Title: Topological quantum matter and quantum computing

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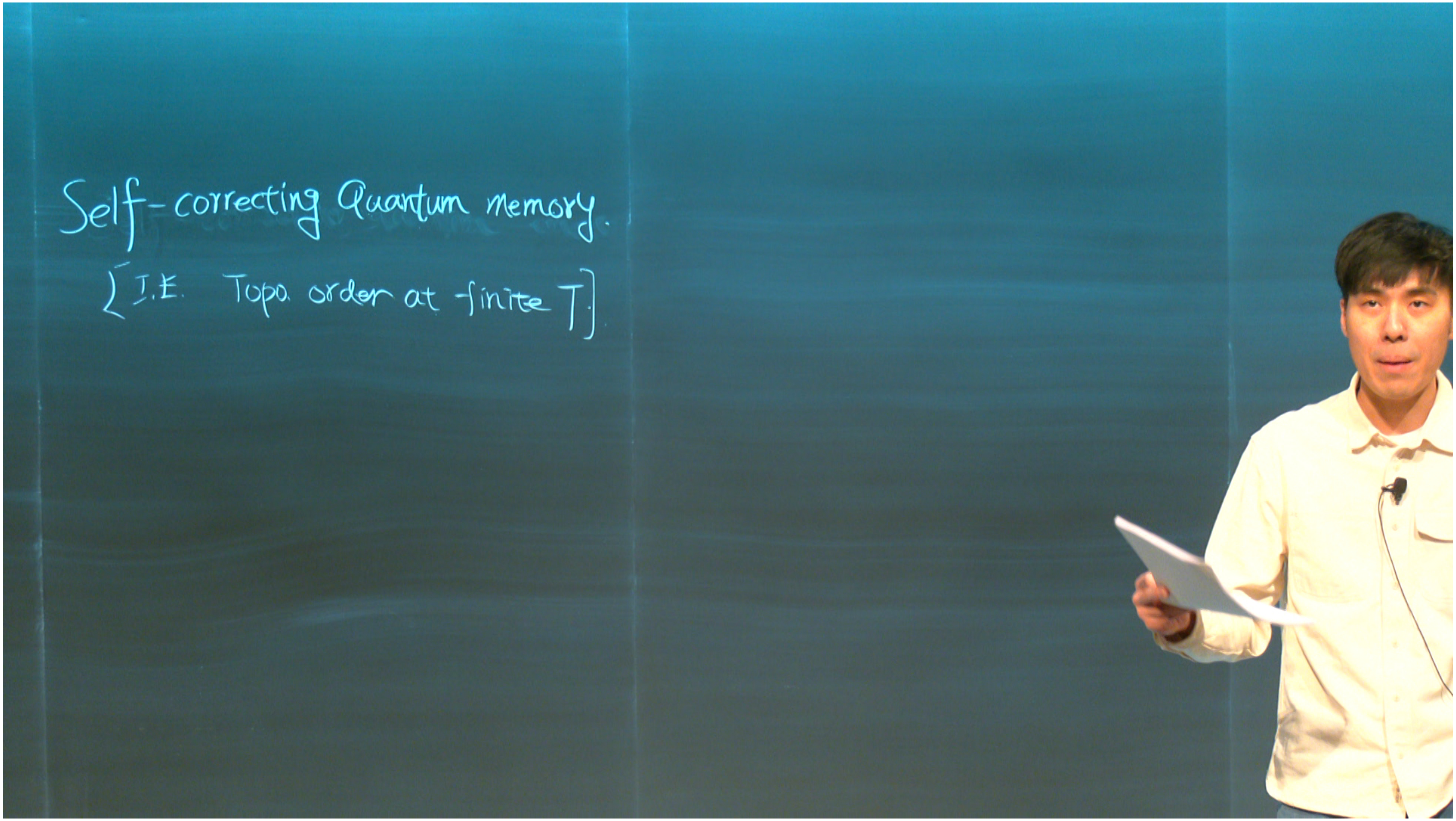

Self-correcting Quantum memory.
[I.E. Topo order at finite T.]

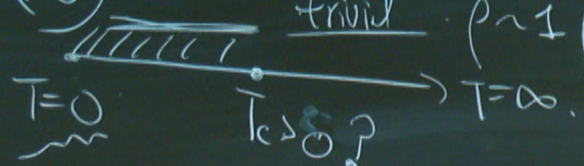
Self-correcting Quantum memory.

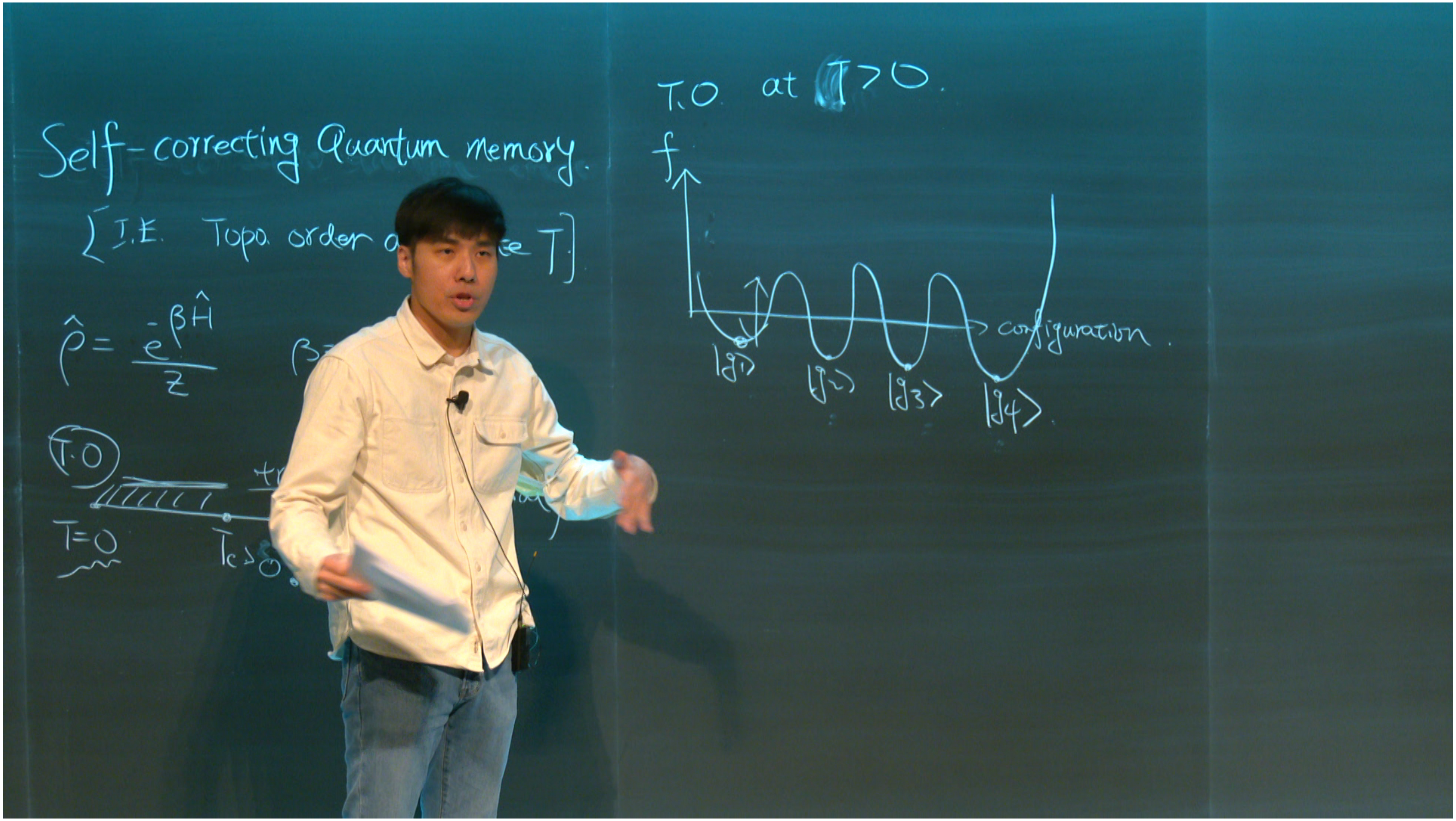
[I.E. Topo. order at finite T]

$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{Z} \qquad \beta = \frac{1}{T}. \qquad T \geq 0$$

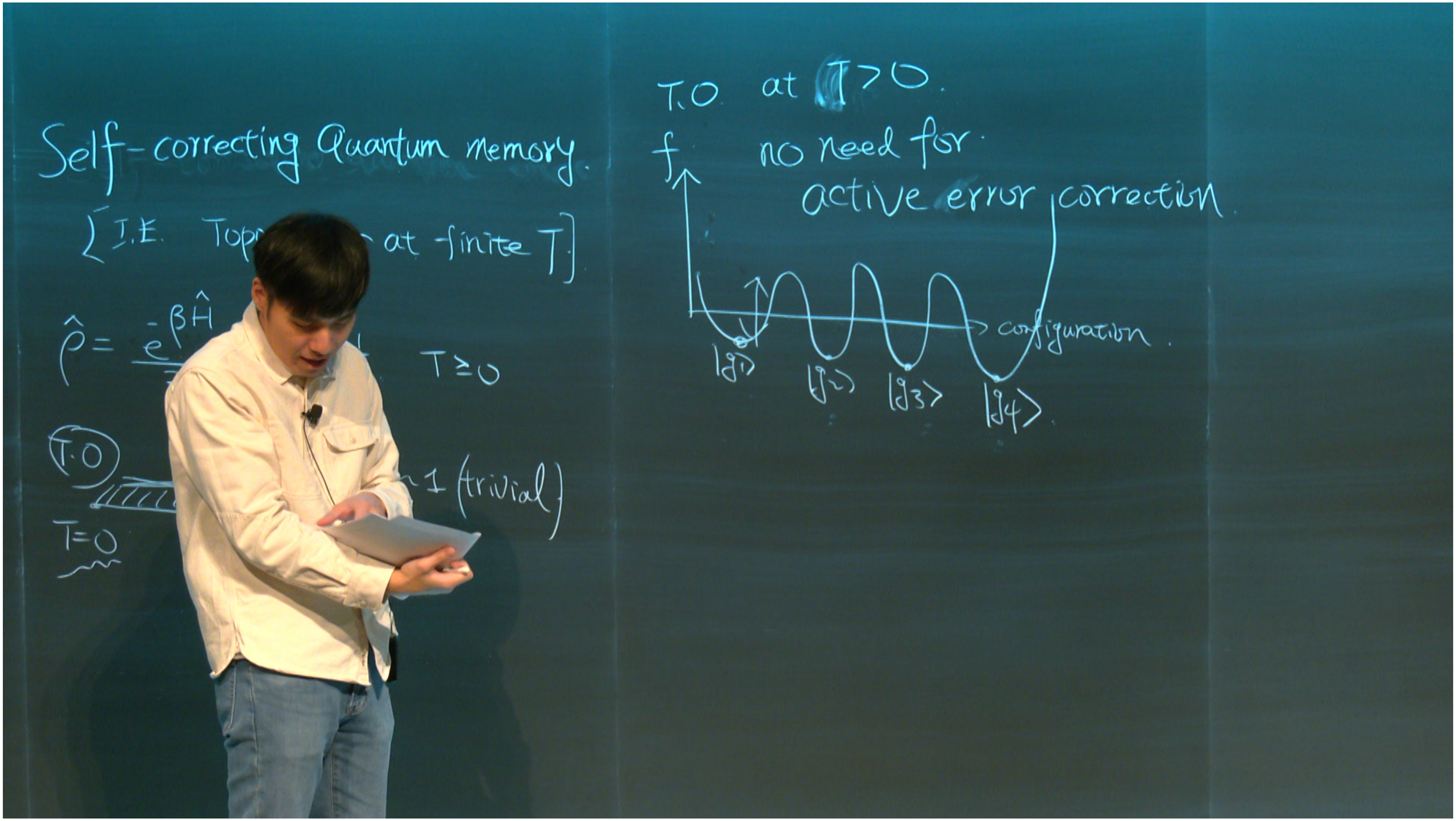
(T.O)

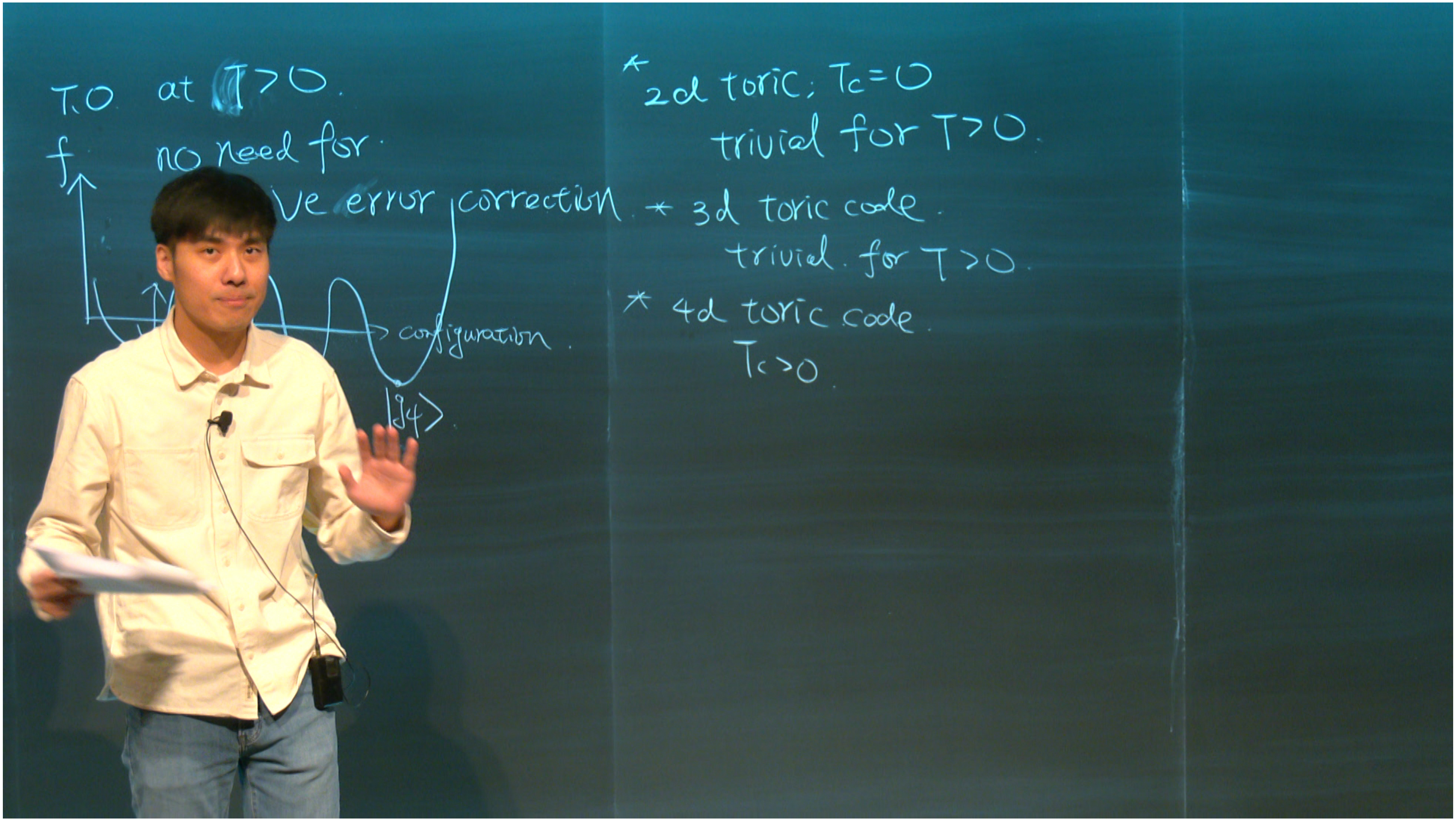
trivial. $\rho \sim 1$ (trivial)

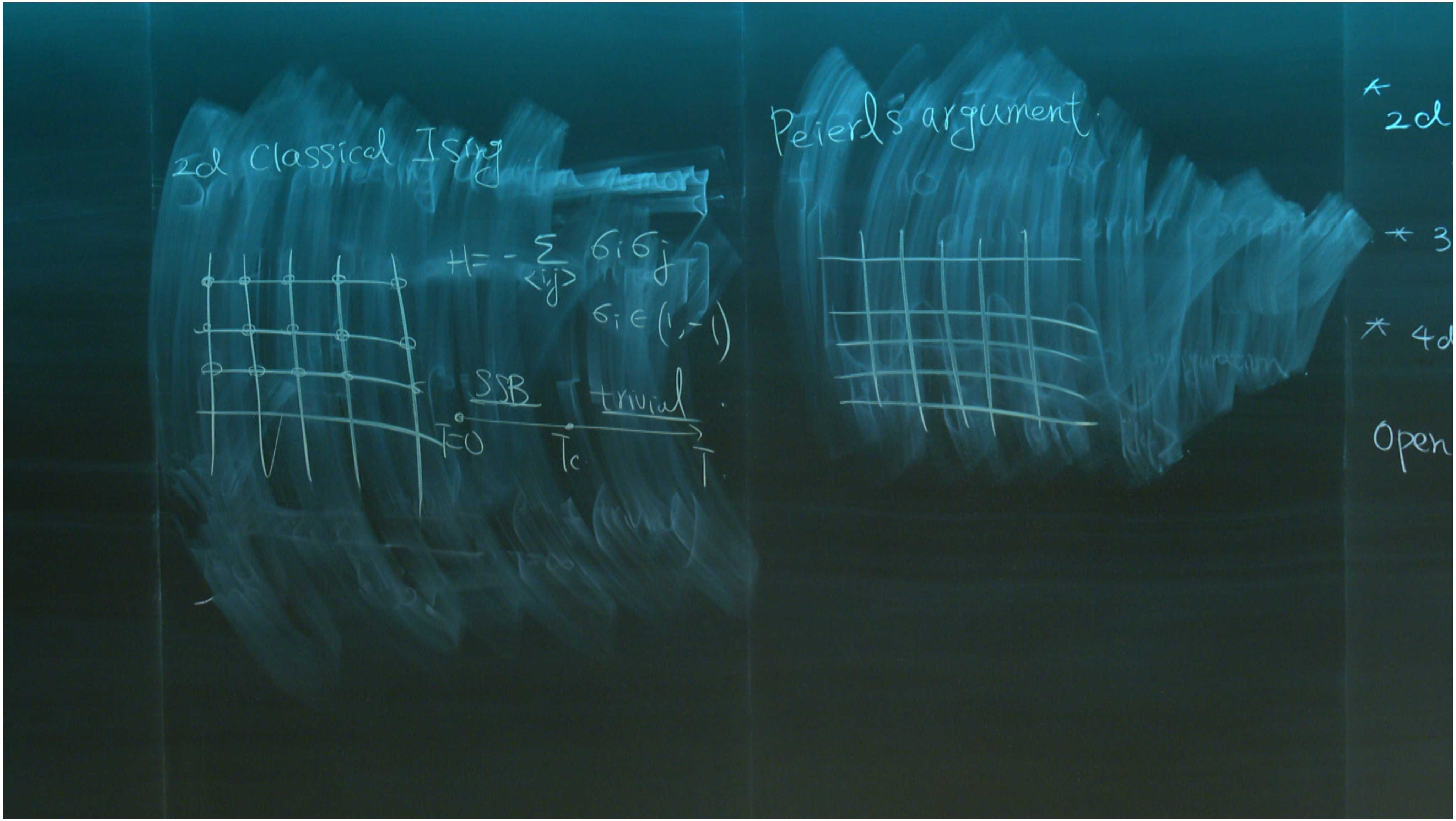


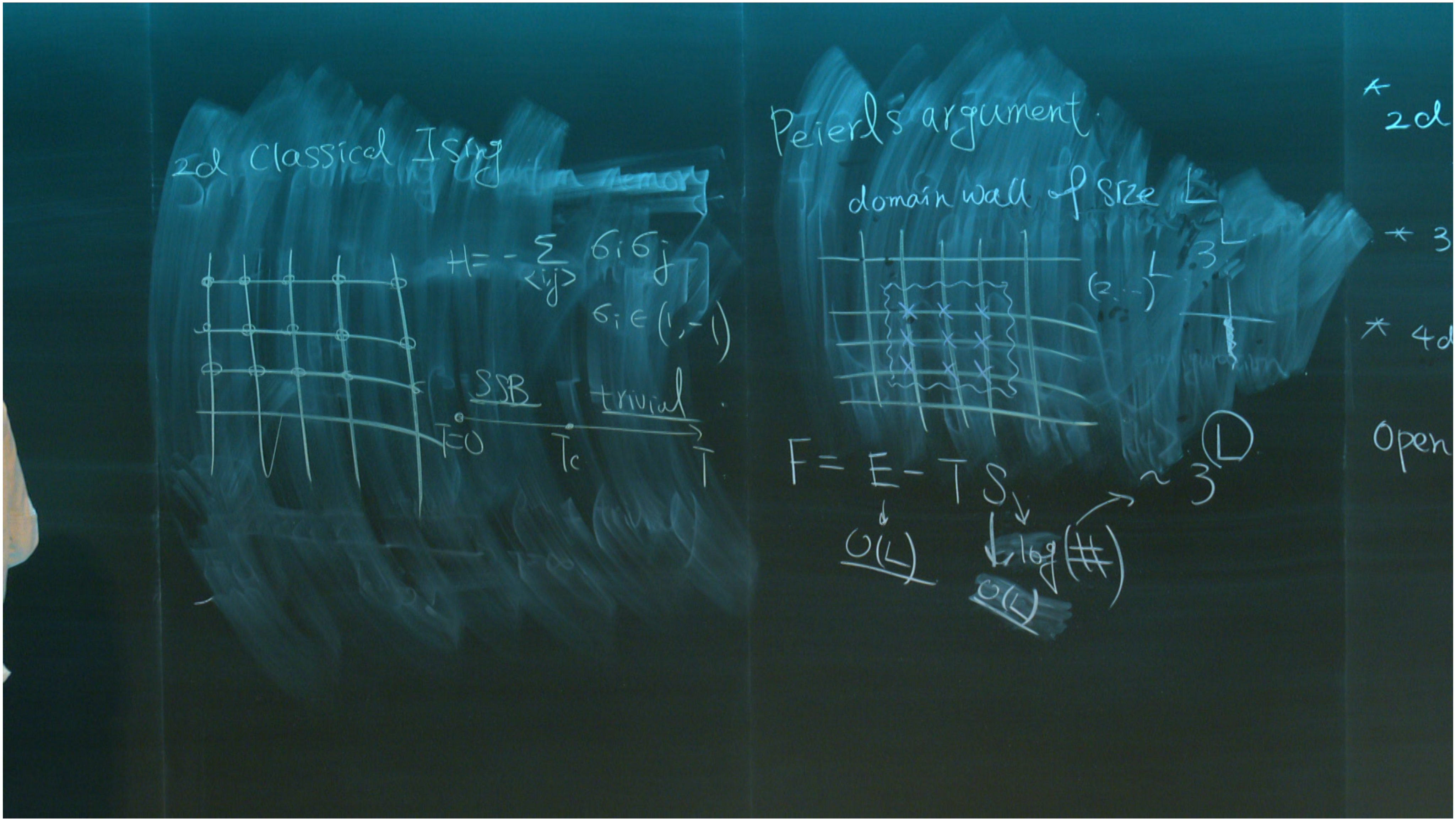


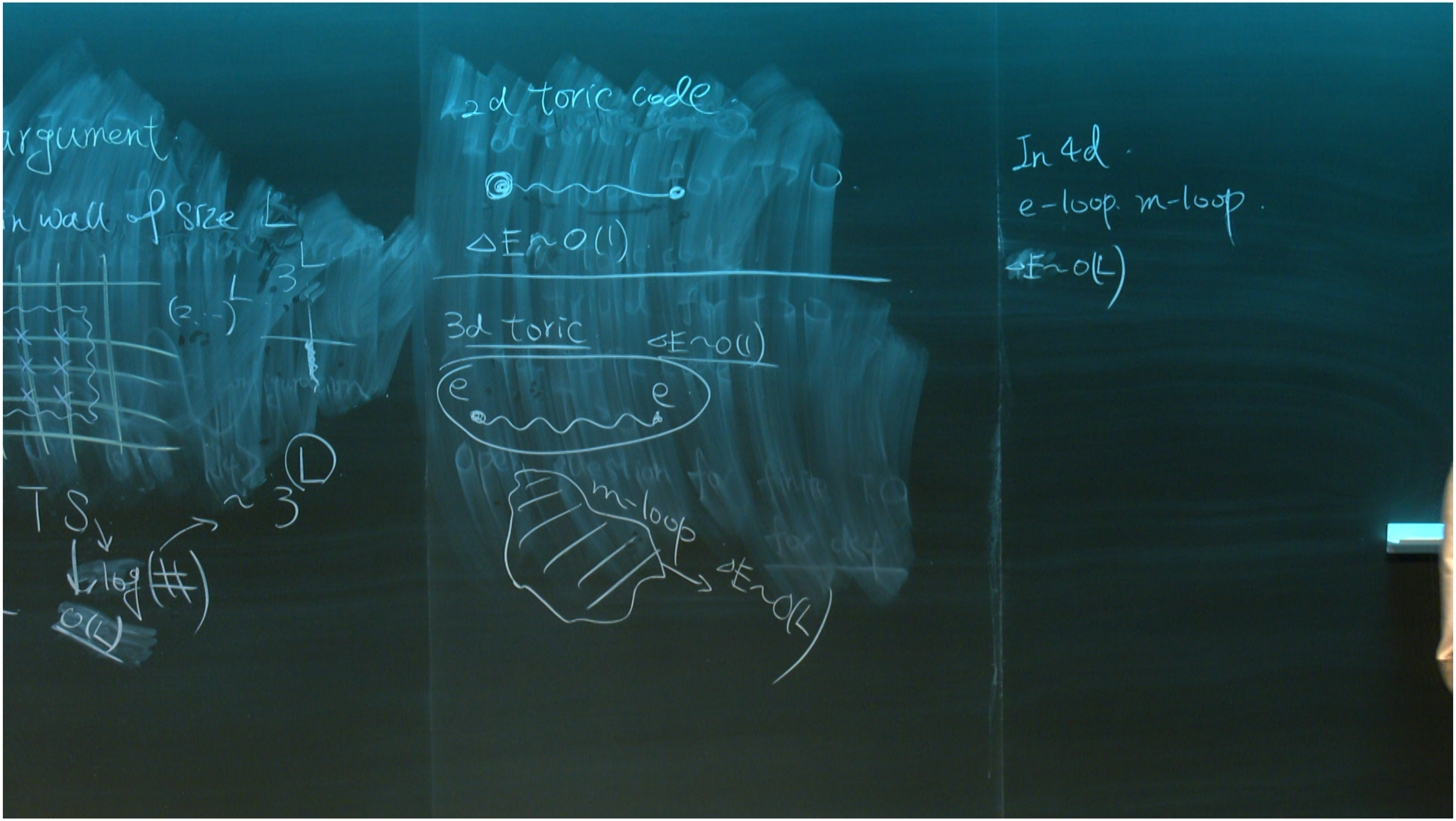
Self-correcting Quantum memory.
T.O at T>0.
configuration
|g1>
|g2>
|g3>
|g4>
T=0

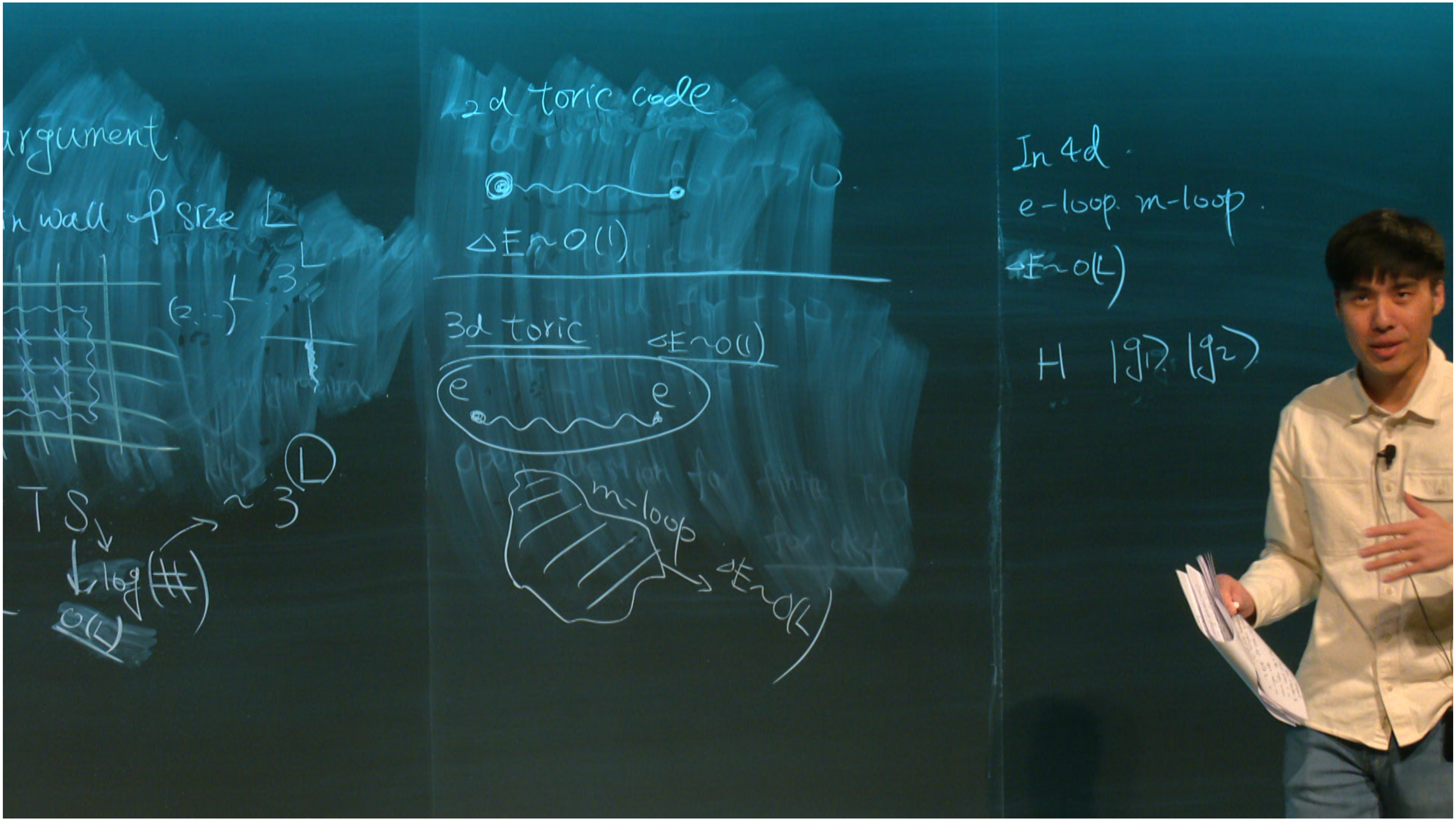

Self-correcting Quantum memory.
at finite T
T ≥ 0
T.O
T=0
1 (trivial)
T.O at T>0.
f
no need for
active error correction
configuration
|g1> |g2> |g3> |g4>


T.O at T>0.
f
no need for
ve error correction
configuration
* 2d toric; Tc=0
trivial for T>0.
* 3d toric code.
trivial. for T>0.
* 4d toric code.
Tc>0

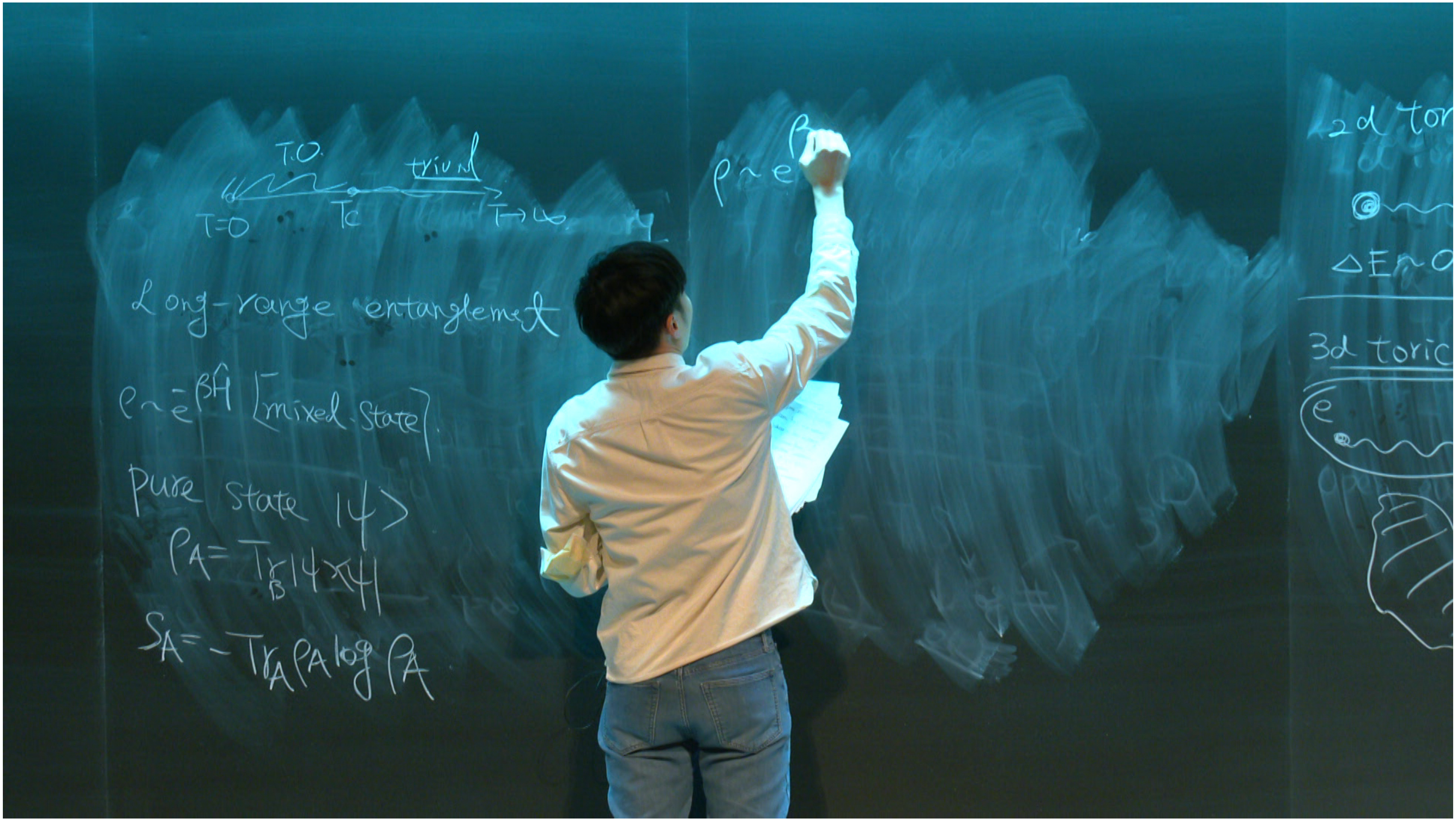

2d Classical Ising
H = - Σ <ij> σi σj
σi ∈ (1, -1)
SSB
trivial
T=0
Tc
T
Peierls argument
* 2d
* 3
* 4d
Open

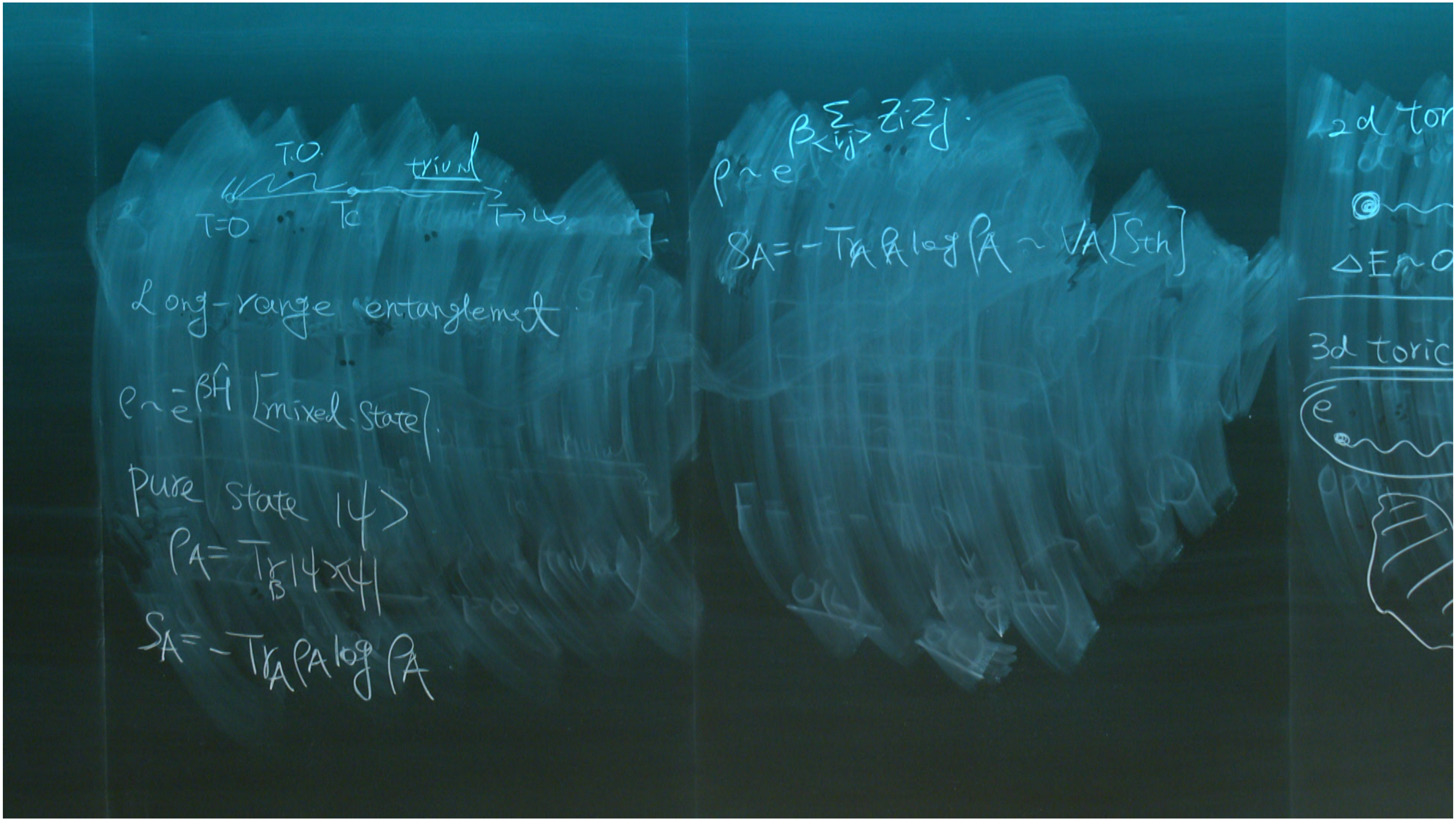

2d Classical Ising
H = - Σ σiσj
<ij>
σi ∈ (1,-1)
SSB
trivial
T=0
Tc
T
Peierls argument
domain wall of size L
3L
F = E - T S
O(L)
log(#)
O(L)
~3L
2d
3
4d
Open

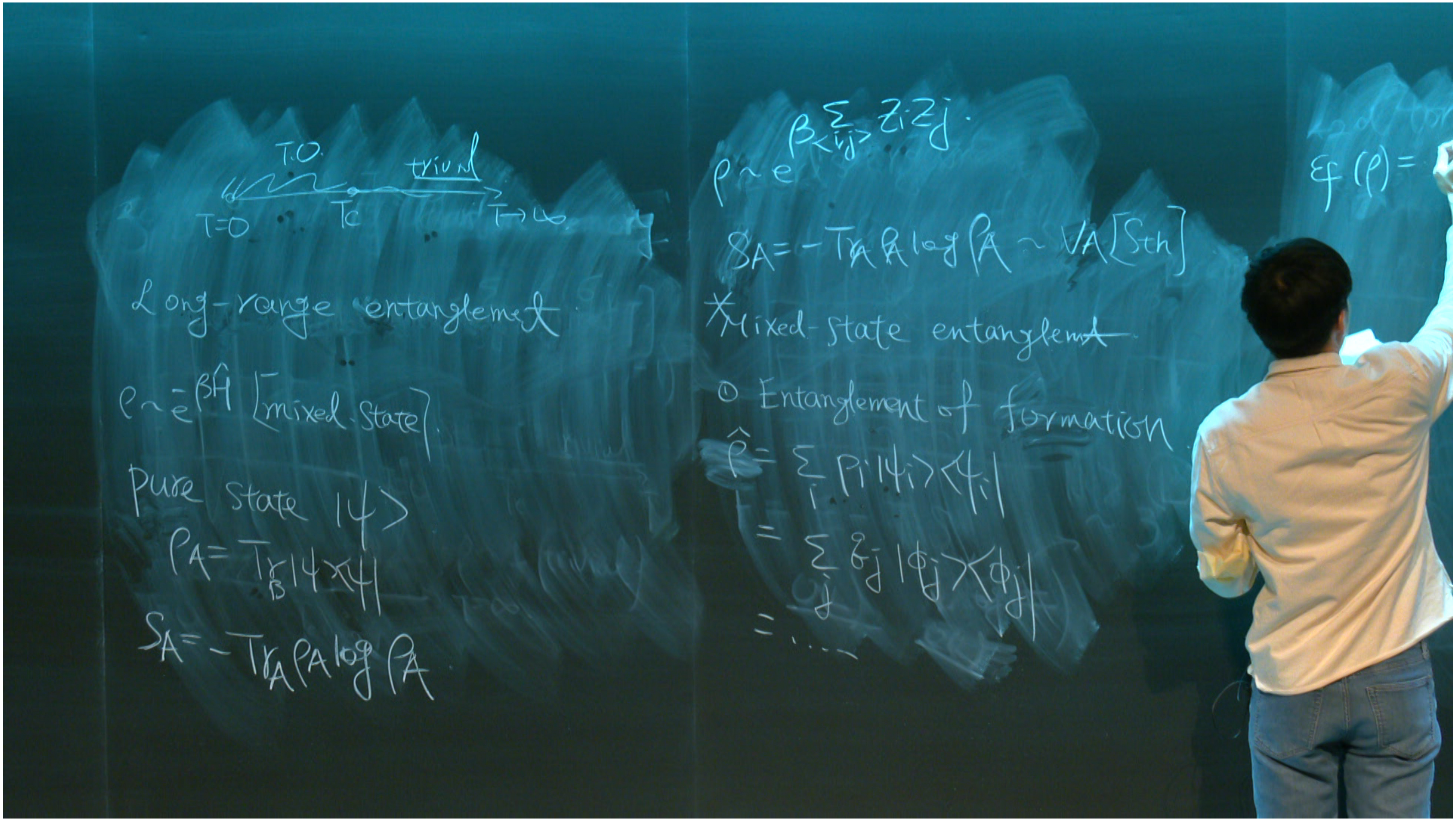

argument
in wall of size L
2d toric code
ΔE ~ O(1)
3d toric
ΔE ~ O(1)
e
e
m-loop
ΔE ~ O(L)
log(#)
O(L)
In 4d
e-loop m-loop
ΔE ~ O(L)



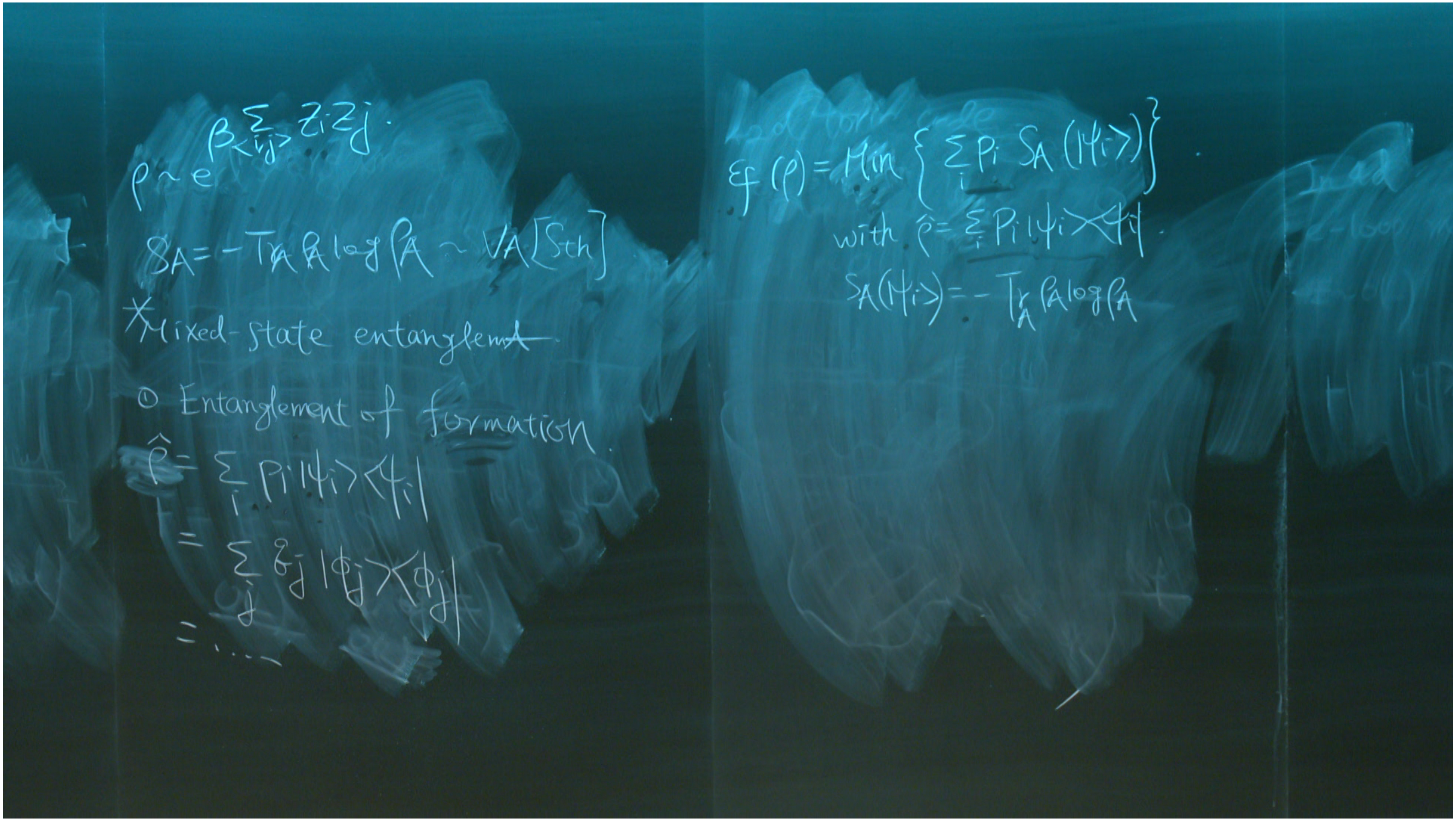
argument
in wall of size L
2d toric code
ΔE ~ O(1)
3d toric
ΔE ~ O(1)
e
e
m-loop
ΔE ~ O(L)
In 4d
e-loop m-loop
ΔE ~ O(L)
H |g1⟩ |g2⟩
~3^L
T S
log(#)
O(L)

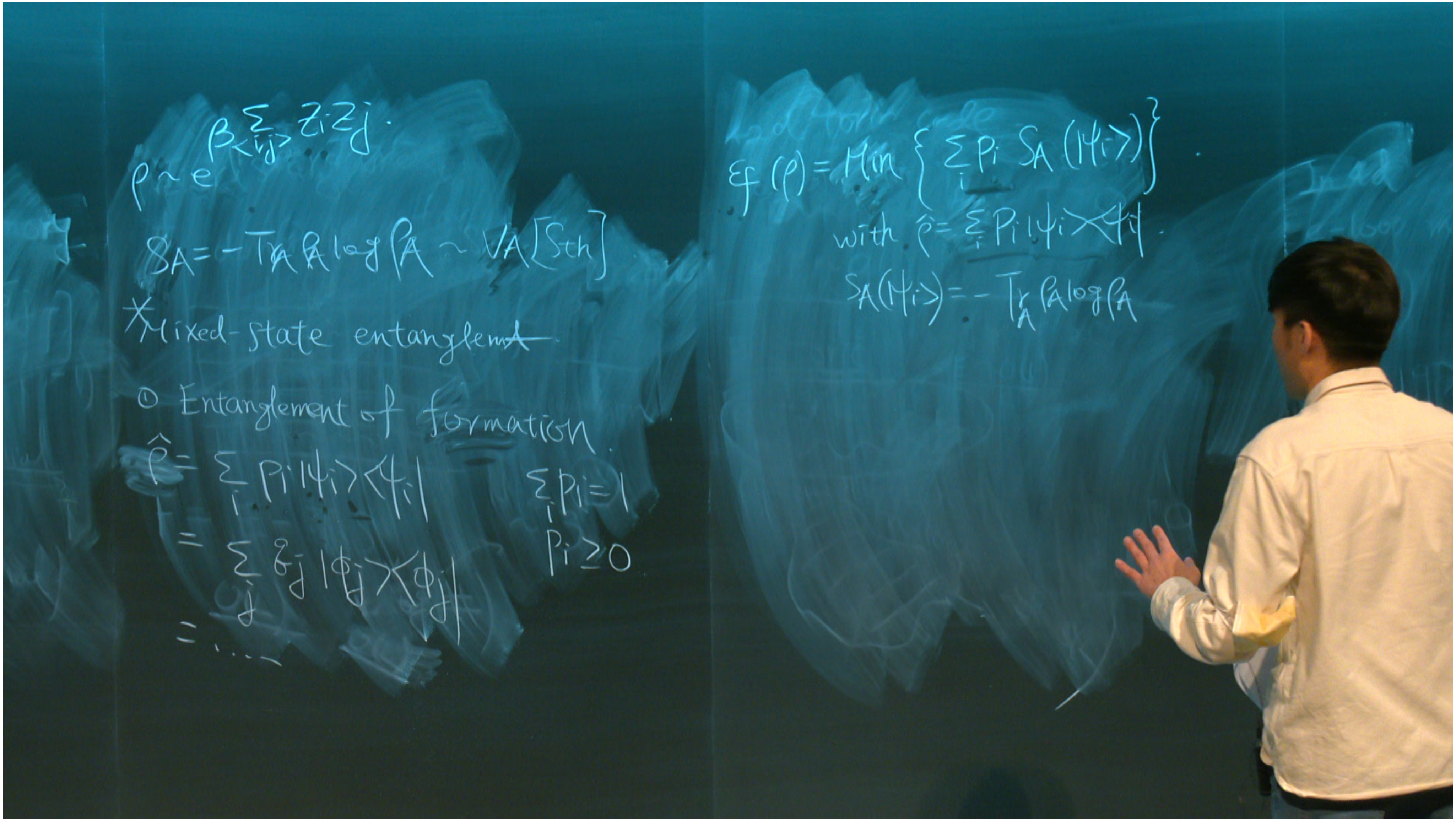

T.O.
trivial
T=0
Tc
T→∞
Long-range entanglement
ρ ~ e^{-βH} [mixed state]
pure state |ψ⟩
ρA = TrB |ψ⟩⟨ψ|
SA = -TrA ρA log ρA
2d tor
3d toric


T.O.
trivial
T=0
Tc
T→∞
Long-range entanglement
ρ ~ e^{-βH} [mixed state]
pure state |ψ>
ρA = TrB |ψ><ψ|
SA = -TrA ρA log ρA
ρ ~ e^{β Σ<ij> Zi Zj}
SA = -TrA ρA log ρA ~ VA [Sth]
3d toric

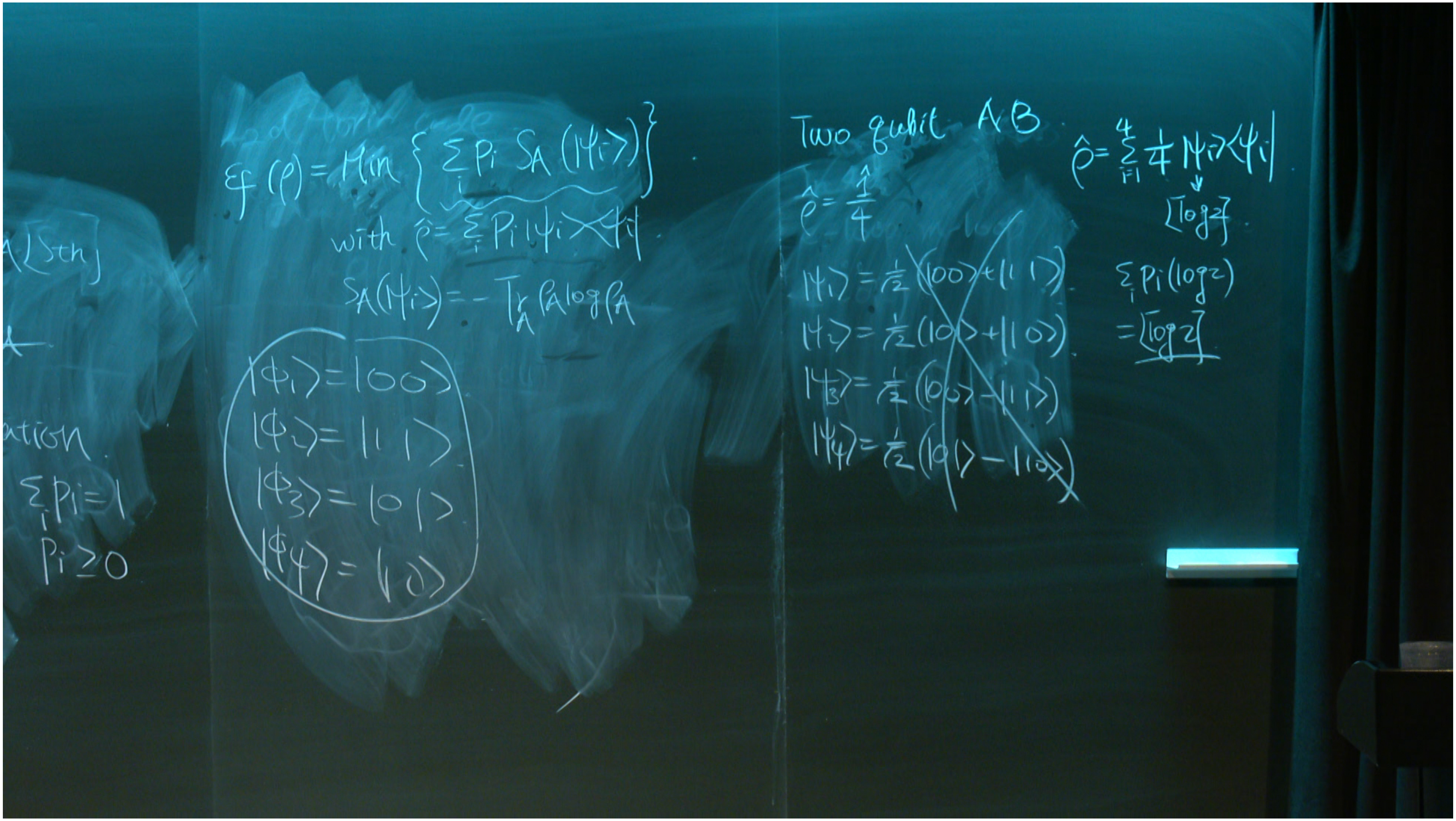


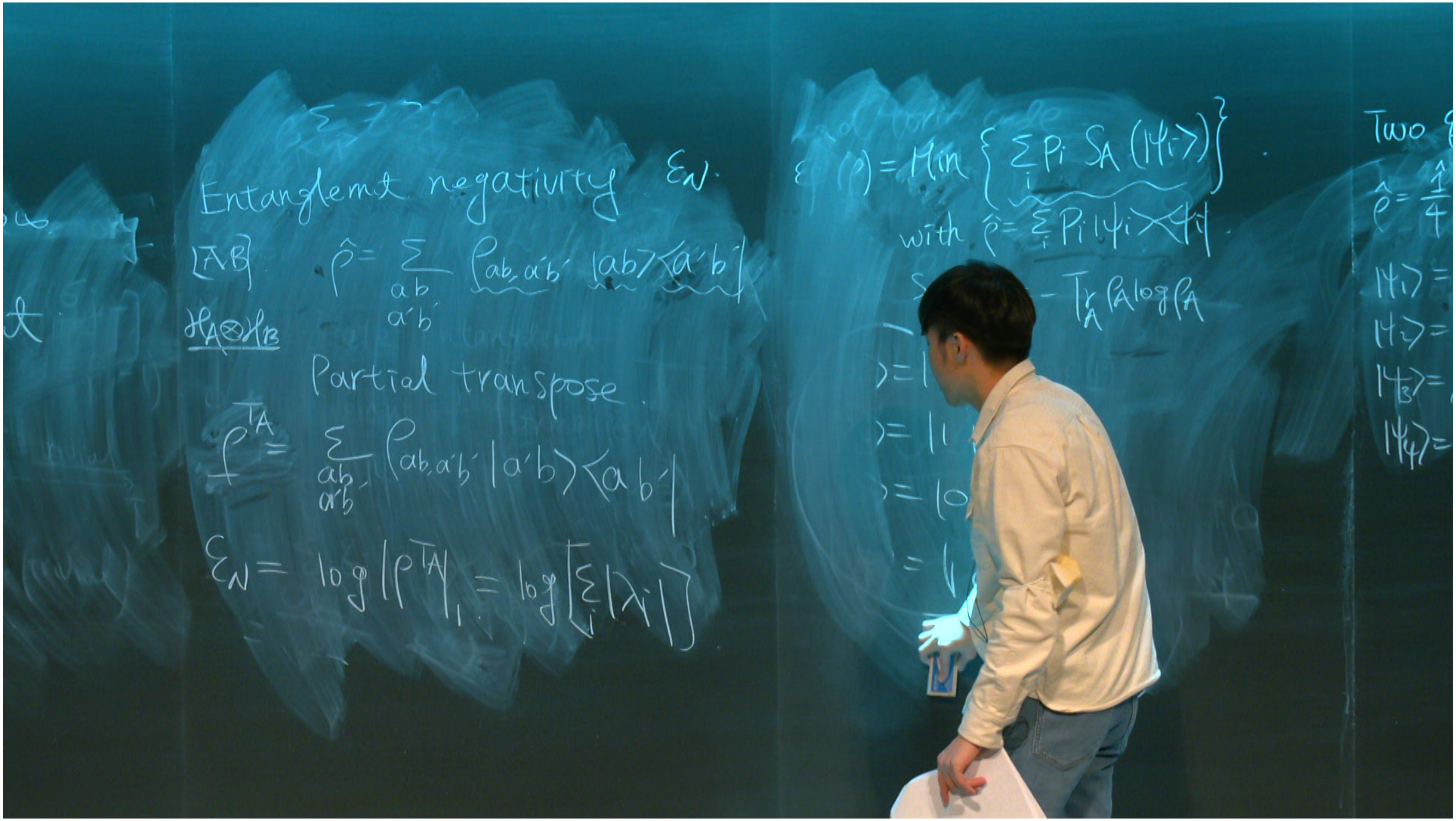
T.O.
trivial
T=0
Tc
T→∞
Long-range entanglement
ρ ~ e^{-βĤ} [mixed state]
pure state |ψ>
ρ_A = Tr_B |ψ><ψ|
S_A = -Tr_A ρ_A log ρ_A
ρ ~ e^{β Σ_<ij> Z_i Z_j}
S_A = -Tr_A ρ_A log ρ_A ~ V_A [S_th]
* Mixed-state entanglement
① Entanglement of formation
ρ̂ = Σ_i p_i |ψ_i><ψ_i|
= Σ_j q_j |φ_j><φ_j|
= ....
E_f (ρ) =

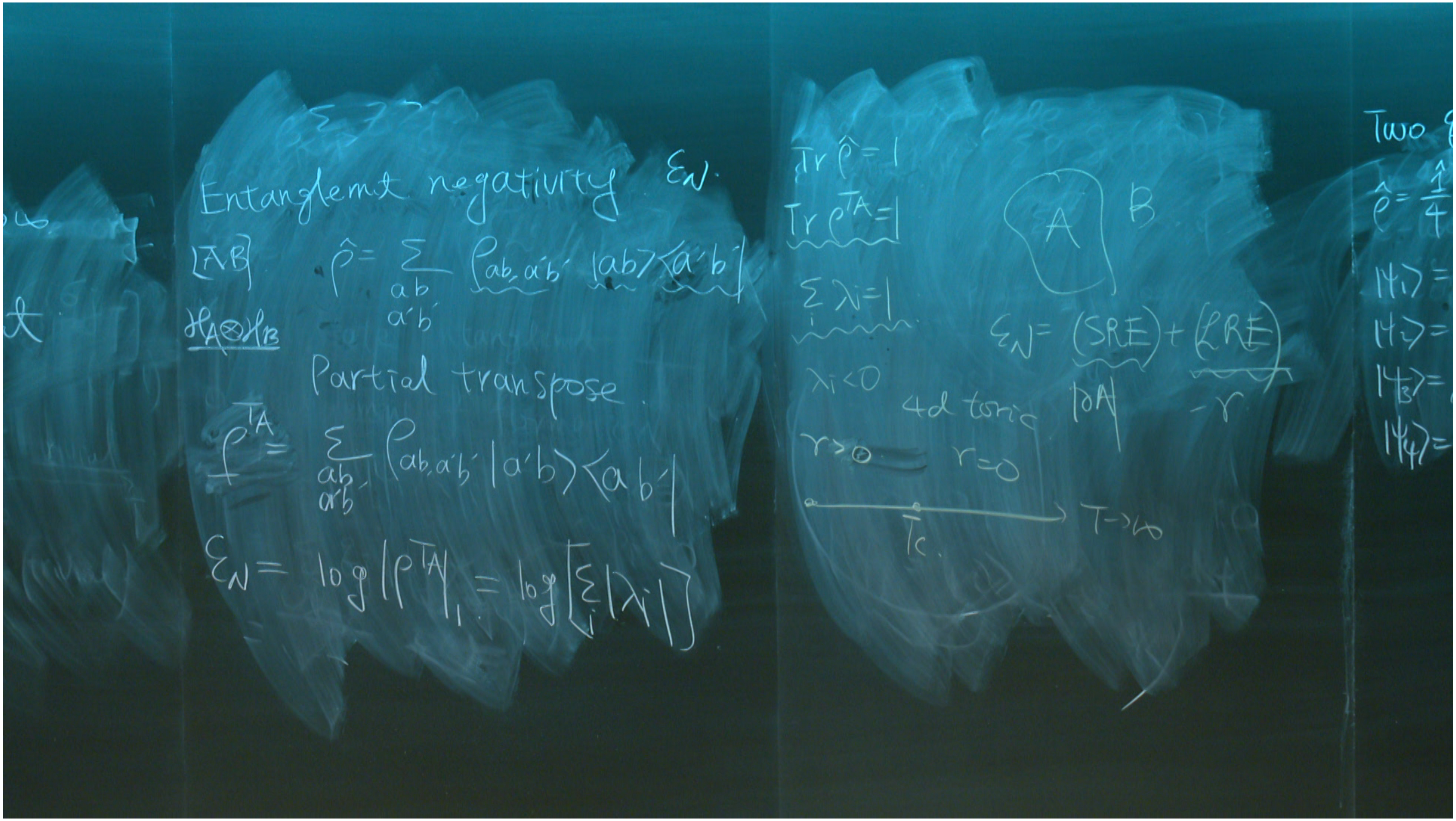

$\rho \sim e^{\beta \sum_{\langle ij \rangle} Z_i Z_j}$
$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A \sim V_A [S_{th}]$
* Mixed-state entanglement
① Entanglement of formation
$\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
$= \sum_j q_j |\phi_j\rangle\langle\phi_j|$
$= \ldots\ldots$
$E_f(\rho) = \mathrm{Min}\left\{ \sum_i p_i S_A(|\psi_i\rangle) \right\}$
with $\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
$S_A(|\psi_i\rangle) = -\mathrm{Tr}_A \rho_A \log \rho_A$

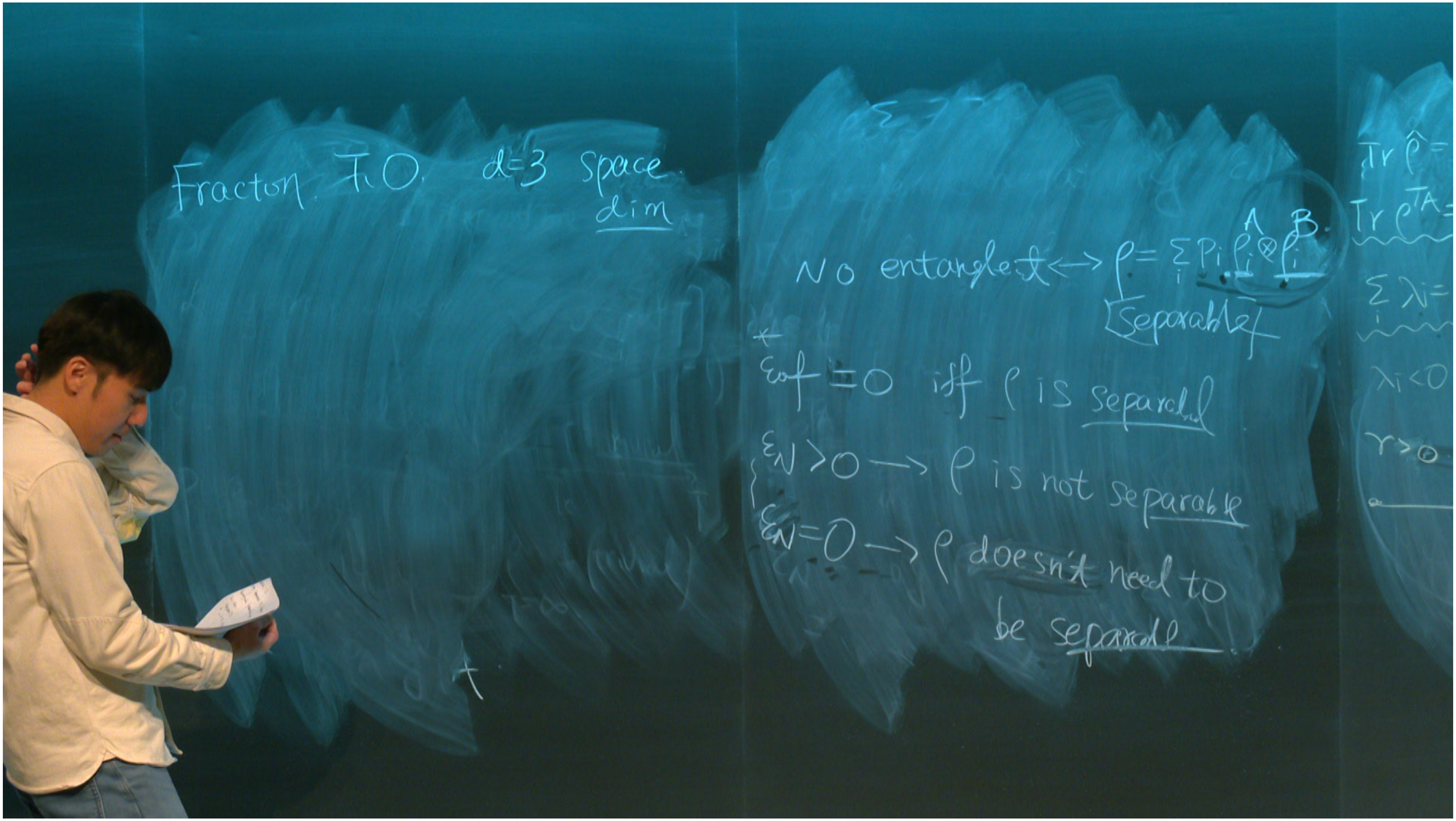


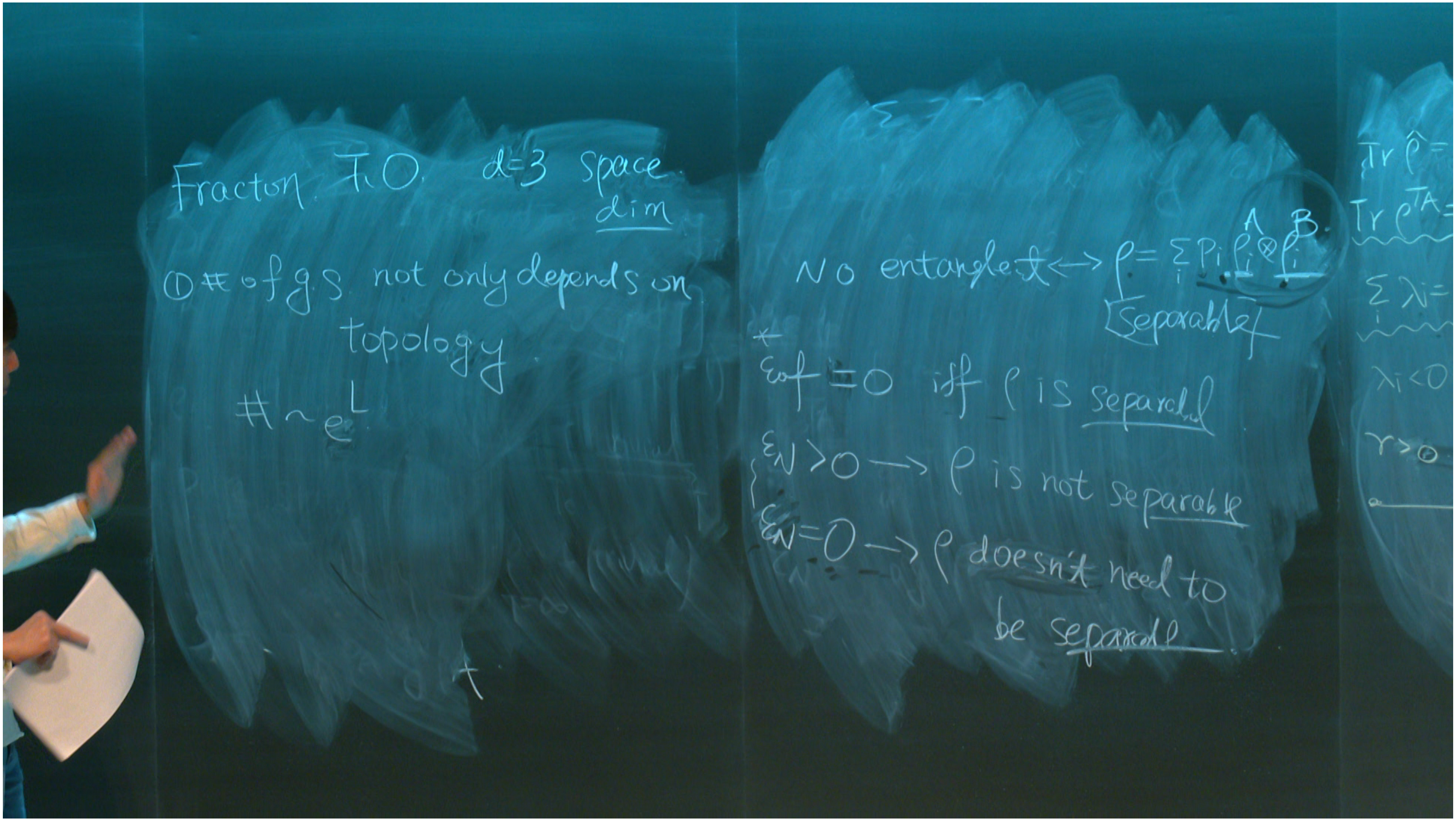
$\rho \sim e^{\beta \sum_{\langle ij \rangle} Z_i Z_j}$
$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A \sim V_A [S_{th}]$
* Mixed-state entanglement
① Entanglement of formation
$\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
$= \sum_j q_j |\phi_j\rangle\langle\phi_j|$
$= \ldots$
$\sum_i p_i = 1$
$p_i \geq 0$
$E_f(\rho) = \mathrm{Min} \{ \sum_i p_i S_A(|\psi_i\rangle) \}$
with $\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
$S_A(|\psi_i\rangle) = -\mathrm{Tr}_A \rho_A \log \rho_A$


Ef (ρ) = Min { Σ pi SA (|ψi>) }
with ρ̂ = Σ pi |ψi><ψi|
SA(|ψi>) = − TrA ρA log ρA
|φ1> = |00>
|φ2> = |11>
|φ3> = |01>
|φ4> = |10>
Σ pi = 1
pi ≥ 0
Two qubit A B
ρ̂ = 1̂/4
|ψ1> = 1/√2 (|00> + |11>)
|ψ2> = 1/√2 (|01> + |10>)
|ψ3> = 1/√2 (|00> − |11>)
|ψ4> = 1/√2 (|01> − |10>)
ρ̂ = Σ 1/4 |ψi><ψi|
[log 2]
Σ pi (log 2)
= [log 2]

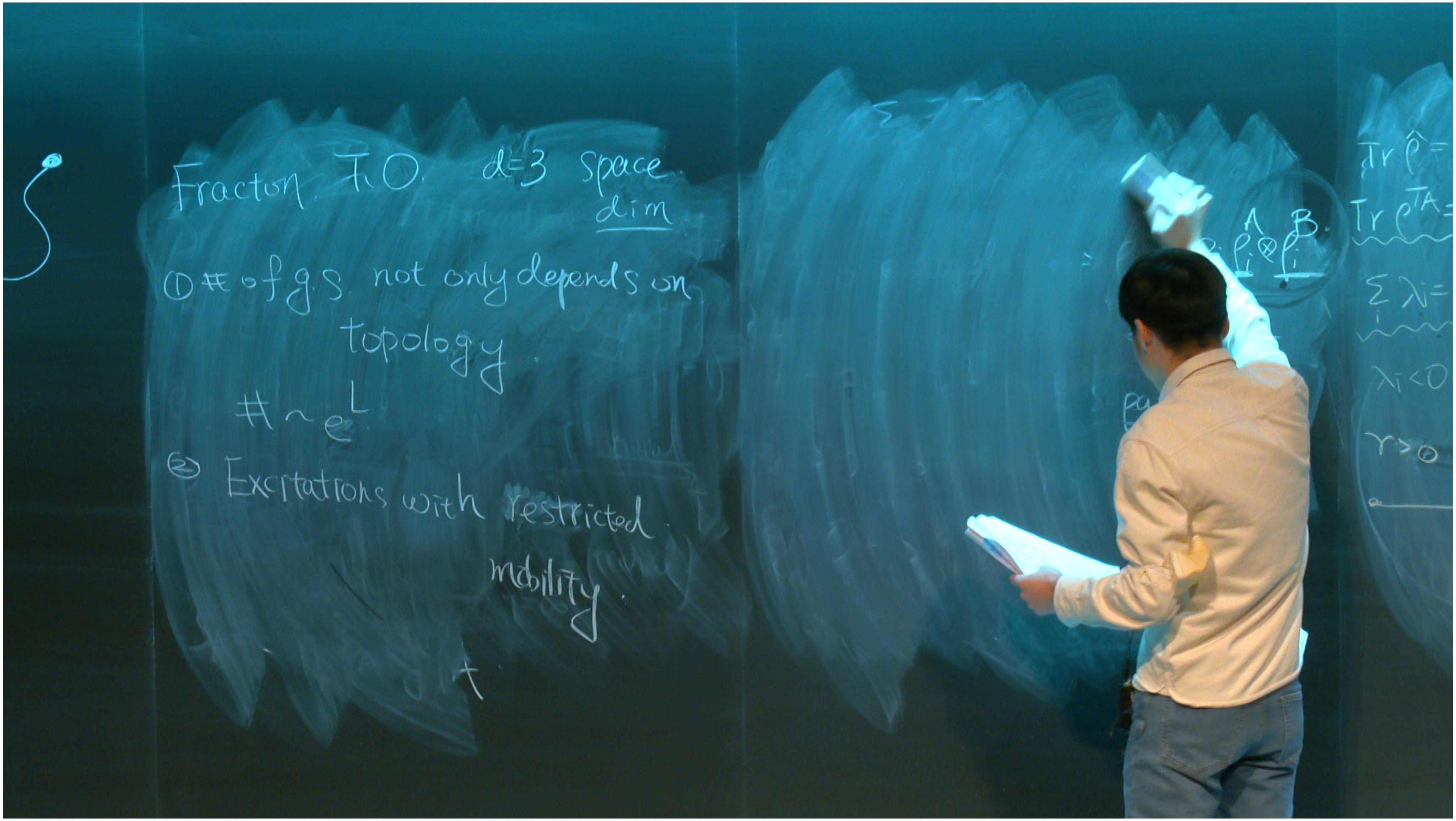

Entanglement negativity $\mathcal{E}_N$
Partial transpose
Two

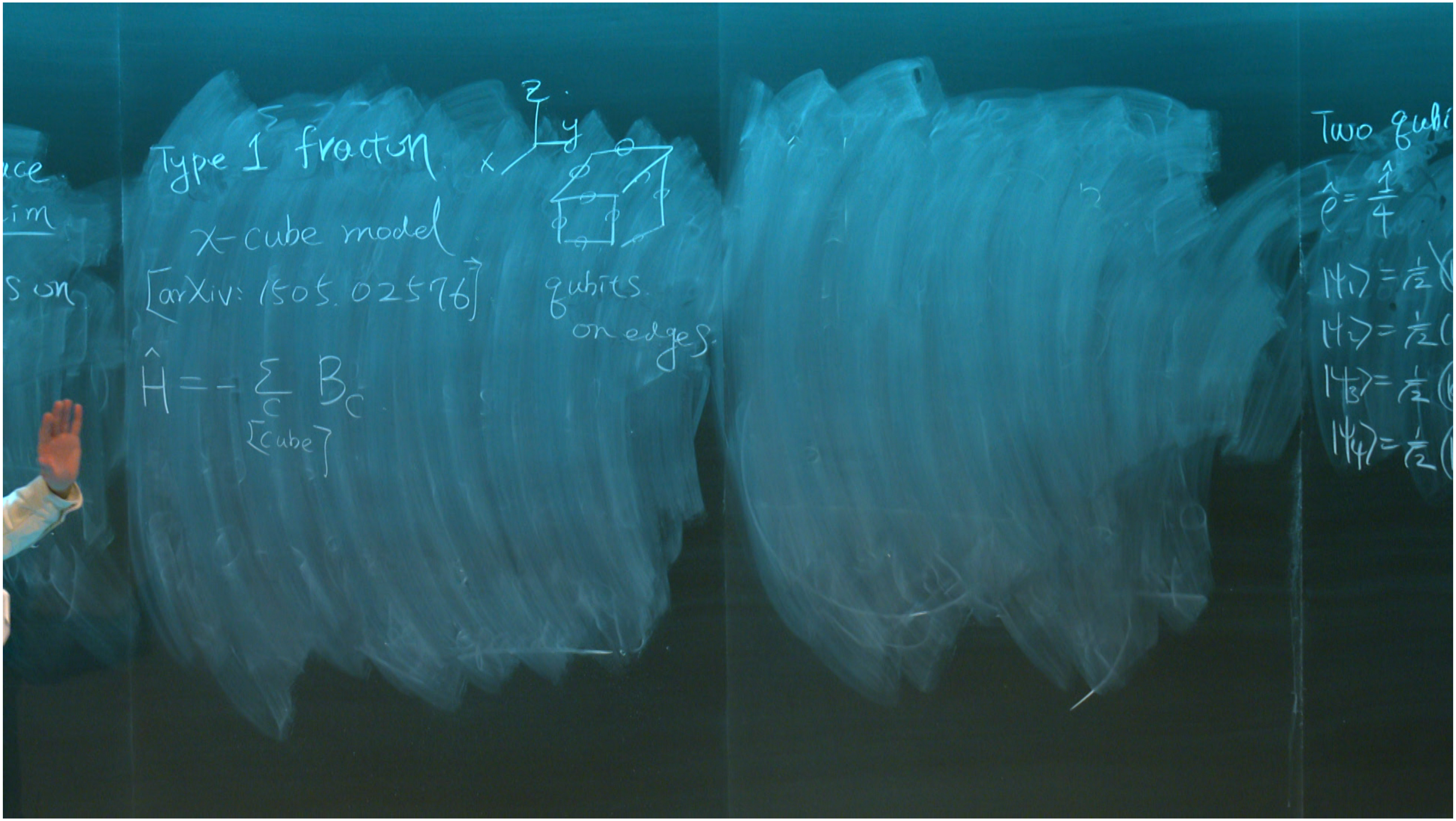

Entanglement negativity $\mathcal{E}_N$
$[A|B]$
$\hat{\rho} = \sum_{ab,a'b'} \rho_{ab,a'b'} |ab\rangle\langle a'b'|$
$\mathcal{H}_A \otimes \mathcal{H}_B$
Partial transpose
$\rho^{T_A} = \sum_{ab,a'b'} \rho_{ab,a'b'} |a'b\rangle\langle ab'|$
$\mathcal{E}_N = \log \|\rho^{T_A}\|_1 = \log\left[\sum_i |\lambda_i|\right]$
$\mathrm{Tr}\,\hat{\rho} = 1$
$\mathrm{Tr}\,\rho^{T_A} = 1$
$\sum_i \lambda_i = 1$
$\lambda_i < 0$
A
B
$\mathcal{E}_N = (\mathrm{SRE}) + (\mathrm{LRE})$
$|\partial A|$
$-\gamma$
4d toric
$\gamma > 0$
$\gamma = 0$
T_c
$T \to \infty$
Two

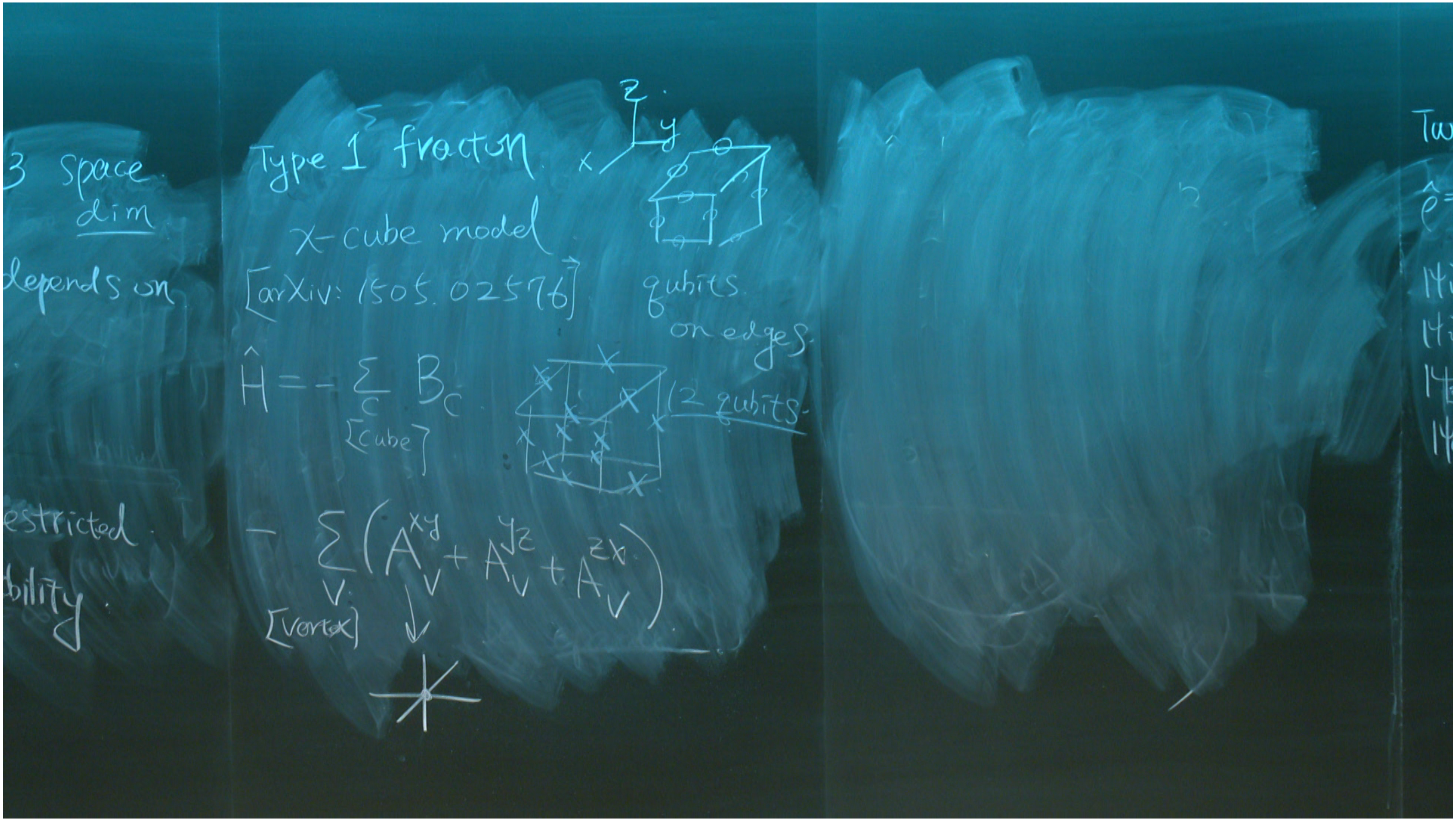

Fracton T.O. d=3 space dim
No entanglement ⟷ ρ = Σ_i p_i ρ_i^A ⊗ ρ_i^B
Separable
E_of = 0 iff ρ is separable
E_N > 0 ⟶ ρ is not separable
E_N = 0 ⟶ ρ doesn't need to be separable
Tr ρ̂ =
Tr ρ^{T_A} =
Σ_i λ_i =
λ_i < 0
γ > 0

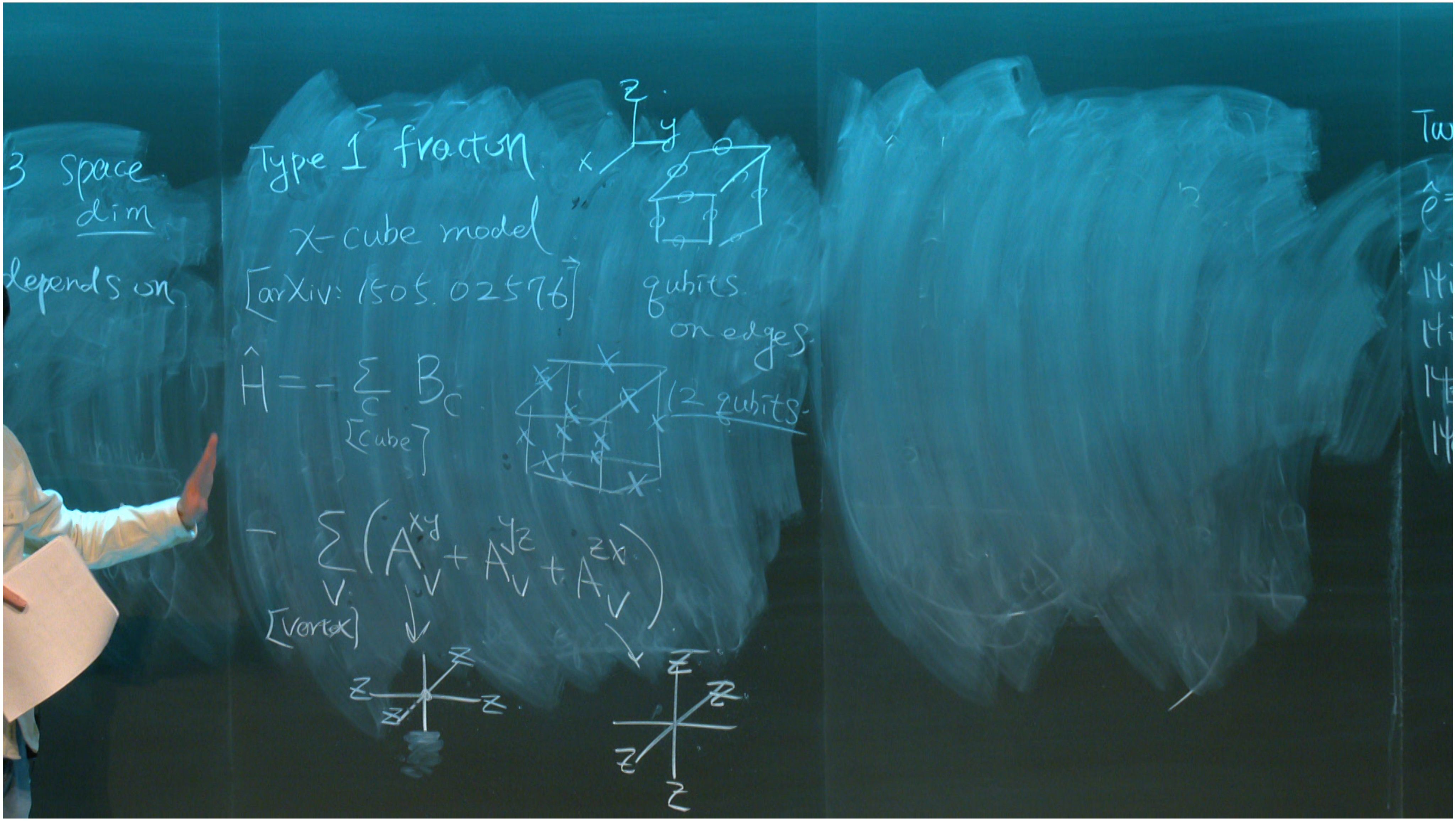


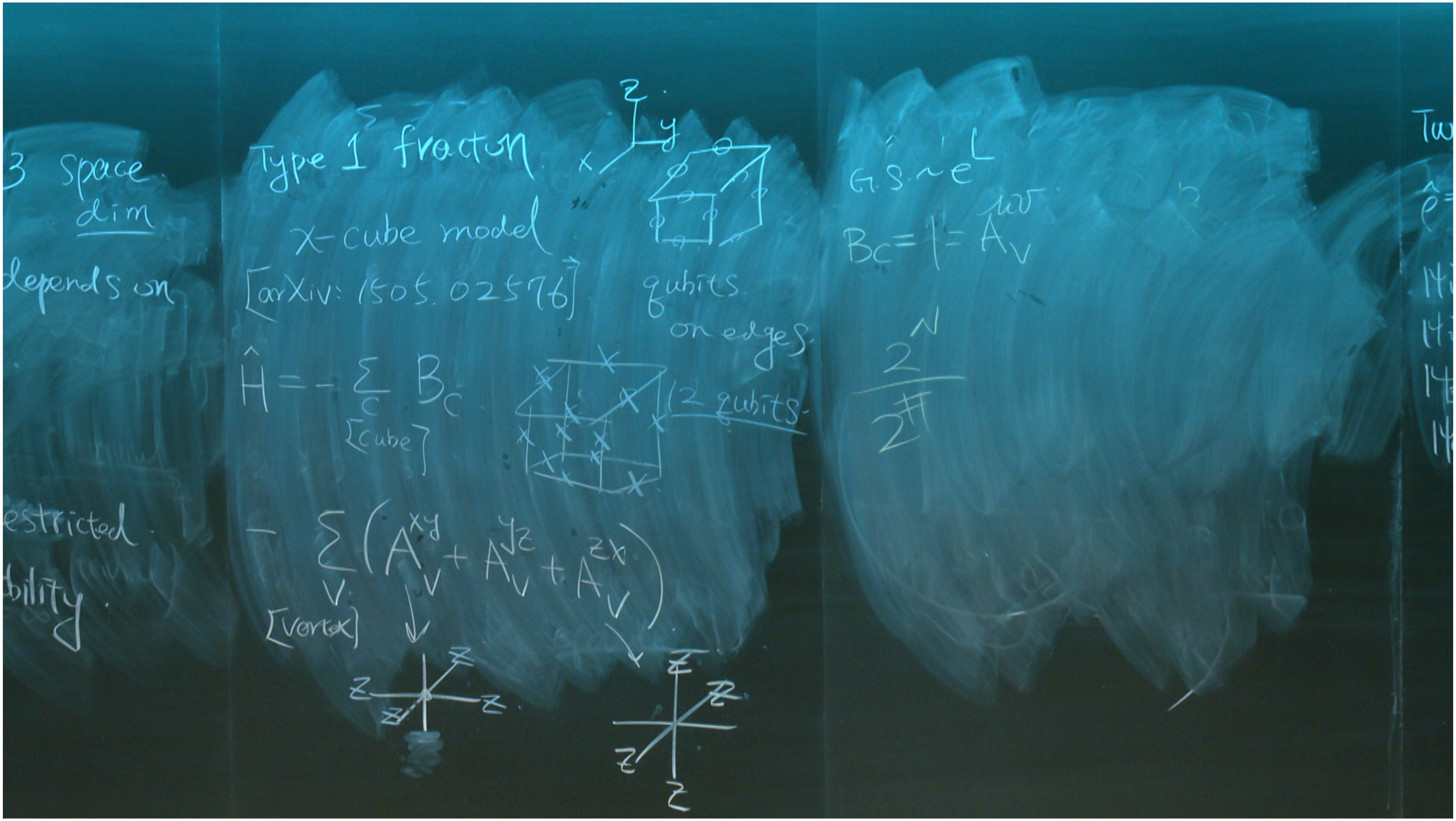
Fraction T. O. d=3 space dim
① # of g.s. not only depends on topology
~ e^L
No entanglement ⟷ ρ = Σ_i p_i ρ_i^A ⊗ ρ_i^B
[separable]
E_of = 0 iff ρ is separable
E_N > 0 ⟶ ρ is not separable
E_N = 0 ⟶ ρ doesn't need to be separable

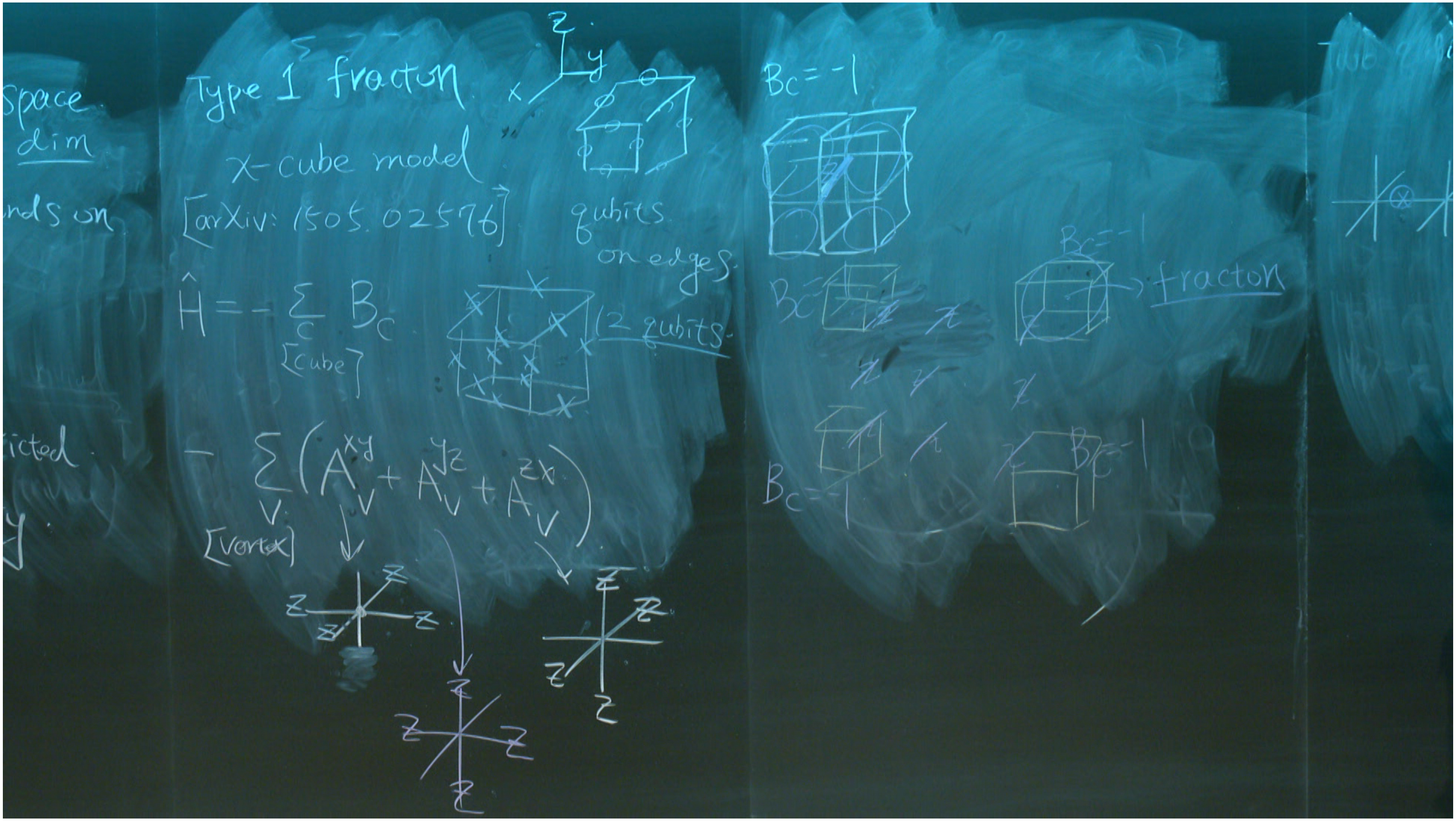

Fracton T.O. d=3 space dim
① # of g.s. not only depends on topology.
~ e^L
② Excitations with restricted mobility.

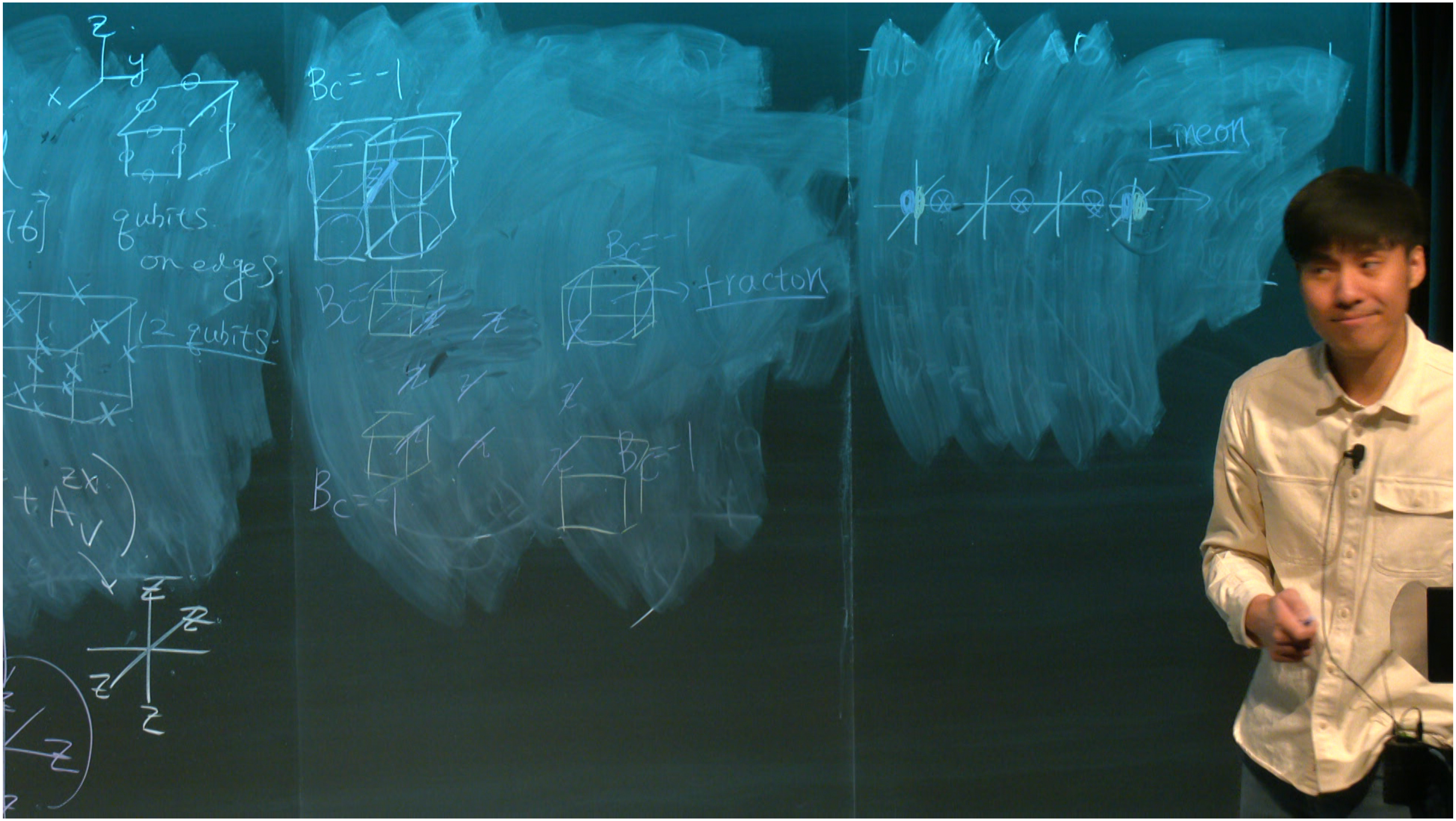

Type 1 fracton
X-cube model
[arXiv: 1505.02576]
qubits on edges
Ĥ = − Σ_c B_c
[cube]
Two qubi


3 space dim
depends on
estricted
bility
Type 1 fracton
X-cube model
[arXiv: 1505.02576]
qubits on edges
12 qubits
{cube}
[vertex]


3 space dim
depends on
Type 1 fracton
X-cube model
[arXiv: 1505.02576]
qubits on edges
12 qubits
[cube]
[vertex]


3 space dim
depends on
estricted
bility
Type 1 fracton
X-cube model
[arXiv: 1505.02576]
qubits on edges
12 qubits
[cube]
[vertex]
G.S. ~ e^L
B_C = 1 = A_V


space
dim
nds on
icted
Type 1 fracton
X-cube model
[arXiv: 1505.02576]
qubits on edges.
$\hat{H} = -\sum_{c} B_c$
[cube]
12 qubits
$- \sum_{v} (A_v^{xy} + A_v^{yz} + A_v^{zx})$
[vertex]
$B_c = -1$
fracton


qubits
on edges
12 qubits
$B_c = -1$
fracton
Lineon