Title: Topological quantum matter and quantum computing

Speakers: Peter Lu

Collection: Symmetries Graduate School 2023

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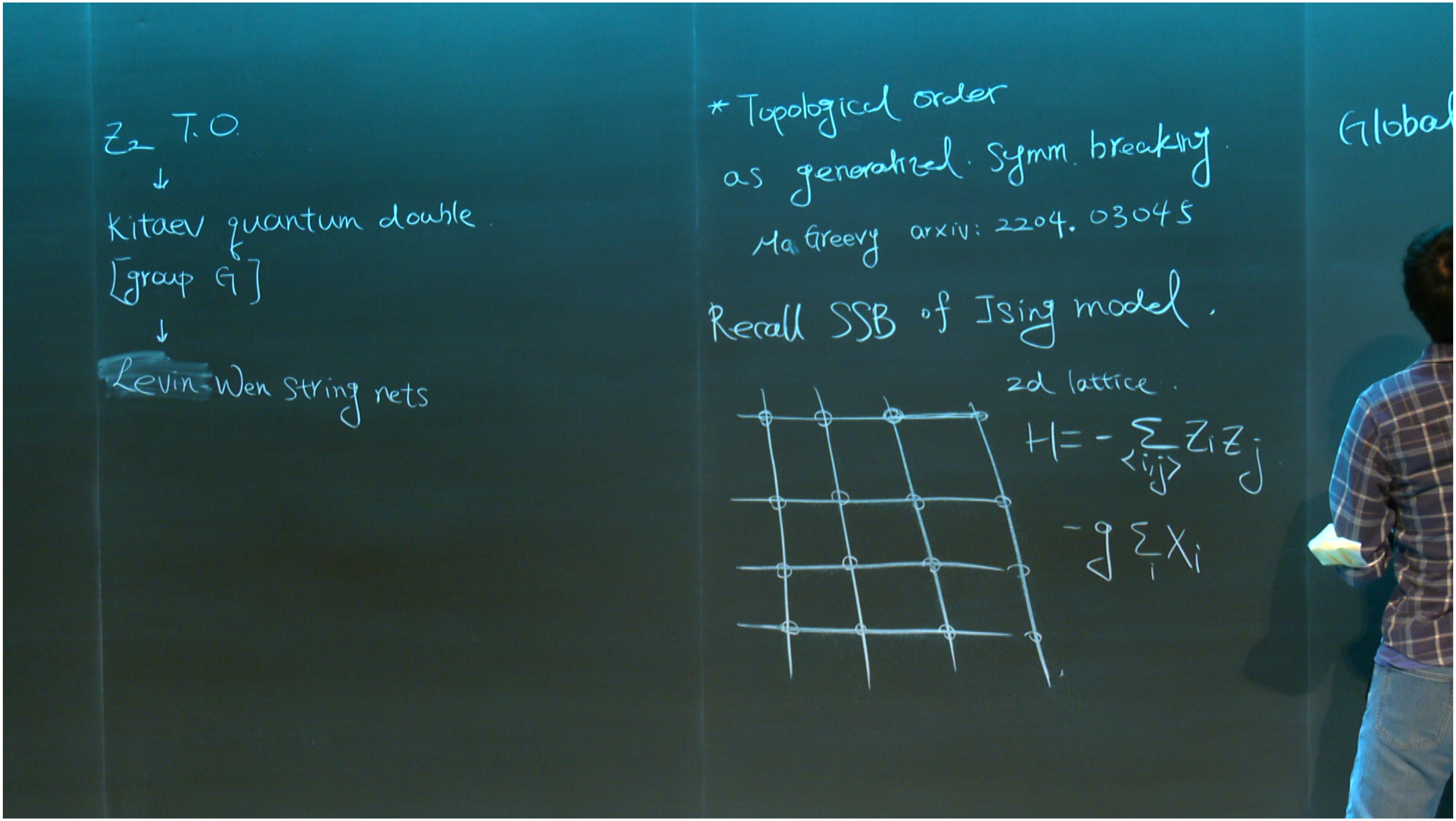
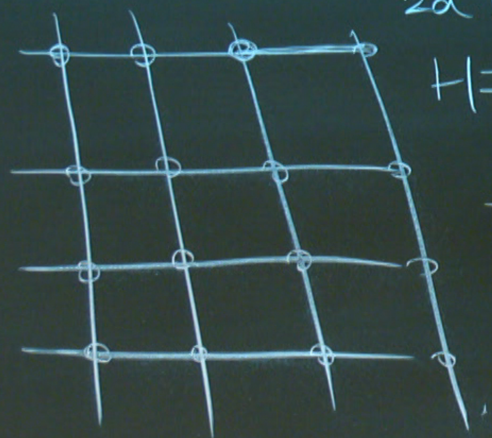

$\mathbb{Z}_2$ T.O.
Kitaev quantum double
[group G]
Levin-Wen String nets
* Topological order
as generalized. Symm. breaking.
Ma Greevy arxiv: 2204.03045
Recall SSB of Ising model.
2d lattice.
$H = -\sum_{\langle i,j \rangle} Z_i Z_j - g \sum_i X_i$
Global

* Topological order
as generalized symm breaking.
Ma, Greevy arxiv: 2204.03045

Recall SSB of Ising model.

2d lattice.



$$H = -\sum_{\langle ij \rangle} Z_i Z_j - g \sum_i X_i$$

Global Z_2 symm $\prod_i X_i$

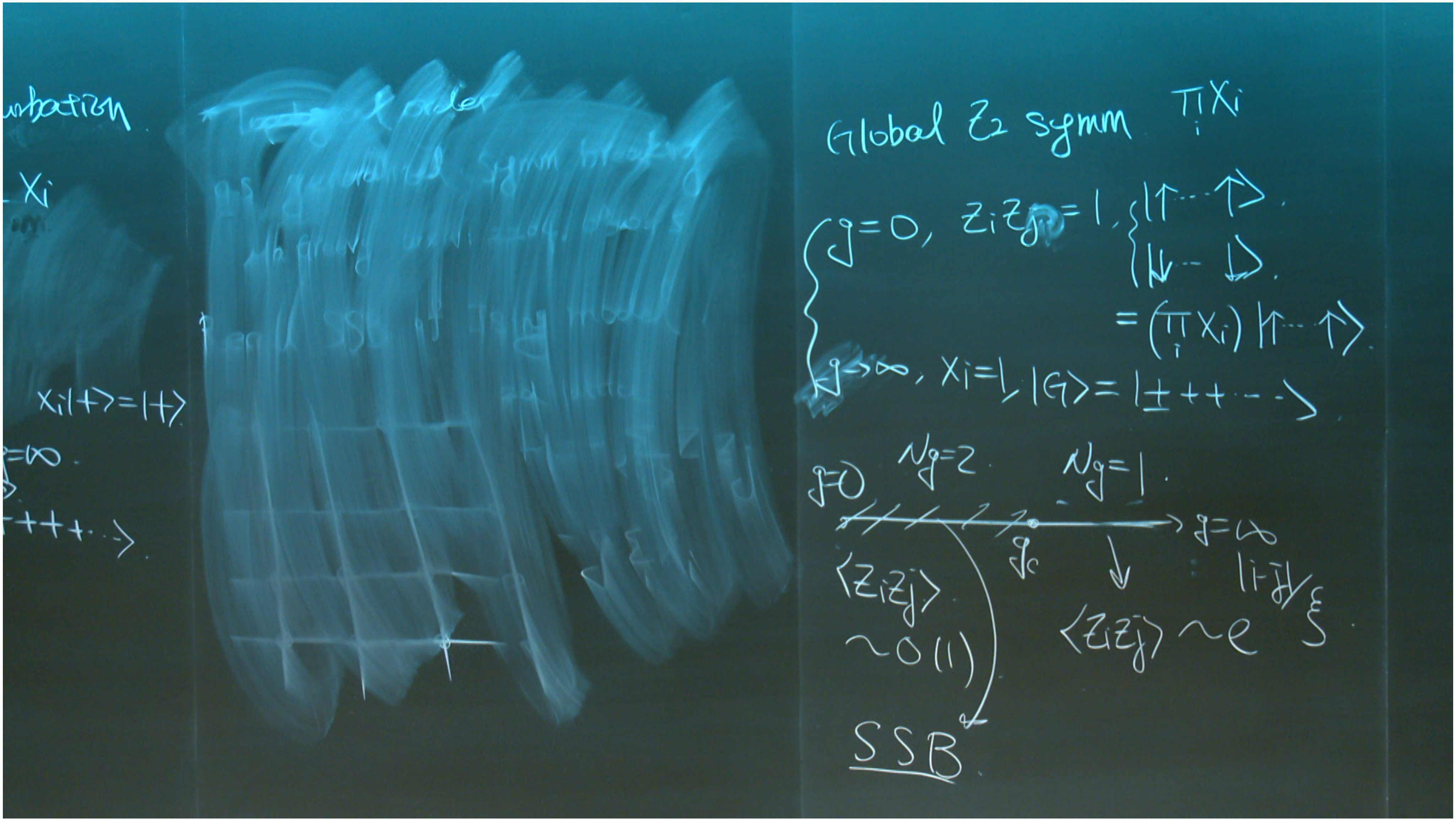
$$\begin{cases} g=0, \ Z_i Z_j = 1, & \begin{cases} |\uparrow \cdots \uparrow\rangle. \\ |\downarrow \cdots \downarrow\rangle. \end{cases} \\ & = (\prod_i X_i) |\uparrow \cdots \uparrow\rangle. \\ g \to \infty, \ X_i = 1, & |G\rangle = |+ + + \cdots\rangle. \end{cases}$$

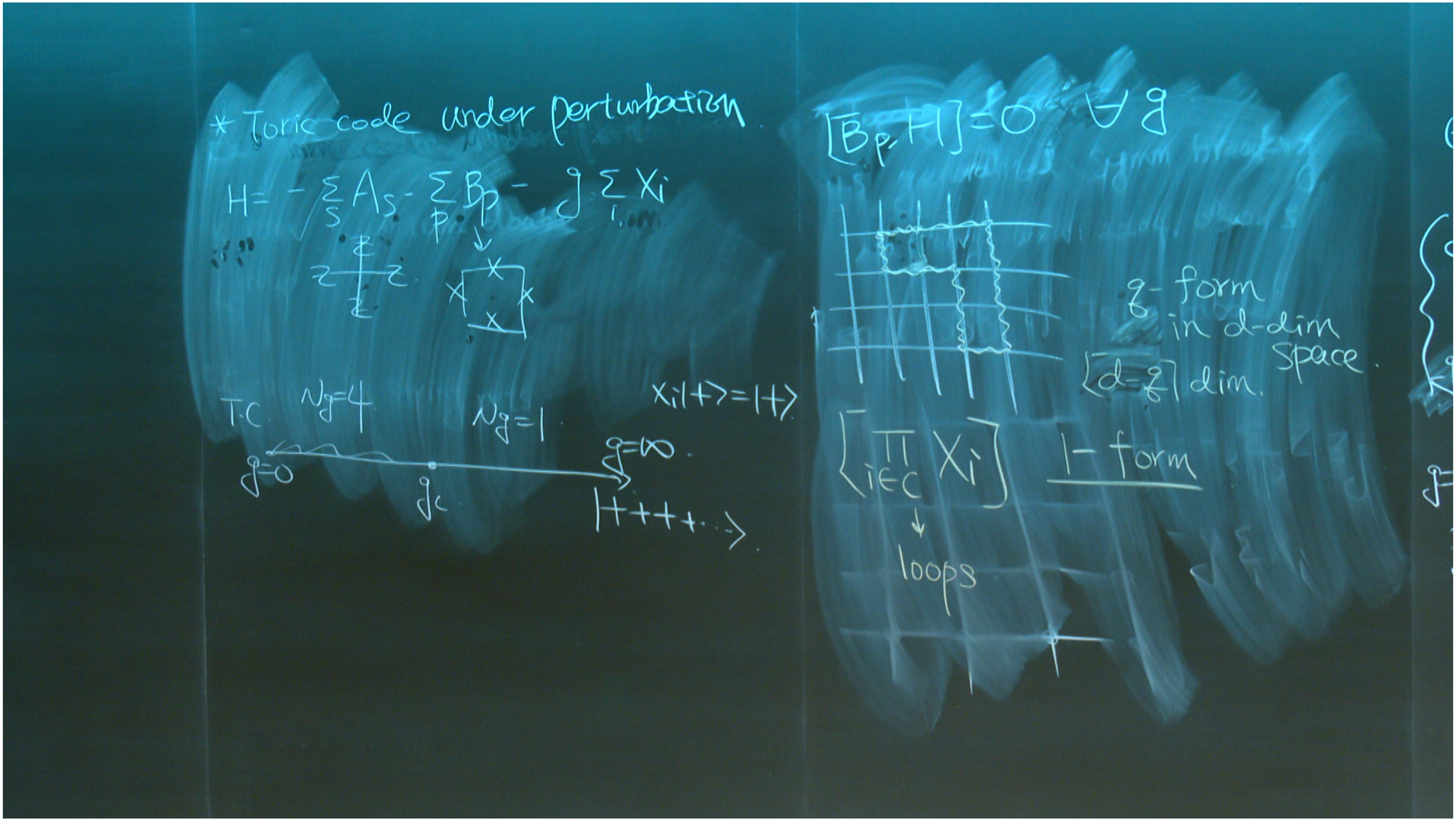
$g=0$ $\quad N_g = 2. \quad N_g = 1.$

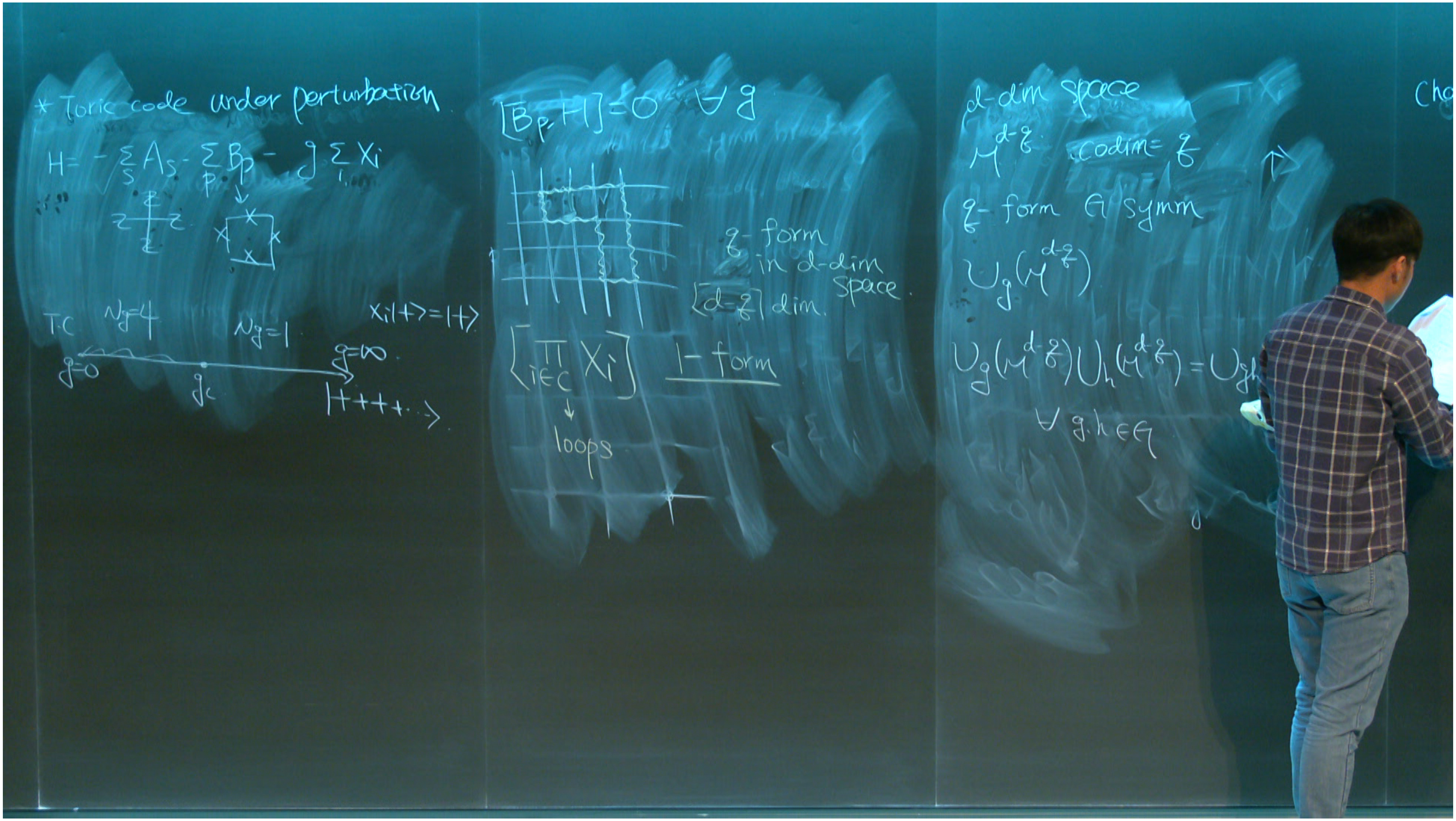
$\longrightarrow g = \infty$, g_c

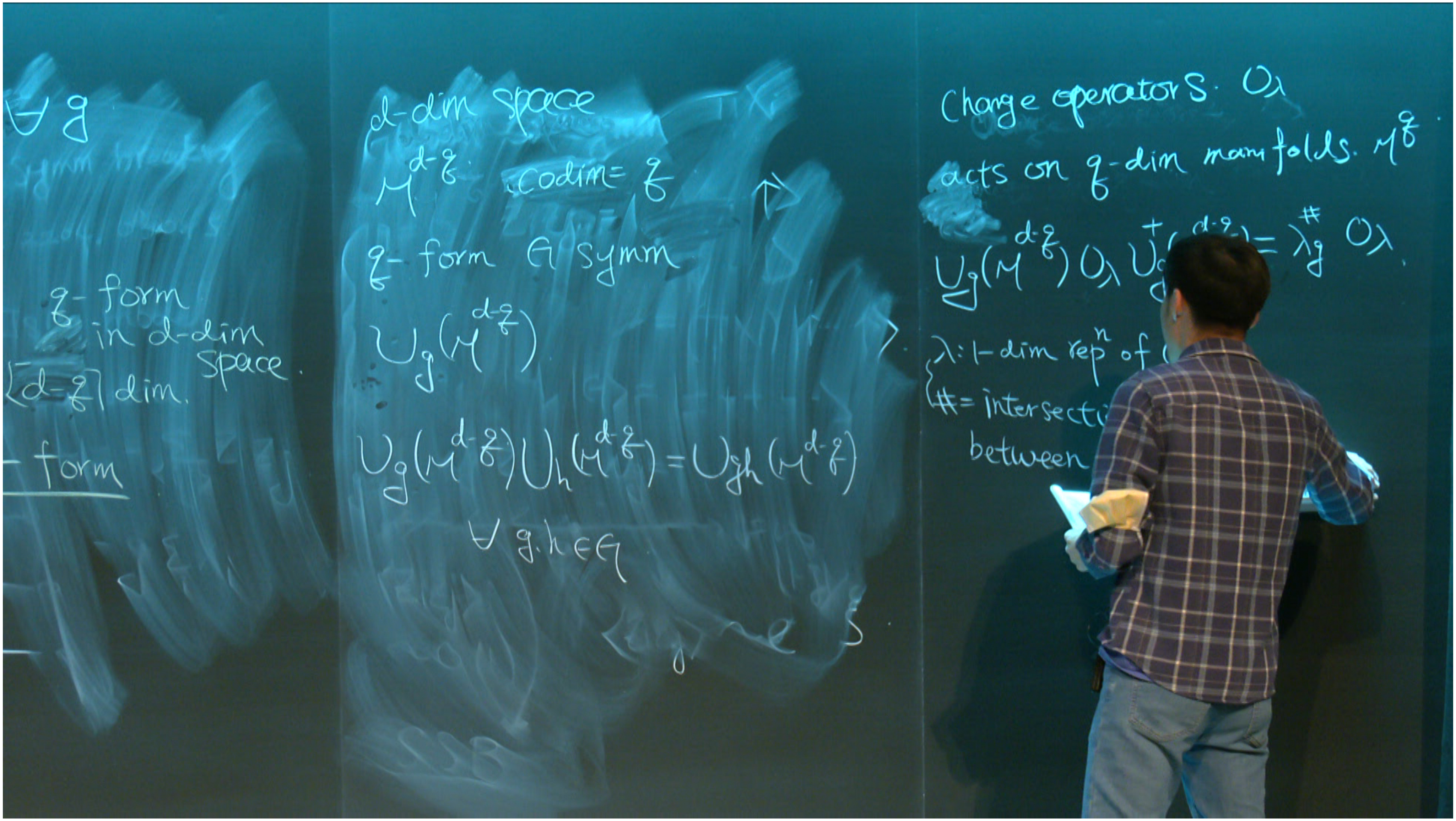
$\langle Z_i Z_j \rangle \sim O(1)$

$\langle Z_i Z_j \rangle \sim e^{-|i-j|/\xi}$

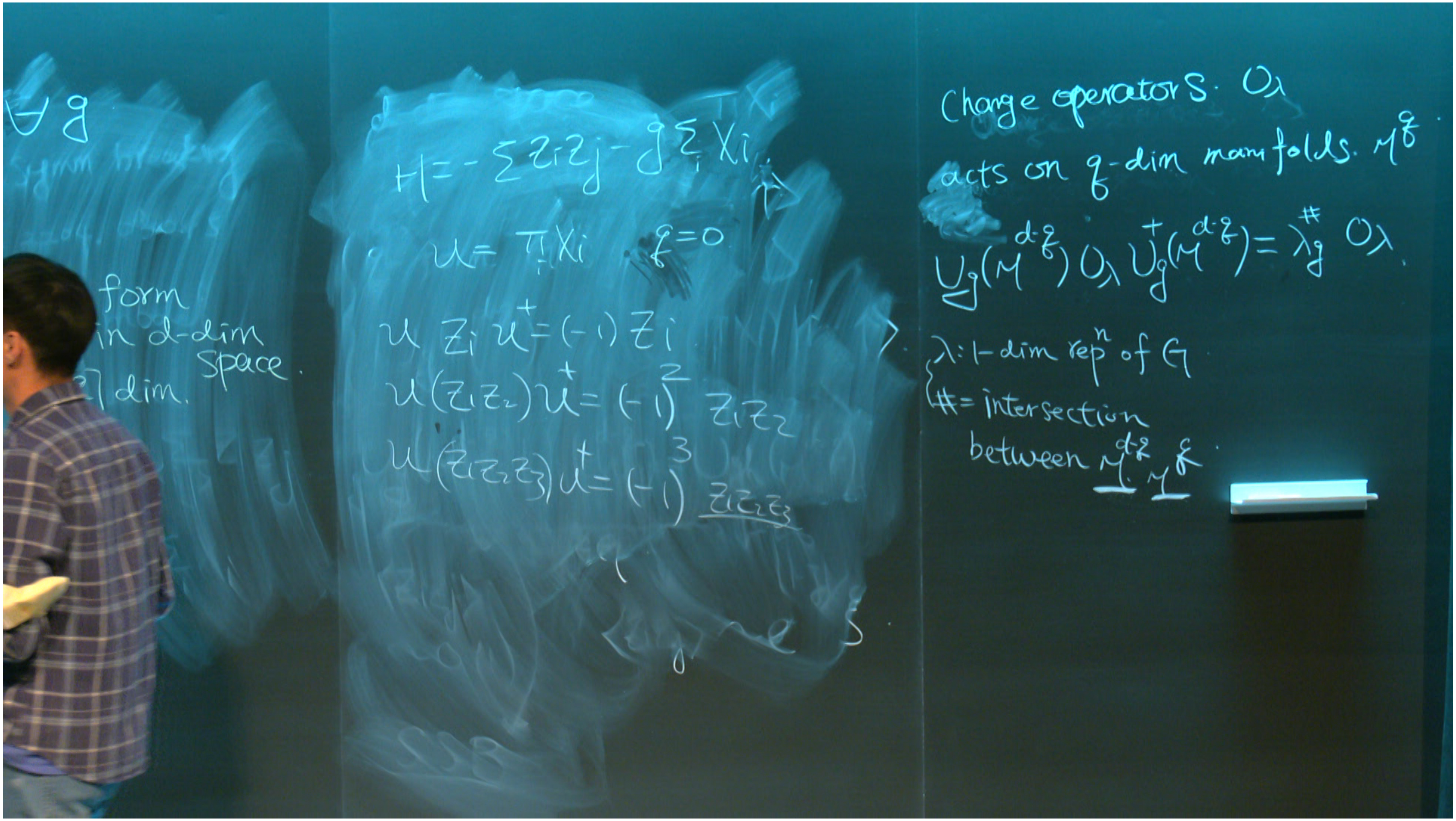

urbation
X_i
X_i|+>=|+>
g=∞
|+++…>
Global Z_2 symm ∏_i X_i
g=0, Z_i Z_j = 1, |↑…↑>
|↓…↓> = (∏_i X_i)|↑…↑>
g→∞, X_i=1, |G> = |+++…>
g=0 N_g=2 N_g=1
g_c g=∞
<Z_i Z_j> ~ O(1)
SSB
<Z_i Z_j> ~ e^{-|i-j|/ξ}

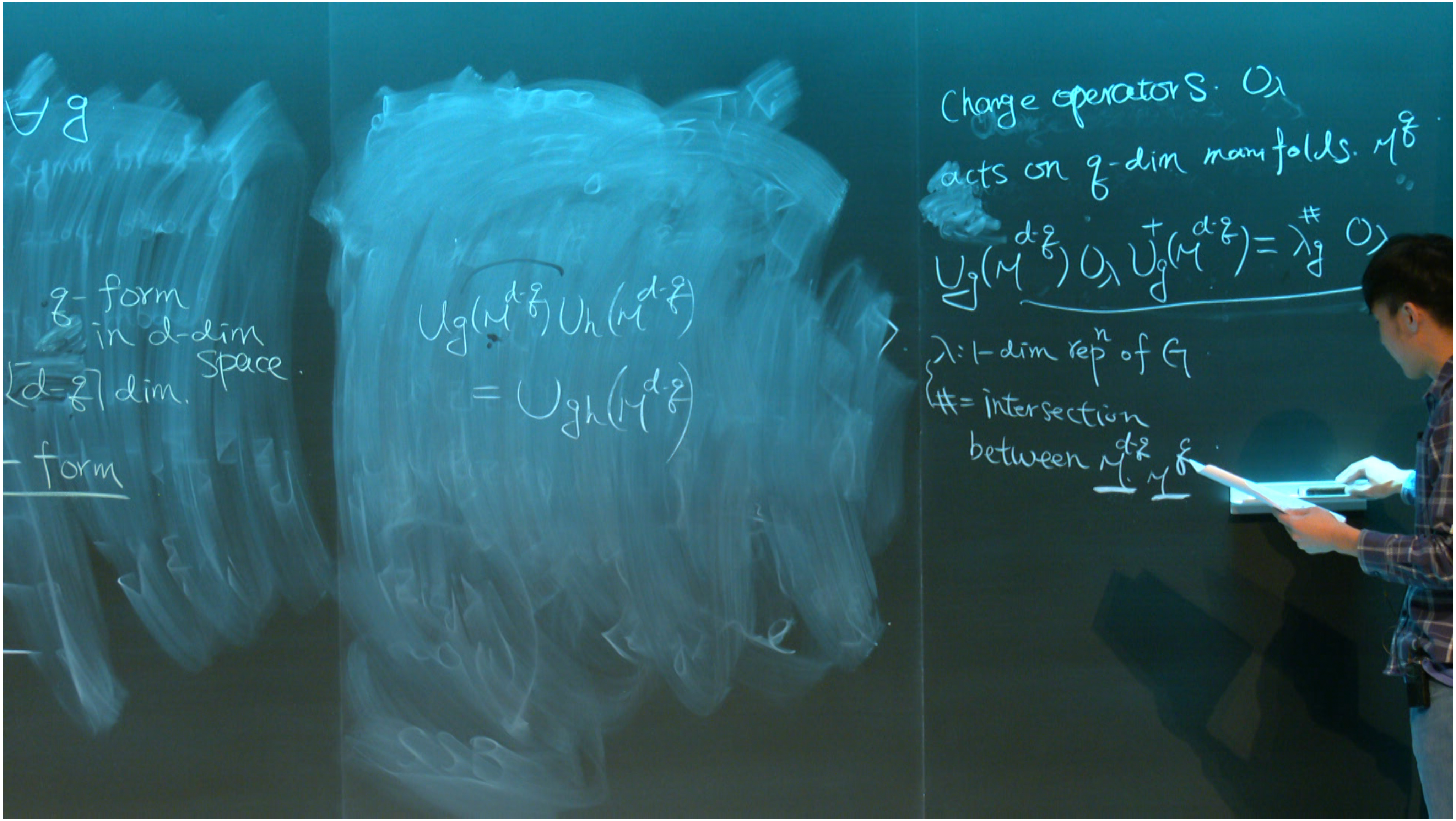

* Toric code under perturbation
H = - Σ A_s - Σ B_p - g Σ X_i
T.C
N_g = 4
N_g = 1
X_i|+> = |+>
g = 0
g_c
g = ∞
|++++...>
[B_p, H] = 0 ∀ g
g - form in d-dim space
[d-g] dim.
[Π_{i∈C} X_i]
loops
1 - form

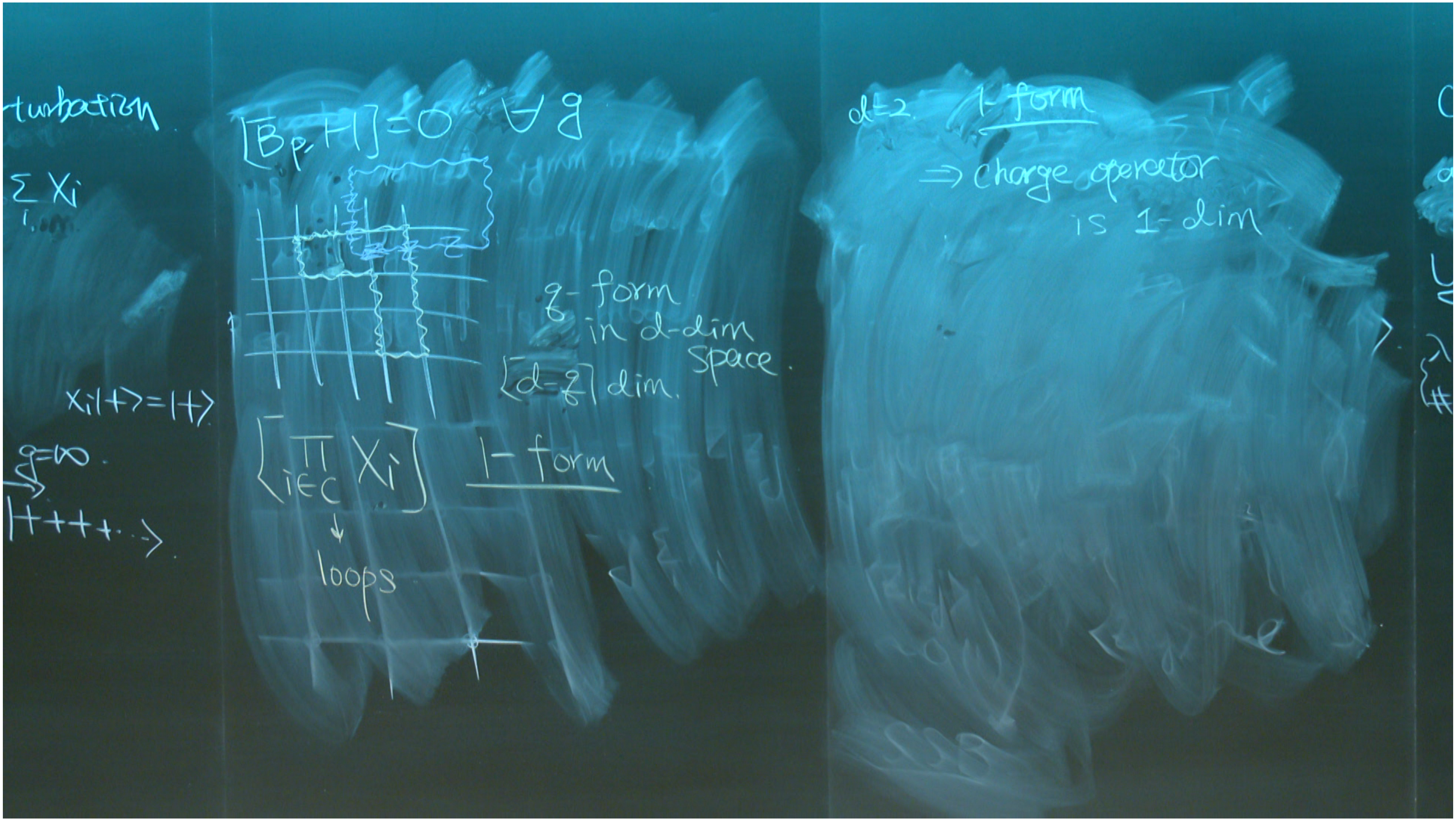

* Toric code under perturbation
H = - Σ A_s - Σ B_p - g Σ X_i
T.C N_g=4 N_g=1
X_i|+>=|+>
g=0 g_c g=∞
|++++...>
[B_p, H] = 0 ∀ g
q-form in d-dim space
(d-q) dim.
Π X_i
i∈C
1-form
loops
d-dim space
M^{d-q} codim = q
q-form G symm
U_g(M^{d-q})
U_g(M^{d-q}) U_h(M^{d-q}) = U_gh
∀ g,h ∈ G



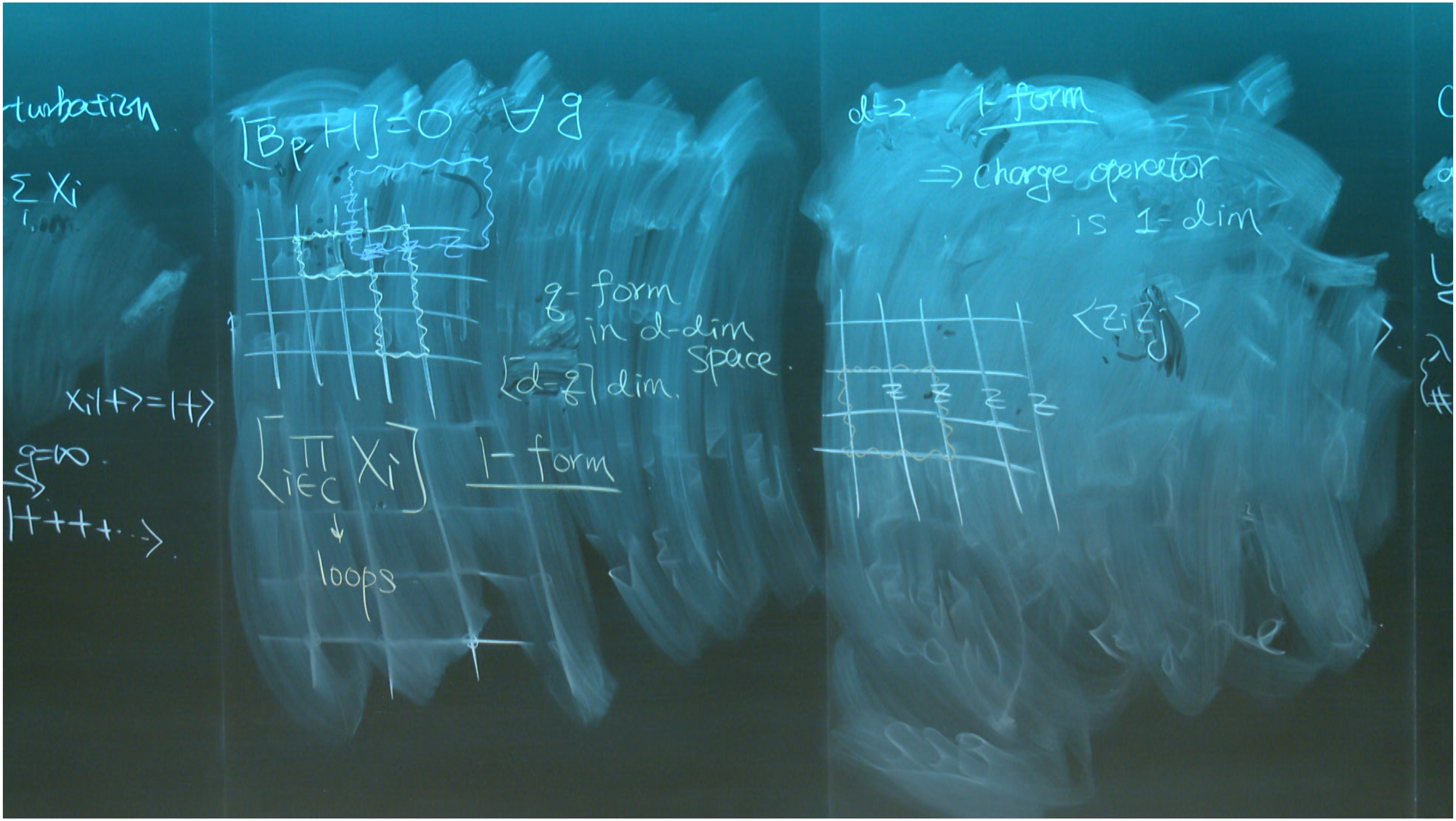
d-dim space
M^{d-q} codim = q
q-form G symm
U_g(M^{d-q})
U_g(M^{d-q}) U_h(M^{d-q}) = U_{gh}(M^{d-q})
∀ g.h ∈ G
q-form
in d-dim space
(d-q) dim.
form
Charge operators: O_λ
acts on q-dim manifolds. M^q
U_g(M^{d-q}) O_λ U_g^†(M^{d-q}) = λ_g^# O_λ
λ: 1-dim rep^n of
#= intersecti
between

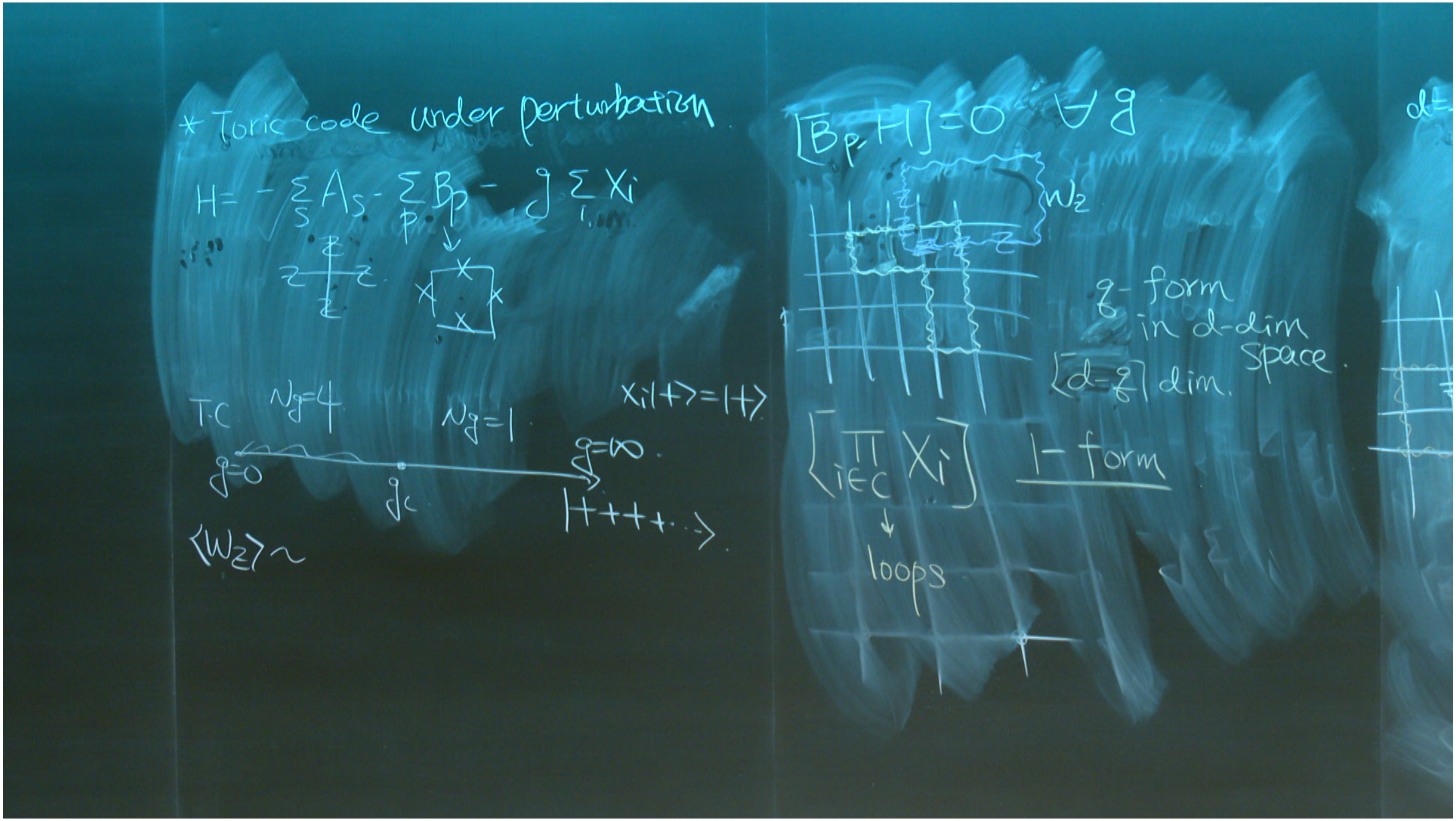

∀ g
form
in d-dim
space.
] dim.
$H = -\sum Z_i Z_j - g \sum_i X_i$
$U = \prod_i X_i$ $g=0$
$U Z_i U^\dagger = (-1) Z_i$
$U(Z_1 Z_2) U^\dagger = (-1)^2 Z_1 Z_2$
$U(Z_1 Z_2 Z_3) U^\dagger = (-1)^3 Z_1 Z_2 Z_3$
Charge operators: O_λ
acts on q-dim manifolds. M^q
$U_g(M^{d-q})\, O_\lambda\, U_g^\dagger(M^{d-q}) = \lambda_g^{\#}\, O_\lambda$.
λ: 1-dim rep^n of G
= intersection between M^{d-q}, M^q

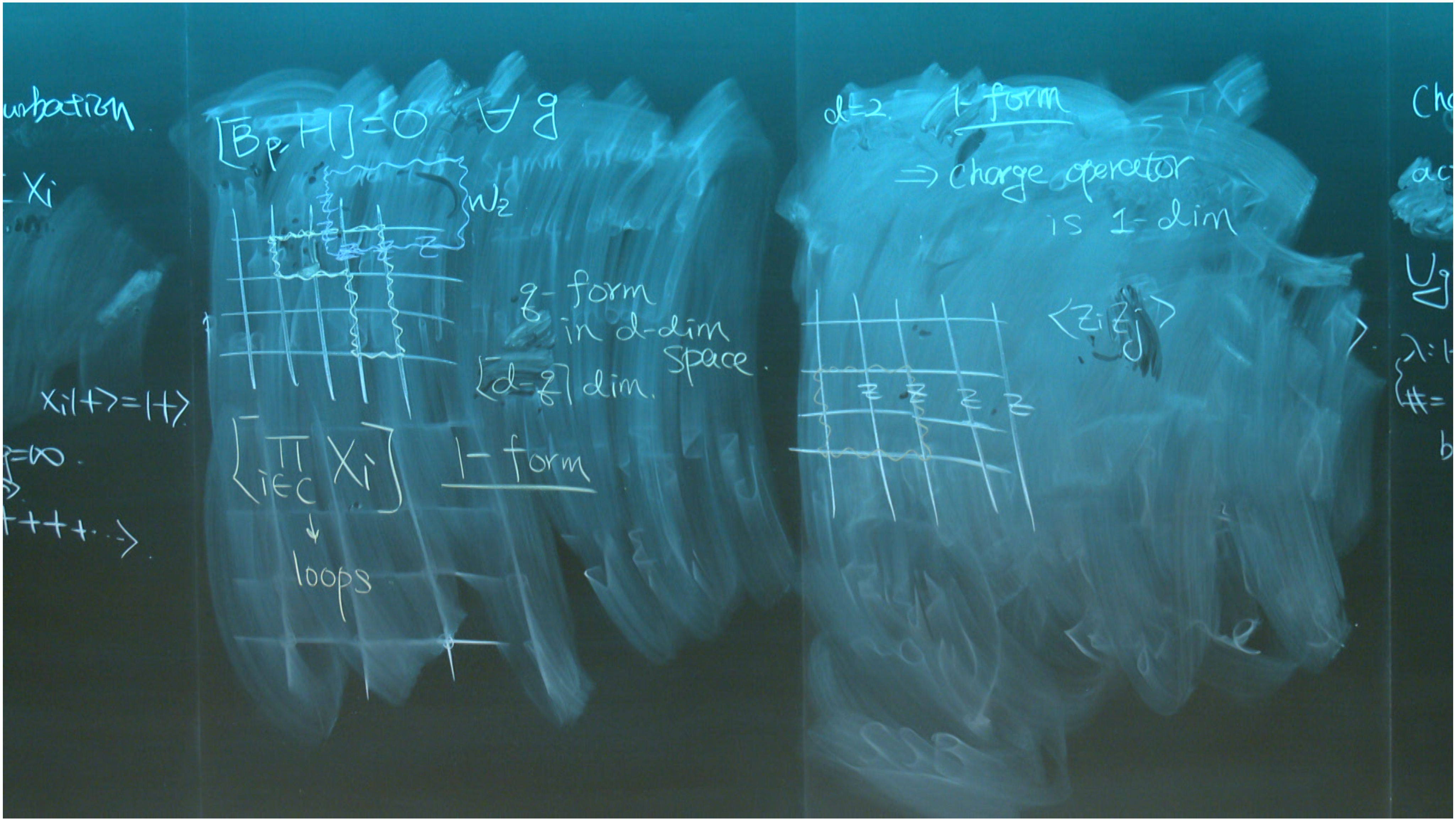

$\forall g$
q- form in d-dim space
$(d-q)$ dim.
- form
$U_g(M^{d-q})\, U_h(M^{d-q}) = U_{gh}(M^{d-q})$
Charge operators: O_λ
acts on q-dim manifolds. M^q
$U_g(M^{d-q})\, O_\lambda\, U_g^\dagger(M^{d-q}) = \lambda_g^{\#}\, O_\lambda$
λ: 1-dim rep^n of G
$\#$ = intersection between M^{d-q}, M^q



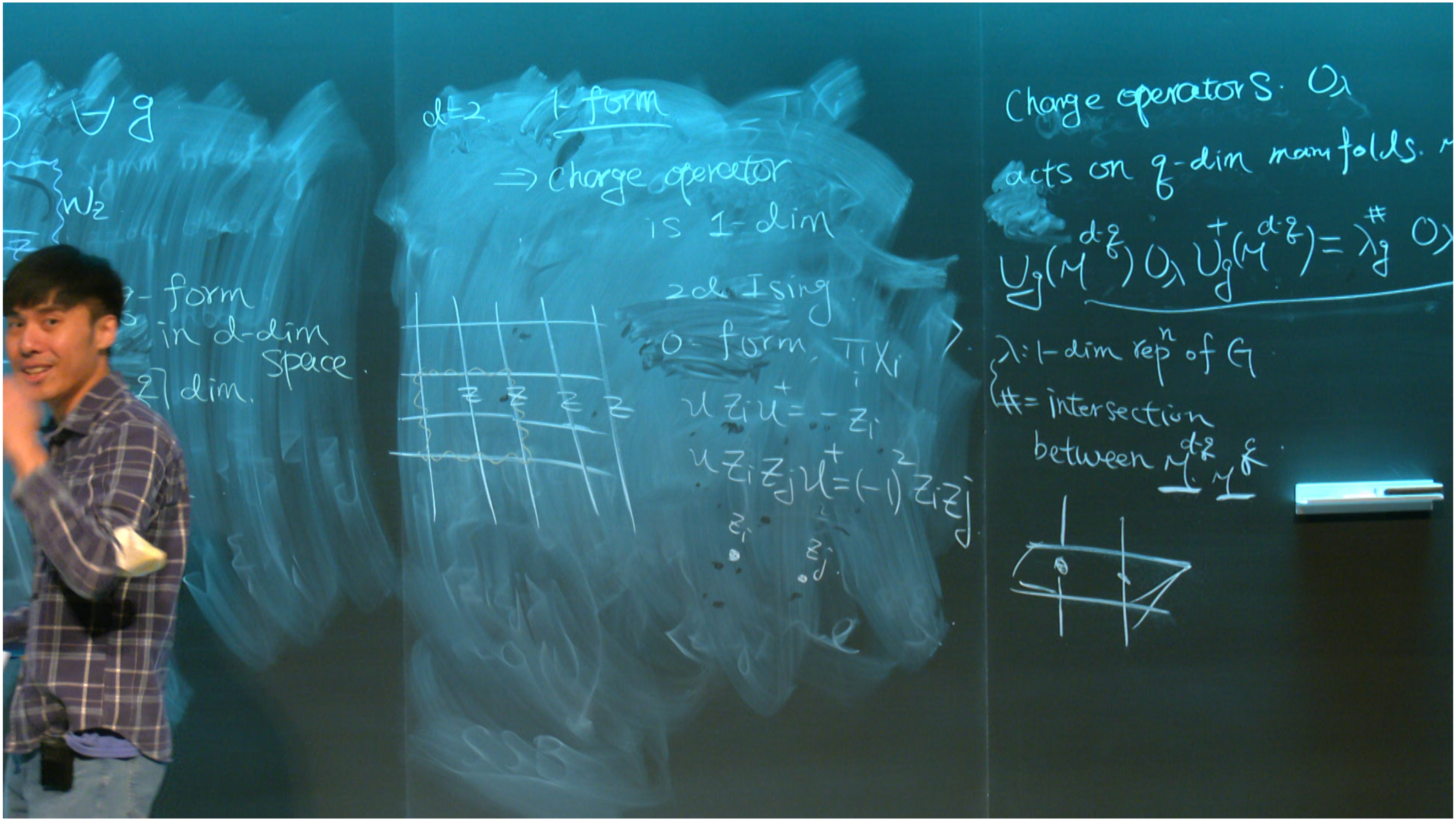
turbation
Σ X_i
X_i|+>=|+>
g=∞
|+++...>
[B_p, H]=0 ∀ g
g- form
in d-dim
space
(d-g) dim
∏_{i∈C} X_i
1- form
loops
d=2
1-form
⇒ charge operator
is 1-dim


turbation
Σ Xi
Xi|+>=|+>
g=∞
|++++...>
[Bp, H]=0 ∀ g
[Π i∈C Xi]
loops
g-form
in d-dim
space.
[d-g] dim.
1-form
d=2. 1-form
⇒ charge operator
is 1-dim
<Zi Zj>
Z Z Z Z


* Toric code under perturbation
H = - Σ A_s - Σ B_p - g Σ X_i
T.C
N_g = 4
N_g = 1
X_i |+> = |+>
g = 0
g_c
g = ∞
|++++...>
<W_z> ~
[B_p, H] = 0 ∀ g
W_z
q-form in d-dim space
[d-q] dim.
[∏ X_i]
1-form
loops



$[B_p, H] = 0 \;\forall g$
W_2
g-form
in d-dim
space.
[d-g] dim.
$\left[\prod_{i\in C} X_i\right]$
loops
1-form
$X_i|+\rangle = |+\rangle$
d=2. 1-form
⇒ charge operator
is 1-dim
$\langle Z_i Z_j\rangle$



d=2. 1-form
⇒ charge operator
is 1-dim
2d Ising
0-form $\prod_i X_i$
$u z_i u^+ = - z_i$
$u z_i z_j u^+ = (-1)^2 z_i z_j$
Charge operators. O_λ
acts on q-dim manifolds.
$U_g(M^{d-q}) O_\lambda U_g^+(M^{d-q}) = \lambda_g^\# O_\lambda$
λ: 1-dim rep^n of G
$\#$= intersection
between M^{d-q}, M^q

* Toric code under perturbation

$$H = -\sum_s A_s - \sum_p B_p - g\sum_i X_i$$

T.C. $N_g = 4$ $N_g = 1$

$X_i|+\rangle = |+\rangle$

$g=0$ g_c $g=\infty$

$|+++\cdots\rangle$

$\langle W_Z\rangle \sim e^{-\alpha|\partial A|}$

perimeter law

$W_Z \sim e^{-\beta|A|}$

Area law

$[B_p, H] = 0 \quad \forall g$

W_Z

$\left[\prod_{i\in C} X_i\right]$ 1-form

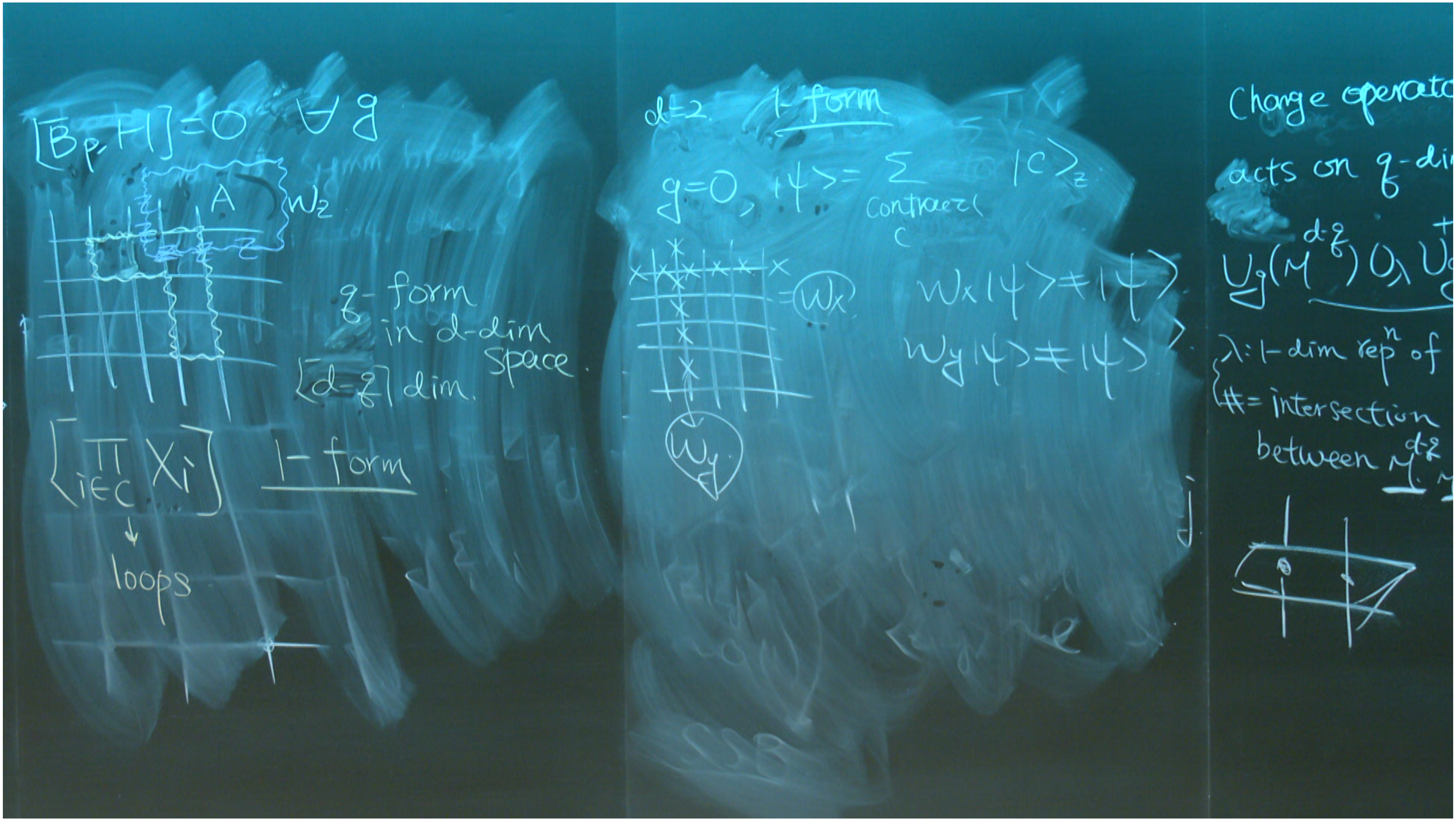
loops

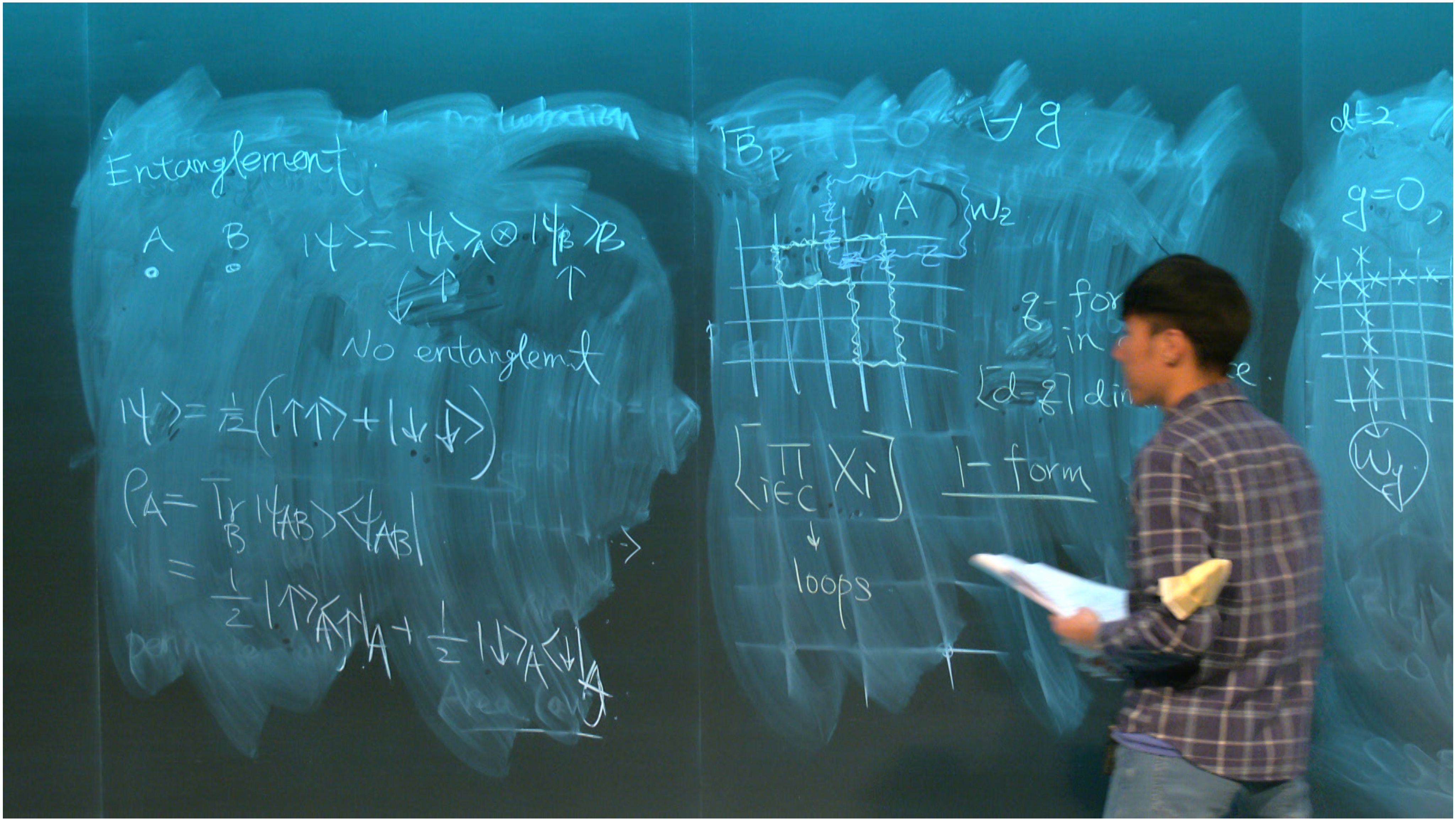
q-form in d-dim space

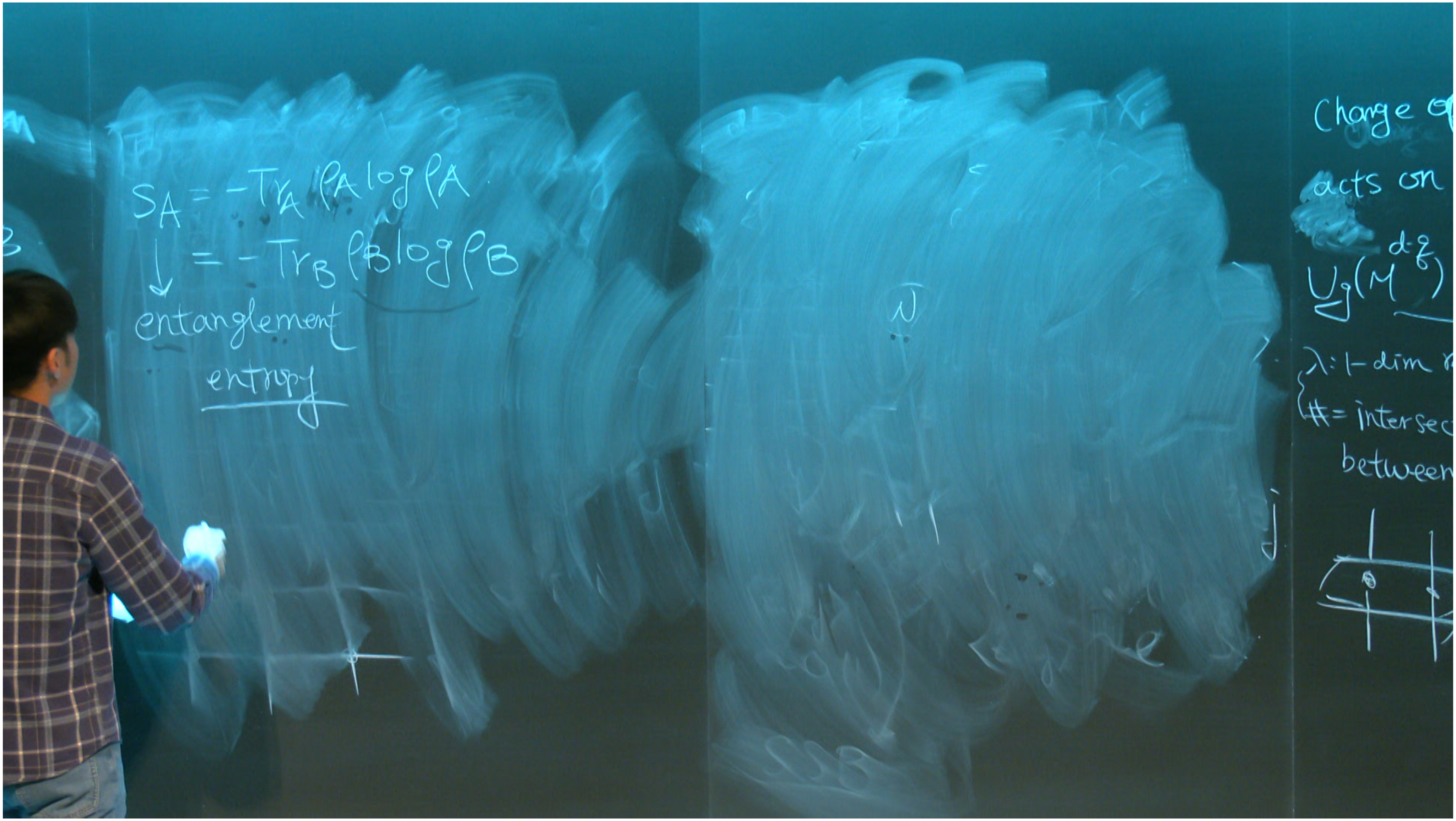
$[d-q]$ dim.

$d=2$

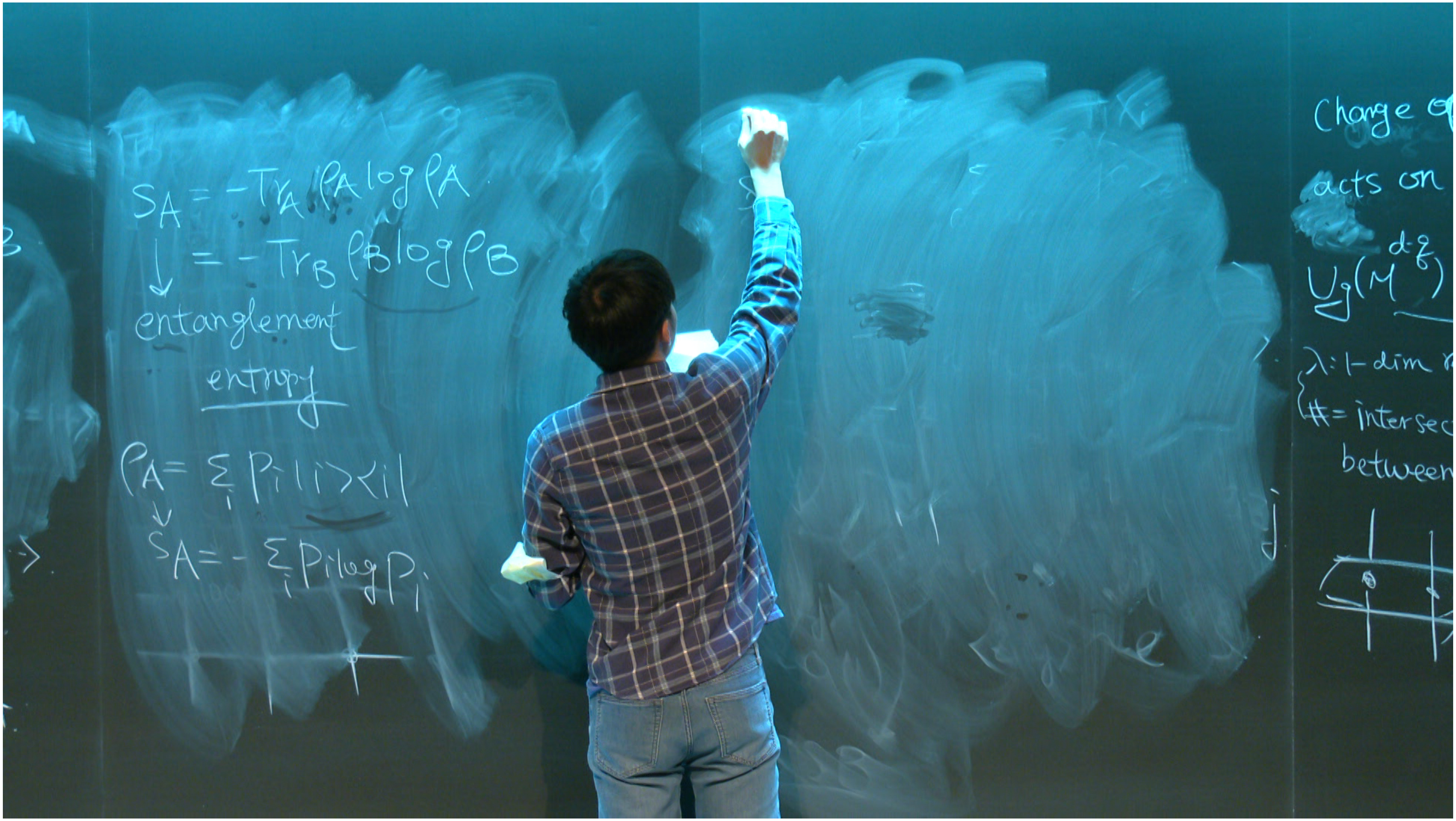
$\Rightarrow$

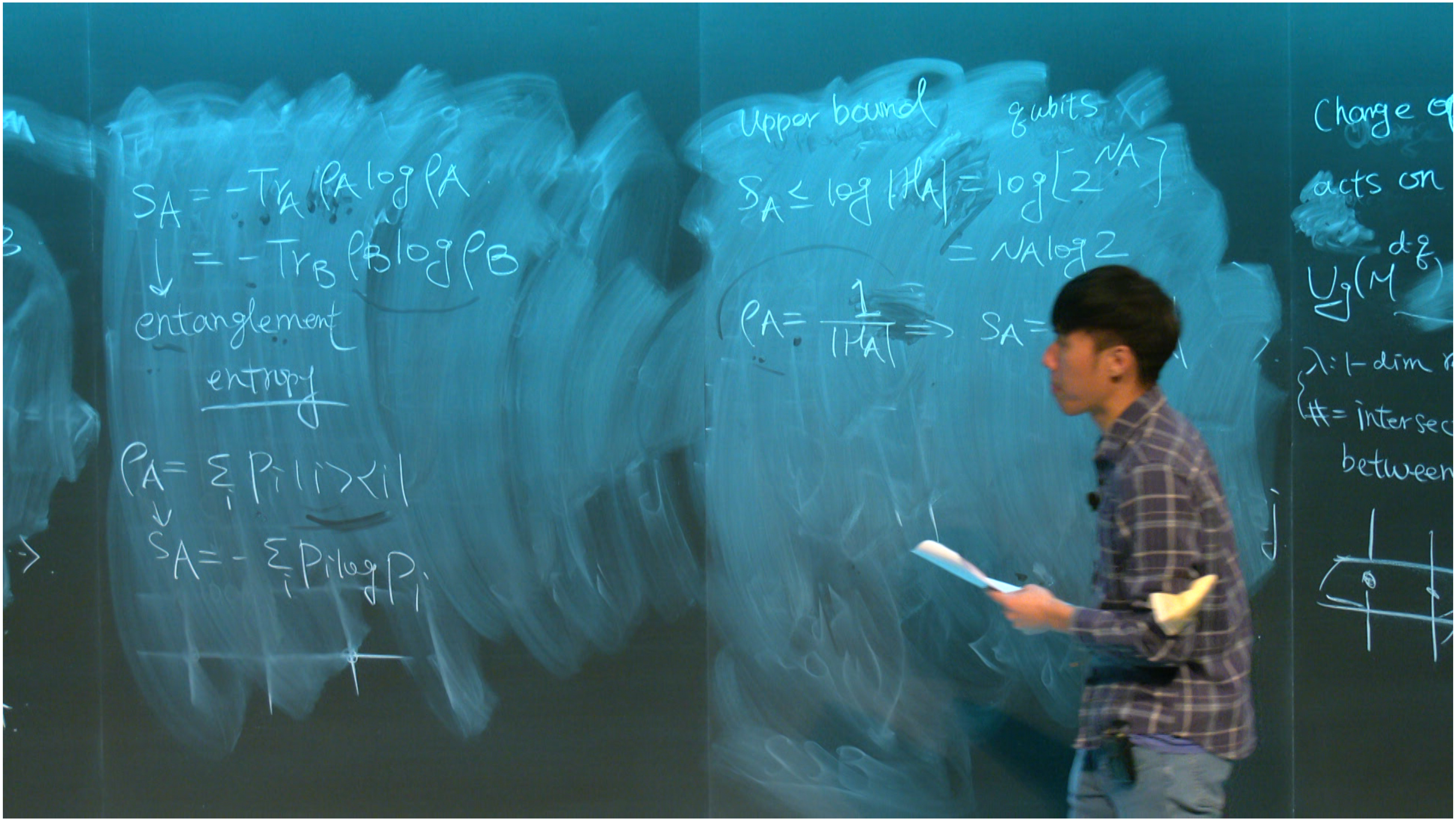

[B_p, H] = 0 ∀ g
A
W_z
q-form
in d-dim
space.
[d-q] dim.
[∏_{i∈C} X_i]
1-form
loops
d=2
1-form
g=0, |ψ> = Σ_{contract C} |c>_z
W_x
W_x|ψ> = |ψ>
W_y|ψ> = |ψ>
W_y
Charge operato
acts on q-di
U_g(M^{d-q}) O_λ U
λ: 1-dim rep^n of
#= intersection
between M^{d-q}


Entanglement
A B
No entanglement
loops
1- form
d=2
g=0

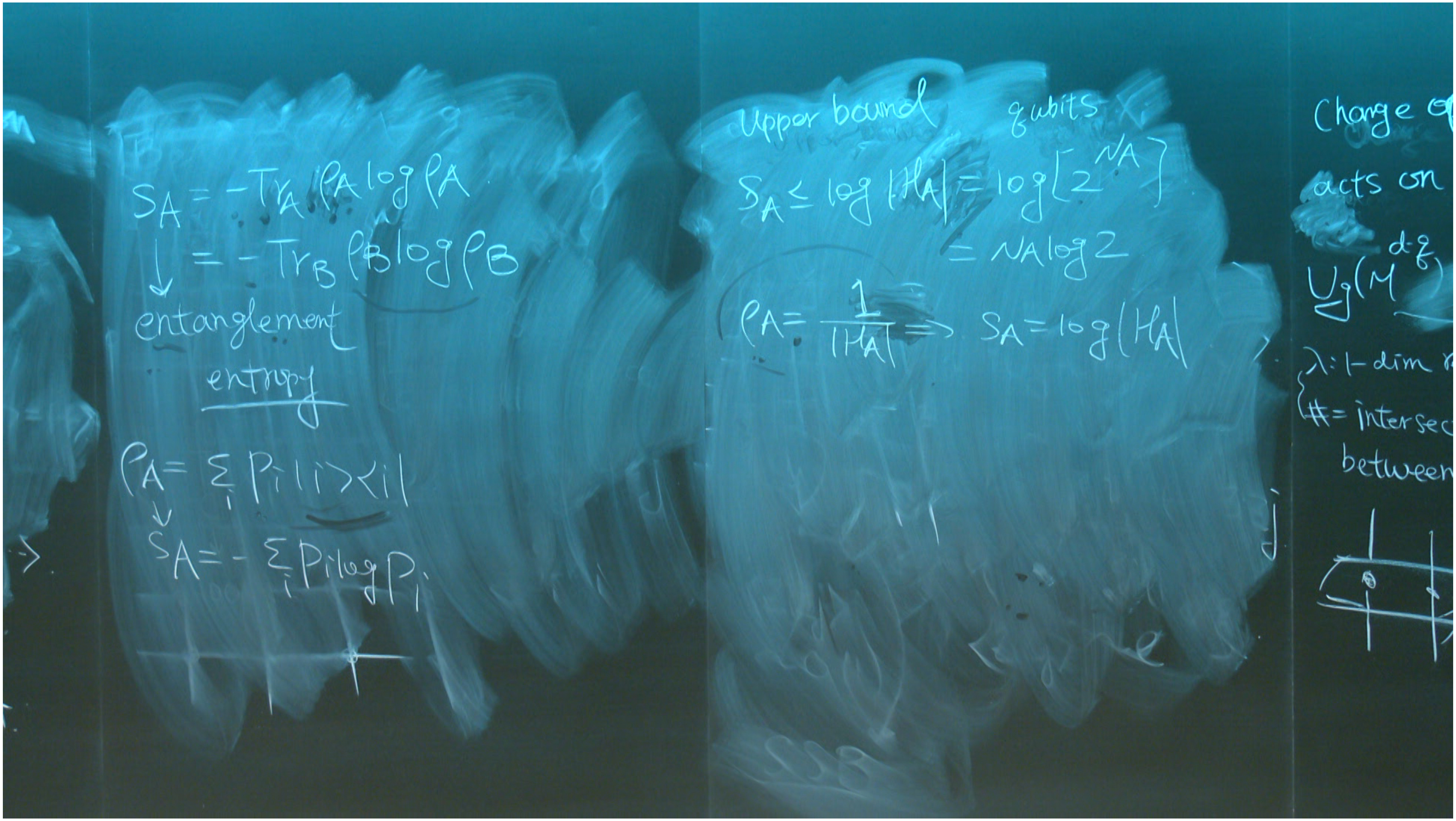


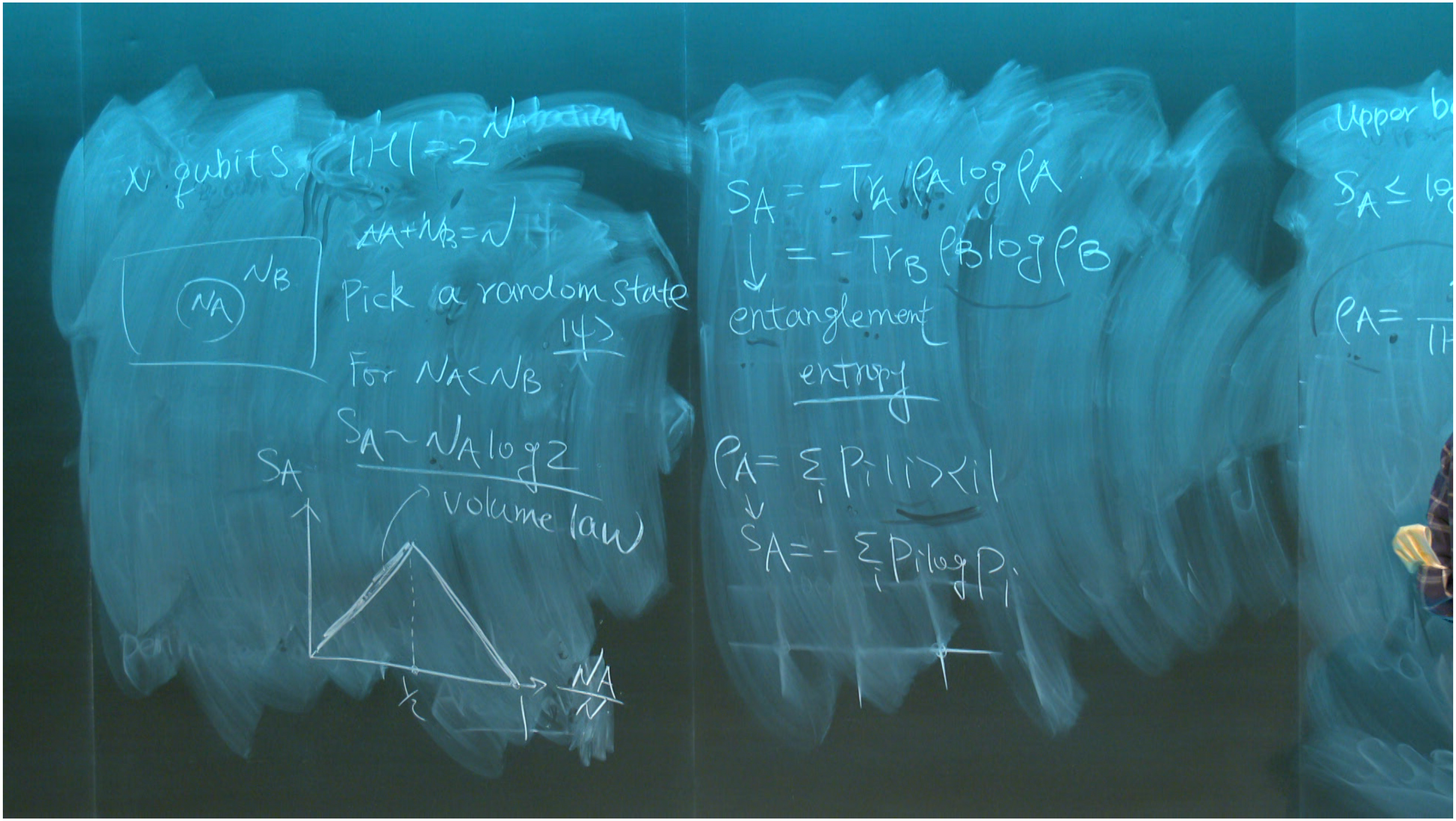
S_A = -Tr_A ρ_A log ρ_A
= -Tr_B ρ_B log ρ_B
entanglement
entropy
acts on

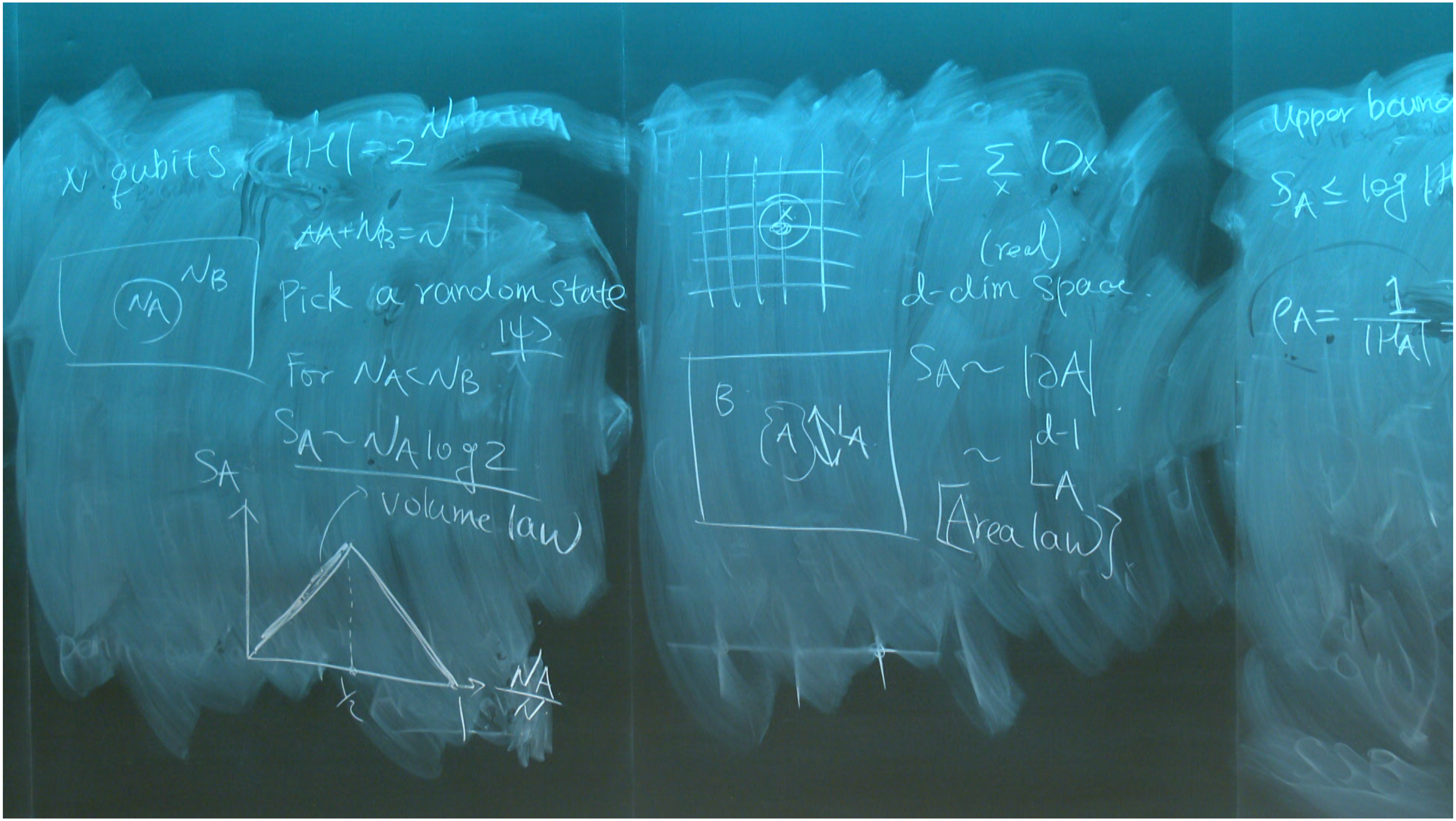

SA = -TrA ρA log ρA
= -TrB ρB log ρB
entanglement
entropy
ρA = Σi Pi |i⟩⟨i|
SA = -Σi Pi log Pi
Change of
acts on
λ: 1-dim
#= intersec
between

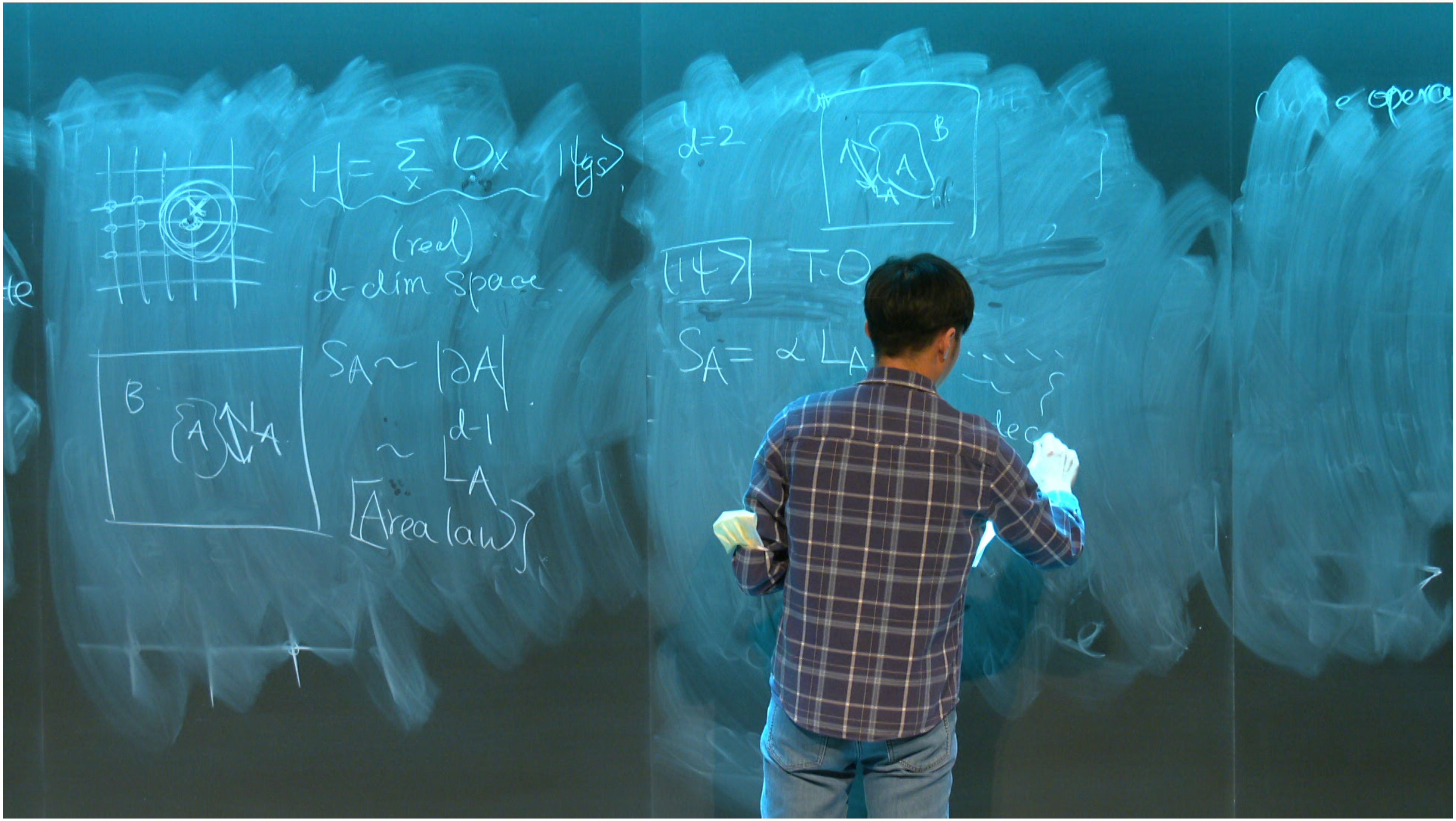


$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$
$= -\mathrm{Tr}_B \rho_B \log \rho_B$
entanglement
entropy
$\rho_A = \sum_i p_i |i\rangle\langle i|$
$S_A = -\sum_i p_i \log p_i$
Upper bound
qubits
$S_A \le \log |\mathcal{H}_A| = \log [2^{N_A}]$
$= N_A \log 2$
$\rho_A = \frac{1}{|\mathcal{H}_A|} \Rightarrow S_A =$
Charge o
acts on
λ: 1-dim
#= intersec
between

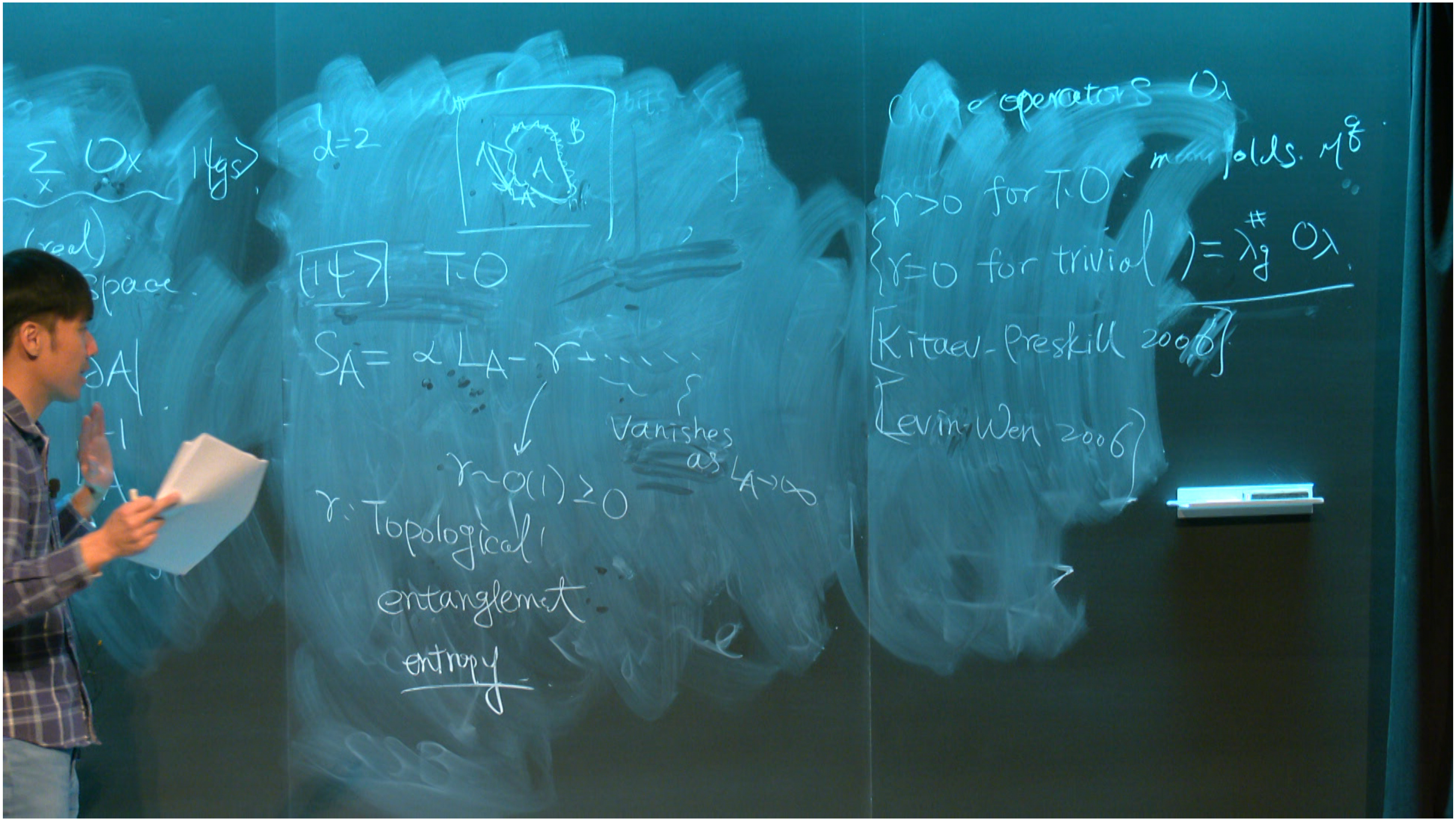

Upper bound
qubits
S_A = -Tr_A ρ_A log ρ_A
= -Tr_B ρ_B log ρ_B
entanglement
entropy
ρ_A = Σ_i P_i |i⟩⟨i|
S_A = - Σ_i P_i log P_i
S_A ≤ log |H_A| = log[2^{N_A}]
= N_A log 2
ρ_A = 1/|H_A| ⇒ S_A = log|H_A|
Change of
acts on
λ: 1-dim
#= intersec
between

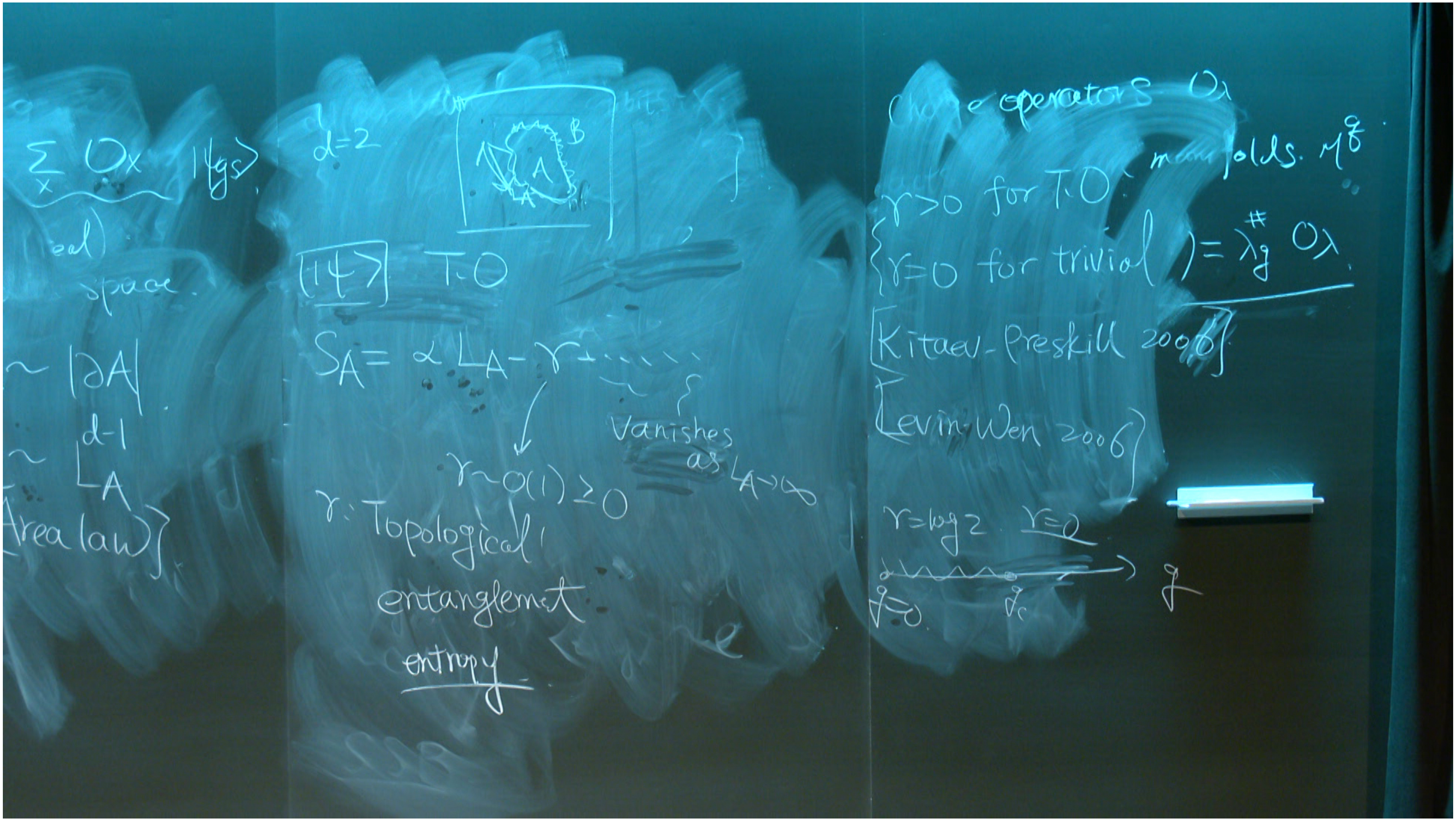

N qubits, $|H| = 2^N$
$N_A + N_B = N$
N_A N_B
Pick a random state $|\psi\rangle$
For $N_A < N_B$
$S_A \sim N_A \log 2$
volume law
S_A
$1/2$
1
N_A/N
$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$
$= -\mathrm{Tr}_B \rho_B \log \rho_B$
entanglement entropy
$\rho_A = \sum_i p_i |i\rangle\langle i|$
$S_A = -\sum_i p_i \log p_i$

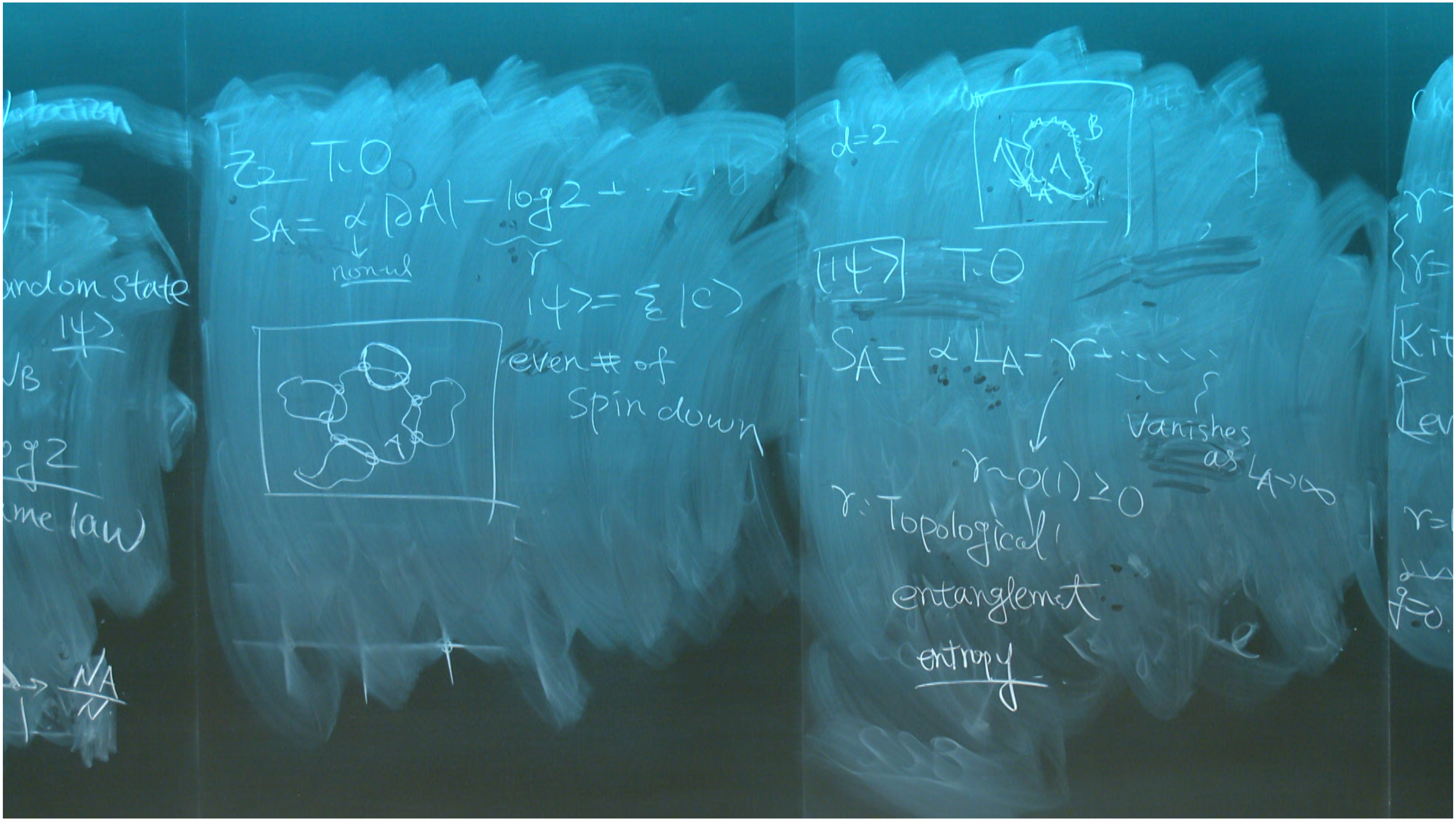

N qubits
|H| = 2^N
N_A + N_B = N
N_A
N_B
Pick a random state |ψ>
For N_A < N_B
S_A ~ N_A log 2
volume law
S_A
1/2
1
N_A/N
H = Σ_x O_x
(real)
d-dim space
B
A
L_A
S_A ~ |∂A|
~ L_A^{d-1}
[Area law]
Upper bound
S_A ≤ log |H
ρ_A = 1/|H_A|

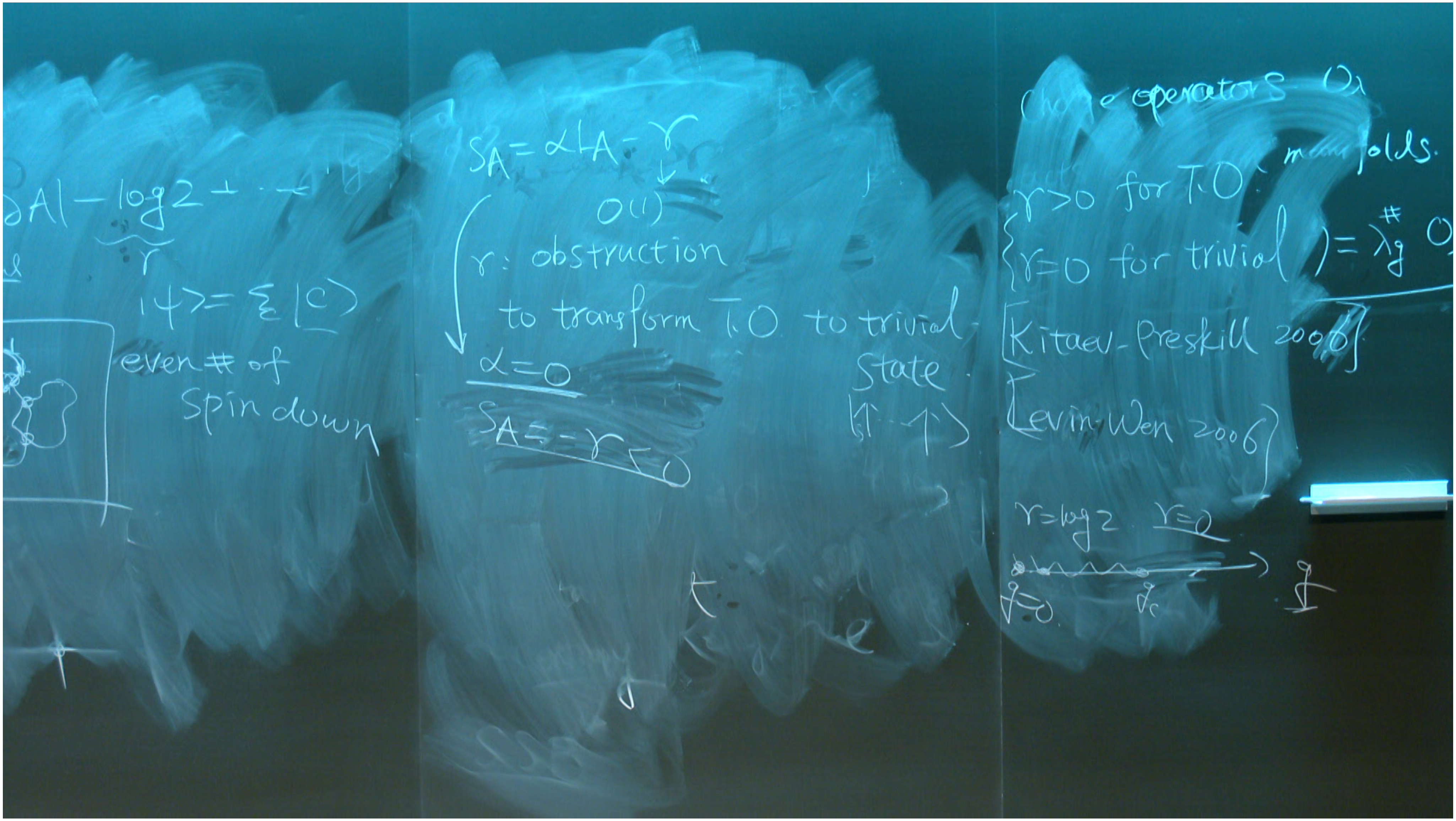


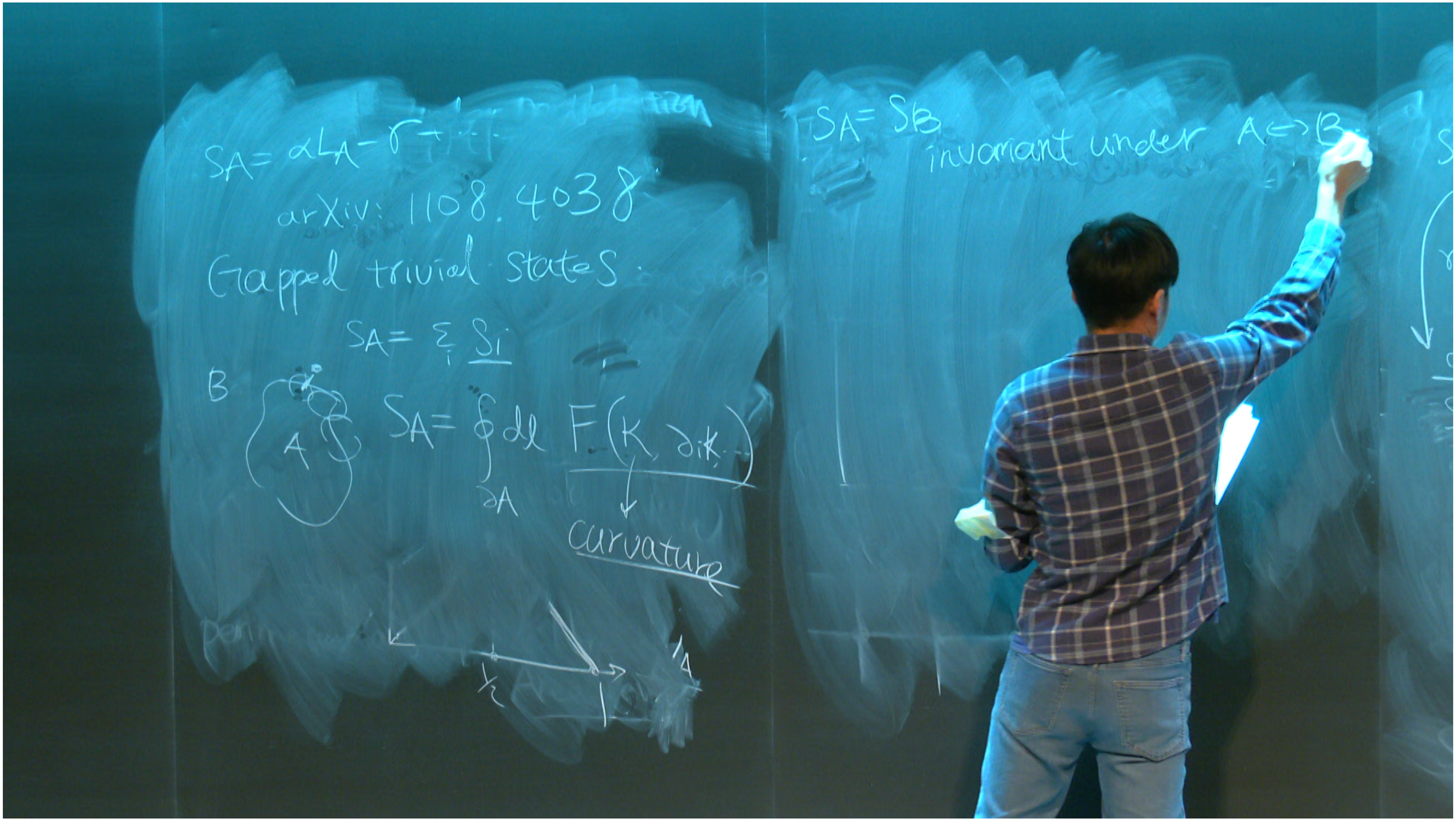
$H = \sum_x O_x$
$|\psi_{gs}\rangle$
(real)
d-dim space
$S_A \sim |\partial A|$
$\sim L_A^{d-1}$
[Area law]
B
A
L_A
d=2
$|\psi\rangle$
$S_A = \alpha L_A$

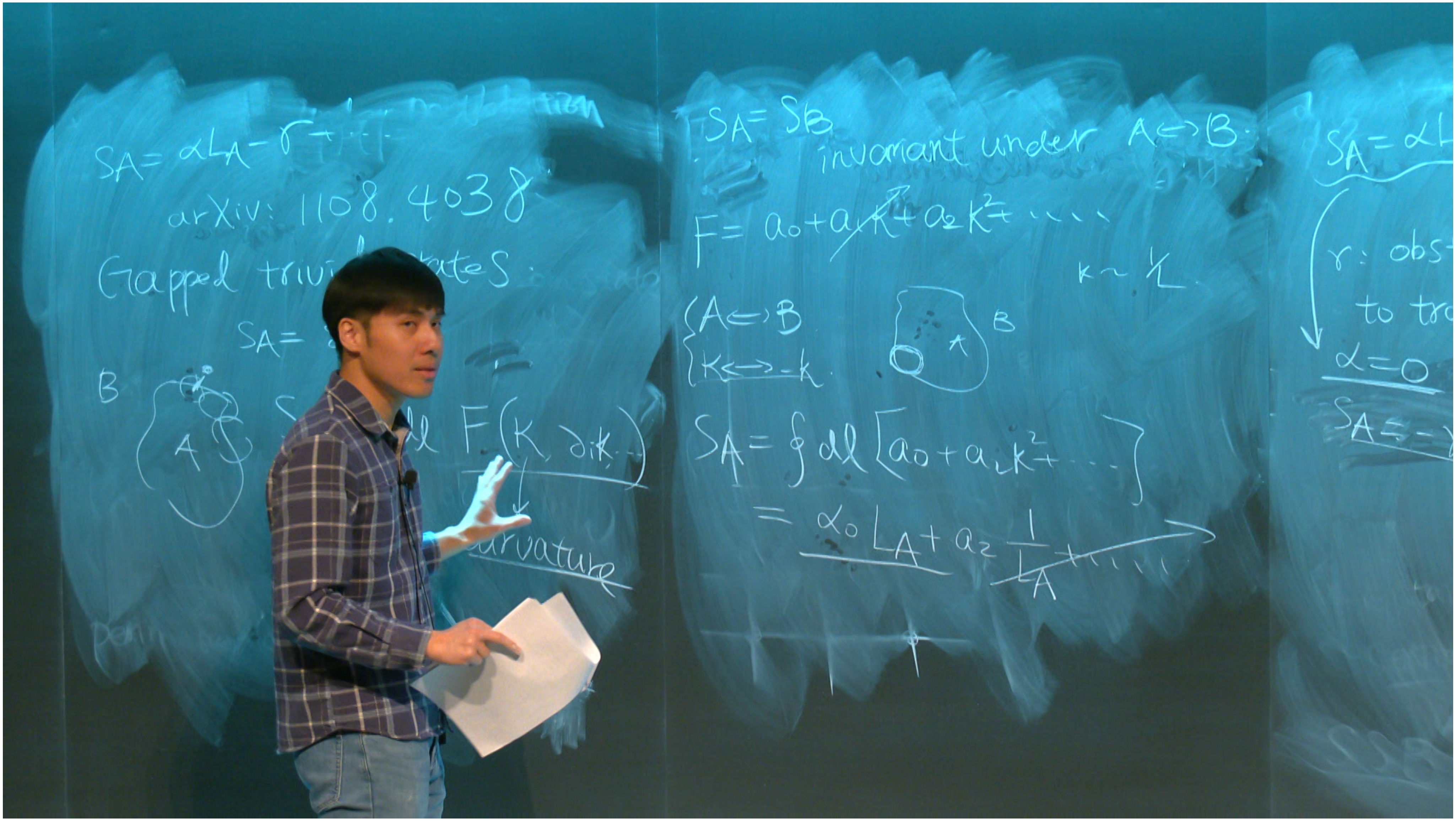

d=2
|Ψ⟩ T.O
S_A = α L_A − γ + ...
vanishes as L_A → ∞
γ ~ O(1) ≥ 0
γ: Topological entanglement entropy
operators O_λ
manifolds M^g
γ > 0 for T.O
γ = 0 for trivial
Kitaev-Preskill 2006
Levin-Wen 2006

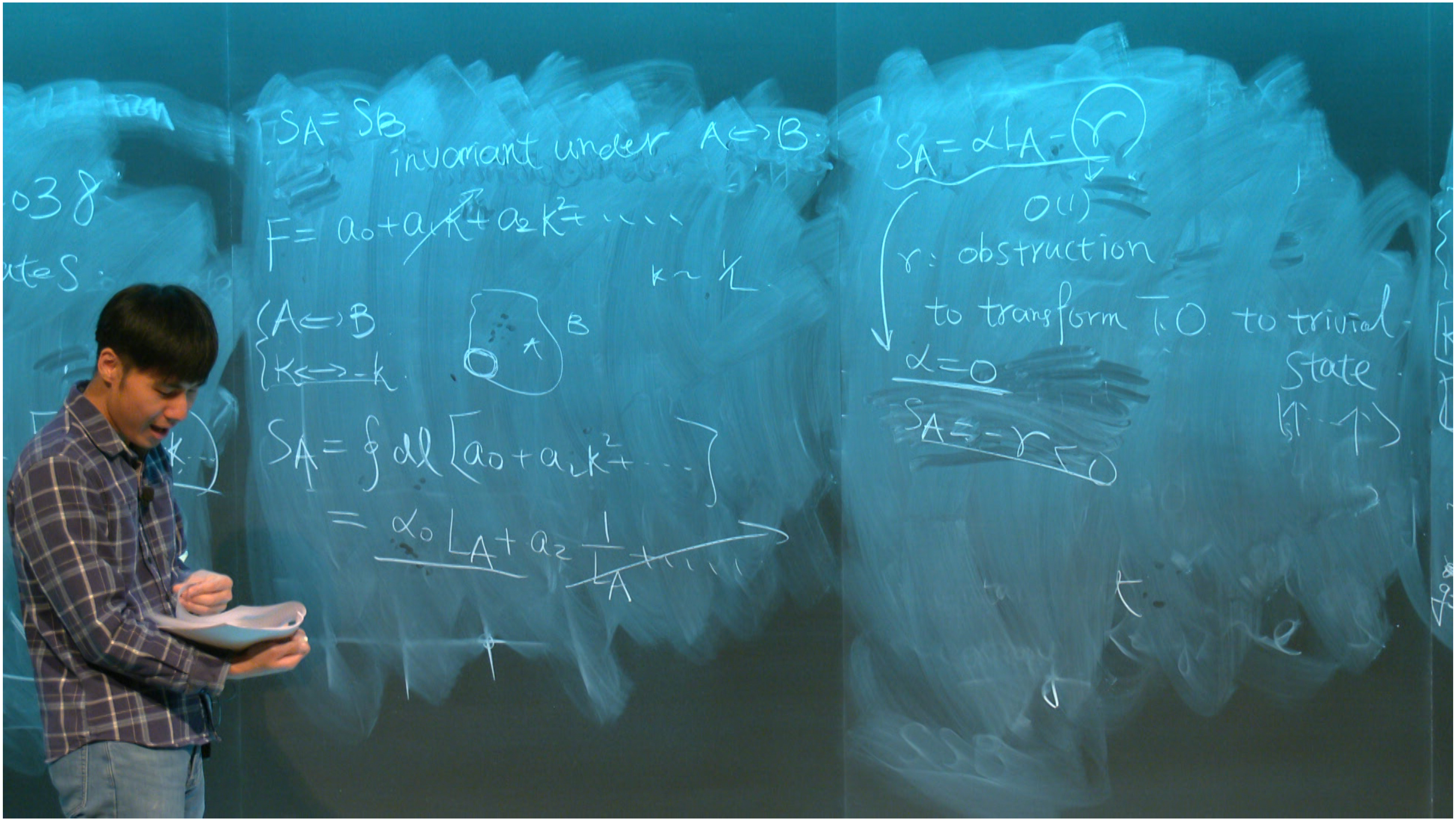

$\sum_x O_x \; |\psi_{gs}\rangle$
space
$\sim |\partial A|$
$d-1$
$\sim L_A$
{Area law}
$d=2$
A
B
L_A
$|\psi\rangle$ T.O
$S_A = \alpha L_A - \gamma + \cdots$
Vanishes as $L_A \to \infty$
$\gamma \sim O(1) \geq 0$
γ: Topological entanglement entropy
operators O_λ
manifolds. M^g
$\gamma > 0$ for T.O
$\gamma = 0$ for trivial
$) = \lambda_g^{\#} O_\lambda$
[Kitaev-Preskill 2006]
[Levin-Wen 2006]
$\gamma = \log 2$
$\gamma = 0$
g
$g=0$
g_c

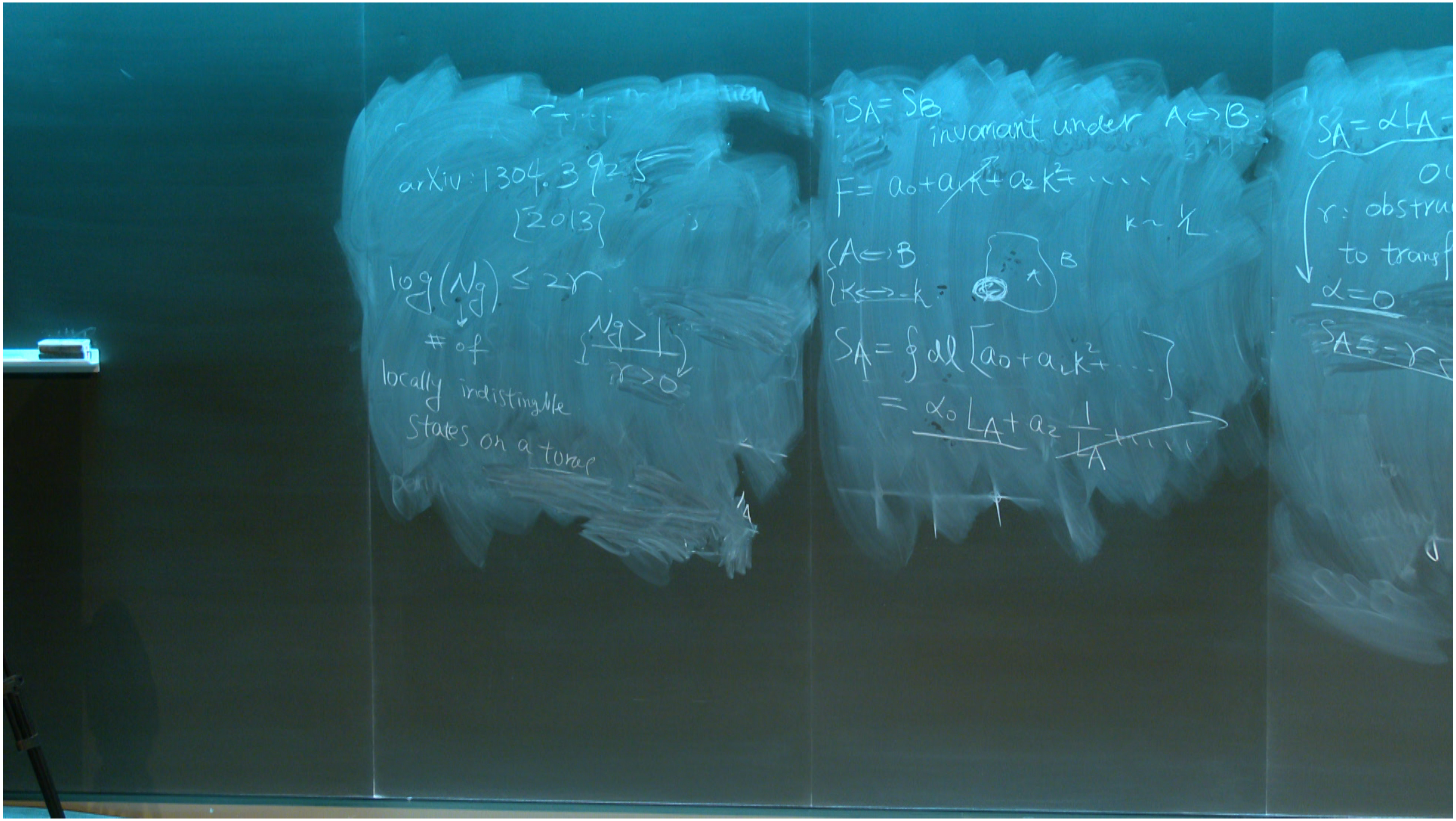

random state
|ψ>
log2
volume law
Z2 T.O
SA = α |∂A| − log2 + ...
non-univ
γ
|ψ> = Σ |C>
even # of
spin down
A
d=2
A
B
LA
|ψ> T.O
SA = α LA − γ + ...
vanishes
as LA → ∞
γ ~ O(1) ≥ 0
γ: Topological
entanglement
entropy
Kit
Lev

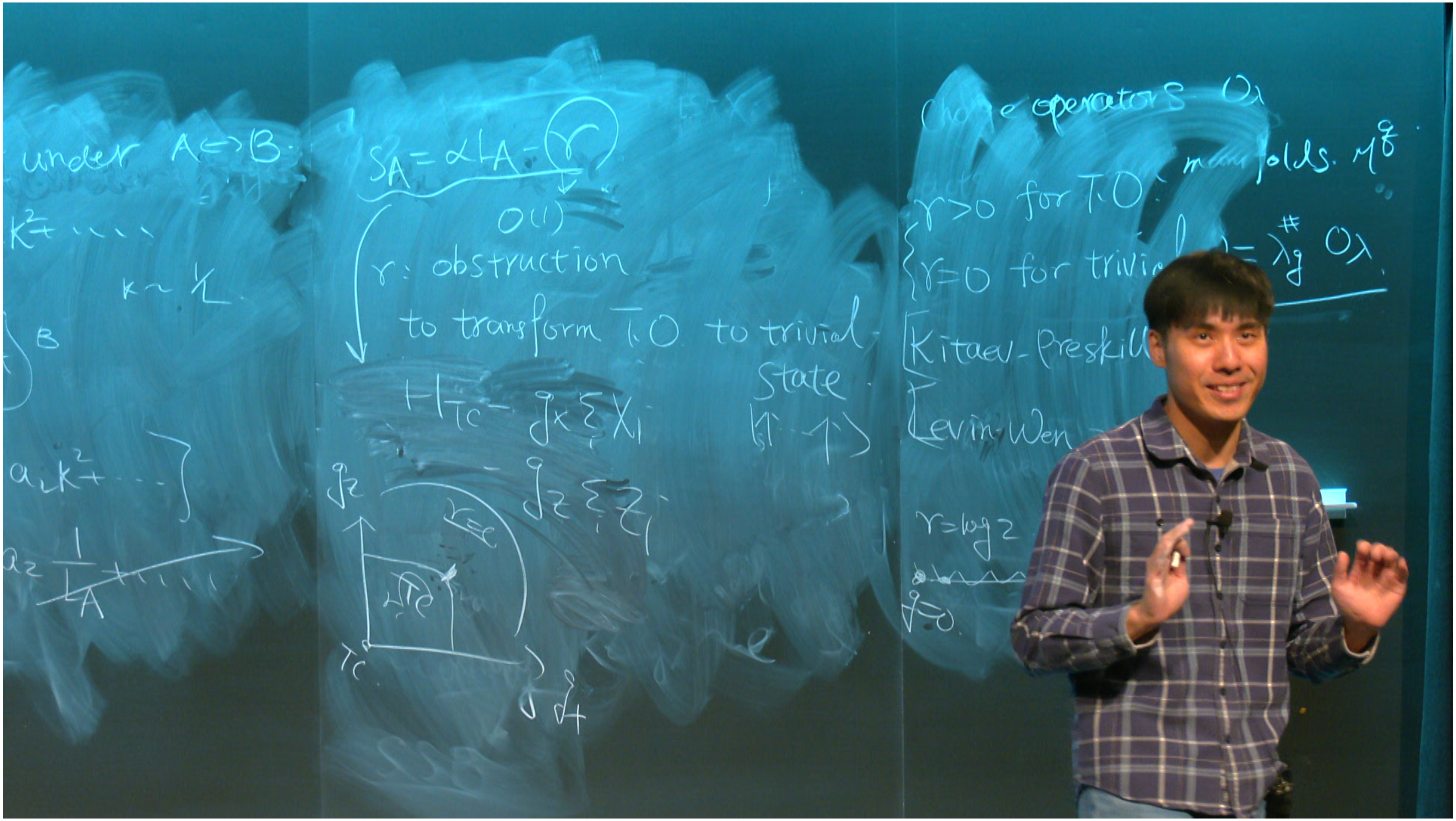

$S_A = \alpha L_A - \gamma$
$O(1)$
γ: obstruction
to transform T.O to trivial
state
$\alpha = 0$
$S_A = -\gamma < 0$
$|\uparrow \cdots \uparrow\rangle$
$|\psi\rangle = \sum |C\rangle$
even # of
spin down
$\gamma > 0$ for T.O
$\gamma = 0$ for trivial
[Kitaev-Preskill 2006]
[Levin-Wen 2006]
$\gamma = \log 2$
$\gamma = 0$
$g = 0$
g_c
g

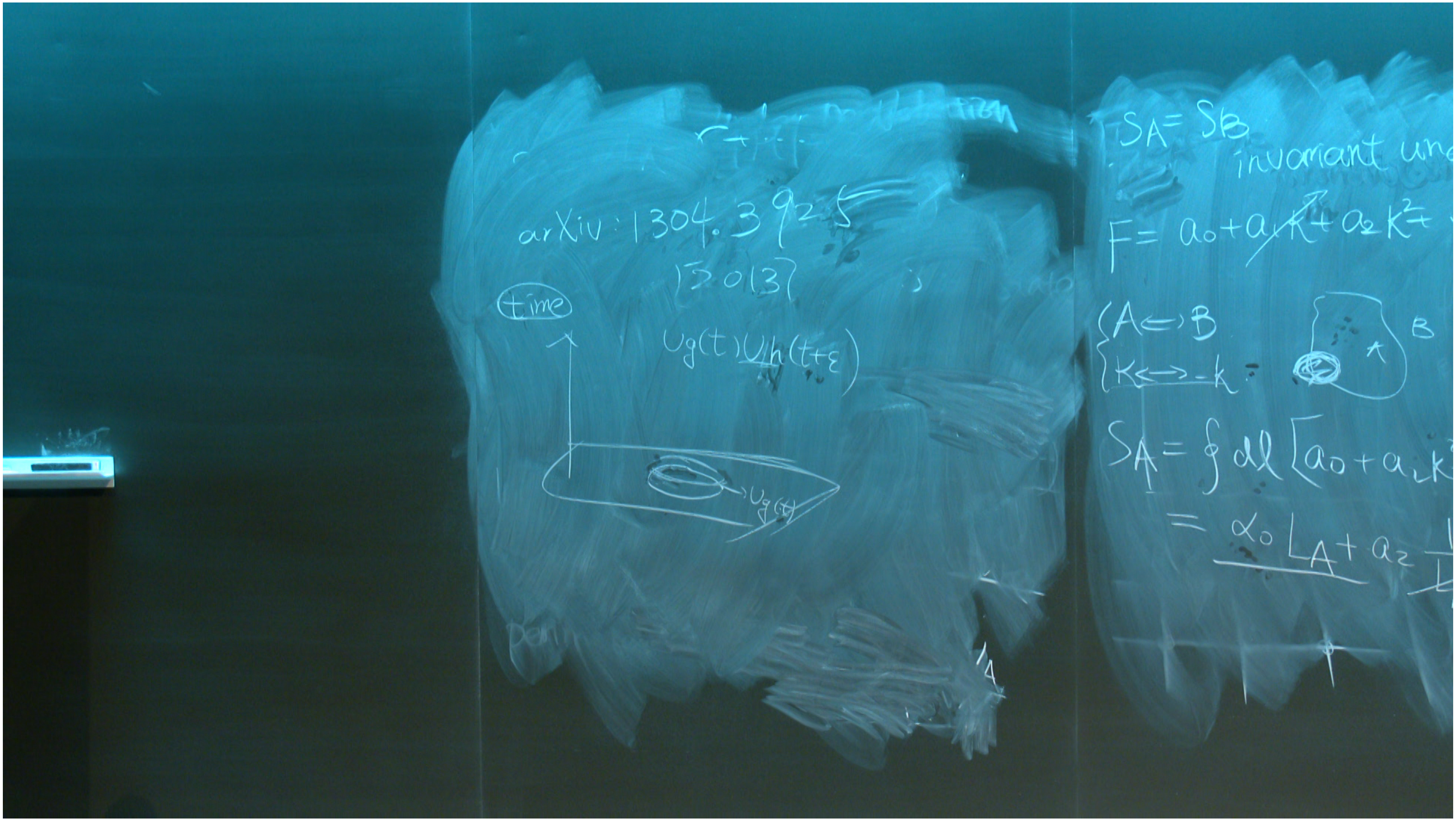

SA = αLA − γ + ...
arXiv: 1108.4038
Gapped trivial states
SA = Σi Si
B
A
SA = ∮∂A dl F(K, ∂iK, ...)
curvature
SA = SB
invariant under A ⟷ B


SA = αLA − γ + ...
arXiv: 1108.4038
Gapped trivial states
SA = SB
invariant under A ↔ B
F = a0 + a1 k + a2 k² + ...
k ~ 1/L
A ↔ B
k ↔ −k
SA = ∮ dl [a0 + a2 k² + ...]
= α0 LA + a2 1/LA + ...
curvature
γ: obs
α = 0


SA = SB invariant under A⟷B
F = a0 + a1 K + a2 K² + ...
K ~ 1/L
A⟷B
K⟷-K
B
A
SA = ∮ dl [a0 + a1 K² + ...]
= α0 LA + a2 1/LA + ...
SA = α LA - γ
O(1)
γ: obstruction
to transform T.O to trivial
state
α = 0
SA = -γ < 0


arXiv: 1304.3925
[2013]
log(Ng) ≤ 2γ
of
locally indistinguishable
states on a torus
Ng > 1
γ > 0
SA = SB invariant under A↔B
F = a0 + a1k + a2k2 + ...
k ~ 1/L
A↔B
k↔-k
B
A
SA = ∮dl [a0 + a2k2 + ...]
= α0 LA + a2 1/LA + ...
SA = αLA -
γ: obstru
to transf
α = 0
SA = -γ


under A⟺B
$S_A = \alpha L_A - \gamma$
O(1)
γ: obstruction
to transform T.O to trivial
state
γ>0 for T.O
γ=0 for trivial
Kitaev-Preskill
Levin-Wen
γ=log 2


arXiv:1304.3925
1503.0137
time
Ug(t)Uh(t+ε)
Ug(t)
SA = SB
invariant un
F = a0 + a1k + a2k² +
A ⟷ B
k ⟷ -k
B
A
SA = ∮dl [a0 + a1k
= α0 LA + a2