

Title: Standard Model Lecture - 230112

Speakers: Latham Boyle

Collection: Standard Model (2022/2023)

Date: January 12, 2023 - 3:45 PM

URL: <https://pirsa.org/23010043>

Standard Model

Lecture #3

Non-Abelian Gauge Theory, Part 2

+

Defining the Standard Model

Last Lecture: GR/YM analogy

$$L_{YM} = \frac{1}{4g^2} \text{Tr}[F_{\mu\nu} F^{\mu\nu}]$$

(renormalizable)

$$L_{GR} = \frac{1}{16\pi G_N} R$$

(non-renormalizable)

This Lecture:

- How to specify a specific YM theory.
- How to specify the standard model.

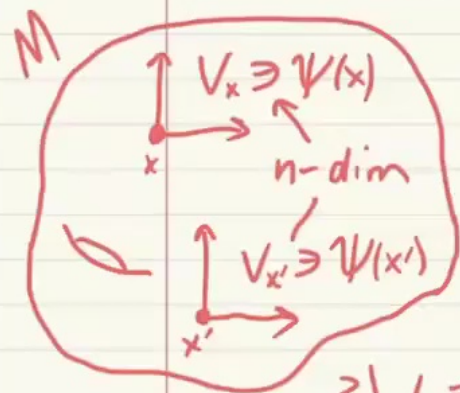
Logic of non-abelian gauge theory (YM theory):

1) Choose gauge group G . e.g. $G = SU(3)$

2) Choose "representation of G ":

i.e. a map from $g \in G \rightarrow U(g) \leftarrow$ matrix
satisfying $U(g_1 g_2) = U(g_1) U(g_2)$

If the U 's are $n \times n$ matrices: an "n-dimensional rep."



"gauge-transformation":

i) over each point x on M , choose $g(x) \in G$

ii) $\psi(x) \rightarrow \tilde{\psi}(x) = U(x) \psi(x)$ " $U(x) = U(g(x))$ "

iii) $A_m(x) \rightarrow \tilde{A}_m(x) = U(x) A_m(x) U^{-1}(x) - (\partial_m U(x)) U^{-1}(x)$

3) $L = L[\psi(x), A_m(x)] \leftarrow$ most general fn that is:
gauge + Lorentz invar (+renormalizable?)

EdinburghJan2023v4

SM Lecture 3

Convention change (math $\rightarrow$ particle physics)

$$A_\mu \rightarrow ig A_\mu \quad \text{and} \quad F_{\mu\nu} \rightarrow ig F_{\mu\nu}$$

Previous lecture's formulae become:

- Covar deriv: $D_\mu = \partial_\mu + ig A_\mu \leftarrow$ YM "gauge field" or "gauge connection"
- Gauge trans: $\psi \rightarrow \tilde{\psi} = U\psi$ where $U(x) \in G$

$$A_\mu \rightarrow \tilde{A}_\mu = U A_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1}$$
- Ricci identity: $[D_\mu, D_\nu] \psi = ig F_{\mu\nu} \psi$
- Field strength: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig [A_\mu, A_\nu]$

$$F_{\mu\nu} \rightarrow \tilde{F}_{\mu\nu} = U F_{\mu\nu} U^{-1}$$
- YM Lagrangian: $L_{YM} = -\frac{1}{4} \text{Tr}[F_{\mu\nu} F^{\mu\nu}]$

Note:
non-Abelian
 gauge theory
 ("YM theory")
 reduces to
Abelian gauge
 theory ("E+M"
 or "Maxwell theory")
 when G is
 abelian:
 $U = e^{i\varphi}, [A_\mu, A_\nu] = 0.$

EdinburghJan2023v4

SM Lecture 3

Adding matter: $\partial_\mu \rightarrow D_\mu$ (Part 1: scalar fields)

- Real scalar field ϕ : $L = -\frac{1}{2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4$
- ↖ "Klein-Gordon Lagrangian"

- Complex scalar field ϕ : $L = -\eta^{\mu\nu}(\partial_\mu\phi)^*(\partial_\nu\phi) - m^2\phi^*\phi - \lambda(\phi^*\phi)^2$

- n complex scalars $\psi = \begin{pmatrix} \psi^1 \\ \vdots \\ \psi^n \end{pmatrix}$: $L = -\eta^{\mu\nu}(\partial_\mu\psi)^\dagger(\partial_\nu\psi) - m^2\psi^\dagger\psi - \lambda(\psi^\dagger\psi)^2$
- ↑ Global sym: $\psi(x) \rightarrow \tilde{\psi}(x) = U\psi(x)$

- replace $\partial_\mu \rightarrow D_\mu$: $\Rightarrow L = -\eta^{\mu\nu}(D_\mu\psi)^\dagger(D_\nu\psi) - m^2\psi^\dagger\psi - \lambda(\psi^\dagger\psi)^2$
- ↑ Gauge sym: $\psi(x) \rightarrow \tilde{\psi}(x) = U(x)\psi(x)$

Adding matter: $\partial_\mu \rightarrow D_\mu$ (Part 1: scalar fields)

- Real scalar field ϕ : $L = -\frac{1}{2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4$
↖ "Klein-Gordon Lagrangian"

- Complex scalar field ϕ : $L = -\eta^{\mu\nu}(\partial_\mu\phi)^*(\partial_\nu\phi) - m^2\phi^*\phi - \lambda(\phi^*\phi)^2$

- n complex scalars $\psi = \begin{pmatrix} \psi^1 \\ \vdots \\ \psi^n \end{pmatrix}$: $L = -\eta^{\mu\nu}(\partial_\mu\psi)^\dagger(\partial_\nu\psi) - m^2\psi^\dagger\psi - \lambda(\psi^\dagger\psi)^2$
↖ Global sym: $\psi(x) \rightarrow \tilde{\psi}(x) = U\psi(x)$

- replace $\partial_\mu \rightarrow D_\mu$:
 $\rightarrow L = -\eta^{\mu\nu}(D_\mu\psi)^\dagger(D_\nu\psi) - m^2\psi^\dagger\psi - \lambda(\psi^\dagger\psi)^2$
↖ Gauge sym: $\psi(x) \rightarrow \tilde{\psi}(x) = U(x)\psi(x)$

Adding matter: $\partial_m \rightarrow D_m$ (Part 2: spinor fields)

- Dirac spinor $\psi = \psi_A^A$: $L = i \bar{\psi}_A \not{\partial}^A_B \psi^B - m \bar{\psi}_A \psi^A$
($A=1,2,3,4$)

where $\bar{\psi}_A = \psi_B^\dagger (\gamma^0)^B_A$, $\not{\partial}^A_B = \partial_m (\gamma^m)^A_B$

-
- n spinors $\psi = \psi^{Aa}$: $L = i \bar{\psi}_{Aa} \not{\partial}^{Aa}_{Bb} \psi^{Bb} - m \bar{\psi}_{Aa} \psi^{Aa}$ $\leftarrow \psi(x) \rightarrow U \psi(x)$
($A=1,\dots,4$) ($a=1,\dots,n$)

where $\bar{\psi}_{Aa} = \psi_{Ba}^\dagger (\gamma^0)^B_A$, $\not{\partial}^{Aa}_{Bb} = \partial_m \delta_b^a (\gamma^m)^A_B$

-
- replace $\partial_m \rightarrow D_m$: $L = i \bar{\psi}_{Aa} \not{D}^{Aa}_{Bb} \psi^{Bb} - m \bar{\psi}_{Aa} \psi^{Aa}$
 $\not{D}^{Aa}_{Bb} = (\partial_m \delta_b^a + A_{mb}^a) (\gamma^m)^A_B$

-
- suppressing indices: $L = i \bar{\psi} \not{D} \psi - m \bar{\psi} \psi$ $\leftarrow$ Gauge sym:
 $\psi(x) \rightarrow U(x) \psi(x)$

Adding matter: $\partial_m \rightarrow D_m$ (Part 2: spinor fields)

- Dirac spinor $\psi = \psi_A^A$: $L = i \bar{\psi}_A \not{\partial}^A_B \psi^B - m \bar{\psi}_A \psi^A$
 $(A=1,2,3,4)$ where $\bar{\psi}_A = \psi_B^\dagger (\gamma^0)^B_A$, $\not{\partial}^A_B = \partial_m (\gamma^m)^A_B$

- n spinors $\psi = \psi^{Aa}$: $L = i \bar{\psi}_{Aa} \not{\partial}^{Aa}_{Bb} \psi^{Bb} - m \bar{\psi}_{Aa} \psi^{Aa}$ ← Global sym: $\psi(x) \rightarrow U \psi(x)$
 $(A=1,\dots,4) (a=1,\dots,n)$ where $\bar{\psi}_{Aa} = \psi_{Ba}^\dagger (\gamma^0)^B_A$, $\not{\partial}^{Aa}_{Bb} = \partial_m \delta^a_b (\gamma^m)^A_B$

- replace $\partial_m \rightarrow D_m$: $L = i \bar{\psi}_{Aa} \not{D}^{Aa}_{Bb} \psi^{Bb} - m \bar{\psi}_{Aa} \psi^{Aa}$
 $\not{D}^{Aa}_{Bb} = (\partial_m \delta^a_b + A_m^a_b) (\gamma^m)^A_B$
 $\not{\partial} = \not{p} = \not{p}_m \gamma^m$

- suppressing indices: $L = i \bar{\psi} \not{D} \psi - m \bar{\psi} \psi$ ← Gauge sym: $\psi(x) \rightarrow U(x) \psi(x)$

Specifying the Standard Model

1) Choose G : $G = SU(3) \times SU(2) \times U(1)$

2) Choose representation of G :

$$(g_3, g_2, g_1)(g'_3, g'_2, g'_1) = (g_3 g'_3, g_2 g'_2, g_1 g'_1)$$

$$U^\dagger U = 1$$

	$SU(3)$	$SU(2)$	$U(1)$		
3 copies "families" "generations"	q_L	3	2	$1/6$	"quarks"
	u_R	3	1	$2/3$	
	d_R	3	1	$-1/3$	
	l_L	1	2	$-1/2$	"leptons"
	ν_R	1	1	0	
	e_R	1	1	-1	
"Higgs doublet" $\rightarrow h$	1	2	$1/2$		scalars (bosons)

Specifying the Standard Model

1) Choose G : $G = SU(3) \times SU(2) \times U(1)$

2) Choose representation of G :

$$(g_3, g_2, g_1)(g'_3, g'_2, g'_1) = (g_3 g'_3, g_2 g'_2, g_1 g'_1)$$

$$U^\dagger U = 1$$

	$SU(3)$	$SU(2)$	$U(1)$		
3 copies "families" "generations"	q_L^i	3	2	$1/6$	"quarks"
	u_R^i	3	1	$2/3$	
	d_R^i	3	1	$-1/3$	
	l_L^i	1	2	$-1/2$	"leptons"
	ν_R^i	1	1	0	
	e_R^i	1	1	-1	
"Higgs doublet" $\rightarrow h$	1	2	$1/2$		scalars (bosons)

$$g \rightarrow U(g)$$

spinors
(fermions)

$$g_i = e^{i\varphi}$$

$$e^{\frac{1}{2}i\varphi}$$

$$e^{\frac{2}{3}i\varphi}$$