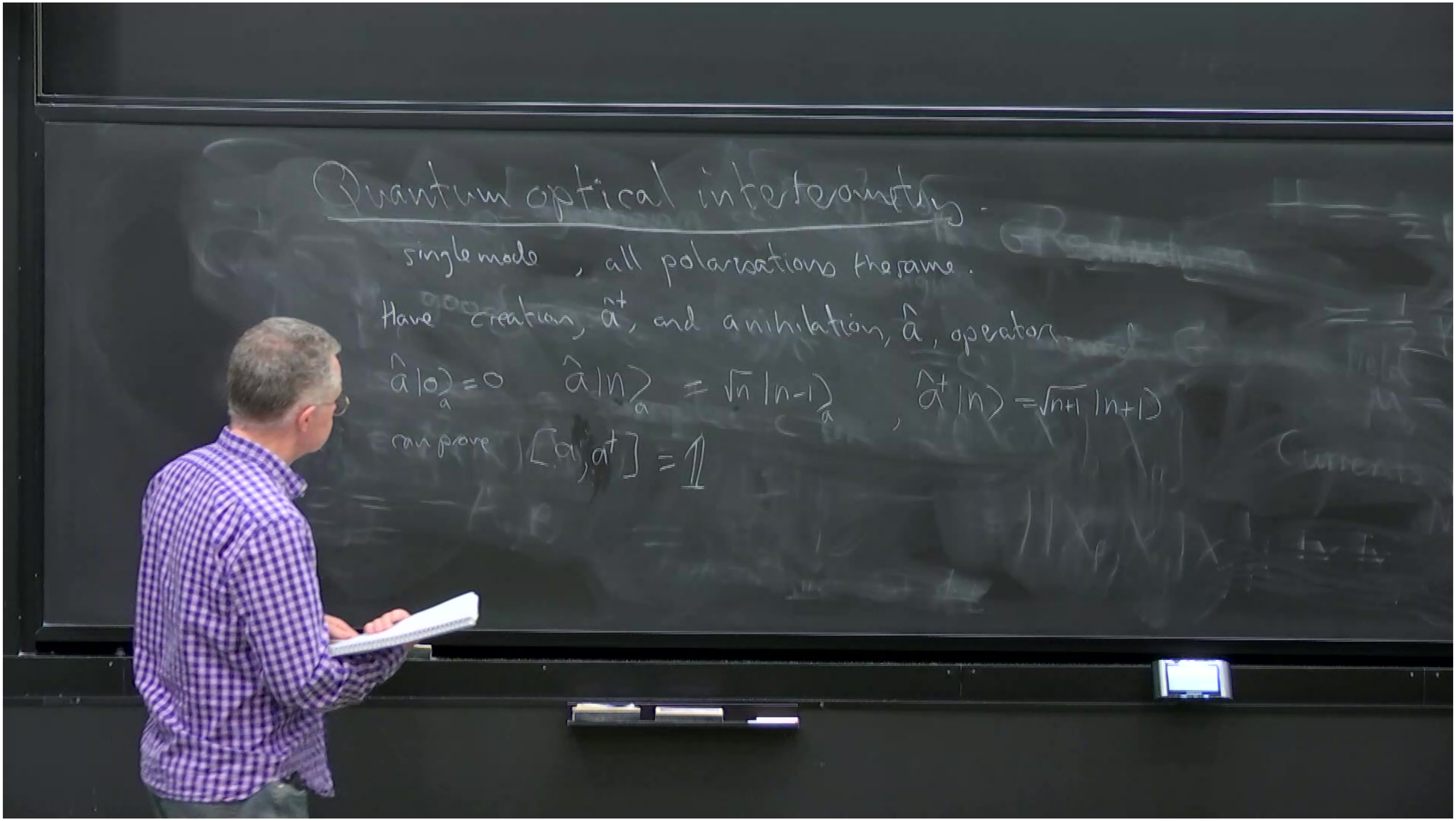
Title: Quantum Foundations Lecture - 230119

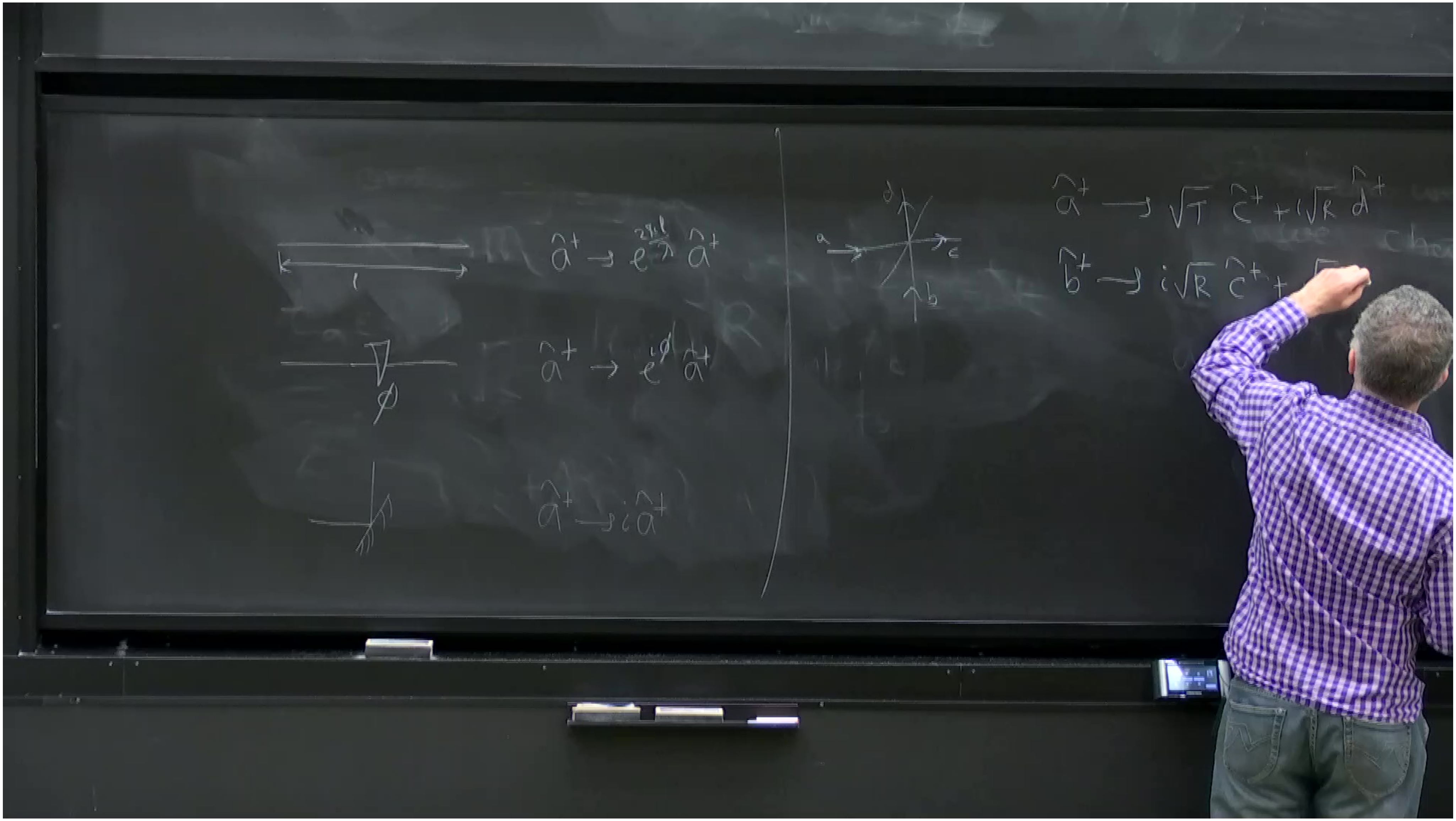
Speakers: Lucien Hardy

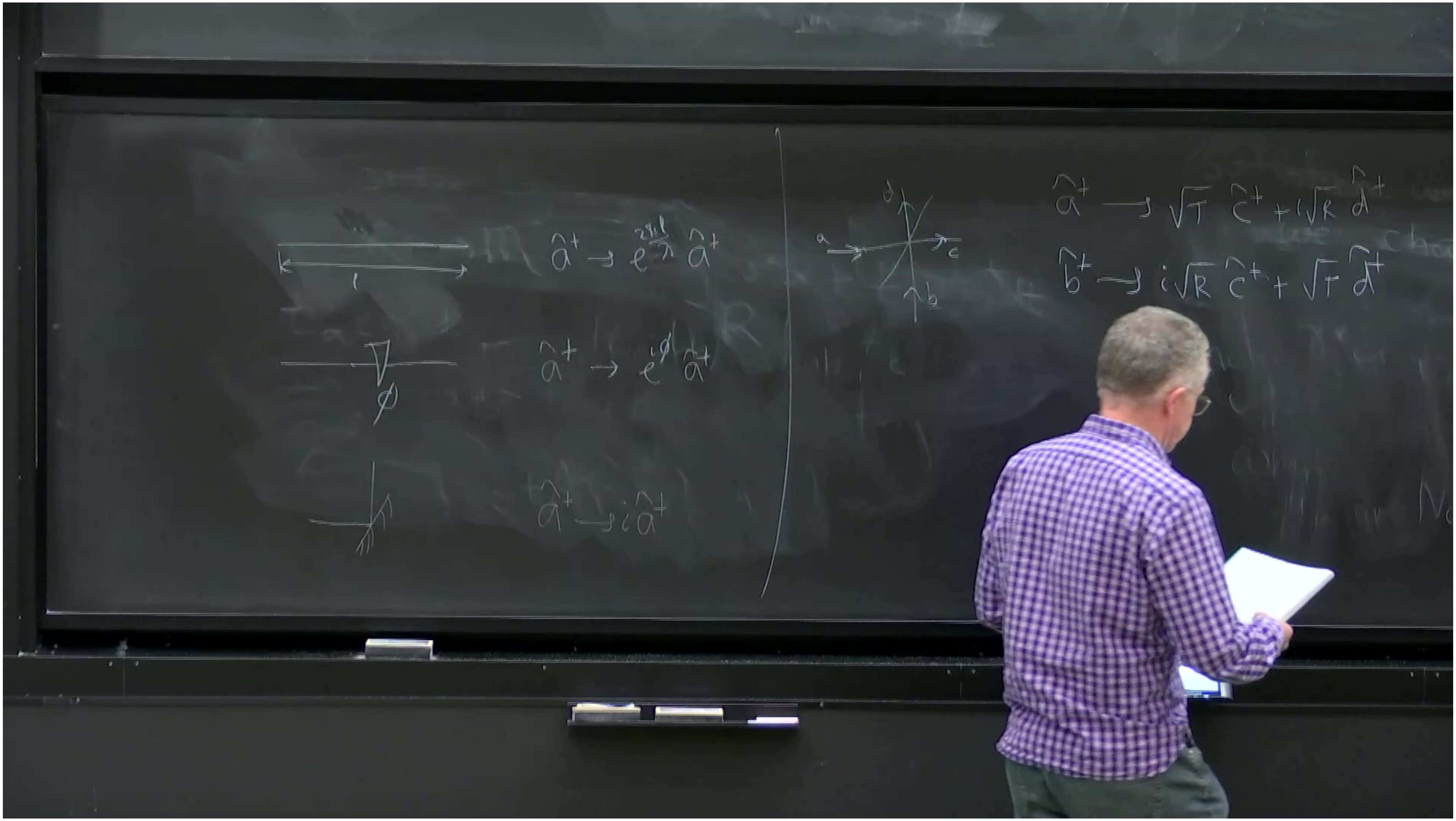
Collection: Quantum Foundations (2022/2023)

Date: January 19, 2023 - 12:15 PM

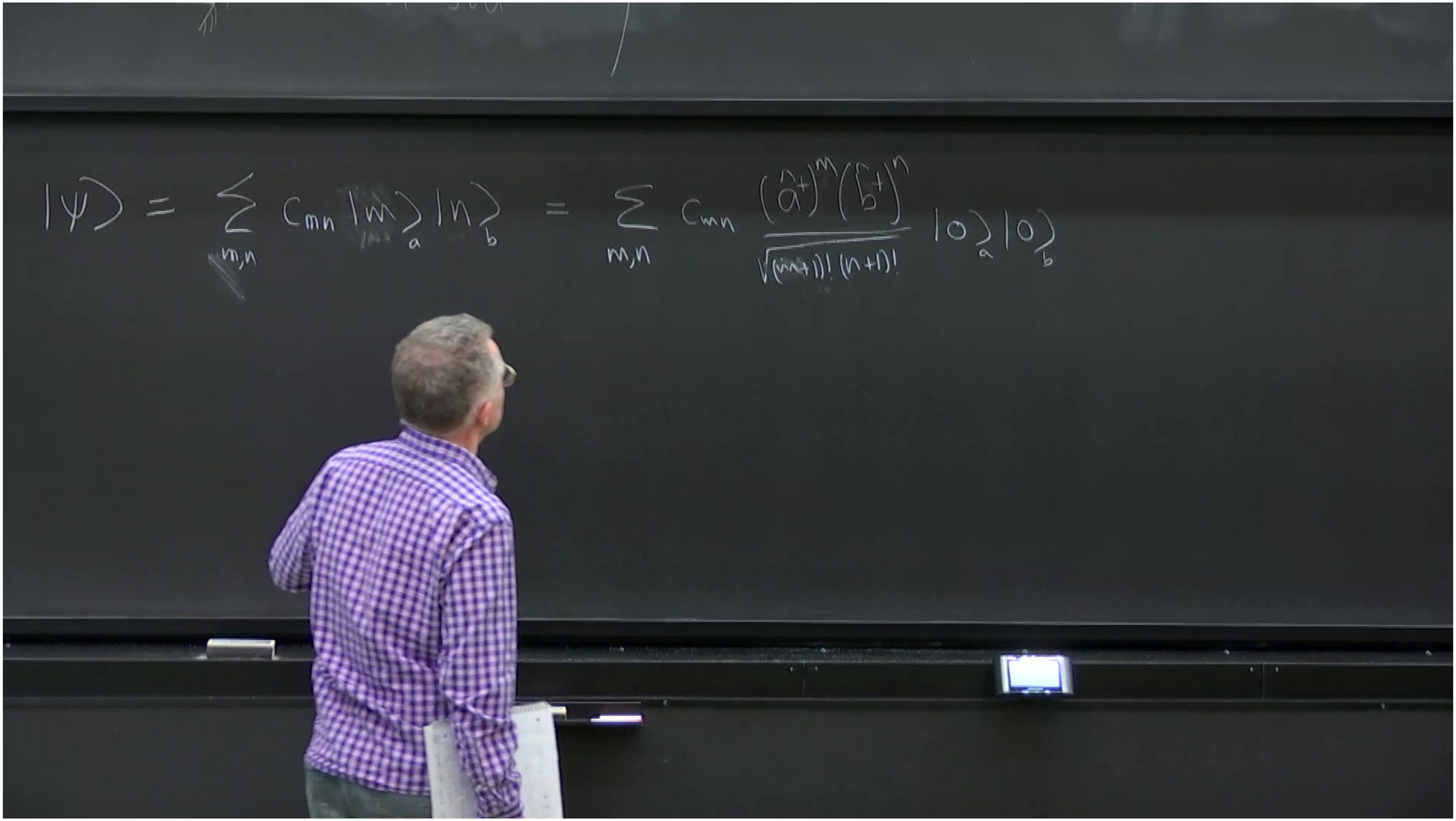
URL: https://pirsa.org/23010048


Quantum optical interferometry.
single mode , all polarisations the same.
Have creation, $\hat{a}^\dagger$, and anihilation, $\hat{a}$, operators
$\hat{a}|0\rangle_a = 0$ $\hat{a}|n\rangle_a = \sqrt{n}\,|n-1\rangle_a$, $\hat{a}^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$
can prove $[a, a^\dagger] = 1$


â† → e^{2πiℓ/λ} â†
â† → e^{iφ} â†
â† → i â†
â† → √T ĉ† + i√R d̂†
b̂† → i√R ĉ† +



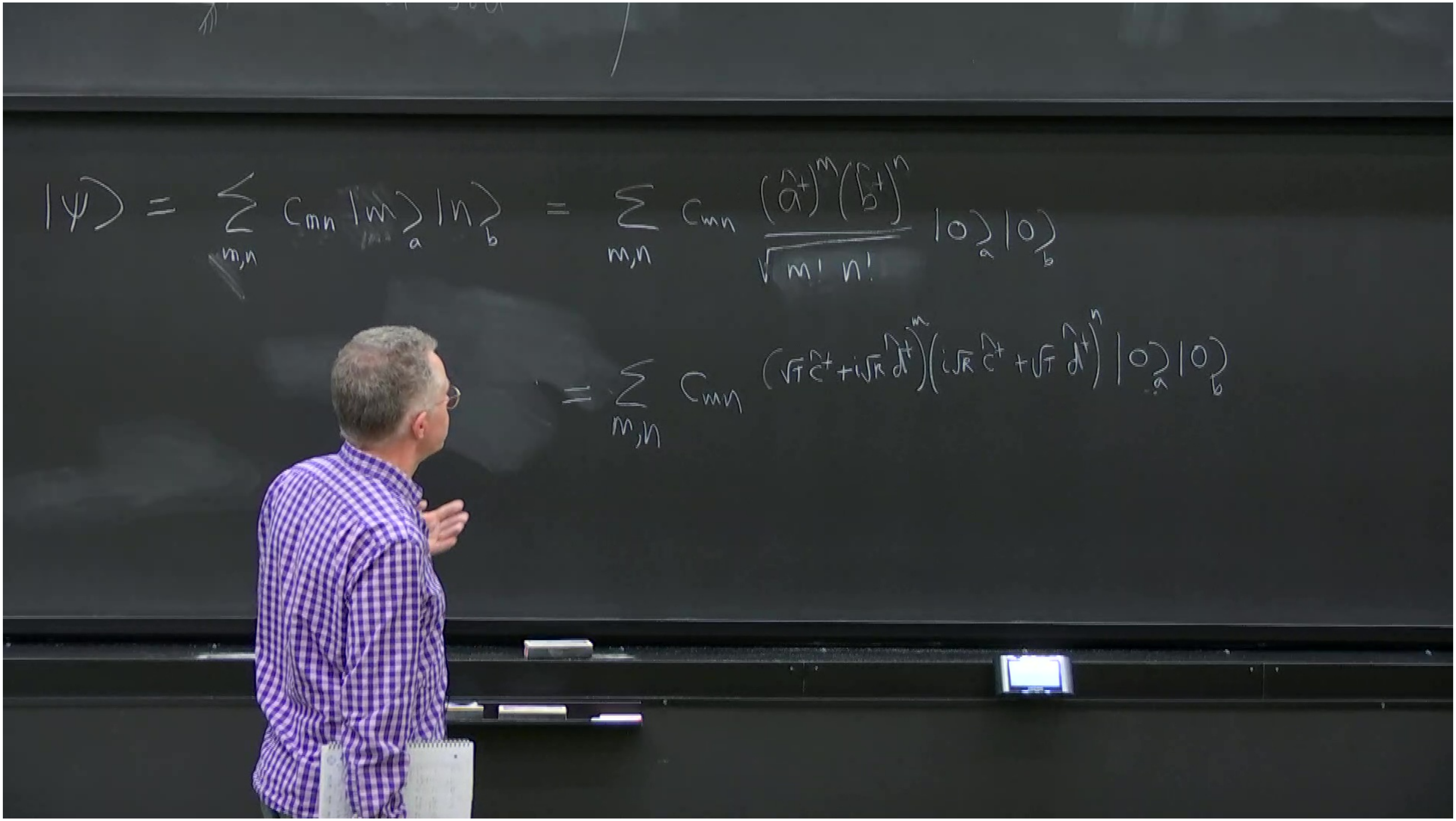
$\hat{a}^+ \to e^{2\pi i l/\lambda}\,\hat{a}^+$
$\hat{a}^+ \to e^{i\phi}\,\hat{a}^+$
$\hat{a}^+ \to i\,\hat{a}^+$
$\hat{a}^+ \to \sqrt{T}\,\hat{c}^+ + i\sqrt{R}\,\hat{d}^+$
$\hat{b}^+ \to i\sqrt{R}\,\hat{c}^+ + \sqrt{T}\,\hat{d}^+$

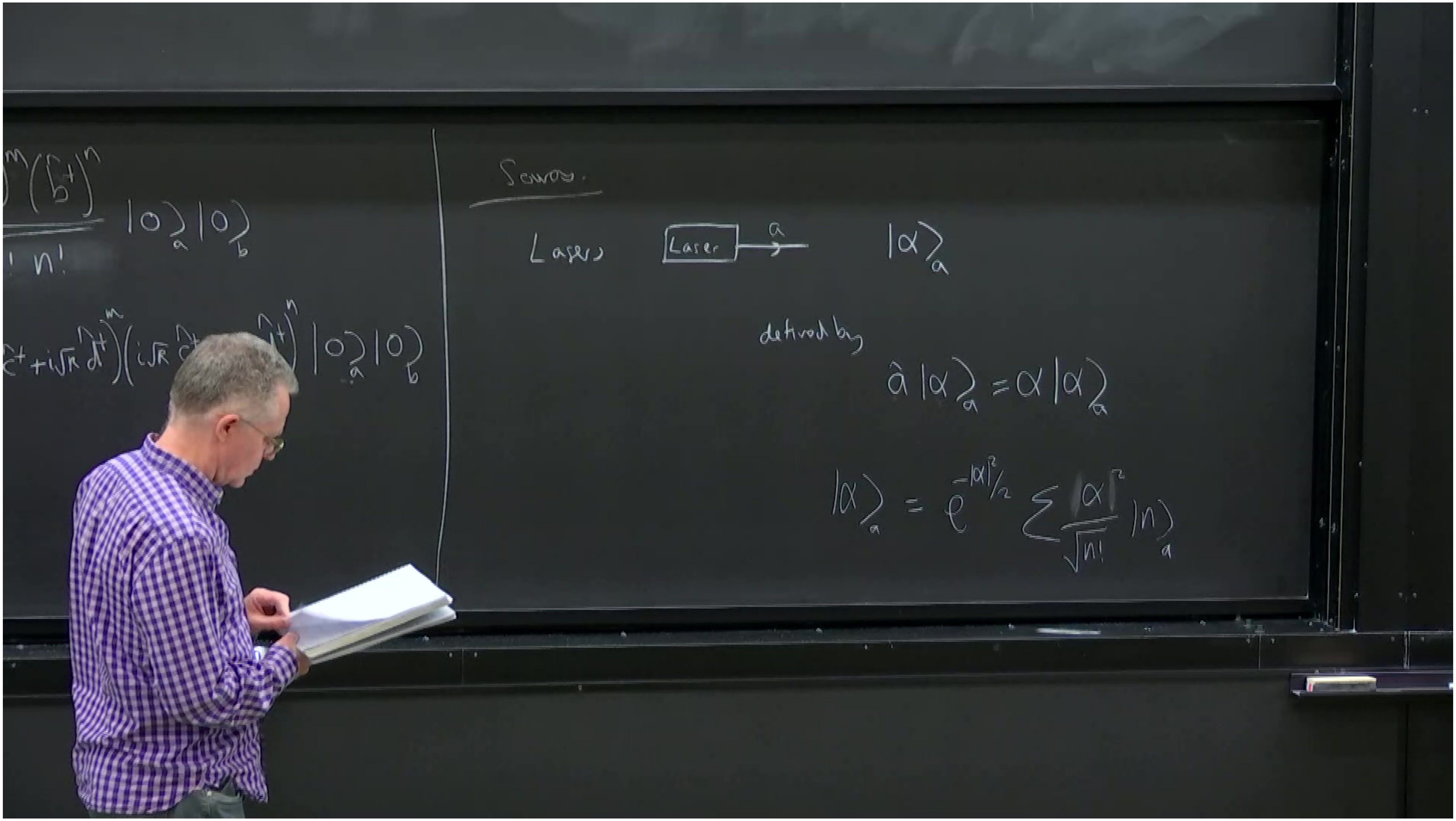

$|\psi\rangle = \sum_{m,n} c_{mn} |m\rangle_a |n\rangle_b = \sum_{m,n} c_{mn} \frac{(\hat{a}^{\dagger})^m (\hat{b}^{\dagger})^n}{\sqrt{(m+1)!\,(n+1)!}} |0\rangle_a |0\rangle_b$

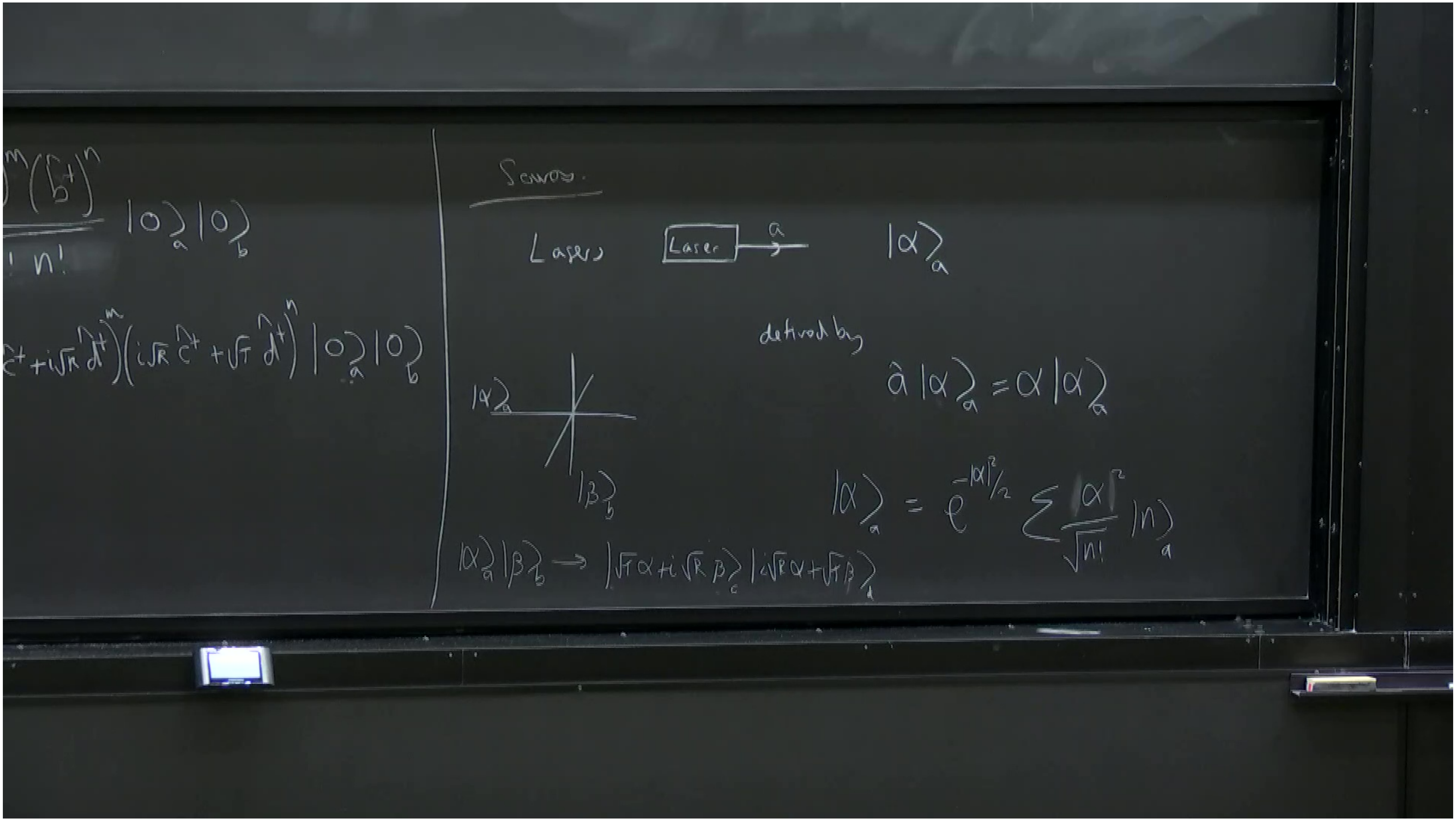
$$|\psi\rangle = \sum_{m,n} c_{mn} |m\rangle_a |n\rangle_b = \sum_{m,n} c_{mn} \frac{(\hat{a}^{+})^m (\hat{b}^{+})^n}{\sqrt{m!\, n!}} |0\rangle_a |0\rangle_b$$

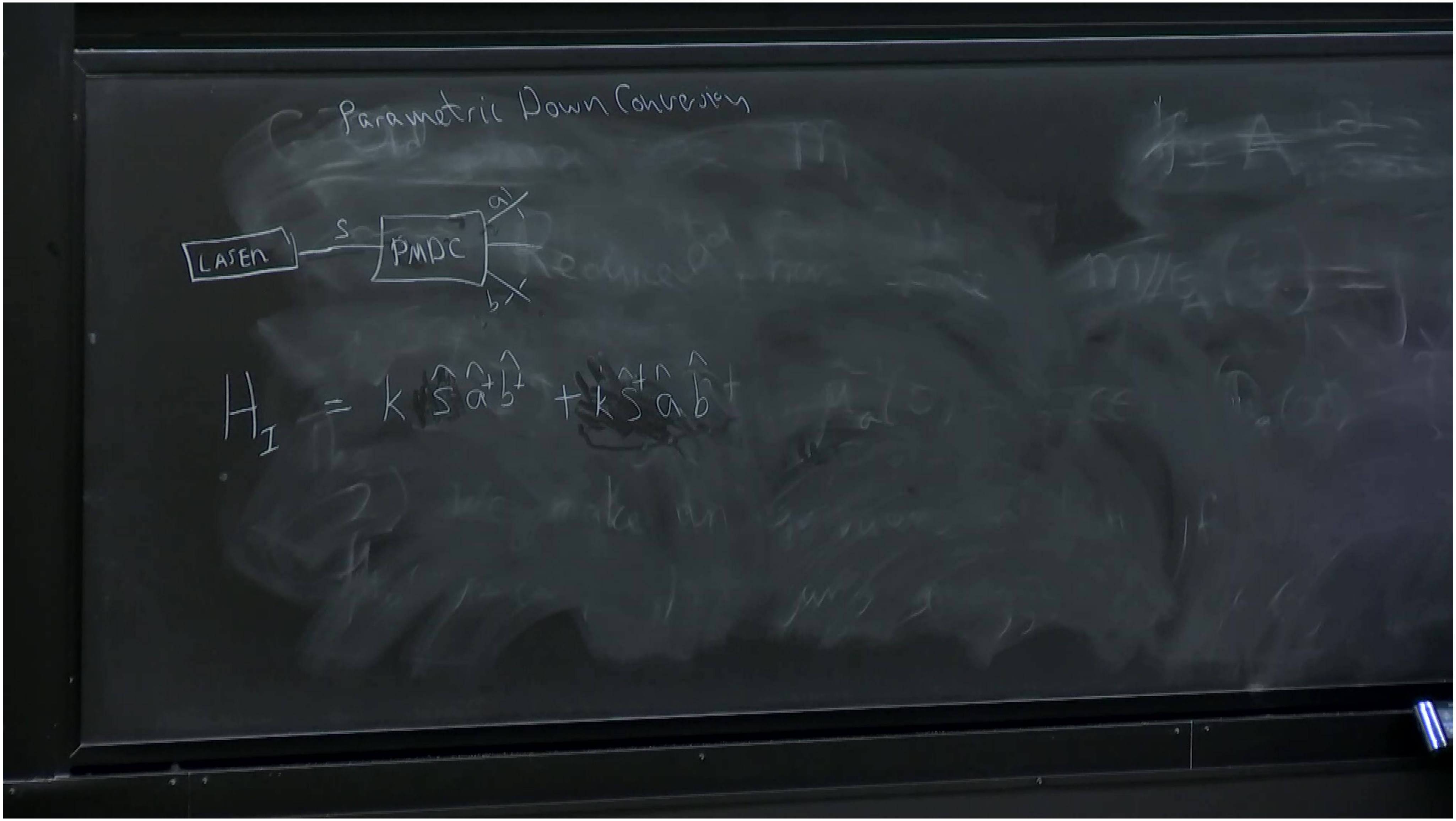
$$|m\rangle_a = \frac{(\hat{a}^{+})^m}{\sqrt{(m+1)!}} |0\rangle_a$$

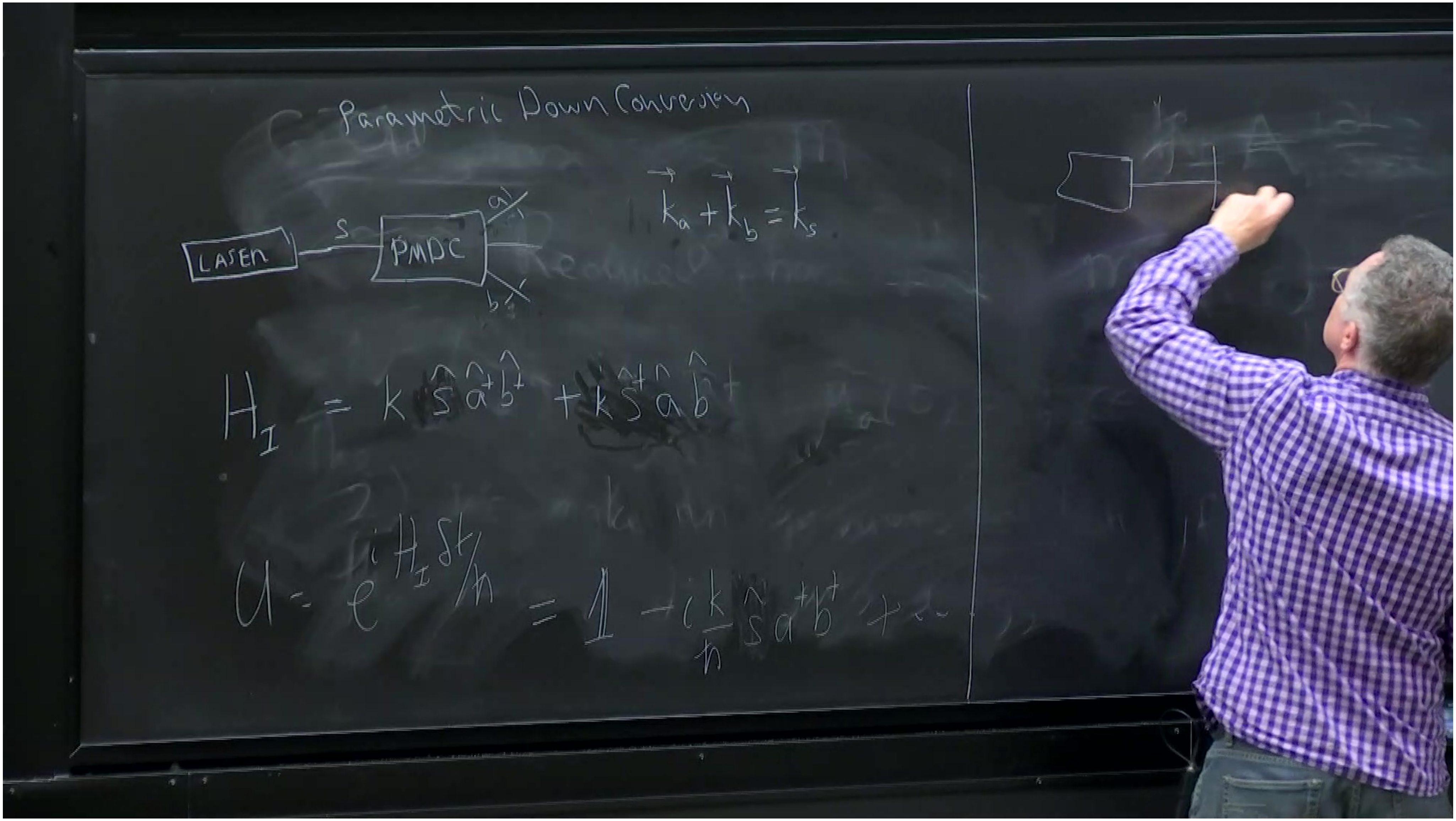
$$|1\rangle = \hat{a}^{+}|0\rangle$$

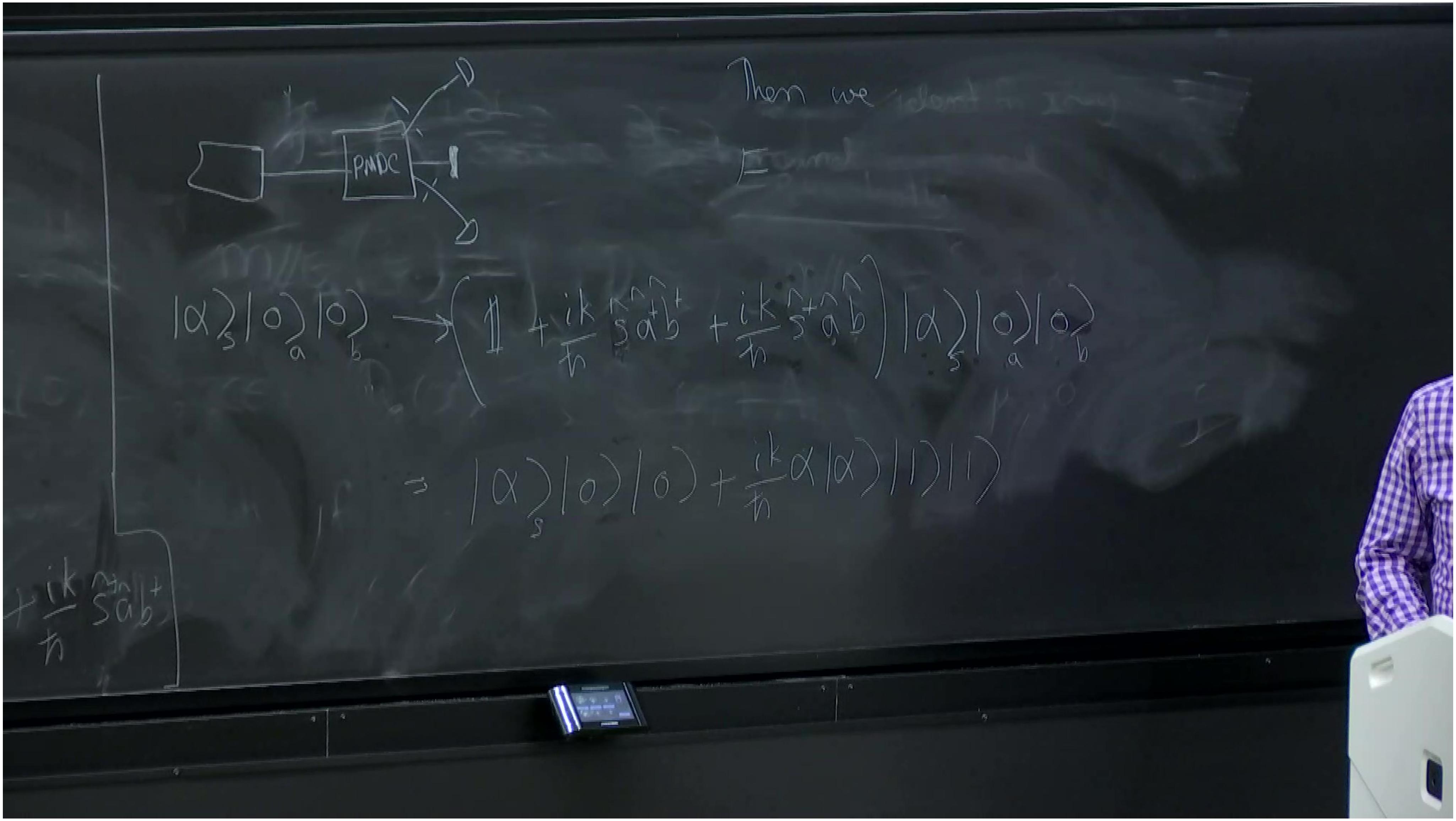

$|\psi\rangle = \sum_{m,n} c_{mn} |m\rangle_a |n\rangle_b = \sum_{m,n} c_{mn} \frac{(\hat{a}^\dagger)^m (\hat{b}^\dagger)^n}{\sqrt{m!\,n!}} |0\rangle_a |0\rangle_b$
$= \sum_{m,n} c_{mn} (\sqrt{T}\hat{c}^\dagger + i\sqrt{R}\hat{d}^\dagger)^m (i\sqrt{R}\hat{c}^\dagger + \sqrt{T}\hat{d}^\dagger)^n |0\rangle_a |0\rangle_b$

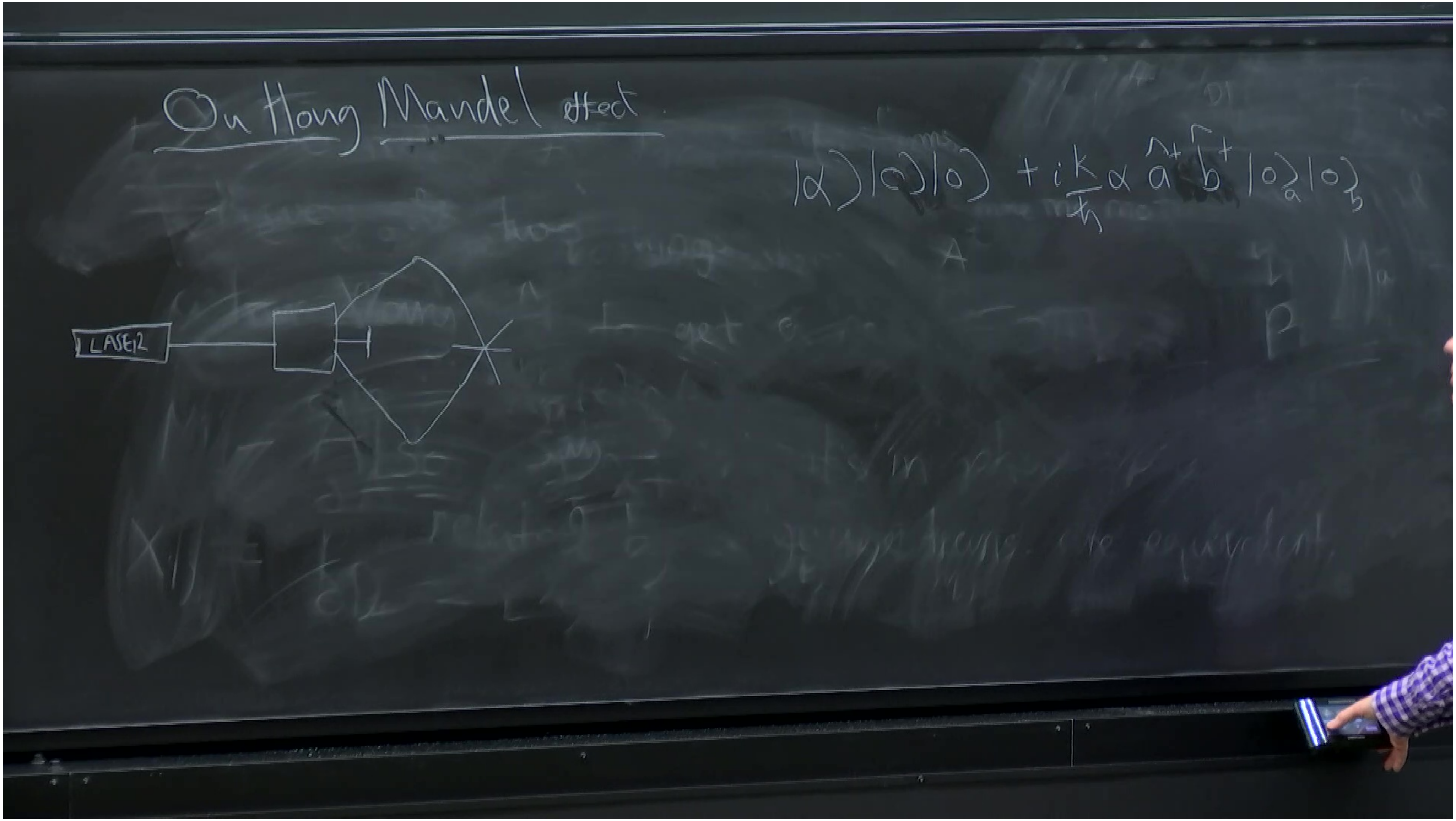

Sources.
Lasers
Laser
a
$|\alpha\rangle_a$
defined by
$\hat{a}|\alpha\rangle_a = \alpha|\alpha\rangle_a$
$|\alpha\rangle_a = e^{-|\alpha|^2/2} \sum \frac{\alpha^n}{\sqrt{n!}} |n\rangle_a$

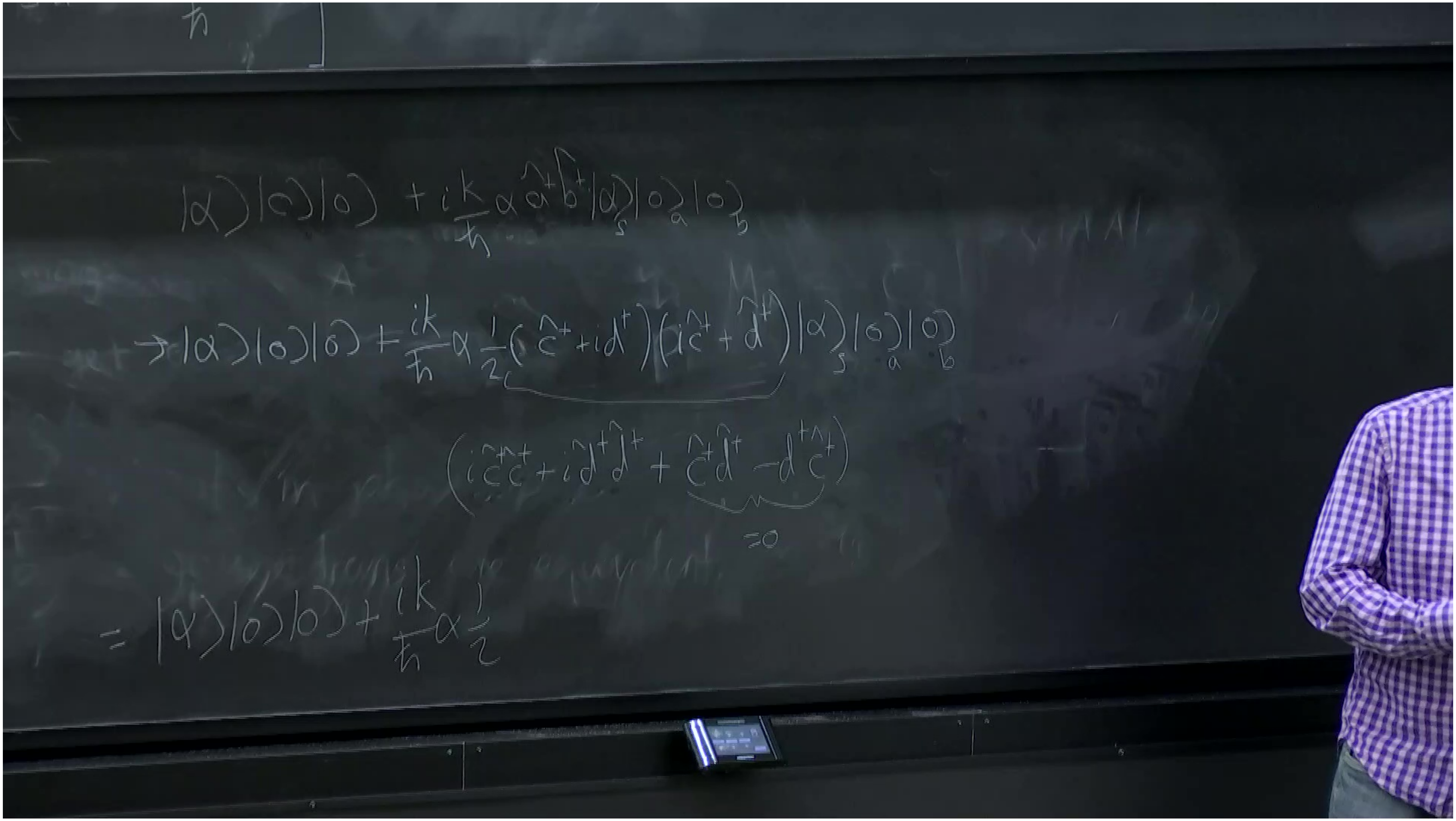

$(\hat{b}^\dagger)^n$
$n!$
$|0\rangle_a |0\rangle_b$
$(i\sqrt{R}\,\hat{c}^\dagger + \sqrt{T}\,\hat{d}^\dagger)^n |0\rangle_a |0\rangle_b$
Sources.
Lasers
Laser
a
$|\alpha\rangle_a$
defined by
$\hat{a}|\alpha\rangle_a = \alpha|\alpha\rangle_a$
$|\alpha\rangle_a$
$|\beta\rangle_b$
$|\alpha\rangle_a = e^{-|\alpha|^2/2} \sum \frac{\alpha^n}{\sqrt{n!}} |n\rangle_a$
$|\alpha\rangle_a |\beta\rangle_b \rightarrow |\sqrt{T}\alpha + i\sqrt{R}\beta\rangle_c \, |i\sqrt{R}\alpha + \sqrt{T}\beta\rangle_d$


Parametric Down Conversion
LASER
s
PMDC
a
b
$H_I = k\hat{s}\hat{a}^\dagger\hat{b}^\dagger + k\hat{s}^\dagger\hat{a}\hat{b}$


Parametric Down Conversion
LASER
s
PMDC
a
b
$\vec{k}_a + \vec{k}_b = \vec{k}_s$
$H_I = k\hat{s}\hat{a}^\dagger\hat{b}^\dagger + k\hat{s}^\dagger\hat{a}\hat{b}$
$U = e^{iH_I\delta t/\hbar} = 1 - i\frac{k}{\hbar}\hat{s}\hat{a}^\dagger\hat{b}^\dagger + \dots$


Then we
PMDC
$|\alpha\rangle_s|0\rangle_a|0\rangle_b \to \left(\mathbb{1} + \frac{ik}{\hbar}\hat{s}\hat{a}^\dagger\hat{b}^\dagger + \frac{ik}{\hbar}\hat{s}^\dagger\hat{a}\hat{b}\right)|\alpha\rangle_s|0\rangle_a|0\rangle_b$
$= |\alpha\rangle_s|0\rangle|0\rangle + \frac{ik}{\hbar}\alpha|\alpha\rangle|1\rangle|1\rangle$


Ou Hong Mandel effect
LASER

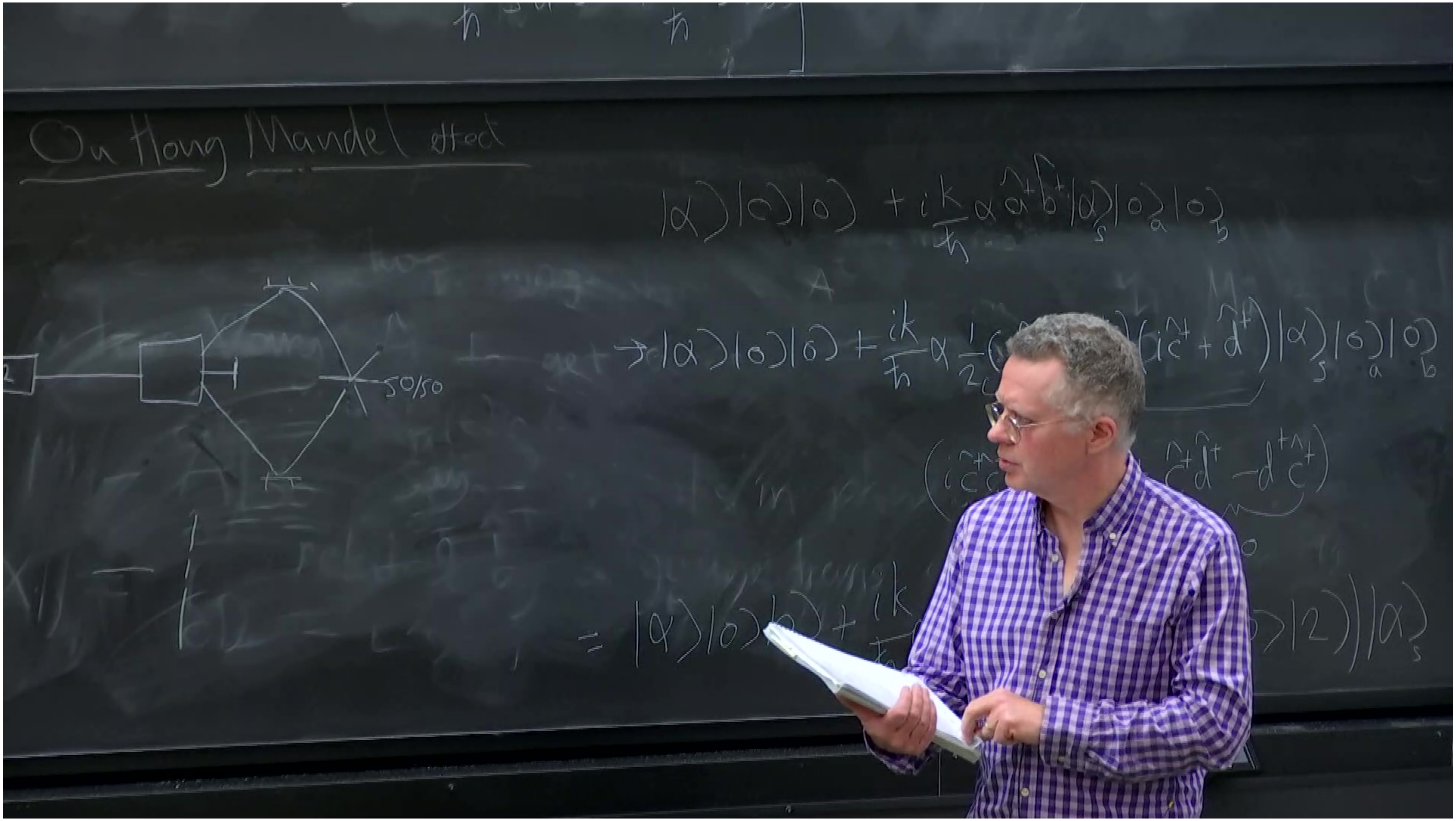

$|\alpha\rangle|0\rangle|0\rangle + \frac{ik}{\hbar}\alpha\,\hat{a}^\dagger\hat{b}^\dagger|\alpha\rangle_s|0\rangle_a|0\rangle_b$
$\rightarrow |\alpha\rangle|0\rangle|0\rangle + \frac{ik}{\hbar}\alpha\,\frac{1}{2}(\hat{c}^\dagger + i\hat{d}^\dagger)(i\hat{c}^\dagger + \hat{d}^\dagger)|\alpha\rangle_s|0\rangle_a|0\rangle_b$
$(i\hat{c}^\dagger\hat{c}^\dagger + i\hat{d}^\dagger\hat{d}^\dagger + \underbrace{\hat{c}^\dagger\hat{d}^\dagger - \hat{d}^\dagger\hat{c}^\dagger}_{=0})$
$= |\alpha\rangle|0\rangle|0\rangle + \frac{ik}{\hbar}\alpha\,\frac{1}{2}$

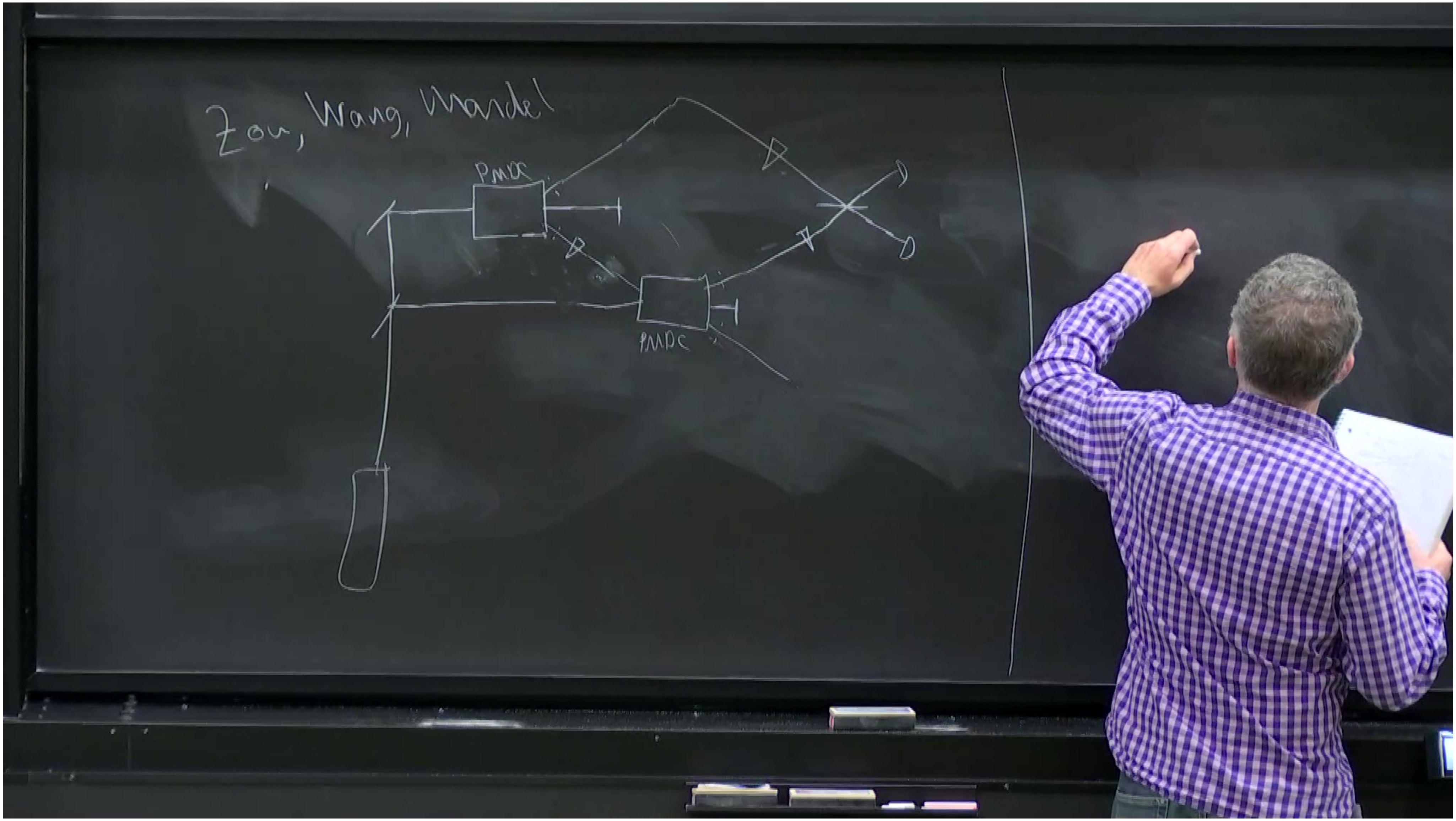
$$|\alpha\rangle|0\rangle|0\rangle + \frac{ik}{\hbar}\alpha\,\hat{a}^\dagger\hat{b}^\dagger|\alpha\rangle_s|0\rangle_a|0\rangle_b$$

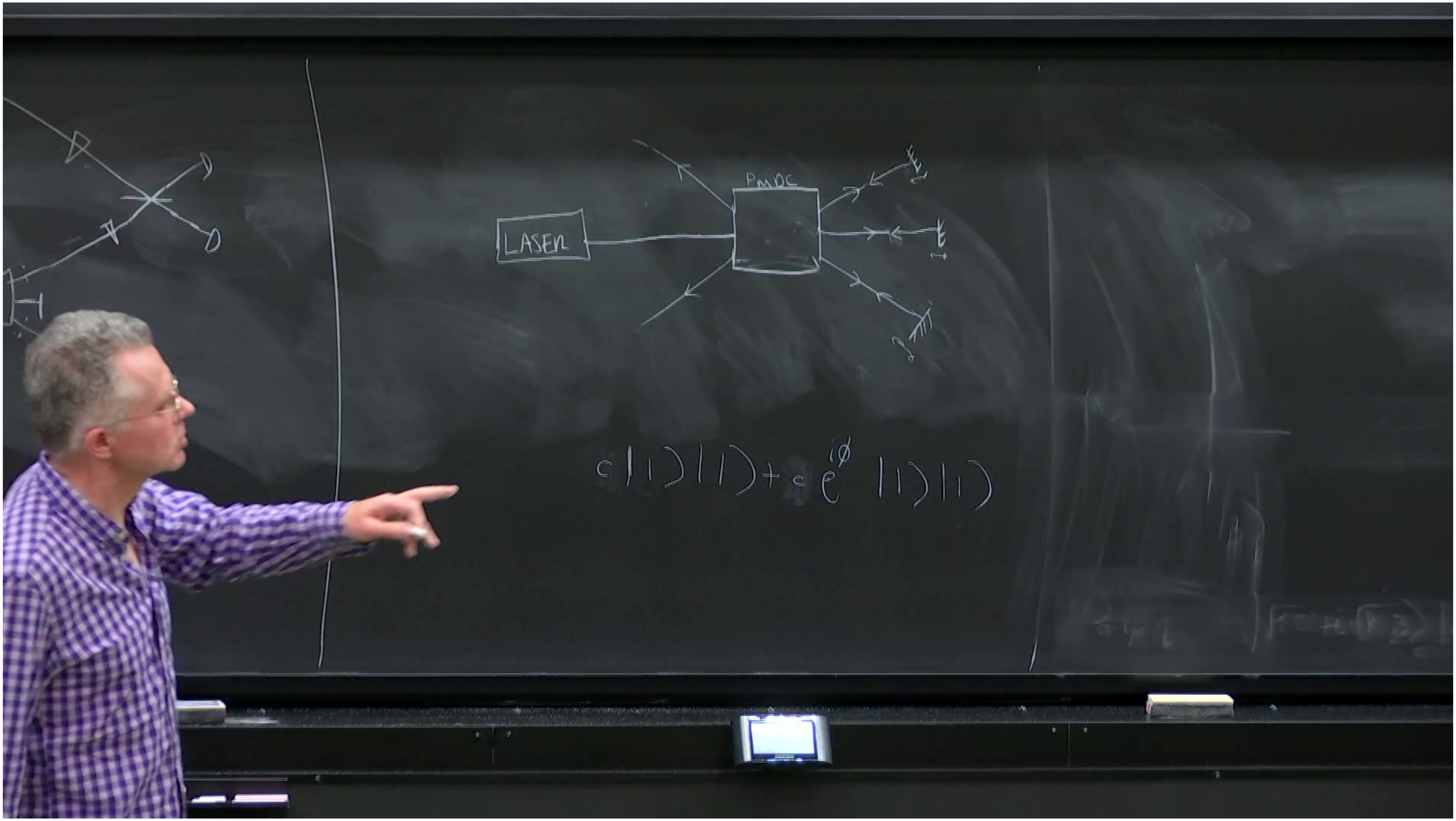
$$\rightarrow |\alpha\rangle|0\rangle|0\rangle + \frac{ik}{\hbar}\alpha\,\frac{1}{2}\underbrace{(\hat{c}^\dagger + i\hat{d}^\dagger)(i\hat{c}^\dagger + \hat{d}^\dagger)}|\alpha\rangle_s|0\rangle_a|0\rangle_b$$

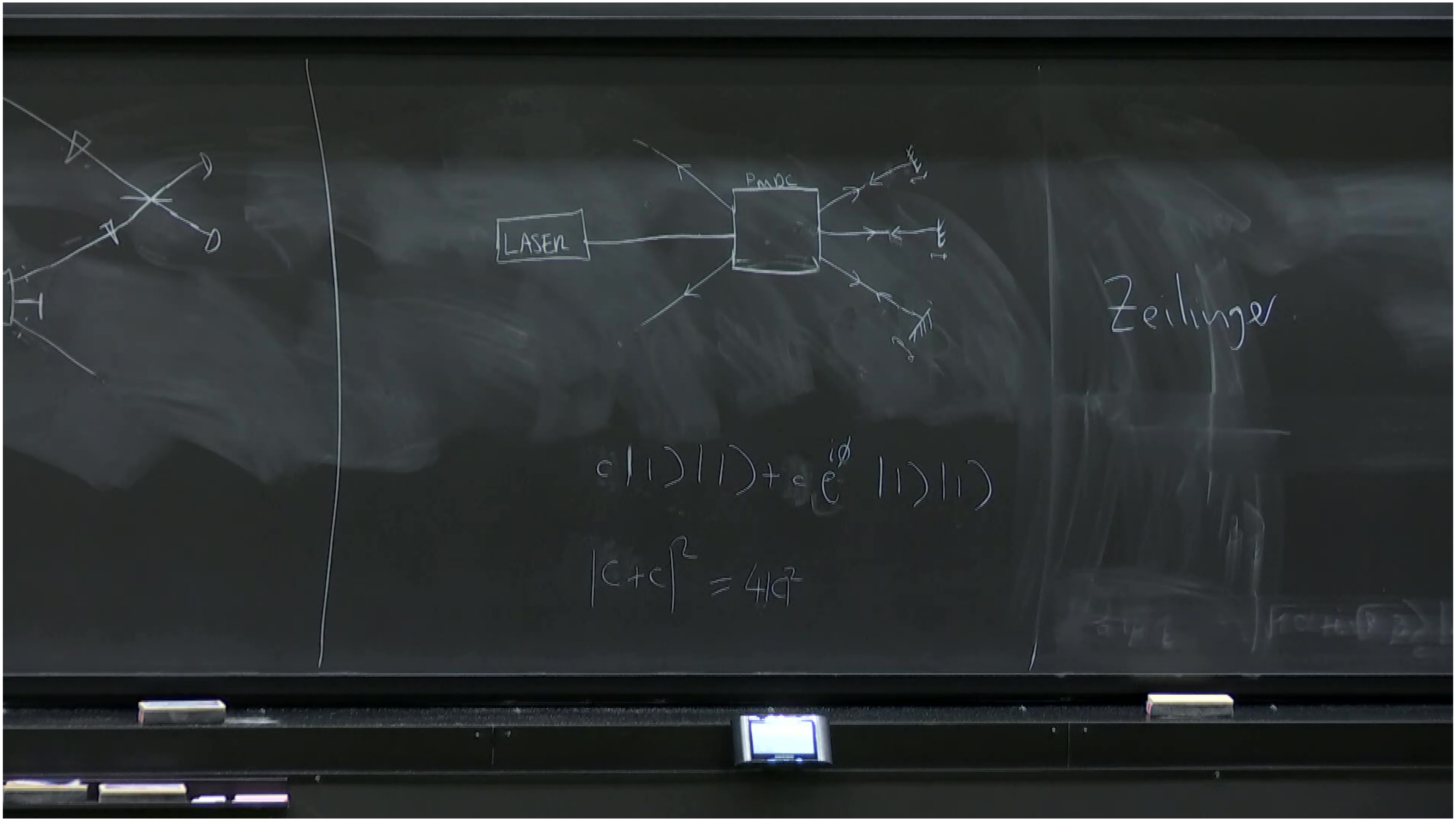
$$\left(i\hat{c}^\dagger\hat{c}^\dagger + i\hat{d}^\dagger\hat{d}^\dagger + \underbrace{\hat{c}^\dagger\hat{d}^\dagger - \hat{d}^\dagger\hat{c}^\dagger}_{=0}\right)$$

$$= |\alpha\rangle|0\rangle|0\rangle + \frac{ik}{\hbar}\alpha\,\frac{1}{2}\left(\sqrt{2}\,|2\rangle|0\rangle + |0\rangle|2\rangle\right)|\alpha\rangle_s$$


On Hong Mandel effect
50/50


Zou, Wang, Mandel
PMDC
PMDC


PMDC
LASER
$c|1\rangle|1\rangle + c\,e^{i\phi}|1\rangle|1\rangle$


LASER
PMDC
Zeilinger
$c|1\rangle|1\rangle + c\,e^{i\phi}|1\rangle|1\rangle$
$|c+c|^2 = 4|c|^2$