Title: Standard Model Lecture - 230126

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Standard Model

Lecture # 9

Some Fundamental SM Patterns

(part 1)

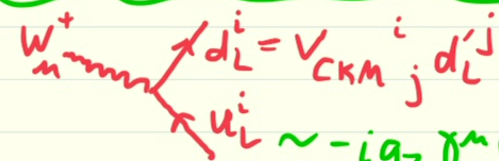
3rd line:

$$\mathcal{L}_{SM} = \dots + i\left[\bar q_L \not\partial q_L + \bar u_R \not\partial u_R + \bar d_R \not\partial d_R + \bar \ell_L \not\partial \ell_L + \bar e_R \not\partial e_R\right] + \dots$$

$$i \bar Q \not D Q = \dots i \bar Q_a \left(i g_3 \not G^i_\mu \lambda^{i\,a}{}_b\right) Q^b \Longrightarrow$$

$-i g_3 \gamma^\mu \lambda^{i\,a}{}_b$

$$i \bar q_L \not D q_L = \dots - \frac{g_2}{2}\left(\bar u_L \ \ \bar d_L\right)\begin{pmatrix} 0 & \not W^+_\mu \\ \not W^-_\mu & 0 \end{pmatrix}\begin{pmatrix} u_L \\ d_L \end{pmatrix} \Longrightarrow$$



$\sim -i g_2 \gamma^\mu V_{CKM}{}^i{}_j$

$$i \bar\psi \not D \psi = \dots i \bar\psi \left(i g_2 T^3 \not W^3 + i g_1 Y \not B\right)\psi$$

$$= \dots - \bar\psi \left[\underbrace{\frac{g_2^2 T^3 - g_1^2 Y}{(g_1^2 + g_2^2)^{1/2}}}_{g_{eff}} \not Z + e Q \not A\right]\psi$$

$-ieQ\gamma^\mu$

$-i g_{eff} \gamma^\mu$

$$\left(Q = T_3 + Y \qquad e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \right)$$

- That's the Standard Model!
- Incredibly successful!
- Shortcomings:

  i) predicts ν's are massless and don't oscillate.

  ii) no DM candidate.

  iii) doesn't explain cosmic matter/anti-matter asymmetry.

Linear Algebra Review:

$$V = \{v_1, \dots, v_m\} \quad (m \text{ dim})$$

$$W = \{w_1, \dots, w_n\} \quad (n \text{ dim})$$

$$V \oplus W = \{v_1, \dots, v_m, w_1, \dots, w_n\} \quad (m+n \text{ dim})$$

$$V \otimes W = \{v_i \otimes w_j\} \quad (m \cdot n \text{ dim})$$

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
q_L	3	2	1/6
u_R	3	1	2/3
d_R	3	1	-1/3
ℓ_L	1	2	-1/2
ν_R	1	1	0
e_R	1	1	-1

$q_L = \mathbb{C}^3 \otimes \mathbb{C}^2 \otimes \mathbb{C}_1$; $q_L^c = \mathbb{C}^{3*} \otimes \mathbb{C}^2 \otimes \mathbb{C}_{-1}$

$u_R = \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_4$; $u_R^c = \mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_{-4}$

$d_R = \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_{-2}$; $d_R^c = \mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_2$

$\ell_L = \mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_{-3}$; $\ell_L^c = \mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_3$

$\nu_R = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$; $\nu_R^c = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$

$e_R = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_{-6}$; $e_R^c = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_6$

$$D_\mu = \partial_\mu + i g_1 y B_\mu$$

$q_L^c \equiv i\gamma^2 q_L^*$

$q_L \to U_3 q_L$ "3"

$q_L^c \to U_3^* q_L^c$ "$\bar{3}$"

$SU(N) \ni U_n \nearrow U_n$ "N", $\searrow U_n^*$ "$\bar{N}$"

h vs. $\tilde{h} = i\sigma_2 h^*$

for $SU(2)$ $2 = \bar{2}$

$$6 + 3 + 3 + 2 + 1 + 1$$

Left: $F = q_L \oplus u_R^c \oplus d_R^c \oplus \ell_L \oplus \nu_R^c \oplus e_R^c$ (16 dim)

Right: $F^c = q_L^c \oplus u_R \oplus d_R \oplus \ell_L^c \oplus \nu_R \oplus e_R$ (16 dim)

Total: $F \oplus F^c$ (32 dim)

$$\varphi = \sum_{k} \left[a_{\underline{k}} e^{-ik\cdot x} + a^{\dagger}_{\underline{k}} e^{ik\cdot x} \right]$$

The 1970's "Grand Unification"

$$
\begin{tikzcd}[column sep=small]
 & \begin{array}{c} SU(5) \\ \text{(Georgi-Glashow)} \end{array} \arrow[dr] & \\
G \arrow[ur] \arrow[dr] \arrow[d, bend right] & & \begin{array}{c} SO(10) \\ \text{(Georgi, Fritzsch, Minkowski)} \end{array} \\
 & \begin{array}{c} SU(4)\times SU(2)_L \times SU(2)_R \\ \text{(Pati-Salam)} \end{array} \arrow[ur] & \\
SU(3)_c \times \underbrace{SU(2)_L \times U(1)_Y} & &
\end{tikzcd}
$$

$$
SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{\langle h \rangle \sim 250\ \text{GeV}} SU(3)_c \times U(1)_Q
$$

Embedding $SU(3)\times SU(2)\times U(1)$ in $SU(5)$:

$SU(3)\times SU(2)\times U(1)$: $(U_3, U_2, \alpha)(U_3', U_2', \alpha') = (U_3U_3', U_2U_2', \alpha\alpha')$

$\Rightarrow (U_3, 1, 1)(1, U_2, 1) = (U_3, U_2, 1) = (1, U_2, 1)(U_3, 1, 1)$

$$\Rightarrow (U_3, U_2, 1) \longrightarrow \left(\begin{array}{c|c} U_2 & \\ \hline & U_3 \end{array}\right)$$

$$\Rightarrow (1, 1, \alpha) \longrightarrow \left(\begin{array}{c|c} \alpha^3 1_2 & \\ \hline & \alpha^{-2} 1_3 \end{array}\right)$$

$$\Rightarrow (U_3, U_2, \alpha) \longrightarrow \left(\begin{array}{c|c} \alpha^3 U_2 & \\ \hline & \alpha^{-2} U_3 \end{array}\right)$$

The Exterior Algebra $\Lambda \mathbb{C}^n$

$\Lambda \mathbb{R}^4 \leftarrow$

$\mathbb{C}^n = \{e_1, \ldots, e_n\}$ $(n \text{ dim})$ $\leftarrow$ 1-forms $\Lambda^1 \mathbb{C}^n$

2-forms $\Lambda^2 \mathbb{C}^n$

$\mathbb{C}^n \otimes \mathbb{C}^n = \{e_i \otimes e_j\}$ $(n^2 \text{ dim}) \supset \Lambda^2 \mathbb{C}^n = \{e_i \otimes e_j - e_j \otimes e_i\}$ $\left(\binom{n}{2} \text{ dim}\right)$

p-forms $\Lambda^p \mathbb{C}^n$

$\underbrace{\mathbb{C}^n \otimes \ldots \otimes \mathbb{C}^n}_{p} = \{\underbrace{e_i \otimes \ldots \otimes e_j}_{p}\}$ $(n^p \text{ dim}) \supset \Lambda^p \mathbb{C}^n = \{\underbrace{e_{[i} \otimes \ldots \otimes e_{j]}}_{p}\}$ $\left(\binom{n}{p} \text{ dim}\right)$

$\Lambda \mathbb{C}^n = \Lambda^0 \mathbb{C}^n \oplus \Lambda^1 \mathbb{C}^n \oplus \ldots \oplus \Lambda^n \mathbb{C}^n$ (2^n dim)

"exterior algebra (over $\mathbb{C}^n$)"

$\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C}^5 \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5$

$\Rightarrow \quad 1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5 \text{ dim}$

SMSB (Bosons)

$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + i g_3 [G_\mu, G_\nu]$

$G_{\mu\nu} = G^i_{\mu\nu} \lambda^i \quad \left(Tr[\lambda^i \lambda^j] = \frac{1}{2}\delta^{ij} \right)$

$W_{\mu\nu} = W^i_{\mu\nu} T^i \quad \left(Tr[T^i T^j] = \frac{1}{2}\delta^{ij} \right)$

$$L_{SM} = -\frac{1}{2} Tr[G_{\mu\nu} G^{\mu\nu}] - \frac{1}{2} Tr[W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$

$$- (D^\mu h)^\dagger (D_\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \text{fermionic terms}$$

$$D_\mu h = \left(\partial_\mu + i g_2 \tfrac{1}{2} \sigma^i W^i_\mu + i g_1 \tfrac{1}{2} \sigma^0 B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ \partial_\mu H \end{pmatrix} + \frac{iv}{2} \begin{pmatrix} g_2 (W^1_\mu - i W^2_\mu) \\ g_1 B_\mu - g_2 W^3_\mu \end{pmatrix} \right] + \dots \longleftarrow \text{check!}$$

"free" (quad.) terms

$$= -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$\left(v^2 \equiv \frac{4m^2}{\lambda} \right)$ $\longleftarrow$ check!

$$- \frac{1}{2} (\partial_\mu H)^2 - m^2 H^2 - \frac{v^2}{8} g_2^2 \left[(W^1_\mu)^2 + (W^2_\mu)^2 \right] - \frac{v^2}{8} (g_2 W^3_\mu - g_1 B_\mu)^2 + \dots$$

rewrite

$$= -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^+_{\mu\nu} W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

check! $\longrightarrow$

$$- \frac{1}{2} (\partial_\mu H)^2 - \frac{1}{2} M_H^2 H^2 - \frac{1}{2} M_W^2 W^+_\mu W^{-\mu} - \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$

$$\left\{ \begin{array}{l} M_H^2 \equiv 2m^2 \\ \cos\theta_W \equiv \dfrac{g_2}{(g_1^2 + g_2^2)^{1/2}} \end{array} \right.$$

$W^\pm_\mu \equiv W^1_\mu \pm i W^2_\mu$

$$\left\{ \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \right\}$$

$$\left. \begin{array}{l} m_W^2 \equiv \frac{1}{4} g_2^2 v^2 \\ m_Z^2 \equiv \frac{1}{4} (g_1^2 + g_2^2) v^2 \end{array} \right\}$$

$\frac{m_W}{m_Z} = \frac{82\,\text{GeV}}{91\,\text{GeV}} = \cos\theta_W$

SMSB (Bosons)

$G_{\mu\nu}=\partial_\mu G_\nu-\partial_\nu G_\mu+ig_3[G_\mu,G_\nu]$

$G_{\mu\nu}=G^i_{\mu\nu}\lambda^i \quad \left(Tr[\lambda^i\lambda^j]=\frac{1}{2}\delta^{ij}\right)$

$W_{\mu\nu}=W^i_{\mu\nu}T^i \quad \left(Tr[T^iT^j]=\frac{1}{2}\delta^{ij}\right)$

$$L_{SM}=-\frac{1}{2}Tr\left[G_{\mu\nu}G^{\mu\nu}\right]-\frac{1}{2}Tr\left[W_{\mu\nu}W^{\mu\nu}\right]-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$h=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H\end{pmatrix}$

$$-(D^\mu h)^\dagger(D_\mu h)+m^2(h^\dagger h)-\frac{\lambda}{4}(h^\dagger h)^2+\text{fermionic terms}$$

$$D_\mu h=\left(\partial_\mu+ig_2\frac{1}{2}\sigma^iW^i_\mu+ig_1\frac{1}{2}\sigma^0B_\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H\end{pmatrix}$$

$$=\frac{1}{\sqrt{2}}\left[\begin{pmatrix}0\\ \partial_\mu H\end{pmatrix}+\frac{iv}{2}\begin{pmatrix}g_2(W^1_\mu-iW^2_\mu)\\ g_1B_\mu-g_2W^3_\mu\end{pmatrix}\right]+\ldots \longleftarrow \text{check!}$$

"free" (quad.) terms

$$=-\frac{1}{4}G^i_{\mu\nu}G^{i\mu\nu}-\frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$\left(v^2\equiv\frac{4m^2}{\lambda}\right)$ $\longleftarrow$ check!

$$-\frac{1}{2}(\partial_\mu H)^2-m^2H^2-\frac{v^2}{8}g_2^2\left[(W^1_\mu)^2+(W^2_\mu)^2\right]-\frac{v^2}{8}(g_2W^3_\mu-g_1B_\mu)^2+\ldots$$

rewrite

$$=-\frac{1}{4}G^i_{\mu\nu}G^{i\mu\nu}-\frac{1}{4}W^+_{\mu\nu}W^{-\mu\nu}-\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$-\frac{1}{2}(\partial_\mu H)^2-\frac{1}{2}M_H^2H^2-\frac{1}{2}M_W^2W^+_\mu W^{-\mu}-\frac{1}{2}M_Z^2Z_\mu Z^\mu$$

check!

$\left\{ M_H^2\equiv 2m^2, \quad \cos\theta_W\equiv\frac{g_2}{(g_1^2+g_2^2)^{1/2}} \right.$

$W^\pm_\mu\equiv W^1_\mu\pm iW^2_\mu$

$$\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix}=\begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix}$$

$m_W^2\equiv\frac{1}{4}g_2^2v^2$

$m_Z^2\equiv\frac{1}{4}(g_1^2+g_2^2)v^2$

$$\frac{m_W}{m_Z}=\frac{82\,\text{GeV}}{91\,\text{GeV}}=\cos\theta_W$$

Physical content of minimal SM (free part):

i) Scalar: $h = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$ where $v^2 = \frac{4m^2}{\lambda}$ and $M_H^2 = 2m^2$

ii) Vectors: Massless: $G_\mu^{i=1,\dots,8}$ (gluons), A_μ (photons)

Massive: $W_\mu^\pm$ $\left(M_W^2 = \frac{1}{4} g_2^2 v^2\right)$, Z_μ $\left(M_Z^2 = \frac{1}{4}(g_2^2 + g_1^2) v^2\right)$

iii) Fermions: Massive (Dirac): $u = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$ $d = \begin{pmatrix} d_L' \\ d_R \end{pmatrix}$ $e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$

$m_u = \frac{v}{\sqrt{2}} y_u$ $m_d = \frac{v}{\sqrt{2}} y_d$ $m_e = \frac{v}{\sqrt{2}} y_e$

Massless (Weyl): ν_L

$\Big\} \times 3$

Next time:
- interactions/ Feynman rules
- shortcomings of minimal SM + possible solutions