Title: Standard Model Lecture - 230119

Speakers: Latham Boyle

Collection: Standard Model (2022/2023)

Date: January 19, 2023 - 3:45 PM

URL: https://pirsa.org/23010044

Standard Model

Lecture # 6

Higgs Mechanism, Electroweak Symmetry Breaking

(Part 1)

$$
\begin{aligned}
L_{SM} = &-\tfrac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}] - \tfrac{1}{2}\mathrm{Tr}[W_{\mu\nu}W^{\mu\nu}] - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&-(D^{\mu}h)^{\dagger}(D_{\mu}h) + m^2(h^{\dagger}h) - \lambda(h^{\dagger}h)^2 \\
&+i\left[\bar{q}_L^{i}\not{D}q_L^{i} + \bar{u}_R^{i}\not{D}u_R^{i} + \bar{d}_R^{i}\not{D}d_R^{i} + \bar{\ell}_L^{i}\not{D}\ell_L^{i} + \left(\bar{\nu}_R^{i}\not{D}\nu_R^{i}\right) + \bar{e}_R^{i}\not{D}e_R^{i}\right] \\
&-\left[\bar{q}_L^{i}Y_u^{ij}\tilde{h}u_R^{j} + \bar{q}_L^{i}Y_d^{ij}h\,d_R^{j} + \left(\bar{\ell}_L^{i}Y_\nu^{ij}\tilde{h}\nu_R^{j}\right) + \bar{\ell}_L Y_e^{ij}h\,e_R^{j} + h.c.\right] \\
&-\left(\bar{\nu}_{R,c}^{i}M^{ij}\nu_R^{j}\right) - \underbrace{\theta\,\mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}]}
\end{aligned}
$$

where: $G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + ig_3[G_\mu, G_\nu]$ etc.

$\tilde{G}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$

$$D_\mu h = \left(\partial_\mu + ig_2 W_\mu + i\left(\tfrac{1}{2}\right)g_1 B_\mu\right)h$$

$$\not{D}q_L^{i} = \gamma^{\mu}\left(\partial_\mu + ig_3 G_\mu + ig_2 W_\mu + i\left(\tfrac{1}{6}\right)g_1 B_\mu\right)q_L^{i}$$

Yukawa matrices (3×3)

$g_1, g_2, g_3, m^2, \lambda,\ Y_u^{ij}, Y_d^{ij}, Y_\nu^{ij}, Y_e^{ij}, M^{ij}$ are constants

Story so far:

i) Met YM theory

ii) Met SM table (gauge group, matter irreps)

iii) Met L_{SM}

Minimal Standard Model

$$\mathcal{L}_{SM} = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] - \frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$-(D_\mu h)^\dagger(D^\mu h) + m^2(h^\dagger h) - \frac{\lambda}{4}(h^\dagger h)^2$$

$$+i\left[\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{\ell}_L \not{D} \ell_L + \bar{e}_R \not{D} e_R\right]$$

$$-\left[\bar{q}_L y_u \tilde{h} u_R + \bar{q}_L V_{CKM} y_d h d_R + \bar{\ell}_L y_e h e_R + h.c.\right]$$

What are the <u>physical</u> particles and interactions?

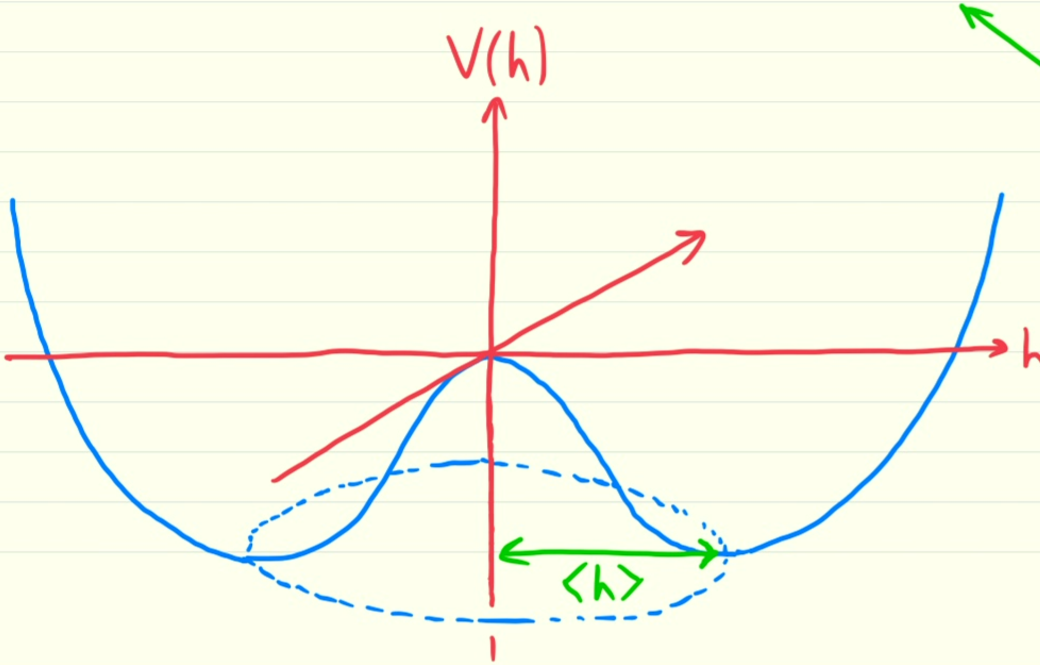
Focus on Higgs terms: "Higgs potential

$$\mathcal{L} = \ldots - (\partial_\mu h)^\dagger (\partial^\mu h) - V(h) \qquad V(h) = -m^2 (h^\dagger h) + \frac{\lambda}{4} (h^\dagger h)^2$$

$$\Rightarrow \mathcal{H} = (\partial_t h)^\dagger (\partial_t h) + (\vec{\nabla} h)^\dagger \cdot (\vec{\nabla} h) + V(h)$$

minimized by

$h(x) = \langle h \rangle = \text{const}$



$\langle h \rangle$ is "vacuum expectation value" ("VEV") of h

$\langle h \rangle \neq 0 \Rightarrow$ "spontaneous symmetry breaking" ("SSB")

In gauge theory:

SSB $\rightarrow$ "Higgs Mechanism"

Perturbative QFT in brief:

i) Split $L = L_{free} + L_{int}$ ← expanded around vacuum

quadratic in fields (unperturbed, "free") (read off phys particles, masses + propagators)

cubic and higher in fields (small perturbations, "interactions") (read off interaction vertices + Feyman rules for vertices)

ii) $L^{scalar}_{free} = -\frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{1}{2}m^2\varphi^2$

$L^{spinor}_{free} = i\bar{\Psi}\not{\partial}\Psi - m\bar{\Psi}\Psi$ → $\Psi = \begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}$ (Dirac), $\Psi = \begin{pmatrix}\psi_R^c\\ \psi_R\end{pmatrix}$ (Majorana)

or $i\bar{\psi}_L\not{\partial}\psi_L$ or $i\bar{\psi}_R\not{\partial}\psi_R$ (Weyl)

$L^{vector}_{free} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\nu$

in all cases, $e^{ik_\mu x^\mu}$ with $k_\mu k^\mu = -m^2$ is exact solution

Minimal Standard Model

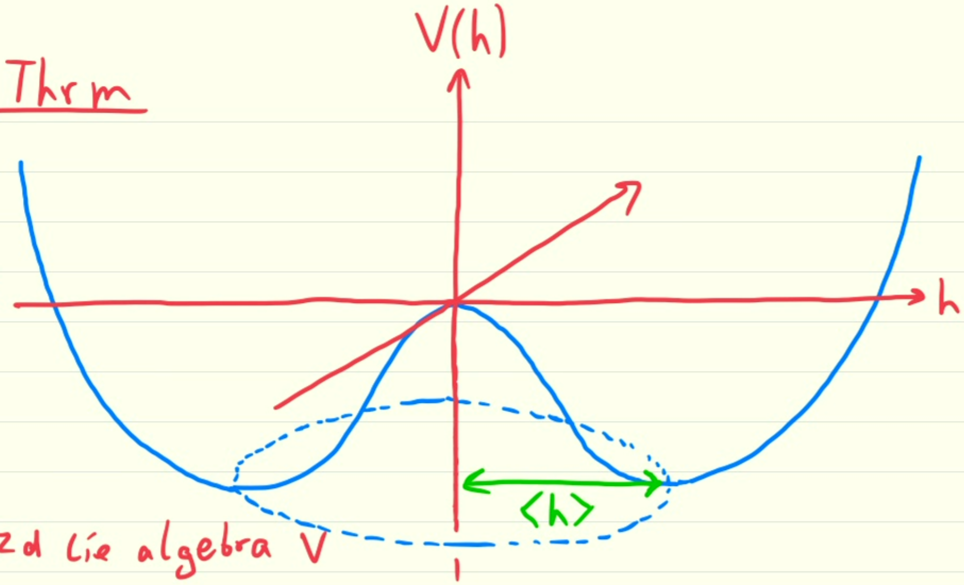
$$\mathcal{L}_{SM} = \left.\begin{array}{l} -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] - \frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ -(D_\mu h)^\dagger(D^\mu h) + m^2(h^\dagger h) - \frac{\lambda}{4}(h^\dagger h)^2 \end{array}\right\} \text{bosonic terms}$$

$$\left.\begin{array}{l} + i\left[\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{\ell}_L \not{D} \ell_L + \bar{e}_R \not{D} e_R\right] \\ -\left[\bar{q}_L y_u \tilde{h} u_R + \bar{q}_L V_{CKM} y_d h d_R + \bar{\ell}_L y_e h e_R + h.c.\right] \end{array}\right\} \text{fermionic terms}$$

What are the physical particles and interactions?

i) Interpret quadratic bosonic terms (SM bosons)
ii) Interpret quadratic fermionic terms (SM fermions)
iii) Interpret remaining non-quadratic terms (SM interactions)

SSB (Global): Goldstone's Thrm

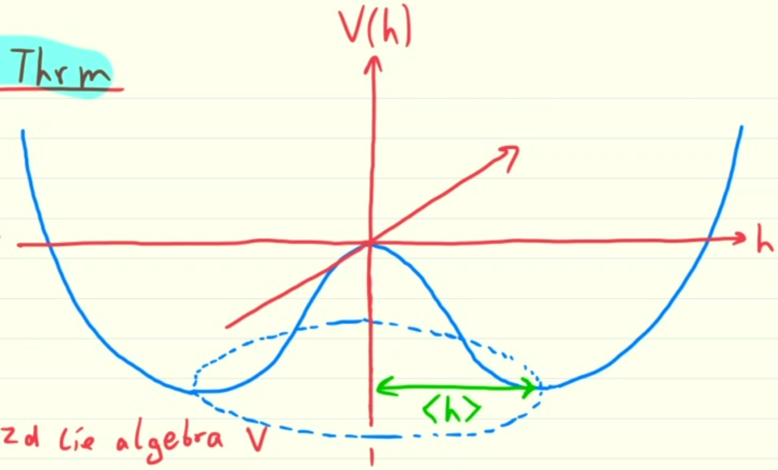


- In SM: $h = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$
- Vacuum manifold: 3-sphere

$(h^\dagger h) = (\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2) = \text{const}$

- Take VEV $\langle h \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
- $su(3)_c \oplus su(2)_L \oplus u(1)_Y = 8+3+1 = 12$d Lie algebra V
- 4 generators act non-triv. on h: $T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $T_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- But 1d subspace acts trivially on $\langle h \rangle$: $Q = Y + T_3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow Q\langle h \rangle = 0$
- So $8+1 = 9$d sub-algebra $W = su(3)_c \oplus u(1)_Q$ annihilates $\langle h \rangle$: $W\langle h \rangle = 0$
  (unbroken generators/gauge sym)
- Remaining 3d quotient space V/W does not: $V/W\langle h \rangle \neq 0$
  (broken generators)
- $\Rightarrow$ 3 massless scalar bosons, one for each broken generator in V/W
  (Goldstone bosons) (Goldstone's Thrm)

SSB (Global): Goldstone's Thrm



- In SM: $h = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$
- Vacuum manifold: 3-sphere

$(h^\dagger h) = (\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2) = \text{const}$

- Take VEV $\langle h \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
- $su(3)_c \oplus su(2)_L \oplus u(1)_Y = 8+3+1 = 12$d Lie algebra V
- 4 generators act non-triv. on h: $T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $T_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- But 1d subspace acts trivially on $\langle h \rangle$: $Q = Y + T_3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow Q\langle h \rangle = 0$
- So $8+1 = 9$d sub-algebra $W = su(3)_c \oplus u(1)_Q$ annihilates $\langle h \rangle$: $W\langle h \rangle = 0$
  (unbroken generators/gauge sym)
- Remaining 3d quotient space V/W does not: $V/W\langle h \rangle \neq 0$
  (broken generators)
- $\Rightarrow$ 3 massless scalar bosons, one for each broken generator in V/W
  (Goldstone bosons) (Goldstone's Thrm)