

Title: Numerical Methods Lecture - 230120

Speakers: Erik Schnetter

Collection: Numerical Methods (2022/2023)

Date: January 20, 2023 - 9:00 AM

URL: <https://pirsa.org/23010011>

The screenshot displays the JupyterLab environment with a Julia 1.8.5 notebook. The notebook content is as follows:

```
[54]: # expansion coefficients
      pmax = 4

[54]: 4

[55]: # evaluation points
      npoints = 11
      x = LinRange(0, pi/2, npoints-1)

[55]: 10-element LinRange{Float64, Int64}:
      0.0, 0.174533, 0.349066, 0.523599, 0.698132, ..., 1.0472, 1.22173, 1.39626, 1.5708

[56]: # target function
      fstar(x) = sin(x)

[56]: fstar (generic function with 1 method)

[57]: # matrix
      A = [x[i]^p for i in 1:npoints, p in 0:pmax]

BoundsError: attempt to access 10-element LinRange{Float64, Int64} at index [11]

Stacktrace:
 [1] throw_boundserror(A::LinRange{Float64, Int64}, I::Tuple{Int64})
    @ Base ./abstractarray.jl:703
 [2] checkbounds
    @ ./abstractarray.jl:668 [inlined]
 [3] getindex(r::LinRange{Float64, Int64}, i::Int64)
    @ Base ./range.jl:932
 [4] (::var"#1#2")::Tuple{Int64, Int64}
    @ Main ./none:0
 [5] iterate
    @ ./generator.jl:47 [inlined]
 [6] collect_to!(dest::Matrix{Float64}, itr::Base.Generator{Base.Iterators.ProductIterator{Tuple{UnitRange{Int64}, UnitRange{Int64}}}, var"#1#2"}, offs::Int64, st::Tuple{Tuple{Int64, Int64}, Tuple{Int64, Int64}})
    @ Base ./array.jl:845
 [7] collect_to_with_first!(dest::Matrix{Float64}, v1::Float64, itr::Base.Generator{Base.Iterators.ProductIterator{Tuple{UnitRange{Int64}, UnitRange{Int64}}}, var"#1#2"}, st::Tuple{Tuple{Int64, Int64}, Tuple{Int64, Int64}})
    @ Base ./array.jl:823
 [8] collect(itr::Base.Generator{Base.Iterators.ProductIterator{Tuple{UnitRange{Int64}, UnitRange{Int64}}}, var"#1#2"})
    @ Base ./array.jl:797
 [9] top-level scope
    @ In[57]:2

[58]: c = (A'*A) \ (A'*b)

UndefVarError: b not defined
```

The left sidebar shows a file explorer with various files, and the bottom status bar indicates the current mode is 'Simple' and the memory usage is 572.92 MB.

symmetry.pl.local

JupyterLab

Mem:587 MB

File Edit View Run Kernel Git Tabs Settings Help

Launcher Linear Algebra 2023.ipynb

Julia 1.8.5

```

npoints = 11
x = LinRange(0, π/2, npoints)

[65]: 11-element LinRange{Float64, Int64}:
      0.0, 0.15708, 0.314159, 0.471239, 0.628319, ..., 1.09956, 1.25664, 1.41372, 1.5708

[66]: # target function
      fstar(x) = sin(x)

[66]: fstar (generic function with 1 method)

[67]: # matrix
      A = [x[i]^p for i in 1:npoints, p in 0:pmax]

[67]: 11×5 Matrix{Float64}:
      1.0  0.0  0.0  0.0  0.0
      1.0  0.15708  0.024674  0.00387578  0.000608807
      1.0  0.314159  0.098696  0.0310063  0.00974091
      1.0  0.471239  0.222066  0.104646  0.0493134
      1.0  0.628319  0.394784  0.24805  0.155855
      1.0  0.785398  0.61685  0.484473  0.380504
      1.0  0.942478  0.888264  0.837169  0.789014
      1.0  1.09956  1.25664  1.41372  1.5708

```

Simple 0 \$ 1 Julia 1.8.5 | ... Mem: 586.59 ... Saving complet... Mode: Comma... Ln 1, Co... Linear Algebra 2023.ipyn...

symmetry.pl.local

JupyterLab

Mem:609 MB

Launcher Linear Algebra 2023.ipynb

Code git Julia 1.8.5

```
[67]: # matrix
      A = [x[i]^p for i in 1:npoints, p in 0:pmax]
```

```
[67]: 11x5 Matrix{Float64}:
      1.0  0.0      0.0      0.0      0.0
      1.0  0.15708  0.024674  0.00387578  0.000608807
      1.0  0.314159  0.098696  0.0310063  0.00974091
      1.0  0.471239  0.222066  0.104646  0.0493134
      1.0  0.628319  0.394784  0.24805   0.155855
      1.0  0.785398  0.61685   0.484473  0.380504
      1.0  0.942478  0.888264  0.837169  0.789014
      1.0  1.09956   1.20903   1.32939   1.46175
      1.0  1.25664   1.57914   1.9844    2.49367
      1.0  1.41372   1.99859   2.82545   3.99438
      1.0  1.5708    2.4674    3.87578   6.08807
```

```
[58]: c = (A'*A) \ (A'*b)
```

UndefVarError: b not defined

Stacktrace:

```
[1] top-level scope
      @ In[58]:1
```

0 1 Julia 1.8.5 | ... Mem: 609.01 ... Saving complet... Mode: Comma... Ln 1, Co... Linear Algebra 2023.ipyn...

symmetry.pl.local

JupyterLab

Mem:634 MB

File Edit View Run Kernel Git Tabs Settings Help

Launcher Linear Algebra 2023.ipynb

Julia 1.8.5

```
[67]: # matrix
A = [x[i]^p for i in 1:npoints, p in 0:pmax]
```

```
[67]: 11x5 Matrix{Float64}:
 1.0  0.0      0.0      0.0      0.0
 1.0  0.15708  0.024674  0.00387578  0.000608807
 1.0  0.314159  0.098696  0.0310063  0.00974091
 1.0  0.471239  0.222066  0.104646  0.0493134
 1.0  0.628319  0.394784  0.24805   0.155855
 1.0  0.785398  0.61685   0.484473  0.380504
 1.0  0.942478  0.888264  0.837169  0.789014
 1.0  1.09956   1.20903   1.32939   1.46175
 1.0  1.25664   1.57914   1.9844    2.49367
 1.0  1.41372   1.99859   2.82545   3.99438
 1.0  1.5708    2.4674    3.87578   6.08807
```

```
[68]: b = fstar.(x)
```

```
[68]: 11-element Vector{Float64}:
 0.0
 0.15643446504023087
 0.3090169943749474
 0.45300040072054675
```

Simple 0 1 Julia 1.8.5 | ... Mem: 633.83 ... Saving complet... Mode: Comma... Ln 1, Co... Linear Algebra 2023.ipy...

symmetry.pl.local

JupyterLab

Mem: 822 MB

Launcher

Linear Algebra 2023.ipynb

Code

Julia 1.8.5

```
0.13813718337625007
0.3090169943749474
0.45399049973954675
0.5877852522924731
0.7071067811865475
0.8090169943749475
0.8910065241883678
0.9510565162951535
0.9876883405951378
1.0
```

[69]: $c = (A^*A) \setminus (A^*b)$

[69]: 5-element Vector{Float64}:
6.82343656696662e-5
0.9964314723758387
0.01935463070333306
-0.2030442461006927
0.028560370414334037

[]:

Julia 1.8.5 | I... Mem: 822.37 ... Saving complet... Mode: Comma... Ln 1, Co... Linear Algebra 2023.ipyn...

symmetry.pl.local

JupyterLab

Mem:1.09 GB

File Edit View Run Kernel Git Tabs Settings Help

Launcher Linear Algebra 2023.ipynb

Julia 1.8.5

```
-0.2030442461006927
0.028560370414334037

[70]: f(c, x) = sum(c[p] * x^p for p in 0:pmax)

[70]: f (generic function with 1 method)

[71]: using WGLMakie

[*]: fig = Figure()
      Axis(fig[1,1])
      xx = LinRange(0, π/2, 201)
      plot!(xx, f(c, xx))
      fig

[ ]:

[ ]:

[ ]:
```

Simple 0 \$ 1 Julia 1.8.5 | Bu... Mem: 1.09 ... Saving complet... Mode: Comma... Ln 1, Co... Linear Algebra 2023.ipy...

symmetry.pl.local

JupyterLab

Mem:1018 MB

File Edit View Run Kernel Git Tabs Settings Help

Launcher Linear Algebra 2023.ipynb

Julia 1.8.5

```
-0.2030442461006927
0.028560370414334037

[77]: f(c, x) = sum(c[p+1] * x^p for p in 0:pmax)

[77]: f (generic function with 1 method)

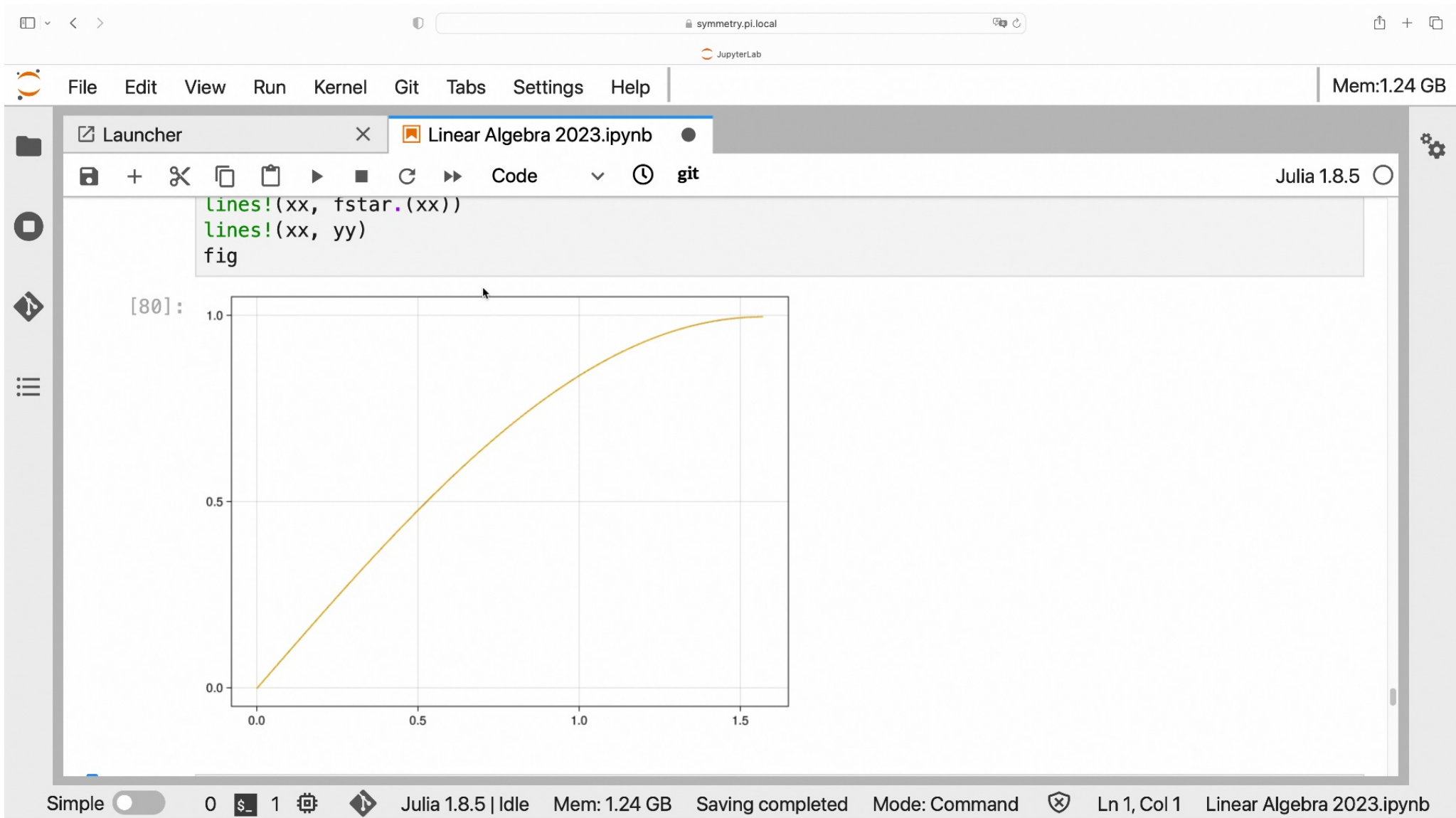
[71]: using WGLMakie

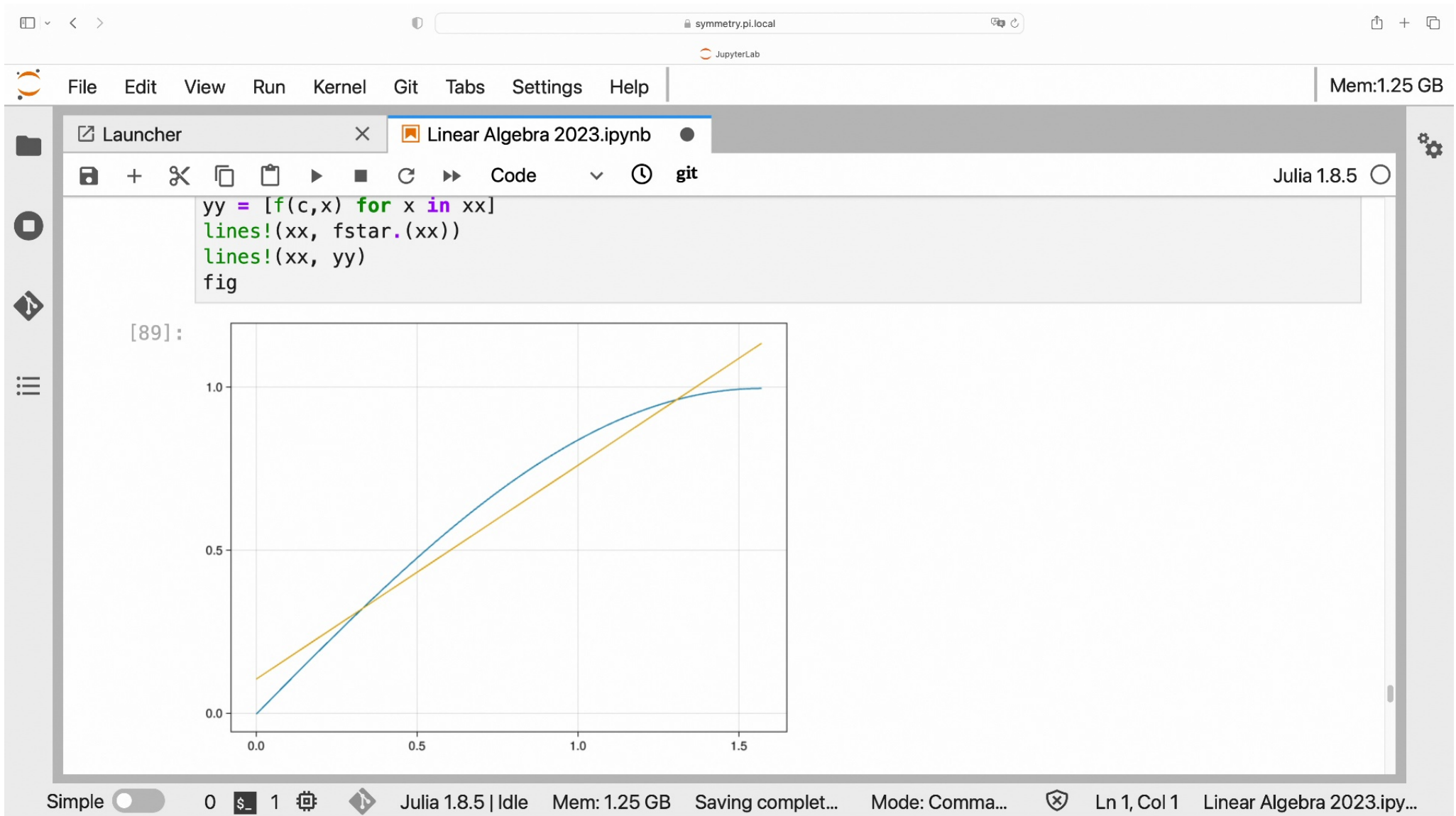
[75]: fig = Figure()
      Axis(fig[1,1])
      xx = LinRange(0, π/2, 201)
      yy = [f(c,x) for x in xx]
      plot!(xx, yy)
      fig

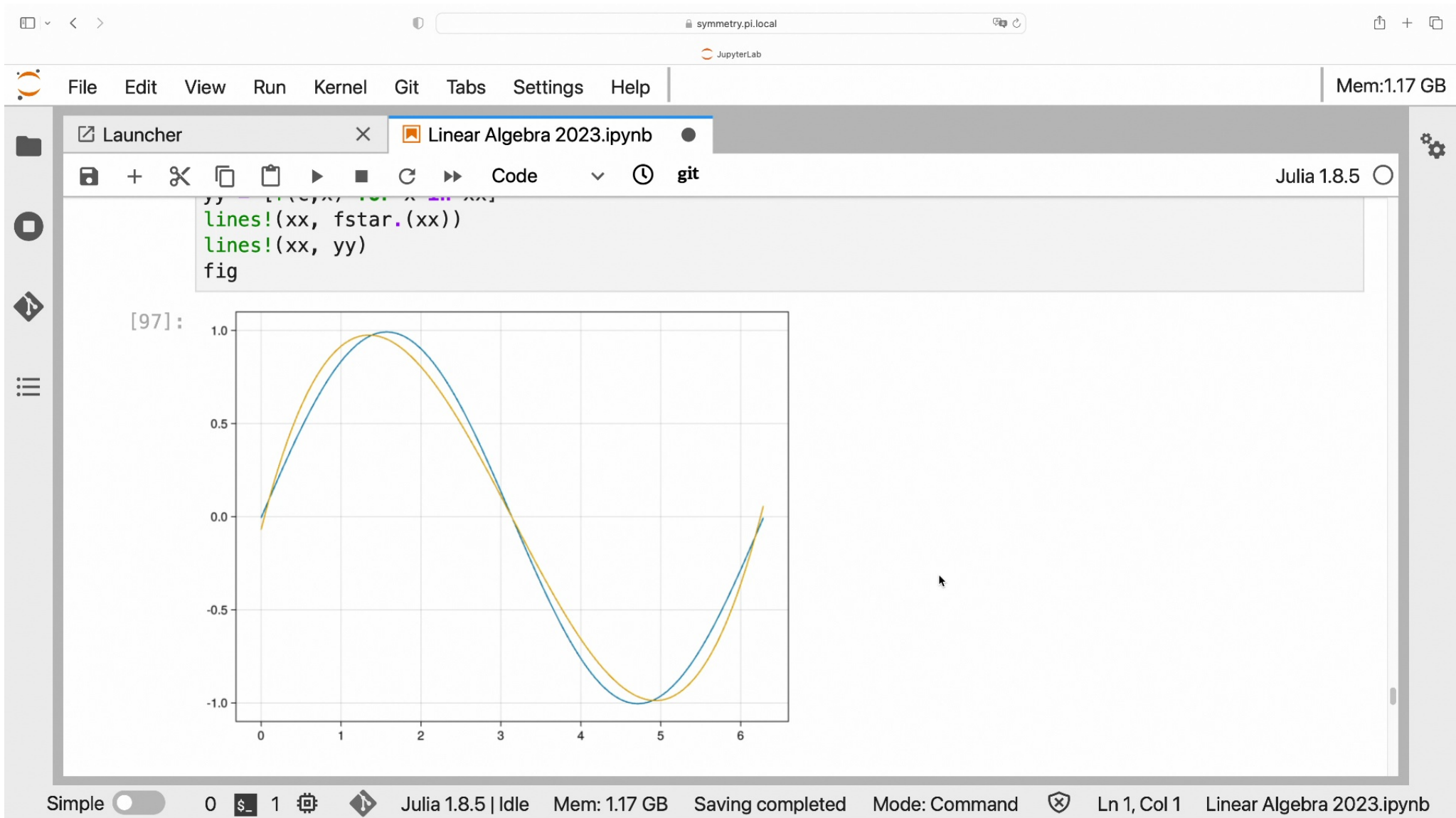
BoundsError: attempt to access 5-element Vector{Float64} at index [0]

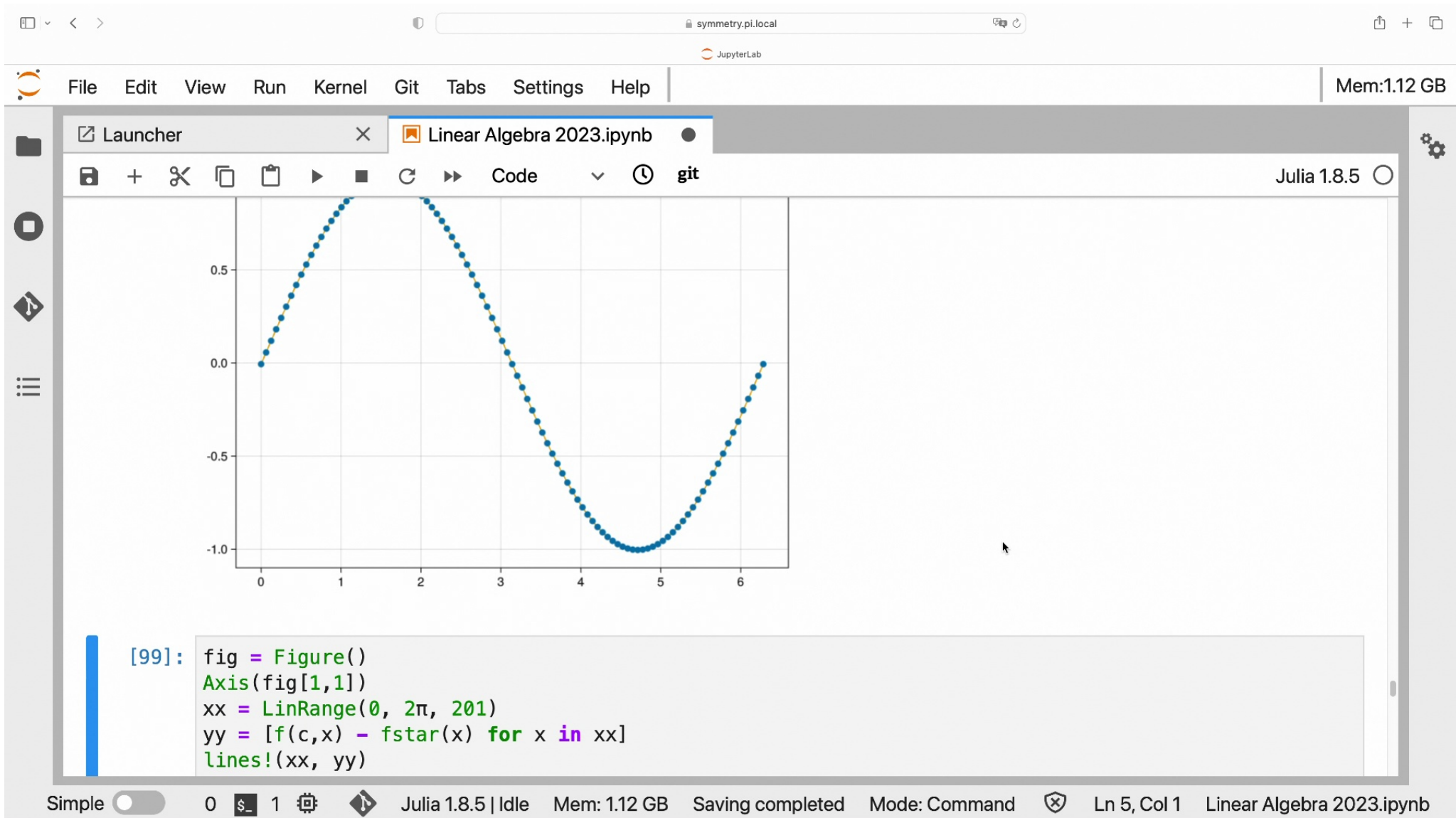
Stacktrace:
 [1] getindex
      @ ./array.jl:924 [inlined]
 [2] #5
```

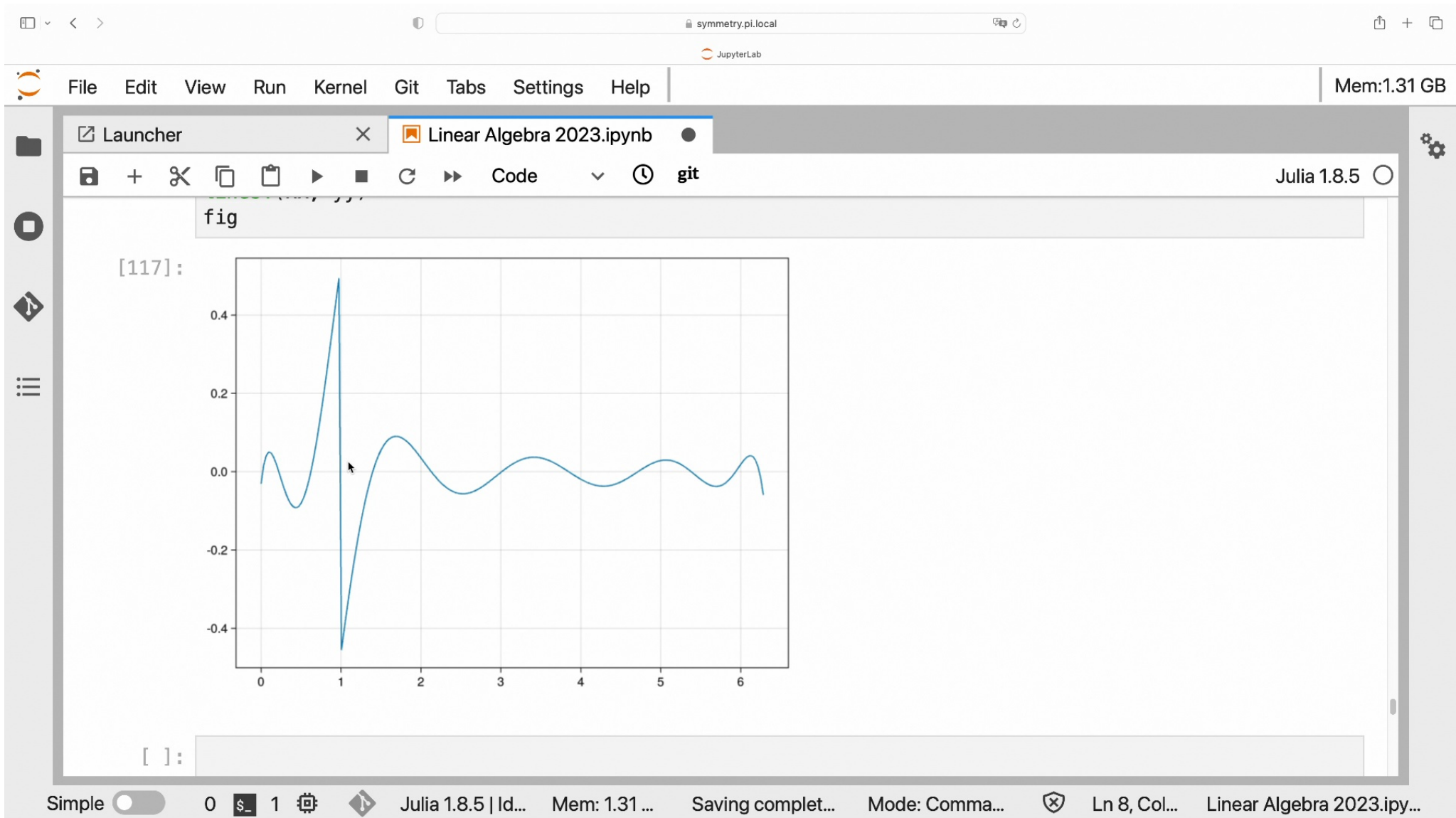
Simple 0 1 Julia 1.8.5 | Idle Mem: 1017.94 MB Saving completed Mode: Edit Ln 2, Col 15 Linear Algebra 2023.ipynb











symmetry.pl.local

JupyterLab

Mem:1.35 GB

FileEditViewRunKernelGitTabsSettingsHelp

LauncherLinear Algebra 2023.ipynb

Julia 1.8.5

-1.39443	-0.241709	0.79938	0.623637	-2.03644	-0.495334
0.274345	0.585941	1.30967	0.199725	0.310771	1.2959
0.390338	-0.243763	-0.0828302	-1.77996	-1.25528	0.165768
0.91283	-0.852078	-0.894069	-0.228502	-0.13873	-0.312594
1.01595	0.170565	1.13128	...	-1.72936	0.403166
-1.50734	-0.337283	0.596123	-0.219408	-1.04985	-0.43931
-2.08594	1.13756	0.912189	-0.161296	-0.929213	1.55072
-0.0980929	1.55993	1.39427	-0.937104	-1.29233	-0.875121
0.557355	0.855606	0.32229	-1.43361	1.46757	0.738437

#

Simple0\$1Julia 1.8.5 | IdleMem: 1.35 GBSaving completedMode: EditLn 1, Col 2Linear Algebra 2023.ipynb

symmetry.pl.local

JupyterLab

Mem:1.42 GB

FileEditViewRunKernelGitTabsSettingsHelp

LauncherLinear Algebra 2023.ipynb

Julia 1.8.5

1.01595	0.170565	1.13128	...	-1.72936	0.403166	-0.749271
-1.50734	-0.337283	0.596123		-0.219408	-1.04985	-0.43931
-2.08594	1.13756	0.912189		-0.161296	-0.929213	1.55072
-0.0980929	1.55993	1.39427		-0.937104	-1.29233	-0.875121
0.557355	0.855606	0.32229		-1.43361	1.46757	0.738437

- QR

Simple

0 1

Julia 1.8.5 | Idle

Mem: 1.42 GB

Saving completed

Mode: Edit

Ln 1, Col 5

Linear Algebra 2023.ipynb

symmetry.pl.local

JupyterLab

Mem:1.21 GB

FileEditViewRunKernelGitTabsSettingsHelp

LauncherLinear Algebra 2023.ipynb

Julia 1.8.5

- QR factorization
- $A = Q * R$

```
[128]: Q = qr(A).Q
      R = qr(A).R;
```

```
[130]: Q * R ≈ A
```

```
[130]: true
```

```
##
```

Simple0\$1Julia 1.8.5 | IdleMem: 1.21 GBSaving completedMode: EditLn 1, Col 4Linear Algebra 2023.ipynb

symmetry.pl.local

JupyterLab

Mem:1.33 GB

FileEditViewRunKernelGitTabsSettingsHelp

LauncherLinear Algebra 2023.ipynb

Julia 1.8.5

- QR factorization
- $A = Q * R$

```
[128]: Q = qr(A).Q
      R = qr(A).R;
```

```
[130]: Q * R ≈ A
```

```
[130]: true
```

Linear Operators and Matrices

```
[133]: using SparseArrays
```

```
[ ]: A = sprandn(10, 10, )
```

Simple0\$1Julia 1.8.5 | IdleMem: 1.33 GBSaving completedMode: EditLn 1, Col 21Linear Algebra 2023.ipynb

symmetry.pl.local

JupyterLab

Mem:1.38 GB

File Edit View Run Kernel Git Tabs Settings Help

Launcher Linear Algebra 2023.ipynb

Julia 1.8.5

```
[140]: eigvals(Matrix(A))
```

```
[140]: 10-element Vector{Float64}:  
  -4.65926389719694e-17  
   1.7883899018346986e-16  
   0.0004885358325410707  
   0.0004885358325412336  
   0.8388575250629489  
   0.8388575250629492  
   0.9719939170088447  
   0.9719939170088452  
   1.438812959105071  
   1.438812959105072
```

```
[*]: ]add Arpack
```

```
[141]: using Arpack
```

```
ArgumentError: Package Arpack not found in current path.  
- Run `import Pkg; Pkg.add("Arpack")` to install the Arpack package.
```

```
Stacktrace:  
 [1] macro expansion
```

Simple 0 1 Julia 1.8.5 | Bu... Mem: 1.38 ... Saving complet... Mode: Comma... Ln 1, Col... Linear Algebra 2023.ipy...

symmetry.pl.local

JupyterLab

Mem: 1.32 GB

File Edit View Run Kernel Git Tabs Settings Help

Launcher Linear Algebra 2023.ipynb

Code git Julia 1.8.5

```
.      .      .      .      .  
.      -0.0691901 .      .      .  
.      -0.104929 .      .      -0.0584544  
.      .      .      ...      .      0.293086  
.      .      .      .      .      -0.36061  
0.05674 .      .      0.00357004 0.0130939 .  
0.208106 .      .      0.0130939 0.258159 .  
.      .      .      .      .      0.00439352
```

[140]: `eigvals(Matrix(A))`

[140]: 10-element Vector{Float64}:
-4.65926389719694e-17
1.7883899018346986e-16
0.0004885358325410707
0.0004885358325412336
0.8388575250629489
0.8388575250629492
0.9719939170088447
0.9719939170088452
1.438812959105071
1.438812959105072

0 1 Julia 1.8.5 | Idle Mem: 1.32 GB Saving completed Mode: Edit Ln 1, Col 1 Linear Algebra 2023.ipynb



The screenshot displays the JupyterLab web application interface. At the top, a browser window shows the URL 'symmetry.pl.local'. The JupyterLab header includes a menu bar with options: File, Edit, View, Run, Kernel, Git, Tabs, Settings, and Help. On the right of the header, the memory usage is indicated as 'Mem:1.60 GB'. Below the header, a tab bar shows two open tabs: 'Launcher' and 'Linear Algebra 2023.ipynb'. The 'Linear Algebra 2023.ipynb' tab is active, showing a code editor with a large grid of small, repeating patterns. The editor's toolbar includes icons for saving, adding, deleting, copying, pasting, running, and other functions. The bottom status bar shows the current mode as 'Simple', the number of cells as '0', the current cell as '1', the kernel as 'Julia 1.8.5 | Idle', the memory usage as 'Mem: 1.60 GB', the status as 'Saving completed', the mode as 'Mode: Edit', and the current line and column as 'Ln 1, Col 1'.

The screenshot displays a JupyterLab environment. The top menu bar includes File, Edit, View, Run, Kernel, Git, Tabs, Settings, and Help. The right side of the top bar shows 'Mem: 1.55 GB'. The left sidebar contains icons for Launcher, Files, Environments, and a menu. The main workspace has two tabs: 'Launcher' and 'Linear Algebra 2023.ipynb'. The 'Linear Algebra 2023.ipynb' tab is active, showing a code cell with the following content:

```
[*]: eigs(A; nev=10)
```

Below the code cell is an empty input field for the output. The bottom status bar indicates 'Julia 1.8.5 | Busy', 'Mem: 1.56 GB', 'Saving completed', 'Mode: Edit', and 'Ln 1, Col 1 Linear Algebra 2023.ipynb'. In the bottom-left corner, there is a small video inset showing a person wearing a mask and gesturing with their hands.

The image shows a JupyterLab web interface. At the top, there's a browser address bar with 'symmetry.pl.local' and a JupyterLab logo. Below that is a menu bar with 'File', 'Edit', 'View', 'Run', 'Kernel', 'Git', 'Tabs', 'Settings', and 'Help'. On the right of the menu bar, it says 'Mem:1.70 GB'. The main area has a toolbar with icons for file operations and execution. The notebook 'Linear Algebra 2023.ipynb' is open, showing a large block of text (possibly a placeholder or a large string) and three code cells. The first code cell is empty, the second contains 'eigs(A; nev=10)', and the third is empty. The status bar at the bottom shows 'Simple', '0', '1', 'Julia 1.8.5 | Bu...', 'Mem: 1.70 ...', 'Saving complet...', 'Mode: Comma...', 'Ln 1, Co...', and 'Linear Algebra 2023.ipy...'.

symmetry.pl.local

JupyterLab

Mem:1.70 GB

FileEditViewRunKernelGitTabsSettingsHelp

LauncherLinear Algebra 2023.ipynb

CodegitJulia 1.8.5

```
[149]: eigs(A; nev=10)
```

```
[149]: ([33.334526522335864, 33.33452652233579, 32.76803232922468, 32.76803232836849, 32.147098306593044,
32.14709830658906, 31.21063725807284, 31.21063725807278, 31.130768969638165, 31.130768969638073],
[-1.1052537336605151e-16 7.219258512980826e-18 ... -4.2164421357702855e-17 3.113571609582585e-16; -1
.6905562555223892e-16 1.2367896265109581e-16 ... 2.20773857970716e-17 -1.2097844626327715e-18; ... ; 5
.695284603563285e-16 3.030319550512735e-17 ... -9.170270374155808e-16 1.5433158417106811e-15; -4.413
729363592221e-16 4.7482891401492066e-15 ... 2.2126535956227744e-16 -5.696207458926047e-15] 10 34
```

Simple0Julia 1.8.5 | IdleMem: 1.70 GBSaving completedMode: CommandLn 1, Col 1Linear Algebra 2023.ipynb