Title: Standard Model Lecture - 230125

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Collection: Standard Model (2022/2023)

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SMSB (Bosons)

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + i g_3 [G_\mu, G_\nu]$$

$$G_{\mu\nu} = G^i_{\mu\nu}\lambda^i \quad \left(\mathrm{Tr}[\lambda^i\lambda^j] = \tfrac{1}{2}\delta^{ij}\right)$$

$$W_{\mu\nu} = W^i_{\mu\nu} T^i \quad \left(\mathrm{Tr}[T^i T^j] = \tfrac{1}{2}\delta^{ij}\right)$$

$$L_{SM} = -\tfrac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}] - \tfrac{1}{2}\mathrm{Tr}[W_{\mu\nu}W^{\mu\nu}] - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$h = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$-(D^\mu h)^\dagger (D_\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4}(h^\dagger h)^2 + \text{fermionic terms}$$

$$D_\mu h = \left(\partial_\mu + i g_2 \tfrac{1}{2}\sigma^i W^i_\mu + i g_1 \tfrac{1}{2}\sigma^0 B_\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial_\mu H \end{pmatrix} + \frac{iv}{2}\begin{pmatrix} g_2(W^1_\mu - iW^2_\mu) \\ g_1 B_\mu - g_2 W^3_\mu \end{pmatrix}\right] + \dots \quad \leftarrow \text{check!}$$

"free" (quad.) terms

$$= -\tfrac{1}{4}G^i_{\mu\nu}G^{i\mu\nu} - \tfrac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} \qquad \left(v^2 \equiv \frac{4m^2}{\lambda}\right)$$

$\leftarrow$ check!

$$-\tfrac{1}{2}(\partial_\mu H)^2 - m^2 H^2 - \frac{v^2}{8}g_2^2\left[(W^1_\mu)^2 + (W^2_\mu)^2\right] - \frac{v^2}{8}(g_2 W^3_\mu - g_1 B_\mu)^2 + \dots$$

rewrite

$$= -\tfrac{1}{4}G^i_{\mu\nu}G^{i\mu\nu} - \tfrac{1}{4}W^+_{\mu\nu}W^{-\mu\nu} - \tfrac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

check! $\rightarrow$

$$-\tfrac{1}{2}(\partial_\mu H)^2 - \tfrac{1}{2}M_H^2 H^2 - \tfrac{1}{2}M_W^2 W^+_\mu W^{-\mu} - \tfrac{1}{2}M_Z^2 Z_\mu Z^\mu$$

$$M_H^2 \equiv 2m^2 \qquad \cos\theta_W \equiv \frac{g_2}{(g_1^2 + g_2^2)^{1/2}}$$

$$W^\pm_\mu \equiv W^1_\mu \pm i W^2_\mu$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

$$m_W^2 \equiv \tfrac{1}{4}g_2^2 v^2 \qquad m_Z^2 \equiv \tfrac{1}{4}(g_1^2 + g_2^2)v^2$$

$$\frac{m_W}{m_Z} = \frac{82\,\text{GeV}}{91\,\text{GeV}} = \cos\theta_W$$

SMSB (Bosons)

$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + ig_3[G_\mu, G_\nu]$

$G_{\mu\nu} = G^i_{\mu\nu}\lambda^i \quad \left(\mathrm{Tr}[\lambda^i\lambda^j] = \tfrac{1}{2}\delta^{ij}\right)$

$W_{\mu\nu} = W^i_{\mu\nu}T^i \quad \left(\mathrm{Tr}[T^iT^j] = \tfrac{1}{2}\delta^{ij}\right)$

$$L_{SM} = -\tfrac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}] - \tfrac{1}{2}\mathrm{Tr}[W_{\mu\nu}W^{\mu\nu}] - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$-(D^\mu h)^\dagger(D_\mu h) + m^2(h^\dagger h) - \tfrac{\lambda}{4}(h^\dagger h)^2 + \text{fermionic terms}$$

$h = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H\end{pmatrix}$

$$D_\mu h = \left(\partial_\mu + ig_2\tfrac{1}{2}\sigma^i W^i_\mu + ig_1\tfrac{1}{2}\sigma^0 B_\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H\end{pmatrix}$$
$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix}0\\ \partial_\mu H\end{pmatrix} + \frac{iv}{2}\begin{pmatrix}g_2(W^1_\mu - iW^2_\mu)\\ g_1B_\mu - g_2W^3_\mu\end{pmatrix}\right] + \ldots \longleftarrow \text{check!}$$

"free" (quad.) terms

$$= -\tfrac{1}{4}G^i_{\mu\nu}G^{i\mu\nu} - \tfrac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$-\tfrac{1}{2}(\partial_\mu H)^2 - m^2H^2 - \frac{v^2}{8}g_2^2\left[(W^1_\mu)^2 + (W^2_\mu)^2\right] - \frac{v^2}{8}(g_2W^3_\mu - g_1B_\mu)^2 + \ldots$$

$\left(v^2 \equiv \frac{4m^2}{\lambda}\right)$ $\longleftarrow$ check!

rewrite

$$= -\tfrac{1}{4}G^i_{\mu\nu}G^{i\mu\nu} - \tfrac{1}{4}W^+_{\mu\nu}W^{-\mu\nu} - \tfrac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$-\tfrac{1}{2}(\partial_\mu H)^2 - \tfrac{1}{2}M_H^2H^2 - \tfrac{1}{2}M_W^2W^+_\mu W^{-\mu} - \tfrac{1}{2}M_Z^2Z_\mu Z^\mu$$

check!

$M_H^2 \equiv 2m^2$

$\cos\theta_W \equiv \frac{g_2}{(g_1^2+g_2^2)^{1/2}}$

$W^\pm_\mu \equiv W^1_\mu \pm iW^2_\mu$

$\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} \equiv \begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix}$

$m_W^2 \equiv \tfrac{1}{4}g_2^2v^2$

$m_Z^2 \equiv \tfrac{1}{4}(g_1^2+g_2^2)v^2$

$\frac{m_W}{m_Z} = \frac{82\,\text{GeV}}{91\,\text{GeV}} = \cos\theta_W$

SMSB (Fermions)

$$L_{SM} = \underset{\uparrow}{\ldots} + i\left[\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{\ell}_L \not{D} \ell_L + \bar{e}_R \not{D} e_R\right]$$

bosonic terms

$$- \left[\bar{q}_L y_u \tilde{h} u_R + \bar{q}_L V_{CKM} y_d h d_R + \bar{\ell}_L y_e h e_R + h.c.\right]$$

$\not{D} = \not{\partial} + \ldots$

$h = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix} + \ldots$

$\tilde{h} = \frac{1}{\sqrt{2}}\begin{pmatrix}v\\0\end{pmatrix} + \ldots$

$q_L = \begin{pmatrix}u_L\\d_L\end{pmatrix}$ $\ell_L = \begin{pmatrix}\nu_L\\e_L\end{pmatrix}$

"free" (quad.) terms

$$= i\left[\bar{u}_L \not{\partial} u_L + \bar{d}_L \not{\partial} d_L + \bar{u}_R \not{\partial} u_R + \bar{d}_R \not{\partial} d_R + \bar{\nu}_L \not{\partial} \nu_L + \bar{e}_L \not{\partial} e_L + \bar{e}_R \not{\partial} e_R\right]$$

check!

$$- \frac{1}{\sqrt{2}} v\left[\bar{u}_L y_u u_R + \bar{d}_L V_{CKM} y_d d_R + \bar{e}_L y_e e_R + h.c.\right] + \ldots$$

rewrite

$$= i\left[\bar{u} \not{\partial} u + \bar{d} \not{\partial} d + \bar{e} \not{\partial} e + \bar{\nu}_L \not{\partial} \nu_L\right]$$

check!

$$- \left[\bar{u} m_u u + \bar{d} m_d d + \bar{e} m_e e\right] + \ldots$$

So minimal SM predicts ν_L is massless Weyl fermion

$u \equiv \begin{pmatrix}u_L\\u_R\end{pmatrix}$ $d \equiv \begin{pmatrix}d_L'\\d_R\end{pmatrix}$ $e \equiv \begin{pmatrix}e_L\\e_R\end{pmatrix}$ $\left(d_L' \equiv V_{CKM}^\dagger d_L\right)$

$m_u \equiv \frac{1}{\sqrt{2}} v y_u$ $m_d \equiv \frac{1}{\sqrt{2}} v y_d$ $m_e \equiv \frac{1}{\sqrt{2}} v y_e$

Physical content of minimal SM (free part):

i) Scalar: $h = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$ where $v^2 = \frac{4m^2}{\lambda}$ and $m_H^2 = 2m^2$

ii) Vectors: Massless: $G_\mu^{i=1,\dots,8}$ (gluons), A_μ (photons)

Massive: $W_\mu^\pm$ $\quad$ Z_μ

$\left(M_W^2 = \frac{1}{4} g_2^2 v^2\right)$ $\quad$ $\left(M_Z^2 = \frac{1}{4}(g_2^2 + g_1^2) v^2\right)$

iii) Fermions: Massive (Dirac): $u = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$ $\quad$ $d = \begin{pmatrix} d_L' \\ d_R \end{pmatrix}$ $\quad$ $e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$

$m_u = \frac{v}{\sqrt{2}} y_u$ $\quad$ $m_d = \frac{v}{\sqrt{2}} y_d$ $\quad$ $m_e = \frac{v}{\sqrt{2}} y_e$

Massless (Weyl): ν_L

$\times 3$

Next time:
- interactions/ Feynman rules
- shortcomings of minimal SM + possible solutions

	$SU(3)$	T_3	Y	$Q = T_3 + Y$
u_L	3	$1/2$	$1/6$	$2/3$
d_L	3	$-1/2$	$1/6$	$-1/3$
u_R	3	0	$2/3$	$2/3$
d_R	3	0	$-1/3$	$-1/3$
ν_L	1	$1/2$	$-1/2$	0
e_L	1	$-1/2$	$-1/2$	-1
ν_R	1	0	0	0
e_R	1	0	-1	-1
H	1	$-1/2$	$1/2$	0

Minimal Standard Model

$$
\begin{aligned}
\mathcal{L}_{SM} = & -\frac{1}{4} G^{i}_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^{i}_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
& - (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 \\
& + i\left[\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{\ell}_L \not{D} \ell_L + \bar{e}_R \not{D} e_R \right] \\
& - \left[\bar{q}_L y_u \tilde{h} u_R + \bar{q}_L V_{CKM} y_d h d_R + \bar{\ell}_L y_e h e_R + h.c. \right]
\end{aligned}
$$

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

previous: "free" (quadratic) terms

now: "interaction" (cubic, quartic) terms

1st line: (1/2)

$$\mathcal{L}_{SM} = -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \ldots$$

$$G^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g_3 f^{imn} G^m_\mu G^n_\nu$$

$$-\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} = -\frac{1}{4} \eta^{\mu\mu'} \eta^{\nu\nu'} \left(\partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g_3 f^{imn} G^m_\mu G^n_\nu\right)\left(\partial_{\mu'} G^i_{\nu'} - \partial_{\nu'} G^i_{\mu'} - g_3 f^{im'n'} G^{m'}_{\mu'} G^{n'}_{\nu'}\right)$$

$$= + \ldots - \frac{1}{4} \eta^{\mu\mu'} \eta^{\nu\nu'} g_3^2 f^{imn} f^{im'n'} G^m_\mu G^n_\nu G^{m'}_{\mu'} G^{n'}_{\nu'}$$

$\Rightarrow$ 4 gluons:

$$= -\frac{i}{4} g_3^2 \eta^{\mu\mu'} \eta^{\nu\nu'} f^{imn} f^{im'n'} + \text{perms}$$

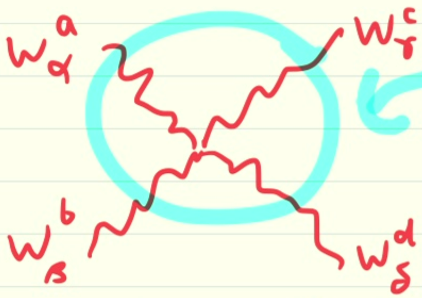
+ 3 gluons:

$$= -g_3 f^{abc} \left[\eta^{\alpha\beta}(k_1-k_2)^\gamma + \eta^{\beta\gamma}(k_2-k_3)^\alpha + \eta^{\gamma\alpha}(k_3-k_1)^\beta\right]$$

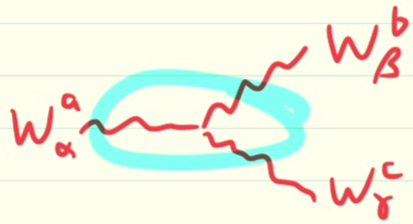
1st line: (2/2)

$$\mathcal{L}_{SM} = -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \ldots$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_2 \epsilon^{ijk} W^j_\mu W^k_\nu$$



$\sim g_2^2$

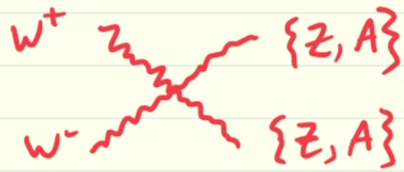


$\sim g_2$

$$W^\pm_\mu = W^1_\mu \pm i W^2_\mu$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

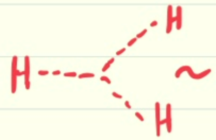
$\Rightarrow$



2nd line:

$$\mathcal{L}_{SM} = \ldots - (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4}(h^\dagger h)^2 \ldots$$

$$h = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H\end{pmatrix}, \quad D_\mu h = \left(\partial_\mu + \frac{i}{2} g_2 W^i_\mu \sigma^i + \frac{i}{2} g_1 B_\mu \sigma^0\right)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H\end{pmatrix}$$

$\sim -\lambda v H^3 \Rightarrow$ 

$\sim -i\lambda v$

$\sim -\lambda H^4 \Rightarrow$ 

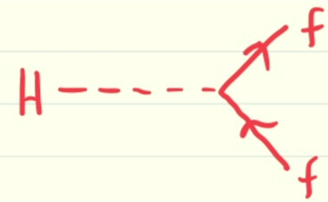
$\sim -i\lambda$

4th Line:

$$h = \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$\mathcal{L}_{SM} = \ldots - \left[\bar{q}_L y_u \tilde{h} u_R + \bar{q}_L V_{CKM} y_d h d_R + \bar{l}_L y_e h e_R + h.c. \right]$$

$$= \ldots - \frac{1}{\sqrt{2}} \left[\bar{u} y_u H u + \bar{d} y_d H d + \bar{e} y_e H e \right]$$

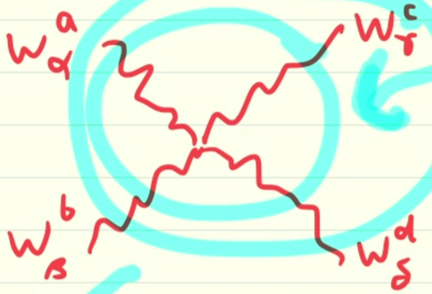


$$-\frac{i}{\sqrt{2}} y_f = -i \frac{m_f}{v}$$

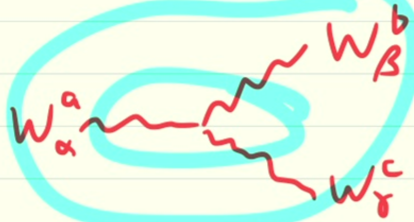
1st line: (2/2)

$$\mathcal{L}_{SM} = -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \dots$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_2 \epsilon^{ijk} W^j_\mu W^k_\nu$$



$\sim g_2^2$

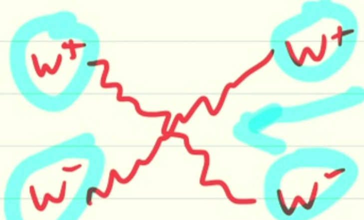


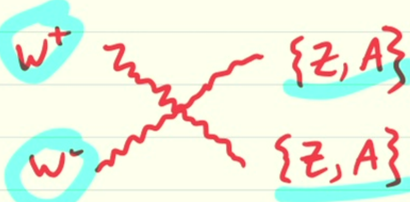
$\sim g_2$

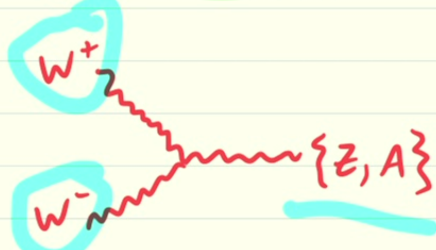
$$W^\pm_\mu = W^1_\mu \pm i W^2_\mu$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

$\Rightarrow$







SMSB (Bosons)

$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + i g_3 [G_\mu, G_\nu]$ $\quad G_{\mu\nu} = G^i_{\mu\nu} \lambda^i \quad \left(Tr[\lambda^i \lambda^j] = \frac{1}{2}\delta^{ij} \right)$

$W_{\mu\nu} = W^i_{\mu\nu} T^i \quad \left(Tr[T^i T^j] = \frac{1}{2}\delta^{ij} \right)$

$$L_{SM} = -\frac{1}{2} Tr\left[G_{\mu\nu} G^{\mu\nu}\right] - \frac{1}{2} Tr\left[W_{\mu\nu} W^{\mu\nu}\right] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$- (D^\mu h)^\dagger (D_\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \text{fermionic terms}$$

$$D_\mu h = \left(\partial_\mu + i g_2 \tfrac{1}{2} \sigma^i W^i_\mu + i g_1 \tfrac{1}{2} \sigma^0 B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ \partial_\mu H \end{pmatrix} + \frac{iv}{2} \begin{pmatrix} g_2 (W^1_\mu - i W^2_\mu) \\ g_1 B_\mu - g_2 W^3_\mu \end{pmatrix} \right] + \ldots \longleftarrow \text{check!}$$

"free" (quad.) terms

$$= -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \qquad \left(v^2 \equiv \frac{4m^2}{\lambda} \right)$$

$\longleftarrow$ check!

$$- \frac{1}{2} (\partial_\mu H)^2 - m^2 H^2 - \frac{v^2}{8} g_2^2 \left[(W^1_\mu)^2 + (W^2_\mu)^2 \right] - \frac{v^2}{8} (g_2 W^3_\mu - g_1 B_\mu)^2 + \ldots$$

rewrite

$$= -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^+_{\mu\nu} W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

check!

$$- \frac{1}{2} (\partial_\mu H)^2 - \frac{1}{2} M_H^2 H^2 - \frac{1}{2} M_W^2 W^+_\mu W^{-\mu} - \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$

$$\left\{ \begin{array}{l} M_H^2 \equiv 2m^2 \\ \cos\theta_W \equiv \dfrac{g_2}{(g_1^2 + g_2^2)^{1/2}} \end{array} \right.$$

$$W^\pm_\mu \equiv W^1_\mu \pm i W^2_\mu \quad \left\{ \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \right\} \begin{array}{l} m_W^2 \equiv \frac{1}{4} g_2^2 v^2 \\ m_Z^2 \equiv \frac{1}{4} (g_1^2 + g_2^2) v^2 \end{array} \left\} \frac{m_W}{m_Z} = \frac{82\,\text{GeV}}{91\,\text{GeV}} = \cos\theta_W \right.$$