Title: Gravitational Physics Lecture - 230130

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Collection: Gravitational Physics (2022/2023)

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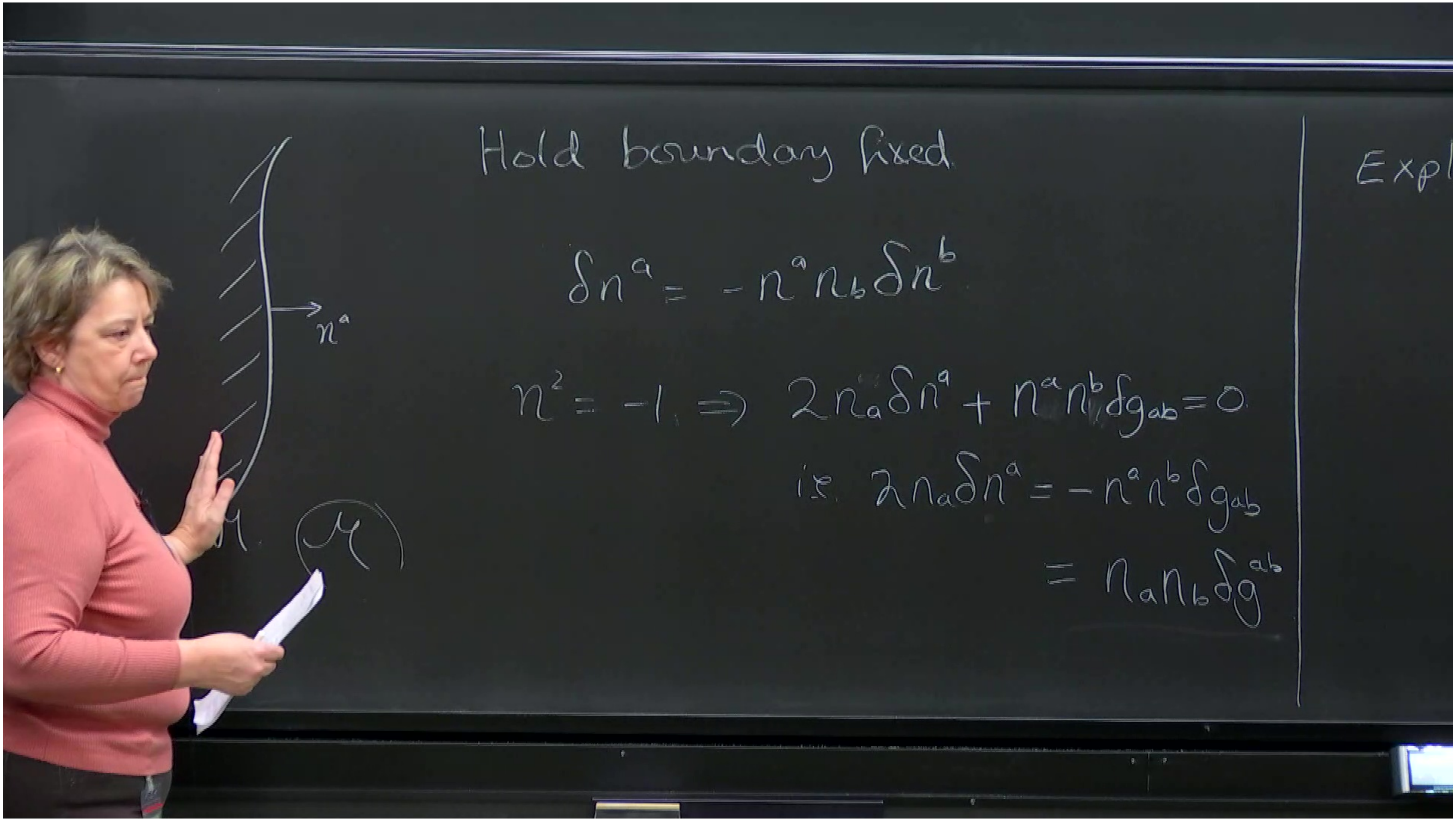


Hold boundary fixed.
$\delta n^a = -n^a n_b \delta n^b$
$n^2 = -1 \Rightarrow 2 n_a \delta n^a + n^a n^b \delta g_{ab} = 0$
i.e. $2 n_a \delta n^a = -n^a n^b \delta g_{ab}$
$= n_a n_b \delta g^{ab}$
n^a
Expl

$\delta g_{ab} = 0.$

$n^a n^b \delta g_{ab}$

$n_a n_b \delta g^{ab}$

Explore boundary term:

have K.

$$\delta K = \delta(\nabla_a n^a)$$

$$= \nabla_a \delta n^a + \delta \Gamma^a_{ab} n^b$$

$$= -\nabla_a(n^a \underline{n_b \delta n^b}) + \frac{1}{2} n^b g^{cd}(\nabla_b \delta g_{cd})$$

$$= -\frac{1}{2}\nabla_a(n^a n_b n_c \delta g^{bc}) - \frac{1}{2} n^b g_{cd} \nabla_b \delta g^{cd}$$

$$-\frac{1}{2} K n_b n_c \delta g$$

ary term:

$(\nabla_a n^a)$

$\nabla_a \delta n^a + \delta\Gamma^a_{ab} n^b$

$\nabla_a(n^a \underline{n_b \delta n^b}) + \frac{1}{2} n^b g^{cd}(\nabla_b \delta g_{cd})$

$\nabla_a(n^a n_b n_c \delta g^{bc}) - \frac{1}{2} n^b g_{cd} \nabla_b \delta g^{cd}$

$- \frac{1}{2} K n_b n_c \delta g^{bc} - \underbrace{(\nabla_n n_b) n_c \delta g^{bc}}_{\text{can set to zero}}$

$- \frac{1}{2} n_b n_c \nabla_n \delta g^{bc} - \frac{1}{2} g_{cd} \nabla_n \delta g^{cd}$

can choose how to extend n off the boundary

ary term:

$(\nabla_a n^a)$

$\nabla_a \delta n^a + \delta\Gamma^a{}_{ab} n^b$.

$-\nabla_a(n^a \underline{n_b \delta n^b}) + \frac{1}{2} n^b g^{cd} (\nabla_b \delta g_{cd})$

$-\frac{1}{2}\nabla_a(n^a n_b n_c \delta g^{bc}) - \frac{1}{2} n^b g_{cd} \nabla_b \delta g^{cd}$

$-\frac{1}{2} K\, n_b n_c \delta g^{bc} - \underbrace{(\nabla_n n_b) n_c \delta g^{bc}}_{\text{can set to zero}}$

$-\frac{1}{2} n_b n_c \nabla_n \delta g^{bc} - \frac{1}{2} g_{cd} \nabla_n \delta g^{cd}$

[Can choose how to extend n off the boundary, so $\nabla_n n \to 0$]

$= -\frac{1}{2} K\, n_b n_c \delta g^{bc}$

$-\frac{1}{2} n_b (h_c{}^a - \delta^a_c) \nabla_a \delta g^{bc}$

$-\frac{1}{2} \nabla_n (\delta g^{-1})^c{}_c$

$$= -\frac{1}{2} K \, n_b n_c \delta g^{bc} - \frac{1}{2} n_b h_c{}^a \nabla_a \delta g^{bc}$$

$$\boxed{+\frac{1}{2} n_b \nabla_a \delta g^{ba} - \frac{1}{2} \nabla_n (\delta g^{-1})^c_c}$$

$$= \boxed{\tfrac{1}{2} \text{BT.}} - \frac{1}{2} K \, n_b n_c \delta g^{bc}$$

$$- \frac{1}{2} \underbrace{h_c{}^a \nabla_a (n_b \delta g^{bc})}_{D_c(n_b \delta g^{bc})} + \frac{1}{2} \delta g^{bc} h_c{}^a (\nabla_a n_b)$$

The total der

$h_c{}^a \nabla_a \delta g^{bc}$

$\delta g^{-1})^c_c$

$\delta g^{bc} h_c{}^a (\nabla_a n_b)$

The total derivative integrates to zero as ∂M has no boundary.

$$\delta \int K\sqrt{g}\, d^3x = \int \tfrac{1}{2}\left[n_b \nabla_a \delta g^{ab} - \nabla_n (\delta g^{-1})^c_c\right]$$

Can set to zero in Lagrange variation.

$$+ \tfrac{1}{2} K_{ab} \delta g^{ab}$$

$$- \tfrac{1}{2} K (n_b n_c + g_{bc}) \delta g^{bc}$$

from $\delta\sqrt{g}$.

...ue integrates
...has no boundary

$$\int \frac{1}{2}\left[n_b \nabla_a \delta g^{ab} - D_n(\delta g^{-1})^{K}_{c}\right]$$

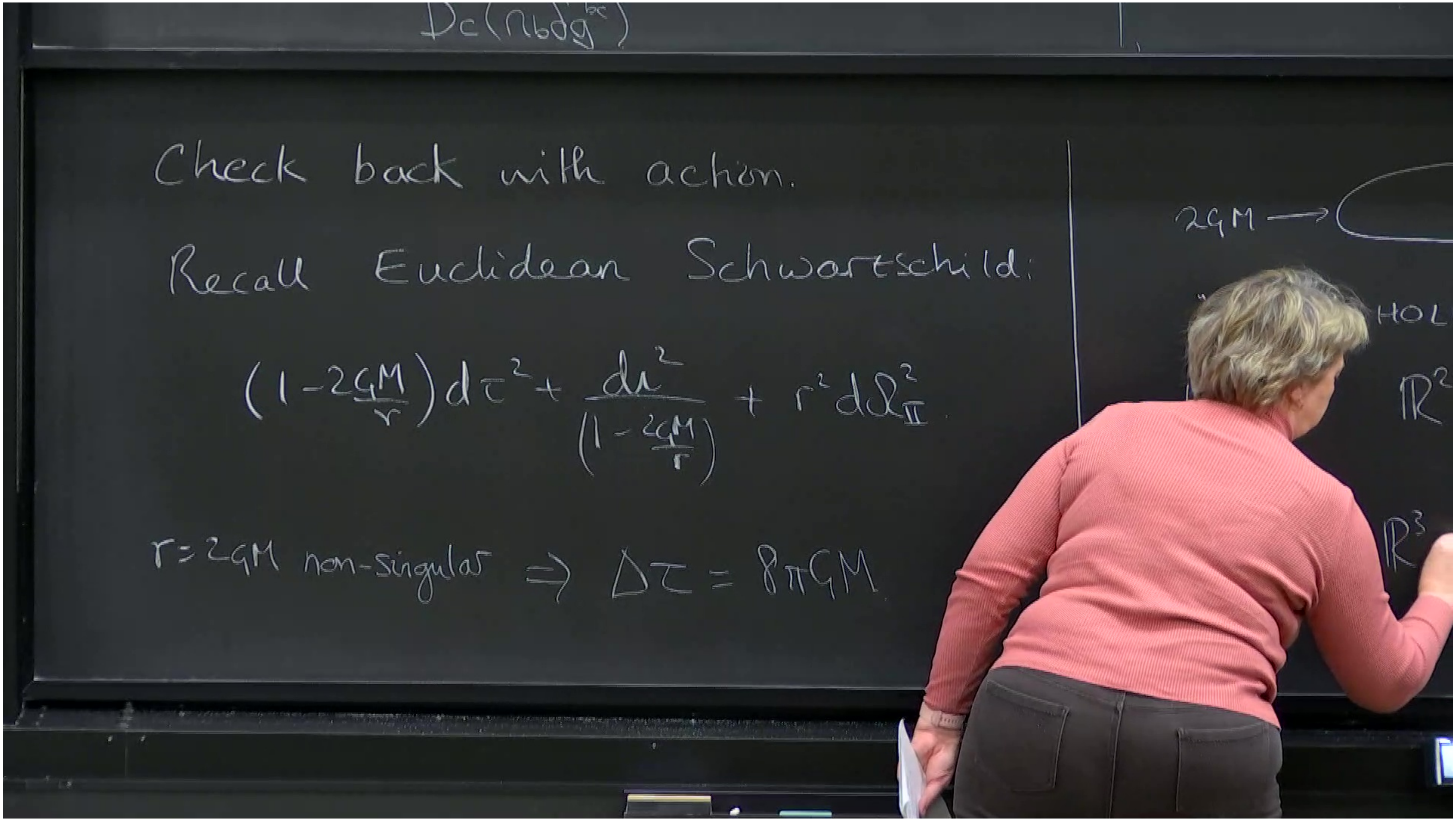
$$\left\{ \begin{array}{l} + \frac{1}{2} K_{ab}\, \delta g^{ab} \\ - \frac{1}{2} K\,(n_b n_c + g_{bc})\,\delta g^{bc} \end{array} \right.$$

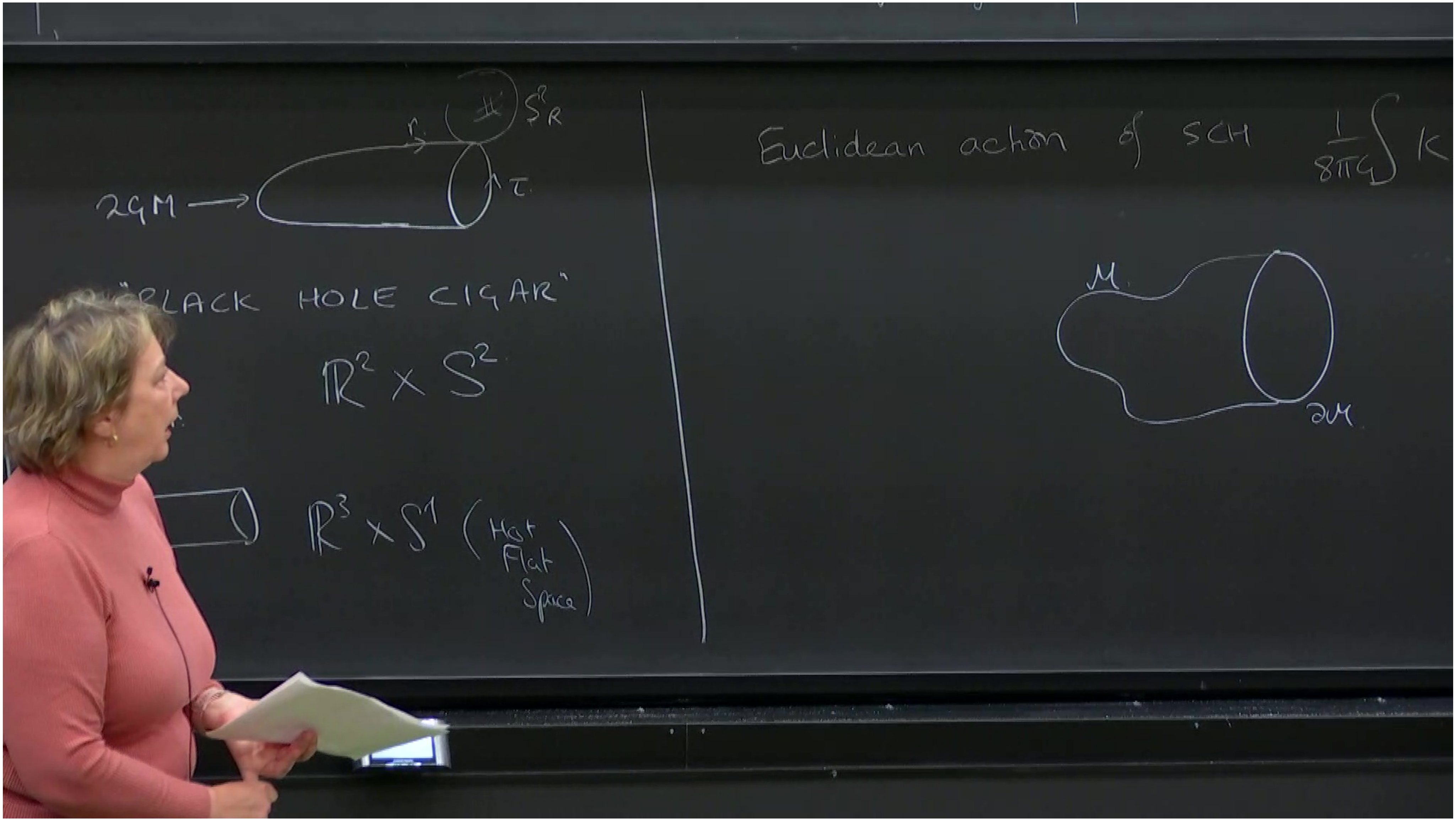
...ange

from $\delta\sqrt{g}$

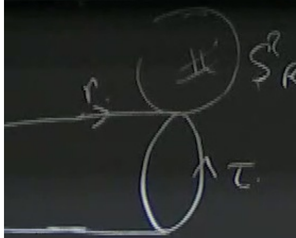
Therefore our full gravitational action is

$$-\frac{1}{16\pi G}\int_{M} R\sqrt{g}\, d^4x \; \underbrace{- \frac{1}{8\pi G}\int_{\partial M} K\sqrt{g}\, d^3x}_{\text{GIBBONS-HAWKING}}$$


Check back with action.
Recall Euclidean Schwartschild:
$(1-\frac{2GM}{r})d\tau^2 + \frac{dr^2}{(1-\frac{2GM}{r})} + r^2 d\Omega_{II}^2$
$r = 2GM$ non-singular $\Rightarrow \Delta\tau = 8\pi GM$
$2GM \longrightarrow$
$\mathbb{R}^2$
$\mathbb{R}^3$



S^2_R
2GM
"BLACK HOLE CIGAR"
$\mathbb{R}^2 \times S^2$
$\mathbb{R}^3 \times S^1$ (Hot Flat Space)
Euclidean action of SCH $\frac{1}{8\pi G}\int K$
M
∂M

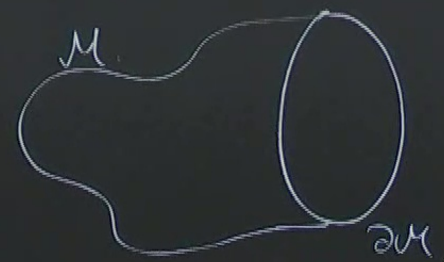


LE CIGAR"

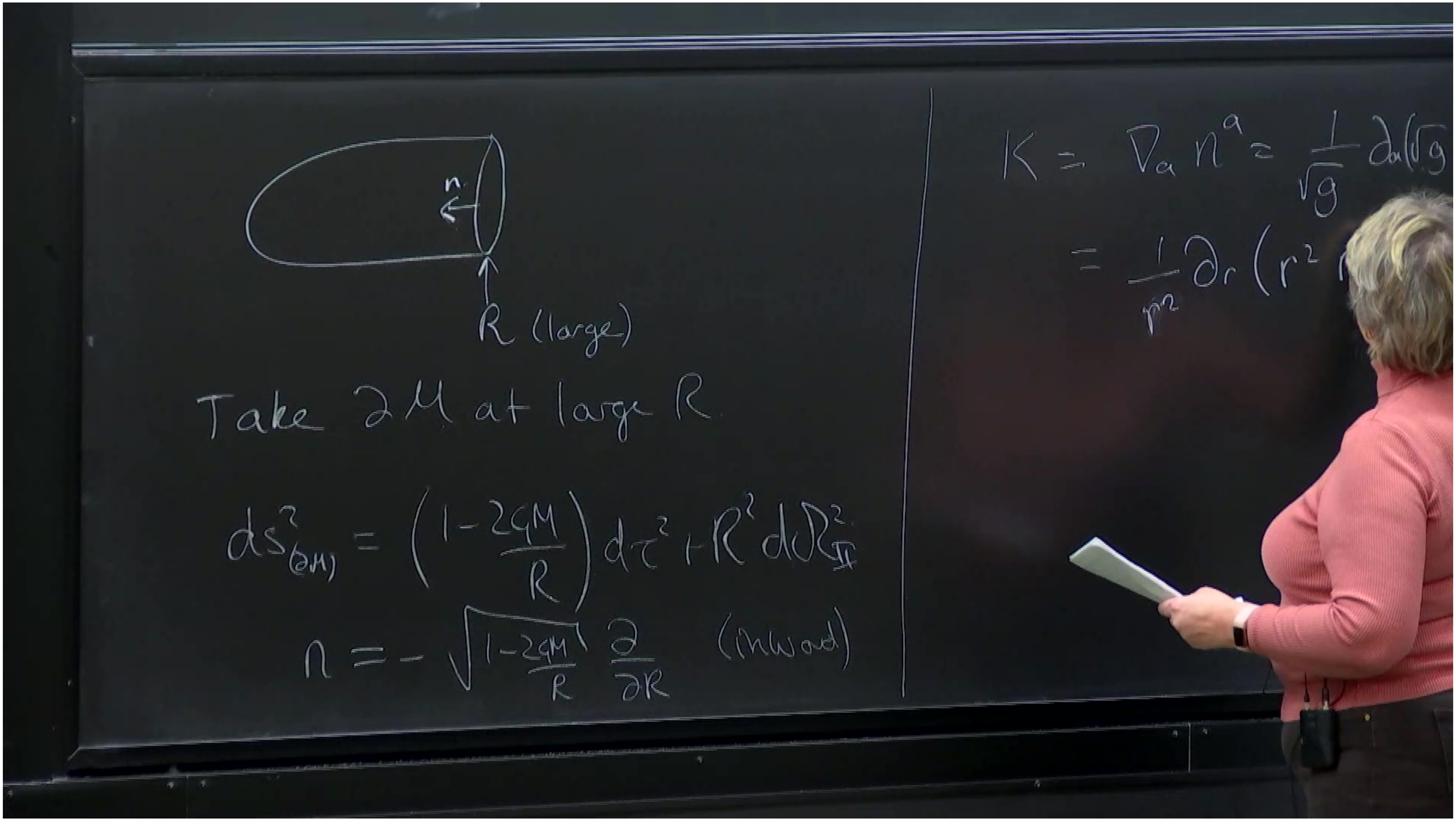
$^2 \times S^2$

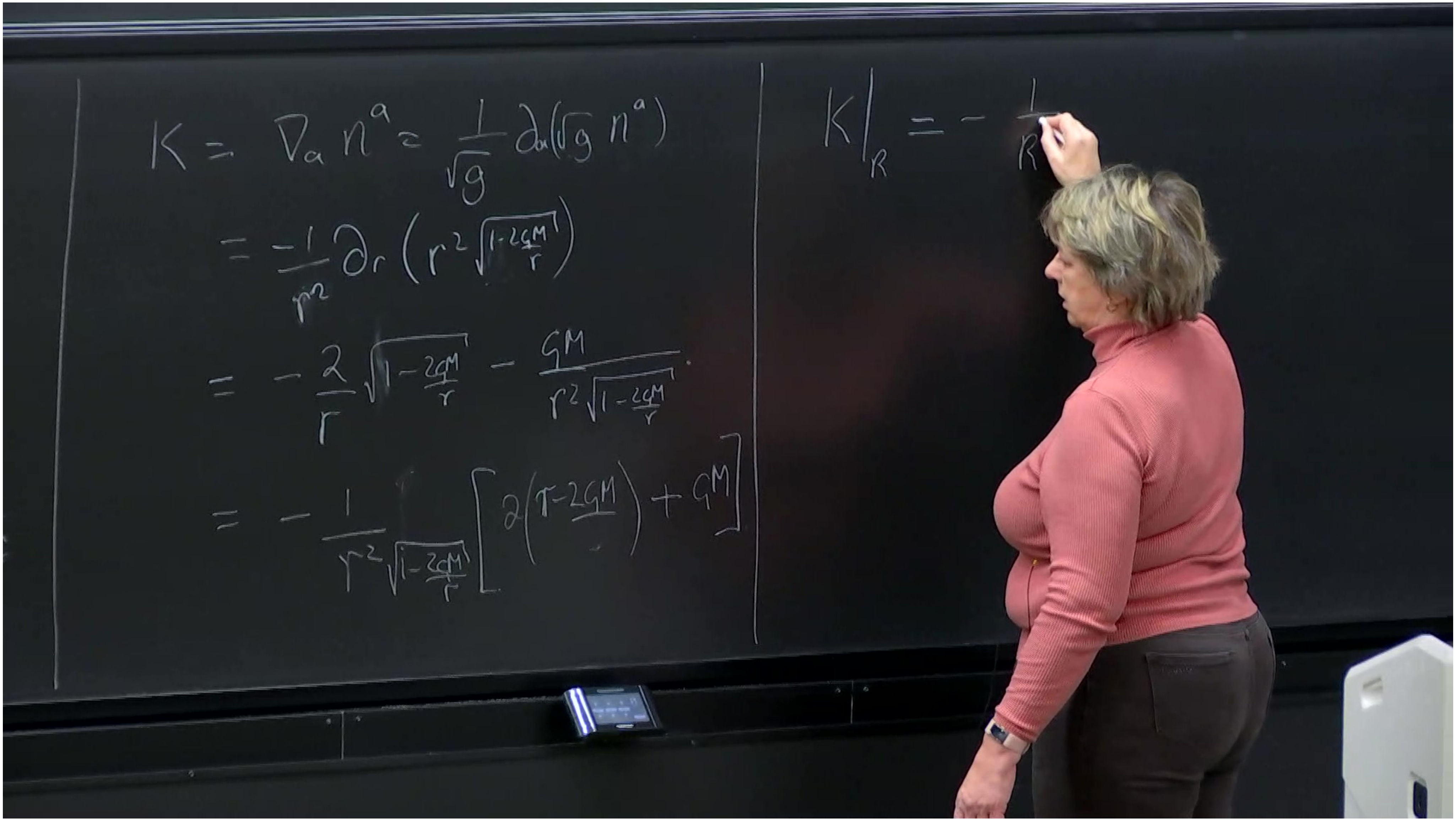
$\times S^1$ (Hot Flat Space)

Euclidean action of SCH $\frac{1}{8\pi G}\int K\sqrt{h}\, d^3x$

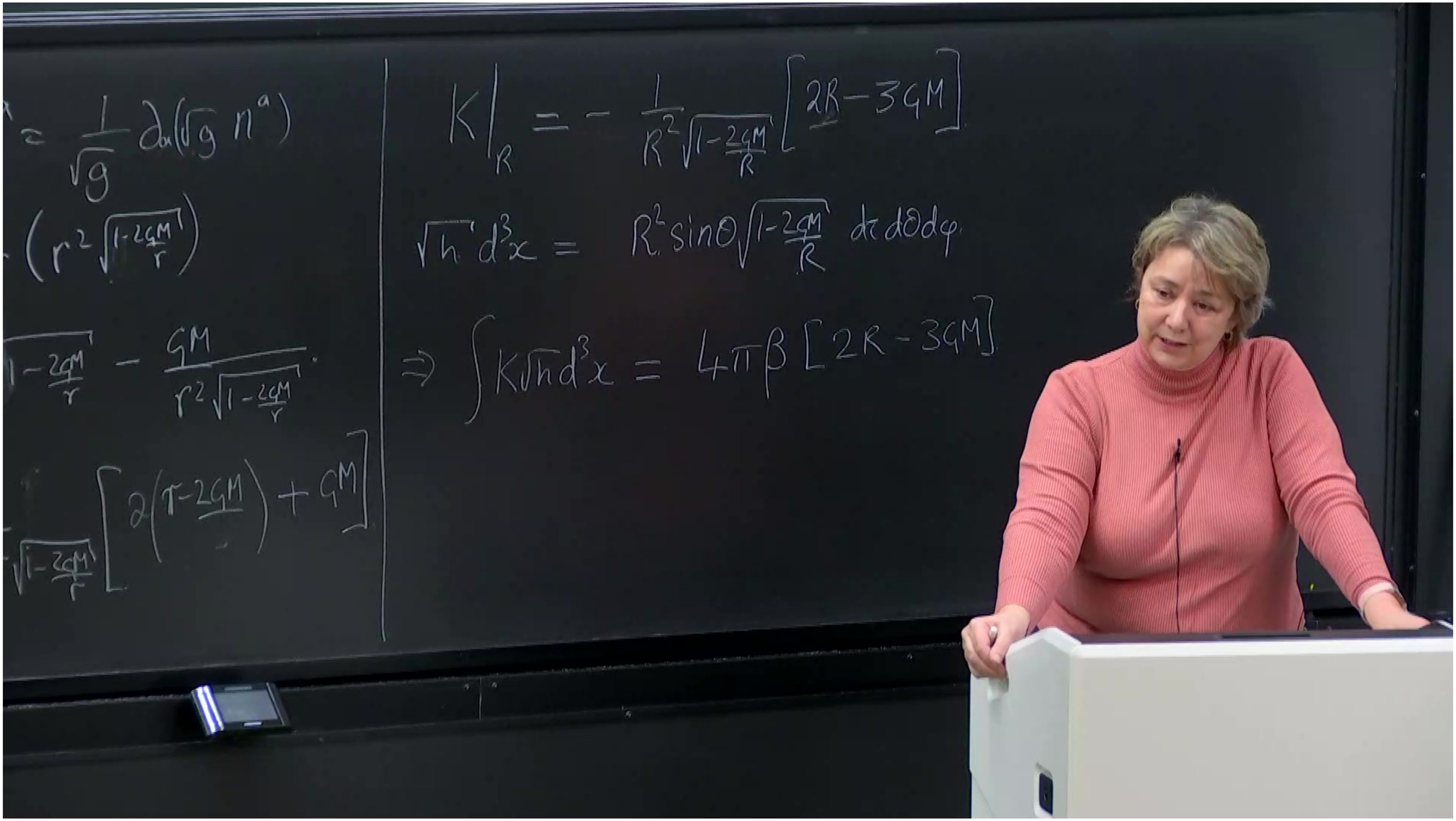


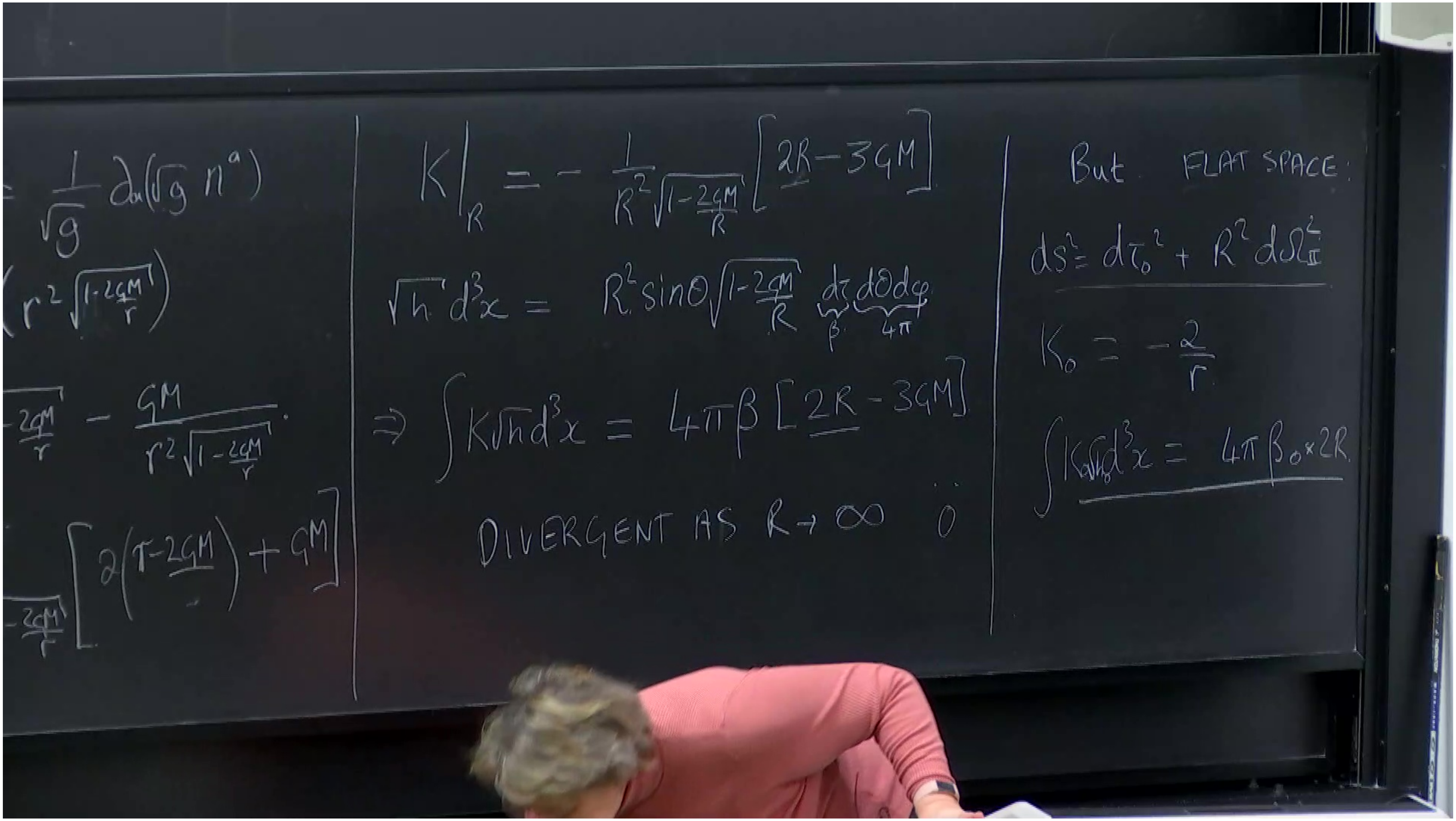
$$\int [dg]\, e^{-S_E}$$

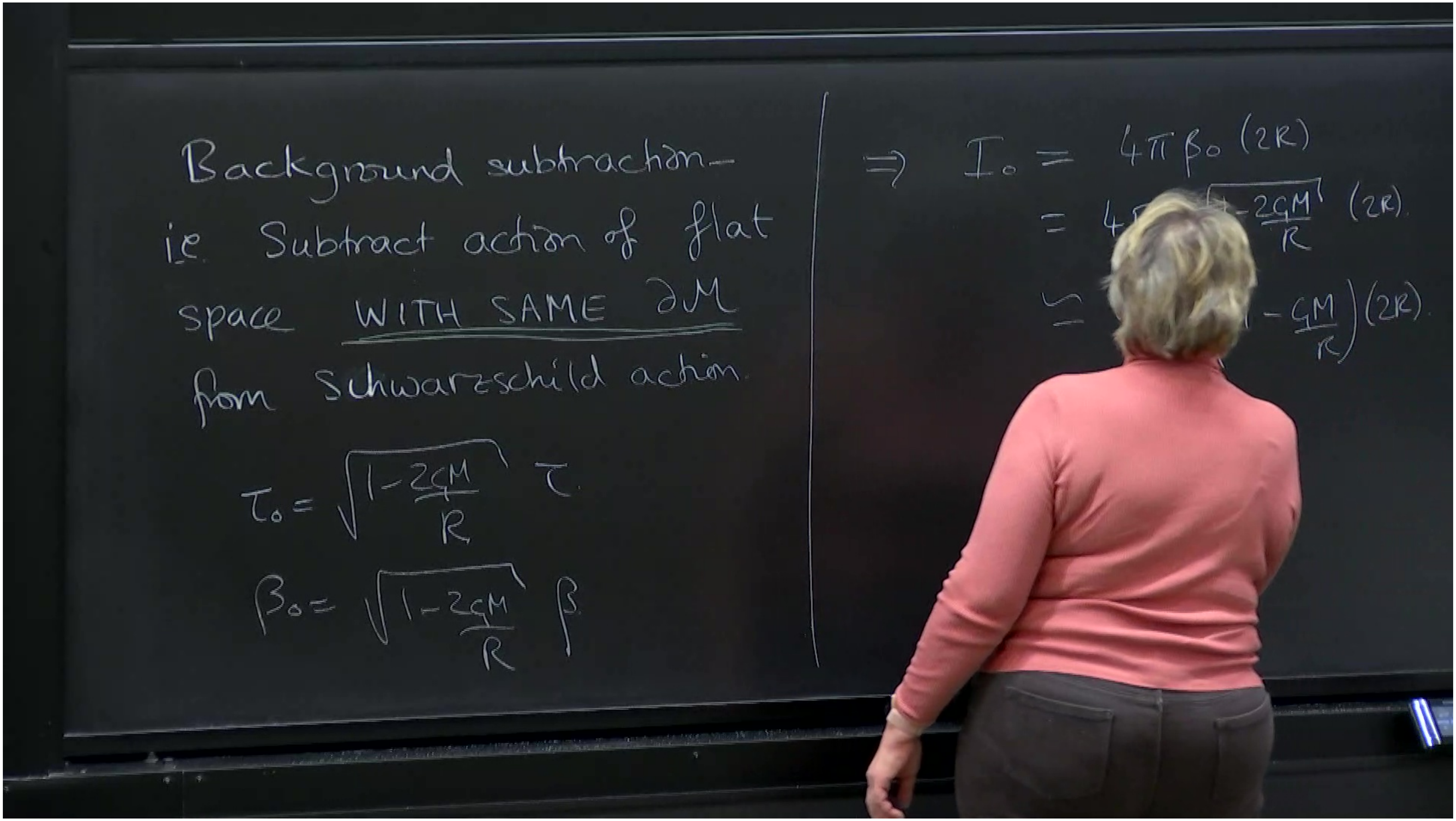

n
R (large)
Take ∂M at large R
$ds^2_{(\partial M)} = \left(1 - \frac{2GM}{R}\right) d\tau^2 + R^2 d\Omega^2_{II}$
$n = -\sqrt{1 - \frac{2GM}{R}} \frac{\partial}{\partial R}$ (inward)
$K = \nabla_a n^a = \frac{1}{\sqrt{g}} \partial_a (\sqrt{g}$
$= \frac{1}{r^2} \partial_r (r^2$



$K = \nabla_a n^a = \frac{1}{\sqrt{g}} \partial_a(\sqrt{g}\, n^a)$
$= -\frac{1}{r^2} \partial_r \left(r^2 \sqrt{1-\frac{2GM}{r}} \right)$
$= -\frac{2}{r}\sqrt{1-\frac{2GM}{r}} - \frac{GM}{r^2\sqrt{1-\frac{2GM}{r}}}$
$= -\frac{1}{r^2\sqrt{1-\frac{2GM}{r}}}\left[2\left(r-2GM\right)+GM\right]$
$K\Big|_R = -\frac{1}{R}$


$K\big|_R = -\frac{1}{R^2\sqrt{1-\frac{2GM}{R}}}\left[2R - 3GM\right]$
$\sqrt{h}\, d^3x = R^2 \sin\theta \sqrt{1-\frac{2GM}{R}}\, d\tau\, d\theta\, d\varphi$
$\Rightarrow \int K\sqrt{h}\, d^3x = 4\pi\beta\left[2R - 3GM\right]$


$K\big|_R = -\frac{1}{R^2\sqrt{1-\frac{2GM}{R}}}\left[2R - 3GM\right]$
$\sqrt{h}\,d^3x = R^2 \sin\theta \sqrt{1-\frac{2GM}{R}}\,\underbrace{d\tau}_{\beta}\,\underbrace{d\theta\, d\varphi}_{4\pi}$
$\Rightarrow \int K\sqrt{h}\,d^3x = 4\pi\beta\left[2R - 3GM\right]$
DIVERGENT AS $R \to \infty$
But FLAT SPACE:
$ds^2 = d\tau_0^2 + R^2 d\Omega_{II}^2$
$K_0 = -\frac{2}{r}$
$\int K_0\sqrt{h_0}\,d^3x = 4\pi\beta_0 \times 2R$


Background subtraction -
i.e. Subtract action of flat
space WITH SAME ∂M
from Schwarzschild action.
τ0 = √(1 - 2GM/R) τ
β0 = √(1 - 2GM/R) β
⇒ I0 = 4π β0 (2R)
= 4π … - 2GM/R (2R)
≃ … - GM/R) (2R)

Background subtraction –

i.e. Subtract action of flat space <u>WITH SAME ∂M</u> from Schwarzschild action.

$$\tau_0 = \sqrt{1 - \frac{2GM}{R}}\ \tau$$

$$\beta_0 = \sqrt{1 - \frac{2GM}{R}}\ \beta$$

$$\Rightarrow \quad I_0 = 4\pi \beta_0 (2R) \,/\, 8\pi G$$

$$= 4\pi \beta \sqrt{1 - \frac{2GM}{R}}\ (2R)/8$$

$$\simeq 4\pi\beta \left(1 - \frac{GM}{R}\right)(2R)/8$$

$$= 4\pi\beta(2R - 2GM)/8$$

$$I_{ren} = I_{sch} - I_0 = \frac{4\pi\beta}{8\pi G}(2R-$$

n—
flat
M
tion.

$$\Rightarrow \quad I_0 = -4\pi\beta_0 (2R) \,/\, 8\pi G$$

$$= -4\pi\beta \sqrt{1-\frac{2GM}{R}}\,(2R)/8\pi G$$

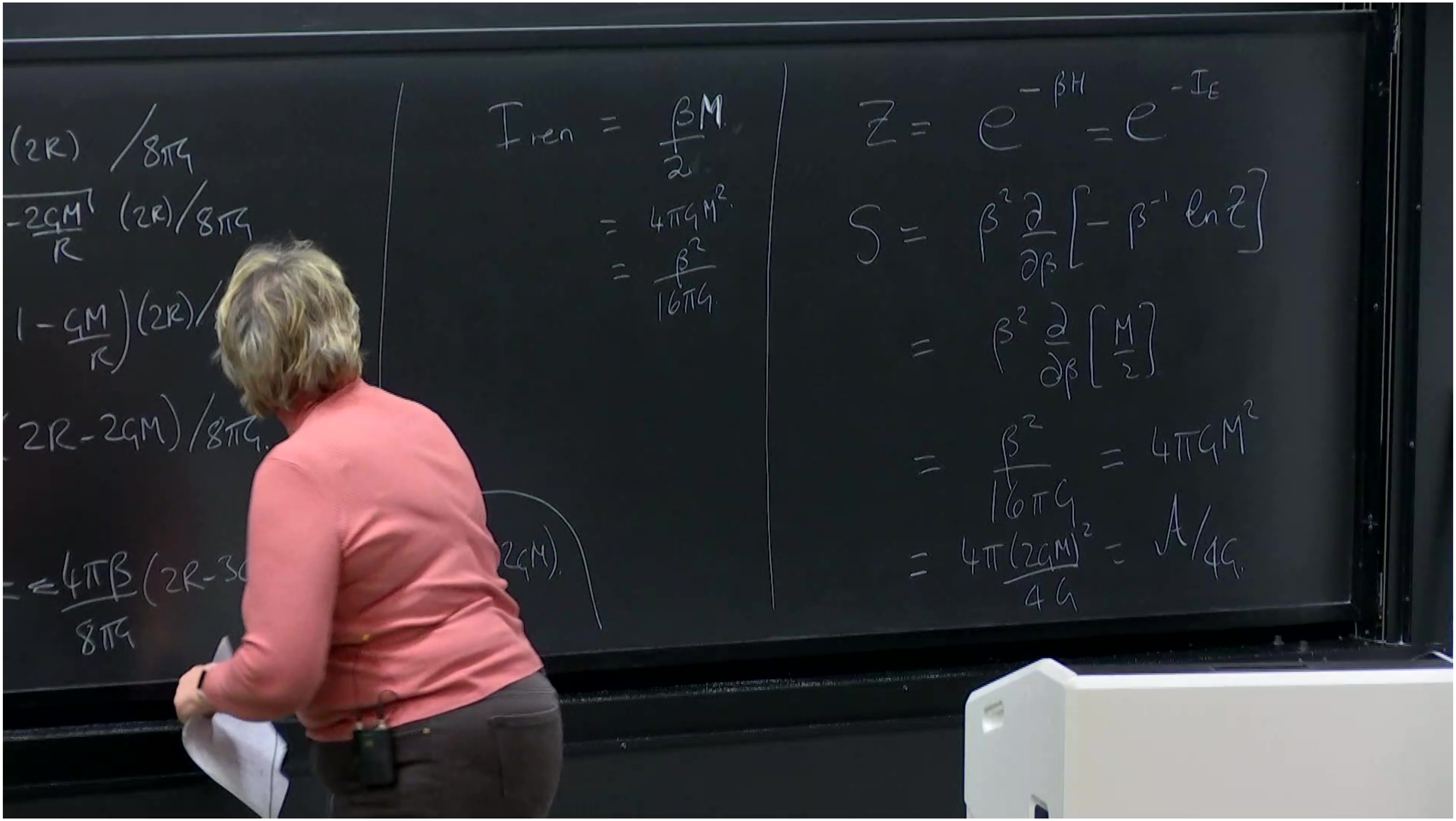
$$\simeq -4\pi\beta\left(1-\frac{GM}{R}\right)(2R)/8\pi G$$

$$= -4\pi\beta\,(2R-2GM)/8\pi G.$$

$$I_{ren} = I_{Sch} - I_0 = -\frac{4\pi\beta}{8\pi G}(2R-3GM) + \frac{4\pi\beta}{8\pi G}(2R-2GM)$$

$$I_{ren} = \frac{\beta M}{2}$$

$$= 4\pi$$


I_ren = βM/2
= 4πGM²
= β²/16πG
Z = e^(-βH) = e^(-I_E)
S = β² ∂/∂β [-β⁻¹ ln Z]
= β² ∂/∂β [M/2]
= β²/16πG = 4πGM²
= 4π(2GM)²/4G = A/4G

Think of SCH.

$$A = 4\pi r_+^2$$

$$\delta A = 8\pi r_+ \delta r_+$$

$$= 8\pi (2GM)\, 2G\, \delta M$$

$$\delta M = \underbrace{\frac{1}{8\pi G M}}_{T} \underbrace{\frac{\delta A}{4G}}_{\delta S}$$