Title: Standard Model Lecture - 230116

Speakers: Latham Boyle

Collection: Standard Model (2022/2023)

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Standard Model + Below

Lecture #4

The Standard Model

Fields and Lagrangian

(Part 1)

Group theory warm-up (1/3)

- n-dimensional Lie algebra X
- basis for X: $\{T^1, \ldots, T^n\}$ ← "generators of X"
- can characterize Lie bracket in X by: $[T^i, T^j] = f^{ijk} T^k$ (math) — anti-hermitian

  f^{ijk}: "structure constants" or $[T^i, T^j] = i f^{ijk} T^k$ (physics) — hermitian
- If $\mathrm{Tr}[T^i T^j] = \frac{1}{2}\delta^{ij} \Rightarrow f^{ijk}$ ← fully anti-symmetric ← check!

- Yang-Mills w/ gauge group G, Lie algebra X

  $A_\mu(x) \in X$ ("Lie-algebra-valued one-form")

  $F_{\mu\nu}(x) \in X$ ("Lie-algebra-valued two-form")
- Expand: $A_\mu(x) = A_\mu^i(x) T^i$, $F_{\mu\nu}(x) = F_{\mu\nu}^i(x) T^i$

  So $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \Rightarrow F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu - g f^{ijk} A^j_\mu A^k_\nu$ — check!

Group theory warm-up (2/3)

- $G_{SM} = SU(3) \times SU(2) \times U(1) \ni (g_3, g_2, g_1)$

  $\hookrightarrow (g_3, g_2, g_1)(g_3', g_2', g_1') = (g_3 g_3', g_2 g_2', g_1 g_1')$

- $\mathfrak{X}_{SM} = su(3) \oplus su(2) \oplus u(1)$

- $u(1) \ni B_\mu(x) \leftarrow$ (valued in $\mathbb{R}$)

- $su(2) \ni W_\mu(x) = W_\mu^i(x) T^i$, $T^i = \frac{1}{2}\sigma^i \leftarrow$ Pauli matrices $(i = 1, 2, 3)$, $[T^i, T^j] = i\epsilon^{ijk} T^k \leftarrow$ check!

- $su(3) \ni G_\mu(x) = G_\mu^i(x) T^i$, $T^i = \lambda^i \leftarrow$ Gell-Mann matrices $(i = 1, \ldots, 8)$

Group theory warm-up (2/3)

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  Pauli matrices $(i = 1, 2, 3)$
- $su(3) \ni G_\mu(x) = G_\mu^i(x) T^i$, $T^i = \lambda^i \leftarrow$ Gell-Mann matrices $(i = 1, \dots, 8)$

Group theory warm-up (3/3)

- Pick Rep(G): $g \in G \rightarrow U(g)$
- acts on $\psi \in V$: $\psi \rightarrow \psi' = U(g)\psi$
- If Rep(G) is <u>reducible</u>: $V = V_1 \oplus V_2 \oplus \ldots$ ← irreducible representations or "irreps"

- $SU(N)$ has infinitely many irreps, but SM uses only three:

"trivial

matter fields:
- i) $U(g) = 1$: $\psi \rightarrow U(g)\psi = \psi$ ← "trivial irrep" or "1"
- ii) $U(g) = g$: $\psi \rightarrow U(g)\psi = g\psi$ ← "fundamental irrep" or "N" ← check!

gauge fields:
- iii) $U(g) = g$: $A_\mu \rightarrow U(g) A_\mu U(g)^{-1}$ ← "adjoint irrep" or "N^2-1"

$A_\mu \in su(N)$

- $U(1) \ni e^{i\varphi}$, Rep: $e^{i\varphi} \rightarrow e^{qi\varphi}$ ← check! ← labelled by $U(1)$ charge: "q"

Group theory warm-up (3/3)

- Pick Rep(G): $g \in G \to U(g)$
- acts on $\psi \in V$: $\psi \to \psi' = U(g)\psi$
- If Rep(G) is <u>reducible</u>: $V = V_1 \oplus V_2 \oplus \ldots$ ← irreducible representations or "irreps"

- SU(N) has infinitely many irreps, but SM uses only three:

matter fields:
- i) $U(g) = 1$: $\psi \to U(g)\psi = \psi$ ← "trivial irrep" or "1" ("trivial")
- ii) $U(g) = g$: $\psi \to U(g)\psi = g\psi$ ← "fundamental irrep" or "N" (check!)

gauge fields:
- iii) $U(g) = g$: $A_\mu \to U(g) A_\mu U(g)^{-1}$ ← "adjoint irrep" or "N^2-1"

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- $U(1) \ni e^{i\varphi}$, Rep: $e^{i\varphi} \to e^{qi\varphi}$ ← check! ← labelled by U(1) charge: "q"