Title: Standard Model Lecture - 230123

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Collection: Standard Model (2022/2023)

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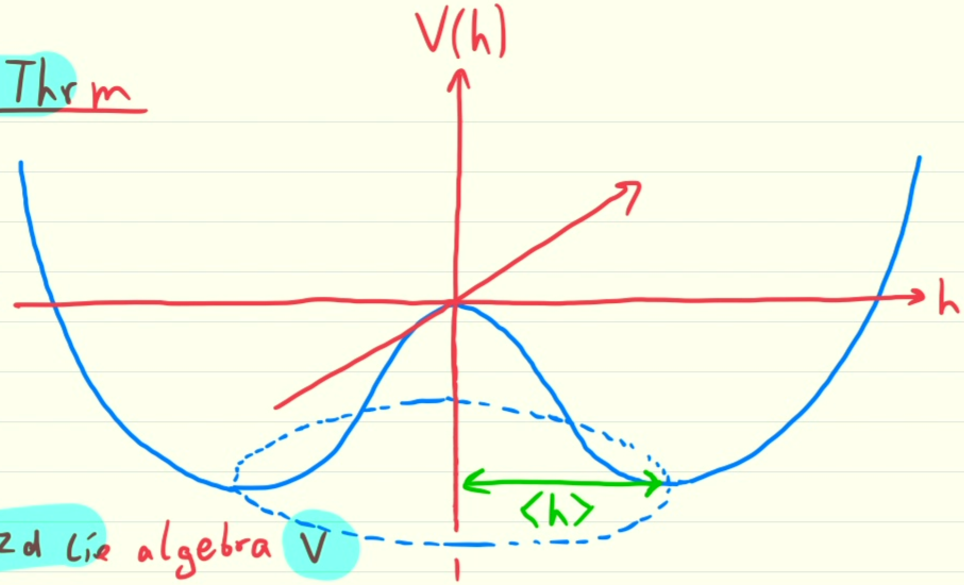
Standard Model

Lecture # 7

Higgs Mechanism, Electroweak Symmetry Breaking

(Part 2)

SSB (Global): Goldstone's Thrm



- In SM: $h = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$
- Vacuum manifold: 3-sphere

$(h^\dagger h) = (\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2) = \text{const}$

- Take VEV $\langle h \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$
- $su(3)_c \oplus su(2)_L \oplus u(1)_Y = 8+3+1 = 12$d Lie algebra V
- 4 generators act non-triv. on h: $T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $T_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- But 1d subspace acts trivially on $\langle h \rangle$: $Q = Y + T_3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow Q\langle h \rangle = 0$
- So $8+1 = 9$d sub-algebra $W = su(3)_c \oplus u(1)_Q$ annihilates $\langle h \rangle$: $W\langle h \rangle = 0$

(unbroken generators/gauge sym)

- Remaining 3d quotient space V/W does not: $V/W\langle h \rangle \neq 0$

(broken generators)

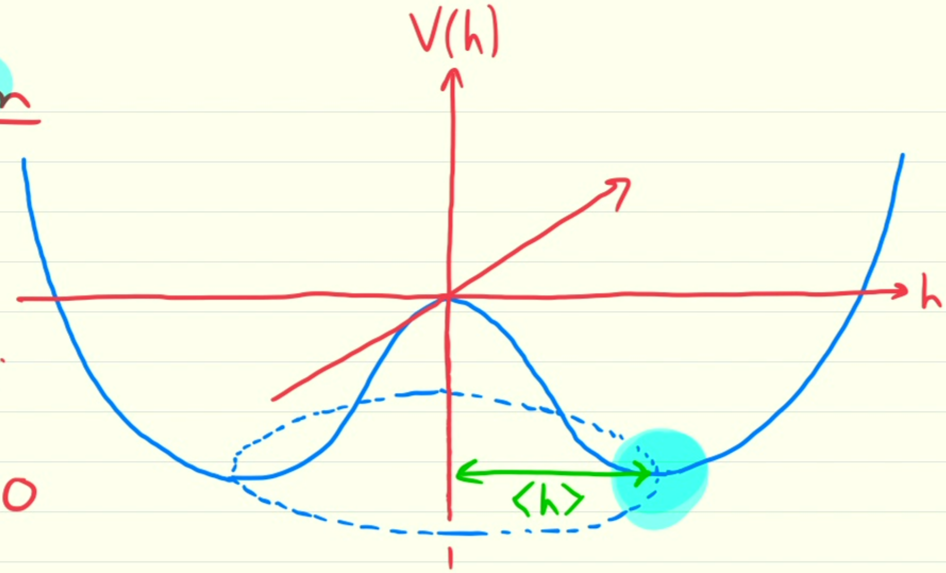
$[v] \in V/W$

$[v] = \{v + w \mid w \in W\}$

- $\Rightarrow$ 3 massless scalar bosons, one for each broken generator in V/W

(Goldstone bosons) (Goldstone's Thrm)

SSB (Gauge): Higgs Mechanism



- Take $h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$ "Higgs VEV" "Higgs boson"

  "unitary gauge"

- V: full $su(3)_c \oplus su(2)_L \oplus u(1)_Y$ Lie alg.

  $8 + 3 + 1 = 12d$

- W: $su(3)_c \oplus u(1)_Q$ sub-alg: $W\langle h\rangle = 0$

  $8 + 1 = 9d$

  (λ^i) $(Q = T_3 + Y)$ "electric charge"

  "gluons" "photon" ← "unbroken"/massless vector bosons

  G^i_μ A_μ (2 d.o.f.: helicity ±1)

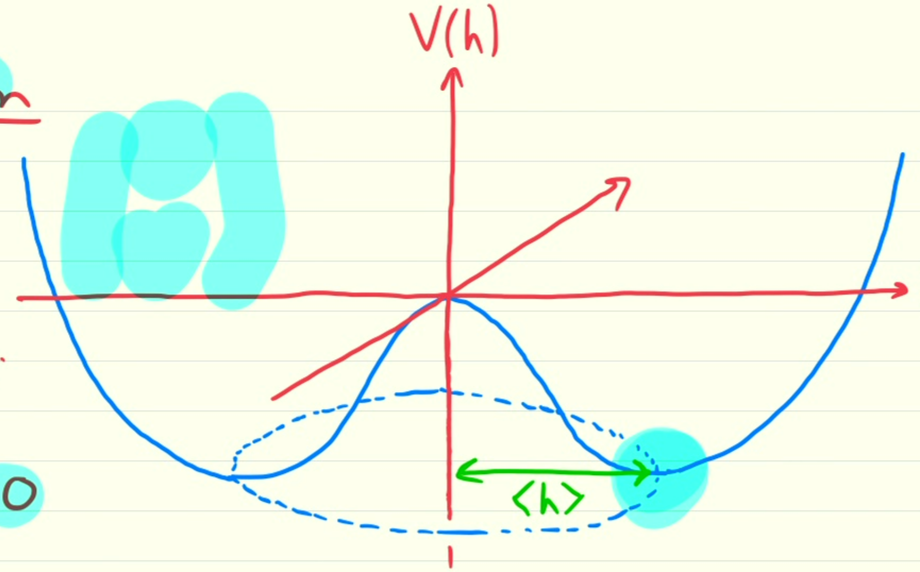
- V/W: remaining $12 - 9 = 3d$ space: $V/W\langle h\rangle \neq 0$ ← broken EW gauge sym.

  → corresponding generators each "eat" a Goldstone boson and become massive ("Higgs mechanism")

  → "Weak vector bosons" ← "broken"/massive vector bosons

  $W^+_\mu, W^-_\mu, Z^0_\mu$ (3 d.o.f.: helicity ±1, 0)

SSB (Gauge): Higgs Mechanism



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- V: full $su(3)_c \oplus su(2)_L \oplus u(1)_Y$ Lie alg.

  $8 + 3 + 1 = 12d$

- W: $su(3)_c \oplus u(1)_Q$ sub-alg: $W\langle h \rangle = 0$

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  W^+_μ, W^-_μ, Z^0_μ (3 d.o.f.: helicity $\pm 1, 0$)

$$
\begin{aligned}
\mathcal{L}_{SM} = & -\tfrac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] - \tfrac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& -(D^{\mu}h)^{\dagger}(D_{\mu}h) + m^2(h^{\dagger}h) - \lambda(h^{\dagger}h)^2 \\
& + i\left[\bar{q}_L^{i}\not{D}q_L^{i} + \bar{u}_R^{i}\not{D}u_R^{i} + \bar{d}_R^{i}\not{D}d_R^{i} + \bar{\ell}_L^{i}\not{D}\ell_L^{i} + \left(\bar{\nu}_R^{i}\not{D}\nu_R^{i}\right) + \bar{e}_R^{i}\not{D}e_R^{i}\right] \\
& - \left[\bar{q}_L^{i}Y_u^{ij}\tilde{h}u_R^{j} + \bar{q}_L^{i}Y_d^{ij}h\,d_R^{j} + \left(\bar{\ell}_L^{i}Y_\nu^{ij}\tilde{h}\nu_R^{j}\right) + \bar{\ell}_L Y_e^{ij}h\,e_R^{j} + h.c.\right] \\
& - \left(\bar{\nu}_{R,c}^{i}M^{ij}\nu_R^{j}\right) - \underline{\theta\,\mathrm{Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right]}
\end{aligned}
$$

Q1: Why $\bar{q}_L^{i}\not{D}q_L^{i} = \bar{q}_L\not{D}q_L$ and not $\bar{q}_L^{i}K^{ij}\not{D}q_L^{j} = \bar{q}_L K\not{D}q_L$?

A1:
- Diagonalize $K = U^{\dagger}\Lambda U$, with Λ diagonal + positive
- redefine $\Lambda^{1/2}U q_L = q_L'$
- drop primes to recover above diagonal form

$$\mathcal{L}_{SM} = -\frac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}] - \frac{1}{2}\mathrm{Tr}[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$- (D^\mu h)^\dagger (D_\mu h) + m^2(h^\dagger h) - \lambda (h^\dagger h)^2$$
$$+ i\left[\bar{q}_L^i \not{D} q_L^i + \bar{u}_R^i \not{D} u_R^i + \bar{d}_R^i \not{D} d_R^i + \bar{\ell}_L^i \not{D} \ell_L^i + \left(\bar{\nu}_R^i \not{D} \nu_R^i\right) + \bar{e}_R^i \not{D} e_R^i\right]$$
$$- \left[\bar{q}_L^i Y_u^{ij} \tilde{h} u_R^j + \bar{q}_L^i Y_d^{ij} h d_R^j + \left(\bar{\ell}_L^i Y_\nu^{ij} \tilde{h} \nu_R^j\right) + \bar{\ell}_L Y_e^{ij} h e_R^j + h.c.\right]$$
$$- \left(\bar{\nu}_{R,c}^i M^{ij} \nu_R^j\right) - \underline{\theta\, \mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}]}$$

Q2: Can we also absorb the Y's by field redefinition?

A2: $Y_u = V_u^\dagger y_u U_u \leftarrow u_R' = U_u u_R,\ q_L' = V_u q_L$

$Y_d = V_d^\dagger y_d U_d \leftarrow d_R' = U_d d_R,\ \cancel{q_L' = V_d q_L}$

$Y_e = V_e^\dagger y_e U_e \leftarrow e_R' = U_e e_R,\ \ell_L' = V_e \ell_L$

$$\boxed{Y = V^\dagger y\, U}$$
"singular value decomposition"

Check!

$$\mathcal{L}_{SM} = -\frac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}] - \frac{1}{2}\mathrm{Tr}[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$- (D^\mu h)^\dagger (D_\mu h) + m^2 (h^\dagger h) - \lambda (h^\dagger h)^2$$
$$+ i\left[\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + \bar{\ell}_L \not{D} \ell_L + (\quad) + \bar{e}_R \not{D} e_R\right]$$
$$- \left[\bar{q}_L y_u \tilde{h} u_R + q_L V_{CKM} y_d h d_R + (\quad) + \bar{\ell}_L y_e h e_R + h.c.\right]$$
$$- (\quad) - \underline{\theta \,\mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}]}$$

$$V_{CKM} = V_u V_d^\dagger$$

Q2: Can we also absorb the Y's by field redefinition?

A2: $Y_u = V_u^\dagger y_u U_u \leftarrow u_R' = U_u u_R,\ q_L' = V_u q_L$

$Y_d = V_d^\dagger y_d U_d \leftarrow d_R' = U_d d_R,\ \cancel{q_L' = V_d q_L}$

$Y_e = V_e^\dagger y_e U_e \leftarrow e_R' = U_e e_R,\ \ell_L' = V_e \ell_L$

SMSB (Bosons)

$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + i g_3 [G_\mu, G_\nu]$

$G_{\mu\nu} = G^i_{\mu\nu} \lambda^i \quad \left(Tr[\lambda^i \lambda^j] = \frac{1}{2}\delta^{ij} \right)$

$W_{\mu\nu} = W^i_{\mu\nu} T^i \quad \left(Tr[T^i T^j] = \frac{1}{2}\delta^{ij} \right)$

$$L_{SM} = -\frac{1}{2} Tr[G_{\mu\nu} G^{\mu\nu}] - \frac{1}{2} Tr[W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$- (D^\mu h)^\dagger (D_\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \text{fermionic terms}$$

$$D_\mu h = \left(\partial_\mu + i g_2 \frac{1}{2} \sigma^i W^i_\mu + i g_1 \frac{1}{2} \sigma^0 B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ \partial_\mu H \end{pmatrix} + \frac{iv}{2} \begin{pmatrix} g_2 (W^1_\mu - i W^2_\mu) \\ g_1 B_\mu - g_2 W^3_\mu \end{pmatrix} \right] + \dots \quad \longleftarrow \text{check!}$$

"free" (quad.) terms

$$= -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \qquad \left(v^2 \equiv \frac{4m^2}{\lambda} \right)$$

$\longleftarrow$ check!

$$- \frac{1}{2} (\partial_\mu H)^2 - m^2 H^2 - \frac{v^2}{8} g_2^2 \left[(W^1_\mu)^2 + (W^2_\mu)^2 \right] - \frac{v^2}{8} (g_2 W^3_\mu - g_1 B_\mu)^2 + \dots$$

rewrite

$$= -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} - \frac{1}{4} W^+_{\mu\nu} W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

check! $\longrightarrow$

$$- \frac{1}{2} (\partial_\mu H)^2 - \frac{1}{2} M_H^2 H^2 - \frac{1}{2} M_W^2 W^+_\mu W^{-\mu} - \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$

$$\left\{ \begin{array}{l} M_H^2 \equiv 2m^2 \\ \cos\theta_W \equiv \dfrac{g_2}{(g_1^2 + g_2^2)^{1/2}} \end{array} \right.$$

$W^\pm_\mu \equiv W^1_\mu \pm i W^2_\mu$

$$\left\{ \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \right\}$$

$$\begin{array}{l} m_W^2 \equiv \frac{1}{4} g_2^2 v^2 \\ m_Z^2 \equiv \frac{1}{4} (g_1^2 + g_2^2) v^2 \end{array}$$

$$\frac{m_W}{m_Z} = \frac{82\,\text{GeV}}{91\,\text{GeV}} = \cos\theta_W$$