

Title: Generating Kitaev spin liquid from a stochastic measurement-only circuit

Speakers: Zhu-Xi Luo

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Abstract: Experimental realizations of long-range entangled states such as quantum spin liquids are challenging due to numerous complications in solid state materials. Digital quantum simulators, on the other hand, have recently emerged as a promising platform to controllably simulate exotic phases. I will talk about a constructive design of long-range entangled states in this setting, and exploit competing measurements as a new source of frustration to generate spin liquid. Specifically, we consider random projective measurements of the anisotropic interactions in the Kitaev honeycomb model. The monitored trajectories can produce analogues of the two phases in the original Kitaev model: (i) a topologically-ordered phase with area-law entanglement and two protected logical qubits, and (ii) a "critical" phase with a logarithmic violation of area-law entanglement and long-range tripartite entanglement. A Majorana parton description permits an analytic understanding of these two phases through a classical loop model. Extensive numerical simulations of the monitored dynamics confirm our analytic predictions. This talk is based on <https://arxiv.org/abs/2207.02877>.

Zoom link: <https://pitp.zoom.us/j/99600719755?pwd=a0pOWlliU0swVDdGYnhxaGFGNkJSdz09>

# Long-range entangled Kitaev spin liquid states

generated from

a stochastic **measurement-only** circuit



arXiv: 2207.02877

Zhu-Xi Luo  
Harvard University



w/ A. Lavasani and S. Vijay

1/11/2023 @ PI

# Long-range entanglement

## What and why

- Definition: A state is LRE if it cannot be converted into a product state by a generalized stochastic local transformation.



# Long-range entanglement

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- ~~Definition: A state is LRE if it cannot be converted into a product state by a generalized stochastic local transformation.~~
- Include: Gapped topological ordered states and gapless states.
- For example, [quantum spin liquids](#), quantum Hall, fractons, ...
- Exclude: conventional symmetry-breaking states, symmetry protected topological states, ...



# Long-range entanglement

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- Include: Gapped topological ordered states and gapless states.
- For example, [quantum spin liquids](#), quantum Hall, fractons, ...
- Exclude: conventional symmetry-breaking states, symmetry protected topological states, ...
- LRE states are fundamentally interesting, rich, and can be useful for quantum computation.



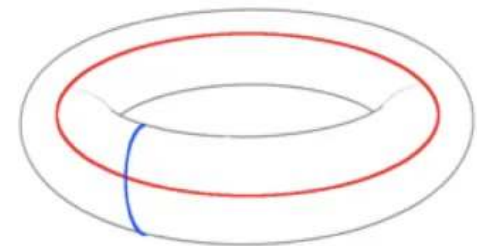
# Protected qubits

## Why LRE is useful

- Big challenge in quantum computation: decoherence.
- Idea: Use LRE states to encode quantum information, since they are robust against local operations (errors). The information is protected by topology.

[Kitaev (97'), Freedman, Larsen, Wang (00')]

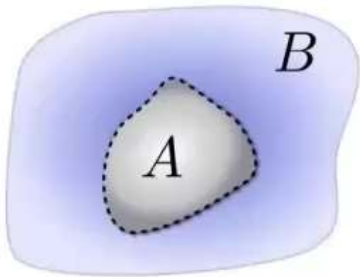
- Example: long cycle qubits in toric code. Degenerate ground states on torus cannot be transformed into each other unless an extensive number of operations are performed.



# Topological entanglement entropy

## Manifestation of LRE

- One computable quantity that makes LRE manifest, is topological entanglement entropy (TEE).
- If entanglement is short-ranged, it typically has area-law  $S_A = \alpha l$
- By construction, TEE eliminates short-range entanglement.

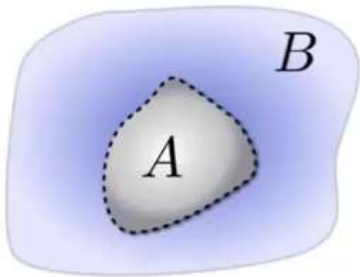


[Kitaev, Preskill (2005),  
Levin, Wen (2005)]

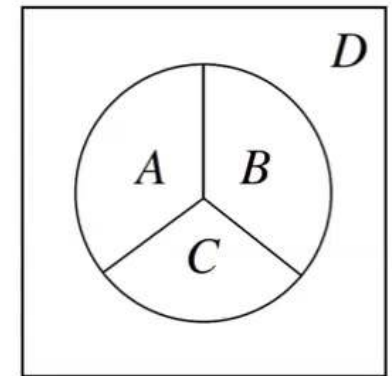
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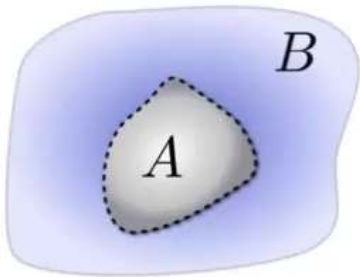


[Kitaev, Preskill (2005),  
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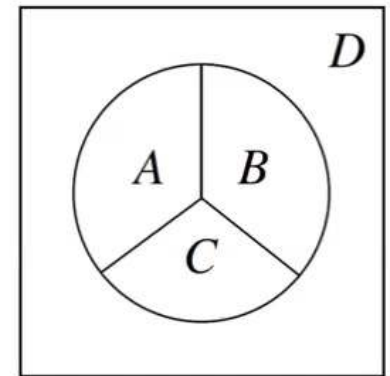
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$$l_{A,B} + l_{A,C} + l_{A,D}$$



[Kitaev, Preskill (2005),  
Levin, Wen (2005)]

## Important example of LRE: Quantum spin liquids

Anderson (1973) (1987)



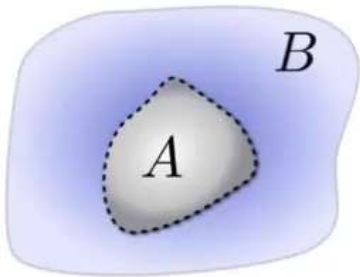
Figure: [phys.org](http://phys.org)

“Liquid” of disordered spins down to very low temperatures

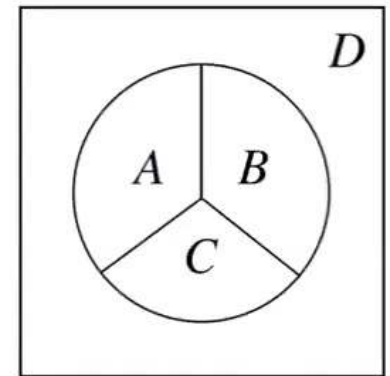
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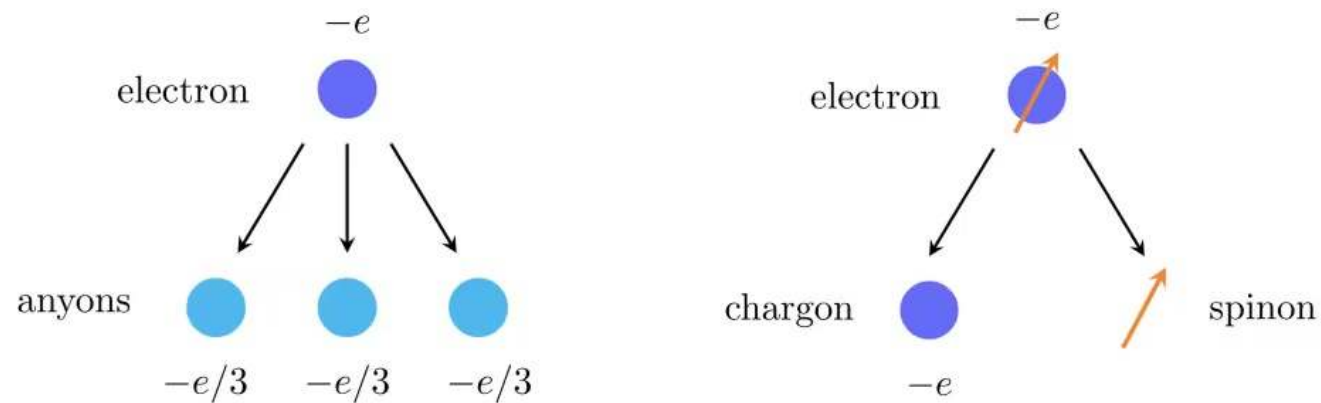
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[Kitaev, Preskill (2005),  
Levin, Wen (2005)]

# Quantum spin liquids

- Beyond the conventional Landau symmetry-breaking paradigm
- Characterized by fractionalized excitations, long-range entanglement, topological orders ...
- Theoretical tools include gauge theory, topological field theory, category theory ...



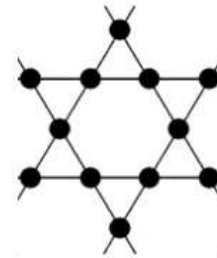
# Realizations of LRE

## Guiding principles of the search

- In search of quantum spin liquids, we often look for **frustrations**.

- Geometry

- e.g. Kagome NN Heisenberg



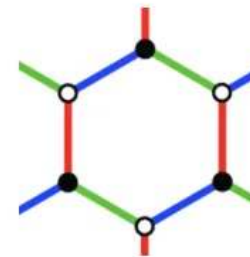
Zhu & White (2015), Hu et al (2015),  
Kaneko, Morita & Imada (2014), ...

- Longer-range interactions

- e.g. triangular NN+NNN Heisenberg

Esser (1989), Sachdev (1992), Lecondinant et al (1997), Ran,  
Hermele, Lee & Wen (2007), Yan, Huse & White (2011), ...

- Anisotropy, e.g. Kitaev honeycomb model



Real solid state materials are complicated.

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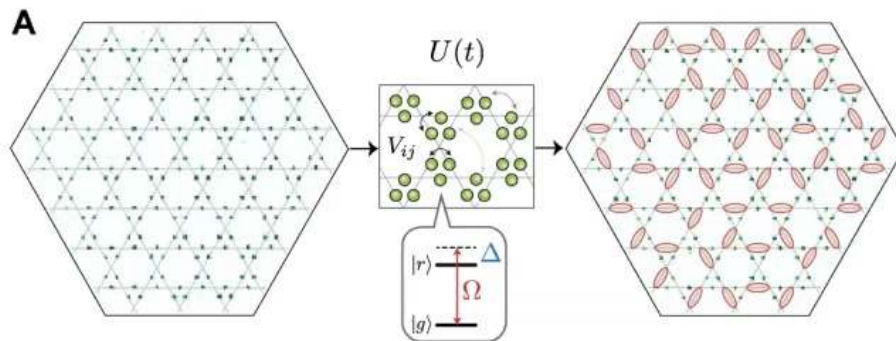
Can we **constructively** engineer the **frustrations**?

# Quantum simulators

## Programmable approach towards LRE

- Platforms: Trapped ions, ultracold atoms, superconducting qubits, ...

Engineer geometric frustration  
in Rydberg atom arrays



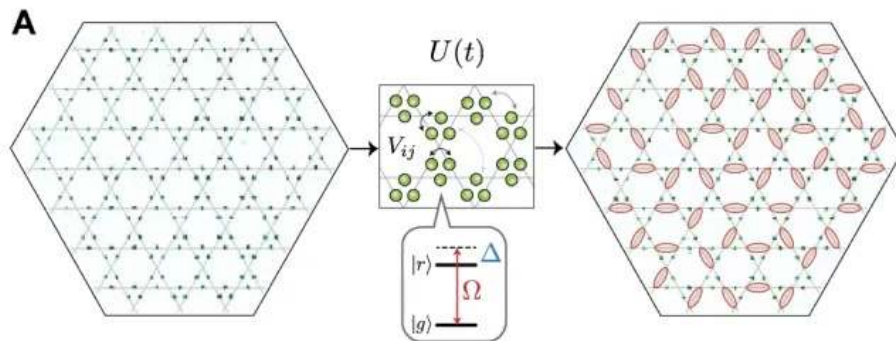
Semighini et al, Science (2021)

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Semighini et al, Science (2021)

This talk explores a new source of frustration:

**Competition between measurements**



# Measurements

Innocent ones



Observer effect



Your see a police officer measuring your speed



And you slow down

# Warmup example

- Measurement can destroy entanglement.

- Consider two qubits A, B in the Bell state

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$$

- Measurement of A in the computational basis,

- Get  $0_A$  or  $1_A$  each with probability  $1/2$
- There is no entanglement left.

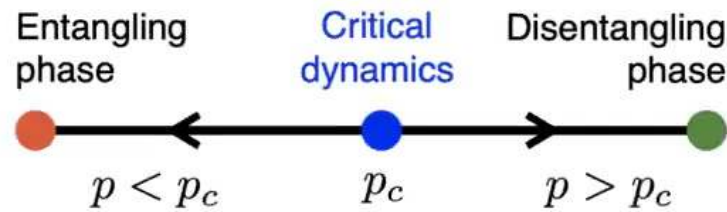
$$\begin{aligned} |\Phi^+\rangle_{AB} &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B), \\ |\Psi^+\rangle_{AB} &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B), \\ |\Psi^-\rangle_{AB} &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B), \\ |\Phi^-\rangle_{AB} &= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B), \end{aligned}$$

## Warmup example II

- Measurement can also create entanglement.
  - Consider three qubits ABC in the state  $|0\rangle_A \otimes \frac{1}{\sqrt{2}}(|00\rangle_{BC} + |11\rangle_{BC})$
  - Measure BC in the Bell basis does nothing  $|0\rangle_A \otimes |\Phi^+\rangle_{BC}$
  - Measure AB in the Bell basis disentangles BC, and maximally entangles AB.
    - Before:  $\frac{1}{2}(|\Phi^+\rangle_{AB} + |\Phi^-\rangle_{AB}) \otimes |0\rangle_C + \frac{1}{2}(|\Psi^+\rangle_{AB} + |\Psi^-\rangle_{AB}) \otimes |1\rangle_C$
    - After: with probability 1/4, get  $|\Phi^+\rangle_{AB} \otimes |0\rangle_C$
  - Measure BC again in the Bell basis can give back the initial entanglement structure.

There is ... a time to tear down and a time to build. (Ecclesiastes 3:3)

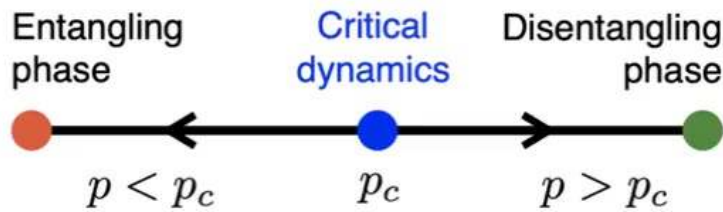
Measurements decrease entanglement



Li, Chen, Fisher, PRB (2018);  
Skinner, Ruhman, Nahum PRX (2019),  
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### Measurements generate entanglement

- Competing measurements induce phase transitions and volume law

Ippoliti et al, PRX (2021); Sang, Hsieh, PRR (2021); Lang, Büchler, PRB (2020),; Lavasani, Alavirad, Barkeshli, Nat Phys (2021)

...

- Measuring stabilizers directly to get toric code

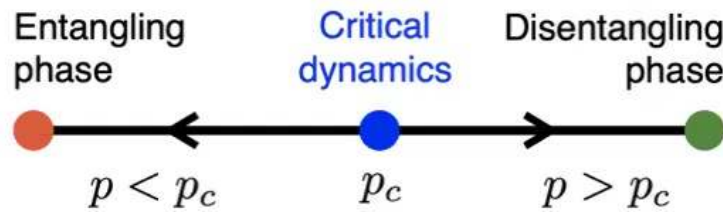
Lavasani, Alavirad, Barkeshli, PRL (2021) ...

- Fixed sequence of measurements to generate protected qubits. Haah, Hastings, Quantum (2021) ...

Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021) ...

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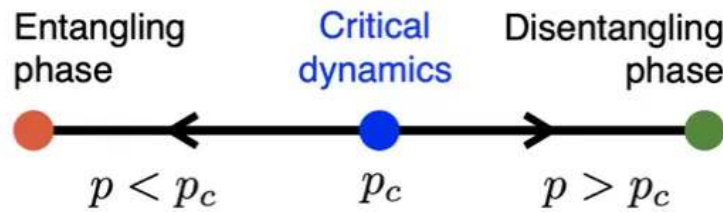
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Measurement-only

Stochastic

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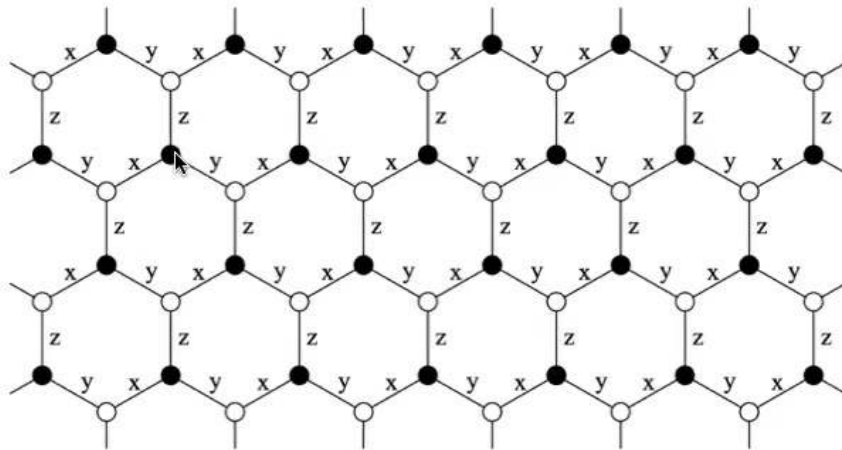
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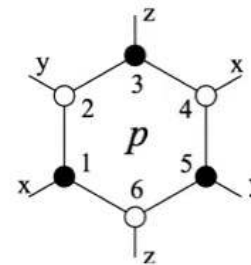
# Kitaev honeycomb model I

$$H = -J_x \sum_{x\text{-links}} X_j X_k - J_y \sum_{y\text{-links}} Y_j Y_k - J_z \sum_{z\text{-links}} Z_j Z_k$$



Conserved quantities: Loops of bond operators

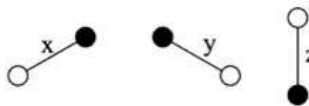
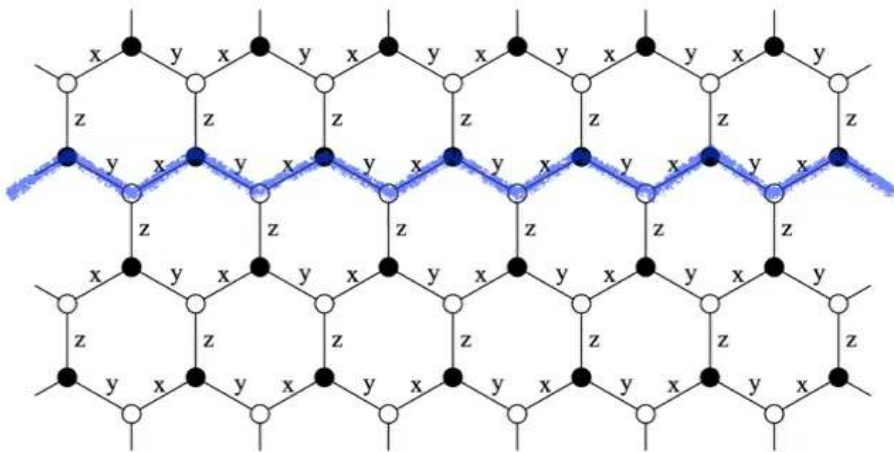
♦ Small loop: **plaquette operators**



$$W_p = X_1 Y_2 Z_3 X_4 Y_5 Z_6$$

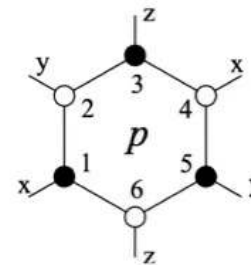
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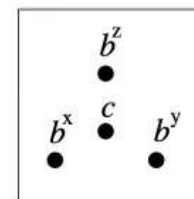
◆ Long loop: **non-contractible cycles**

# Kitaev honeycomb model II

$$X_i = ib_i^x c_i, \quad Y_i = ib_i^y c_i, \quad Z_i = ib_i^z c_i, \quad b_i^x b_i^y b_i^z c_i = 1$$

$$H = \frac{i}{2} \sum_{\langle ij \rangle} J_{a_{ij}} u_{ij}^a c_i c_j, \quad u_{ij}^a = ib_i^a b_j^a$$

$\mathbb{Z}_2$  gauge field, commutes with Hamiltonian.



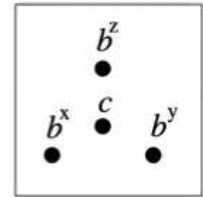
Entanglement entropy

$$S_A = (\alpha + \log 2)L - \log 2$$

$$S_{topo} = \log 2$$

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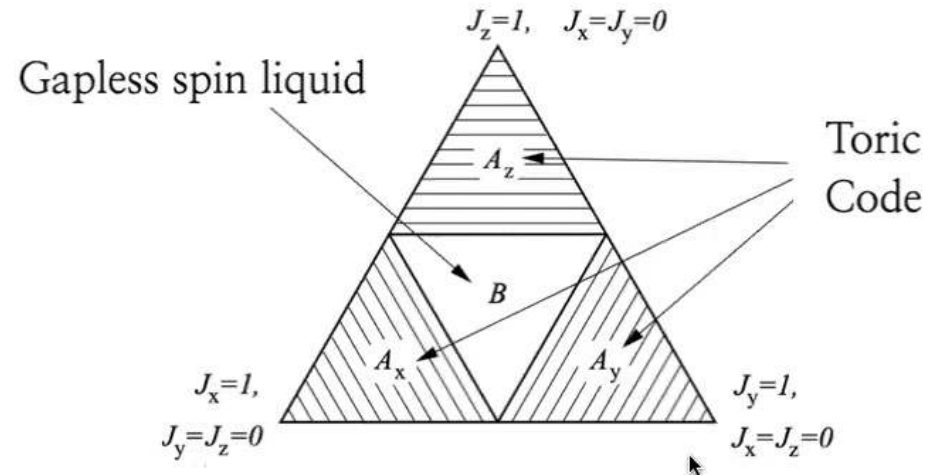
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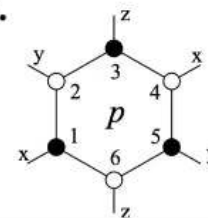
# Our setup

Inspired by the Kitaev honeycomb model

- Honeycomb lattice, qubits live on vertices. Start from maximally mixed initial state

$$\rho = \mathbb{1}/2^N$$

- At each time step,  $N$  consecutive measurements. Each measurement chooses one site and choose the measure one of the bond operators  $X_i X_j$ ,  $Y_i Y_k$ ,  $Z_i Z_l$  with probabilities  $p_x$ ,  $p_y$ ,  $p_z$
- $W_p$ , the product of bond operators around an elementary plaquette, commute with all bond measurements and are measured with constant rate.



$$W_p = X_1 Y_2 Z_3 X_4 Y_5 Z_6$$

# Stabilizer formalism

- Maximum  $L = 40$ ,  $N = 2L^2$ .
- Efficient description of stabilizer states: requires only  $2n^2$  bits to specify a  $n$  qubit state.
- Stabilizer group  $G = \langle g_1, g_2, \dots, g_n \rangle$ .  $\forall U \in G, U|\psi\rangle = |\psi\rangle$ .
- Decompose  $G = G_A \cdot G_B \cdot G_{AB}$ . Then the entanglement is  $S_A = \frac{1}{2} |G_{AB}|$ .
- Example: Bell state  $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ ,  $G = \langle XX, ZZ \rangle = G_{AB}$ ,  $S_A = 1$ .
- Example: GHZ state  $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ ,  $G = \langle XXX, ZZI, IZZ \rangle$ . Choose  $A=12$  and  $B=3$ ,  
$$G_{AB} = \langle XXX, IZZ \rangle, \quad S_A = 1.$$

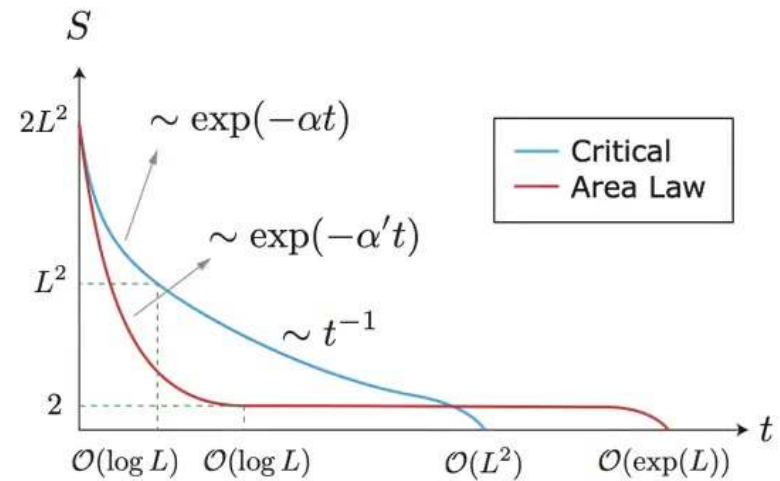
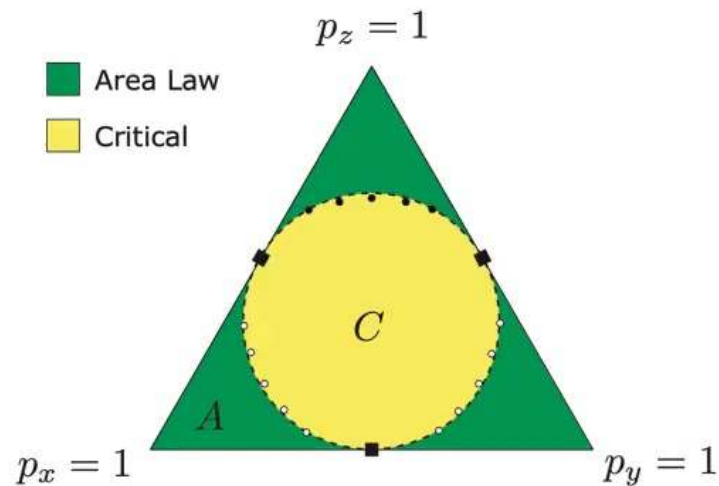
[Gottesman (1996, 1997); Gottesman & Knill (2004); Aaronson & Gottesman (2004); Fattal et al (2004)]

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- In our setup:  $S_A = \frac{1}{2}n_p + \frac{1}{2}n_c - 1$ 
  - # of plaquettes on the boundary
  - # of Majorana dimers/string operators with one end in A and another in B
- Quantities are average over an ensemble of randomly generated quantum trajectories.

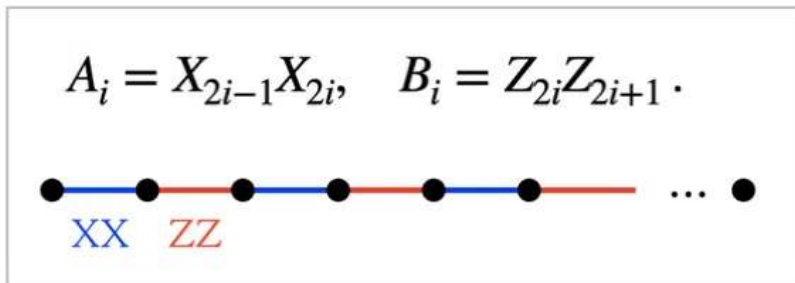
# Preview of results

- **A phase** with area-law entanglement and two protected qubits.
- **C phase** with logarithmic violation of area law and TEE.
- Purification dynamics analyzed.

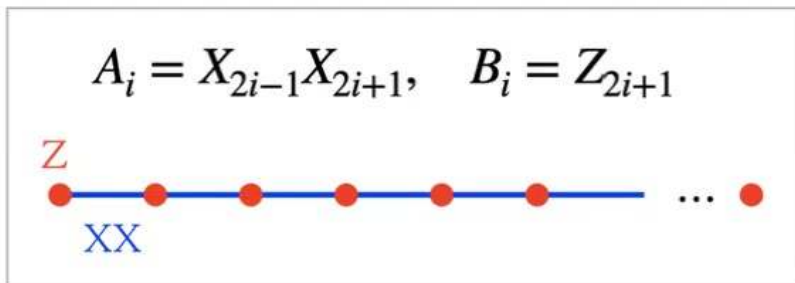


# Quasi-1d limit

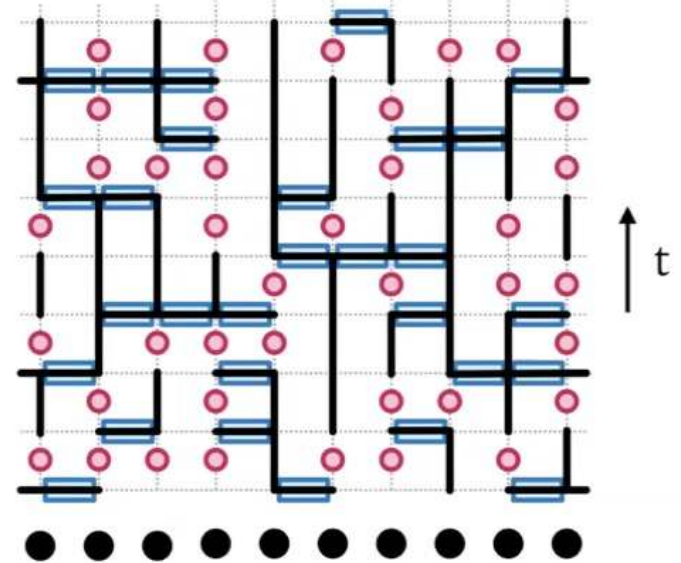
- Phase diagram boundary  $p_y = 0$ .  $L$  decoupled rows. Focus on one row with  $2L$  spins.



↓ Loc. uni.



- Projective transverse Ising model

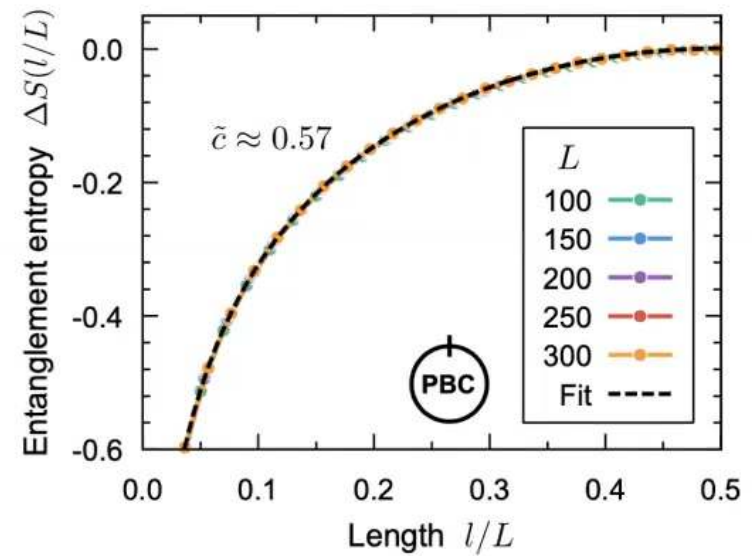
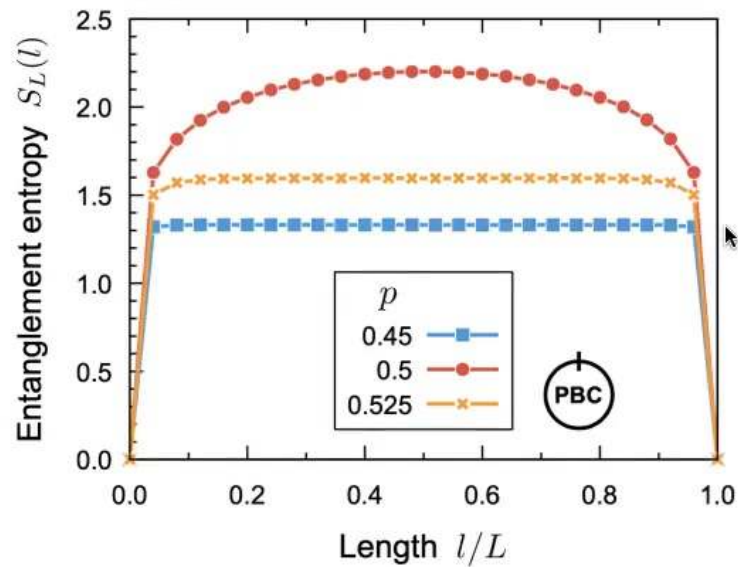


- Maps to 2d classical percolation,  $p_c = 0.5$ .

Ippoliti et al, PRX (2021); Sang, Hsieh, PRR (2021); Lang, Büchler, PRB (2020); Lavasani, Alavirad, Barkeshli, Nat Phys (2021)...

# Quasi-1d limit

Known results



$$\Delta S(l/L) \equiv S_L(l) - S_L(L/2) \sim \frac{\tilde{c}}{3} \log_2 \left[ \sin \left( \pi \frac{l}{L} \right) \right]$$

$$\tilde{c} \approx 0.57$$

[Lang, Büchler, PRB (2020)]

# Parton description

$$X_i = ib_i^x c_i, \quad Y_i = ib_i^y c_i, \quad Z_i = ib_i^z c_i, \quad b_i^x b_i^y b_i^z c_i = 1$$

- The projection into physical Hilbert space **commutes** with spin measurements,

$$\frac{1 + Z_i}{2} |\Psi\rangle \sim P \frac{1 + ib_i^z c_i}{2} |\psi_f\rangle, \quad P = \prod_j \frac{1 + b_j^x b_j^y b_j^z c_j}{2}$$

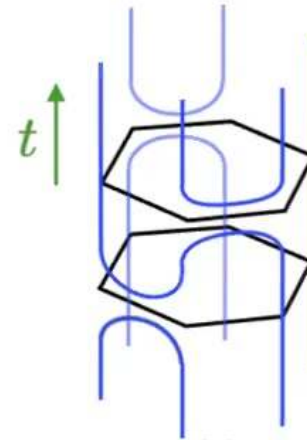
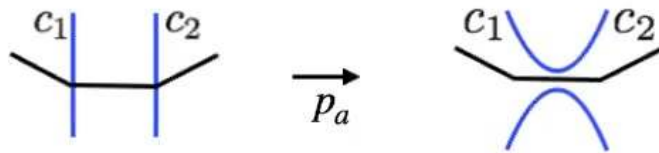
- Problem reduces to evolution of Majorana partons, followed by a final projection.
- Plaquette operators are all measured after a short time, so decompose

$$\rho_f(t) = \rho_c(t) \otimes \rho_b$$

- Measurements of bond operators **commute** with  $\rho_b$ . Problem reduces to the evolution of **Majorana c partons**.

# Loop representation

- Evolution is described by a loop model on the 3d lattice.  $c$ -Majoranas live on endpoints of loops.
- Measurements reconnect the loops.



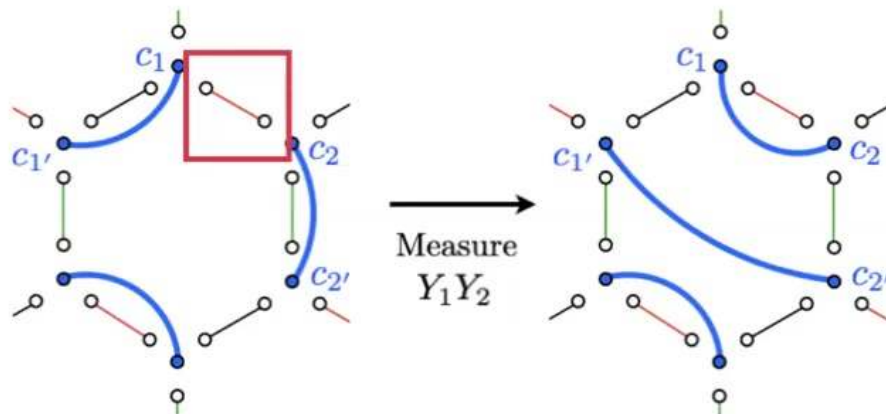
- The loop model has two phases:

[Nahum, Chalker, Serna, Ortuno, Somoza PRL(2011), PRB (2013); Nahum, Serna, Somoza Ortuno, PRB (2013).]

- Short loops,  $Q(r) \sim \exp(-r/\xi) \Rightarrow$  Get area law entanglement, A phase.
- Long loops,  $Q(r) \sim r^{-2} \Rightarrow$  Get  $R \log R$  entanglement, C phase.

# Steady state

- Bond measurements reconnect different dimer coverings of the honeycomb lattice.



$$\begin{aligned} & \Pi \frac{1 + ic_1 c_1'}{2} \frac{1 + ic_2 c_2'}{2} \frac{1 + ib_1^y b_2^y}{2} \Pi \\ &= \frac{1 - ic_1 c_2}{2} \frac{1 + ic_1' c_2'}{2} \frac{1 + ib_1^y b_2^y}{2} \end{aligned}$$

- In terms of spins, a dimer corresponds to the product of bond operators.
- Different length distributions of dimers gives entanglement scalings.

# Purification dynamics

Different degrees of freedom disentangle from the environment with different rates.

(i) Plaquette operators ( $L^2 - 1$ )

◆ Leads to the exponential drop of  $S$

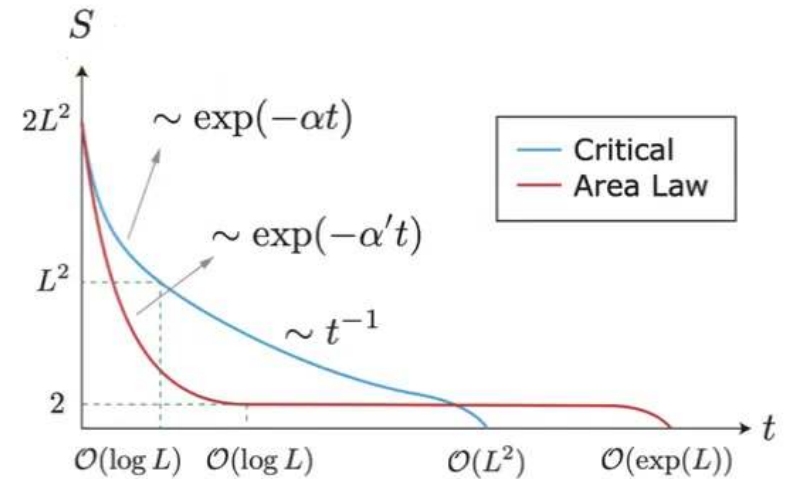
(ii) Nontrivial-cycle stabilizers (2)

◆ In the A phase, survives until exponential time

◆ In the C phase, survives up to polynomial time

(iii) Bond operators of specific type ( $L^2 - 1$ )

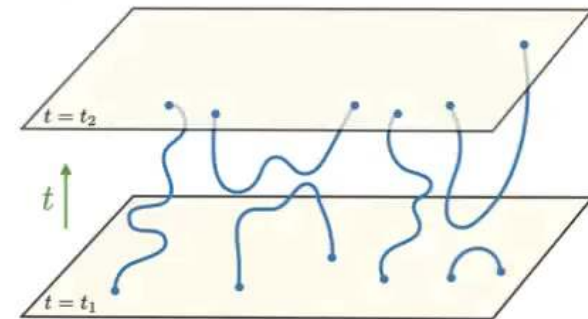
◆  $\rho_f = |\Psi_b\rangle\langle\Psi_b| \otimes \mathbf{1}$



# Purification dynamics continued

(iii) Bond operators of specific type ( $L^2 - 1$ )

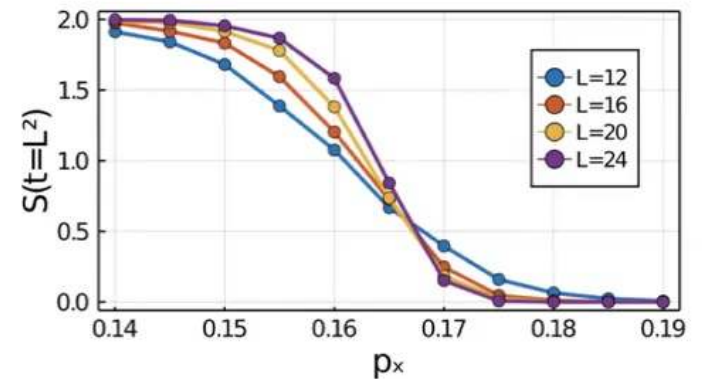
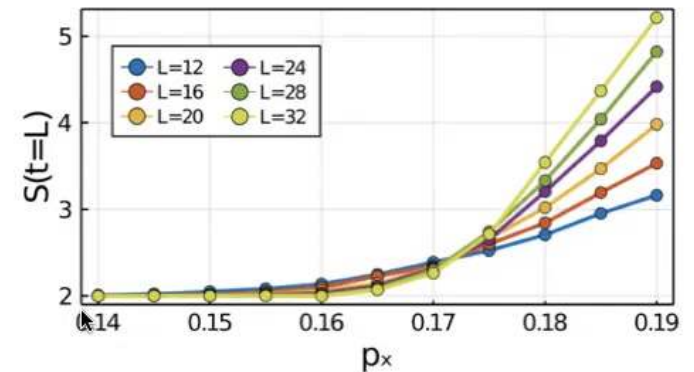
- ◆  $S$  measures the probability of non-returning walk, which is the number of loops connecting top and bottom time slices.
- ◆ In C phase, the probability is  $t^{-1}$ . Spanning number  $\sim L^2/t$ .
- ◆ In A phase, spanning number decays exponentially.



# Purification dynamics continued

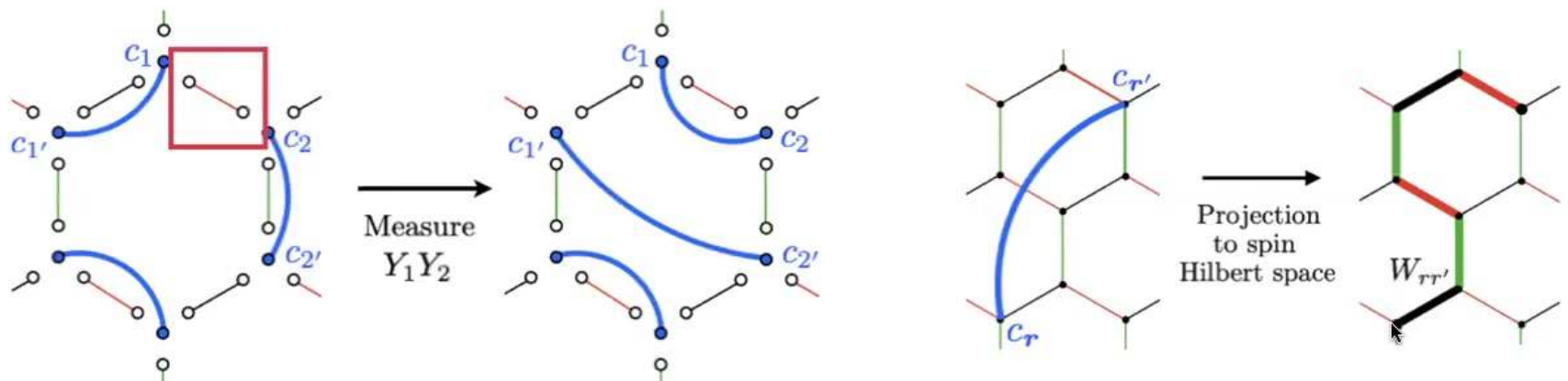
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# Steady state

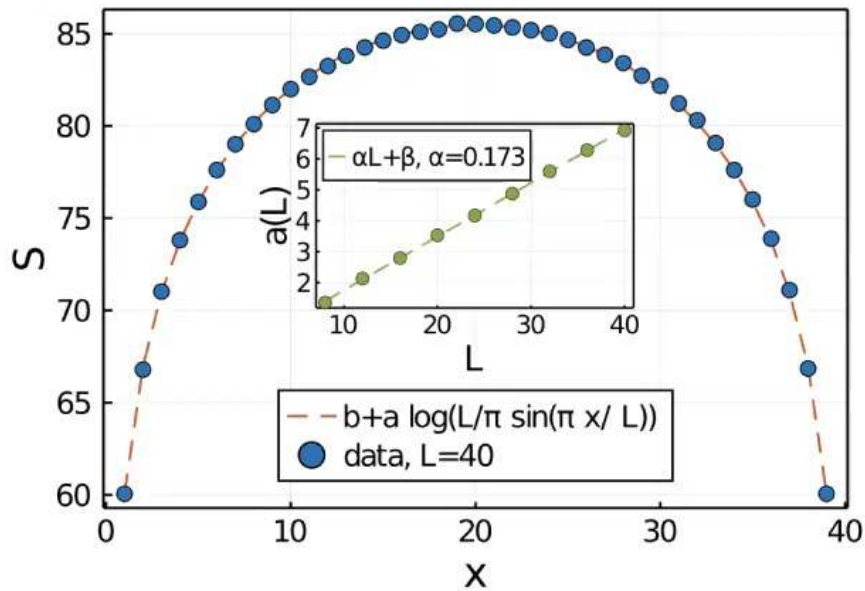
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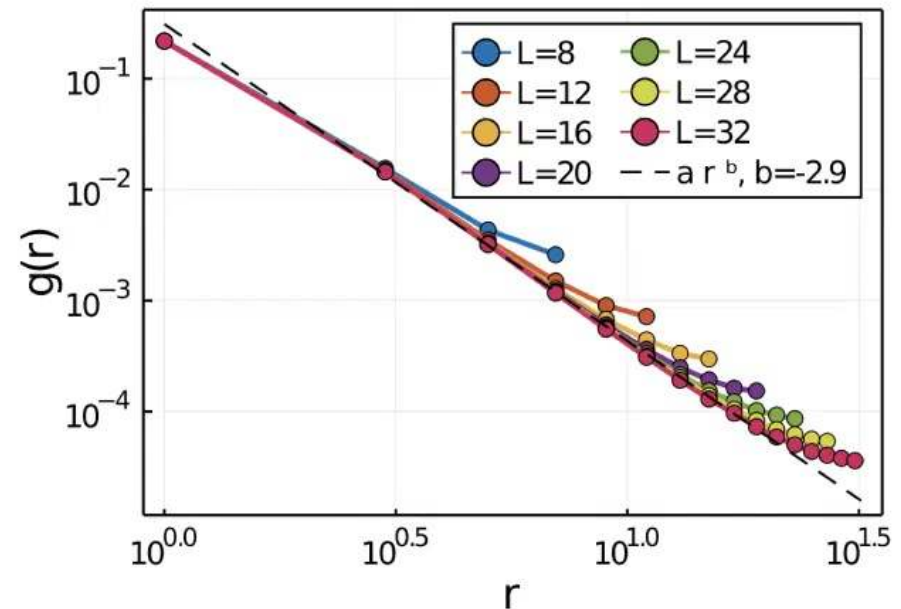
# Entanglement in the C phase

Entanglement



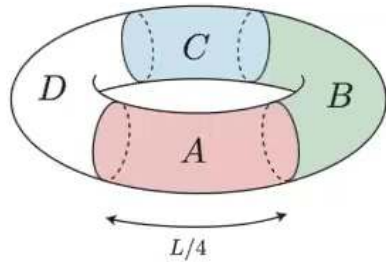
$$S(x) = b(L) + a(L) \log \left[ \frac{L}{\pi} \sin \left( \frac{\pi x}{L} \right) \right]$$

Wilson line correlator

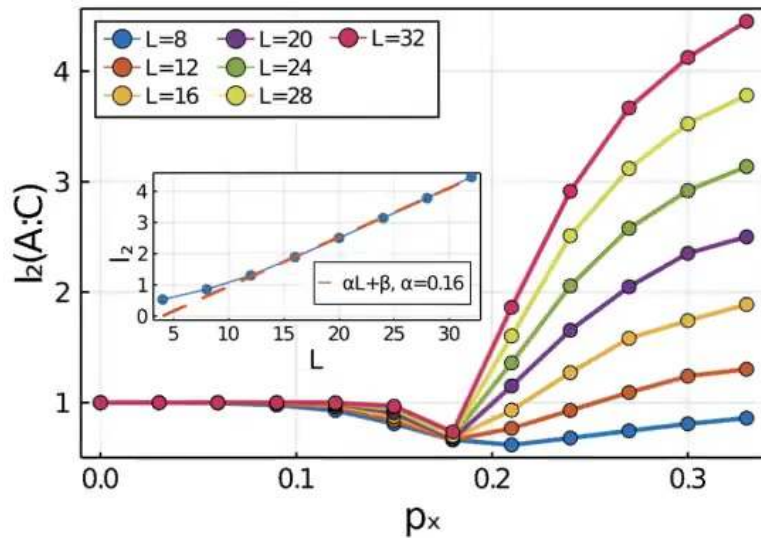


$$g(\mathbf{r} - \mathbf{r}') \equiv \overline{\langle W_{\mathbf{r}\mathbf{r}'} \rangle^2}$$

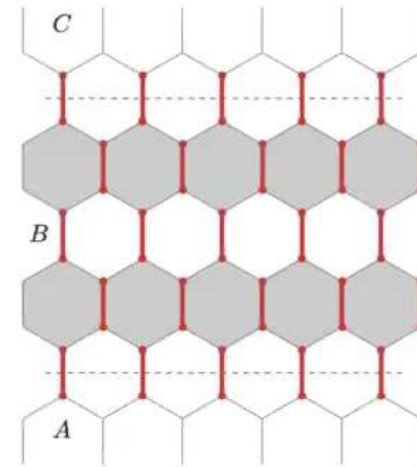
# Mutual information



$$I_2(A : C) = S_A + S_C - S_{AC}$$



A phase:

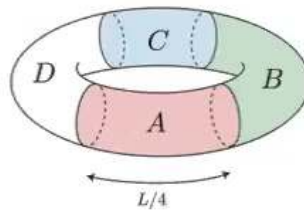
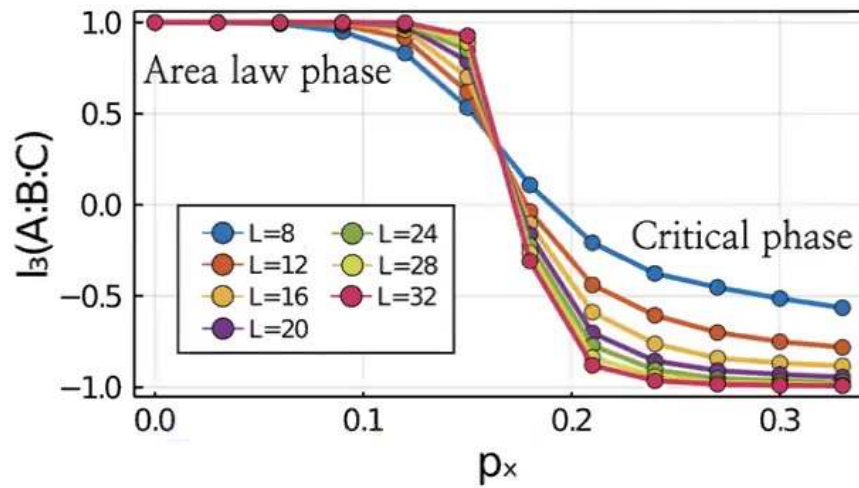


C phase:

$$\int_{-L/2}^{L/2} dy' \int_0^{L/4} dx' \int_{-L/2}^{L/2} dy \int_{x'+L/4}^{x'+L/2} \frac{dx}{(x^2 + y^2)^{3/2}}$$

# Long-range entanglement

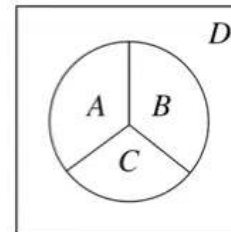
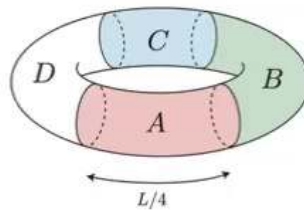
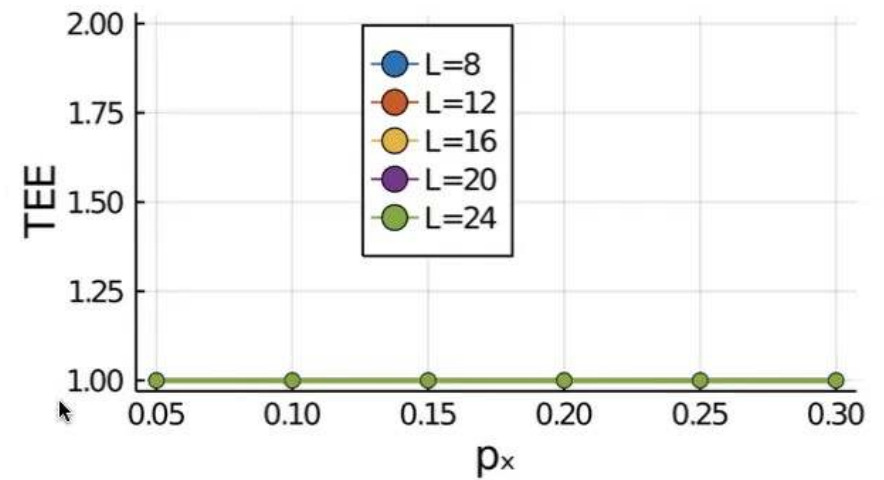
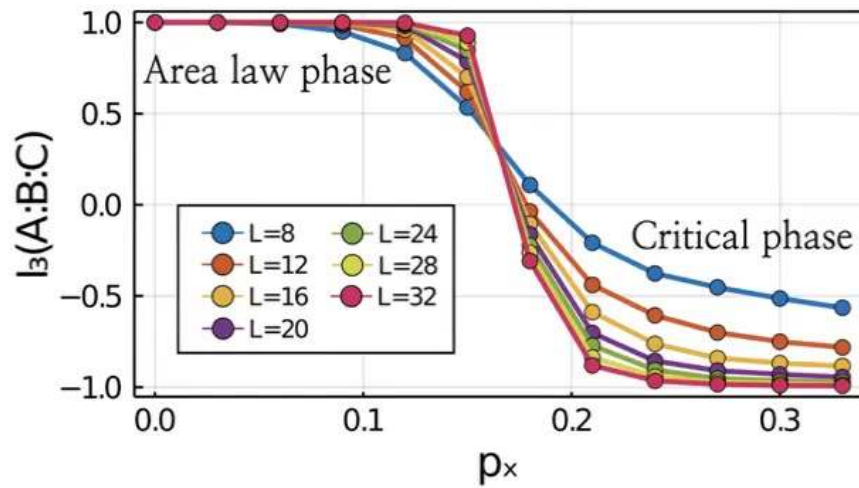
$$I_3(A : B : C) = I_2(A : B) + I_2(A : C) - I_2(A : BC)$$



# Long-range entanglement

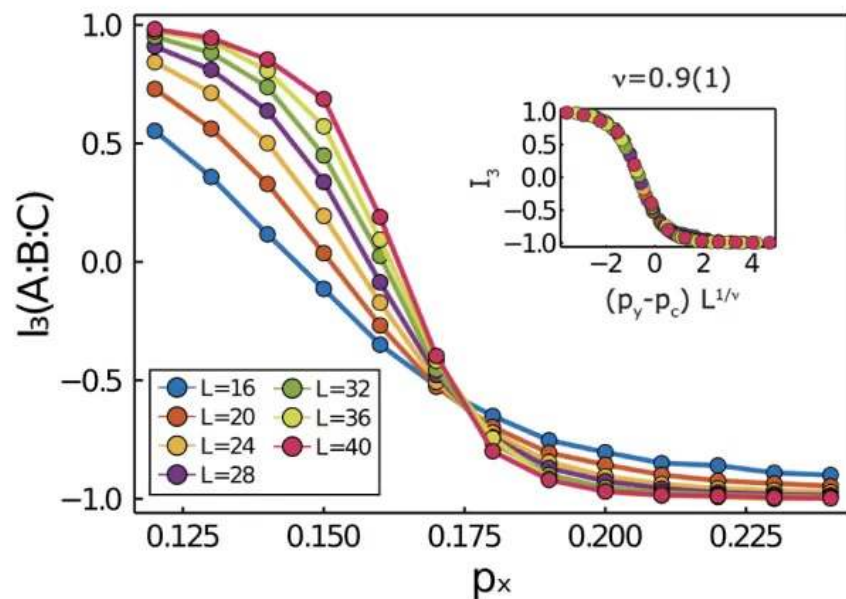
$$I_3(A : B : C) = I_2(A : B) + I_2(A : C) - I_2(A : BC)$$

$$S_{\text{topo}} \equiv S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC}$$



# Phase transition

$I_3$  as the order parameter



$$I_3(p, L) = F((p - p_c)L^{1/\nu})$$

- At  $p_x = p_y$  way from diagram boundary,  
 $p_c = 0.172(5)$  and  $\nu = 0.9(1)$
- Consistent with previous numerical results for the loop model.

[Ortuño, Somoza, Chalker, PRL (2009); Serna, (2021).]

- On the phase boundary,

$$p_c = 4/3, \quad \nu = 4/3$$

# Analogues in the equilibrium?

- **A phase**: Area law entanglement, TEE, two protected qubits due to two long cycles, arises in the corner regions of the phase diagram of the original Kitaev honeycomb model  $\Rightarrow$  **2d  $\mathbb{Z}_2$  toric code** on a torus in a specific superselection sector.
- **C phase**: Resembles a phase where **Majoranas form a Fermi surface** because of the scaling of EE and the bipartite mutual information. Not present in the original Kitaev honeycomb because of the coexistence of time reversal and translation symmetries.

[Knolle, Moessner, Perkins, PRL (2019); Motrunich, Gamle, Huse, PRB (2002)]

[Lahtinen, Ludwig, Treibst (2014), Schalker, Read, Kagalovsky (2001), Self et al (2019)]

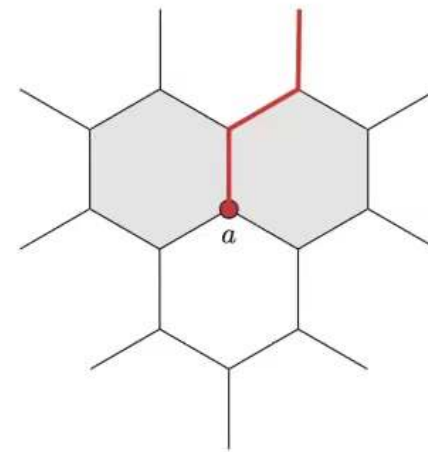
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  - Breaking translations by **randomness in spin exchange?**  
[Knolle, Moessner, Perkins, PRL (2019); Motrunich, Gamle, Huse, PRB (2002)]
  - Breaking time reversal?  
[Lahtinen, Ludwig, Treibst (2014), Schalker, Read, Kagalovsky (2001), Self et al (2019)]

# Perturbations I

## Adding random single qubit measurements

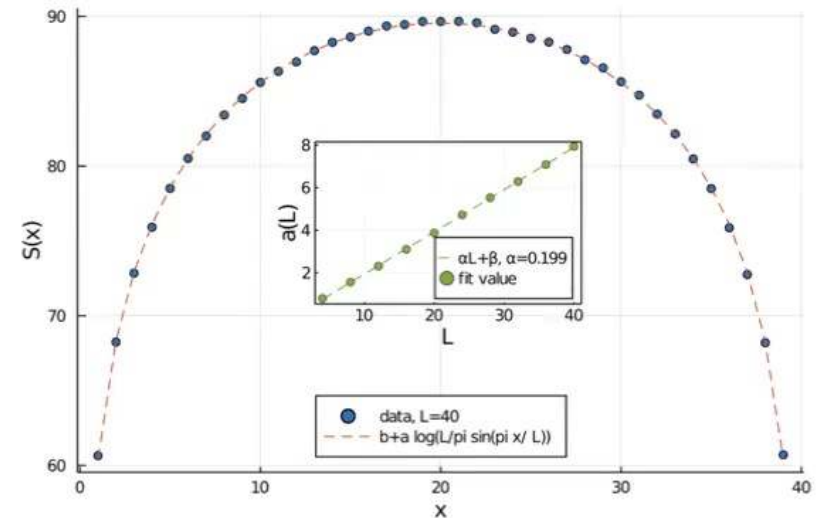
- Modified model: Measure  $Z$  with probability  $p_s$ , measure plaquette operators with probability  $p_{plaq}$  and measure bond operators with  $1 - p_s - p_{plaq}$ . Impose  $p_{plaq}$  later.
- When  $p_s \ll p_{plaq}$ , both phases survive.
- Plaquette operators are being measured with constant rate  $q$ . The two phases are stable if  $p_s \ll q$ .
- In the other limit, volume-law phase kick in.



# Perturbations II

## Adding three-qubit measurements

- Translates into next-nearest neighbor coupling of c-Majoranas and next-nearest neighbor moves in the loop model
- Preserves free-fermion nature and keeping extensive conservation laws
- Flow into another critical phase with long-range correlations.



# Summary

- We used frustrated decoherence to fight against decoherence.
- We examined measurement-only circuit dynamics based on Kitaev honeycomb setup, realized two LRE phases and protected qubits.
  - Decoding scheme?
  - Better understanding of the critical phase?
  - Generate gapless spin liquids?
  - Generate new exotic states?
  - ...