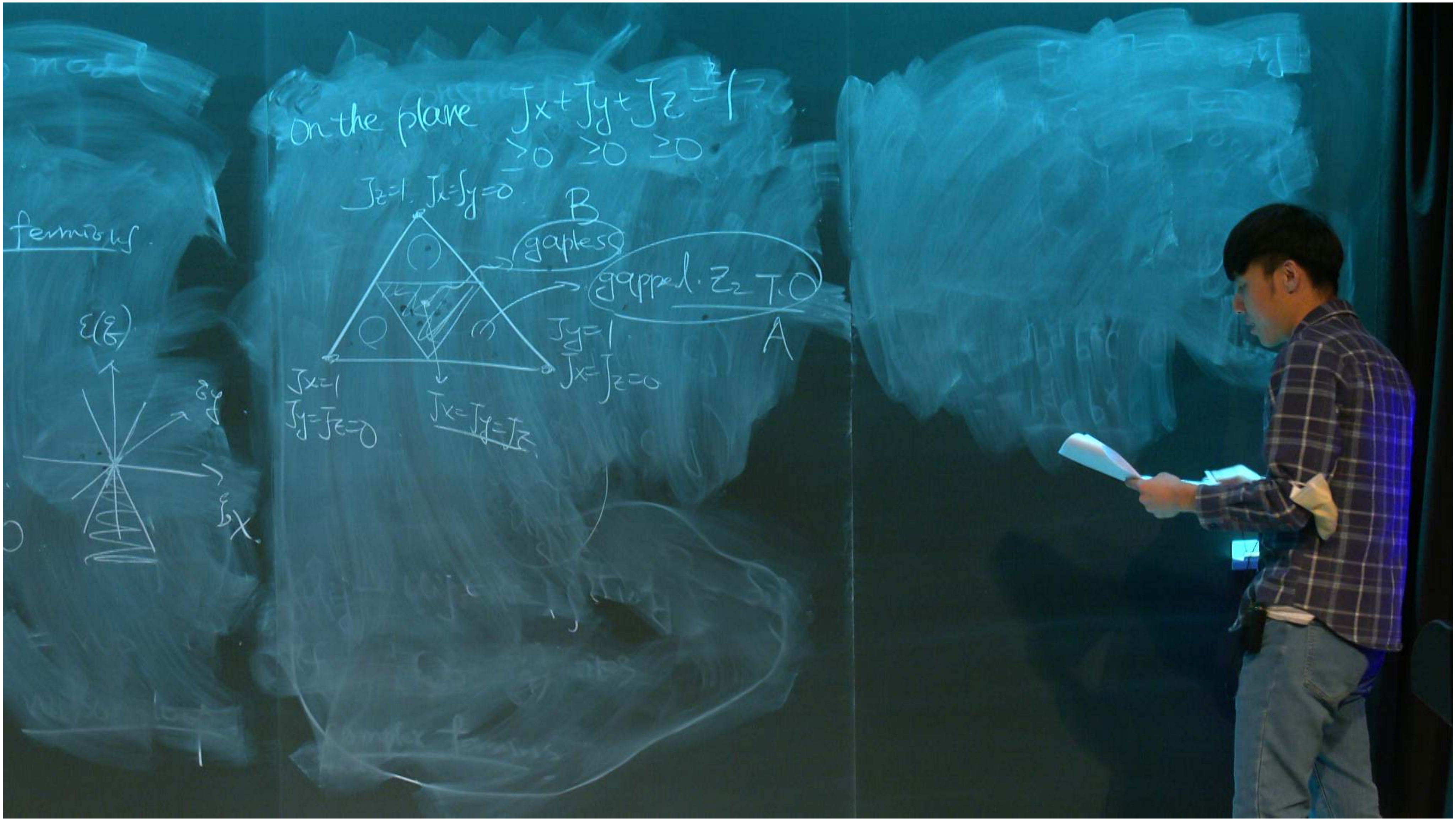
$$\hat{u}_{ij} \to u_{ij} \in \{\pm 1\}$$


[Lieb]
Vertex-free configurations
$w_p = 1$; $u_{ij} = 1$
Kitaev honeycomb model
Parton
At sit
loop

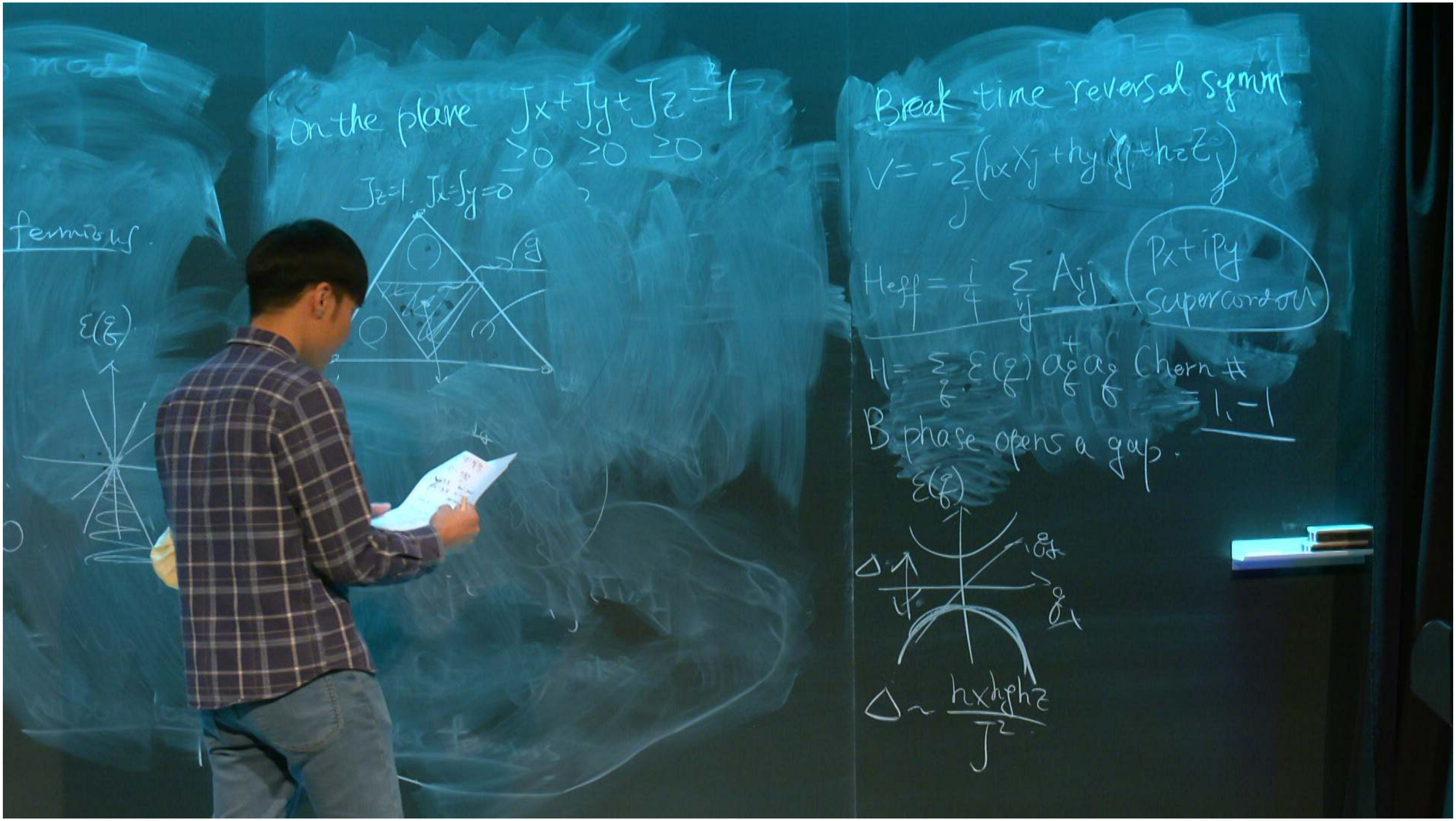

[Lieb]
Vertex-free configurations.
$W_p = 1$, $u_{ij} = 1$
Kitaev honeycomb model
$H = \sum_{q} \varepsilon(q)\, a_q^\dagger a_q$
complex fermions.
$|J_x| \le |J_y| + |J_z|$
$|J_y| \le |J_z| + |J_x|$
$|J_z| \le |J_x| + |J_y|$
Gapless $\Rightarrow$ $\varepsilon(q^*) = 0$
q_y
q_x
Parton
At sit
$H = -$
$= -$
$= \sum$
$\hat{u}_{ij} \rightarrow$

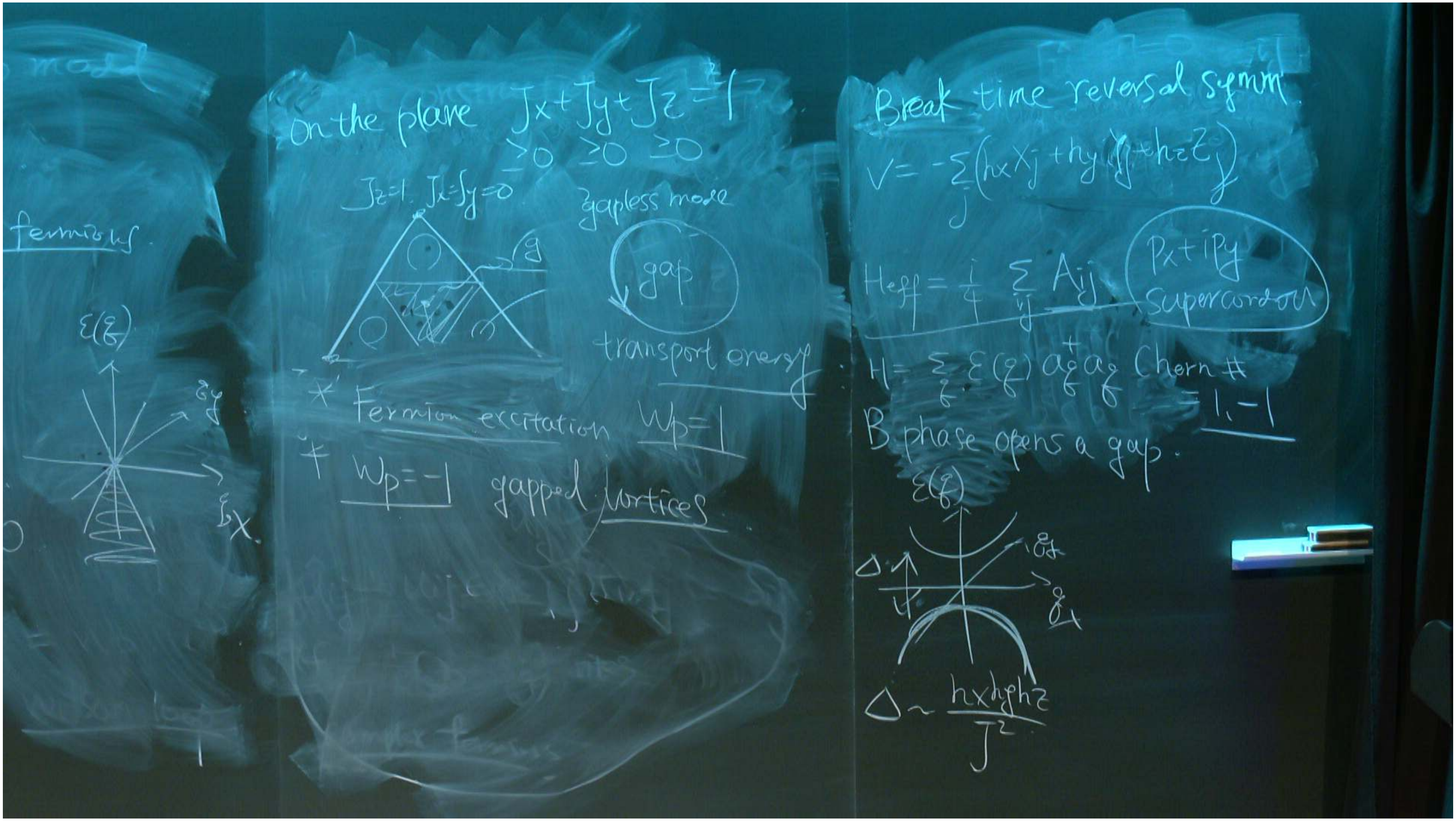

[Lieb]
Vertex-free configurations
w_p = 1, u_ij = 1
Kitaev honeycomb model
H = Σ_q ε(q) a_q^† a_q
complex fermions
|Jx| ≤ |Jy| + |Jz|
|Jy| ≤ |Jz| + |Jx|
|Jz| ≤ |Jx| + |Jy|
Gapless ⇒ ε(q*) = 0
ε(q)
q_y
q_x

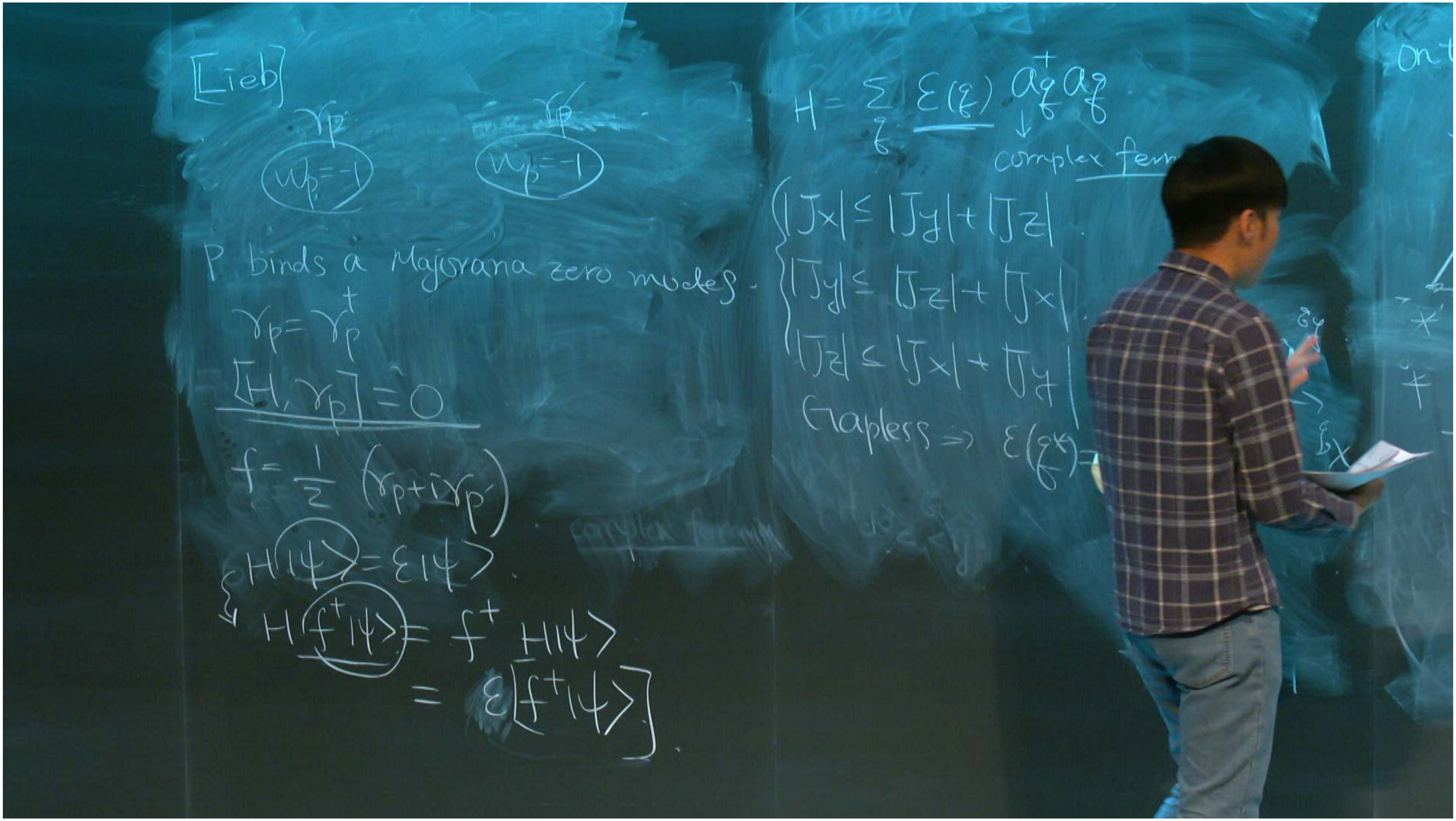

On the plane $J_x + J_y + J_z = 1$
$\geq 0 \geq 0 \geq 0$
$J_z=1, J_x=J_y=0$
gapless
$J_y=1$
$J_x=J_z=0$
$J_x=1$
$J_y=J_z=0$
$J_x=J_y=J_z$

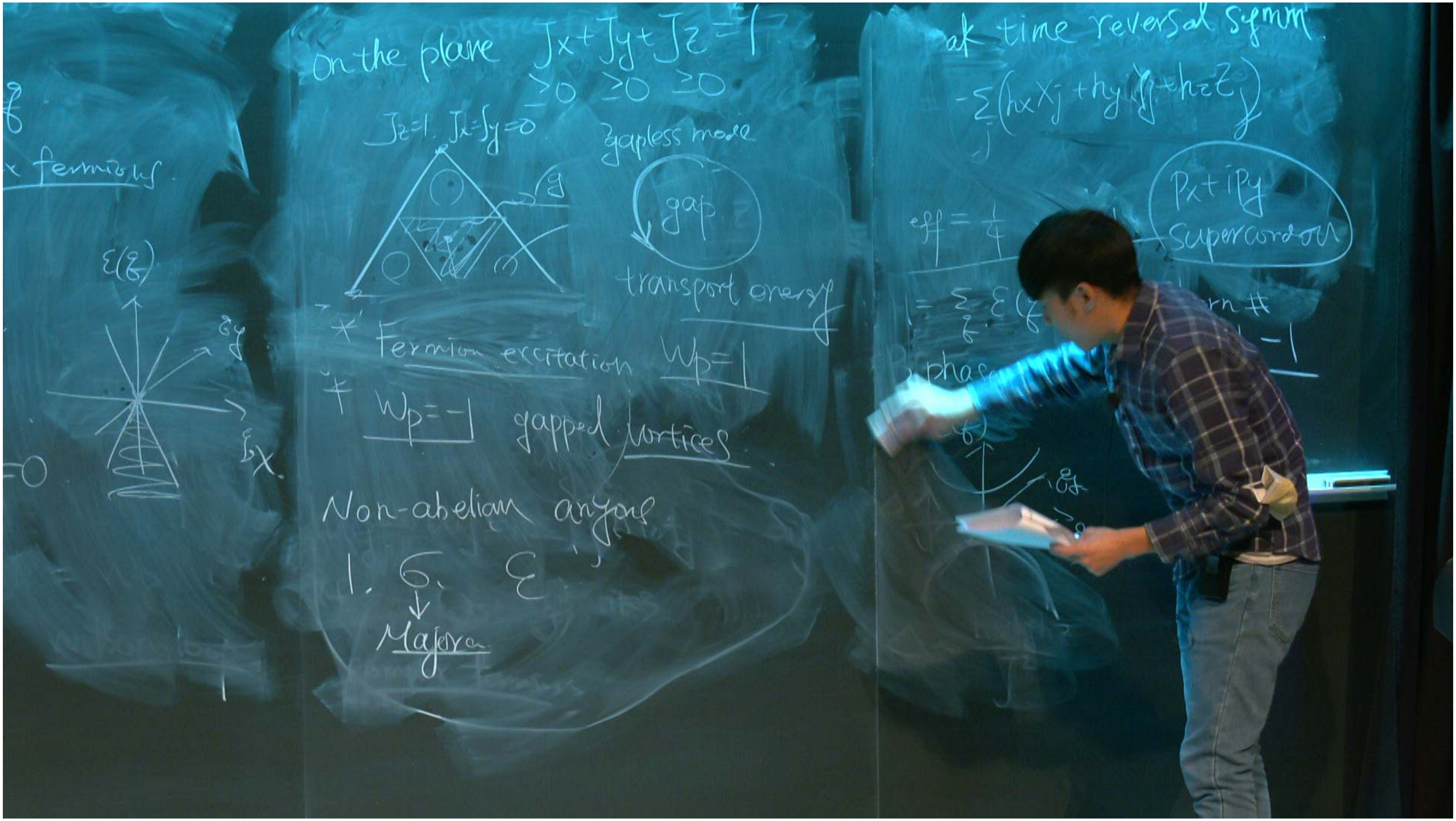


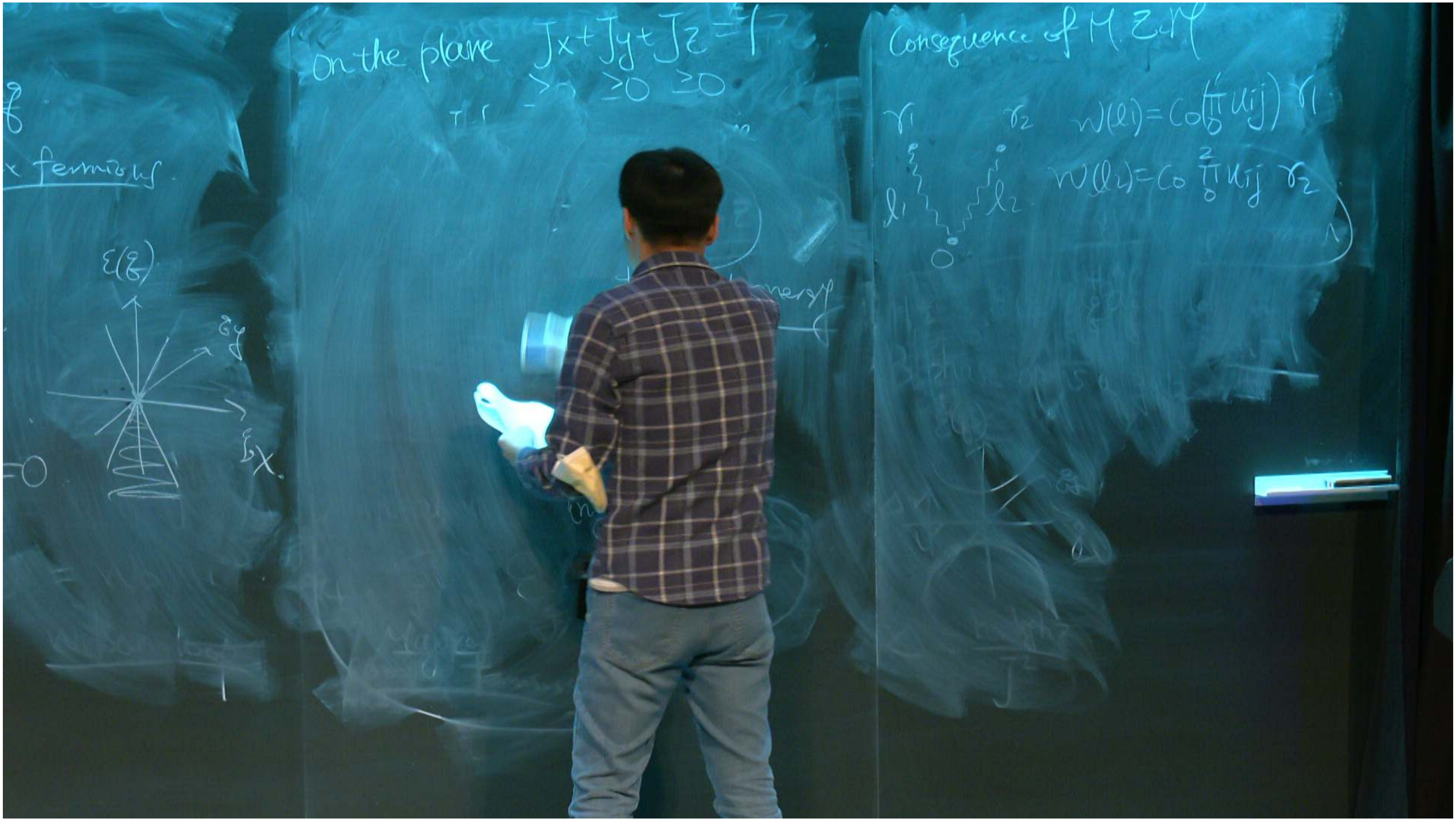
fermions
On the plane Jx+Jy+Jz=1
≥0 ≥0 ≥0
Jz=1. Jx=Jy=0
B
gapless
gapped. Z2 T.O
A
Jy=1
Jx=Jz=0
Jx=1
Jy=Jz=0
Jx=Jy=Jz

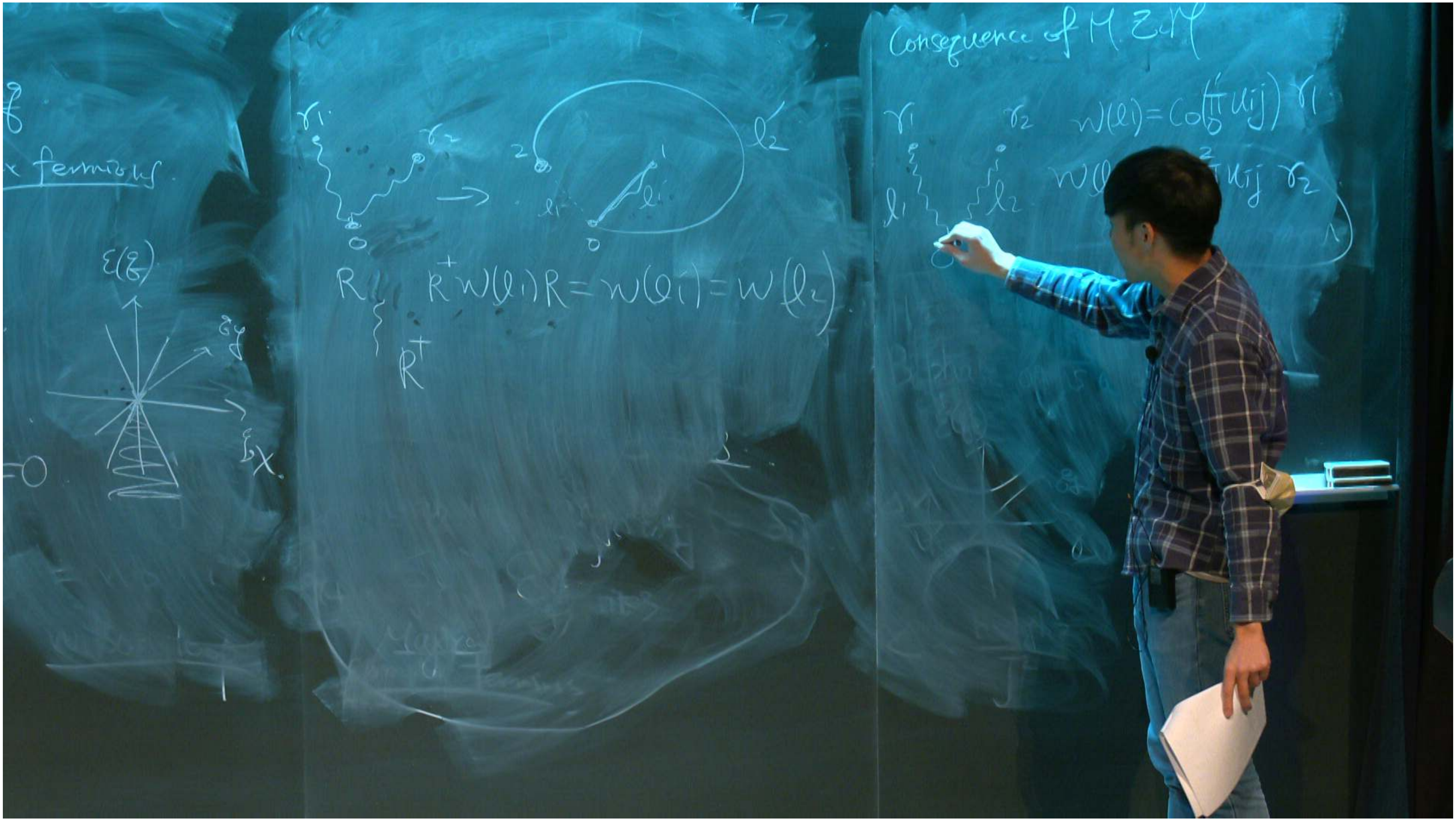

fermions
ε(q)
On the plane Jx + Jy + Jz = 1
≥0 ≥0 ≥0
Jz=1, Jx=Jy=0
Break time reversal symm.
V = -Σj (hx Xj + hy Yj + hz Zj)
Heff = i/4 Σij Aij
Px+iPy superconductor
H = Σq ε(q) a†q aq
Chern # = 1, -1
B phase opens a gap.
ε(q)
Δ
Δ ~ hxhyhz / J²

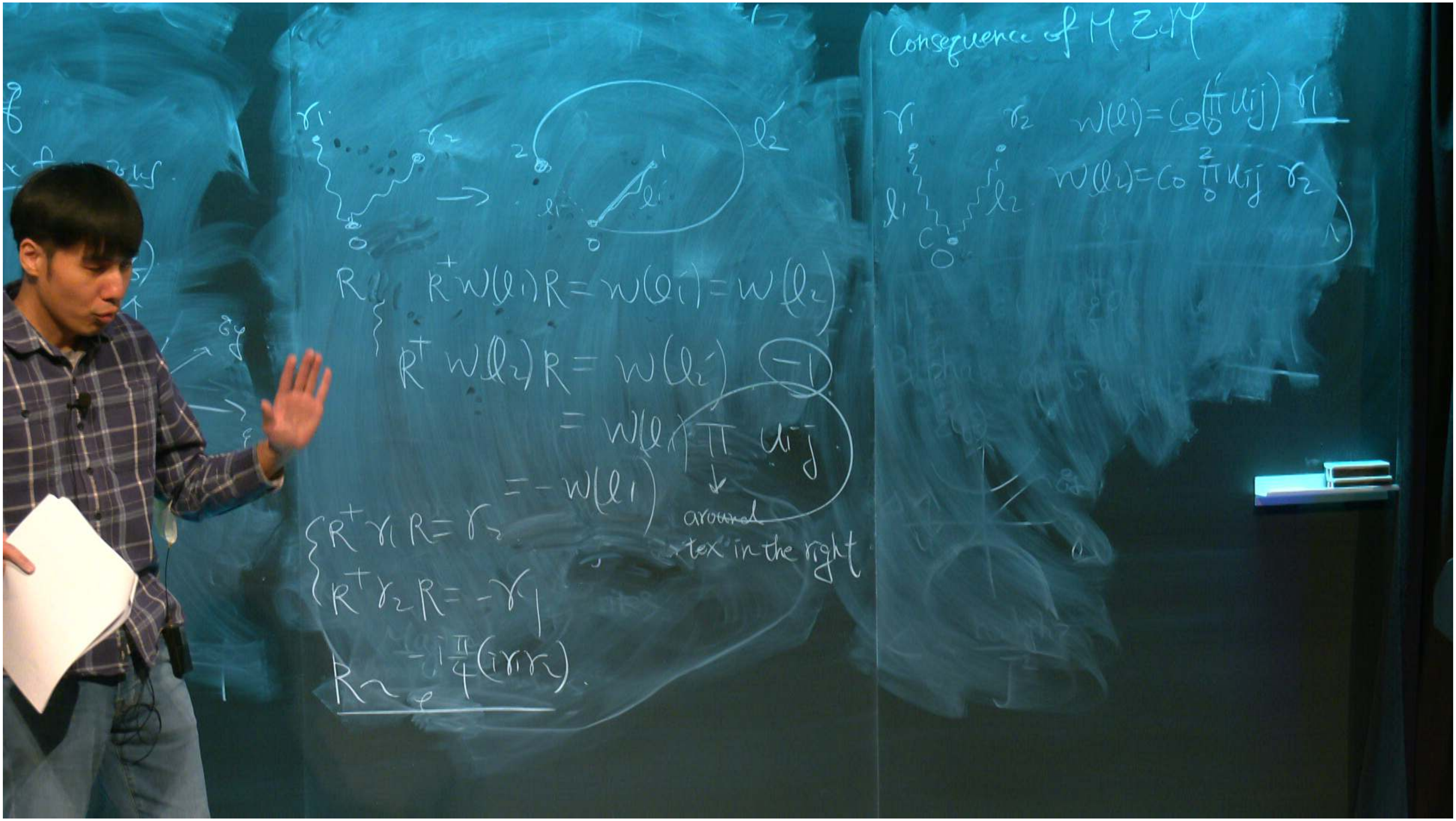

fermions.
On the plane Jx+Jy+Jz=1
≥0 ≥0 ≥0
Jz=1. Jx=Jy=0
gapless mode
gap
transport energy
Fermion excitation Wp=1
Wp=-1 gapped vortices
Break time reversal symm.
V=-Σ(hx Xj + hy Yj + hz Zj)
Heff = i/4 Σ Aij
Px+iPy superconductor
H= Σ ε(q) aq+ aq Chern #
=1, -1
B phase opens a gap.
Δ ~ hxhyhz/J²


[Lieb]
P binds a Majorana zero modes.
complex ferm
Gapless


On the plane Jx+Jy+Jz=1
≥0 ≥0 ≥0
Jz=1, Jx=Jy=0
gapless mode
gap
transport energy
Fermion excitation Wp=1
Wp=-1 gapped vortices
Non-abelian anyons
1, σ, ε
Majorana
time reversal symm
px+ipy
superconductor
fermions
ε(ξ)


On the plane Jx+Jy+Jz=1
Consequence of MZM
fermions


Consequence of M.Z.M
$R^{\dagger} W(l_1) R = W(l_1') = W(l_2)$
fermions



Consequence of MZM
$W(l_1) = C_0 \left(\prod u_{ij}\right) \gamma_1$
$W(l_2) = C_0 \prod u_{ij} \gamma_2$
$R^\dagger W(l_1) R = W(l_1') = W(l_2)$
$R^\dagger W(l_2) R = W(l_2') \; (-1)$
$= W(l_1) \prod u_{ij}$
$= -W(l_1)$
around
tex in the right
$R^\dagger \gamma_1 R = \gamma_2$
$R^\dagger \gamma_2 R = -\gamma_1$
$R \sim e^{-i\frac{\pi}{4}(i\gamma_1\gamma_2)}$