Title: Topological quantum matter and quantum computing

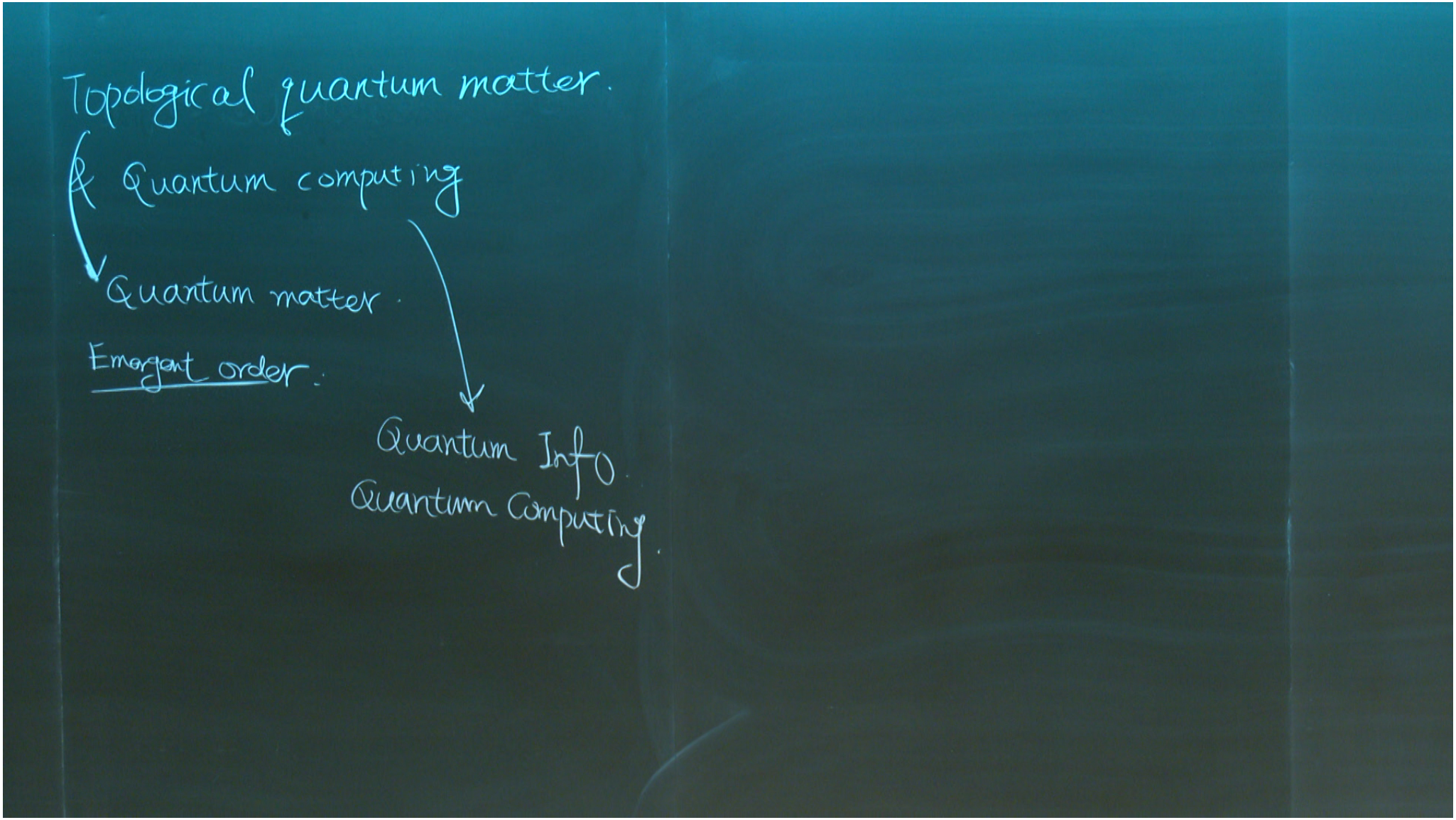
Speakers: Peter Lu

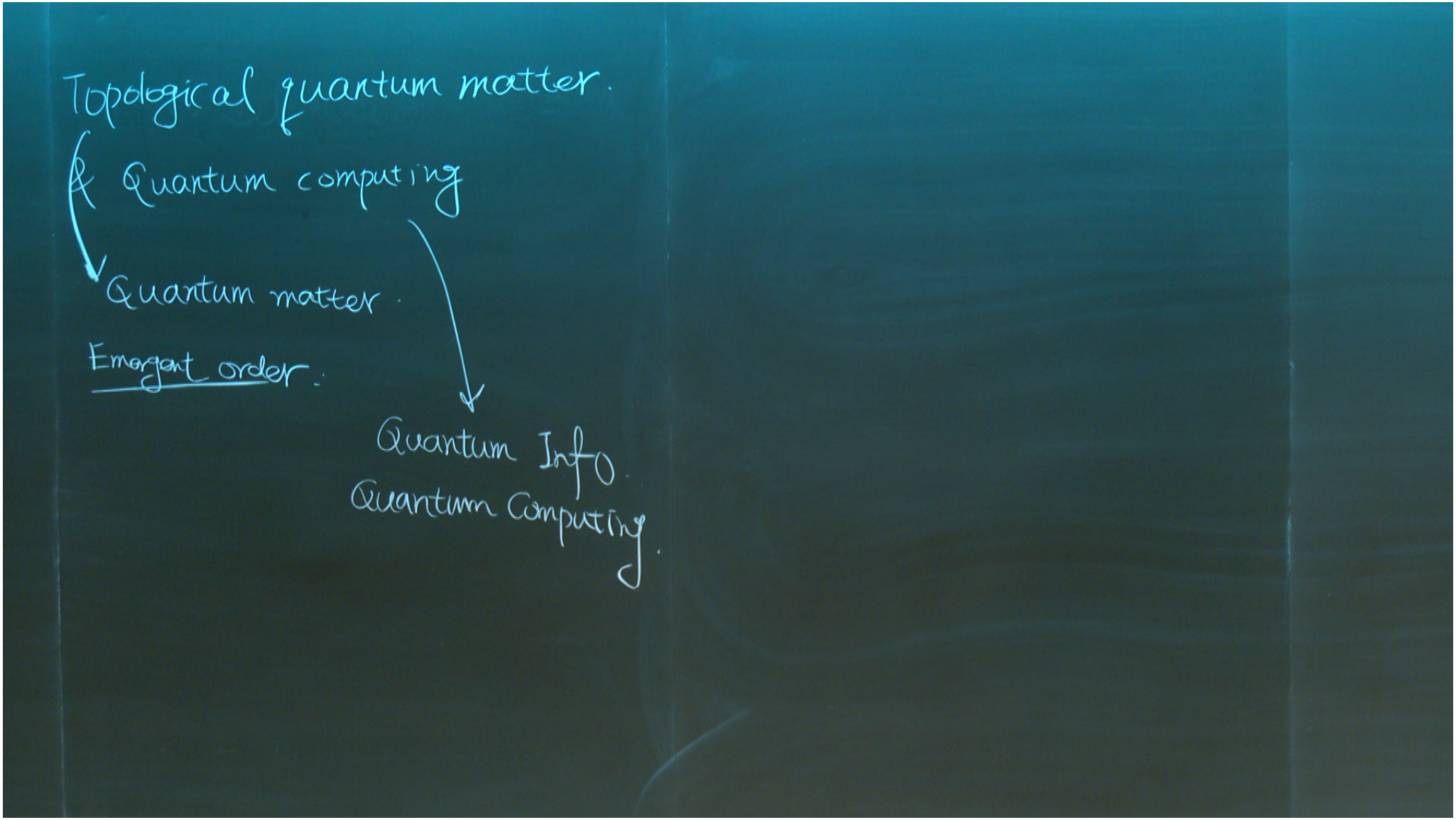
Collection: Symmetries Graduate School 2023

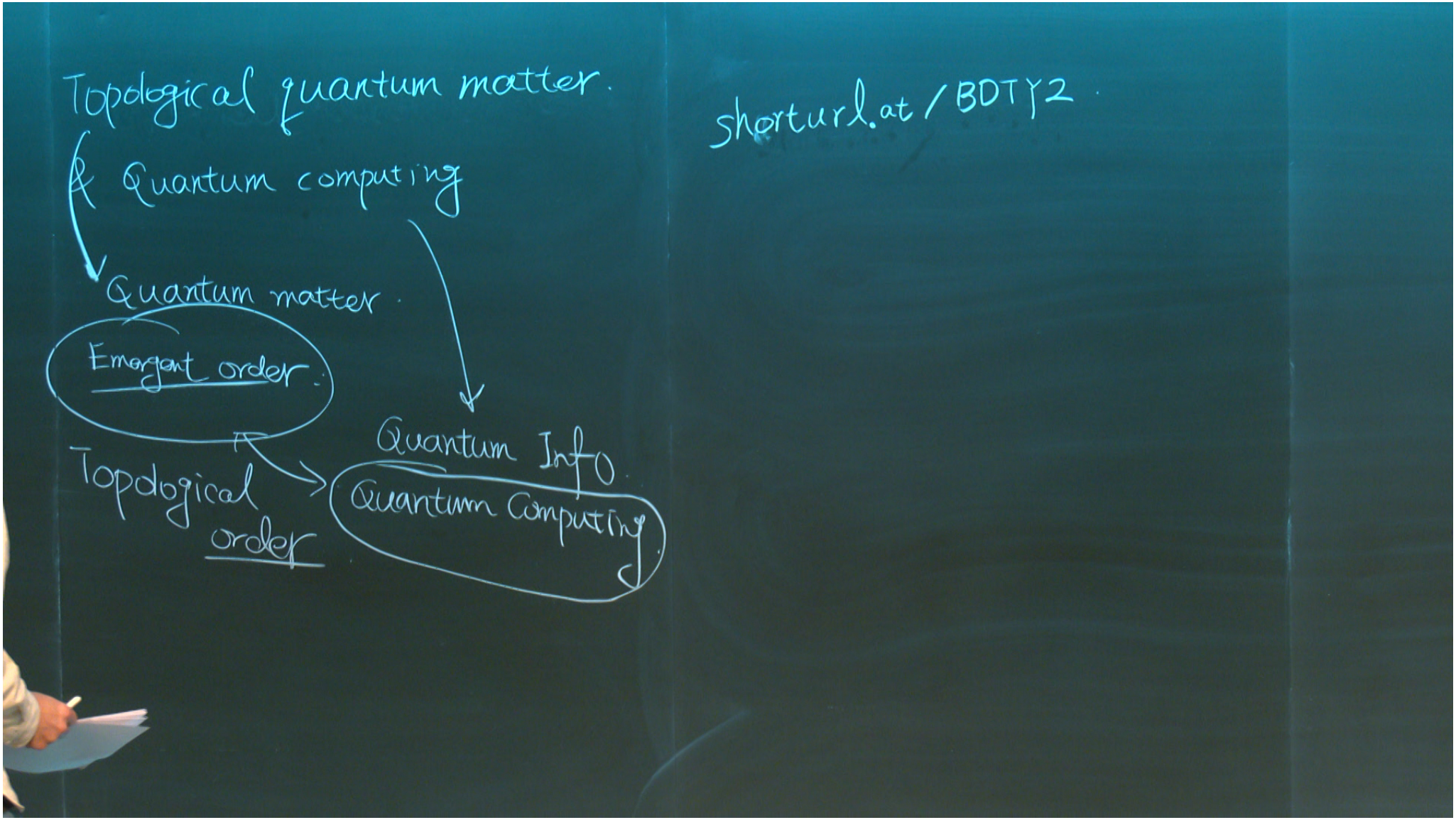
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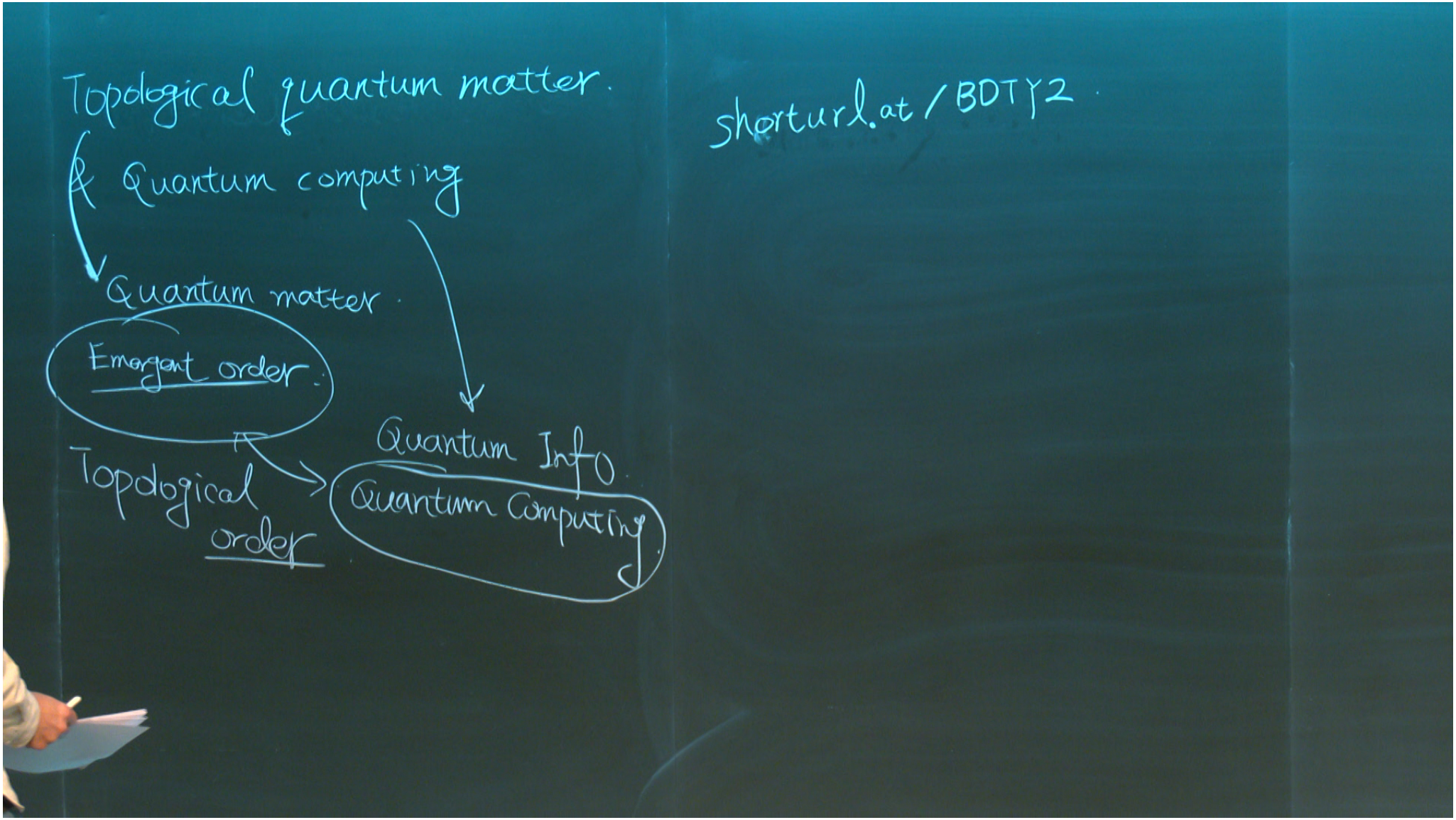
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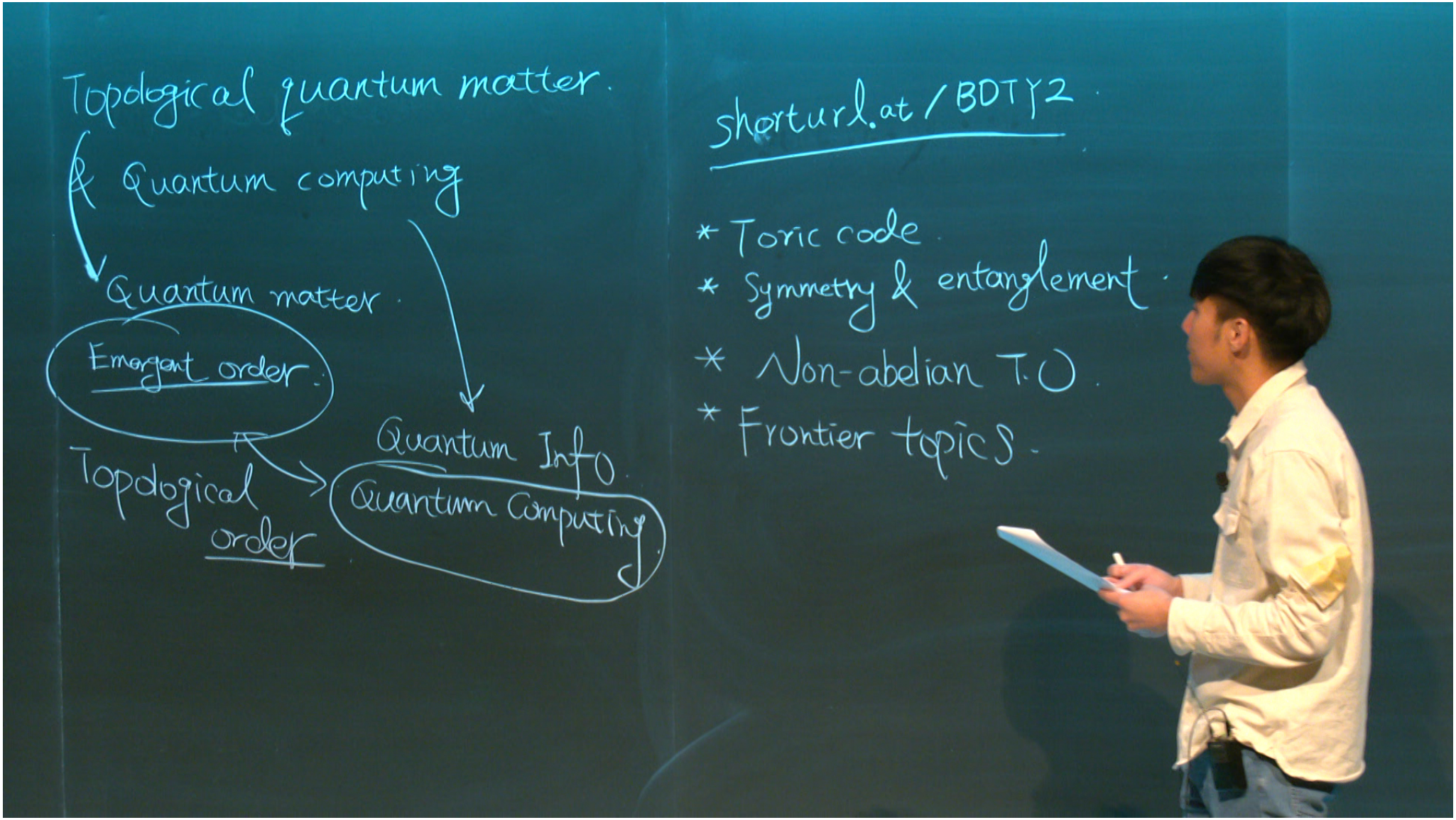
Abstract: Interacting quantum particles can form non-trivial states of matter characterized by topological order, which features several unconventional properties such as topological degeneracy and fractionalized quasiparticles. In addition, it also provides a promising platform for realizing quantum computing in a robust manner. In this series of lectures, I will introduce the basics of topological order and its connection to quantum computing from various aspects involving lattice models, symmetry, and entanglement structure. Several frontier topics such as fracton topological phases, self-correcting quantum memory, state preparation, and quantum LDPC codes will be briefly discussed.

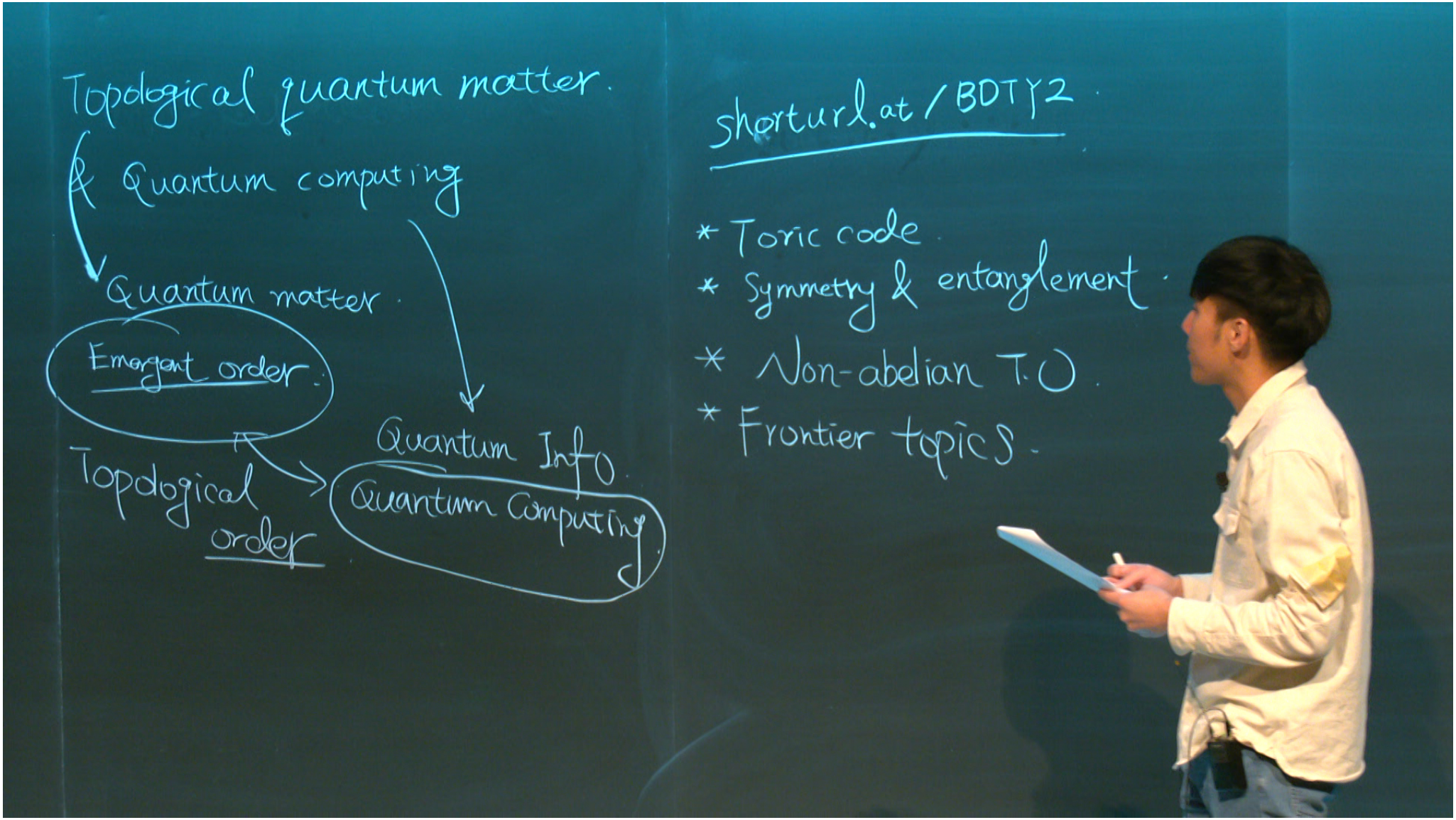

Topological quantum matter.
& Quantum computing
Quantum matter.
Emergent order.
Quantum Info.
Quantum Computing.

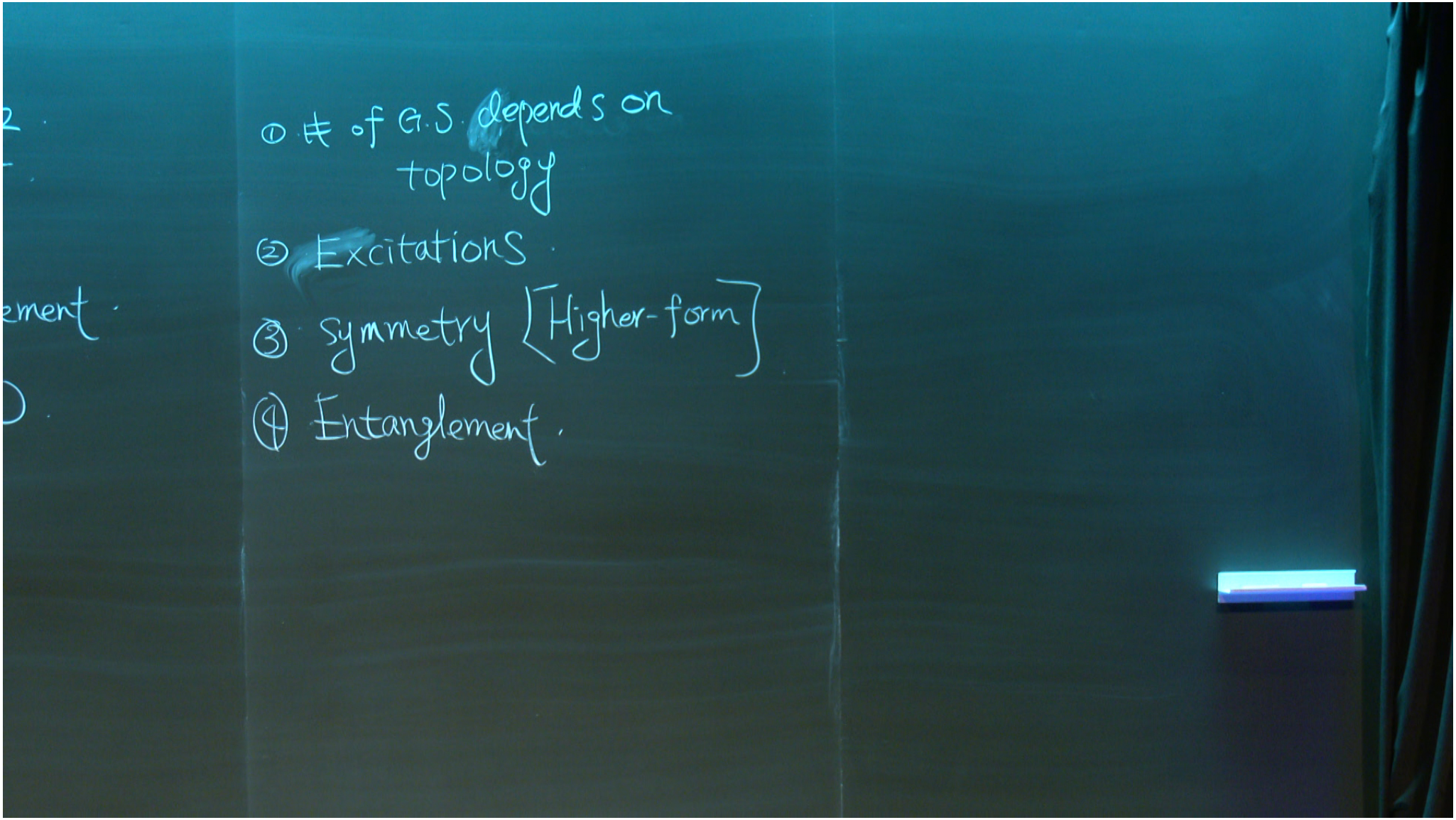

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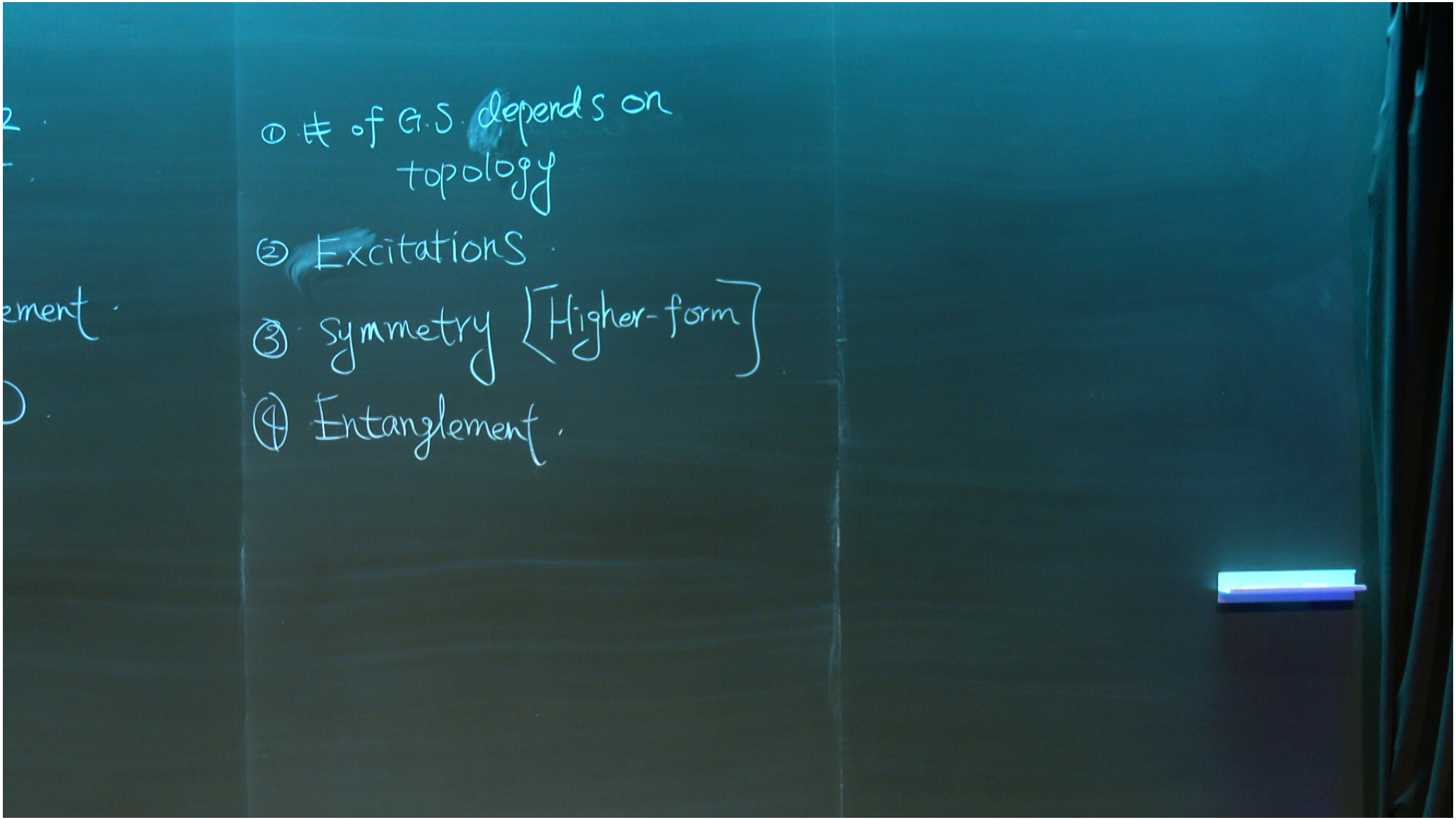

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shorturl.at/BDTY2
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* Frontier topics.


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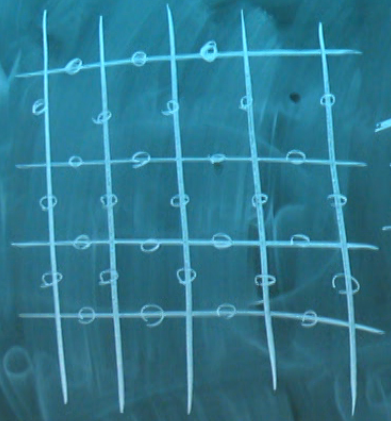

① # of G.S. depends on topology
② Excitations
③ Symmetry [Higher-form]
④ Entanglement


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$\mathbb{Z}_2$ toric code in 2d space

[Kitaev, 2003]

spin-½ [qubits]



$\underline{Z} = 6^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

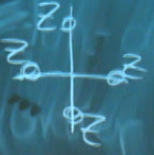
$\underline{X} = 6^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

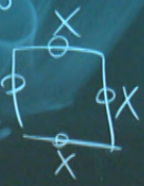
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$$H = -\sum_s A_s - \sum_p B_p$$

s ↓ star

p ↓ plaquette

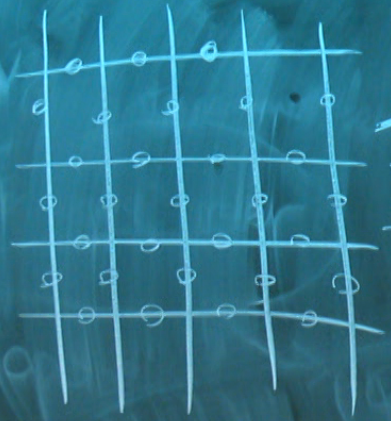
$A_s =$ 

$B_p =$ 

Z_2 toric code in 2d space

[Kitaev, 2003]

spin-½ [qubits]



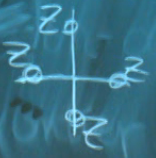
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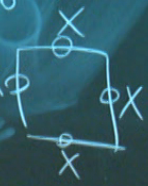
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star

$A_s =$ 

plaquette

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Z_2 toric code in 2d space

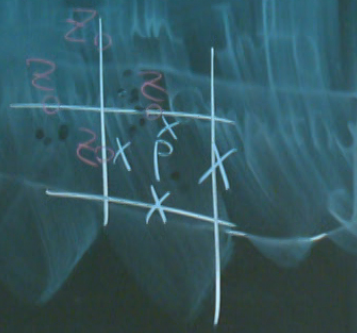
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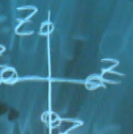
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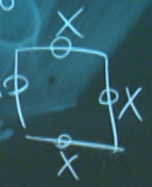


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$$H = -\sum_s A_s - \sum_p B_p$$

s ↓ star p ↓ plaquette

$A_s =$ 

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$[A_s, A_{s'}] = 0 = [B_p, B_{p'}]$

$= [A_s, B_p]$

Z_2 toric code in 2d space

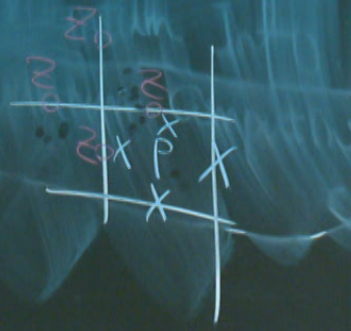
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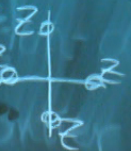
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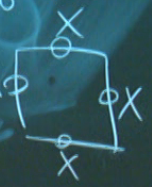


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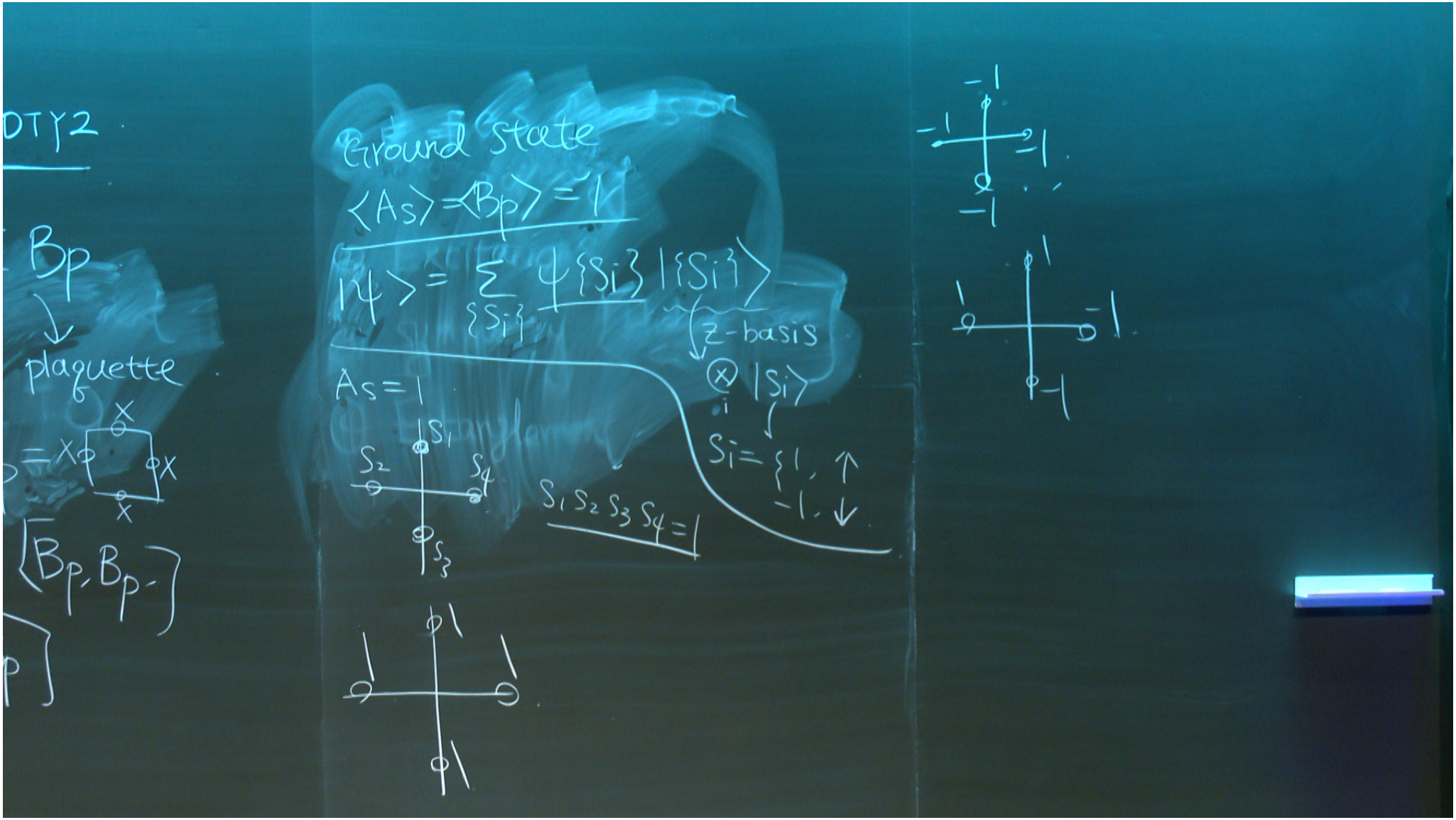
s ↓ star, p ↓ plaquette

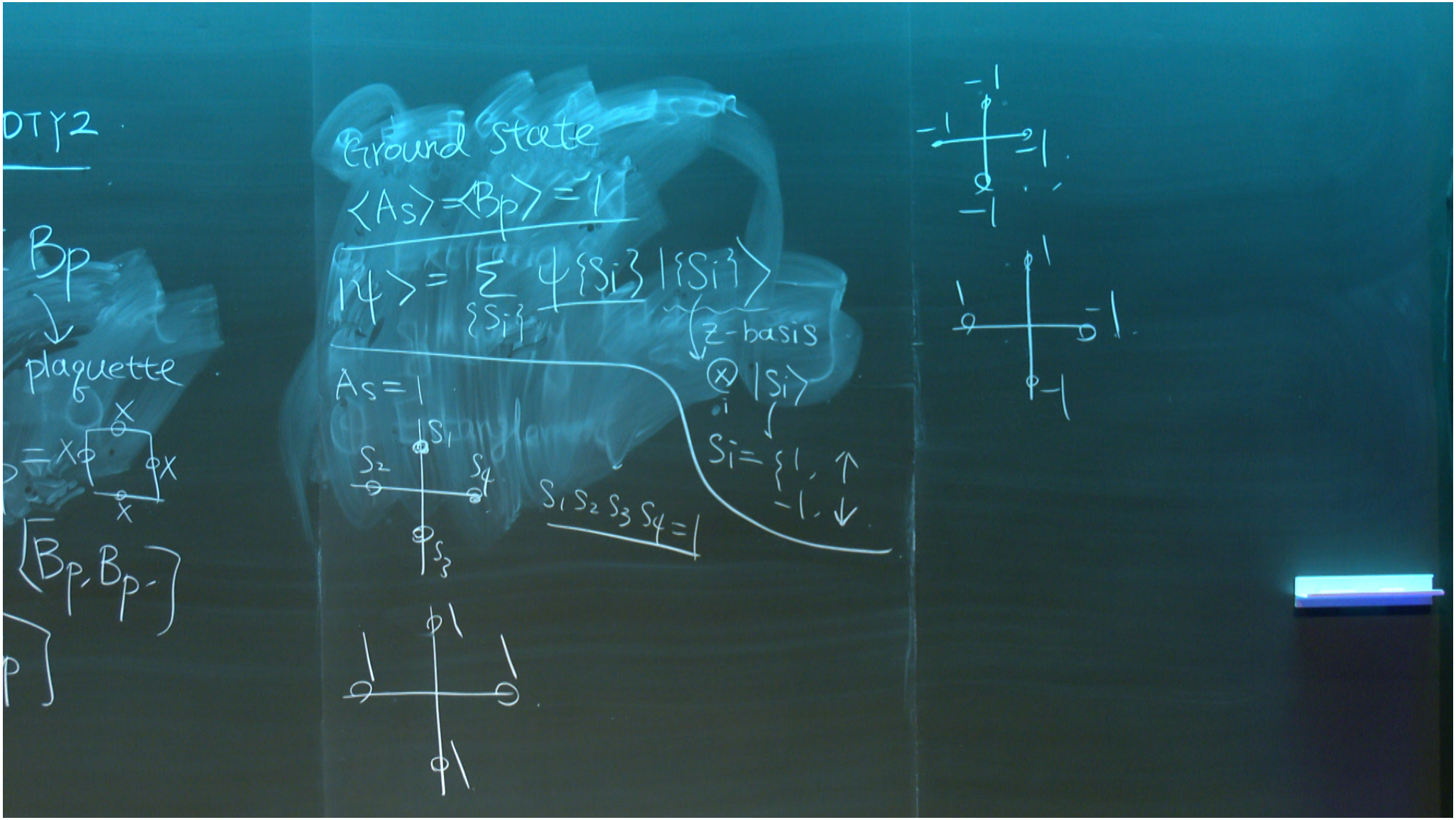
$A_s =$ 

$B_p =$ 

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$= [A_s, B_p]$


Ground state
$\langle A_s \rangle = \langle B_p \rangle = 1$
$|\psi\rangle = \sum_{\{s_i\}} \psi\{s_i\} |\{s_i\}\rangle$
z-basis
$\otimes_i |s_i\rangle$
$A_s = 1$
$s_1 s_2 s_3 s_4 = 1$
$s_i = \{1, \uparrow$
$-1, \downarrow$
B_p
plaquette


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Z_2 toric code in 2d space

[Kitaev, 2003]

$S_1 S_2 S_3 S_4 = 1$

even # of spin down

↑ = — = $S=1$

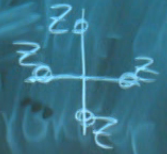
↓ = — = $S=-1$

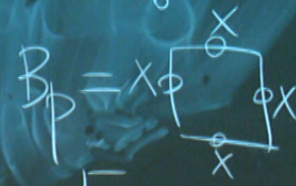
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p ↓ plaquette

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$[A_s, A_{s'}] = 0 = [B_p, B_{p'}]$

$= [A_s, B_p]$

Grou

$\langle A_s$

$|\psi\rangle$

$A_s =$

S_2

Z_2 toric code in 2d space

[Kitaev, 2003]

$S_1 S_2 S_3 S_4 = 1$

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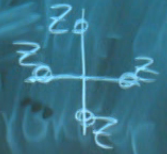
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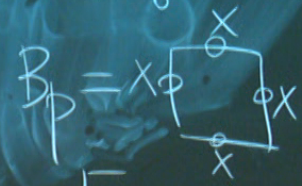
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$$H = -\sum_s A_s - \sum_p B_p$$

s: star

p: plaquette

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Grou

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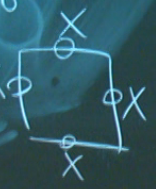
$A_s =$

S_2

/BDTY2

$-\sum_p B_p$

plaquette

$B_p =$ 

$) = [B_p, B_{p'}]$

$s, B_p]$

Ground state

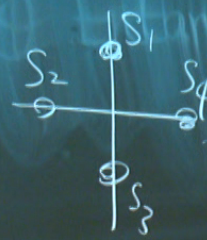
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$|\psi\rangle = \sum_{\{s_i\}} \psi\{s_i\} |\{s_i\}\rangle$

(z-basis

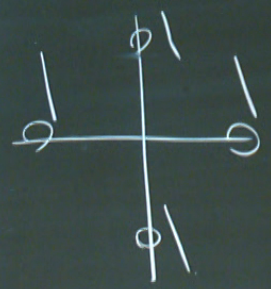
$\bigotimes_i |s_i\rangle$

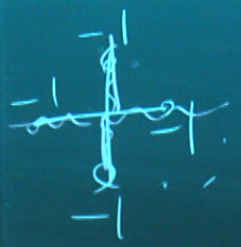
$A_s = 1$

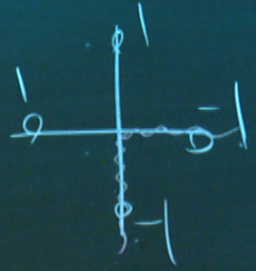


$s_1 s_2 s_3 s_4 = 1$

$s_i = \{1, \uparrow$
$-1, \downarrow$



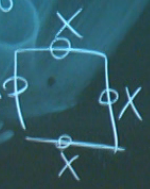




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$-\sum_p B_p$

plaquette

$B_p =$ 

$) = [B_p, B_{p'}]$

$s, B_p]$

Ground state

$\langle A_s \rangle = \langle B_p \rangle = 1$

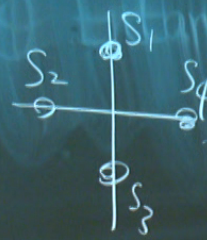
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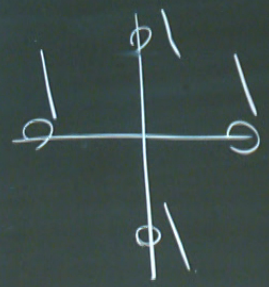
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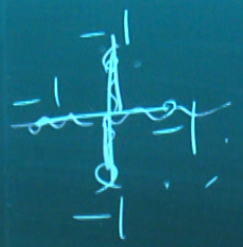
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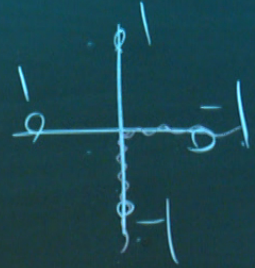
$A_s = 1$

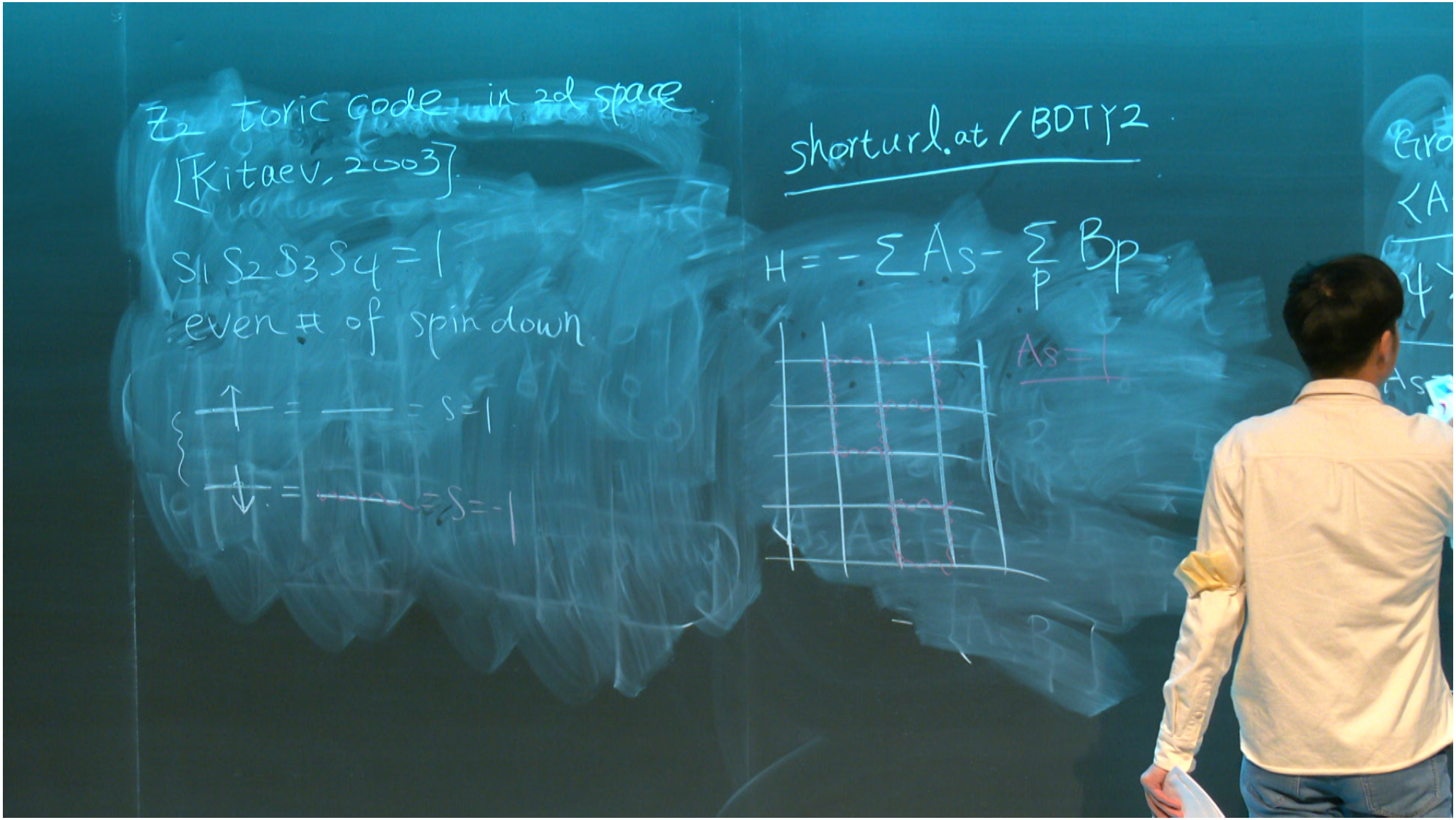


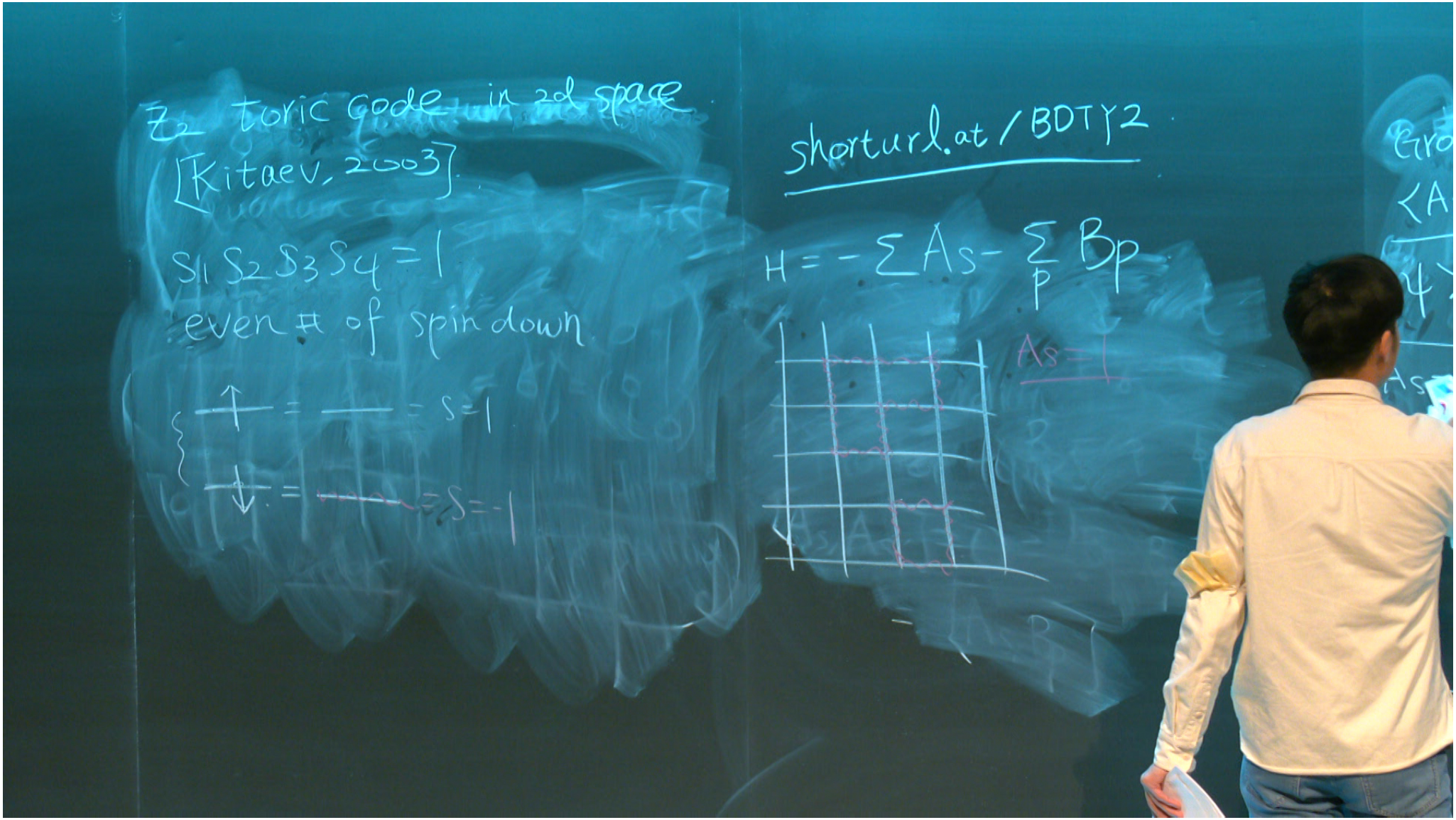
$s_1 s_2 s_3 s_4 = 1$

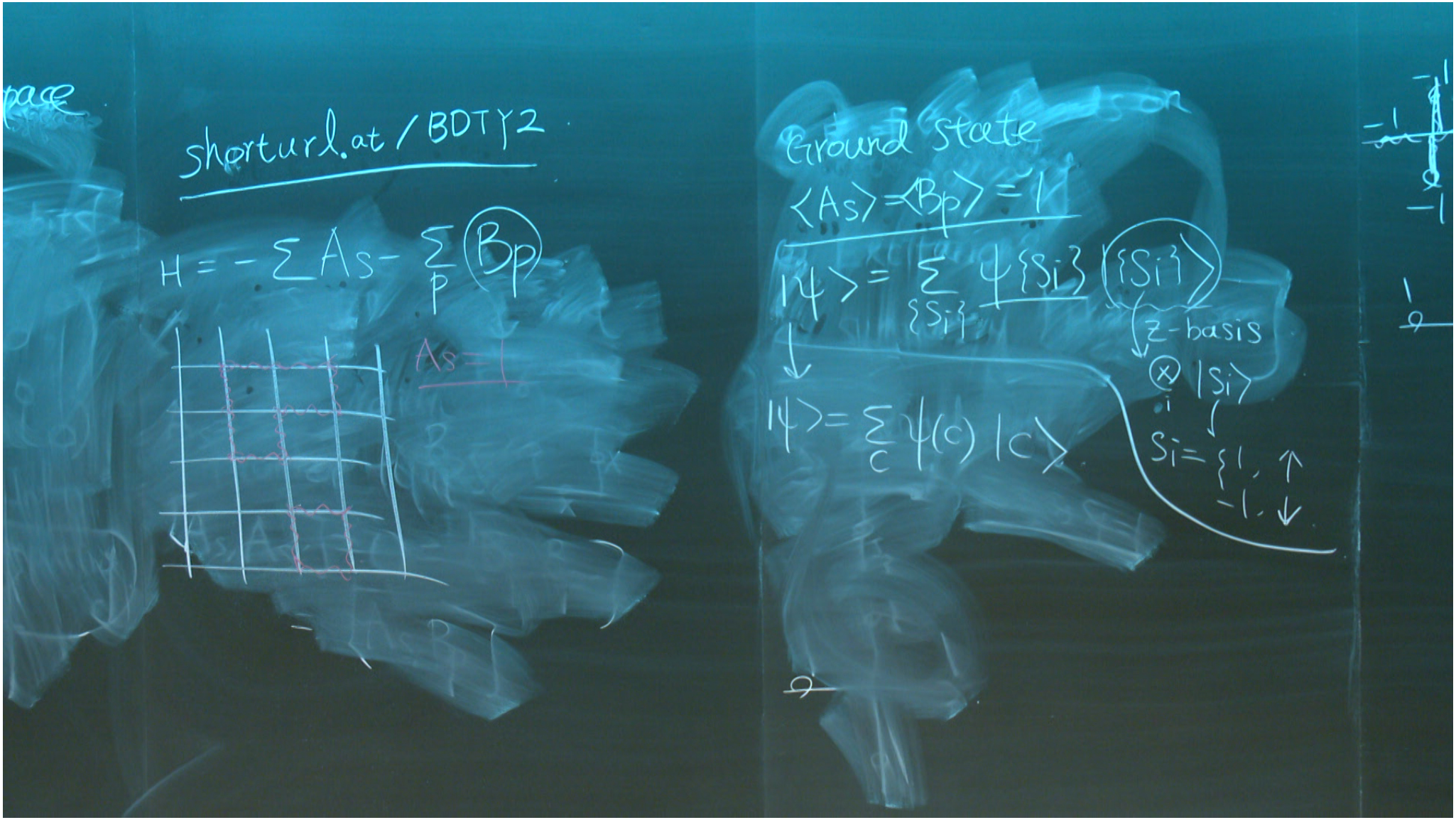


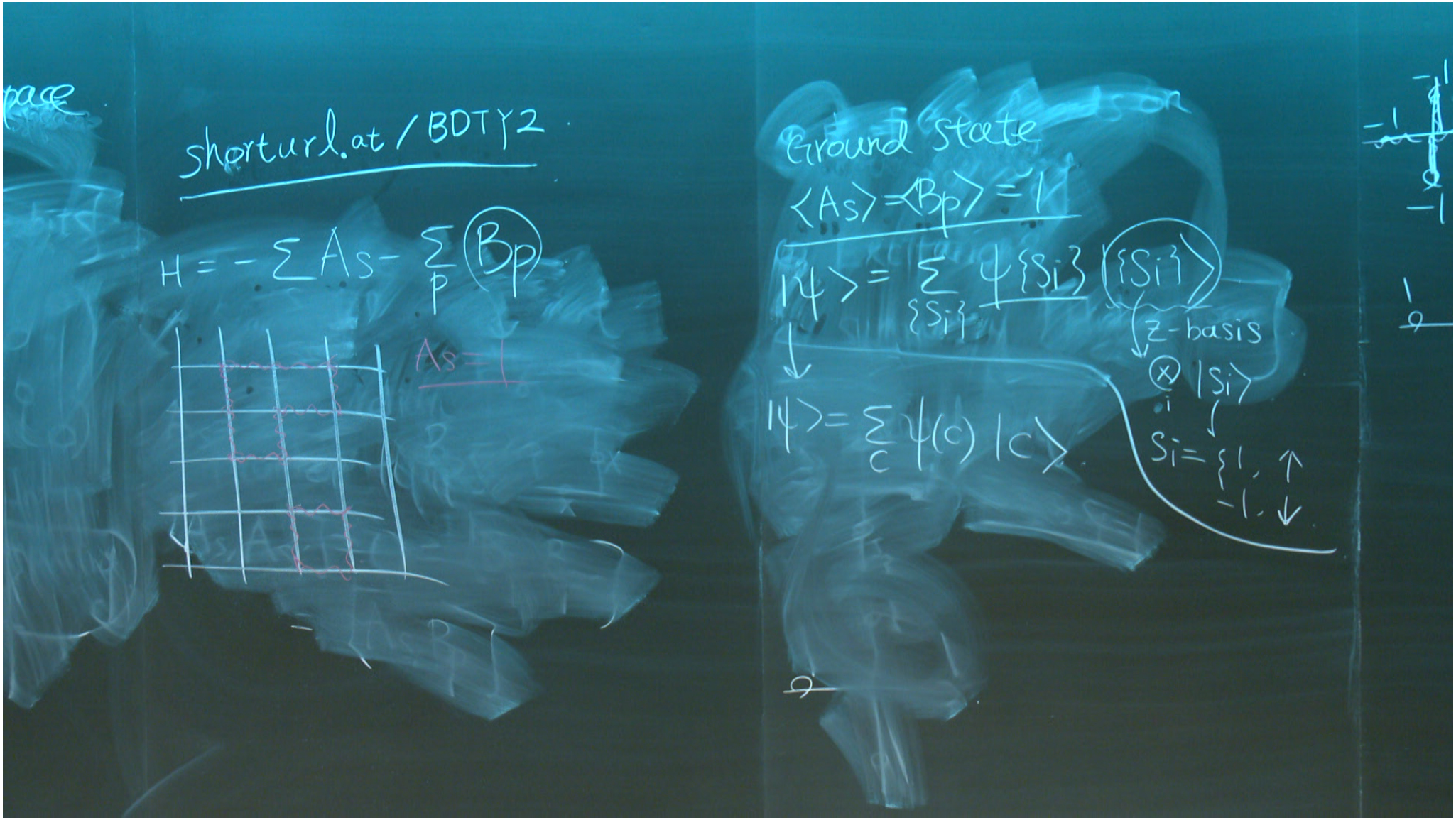


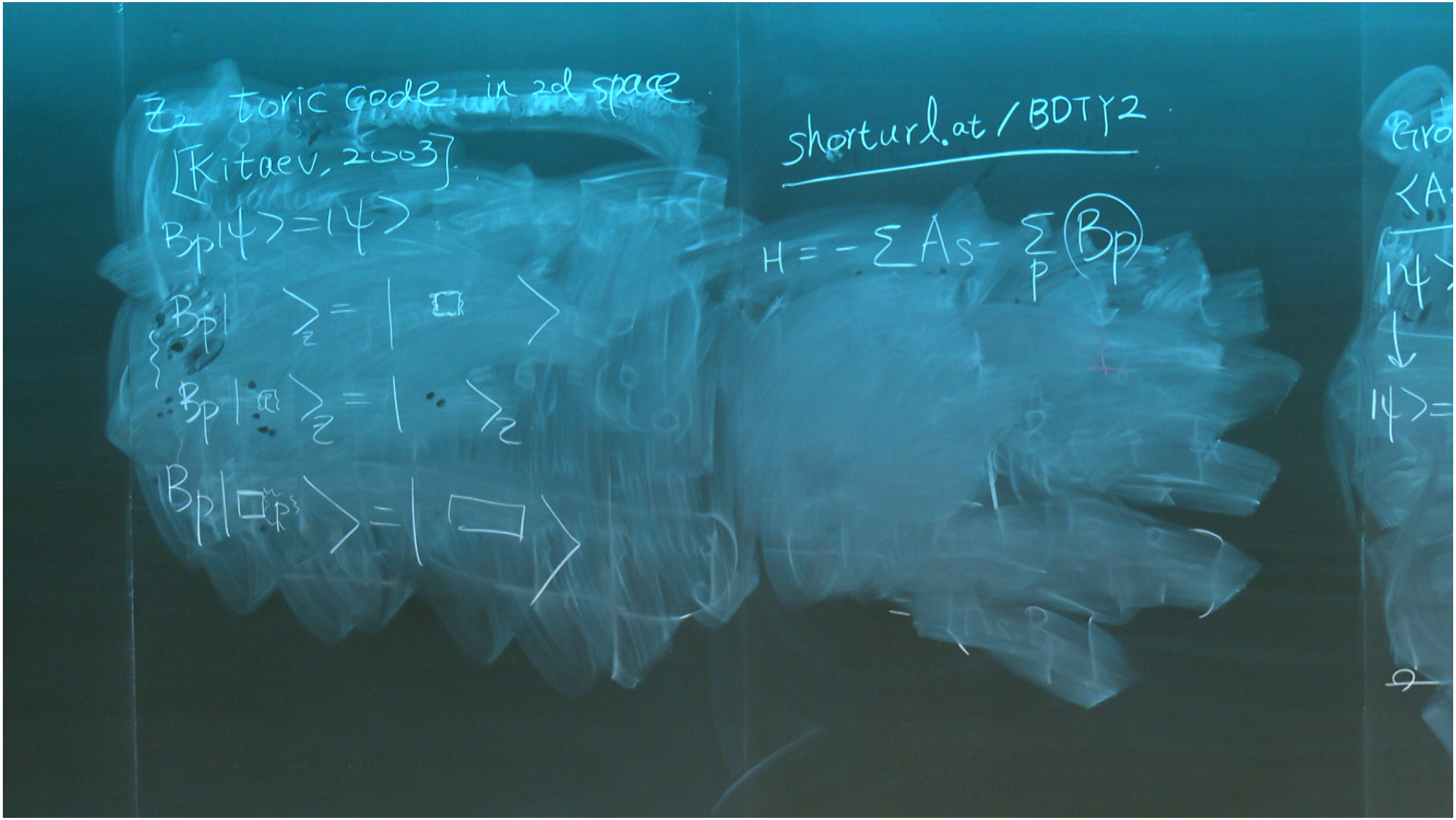


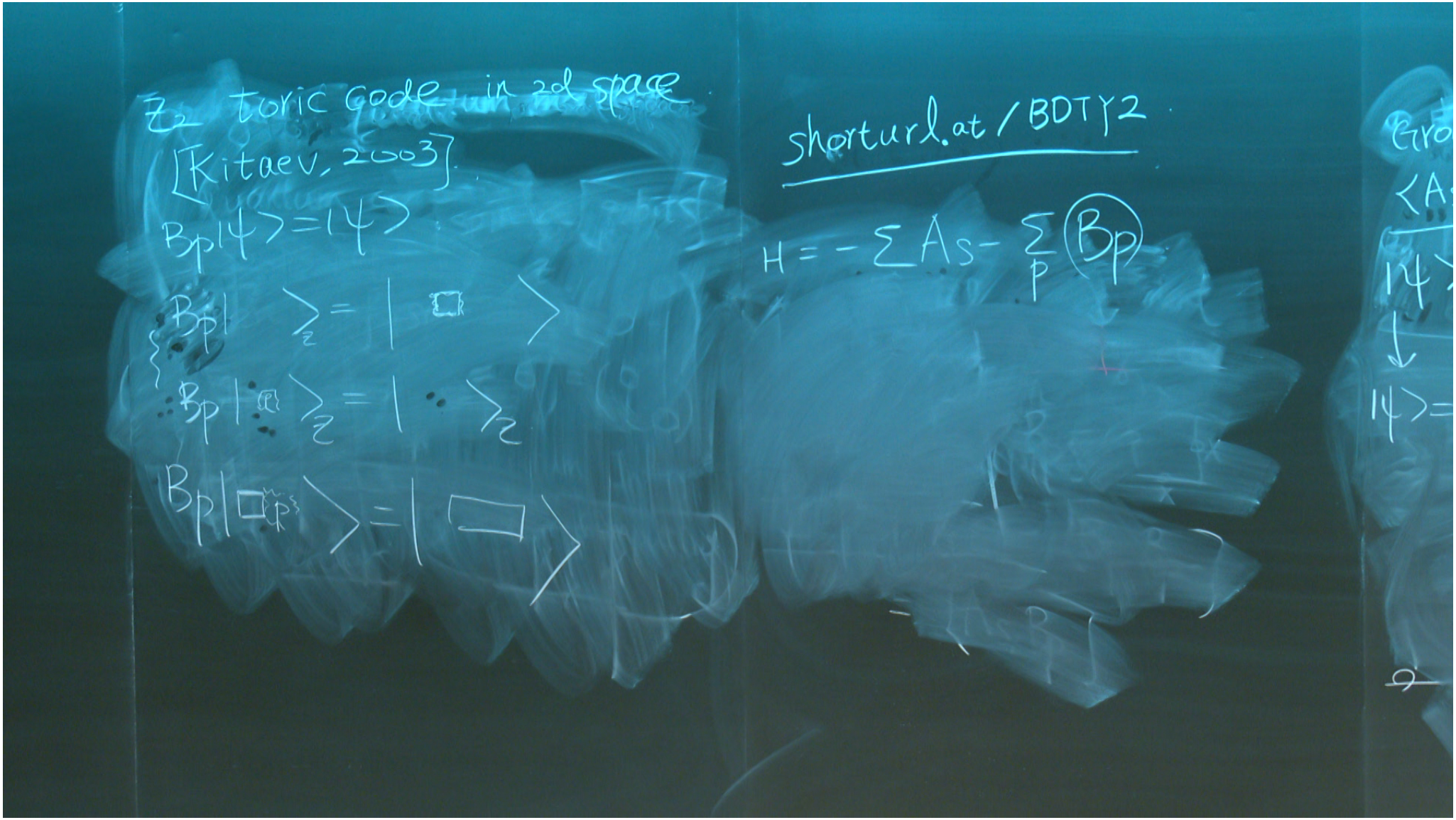

Z_2 toric code in 2d space
[Kitaev, 2003]
$s_1 s_2 s_3 s_4 = 1$
even # of spin down
$s = 1$
$s = -1$
shorturl.at/BDTY2
$H = -\sum A_s - \sum_p B_p$
$A_s = 1$

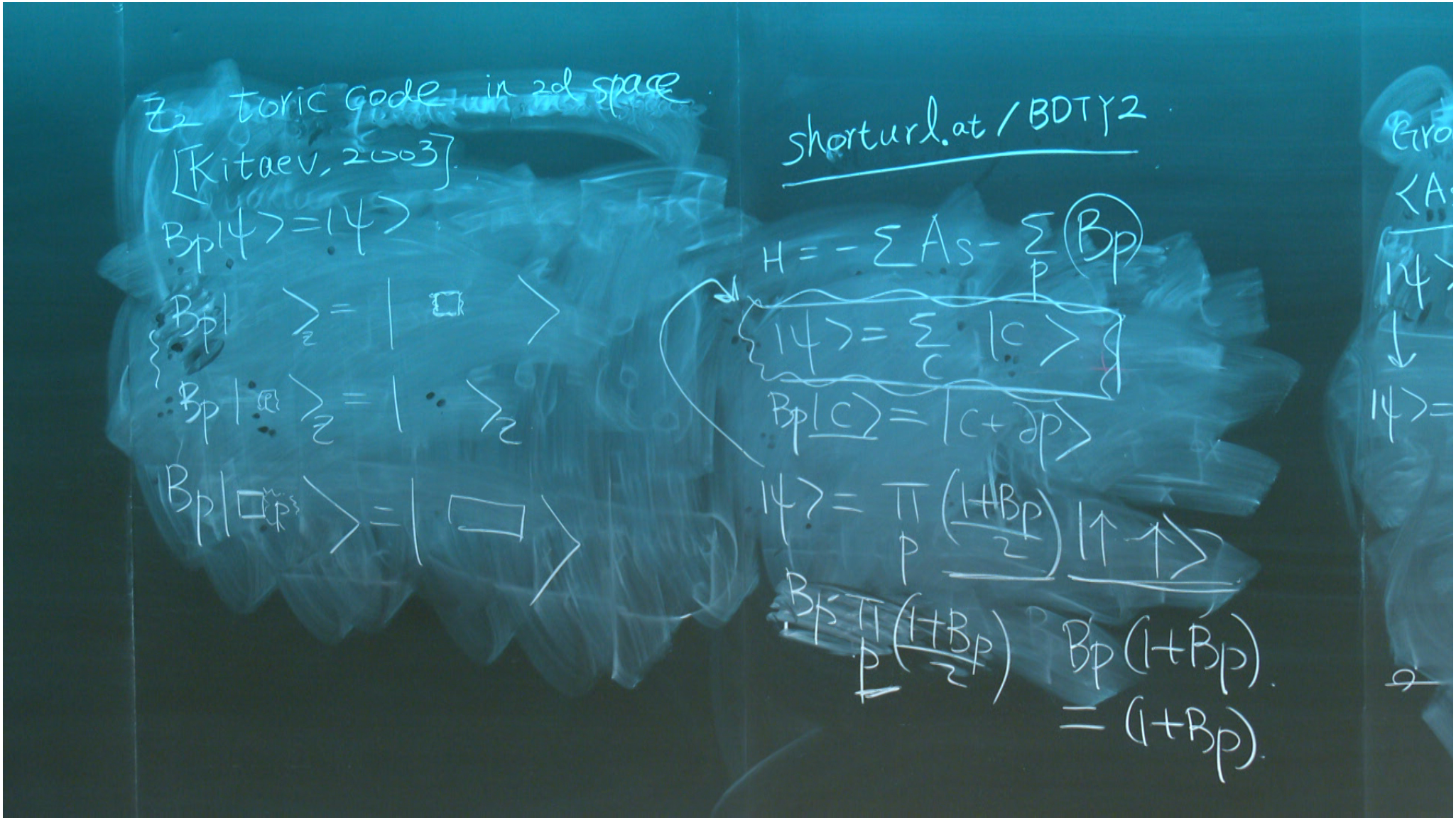

Z_2 toric code in 2d space
[Kitaev, 2003]
$s_1 s_2 s_3 s_4 = 1$
even # of spin down
$s=1$
$s=-1$
shorturl.at/BDTY2
$H = -\sum A_s - \sum_p B_p$
$A_s = 1$

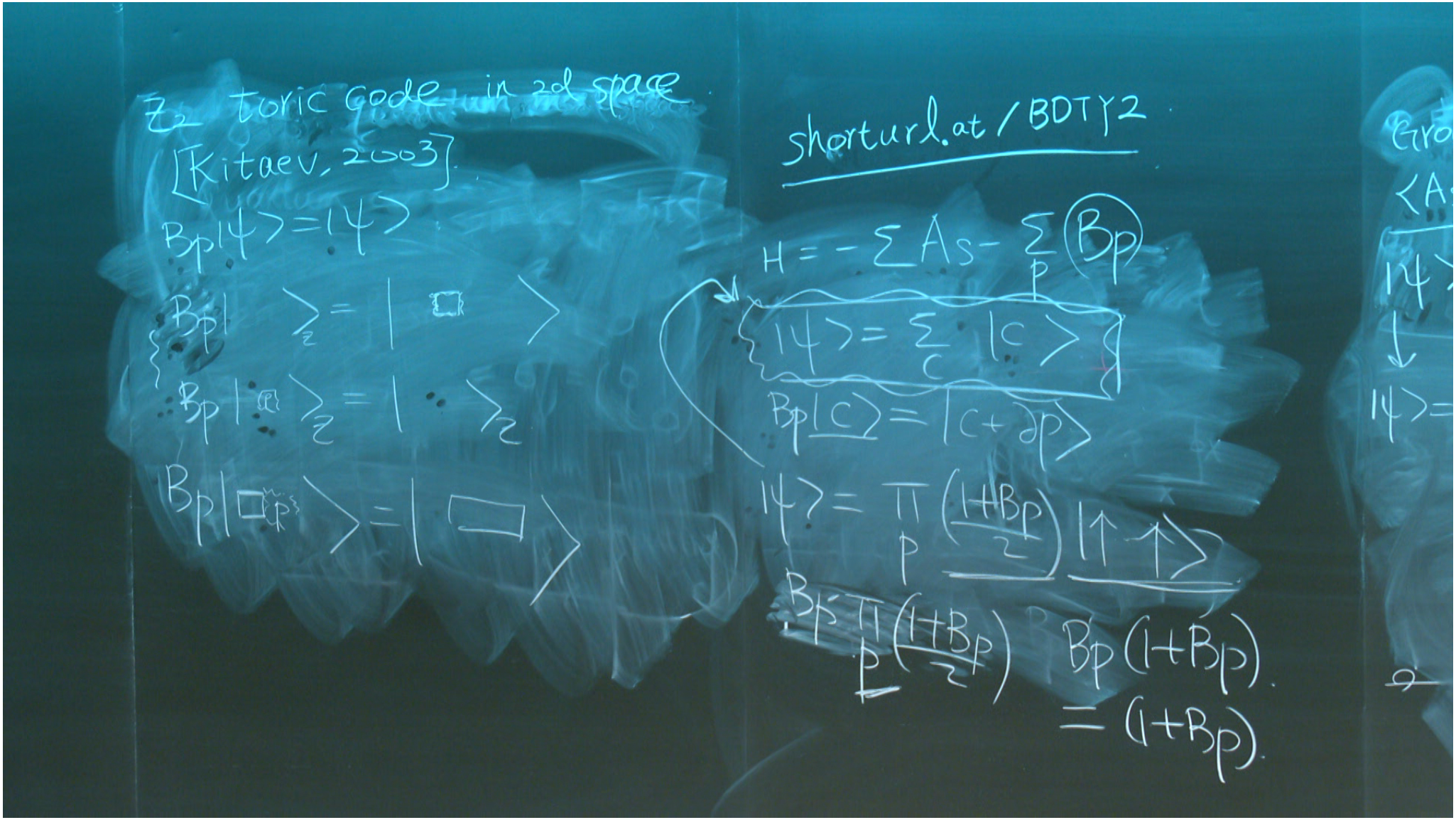

shorturl.at/BDTY2
H = -ΣAs - Σp Bp
As = 1
Ground state
⟨As⟩ = ⟨Bp⟩ = 1
|ψ⟩ = Σ{Si} ψ{Si} |{Si}⟩
z-basis
⊗i |Si⟩
Si = {1, ↑ , -1, ↓
|ψ⟩ = Σc ψ(c) |c⟩

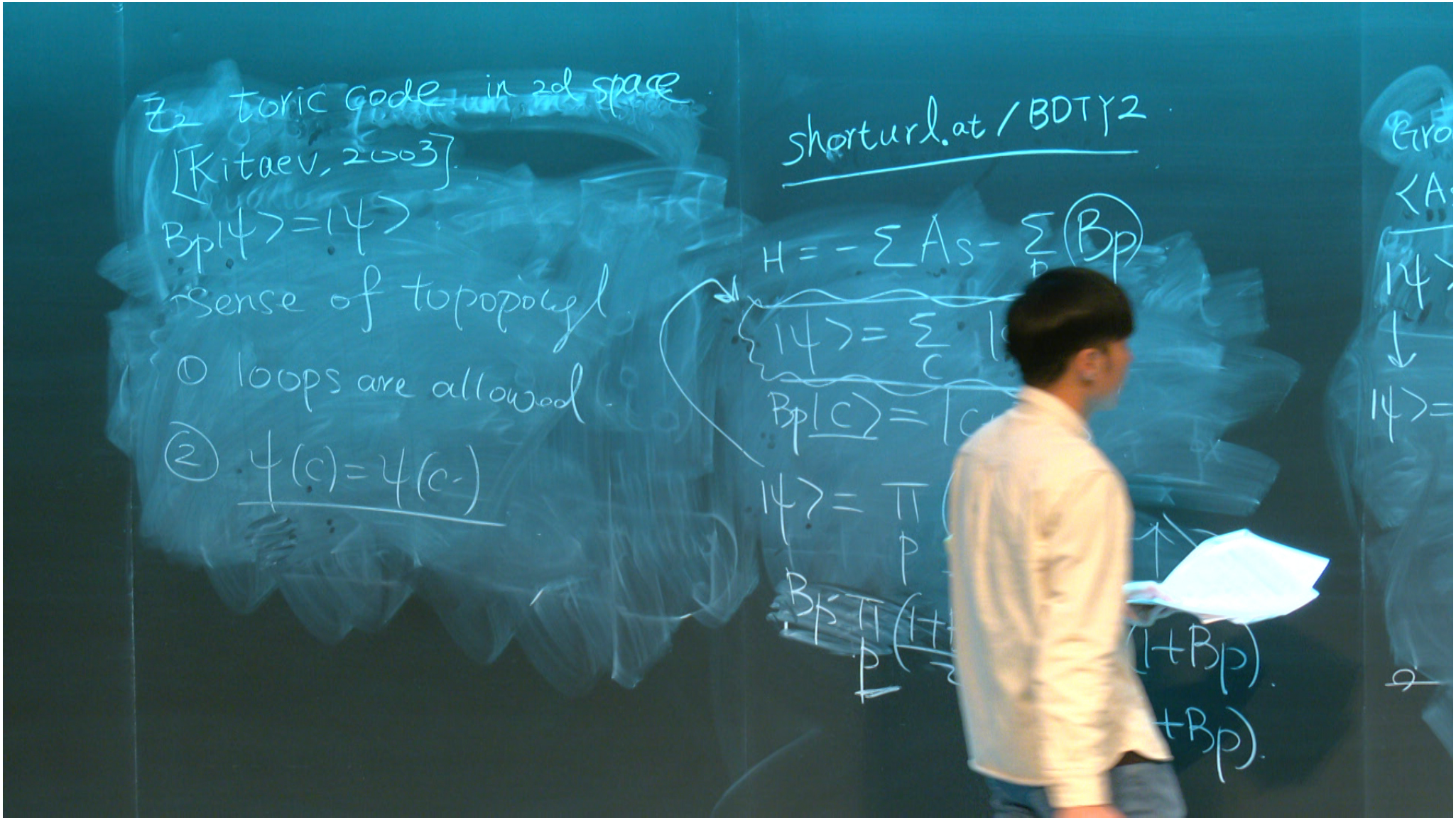

shorturl.at/BDTY2
H = -ΣAs - Σp Bp
As = 1
Ground state
⟨As⟩ = ⟨Bp⟩ = 1
|ψ⟩ = Σ{Si} ψ{Si} |{Si}⟩
z-basis
⊗i |Si⟩
Si = {1, ↑, -1, ↓}
|ψ⟩ = Σc ψ(c) |c⟩

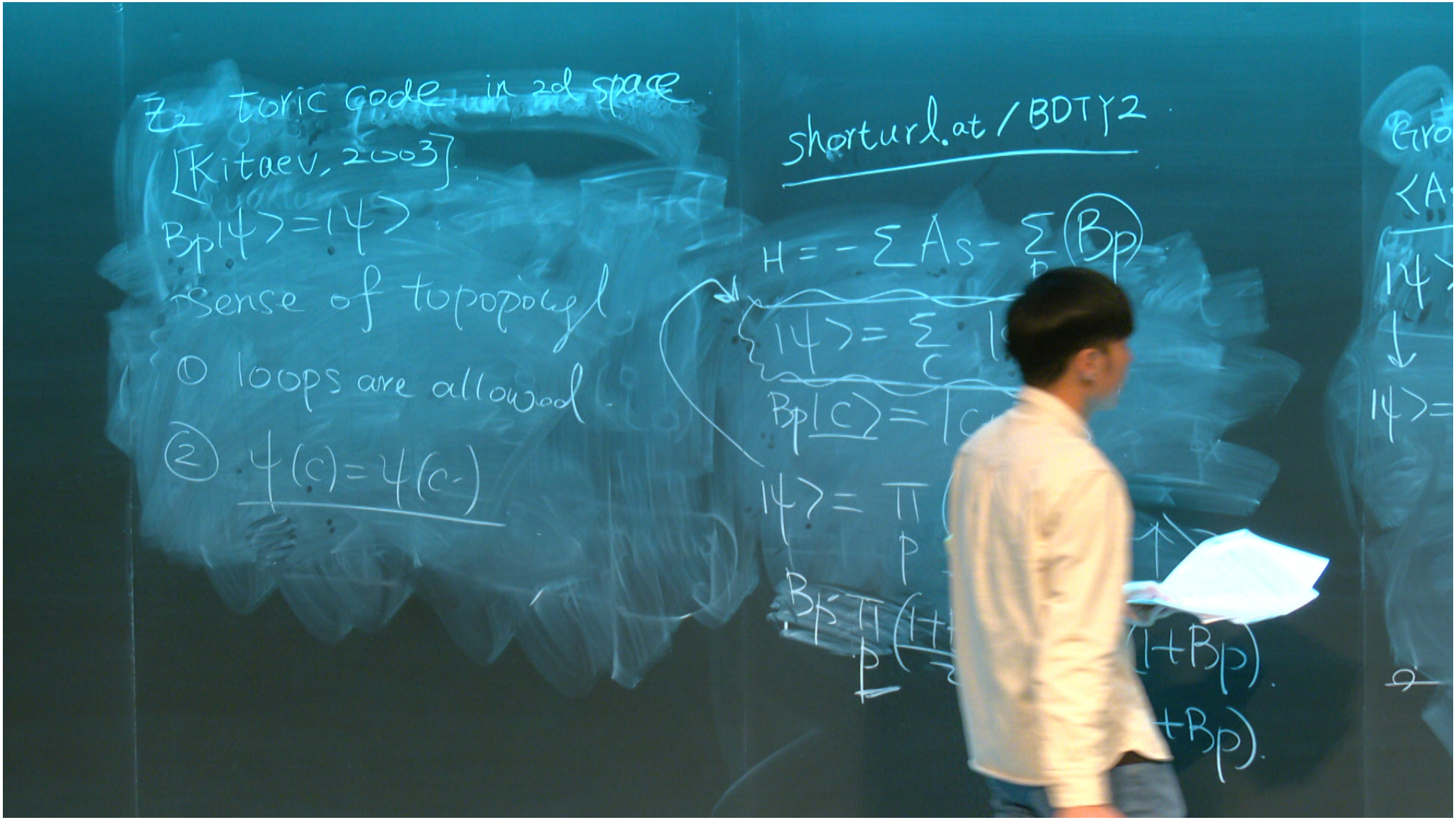

Z_2 toric code in 2d space
[Kitaev, 2003]
$B_p|\psi\rangle = |\psi\rangle$
$B_p|\ \rangle_2 = |\ \rangle$
$B_p|\ \rangle_2 = |\ \rangle_2$
$B_p|\ \rangle = |\ \rangle$
shorturl.at/BDTY2
$H = -\sum A_s - \sum_p B_p$
$|\psi\rangle$
$|\psi\rangle =$

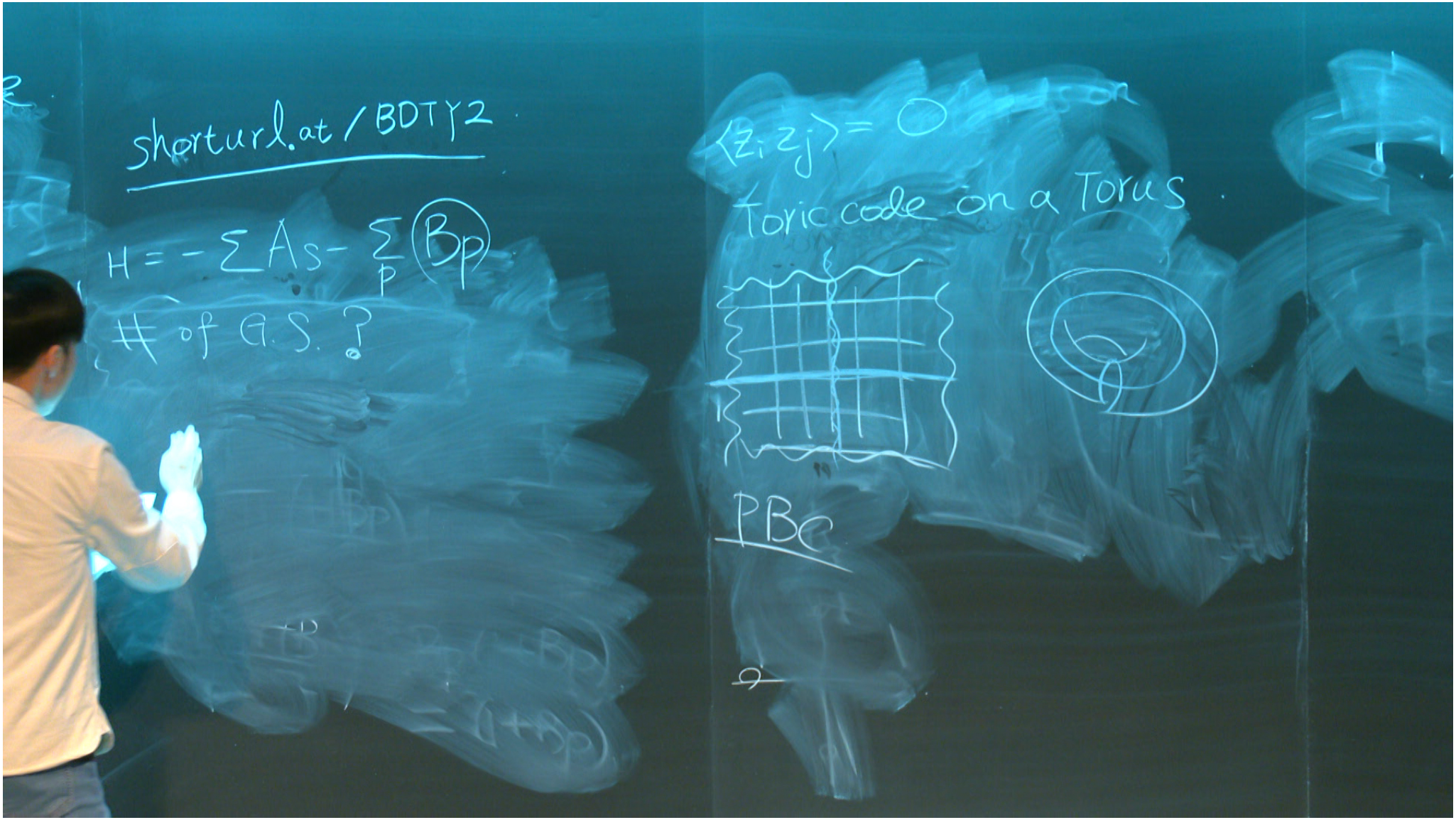

Z_2 toric code in 2d space
[Kitaev, 2003]
$B_p|\psi\rangle = |\psi\rangle$
$B_p|\ \rangle_2 = |\ \rangle$
$B_p|\ \rangle_2 = |\ \rangle_2$
$B_p|\ \rangle = |\ \rangle$
shorturl.at/BDTY2
$H = -\sum A_s - \sum_p B_p$
$|\psi\rangle$
$|\psi\rangle =$

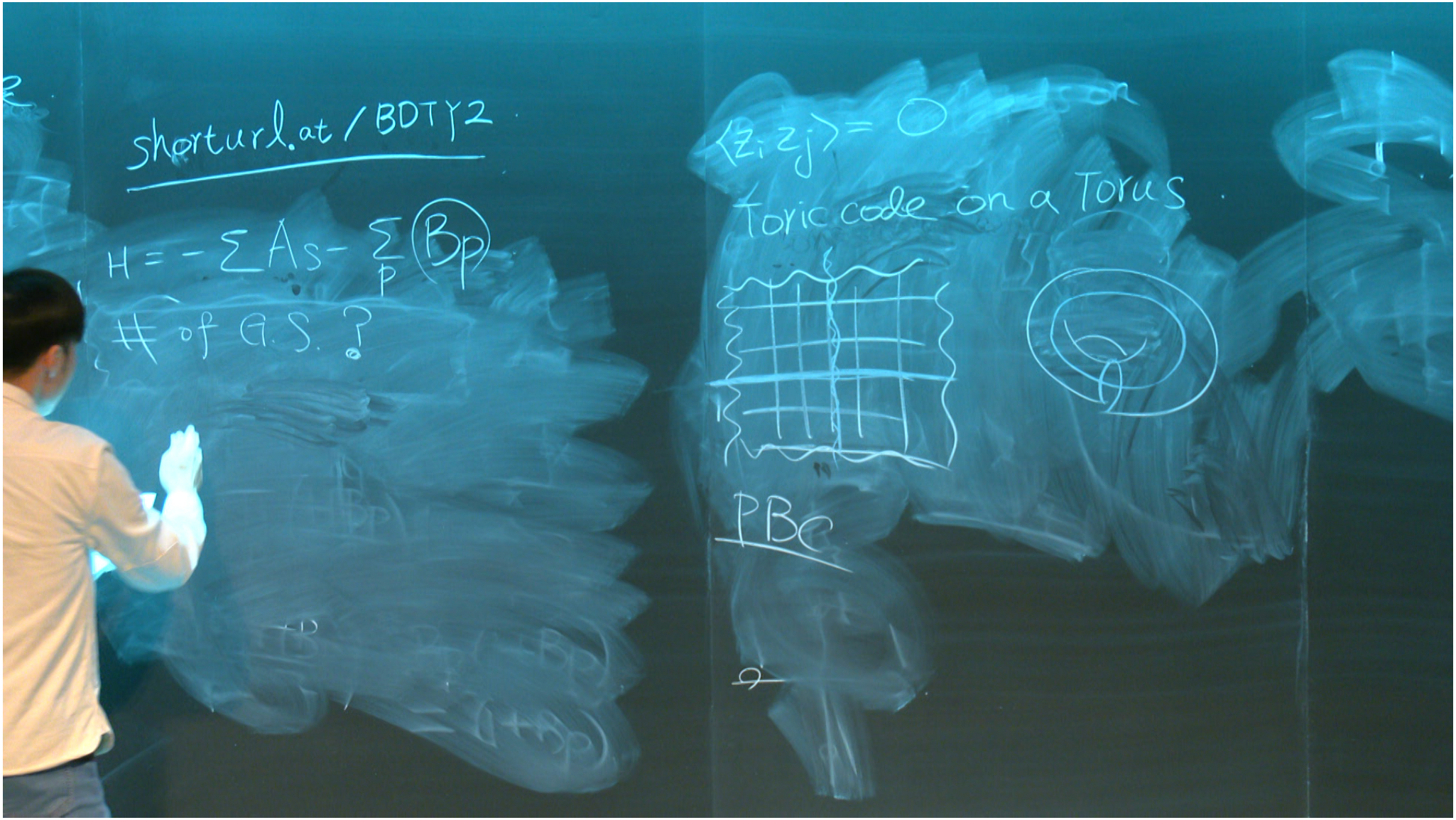

Z_2 toric code in 2d space
[Kitaev, 2003]
$B_p|\psi\rangle = |\psi\rangle$
$B_p|\quad\rangle_2 = |\quad\rangle$
$B_p|\quad\rangle_2 = |\quad\rangle_2$
$B_p|\quad\rangle = |\quad\rangle$
shorturl.at/BDTY2
$H = -\sum_s A_s - \sum_p B_p$
$|\psi\rangle = \sum_C |C\rangle$
$B_p|C\rangle = |C + \partial p\rangle$
$|\psi\rangle = \prod_p \left(\frac{1+B_p}{2}\right)|\uparrow\uparrow\rangle$
$B_p \prod_p \left(\frac{1+B_p}{2}\right) B_p(1+B_p)$
$= (1+B_p)$.

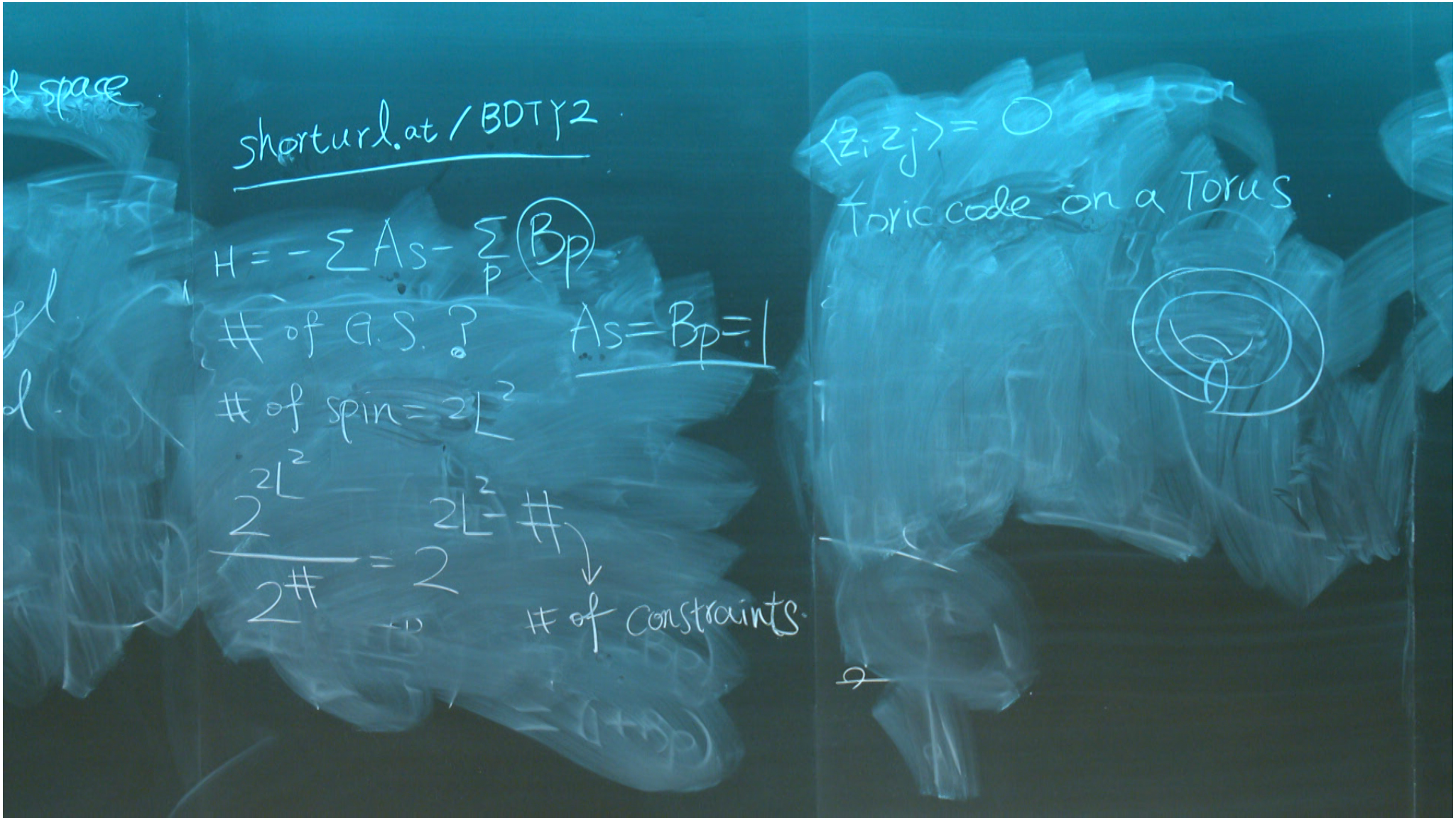

Z_2 toric code in 2d space
[Kitaev, 2003]
$B_p|\psi\rangle = |\psi\rangle$
$B_p|\quad\rangle_2 = |\quad\rangle$
$B_p|\quad\rangle_2 = |\quad\rangle_2$
$B_p|\quad\rangle = |\quad\rangle$
shorturl.at/BDTY2
$H = -\sum_s A_s - \sum_p B_p$
$|\psi\rangle = \sum_C |C\rangle$
$B_p|C\rangle = |C + \partial p\rangle$
$|\psi\rangle = \prod_p \left(\frac{1+B_p}{2}\right)|\uparrow\uparrow\rangle$
$B_p \prod_p \left(\frac{1+B_p}{2}\right)$
$B_p(1+B_p)$
$= (1+B_p)$.

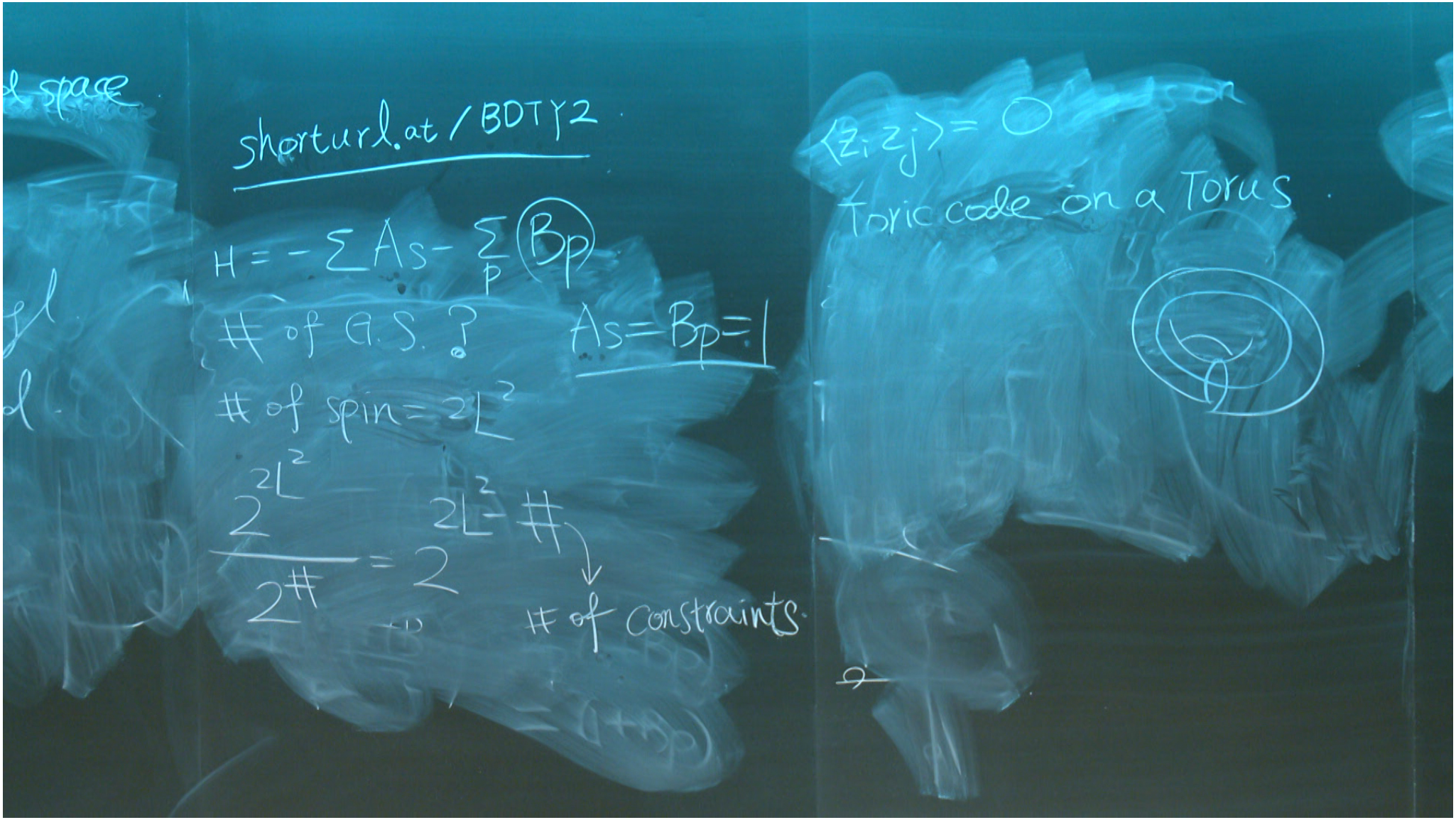

Z2 toric code in 2d space
[Kitaev, 2003]
Bp|ψ⟩=|ψ⟩
sense of topological
① loops are allowed
② ψ(c)=ψ(c')
shorturl.at/BDTY2
H=-ΣAs-ΣBp
|ψ⟩=Σ
Bp|c⟩=|c
|ψ⟩=Π
Bp Π(1+
(1+Bp)
+Bp)

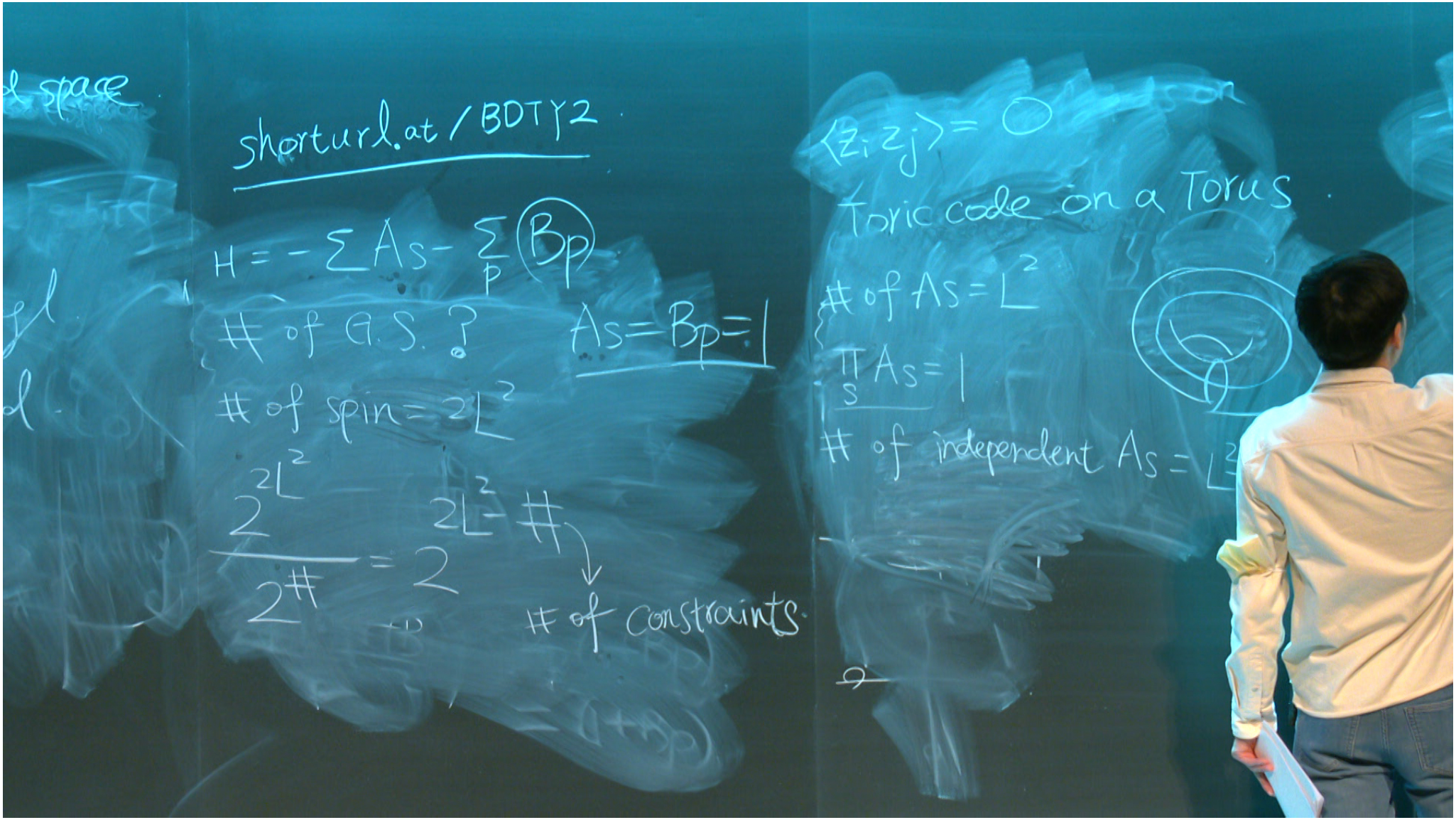

Z2 toric code in 2d space
[Kitaev, 2003]
Bp|ψ⟩=|ψ⟩
sense of topological
① loops are allowed
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shorturl.at/BDTY2
H=-ΣAs-ΣBp
|ψ⟩=Σ
Bp|c⟩=|c
|ψ⟩=Π
Bp
Π(1+
(1+Bp)
+Bp)

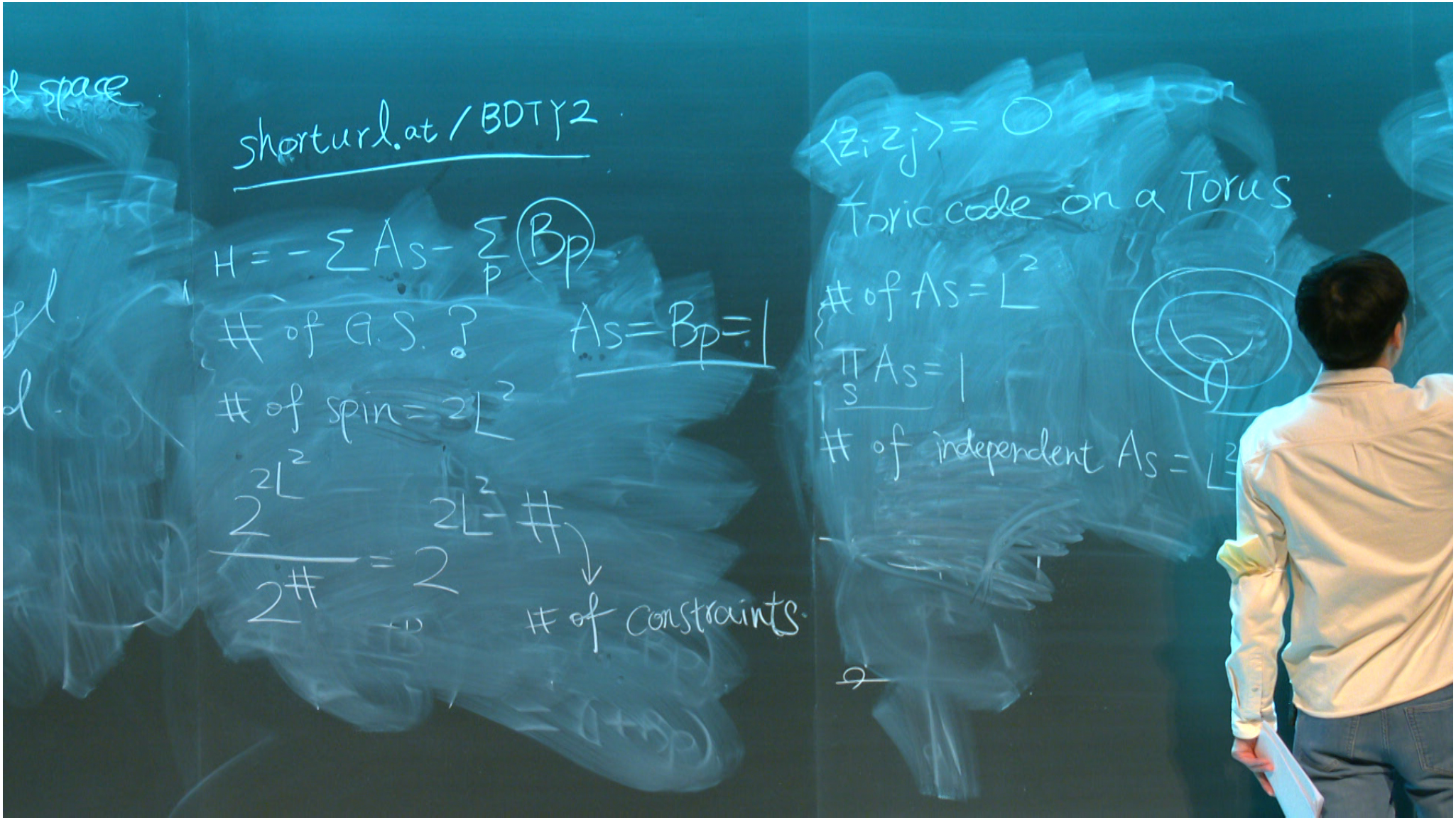

shorturl.at/BDTY2
H = - Σ As - Σ Bp
p
# of G.S. ?
⟨zi zj⟩ = 0
Toric code on a Torus
PBC

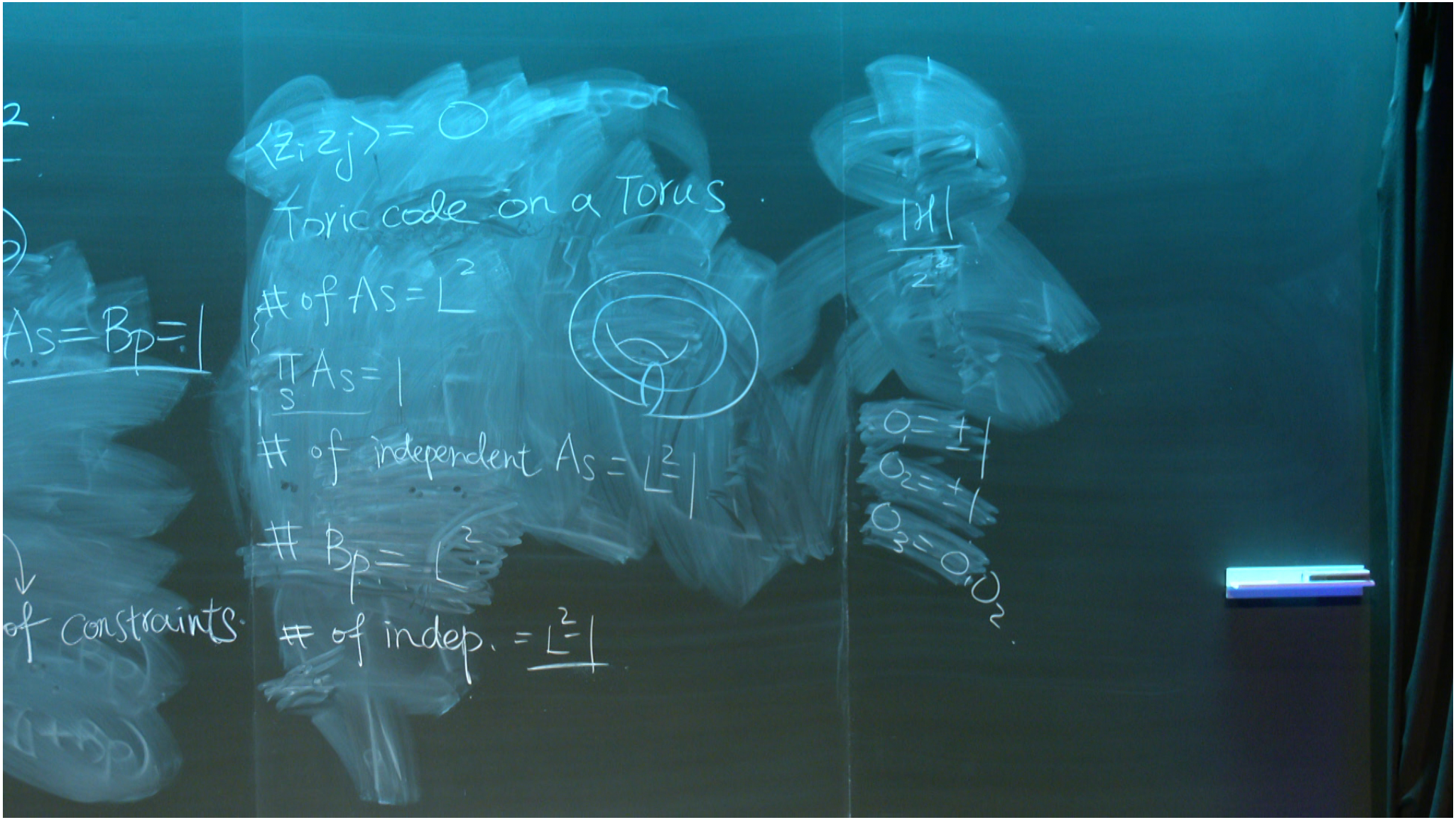

shorturl.at/BDTY2
H = -ΣAs - Σp Bp
# of G.S. ?
⟨zi zj⟩ = 0
Toric code on a Torus
PBC

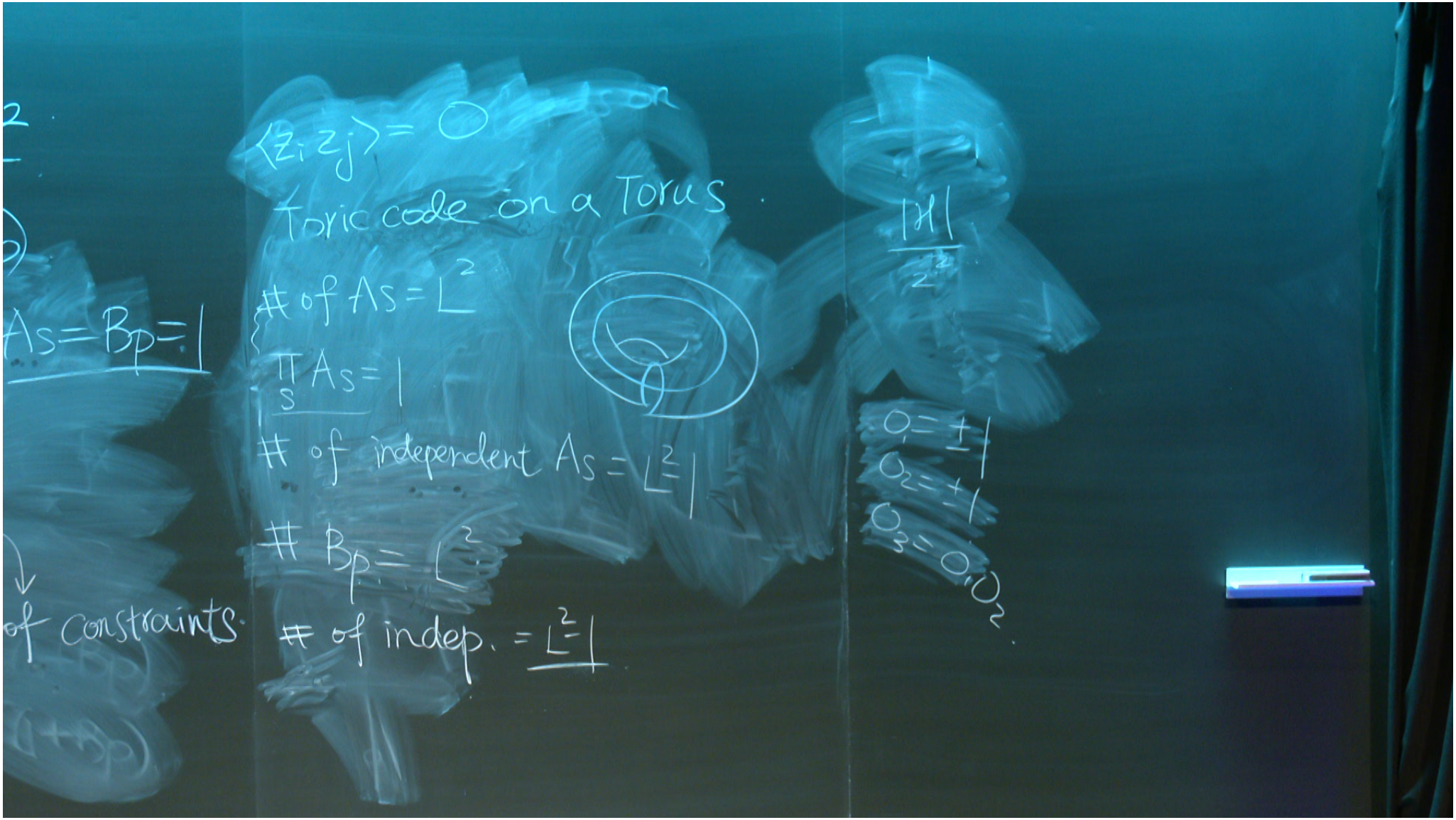

space
shorturl.at/BDTY2
$H = -\sum A_s - \sum_p B_p$
# of G.S. ?
$A_s = B_p = 1$
# of spin $= 2L^2$
$\frac{2^{2L^2}}{2^{\#}} = 2$
$2L^2 - \# =$
# of constraints
$\langle z_i z_j \rangle = 0$
Toric code on a Torus


space
shorturl.at/BDTY2
$H = -\sum A_s - \sum_p B_p$
# of G.S. ?
$A_s = B_p = 1$
# of spin $= 2L^2$
$\frac{2^{2L^2}}{2^{\#}} = 2^{2L^2 - \#}$
# of constraints
$\langle z_i z_j \rangle = 0$
Toric code on a Torus


shorturl.at/BDTY2
$H = -\sum A_s - \sum_p B_p$
# of G.S. ?
$A_s = B_p = 1$
# of spin $= 2L^2$
$\frac{2^{2L^2}}{2^{\#}} = 2$
$2L^2 - \# =$
# of constraints
$\langle z_i z_j \rangle = 0$
Toric code on a Torus
of $A_s = L^2$
$\prod_s A_s = 1$
# of independent $A_s = L^2$


shorturl.at/BDTY2
$H = -\sum A_s - \sum_p B_p$
# of G.S. ?
$A_s = B_p = 1$
# of spin $= 2L^2$
$\frac{2^{2L^2}}{2^{\#}} = 2$
$2L^2 - \#$
# of constraints
$\langle z_i z_j \rangle = 0$
Toric code on a Torus
of $A_s = L^2$
$\prod_s A_s = 1$
# of independent $A_s = L^2$


$\langle z_i z_j \rangle = 0$
Toric code on a Torus
# of As $= L^2$
$\prod_s A_s = 1$
# of independent As $= L^2 - 1$
$B_p = L^2$
of indep. $= L^2 - 1$
$A_s = B_p = 1$
of constraints.
$\frac{|H|}{2^2}$
$O_1 = \pm 1$
$O_2 = \pm 1$
$O_3 = O_1 O_2$


$\langle z_i z_j \rangle = 0$
Toric code on a Torus
# of As = L^2
$\prod_s A_s = 1$
# of independent As = $L^2 - 1$
$B_p = L^2$
of indep. = $L^2 - 1$
$|\mathcal{H}|$
2^2
$O_1 = \pm 1$
$O_2 = \pm 1$
$O_3 = O_1 O_2$
$A_s = B_p = 1$
of constraints.

shorturl.at/BDTY2

$H = -\sum A_s - \sum_p B_p$

# of G.S. ? $A_s = B_p = 1$

# of spin $= 2L^2$

$\frac{2^{2L^2}}{2^{\#}} = 2^{2L^2 - \#}$ → # of constraints.

$2^{2L^2 - (L^2 - 1) - (L^2 - 1)} = 2^2 = 4$

$\langle z_i z_j \rangle = 0$

Toric code on a Torus.

of $A_s = L^2$

$\prod_s A_s = 1$

# of independent $A_s = L^2 - 1$

$B_p = L^2$

of indep. $= L^2 - 1$

$\frac{|\mathcal{H}|}{2^2}$

$O_1 = \pm 1$

$O_2 = \pm 1$

$O_3 = O_1 O_2$

shorturl.at/BDTY2

$H = -\sum A_s - \sum_p B_p$

# of G.S. ? $A_s = B_p = 1$

# of spin $= 2L^2$

$\frac{2^{2L^2}}{2^{\#}} = 2^{2L^2 - \#}$ → # of constraints.

$2L^2 - (L^2 - 1) - (L^2 - 1) = 2^2 = 4$

$\langle z_i z_j \rangle = 0$

Toric code on a Torus.

of $A_s = L^2$

$\prod_s A_s = 1$

# of independent $A_s = L^2 - 1$

$B_p = L^2$

of indep. $= L^2 - 1$

$\frac{|H|}{2^2}$

$O_1 = \pm 1$

$O_2 = \pm 1$

$O_3 = O_1 O_2$

Z_2 toric code in 2d space

[Kitaev, 2003]

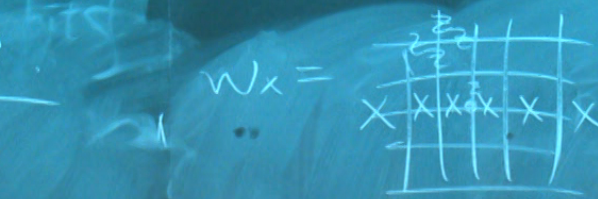
$|0,0\rangle = \prod_p (1+B_p)\,|\uparrow\cdots\uparrow\rangle$

$= \sum_C |C\rangle$

contract.

shorturl.at/BDTY2

$A_s = B_p = 1$

$W_x =$ 

$A_s W_x \prod_p (1+B_p)|\uparrow\cdots\uparrow\rangle$

$= W_x A_s \prod_p (1+B_p)|\uparrow\cdots\uparrow\rangle$

$W_y =$ 

$|1,0\rangle = W_x|0,0\rangle$

$= W_x \prod_p (1+B_p)|0,0\rangle$

$= \sum_C |C\rangle$

C winds around γ

$\langle z_i z_j$

Toric

of

$\prod_s A_s$

of

B_p

of

Z_2 toric code in 2d space

[Kitaev, 2003]

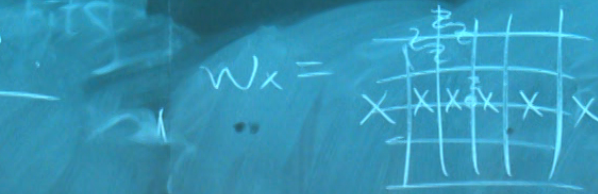
$|0,0\rangle = \prod_p (1+B_p)\,|\uparrow\cdots\uparrow\rangle$

$= \sum_C |C\rangle$

contract.

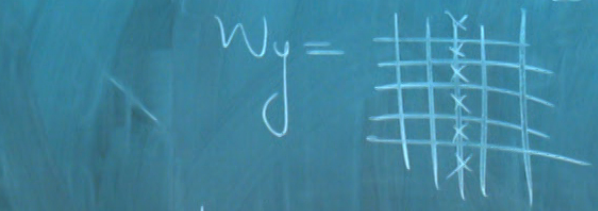
shorturl.at/BDTY2

$A_s = B_p = 1$

$W_x =$ 

$A_s W_x \prod_p (1+B_p)|\uparrow\cdots\uparrow\rangle$

$= W_x A_s \prod_p (1+B_p)|\uparrow\cdots\uparrow\rangle$

$W_y =$ 

$|1,0\rangle = W_x|0,0\rangle$

$= W_x \prod_p (1+B_p)|0,0\rangle$

$= \sum_C |C\rangle$

C winds around γ

$\langle z_i z_j$

Toric

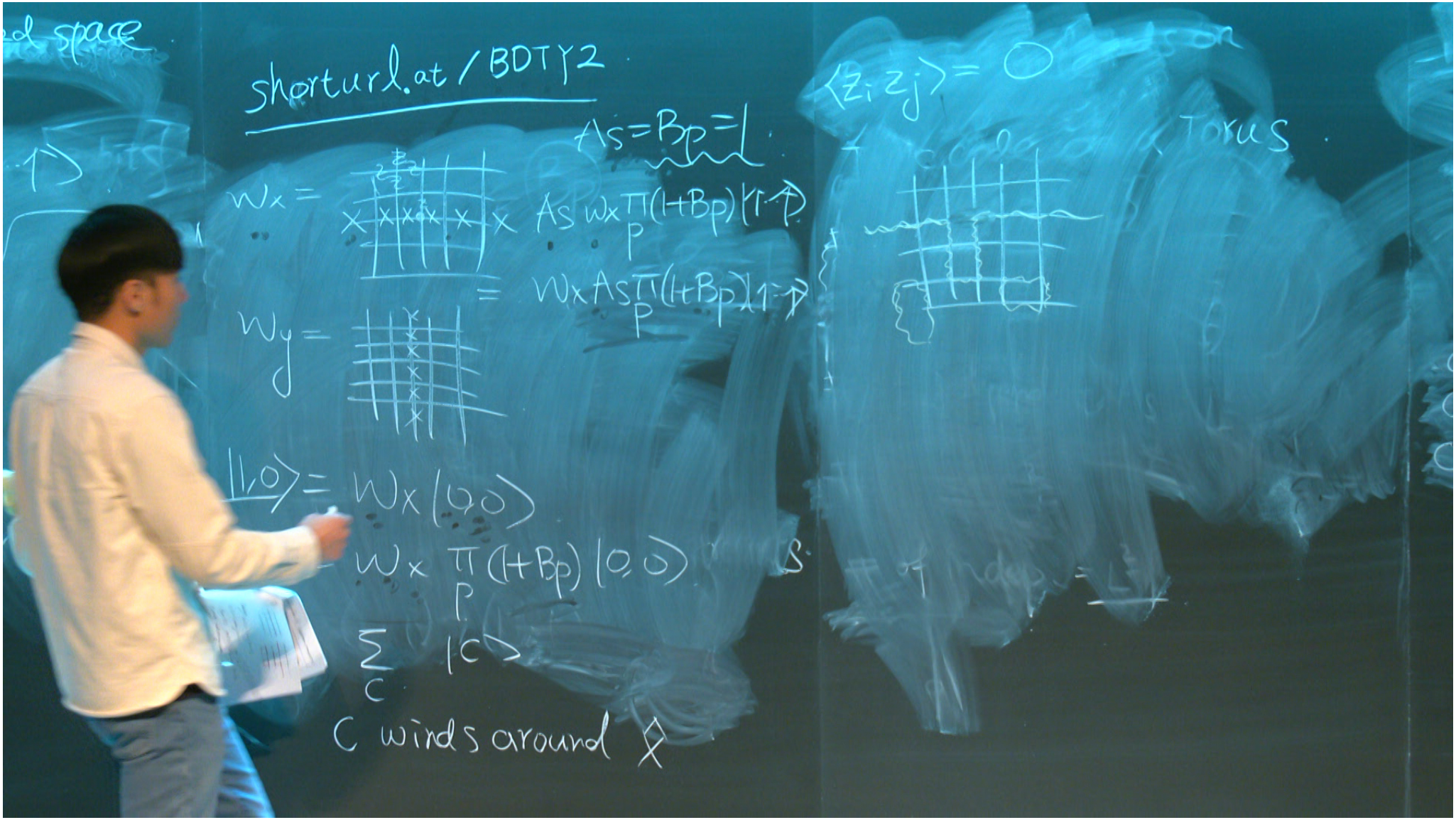
of

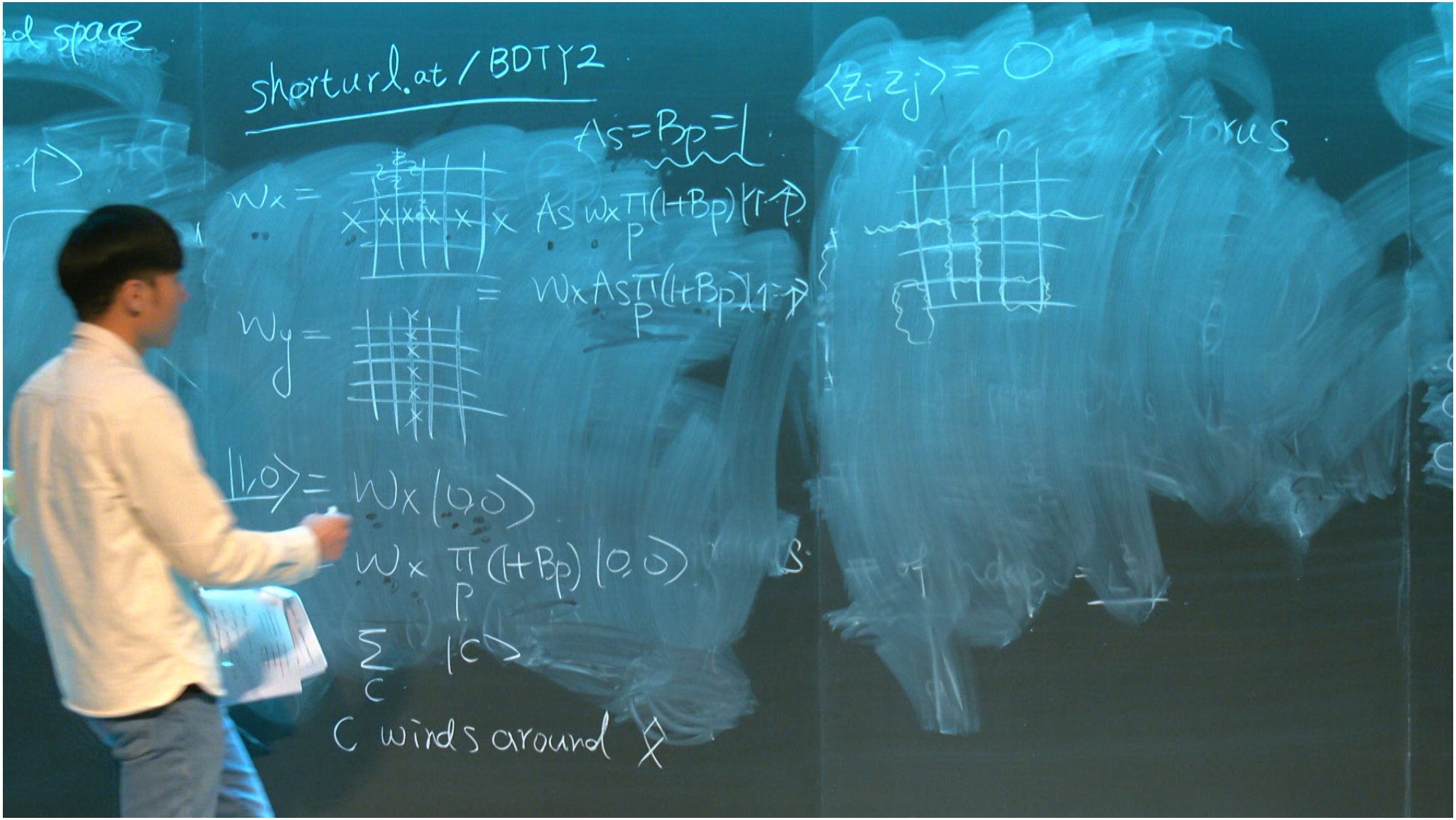
$\prod_s A_s$

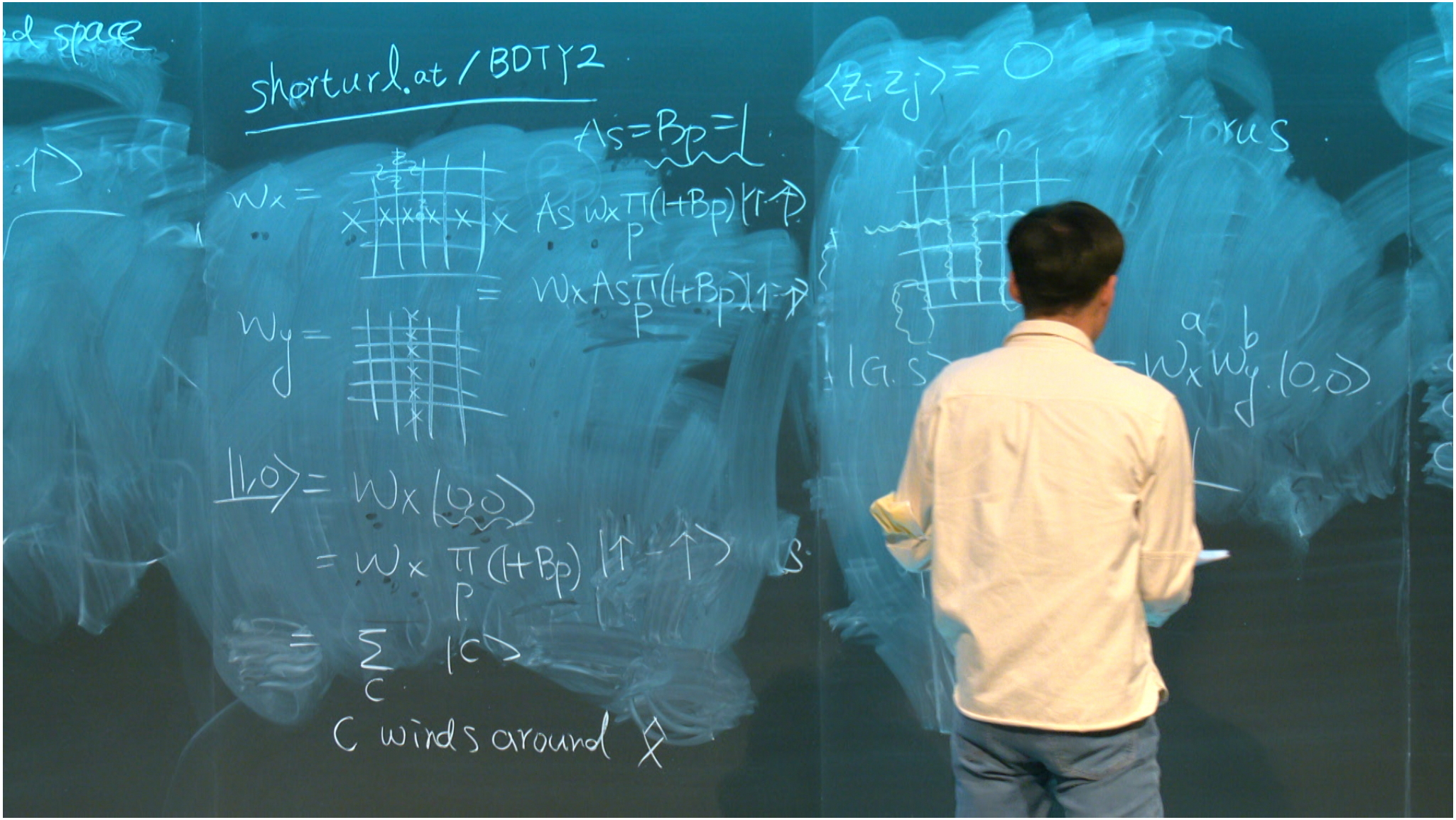
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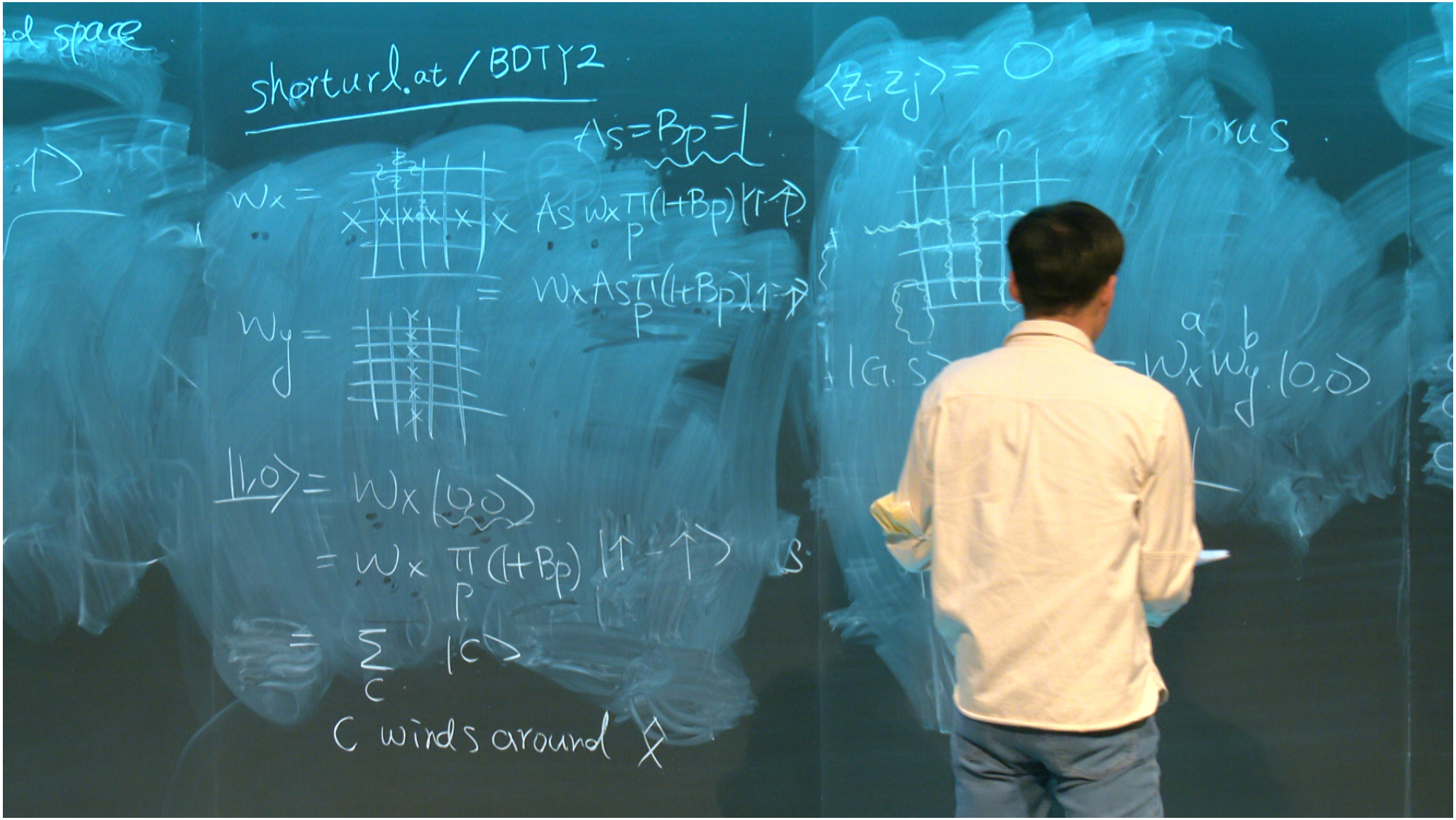
B_p

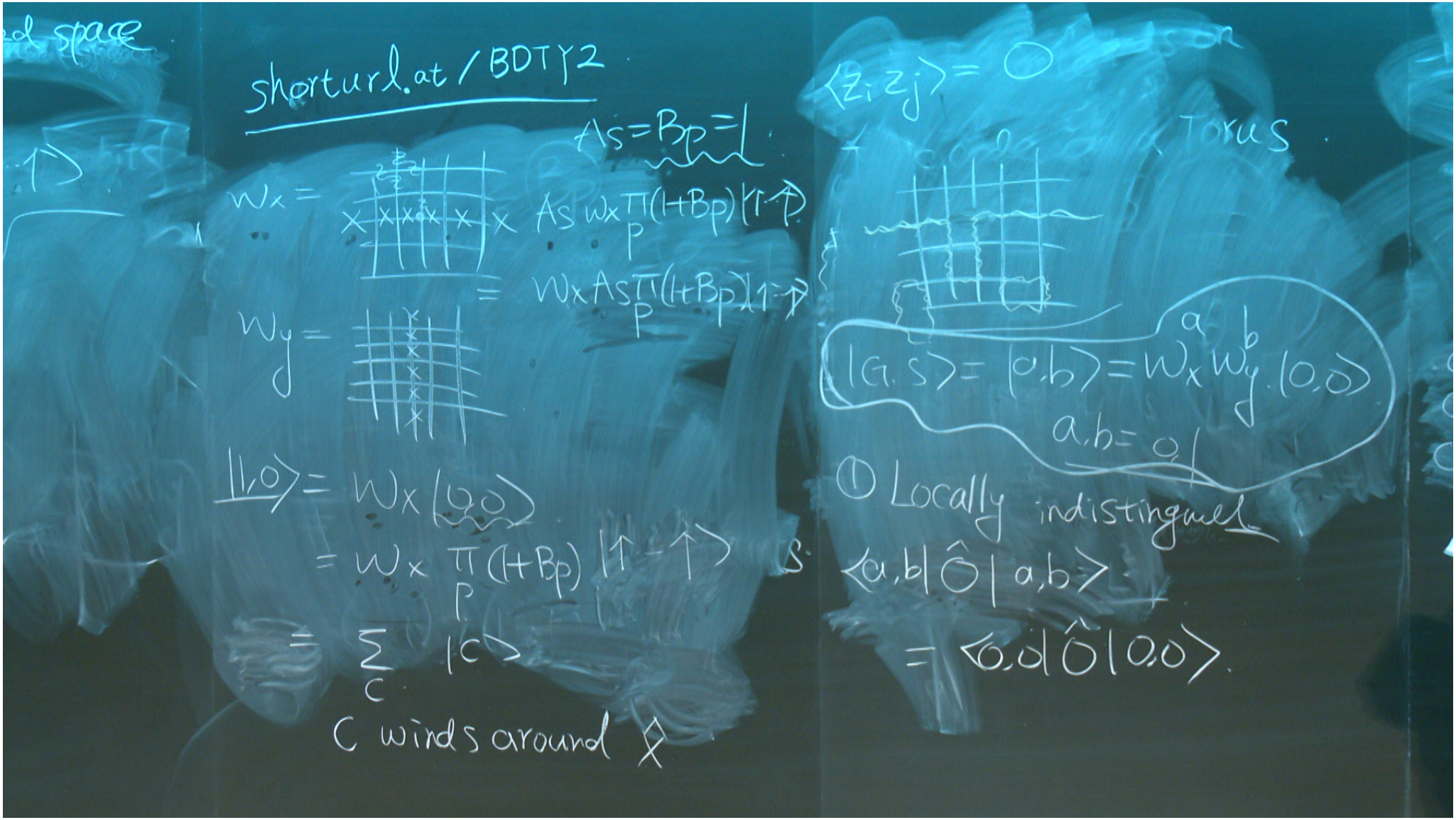
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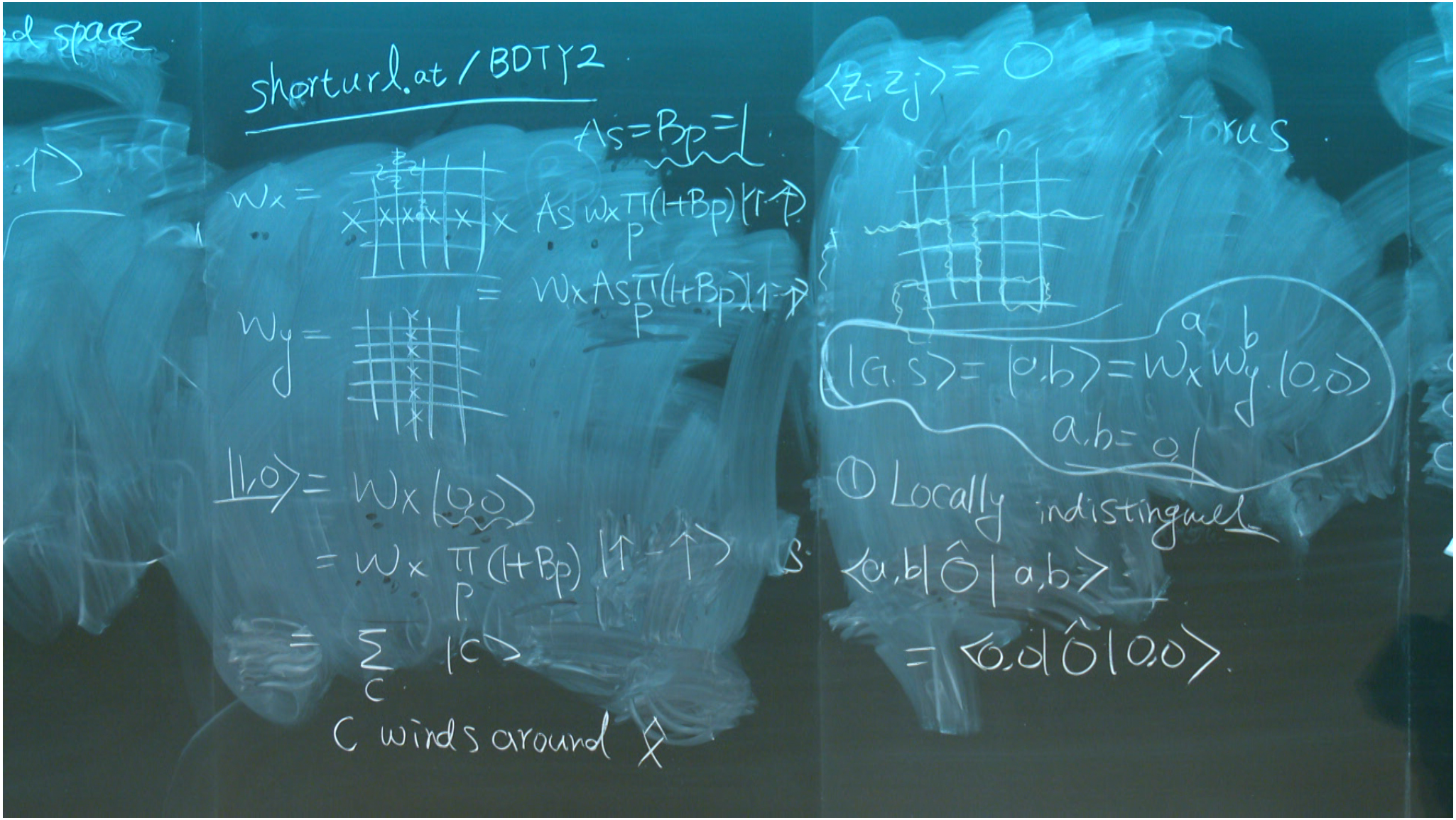

shorturl.at/BDTY2
As = Bp = 1
Wx =
As Wx Π_p (1+Bp) |1⟩
= Wx As Π_p (1+Bp) |1⟩
Wy =
|1,0⟩ = Wx |0,0⟩
Wx Π_p (1+Bp) |0,0⟩
Σ_C |C⟩
C winds around
⟨zi zj⟩ = 0
Torus

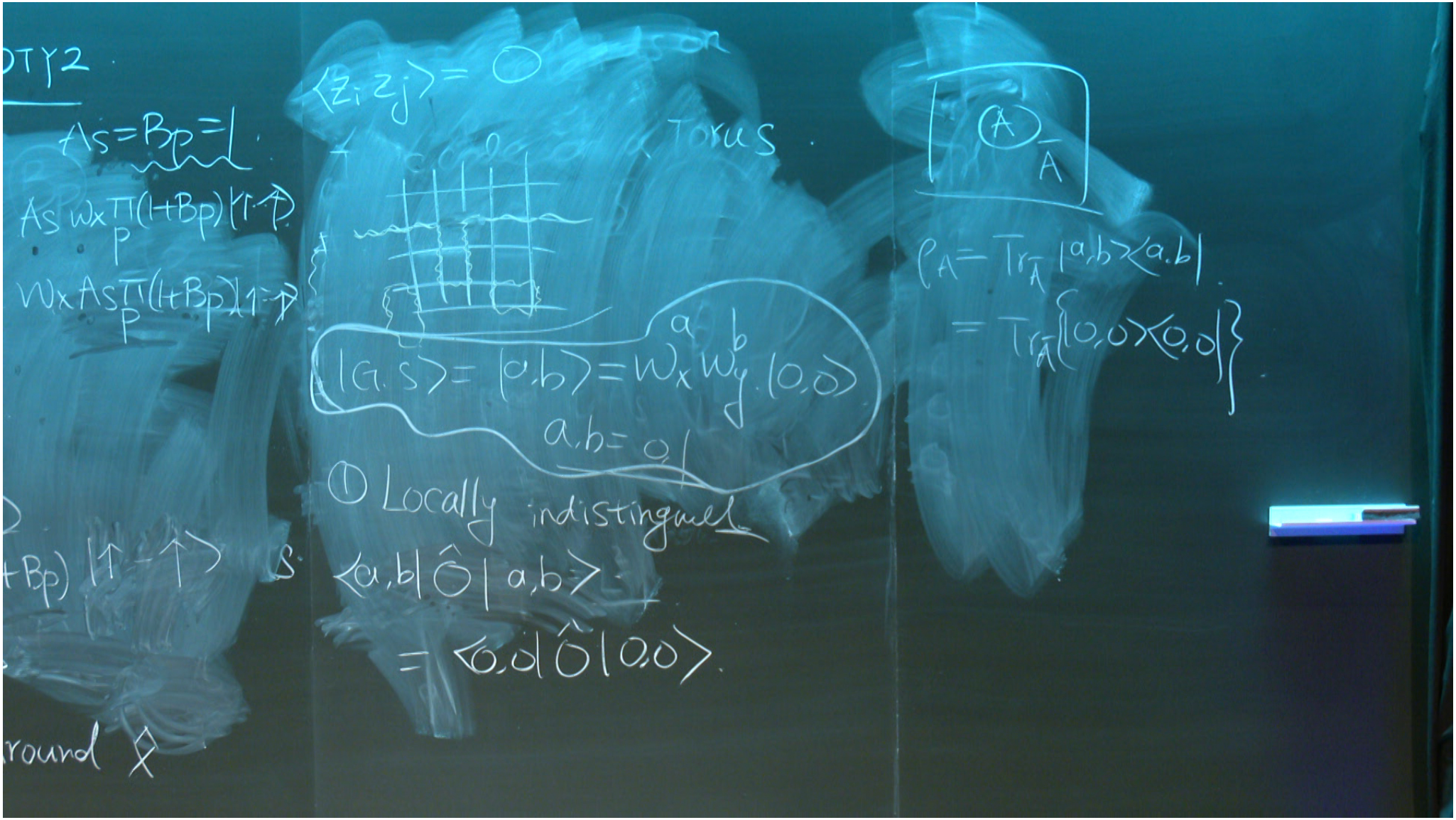

shorturl.at/BDTY2
As = Bp = 1
Wx =
As Wx Π (1+Bp) |1⟩
p
= Wx As Π (1+Bp) |1⟩
p
Wy =
|1,0⟩ = Wx |0,0⟩
Wx Π (1+Bp) |0,0⟩
p
Σ |C⟩
C
C winds around
⟨zi zj⟩ = 0
Torus

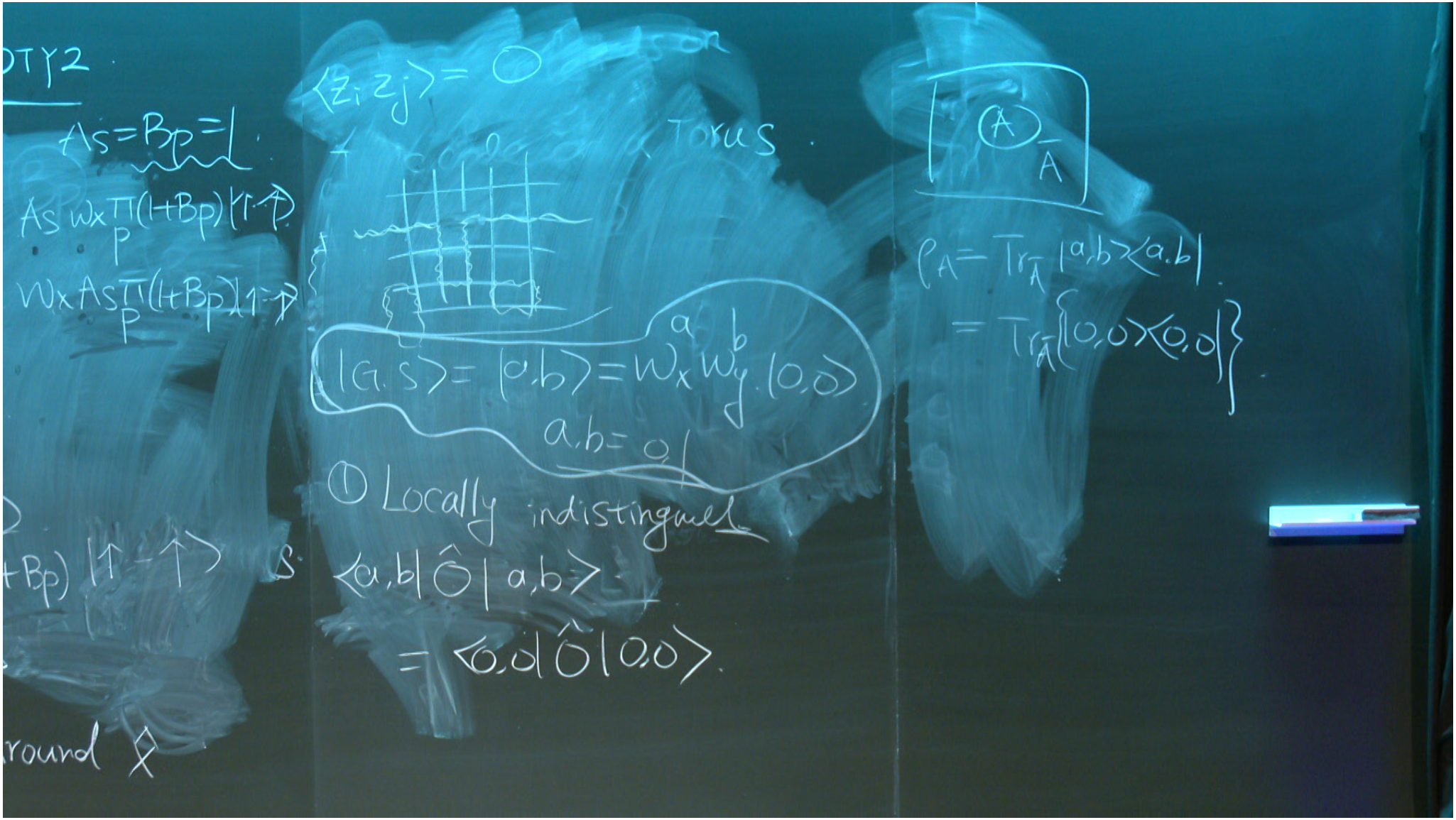

shorturl.at/BDTY2
$\langle z_i z_j \rangle = 0$
Torus
$A_s = B_p = 1$
$W_x =$
$A_s W_x \prod_p (1+B_p)|\uparrow\rangle$
$= W_x A_s \prod_p (1+B_p)|\uparrow\rangle$
$W_y =$
$|\psi_0\rangle = W_x(|\psi_0\rangle)$
$= W_x \prod_p (1+B_p)|\uparrow\rangle$
$= \sum_C |C\rangle$
C winds around


shorturl.at/BDTY2
As = Bp = 1
Wx =
As Wx Π (1+Bp) |1⟩
p
= Wx As Π (1+Bp) |1⟩
p
Wy =
|1,0⟩ = Wx |0,0⟩
= Wx Π (1+Bp) |↑ ... ↑⟩
p
= Σ |c⟩
c
c winds around
⟨zi zj⟩ = 0
Torus
|G.S⟩
Wx^a Wy^b |0,0⟩


d space
shortcut.at / BDTY2
$A_s = B_p = 1$
$W_x =$
$A_s W_x \prod_p (1+B_p)|1\rangle$
$= W_x A_s \prod_p (1+B_p)|1\rangle$
$W_y =$
$|1,0\rangle = W_x|0,0\rangle$
$= W_x \prod_p (1+B_p)|\uparrow\uparrow\rangle$
$= \sum_C |C\rangle$
C winds around
$\langle z_i z_j \rangle = 0$
Torus
$|a,s\rangle = |a,b\rangle = W_x^a W_y^b |0,0\rangle$
$a,b = 0,1$
① Locally indistinguish
$\langle a,b|\hat{O}|a,b\rangle$
$= \langle 0,0|\hat{O}|0,0\rangle$

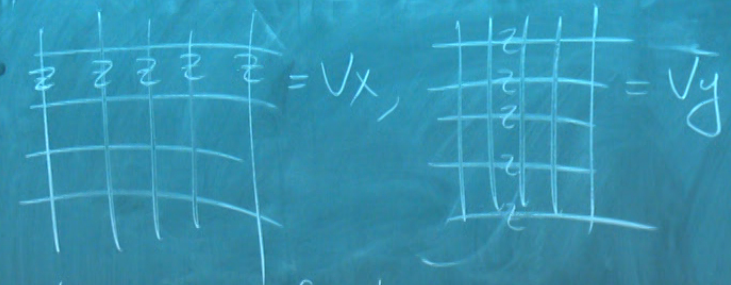

d space
shortcut.at / BDTY2
$A_s = B_p = 1$
$W_x =$
$A_s W_x \prod_p (1+B_p)|1\rangle$
$= W_x A_s \prod_p (1+B_p)|1\rangle$
$W_y =$
$|1,0\rangle = W_x|0,0\rangle$
$= W_x \prod_p (1+B_p)|\uparrow\uparrow\rangle$
$= \sum_C |C\rangle$
C winds around
$\langle z_i z_j \rangle = 0$
Torus
$|a,b\rangle = |a,b\rangle = W_x^a W_y^b |0,0\rangle$
$a,b = 0,1$
① Locally indistinguish
$\langle a,b|\hat{O}|a,b\rangle$
$= \langle 0,0|\hat{O}|0,0\rangle$


$\langle z_i z_j \rangle = 0$
Torus
$|G.S\rangle = |a,b\rangle = W_x^a W_y^b |0,0\rangle$
$a,b = 0,1$
① Locally indistinguish
$\langle a,b|\hat{O}|a,b\rangle$
$= \langle 0,0|\hat{O}|0,0\rangle$
$\rho_A = Tr_{\bar{A}} |a,b\rangle\langle a,b|$
$= Tr_{\bar{A}} \{|0,0\rangle\langle 0,0|\}$


$\langle z_i z_j \rangle = 0$
Torus
$|G.S\rangle = |a,b\rangle = W_x^a W_y^b |0,0\rangle$
$a,b = 0,1$
① Locally indistinguish
$\langle a,b|\hat{O}|a,b\rangle$
$= \langle 0,0|\hat{O}|0,0\rangle$
$\rho_A = Tr_{\bar{A}} |a,b\rangle\langle a,b|$
$= Tr_{\bar{A}} \{|0,0\rangle\langle 0,0|\}$

Z_2 toric code in 2d space

[Kitaev, 2003]



$= V_x$, $= V_y$

$|a,b\rangle = W_x^a W_y^b |a,b\rangle$

$|0,0\rangle \Rightarrow V_x = V_y = 1$

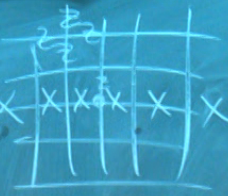
$|0,1\rangle \Rightarrow V_x = -1, V_y = 1$

$|1,0\rangle \Rightarrow V_x = 1, V_y = -1$

$|1,1\rangle \Rightarrow V_x = V_y = -1$

shorturl.at/BDTY2

$A_s = B_p = 1$

$W_x =$  $A_s W_x \prod_p (1+B_p) |\uparrow\uparrow\rangle$

$= W_x A_s \prod_p (1+B_p) |\uparrow\uparrow\rangle$

$W_y =$ 

$|1,0\rangle = W_x |0,0\rangle$

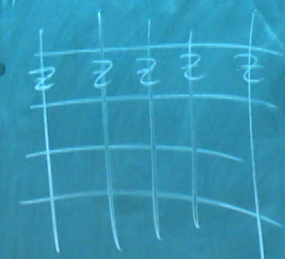
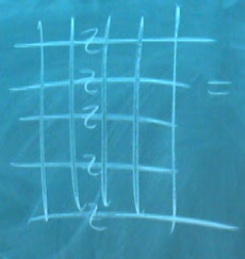
$= W_x \prod_p (1+B_p) |\uparrow\uparrow\rangle$

$= \sum_C |C\rangle$

C winds around χ

Z_2 toric code in 2d space

[Kitaev, 2003]


$= V_x$,

$= V_y$

$|a,b\rangle = W_x^a W_y^b |a,b\rangle$

$|0,0\rangle$ $V_x = V_y = 1$

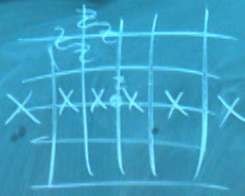
$|0,1\rangle \Rightarrow V_x = -1, V_y = 1$

$|1,0\rangle \Rightarrow V_x = 1, V_y = -1$

$|1,1\rangle \Rightarrow V_x = V_y = -1$

shorturl.at/BDTY2

$A_s = B_p = 1$

$W_x =$

$A_s W_x \prod_p (1+B_p) |\uparrow\uparrow\rangle$

$= W_x A_s \prod_p (1+B_p) |\uparrow\uparrow\rangle$

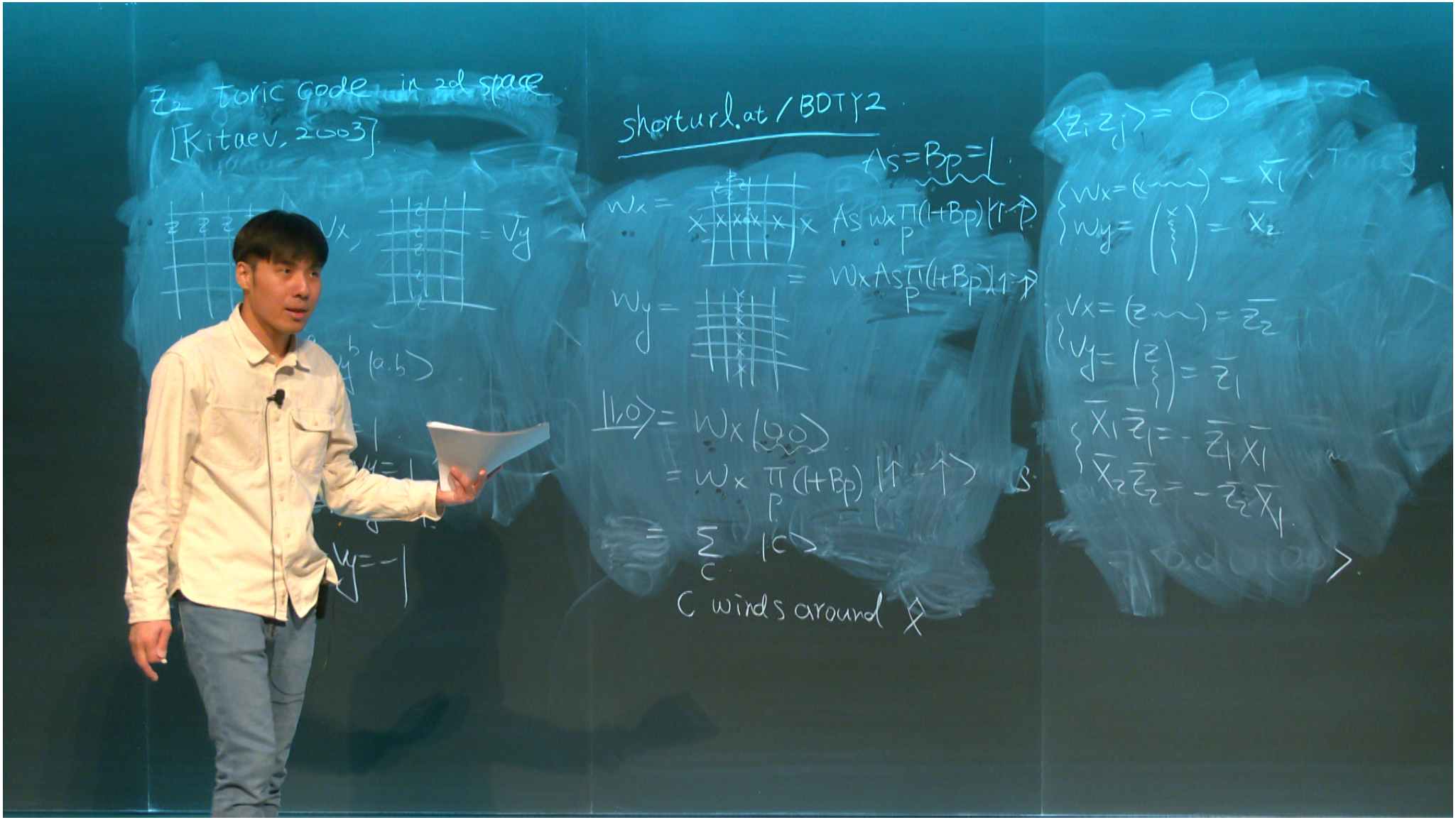
$W_y =$

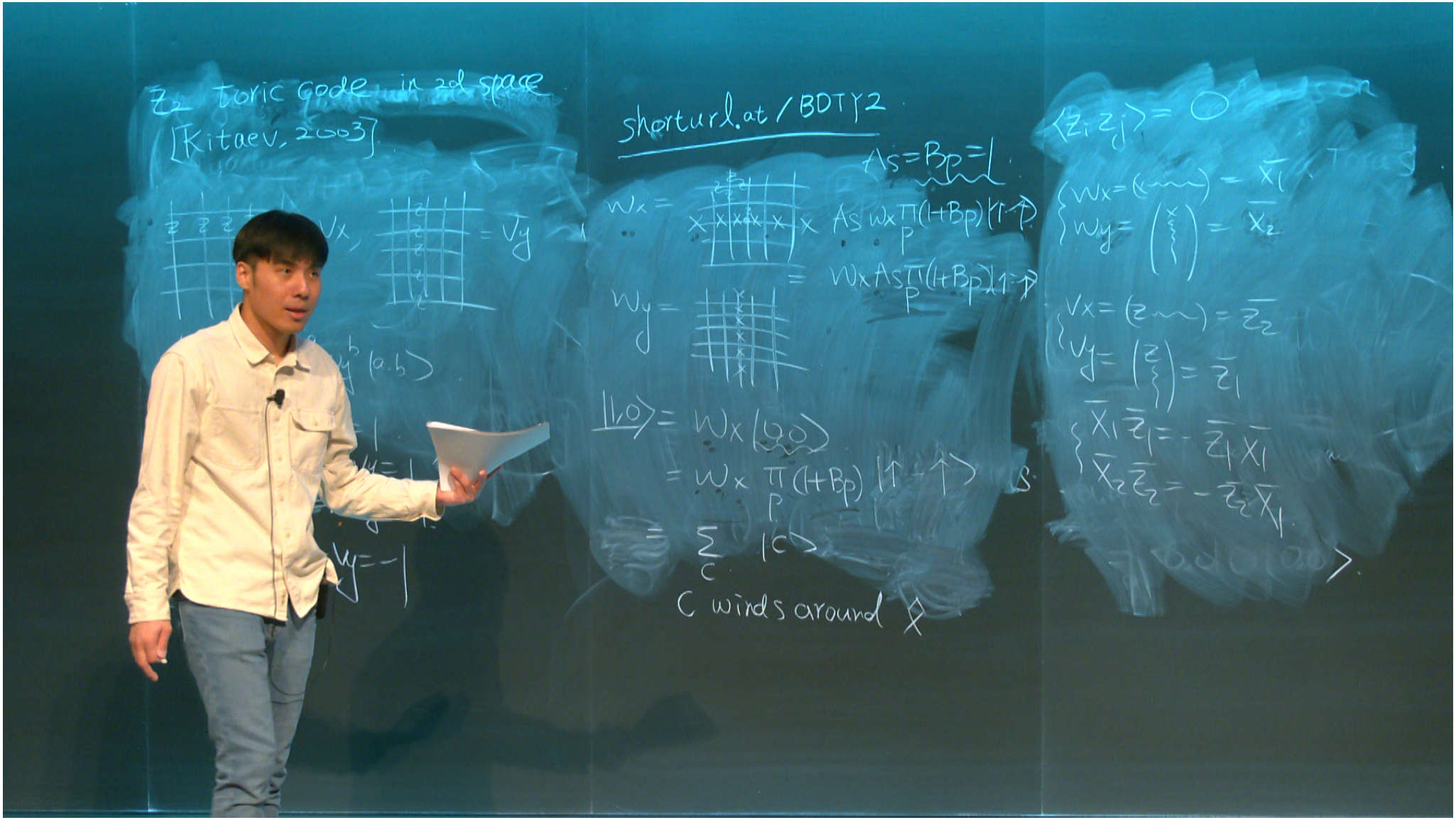

$|1,0\rangle = W_x |0,0\rangle$

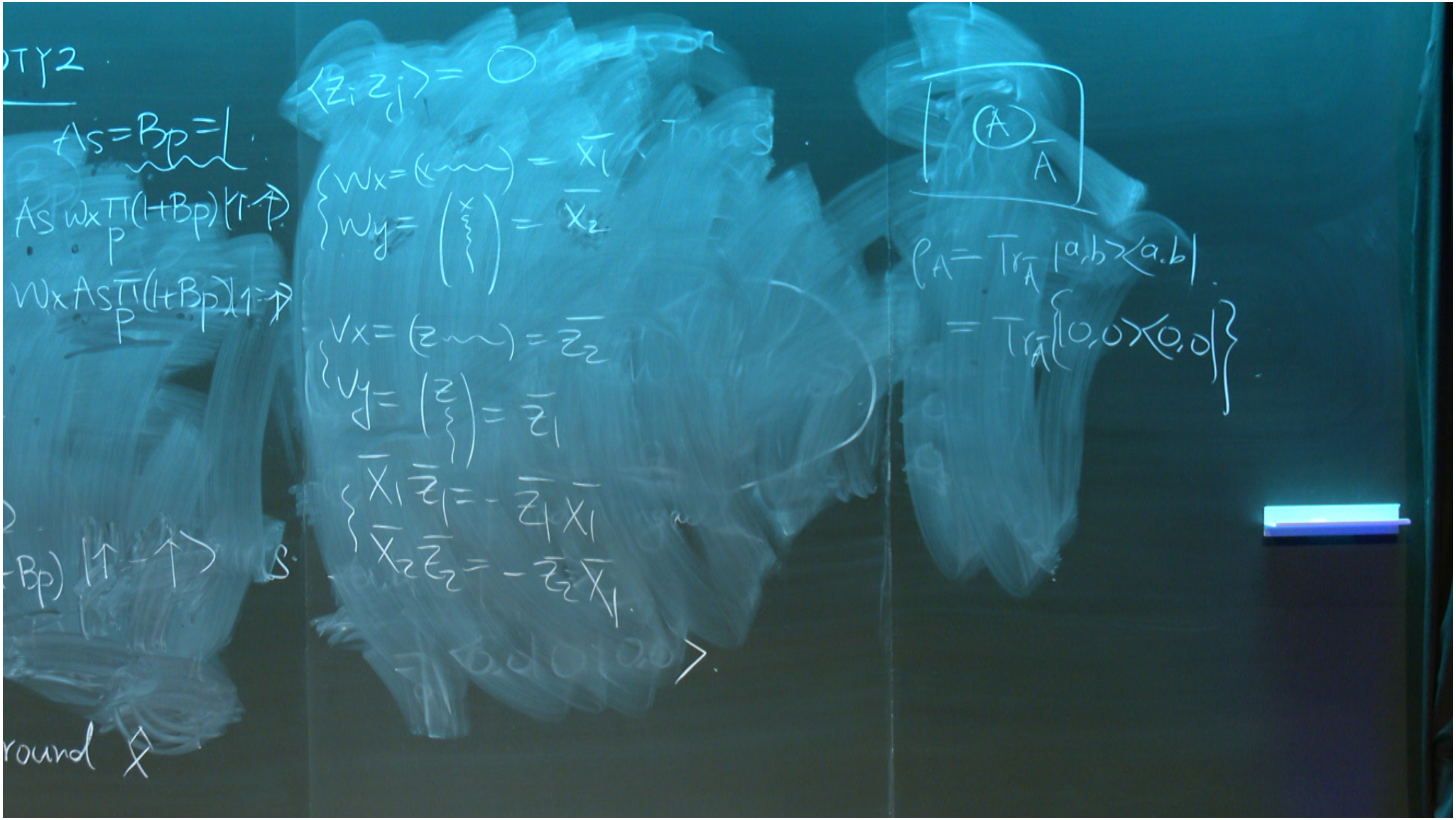
$= W_x \prod_p (1+B_p) |\uparrow\downarrow\rangle$

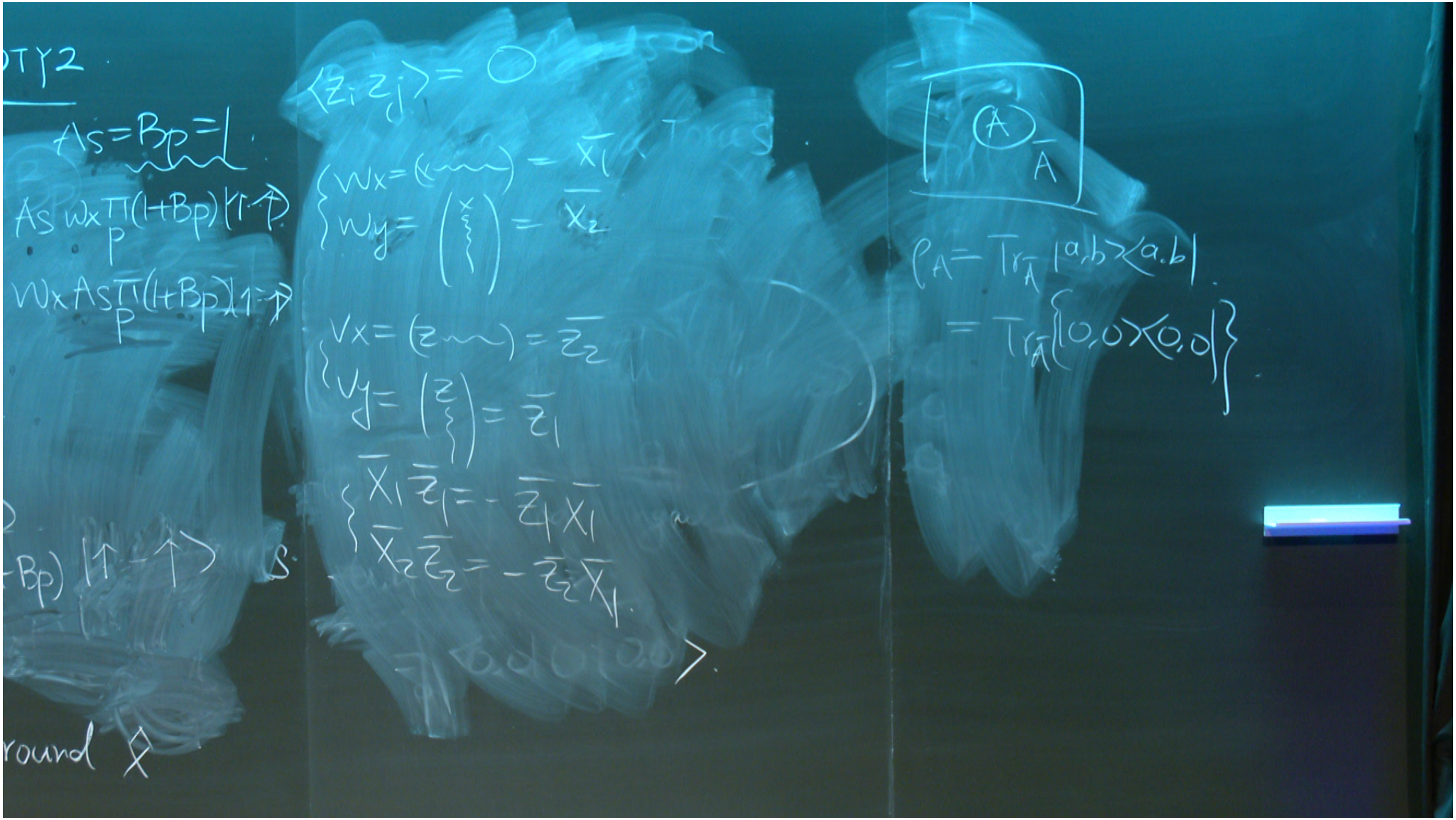
$= \sum_C |C\rangle$

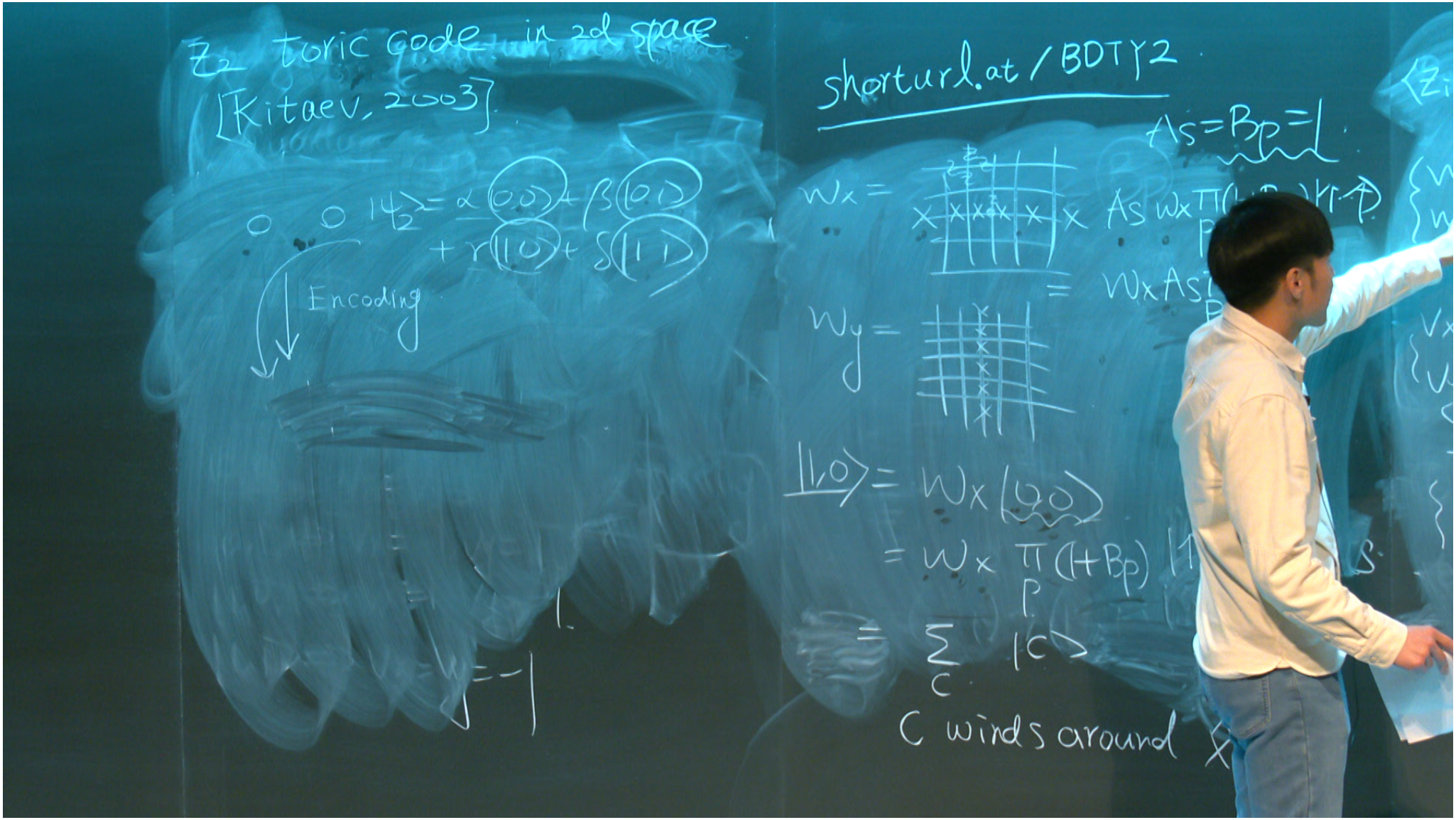
C winds around χ

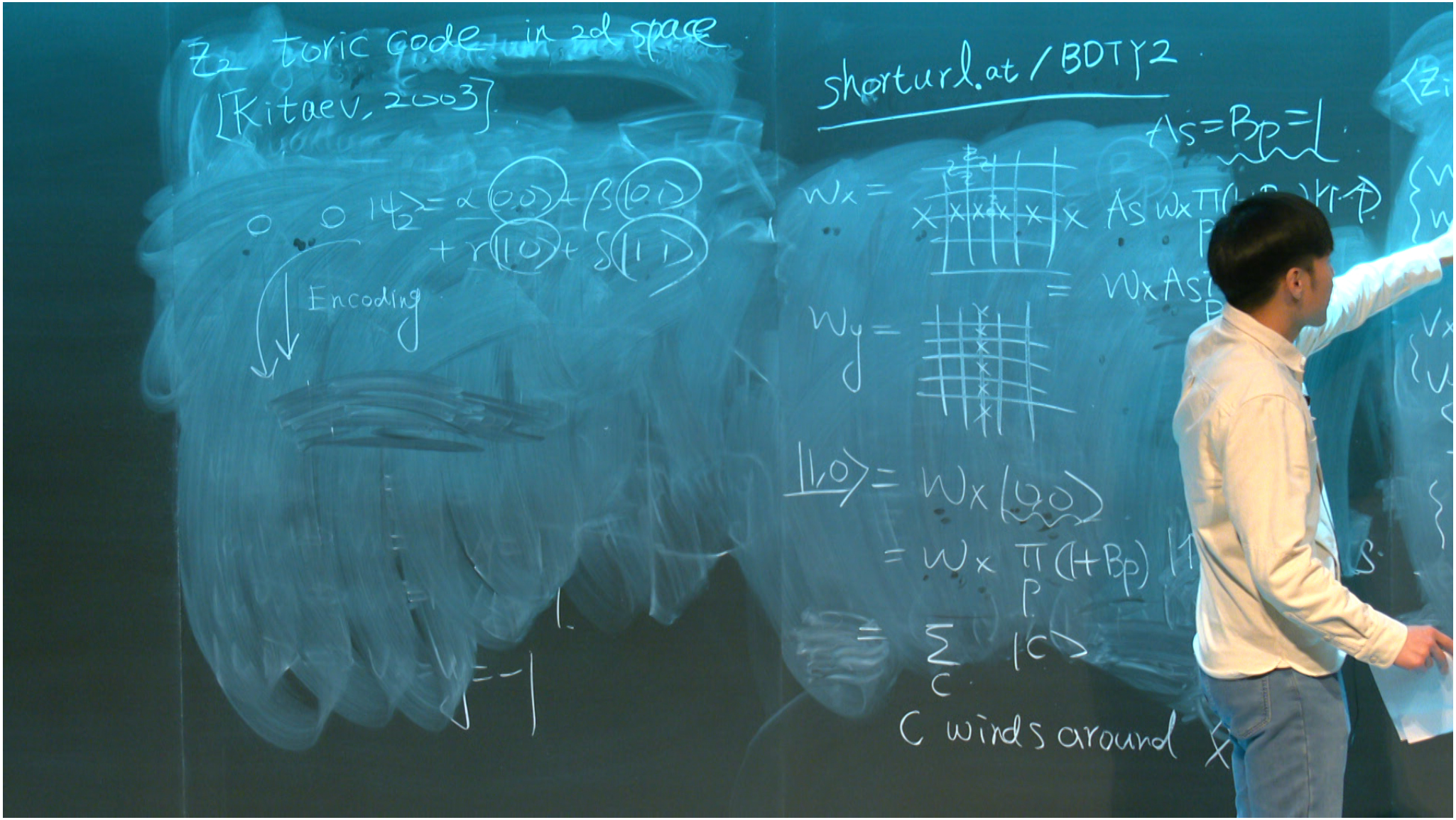

Z_2 toric code in 2d space
[Kitaev, 2003]
shorturl.at/BDTY2
$A_s = B_p = 1$
$\langle z_i z_j \rangle = 0$
$W_x =$
$W_y =$
$|1,0\rangle = W_x |0,0\rangle$
$= W_x \prod_p (1+B_p) |\uparrow \cdots \uparrow\rangle$
$= \sum_C |C\rangle$
C winds around

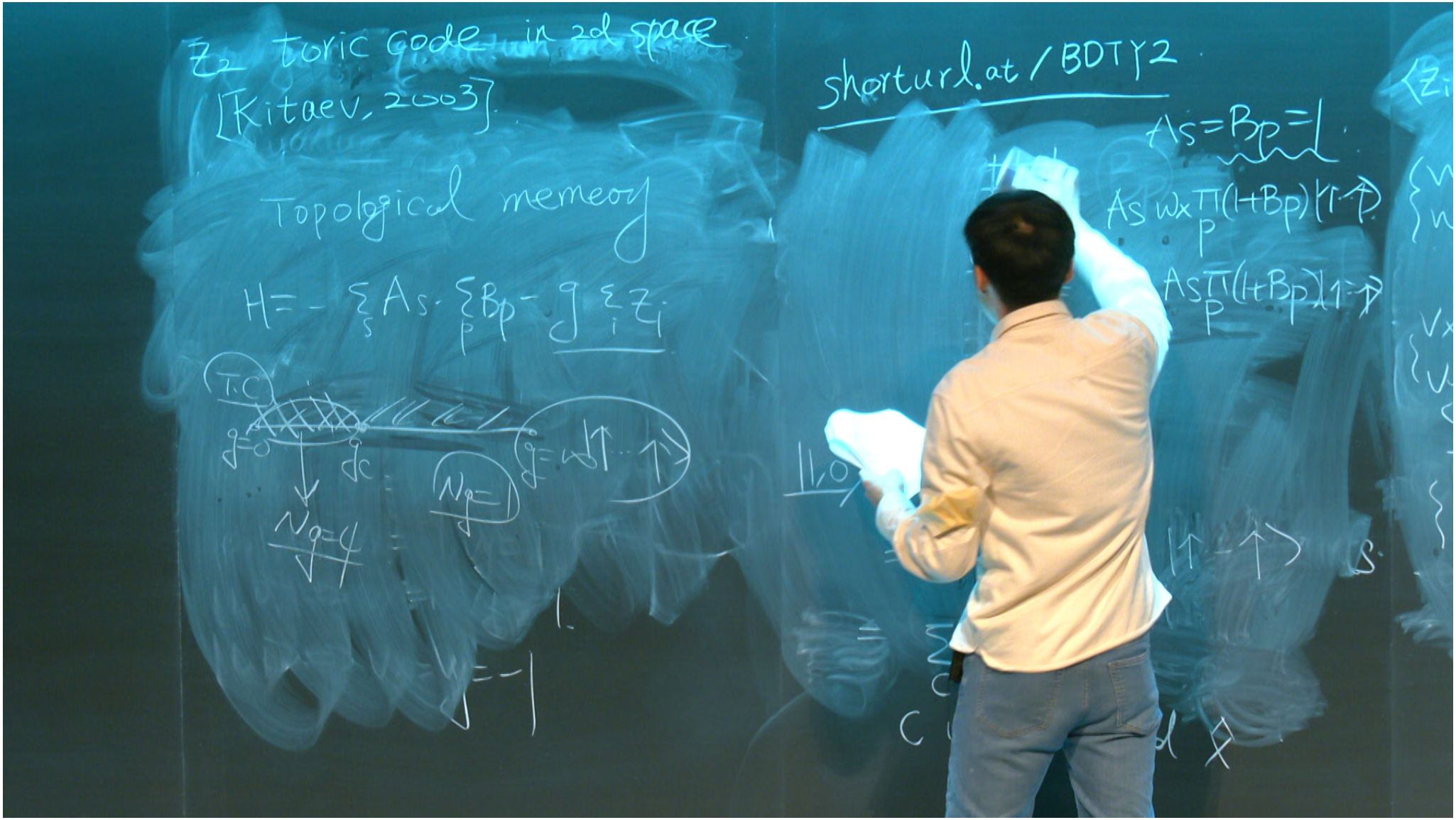

Z_2 toric code in 2d space
[Kitaev, 2003]
shorturl.at/BDTY2
$A_s = B_p = 1$
$\langle z_i z_j \rangle = 0$
$W_x =$
$W_y =$
$|1,0\rangle = W_x |0,0\rangle$
$= W_x \prod_p (1+B_p) |\uparrow \cdots \uparrow\rangle$
$= \sum_C |C\rangle$
C winds around

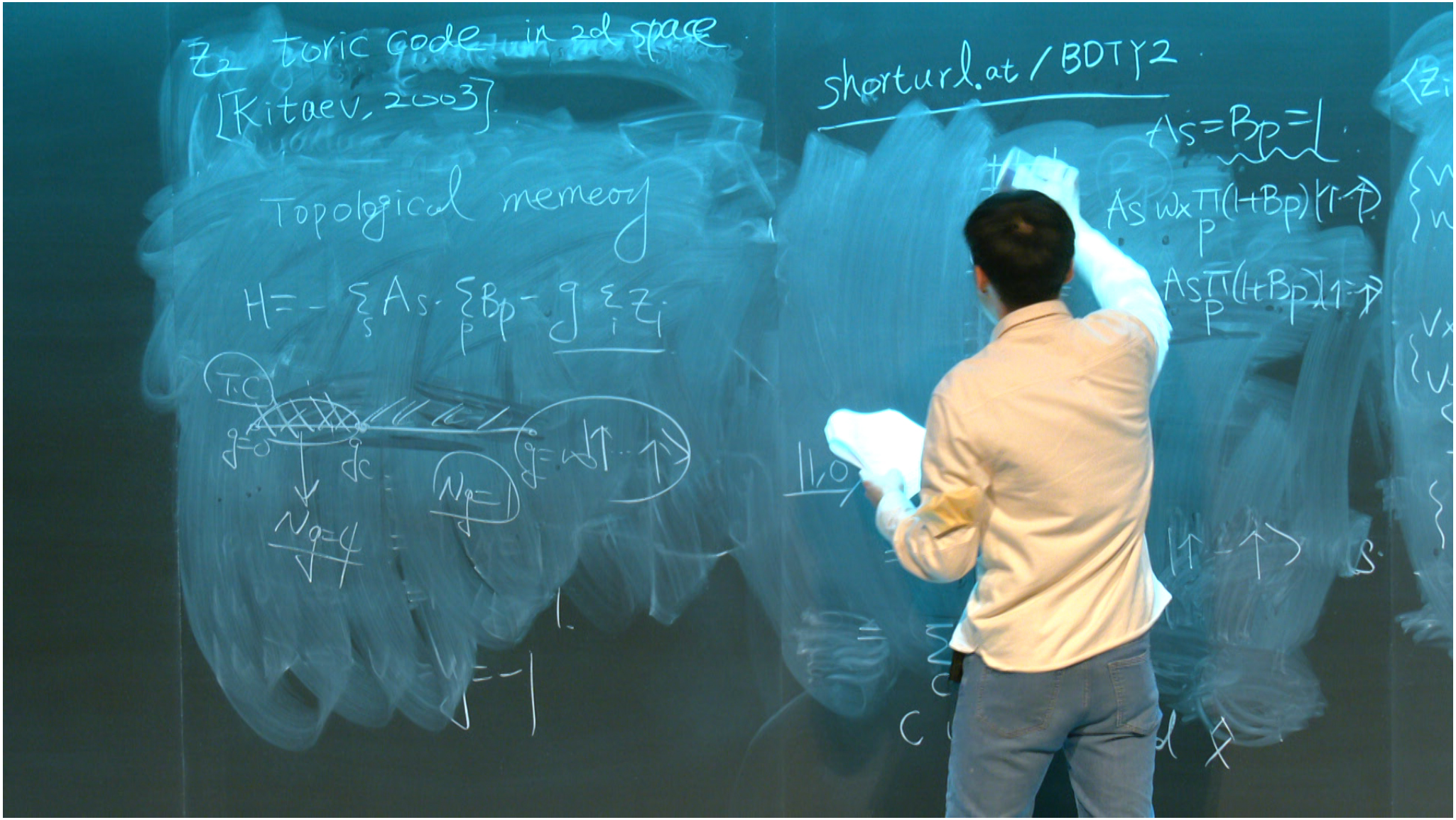

$\langle z_i z_j \rangle = 0$
$W_x = \bar{x}_1$
$W_y = \bar{x}_2$
$V_x = \bar{z}_2$
$V_y = \bar{z}_1$
$\bar{x}_1 \bar{z}_1 = -\bar{z}_1 \bar{x}_1$
$\bar{x}_2 \bar{z}_2 = -\bar{z}_2 \bar{x}_1$
A
$\bar{A}$
$\rho_A = \mathrm{Tr}_{\bar{A}} |a,b\rangle\langle a,b|$
$= \mathrm{Tr}_{\bar{A}} \{|0,0\rangle\langle 0,0|\}$


$\langle z_i z_j \rangle = 0$
$\rho_A = Tr_{\bar{A}} |a,b\rangle\langle a,b|$
$= Tr_{\bar{A}} \{|0,0\rangle\langle 0,0|\}$


Z_2 toric code in 2d space
[Kitaev, 2003]
shorturl.at/BDTY2
$|\psi_2\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
Encoding
$A_s = B_p = 1$
$W_x =$
$W_y =$
$|1,0\rangle = W_x|0,0\rangle$
$= W_x \prod_p (1+B_p)|1$
$= \sum_C |C\rangle$
C winds around


Z_2 toric code in 2d space
[Kitaev, 2003]
shorturl.at/BDTY2
$|\psi_2\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
Encoding
$A_s = B_p = 1$
$W_x =$
$W_y =$
$|1,0\rangle = W_x|00\rangle$
$= W_x \prod_p (1+B_p)|1$
$= \sum_C |C\rangle$
C winds around


Z_2 toric code in 2d space
[Kitaev, 2003]
Topological memory
$H = -\sum_s A_s - \sum_p B_p - g \sum_i Z_i$
T.C
$N_q = 4$
$N_q = 1$
shorturl.at/BDTY2
$A_s = B_p = 1$
$A_s \omega_x \prod_p (1+B_p) |1\rangle$
$A_s \prod_p (1+B_p) |1\rangle$
$|1,0\rangle$
$|\uparrow - \uparrow\rangle$


Z_2 toric code in 2d space
[Kitaev, 2003]
Topological memory
$H = -\sum_s A_s - \sum_p B_p - g \sum_i Z_i$
T.C
$N_q = 4$
$N_q = 1$
shorturl.at/BDTY2
$A_s = B_p = 1$
$A_s \omega_x \prod_p (1+B_p) |1\rangle$
$A_s \prod_p (1+B_p) |1\rangle$
$|1,0\rangle$
$|\uparrow - \uparrow\rangle$

Z_2 toric code in 2d space
[Kitaev, 2003].

Topological memory

$$H = -\sum_s A_s - \sum_p B_p - g\left(\sum_i Z_i\right) - g_x \sum_i X_i$$



shorturl.at/BDTY2

tunneling between G.S

$(Z \cdots Z)$ g^L $g \le O(1)$

$|1,0\rangle$

$= W_x \prod_p (1+B_p)\,|\uparrow\cdots\uparrow\rangle$

$= \sum_C |C\rangle$

C winds around

Z_2 toric code in 2d space
[Kitaev, 2003].

Topological memory

$$H = -\sum_s A_s - \sum_p B_p - g\left(\sum_i Z_i\right) - g_x \sum_i X_i$$



$N_g = 4$

$N_g = 1$

shorturl.at/BDTY2

tunneling between G.S

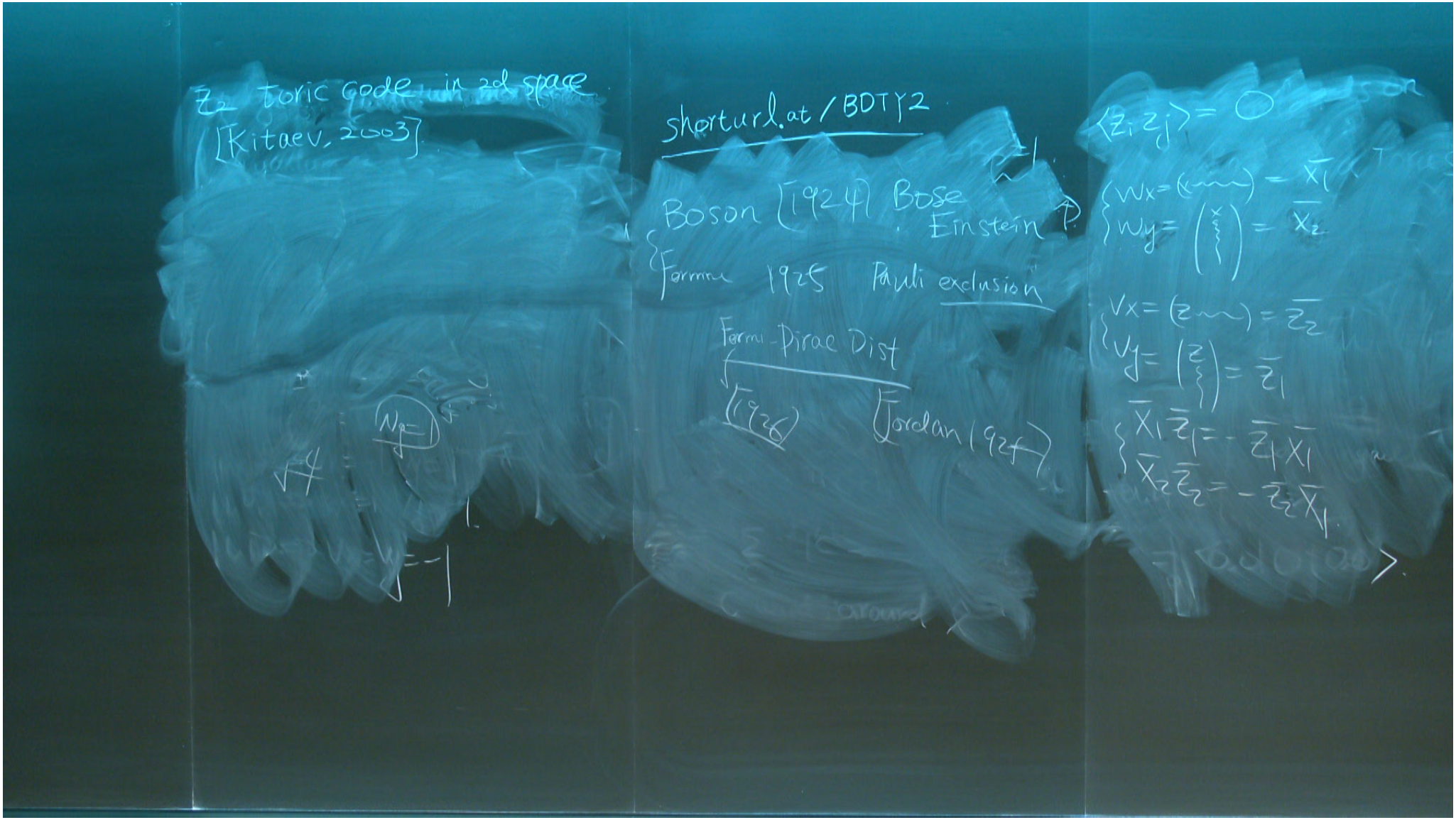
g^L $g \le O(1)$

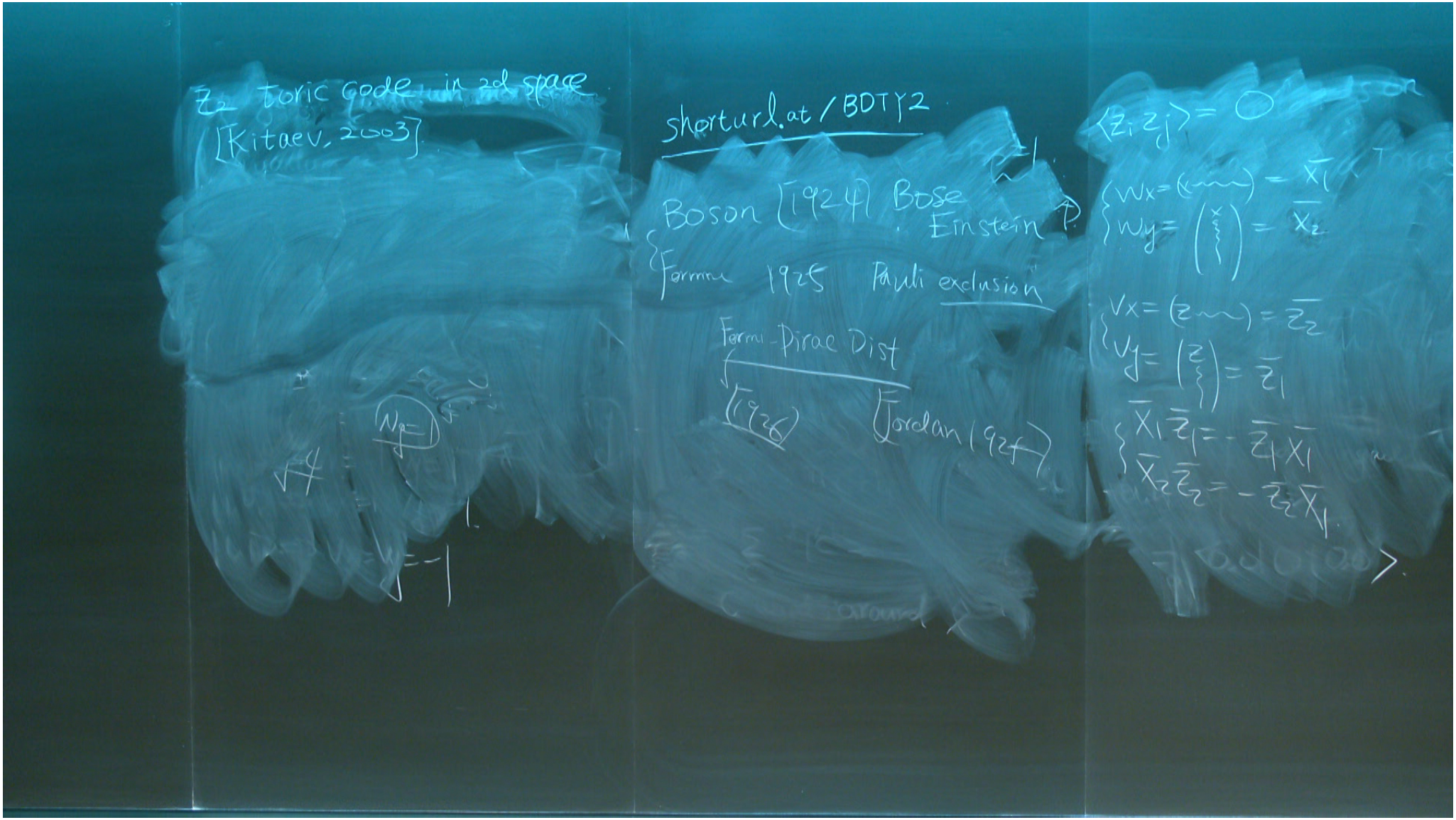
$|1,0\rangle$

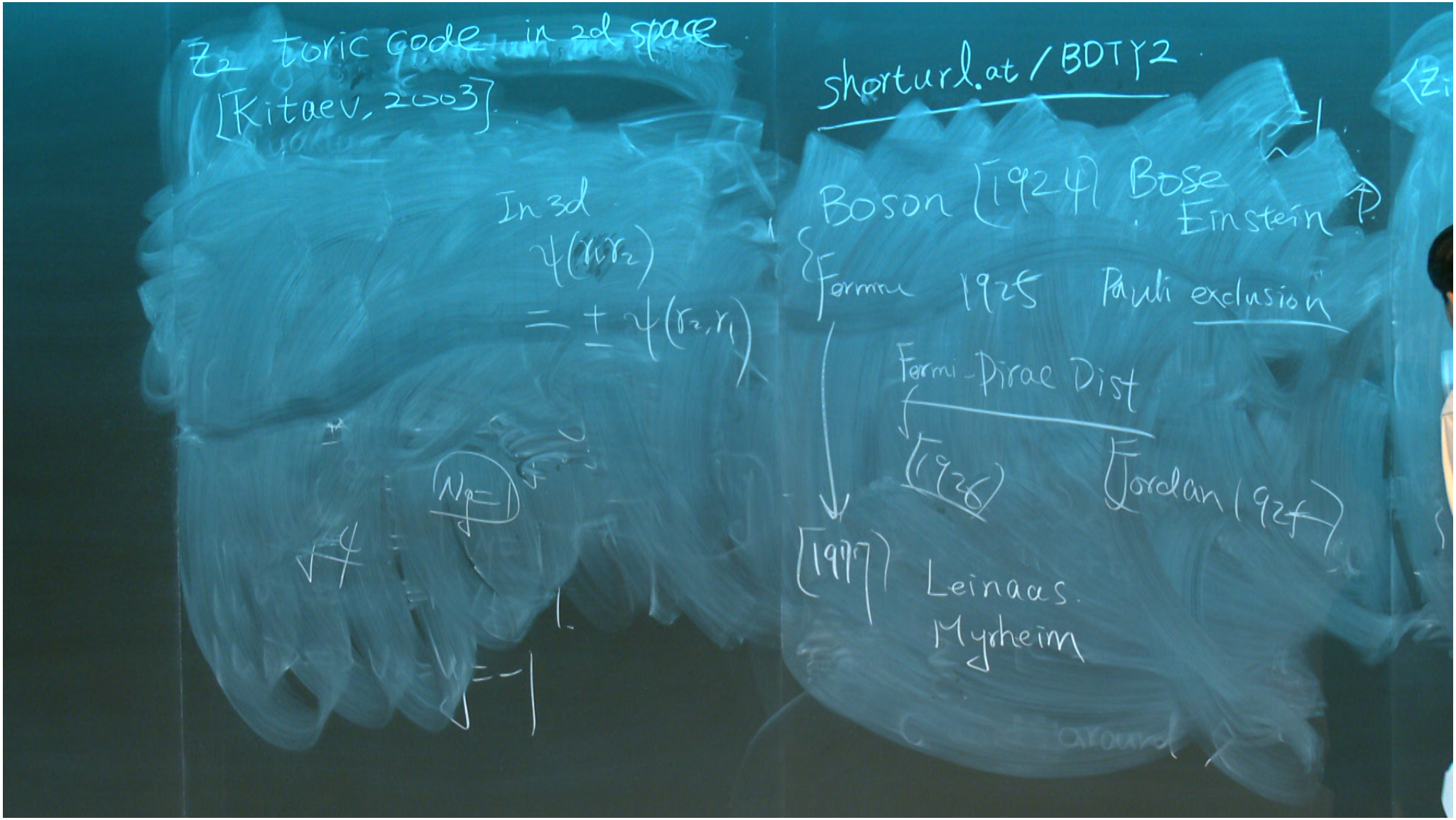
$= W_x \prod_p (1+B_p)\,|\uparrow\cdots\uparrow\rangle$

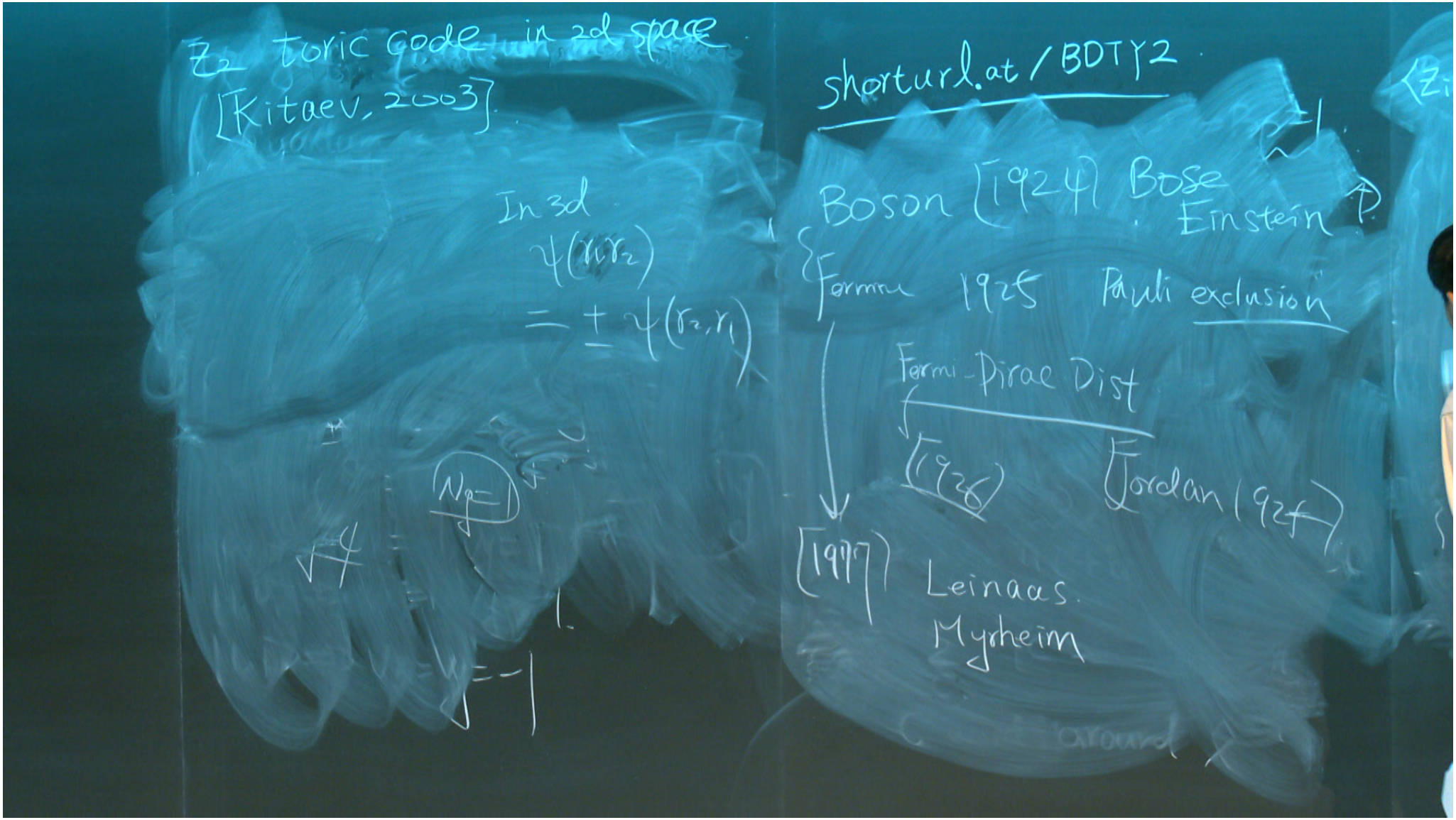
$= \sum_C |C\rangle$

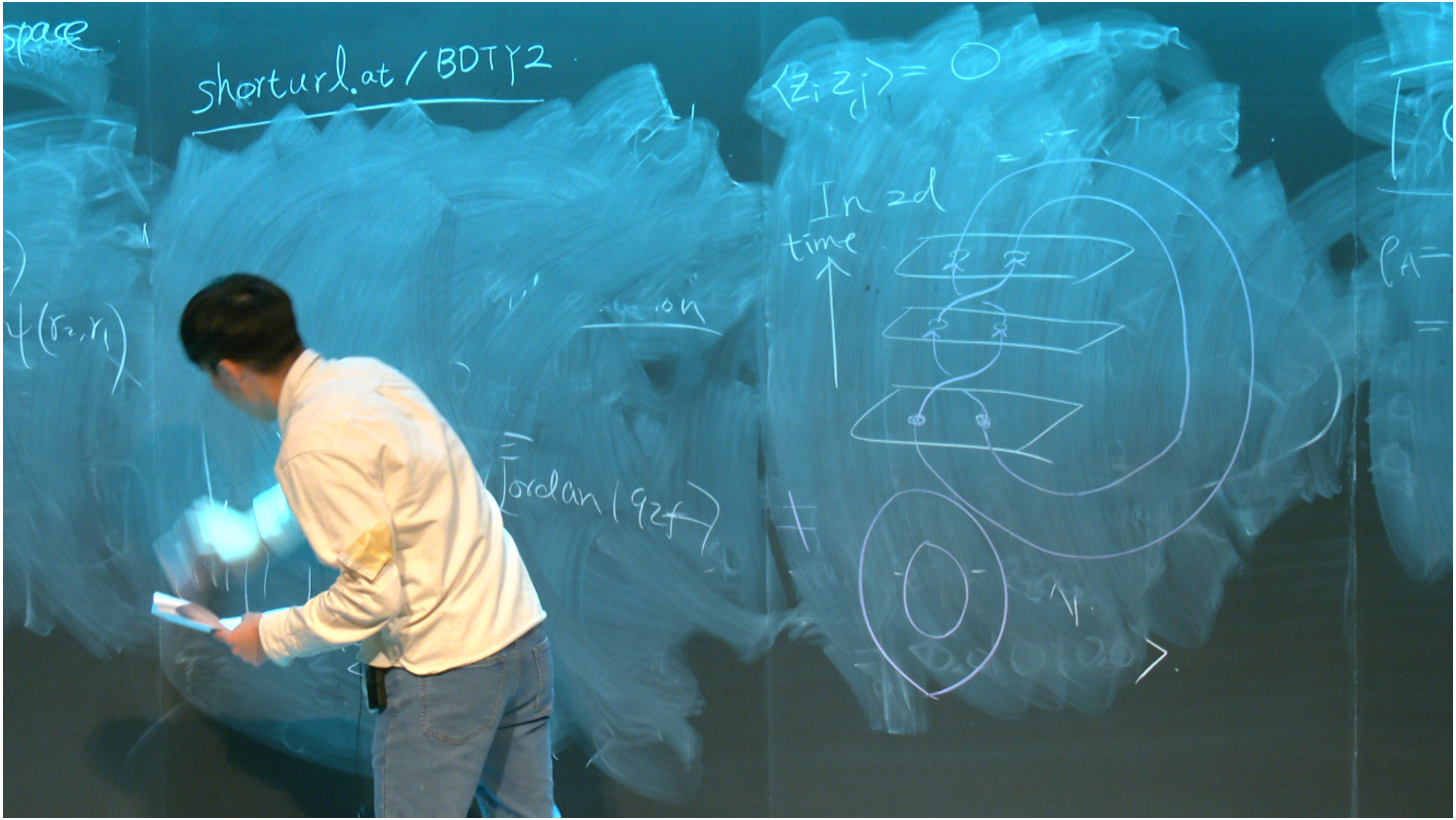
C winds around

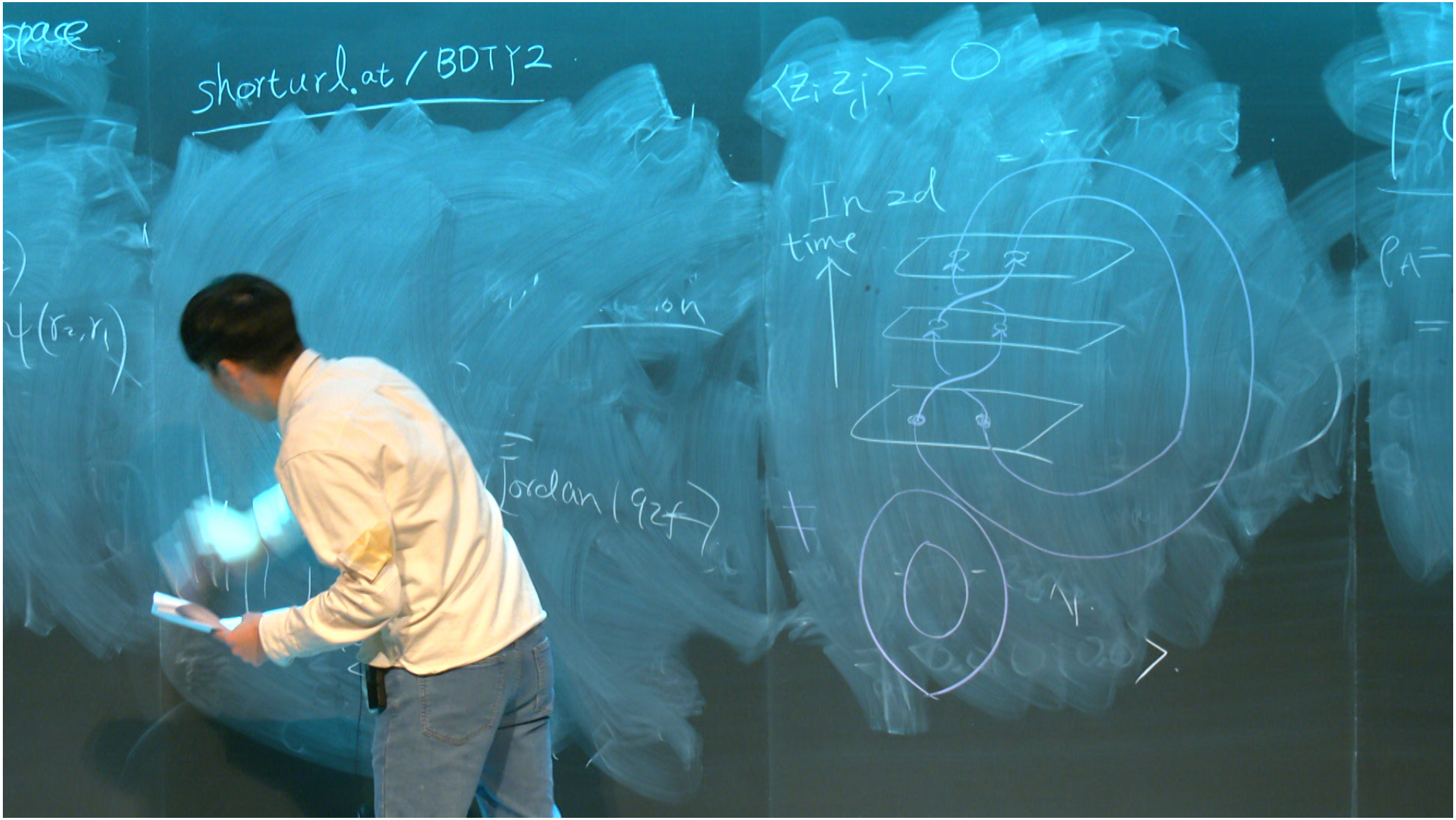

$\mathbb{Z}_2$ toric code in 2d space
[Kitaev, 2003]
shorturl.at/BDTY2
Boson [1924] Bose Einstein
Fermi 1925 Pauli exclusion
Fermi-Dirac Dist
[1926] [Jordan (1924)
$\langle z_i z_j \rangle = 0$
$W_x = \overline{x_1}$
$W_y = \overline{x_2}$
$V_x = \overline{z_2}$
$V_y = \overline{z_1}$
$\overline{x_1}\overline{z_1} = -\overline{z_1}\overline{x_1}$
$\overline{x_2}\overline{z_2} = -\overline{z_2}\overline{x_1}$

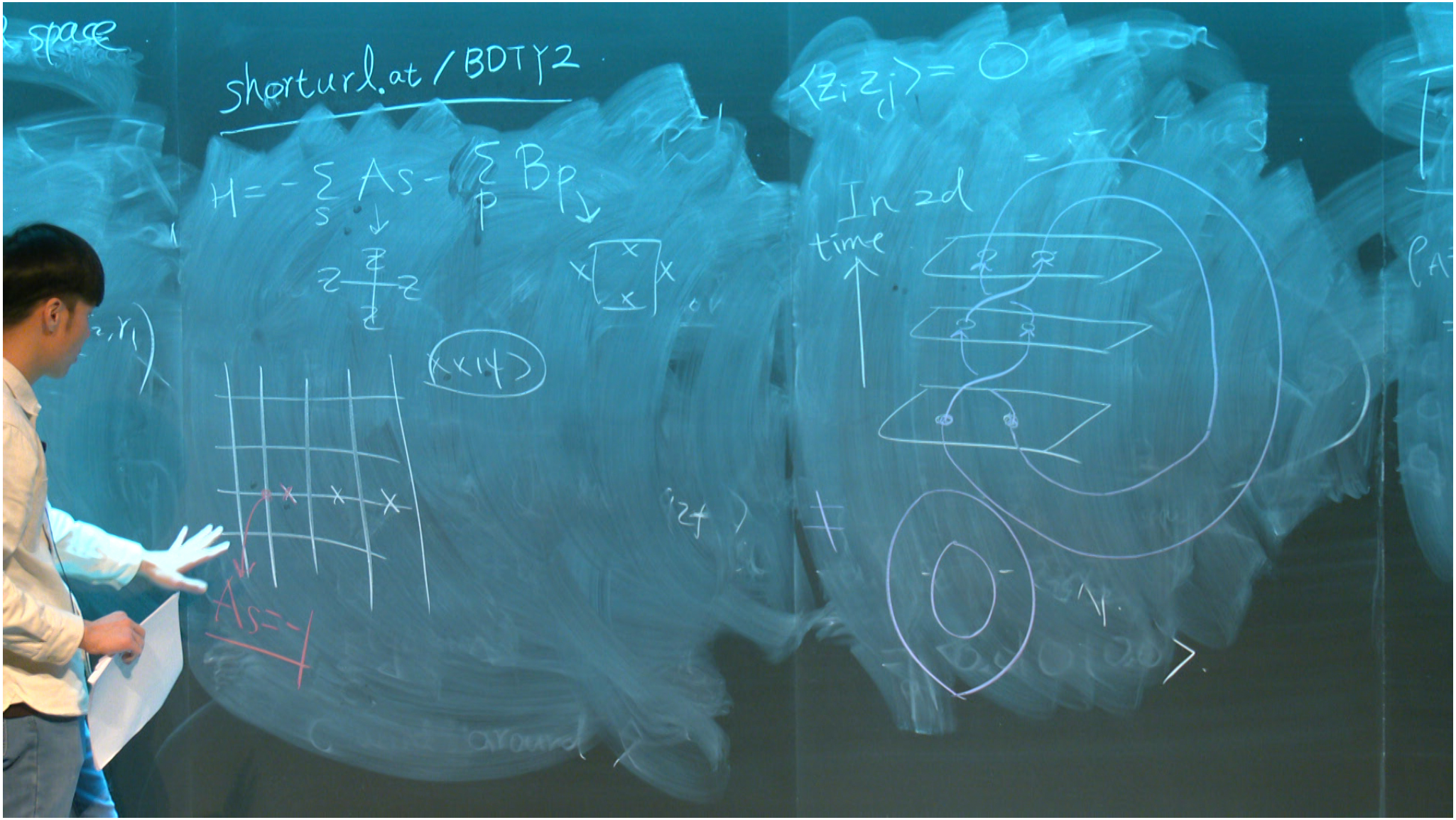

Z_2 toric code in 2d space
[Kitaev, 2003]
shorturl.at/BDTY2
Boson [1924] Bose Einstein
Fermi 1925 Pauli exclusion
Fermi-Dirac Dist
[1926] [Jordan (1927)
$\langle z_i z_j \rangle = 0$
$W_x = (\sim) = \bar{x}_1$
$W_y = \begin{pmatrix} x \\ \vdots \end{pmatrix} = \bar{x}_2$
$V_x = (\sim) = \bar{z}_2$
$V_y = \begin{pmatrix} z \\ \vdots \end{pmatrix} = \bar{z}_1$
$\bar{x}_1 \bar{z}_1 = -\bar{z}_1 \bar{x}_1$
$\bar{x}_2 \bar{z}_2 = -\bar{z}_2 \bar{x}_1$

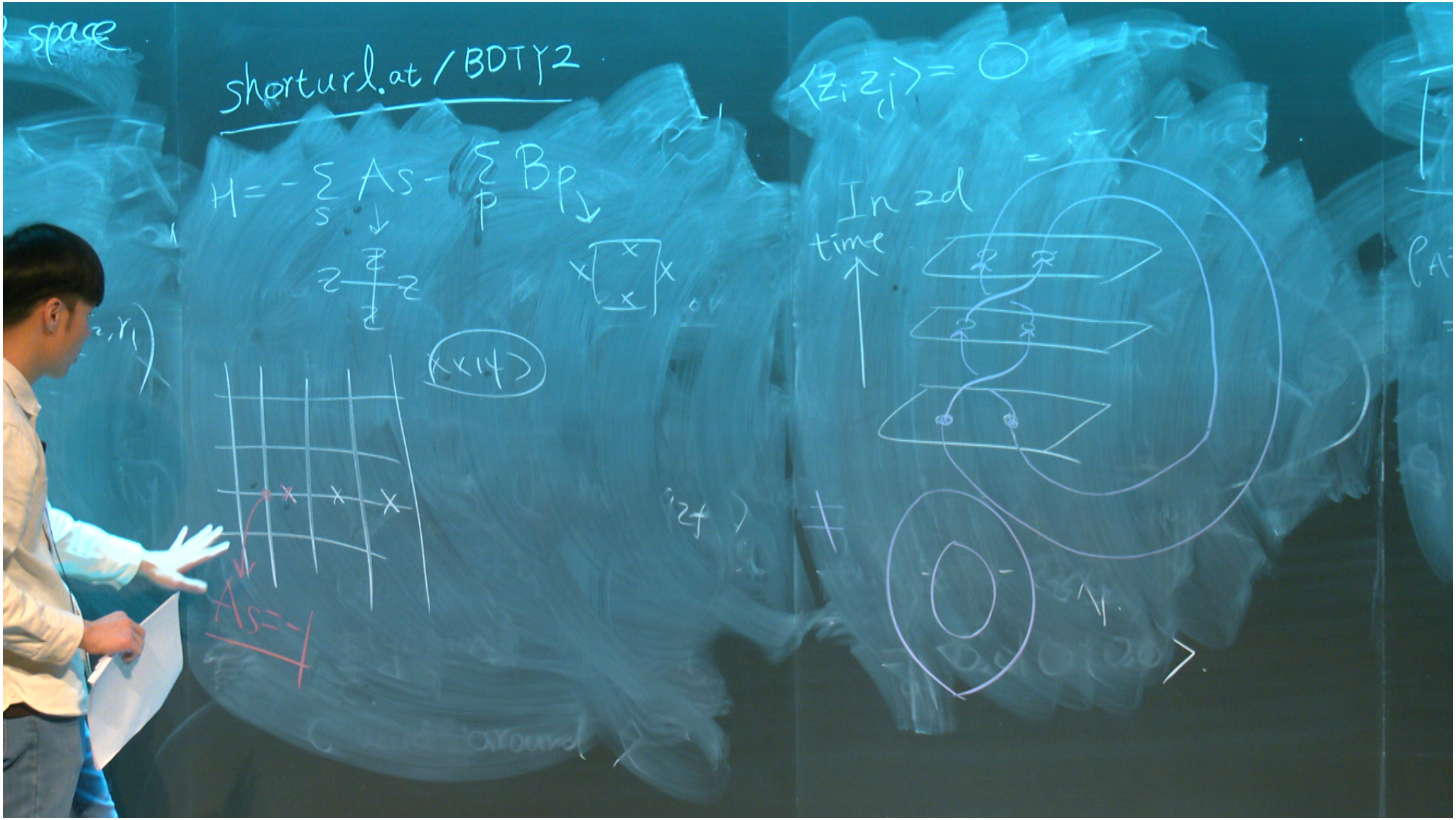

Z_2 toric code in 2d space
[Kitaev, 2003]
In 3d
$\psi(r_1 r_2)$
$= \pm \psi(r_2, r_1)$
$N_p = 1$
$= -1$
shorturl.at/BDTY2
Boson [1924] Bose Einstein
Fermion 1925 Pauli exclusion
Fermi-Dirac Dist
[1926]
Jordan (92f)
[1997] Leinaas Myrheim

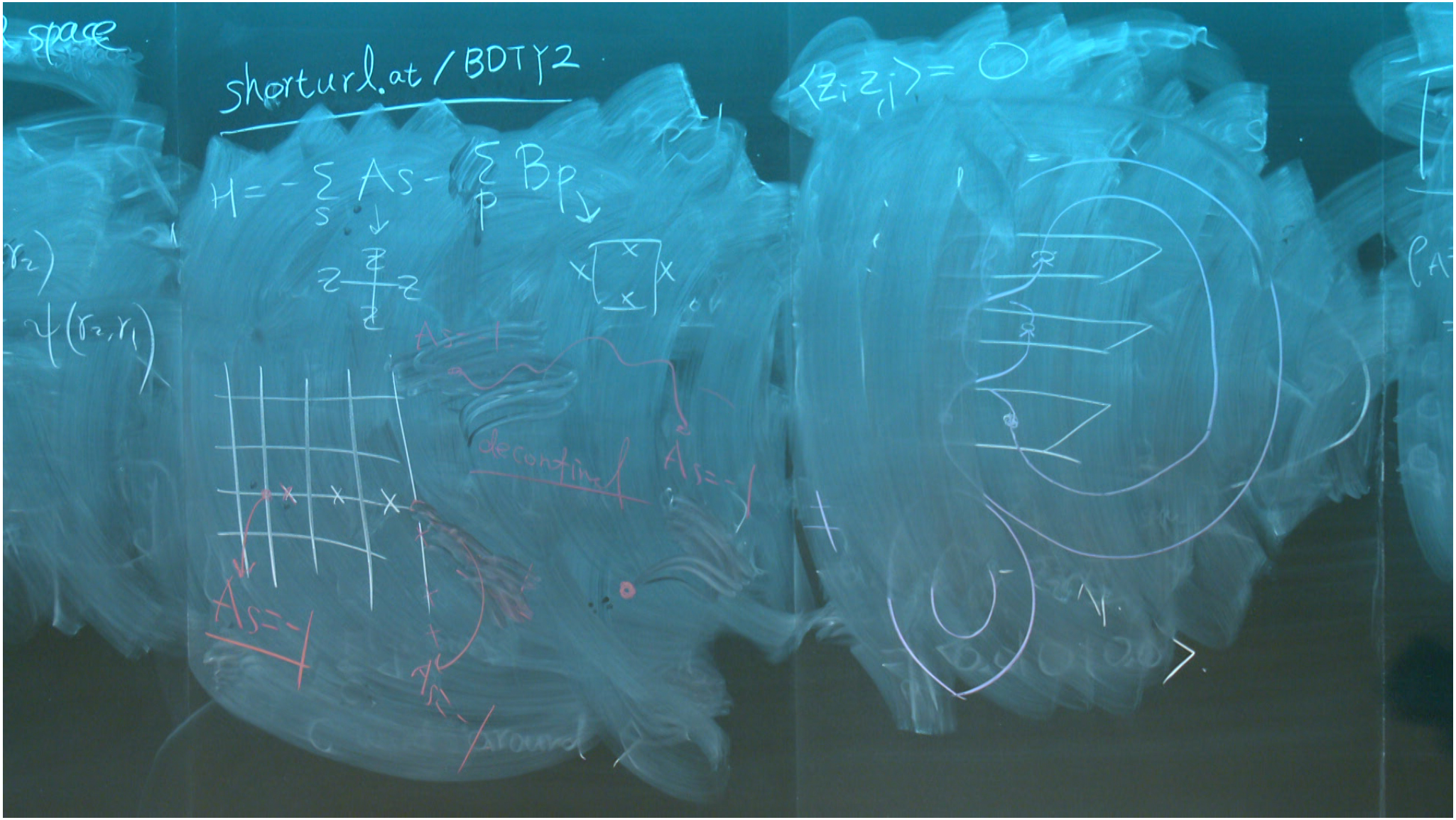

Z_2 toric code in 2d space
[Kitaev, 2003]
In 3d
$\psi(r_1 r_2)$
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Boson [1924] Bose Einstein
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[1997]
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Myrheim

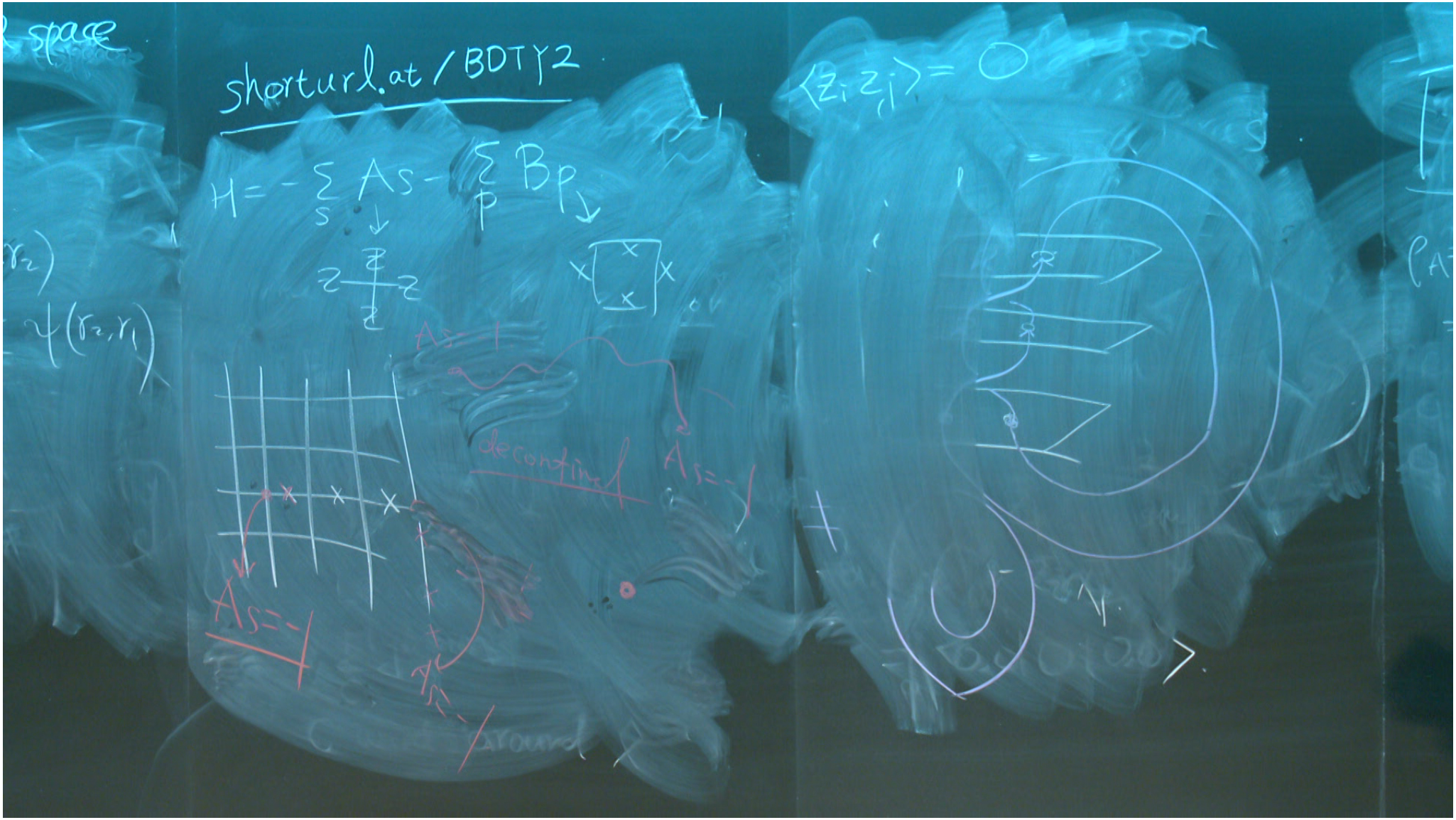

shorturl.at/BDTY2
In 2d
time
Torus

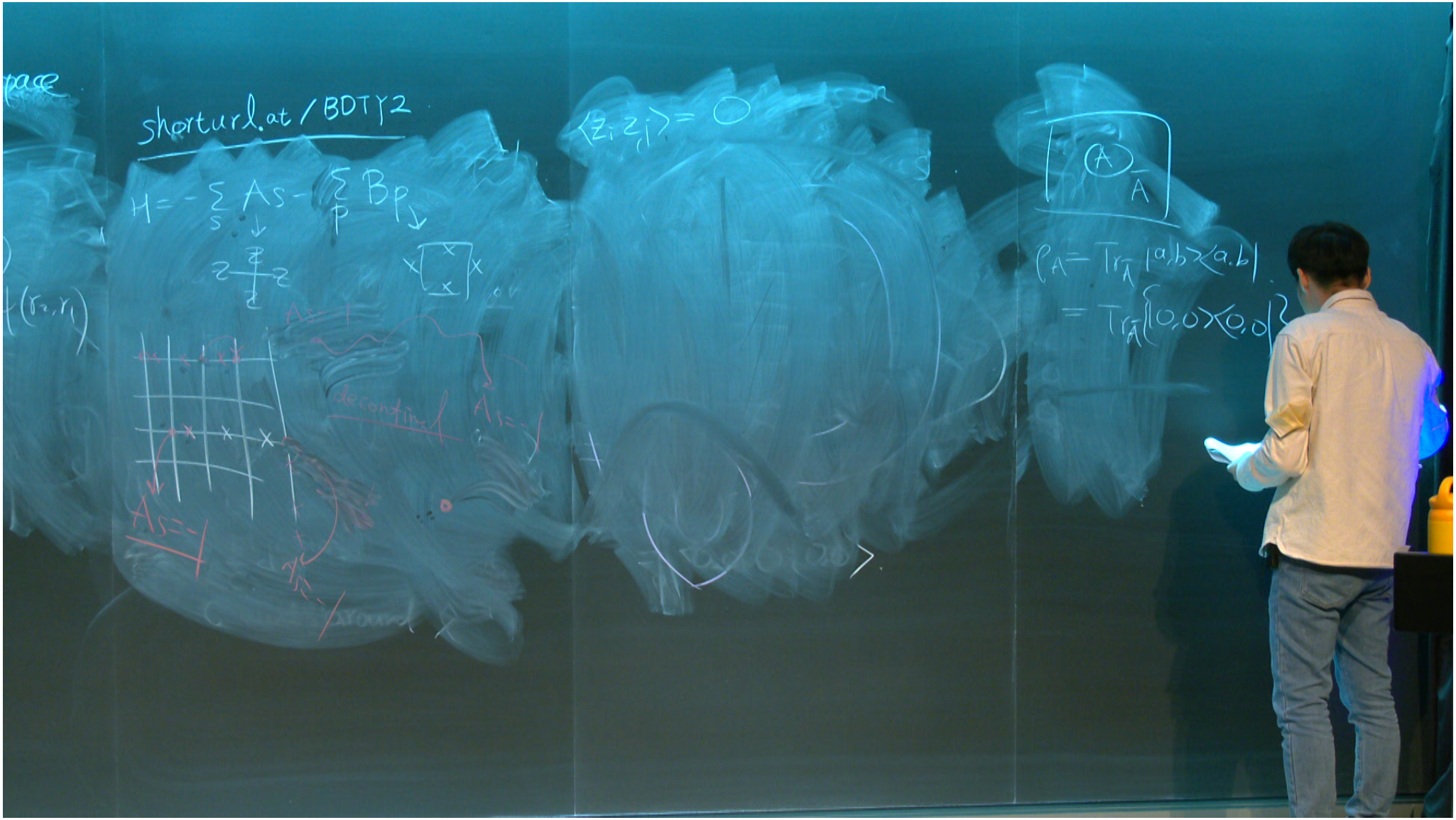

shorturl.at/BDTY2
In 2d
time
Torus

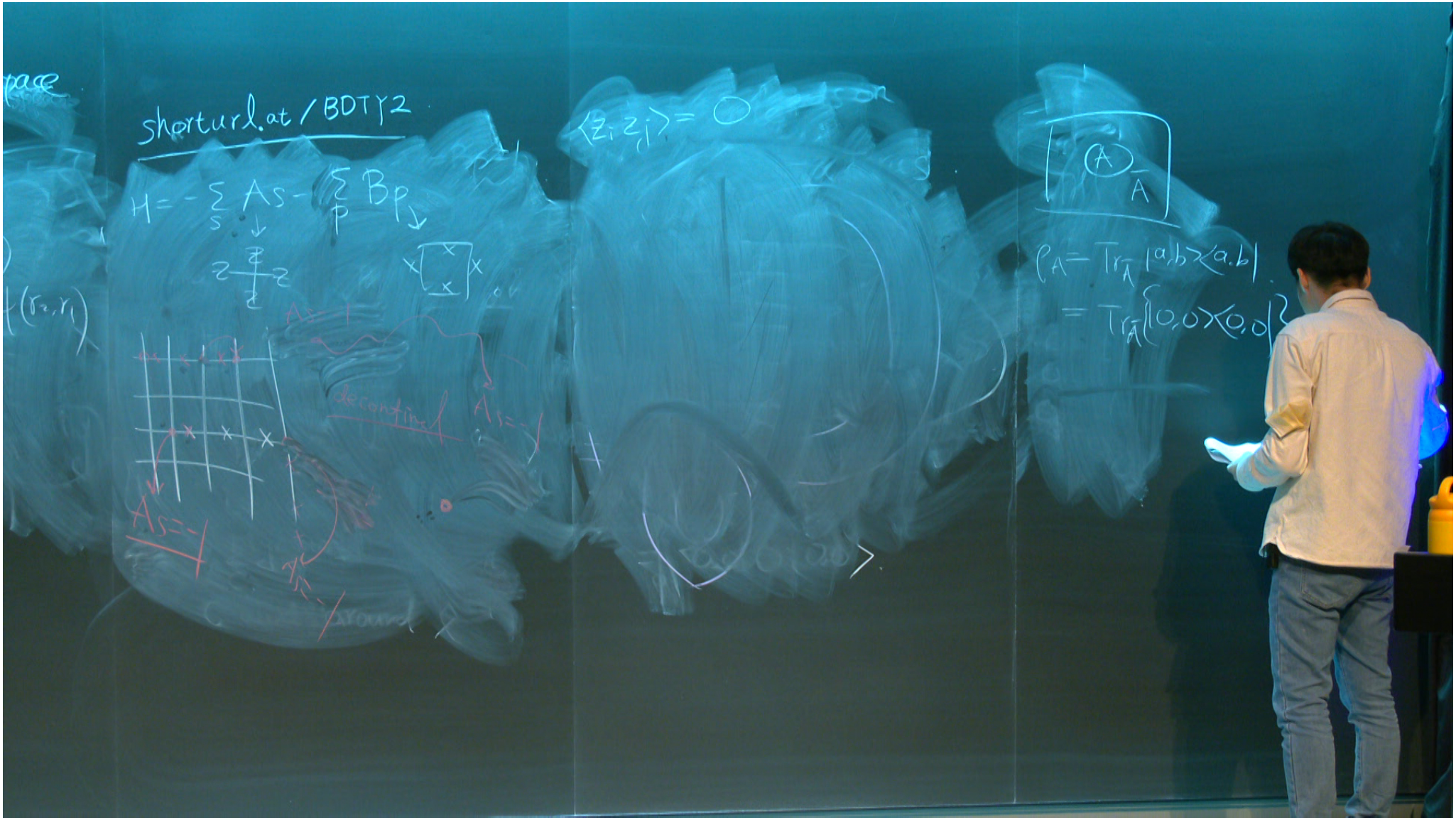

space
shorturl.at/BDTY2
$\langle z_i z_j \rangle = 0$
$H = -\sum_s A_s - \sum_p B_p$
In 2d
time
$A_s = -1$

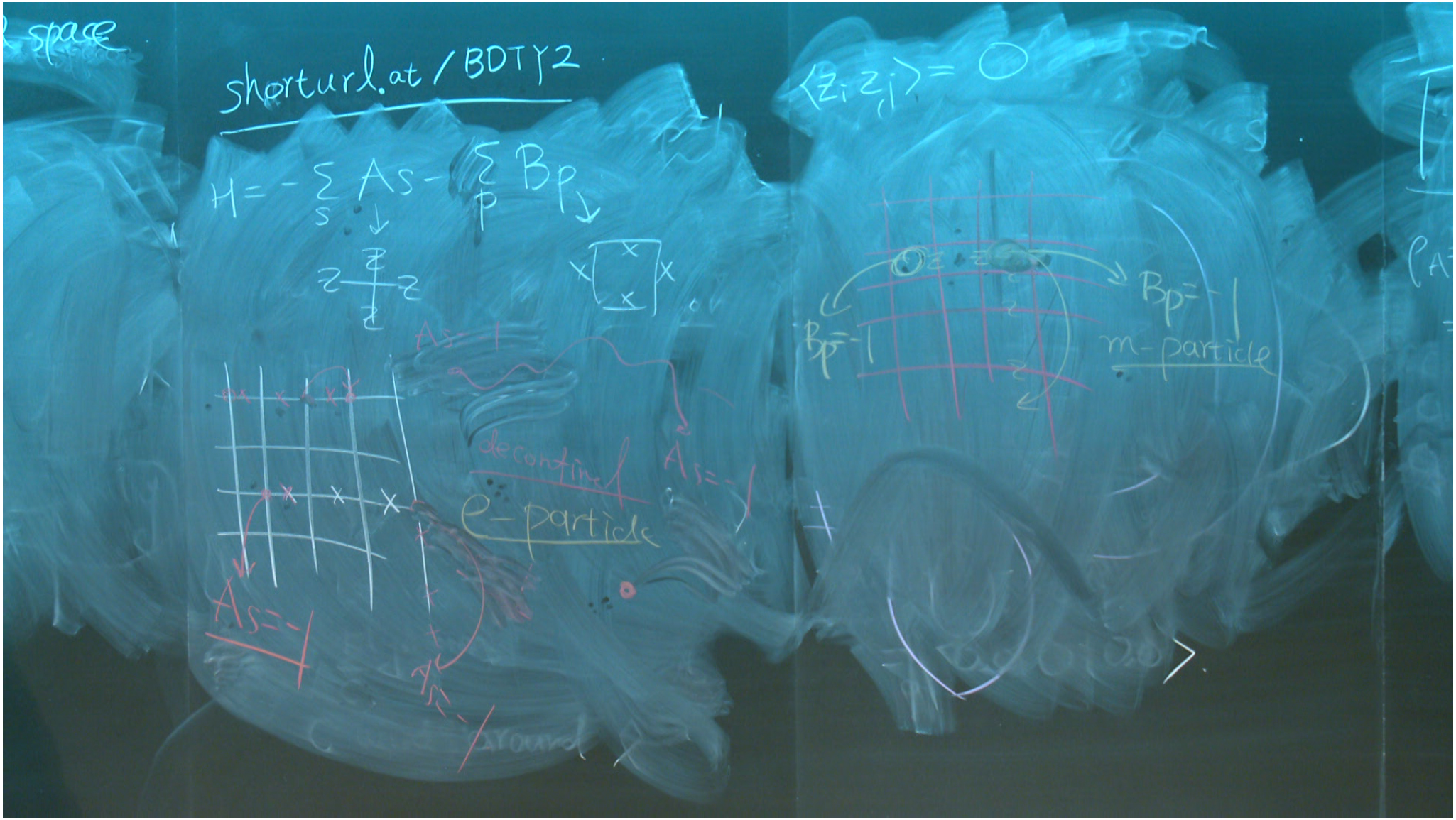

shorturl.at/BDTY2
$\langle z_i z_j \rangle = 0$
$H = -\sum_s A_s - \sum_p B_p$
In 2d
time
$A_s = +1$

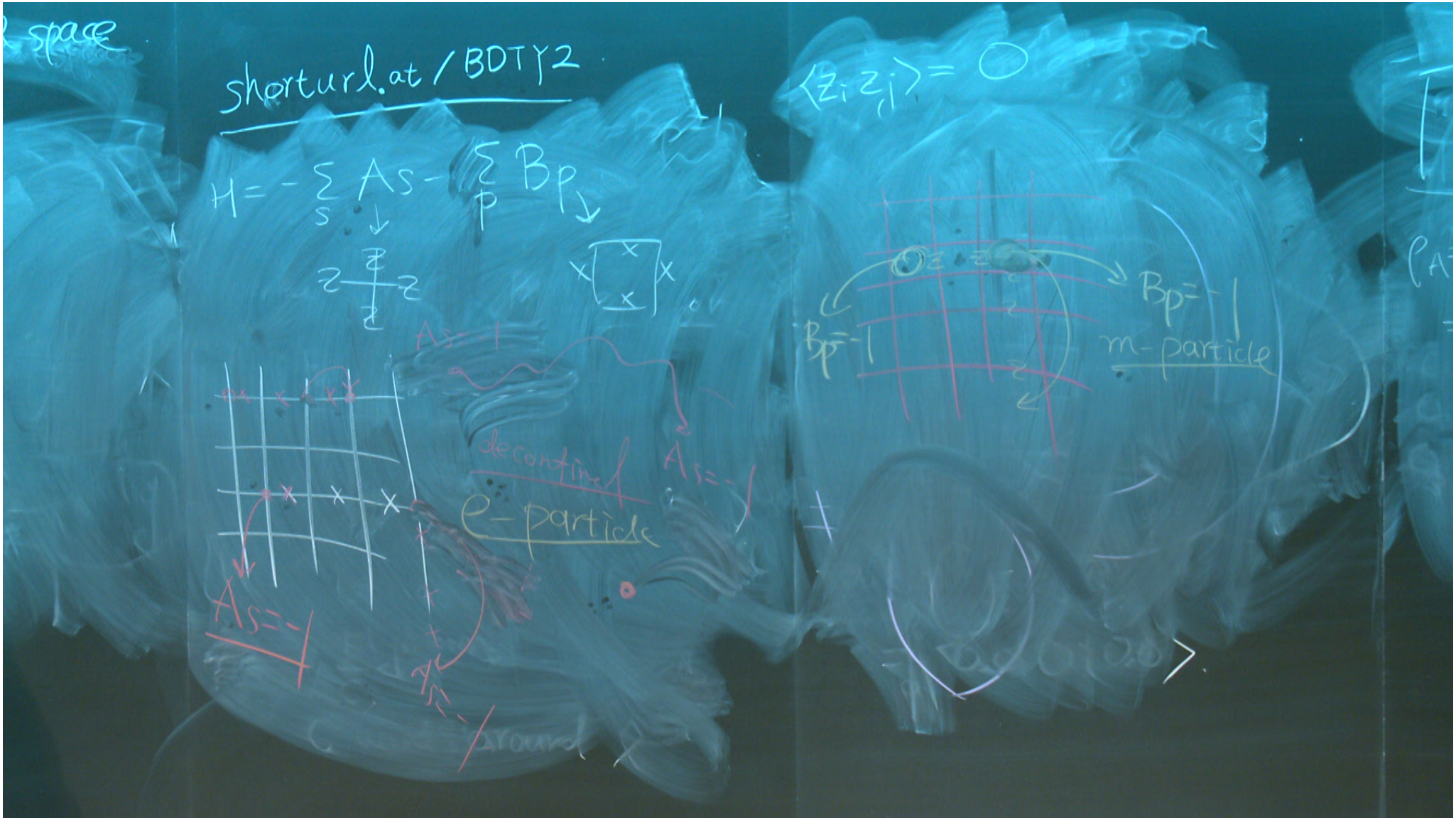

space
shorturl.at/BDTY2
$\langle z_i z_j \rangle = 0$
$H = -\sum_s A_s - \sum_p B_p$
$\psi(r_2, r_1)$
deconfined
$A_s = -1$
$A_s = -1$
around

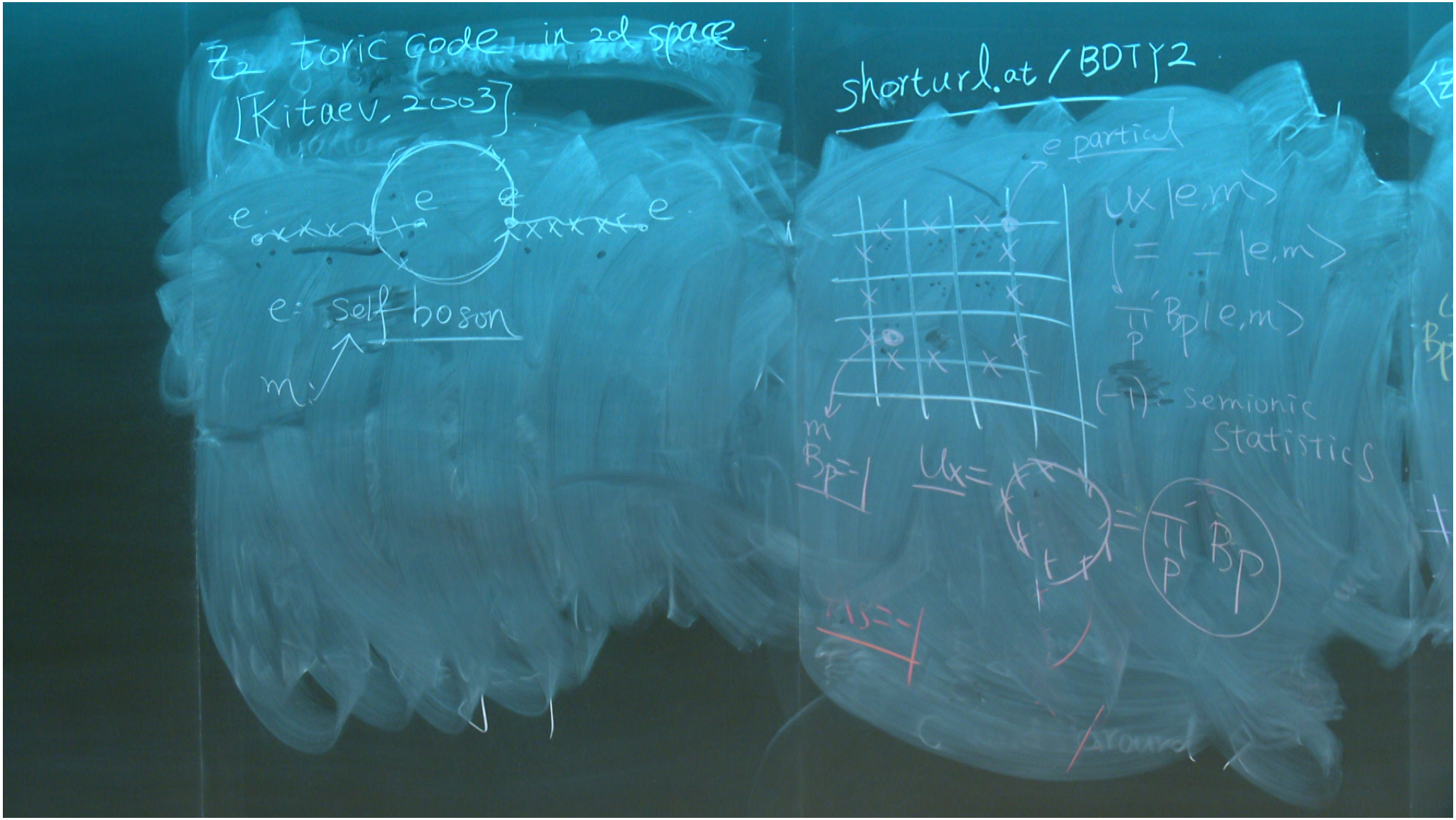

space
shorturl.at/BDTY2
$\langle z_i z_j \rangle = 0$
$H = -\sum_s A_s - \sum_p B_p$
$\psi(r_2, r_1)$
deconfined
$A_s = -1$
$A_s = -1$
around

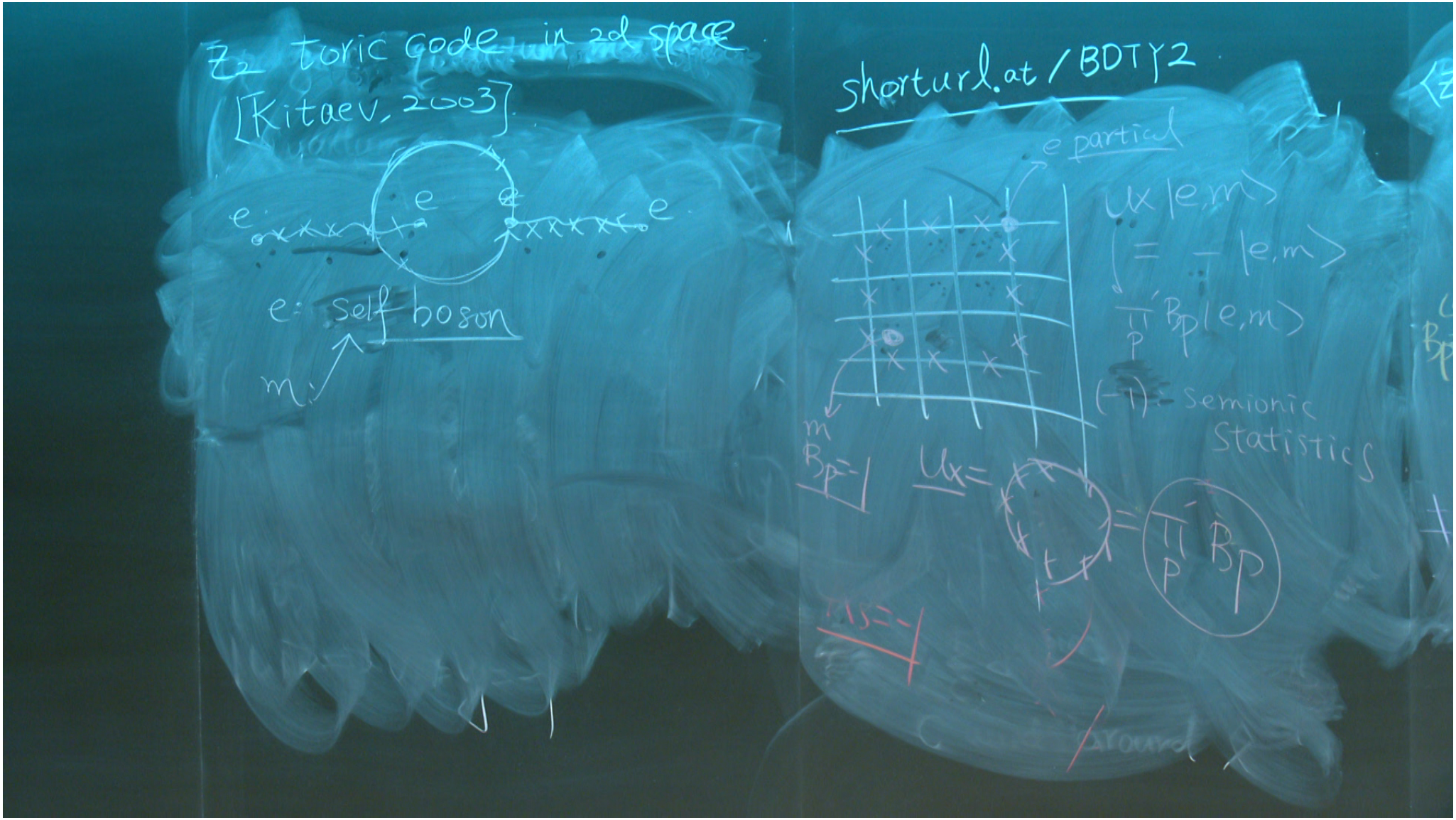

shorturl.at/BDTY2
$\langle z_i z_j \rangle = 0$
$H = -\sum_s A_s - \sum_p B_p$
$\rho_A = \mathrm{Tr}_{\bar{A}} |a,b\rangle\langle a,b|$
$= \mathrm{Tr}_{\bar{A}} \{|0,0\rangle\langle 0,0|\}$
A
$\bar{A}$
deconfined
$A_s = -1$

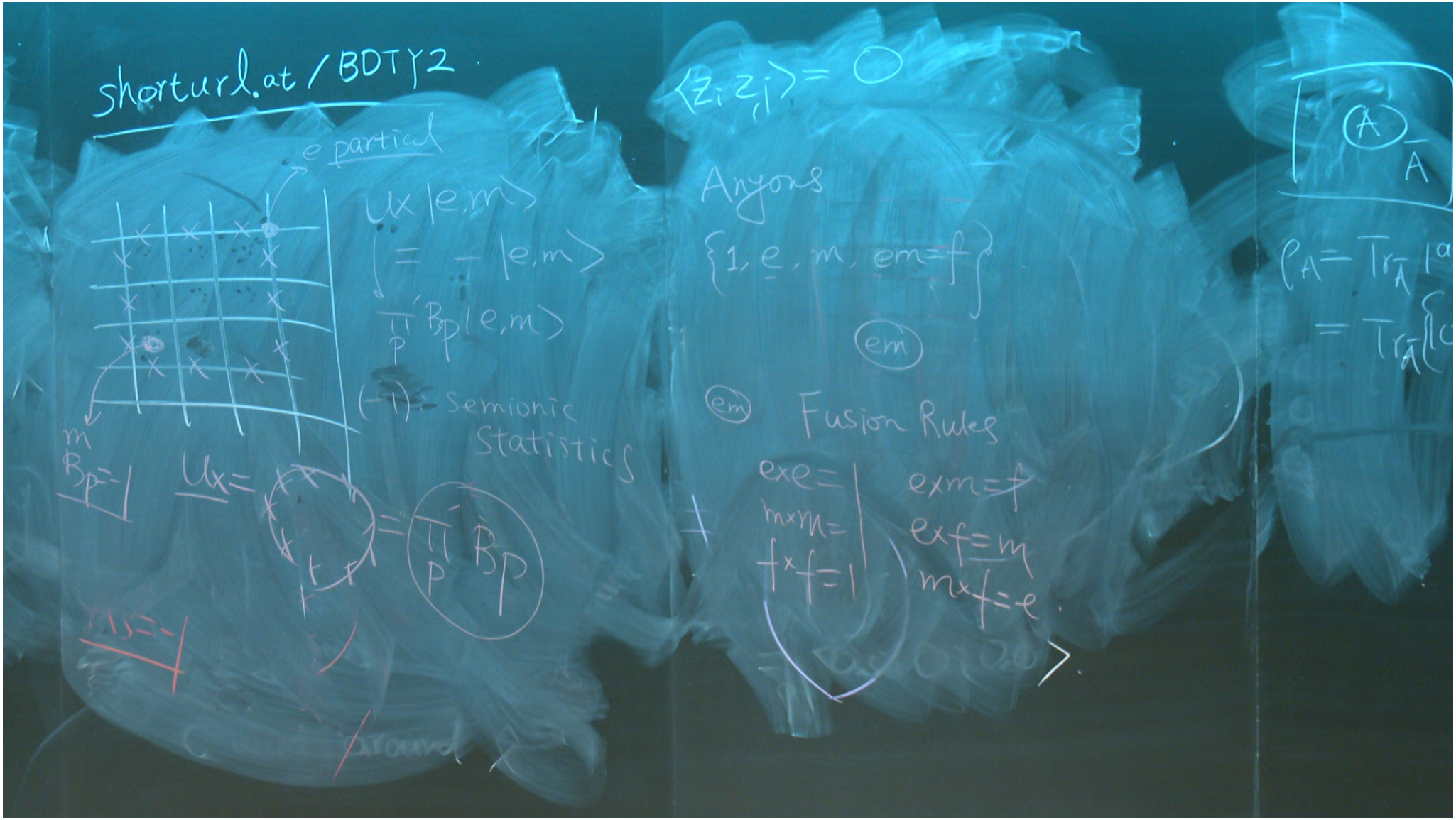

shorturl.at/BDTY2
$\langle z_i z_j \rangle = 0$
$H = -\sum_s A_s - \sum_p B_p$
$\rho_A = \mathrm{Tr}_{\bar{A}} |a,b\rangle\langle a,b|$
$= \mathrm{Tr}_{\bar{A}} \{|0,0\rangle\langle 0,0|\}$
deconfined
A
$\bar{A}$

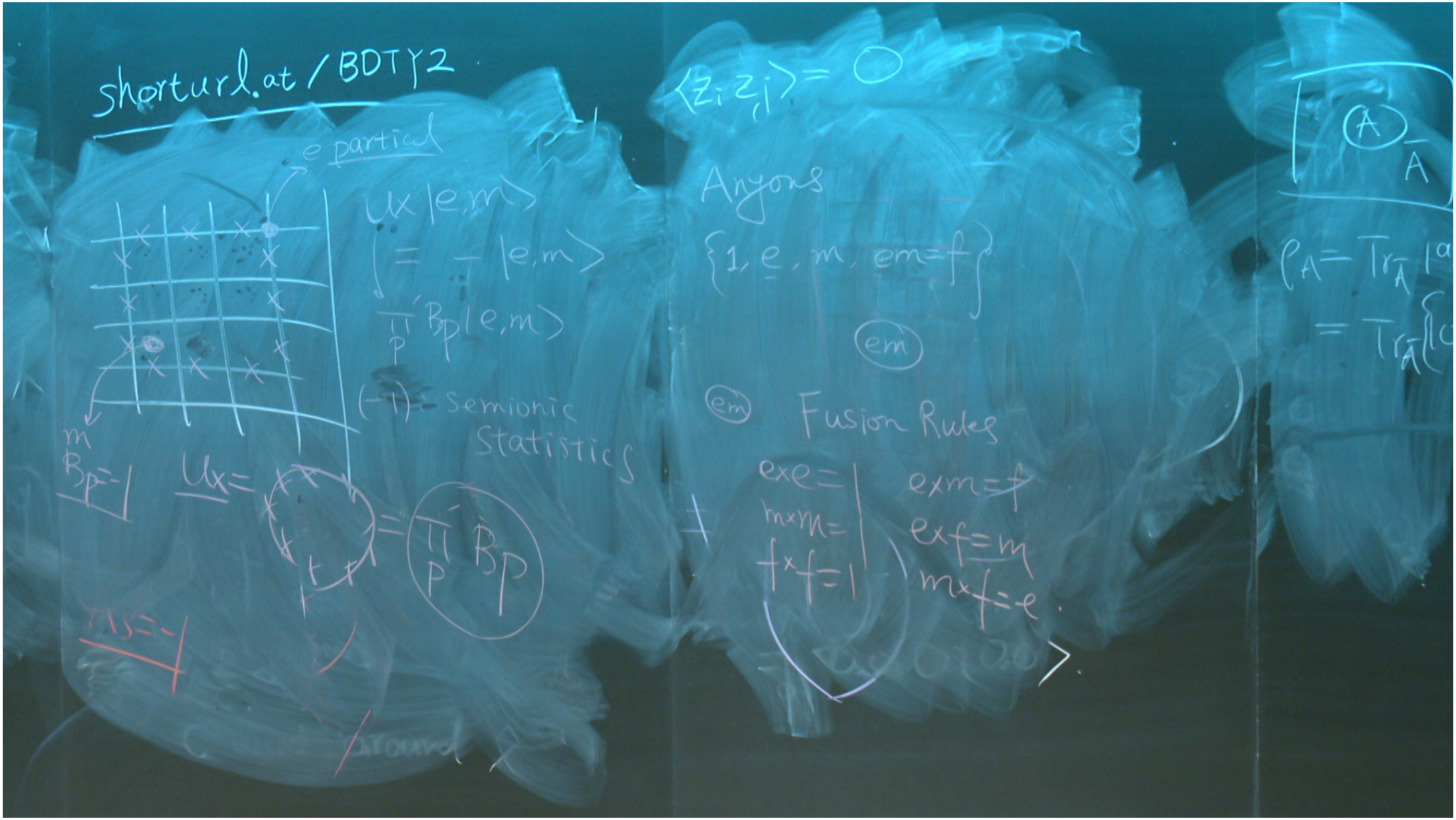

space
shorturl.at/BDTY2
$\langle z_i z_j \rangle = 0$
$H = -\sum_s A_s - \sum_p B_p$
$B_p = -1$
$B_p = -1$
m-particle
$A_s = -1$
deconfined
e-particle
$A_s = -1$
$A_s = -1$
around


space
shorturl.at/BDTY2
$\langle z_i z_j \rangle = 0$
$H = -\sum_s A_s - \sum_p B_p$
$B_p = -1$
$B_p = -1$
m-particle
$A_s = -1$
deconfined
e-particle
$A_s = -1$
$A_s = -1$
around


Z_2 toric code in 2d space
[Kitaev, 2003]
e
e
e
e
e: self boson
m
shorturl.at/BDTY2
e particle
$U_x|e,m\rangle$
$= -|e,m\rangle$
$\prod_p B_p|e,m\rangle$
(-1)-semionic
Statistics
m
$B_p=1$
$U_x=$
$\prod_p B_p$
$\prod B_p=-1$
around


Z_2 toric code in 2d space
[Kitaev, 2003]
e
e
e
e
e: self boson
m
shorturl.at/BDTY2
e particle
$U_x|e,m\rangle = -|e,m\rangle$
$\prod_p B_p|e,m\rangle$
$(-1)^{\cdots}$ Semionic Statistics
m
$B_p=1$
$U_x=$
$\prod_p B_p$
$\cdots=-1$


shorturl.at/BDTY2
e particle
m
Semionic Statistics
Anyons
{1, e, m, em=f}
em
Fusion Rules
exe=1
mxm=1
fxf=1
exm=f
exf=m
mxf=e


shorturl.at/BDTY2
e particle
m
$B_p = 1$
$U_x =$
$\prod_p B_p$
$U_x |e,m\rangle$
$= -|e,m\rangle$
$\prod_p B_p |e,m\rangle$
(-1) = Semionic Statistics
Anyons
{1, e, m, em=f}
em
Fusion Rules
exe =
mxm =
fxf = 1
exm = f
exf = m
mxf = e
$\langle z_i z_j \rangle$ =
A
$\rho_A = \mathrm{Tr}_{\bar{A}}$