Title: Standard Model Lecture - 230109

Speakers: Latham Boyle

Collection: Standard Model (2022/2023)

Date: January 09, 2023 - 9:00 AM

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SM Lecture 1

The Standard Model

(Latham Boyle)

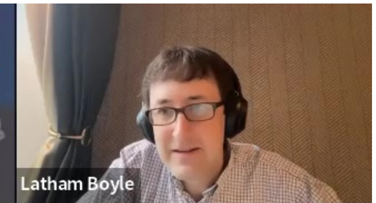
What are:

- the fundamental building blocks of nature?
- the fundamental rules they obey?

The greatest puzzle!

Best current understanding:

- General Relativity (1915)
- Standard Model of Particle Physics (1970s)



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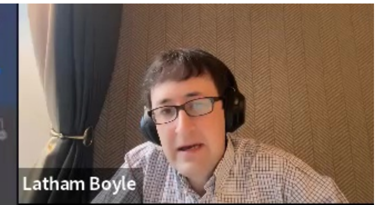


Mathematical Preliminaries:

1) Vector Space V: $v_1 + v_2 \in V$ $\lambda v \in V$

(over field $\mathbb{F}$) $(v_1, v_2 \in V)$ $(\lambda \in \mathbb{F},\ v \in V)$

e.g. $\mathbb{F} = \mathbb{R}, \mathbb{C}$

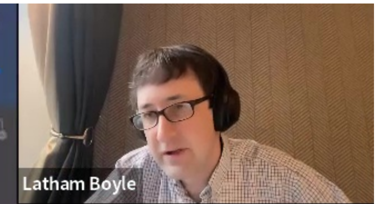
2) Algebra A: vector space with

bilinear product: $A \otimes A \to A$

3) <u>Example 1:</u> $M_n(\mathbb{C})$: $n \times n$ complex matrices

non-commutative but associative

$(AB \neq BA)$ $(AB)C = A(BC)$



Mathematical Preliminaries:

1) Vector Space V: (over field $\mathbb{F}$)

$v_1 + v_2 \in V$ $\quad (v_1, v_2 \in V)$

$\lambda v \in V$ $\quad (\lambda \in \mathbb{F},\ v \in V)$

$\lambda_1 v_1 + \lambda_2 v_2 \in V$

e.g. $\mathbb{F} = \mathbb{R}, \mathbb{C}$

2) Algebra A: vector space with bilinear product: $A \otimes A \to A$

3) Example 1: $M_n(\mathbb{C})$: $n \times n$ complex matrices

non-commutative $(AB \neq BA)$ but associative $(AB)C = A(BC)$

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Mathematical Preliminaries:

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non-commutative but associative

$(AB \neq BA)$ $\quad (AB)C = A(BC)$

non-Abelian

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4) Example 2: $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ (normed division algebras)

$\mathbb{R}$: basis: $\{1\} \Rightarrow r = a_0 1$

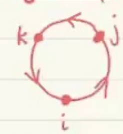
$\mathbb{C}$: adjoin "i" $\Rightarrow$ basis: $\{1\} \cup \{1\} i = \{1, i\} \Rightarrow z = a_0 1 + a_1 i$

new rule: $i^2 = -1$

$\mathbb{H}$: adjoin "j" $\Rightarrow$ basis: $\{1,i\} \cup \{1,i\} j = \{1, i, j, ij \equiv \text{"}k\text{"}\} \Rightarrow q = a_0 1 + a_1 i + a_2 j + a_3 k$

new rules: $j^2 = -1$, $\{i,j\} \equiv ij + ji = 0$, associative $\Rightarrow$

$[a,b,c] \equiv (ab)c - a(bc) = 0$



$\mathbb{O}$: adjoin "ℓ" $\Rightarrow$ basis: $\{1,i,j,k\} \cup \{1,i,j,k\}\ell = \{1, i, j, k, \ell, i\ell, j\ell, k\ell\}$

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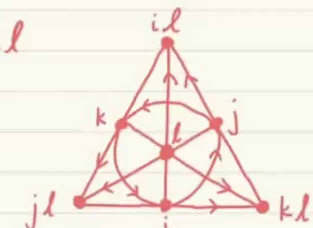
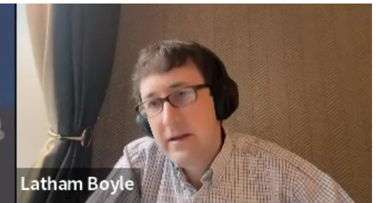


("Fano plane")



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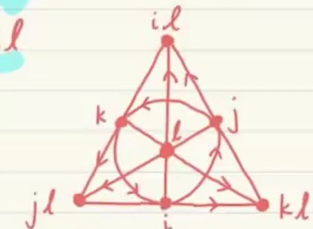

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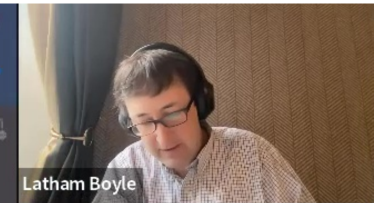

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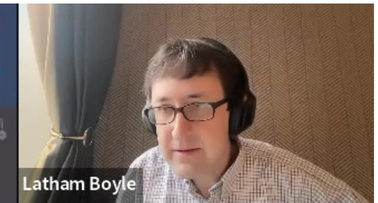

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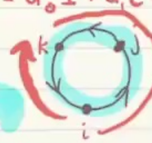

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  $k\ell \cdot i = -j\ell$

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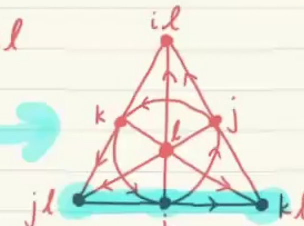

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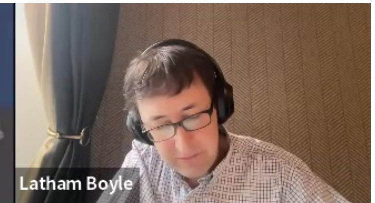

$[a,b]$ (Lie bracket) $\left([a,b]=ab-ba\right)$

5) Lie algebras: $[a,b]=-[b,a]$ (anti symmetric)

$[[a,b],c]+[[b,c],a]+[[c,a],b]=0$ (Jacobi)

Classical simple lie algebras:

$so(n)=\{x\in M_n(\mathbb{R}) \mid x^t=-x,\ tr(x)=0\}$ $\quad n=2,3,4,\ldots$

$su(n)=\{x\in M_n(\mathbb{C}) \mid x^\dagger=-x,\ tr(x)=0\}$ $\quad n=2,3,4,\ldots$

$sp(n)=\{x\in M_n(\mathbb{H}) \mid x^\dagger=-x\}$

5 exceptional Lie algebras:

$g_2, f_4, e_6, e_7, e_8 \leftarrow$ all related to octonions $\mathbb{O}$

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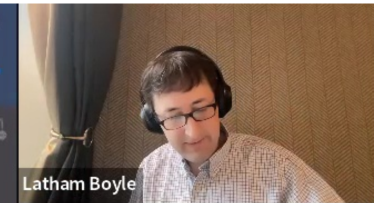

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$g_2, f_4, e_6, e_7, e_8 \leftarrow$ all related to octonions $\mathbb{O}$

$z=a_0+a_1 i$

$q=a_0+a_1 i+a_2 j+a_3 k$

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$[a,b]$ (Lie bracket) $\quad ([a,b] = ab - ba)$

5) Lie algebras: $[a,b] = -[b,a]$ (antisymmetric)

$$[[a,b],c] + [[b,c],a] + [[c,a],b] = 0 \quad \text{(Jacobi)}$$

Classical simple Lie algebras:

$$so(n) = \{x \in M_n(\mathbb{R}) \mid x^t = -x,\ tr(x) = 0\} \qquad n = 2, 3, 4, \dots$$

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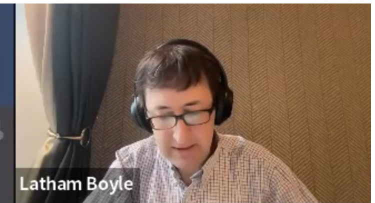
$$sp(n) = \{x \in M_n(\mathbb{H}) \mid x^\dagger = -x\} \qquad \underline{\underline{X}} = i\tilde{x} \quad [\tilde{x}, \tilde{y}] = i\tilde{z}$$

5 exceptional Lie algebras:

$g_2, f_4, e_6, e_7, e_8 \leftarrow$ all related to octonions $\mathbb{O}$

$z = a_0 + a_1 i$

$q = a_0 + a_1 i + a_2 j + a_3 k$



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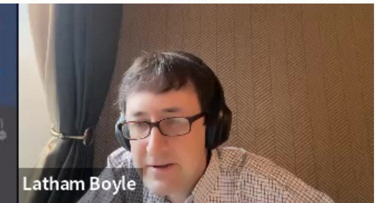


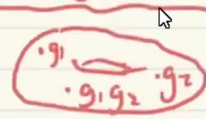
6) Group G:

i) Associative product: $\forall\, g_1, g_2, g_3 \in G$: $g_1 g_2 \in G$, $(g_1 g_2) g_3 = g_1 (g_2 g_3)$

ii) Identity: $\exists$ "1" s.t. $1g = g1 = g$ $(\forall\, g \in G)$

iii) Inverses: $\forall\, g$, $\exists\, g^{-1}$ s.t. $g g^{-1} = g^{-1} g = 1$

Lie group G:



$g = e^{x} \approx 1 + x$

Classical Simple Lie Groups:

$SO(n) = \{ g \in M_n(\mathbb{R}) \mid g^T g = 1,\ \det(g) = 1 \}$

$SU(n) = \{ g \in M_n(\mathbb{C}) \mid g^\dagger g = 1,\ \det(g) = 1 \}$

$Sp(n) = \{ g \in M_n(\mathbb{H}) \mid g^\dagger g = 1 \}$

Exceptional Lie Groups:

G_2, F_4, E_6, E_7, E_8

(related to octonions $\mathbb{O}$)

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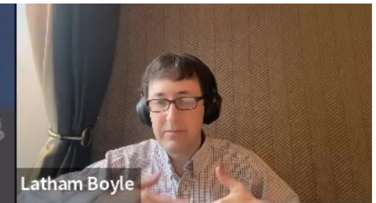

Bonus:

Holonomy(g)	dim(M)	type	
$O(n)$	n	Non-orientable	$\leftarrow \mathbb{R}$
$SO(n)$	n	Orientable	
$U(n)$	$2n$	Kähler	$\leftarrow \mathbb{C}$
$SU(n)$	$2n$	Calabi-Yau	
$Sp(n)\cdot Sp(1)$	$4n$	Quaternionic Kähler	$\leftarrow \mathbb{H}$
$Sp(n)$	$4n$	Hyperkähler	
G_2	7	G_2 Manifold	$\leftarrow \mathbb{O}$
$Spin(7)$	8	$Spin(7)$	

Also: Singularities/catastrophes, Jordan algebras, Finite simple groups (the Monster), Supersymmetric YM theories + classical string theories, Hopf fibrations, ...