Title: Standard Model Lecture - 230130

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Standard Model

Lecture #10

Some Fundamental SM Patterns

(part 2)

SU(5) overview:

($\Lambda\mathbb{C}^5$ transforms under SU(5)

If: $SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$

Then: $\Lambda\mathbb{C}^5 \longrightarrow F \oplus F^c$

Embedding $SU(3)\times SU(2)\times U(1)$ in $SU(5)$:

$SU(3)\times SU(2)\times U(1)$: $(U_3, U_2, \alpha)(U_3', U_2', \alpha') = (U_3U_3', U_2U_2', \alpha\alpha')$

$$\Rightarrow (U_3, 1, 1)(1, U_2, 1) = (U_3, U_2, 1) = (1, U_2, 1)(U_3, 1, 1)$$

$$\Rightarrow (U_3, U_2, 1) \longrightarrow \left(\begin{array}{c|c} U_2 & \\ \hline & U_3 \end{array}\right)$$

$$\Rightarrow (1, 1, \alpha) \longrightarrow \left(\begin{array}{c|c} \alpha^3 1_2 & \\ \hline & \alpha^{-2} 1_3 \end{array}\right)$$

$$\Rightarrow (U_3, U_2, \alpha) \longrightarrow \left(\begin{array}{c|c} \alpha^3 U_2 & \\ \hline & \alpha^{-2} U_3 \end{array}\right)$$

The Exterior Algebra $\Lambda \mathbb{C}^n$

$\Lambda \mathbb{R}^4 \leftarrow$

$\mathbb{C}^n = \{e_1, \dots, e_n\}$ (n dim) $\leftarrow$ 1-forms $\Lambda^1 \mathbb{C}^n$

2-forms $\Lambda^2 \mathbb{C}^n$

$\mathbb{C}^n \otimes \mathbb{C}^n = \{e_i \otimes e_j\}$ (n^2 dim) $\supset \Lambda^2 \mathbb{C}^n = \{e_i \otimes e_j - e_j \otimes e_i\}$ ($\binom{n}{2}$ dim)

p-forms $\Lambda^p \mathbb{C}^n$

$\underbrace{\mathbb{C}^n \otimes \dots \otimes \mathbb{C}^n}_{p} = \{\underbrace{e_i \otimes \dots \otimes e_j}_{p}\}$ (n^p dim) $\supset \Lambda^p \mathbb{C}^n = \{\underbrace{e_{[i} \otimes \dots \otimes e_{j]}}_{p}\}$ ($\binom{n}{p}$ dim)

$\Lambda \mathbb{C}^n = \Lambda^0 \mathbb{C}^n \oplus \Lambda^1 \mathbb{C}^n \oplus \dots \oplus \Lambda^n \mathbb{C}^n$ (2^n dim)

"exterior algebra (over $\mathbb{C}^n$)"

$\Lambda \mathbb{C}^5 = \Lambda^0 \mathbb{C}^5 \oplus \Lambda^1 \mathbb{C}^5 \oplus \Lambda^2 \mathbb{C}^5 \oplus \Lambda^3 \mathbb{C}^5 \oplus \Lambda^4 \mathbb{C}^5 \oplus \Lambda^5 \mathbb{C}^5$

$\Rightarrow$ $1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5$ dim

Duality: $\Lambda^p \mathbb{C}^n$ and $(\Lambda^{n-p}\mathbb{C}^n)^*$ are equiv irreps of $SU(N)$

$U \in SU(N)$: $UU^\dagger = 1 \Rightarrow U^i_j \bar{U}^j_k = \delta^i_k$

$\det(U) = 1 \Rightarrow \epsilon_{i_1 \cdots i_n} U^{i_1}_{j_1} \cdots U^{i_n}_{j_n} = \epsilon_{j_1 \cdots j_n}$

$\Rightarrow \epsilon_{i_1 \cdots i_n} U^{i_1}_{j_1} \cdots U^{i_{n-1}}_{j_{n-1}} = \epsilon_{j_1 \cdots j_n} \bar{U}^{j_n}_{i_n}$ etc.

$\Rightarrow$ e.g. $\varphi^{i_1 i_2} \to U^{i_1}_{j_1} U^{i_2}_{j_2} \varphi^{j_1 j_2}$

$$\bar{\Psi}_{j_1 j_2 j_3} = \epsilon_{j_1 \cdots j_5} \varphi^{j_4 j_5} \to \epsilon_{j_1 \cdots j_5} U^{j_4}_{i_4} U^{j_5}_{i_5} \varphi^{i_4 i_5}$$
$$= \epsilon_{i_1 \cdots i_5} \bar{U}^{i_1}_{j_1} \bar{U}^{i_2}_{j_2} U^{i_3}_{j_3} \varphi^{i_4 i_5}$$
$$= \bar{U}^{i_1}_{j_1} \bar{U}^{i_2}_{j_2} \bar{U}^{i_3}_{j_3} \bar{\Psi}_{i_1 i_2 i_3}$$

Duality: $\Lambda^p \mathbb{C}^n$ and $(\Lambda^{n-p}\mathbb{C}^n)^*$ are equiv irreps of $SU(N)$

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$\Rightarrow$ e.g. $\varphi^{i_1 i_2} \to U^{i_1}_{j_1} U^{i_2}_{j_2} \varphi^{j_1 j_2}$

$$\begin{aligned}\bar{\Psi}_{j_1 j_2 j_3} = \epsilon_{j_1 \dots j_5} \varphi^{j_4 j_5} &\to \epsilon_{j_1 \dots j_5} U^{j_4}_{i_4} U^{j_5}_{i_5} \varphi^{i_4 i_5} \\ &= \epsilon_{i_1 \dots i_5} \bar{U}^{i_1}_{j_1} \bar{U}^{i_2}_{j_2} \bar{U}^{i_3}_{j_3} \varphi^{i_4 i_5} \\ &= \bar{U}^{i_1}_{j_1} \bar{U}^{i_2}_{j_2} \bar{U}^{i_3}_{j_3} \bar{\Psi}_{i_1 i_2 i_3}\end{aligned}$$

Recall: $U_5 = \left(\begin{array}{c|c} \alpha^3 U_2 & \\ \hline & u^{-2} U_3 \end{array}\right) \begin{pmatrix} V \\ W \end{pmatrix}$ $\quad \Lambda \mathbb{C}^5$

$\Lambda^0 \mathbb{C}^5 \to \Lambda^0 \mathbb{C}^5 = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0 \quad (\nu_R^c)$

$\Lambda^1 \mathbb{C}^5 = \mathbb{C}^5 \to U_5 \mathbb{C}^5 = (\underbrace{\mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_3}_{V}) \oplus (\underbrace{\mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_{-2}}_{W}) \quad (\ell_L^c \oplus d_R)$

$\Lambda^2 \mathbb{C}^5 = \Lambda^2 (V \oplus W) = \Lambda^2 V \oplus \Lambda^2 W \oplus V \otimes W$

$= (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_6) \oplus (\mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_{-4}) \oplus (\mathbb{C}^3 \otimes \mathbb{C}^2 \otimes \mathbb{C}_1) \quad (e_R^c \oplus u_R^c \oplus q_L)$

$\Lambda^3 \mathbb{C}^5 = (\Lambda^2 \mathbb{C}^5)^* = (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_{-6}) \oplus (\mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_4) \oplus (\mathbb{C}^{3*} \otimes \mathbb{C}^2 \otimes \mathbb{C}_{-1}) \quad (e_R \oplus u_R \oplus q_L^c)$

$\Lambda^4 \mathbb{C}^5 = (\Lambda^1 \mathbb{C}^5)^* = (\mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_{-3}) \oplus (\mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_2) \quad (\ell_L \oplus d_R^c)$

$\Lambda^5 \mathbb{C}^5 = (\Lambda^0 \mathbb{C}^5)^* = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0 \quad (\nu_R)$

$\Rightarrow \boxed{\Lambda \mathbb{C}^5 = \Lambda^{even} \mathbb{C}^5 \oplus \Lambda^{odd} \mathbb{C}^5 = F \oplus F^c}$

Recall: $U_5 = \left(\begin{array}{c|c} \alpha^3 U_2 & \\ \hline & \alpha^{-2} U_3 \end{array}\right) \begin{pmatrix} V \\ W \end{pmatrix}$

$\Lambda^0 \mathbb{C}^5 \to \Lambda^0 \mathbb{C}^5 = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$ $\quad (\nu_R^c)$

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$= (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_6) \oplus (\mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_{-4}) \oplus (\mathbb{C}^3 \otimes \mathbb{C}^2 \otimes \mathbb{C}_1)$ $\quad (e_R^c \oplus u_R^c \oplus q_L)$

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$\Lambda^4 \mathbb{C}^5 = (\Lambda^1 \mathbb{C}^5)^* = (\mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_{-3}) \oplus (\mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_2)$ $\quad (\ell_L \oplus d_R^c)$

$\Lambda^5 \mathbb{C}^5 = (\Lambda^0 \mathbb{C}^5)^* = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$ $\quad (\nu_R)$

$$\Rightarrow \boxed{\Lambda \mathbb{C}^5 = \Lambda^{\text{even}} \mathbb{C}^5 \oplus \Lambda^{\text{odd}} \mathbb{C}^5 = F \oplus F^c}$$

Duality: $\Lambda^p \mathbb{C}^n$ and $(\Lambda^{n-p}\mathbb{C}^n)^*$ are equiv irreps of $SU(n)$

$U \in SU(N)$: $UU^\dagger = 1 \Rightarrow U^i_j \bar{U}^j_k = \delta^i_k$

$\det(U) = 1 \Rightarrow \epsilon_{i_1\cdots i_n} U^{i_1}_{j_1}\cdots U^{i_n}_{j_n} = \epsilon_{j_1\cdots j_n}$

$\Rightarrow \epsilon_{i_1\cdots i_n} U^{i_1}_{j_1}\cdots U^{i_{n-1}}_{j_{n-1}} = \epsilon_{j_1\cdots j_n} \bar{U}^{j_n}_{i_n}$ etc.

$\Rightarrow$ e.g. $\varphi^{i_1 i_2} \to U^{i_1}_{j_1} U^{i_2}_{j_2} \varphi^{j_1 j_2}$

$$\bar{\Psi}_{j_1 j_2 j_3} = \epsilon_{j_1\cdots j_5}\varphi^{j_4 j_5} \to \epsilon_{j_1\cdots j_5} U^{j_4}_{i_4} U^{j_5}_{i_5} \varphi^{i_4 i_5}$$
$$= \epsilon_{i_1\cdots i_5} \bar{U}^{i_1}_{j_1} \bar{U}^{i_2}_{j_2} \bar{U}^{i_3}_{j_3} \varphi^{i_4 i_5}$$
$$= \bar{U}^{i_1}_{j_1} \bar{U}^{i_2}_{j_2} \bar{U}^{i_3}_{j_3} \bar{\Psi}_{i_1 i_2 i_3}$$

Note:

Since $(U_3, U_2, \alpha) \longrightarrow \left(\begin{array}{c|c} \alpha^3 U_2 & \\ \hline & \alpha^{-2} U_3 \end{array}\right)$

$\Rightarrow$ If $\alpha^6 = 1$: $(U_3, U_2, \alpha) = (\alpha^2 1_3, \alpha^{-3} 1_2, \alpha) \longrightarrow \left(\begin{array}{c|c} 1_2 & \\ \hline & 1_3 \end{array}\right)$

So real embedding $SU(5) \supset [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$

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Lecture #10

Some Fundamental SM Patterns

(part 2)

SO(10) [Spin(10)] GUT:

Consider $\Lambda\mathbb{C}^n = \Lambda^0\mathbb{C}^n \oplus \Lambda^1\mathbb{C}^n \oplus \ldots \oplus \Lambda^n\mathbb{C}^n$

Define two operators on $\Lambda\mathbb{C}^n$:

$a_j^*: \psi \to e_j \wedge \psi \quad (\forall\, \psi \in \Lambda\mathbb{C}^n) \qquad j = 1, \ldots, n$

$a_j: (e_{i_1} \wedge \ldots \wedge e_{i_p}) \to (-1)^{k-1} e_{i_1} \wedge \ldots \wedge e_{i_{k-1}} \wedge e_{i_{k+1}} \wedge \ldots \wedge e_{i_p}$ if $e_{i_k} = e_j$

or 0 if e_j doesn't appear

Check: $\{a_i, a_j\} = \{a_i^*, a_j^*\} = 0$ and $\{a_i, a_j^*\} = \delta_{ij}$

$\Rightarrow a_i^*$ and a_j are like fermion creation/annihilation operators.

Now define: $\left.\begin{array}{l} \gamma^{2i} = i(a_i + a_i^*) \\ \gamma^{2i-1} = (a_i - a_i^*) \end{array}\right\} \Rightarrow \{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu} \quad (\mu, \nu = 1, \ldots, 2n)$

$\Longrightarrow \Lambda\mathbb{C}^n$ is irrep of Clifford Alg.: Dirac spinor (dim 2^n)

$J^{\mu\nu} \propto [\gamma^\mu, \gamma^\nu] \leftarrow$ generators of $so(2n) = spin(2n)$

$[J^{\mu\nu}, J^{\alpha\beta}] \propto \eta^{\mu\alpha} J^{\nu\beta} + \eta^{\nu\beta} J^{\mu\alpha} - \eta^{\mu\beta} J^{\nu\alpha} - \eta^{\nu\alpha} J^{\mu\beta}$

$\Rightarrow \underset{\text{Dirac spinor}}{\Lambda \mathbb{C}^n} = \underset{\text{LH Weyl spinor}}{\Lambda^{even} \mathbb{C}^n} \oplus \underset{\text{RH Weyl spinor}}{\Lambda^{odd} \mathbb{C}^n}$ of $so(2n) = spin(2n)$

Spin(10): $\Lambda \mathbb{C}^5 = \Lambda^{even} \mathbb{C}^5 \oplus \Lambda^{odd} \mathbb{C}^5$

$32 = 16 \oplus \overline{16}$ — The unique irrep of $C\ell(10)$

SU(5): $(1 \oplus 10 \oplus \overline{5}) \oplus (5 \oplus \overline{10} \oplus 1)$ — requires ν_R

$SU(3) \times SU(2) \times U(1)$: $F \oplus F^*$

SM table gets much simpler!

	Spin(10)
ψ_L	16

Pati-Salam: $SU(4)\times SU(2)_L \times SU(2)_R$

	$SU(4)$	$SU(2)_L$	$SU(2)_R$
Q_L	4	2	1
Q_R	4	1	2

$Q_L : \mathbb{C}^4\otimes\mathbb{C}^2\otimes\mathbb{C}^1 \quad Q_L^c : \mathbb{C}^{4*}\otimes\mathbb{C}^2\otimes\mathbb{C}$

$Q_R : \mathbb{C}^4\otimes\mathbb{C}^1\otimes\mathbb{C}^2 \quad Q_R^c : \mathbb{C}^{4*}\otimes\mathbb{C}\otimes\mathbb{C}^2$

- leptons as 4th color of quark
- left-right symmetric

$$SU(3)\times SU(2)\times U(1) \longrightarrow SU(4)\times SU(2)_L \times SU(2)_R$$

$$(U_3,\ U_2,\ \alpha) \longrightarrow \left[\begin{pmatrix}\alpha^p U_3 & \\ & \alpha^{-3p}\end{pmatrix},\ U_2,\ \begin{pmatrix}\alpha^q & \\ & \alpha^{-q}\end{pmatrix}\right]$$

If $p=1,\ q=3 \Rightarrow Q_L\oplus Q_R^c \longrightarrow F$

$$\left.\begin{array}{l} Spin(4) = SU(2)\times SU(2) \\ Spin(6) = SU(4) \end{array}\right\} \rightarrow \text{Pati-Salam} = \begin{pmatrix} Spin(4) & \\ & Spin(6)\end{pmatrix} \subset Spin(10)$$