Title: Numerical Methods Lecture - 230119

Speakers: Erik Schnetter

Collection: Numerical Methods (2022/2023)

Date: January 19, 2023 - 9:15 AM

URL: https://pirsa.org/23010005

symmetry.pi.local

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Mem:443 MB

Untitled6.ipynb

Code git Julia 1.8.5

[2]:
```
versioninfo()
```

```
Julia Version 1.8.5
Commit 17cfb8e65ea (2023-01-08 06:45 UTC)
Platform Info:
  OS: Linux (x86_64-linux-gnu)
  CPU: 80 × Intel(R) Xeon(R) Gold 6148 CPU @ 2.40GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-13.0.1 (ORCJIT, skylake-avx512)
  Threads: 1 on 80 virtual cores
Environment:
  LD_LIBRARY_PATH = /cm/shared/apps/slurm/19.05.8/lib64/slurm:/cm/shared/apps/slurm/19.05.8/lib64
  LD_LIBRARY_PATH_modshare = /cm/shared/apps/slurm/19.05.8/lib64:1:/cm/shared/apps/slurm/19.05.8/lib64/slurm:1
```

Linear Algebra

Vectors and Matrices

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 443.09 MB Saving completed Mode: Command Ln 1, Col 14 Untitled6.ipynb

```
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-13.0.1 (ORCJIT, skylake-avx512)
  Threads: 1 on 80 virtual cores
Environment:
  LD_LIBRARY_PATH = /cm/shared/apps/slurm/19.05.8/lib64/slurm:/cm/shared/apps/slurm/19.05.8/lib64
  LD_LIBRARY_PATH_modshare = /cm/shared/apps/slurm/19.05.8/lib64:1:/cm/shared/apps/slurm/19.05.8/lib64/slurm:1
```

Linear Algebra

Vectors and Matrices

```julia
[3]: x = [1, 2, 3]
```

```
[3]: 3-element Vector{Int64}:
 1
 2
 3
```

```
[ ]:
```

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help Mem:448 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
[4]: 4-element Vector{Float64}:
      0.0
      0.0
      0.0
      0.0

[5]: x = ones(4)

[5]: 4-element Vector{Float64}:
      1.0
      1.0
      1.0
      1.0

[6]: x = fill(2, 4)

[6]: 4-element Vector{Int64}:
      2
      2
      2
      2

[ ]:
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 447.81 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help Mem:454 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
 1.0

[6]: x = fill(2, 4)

[6]: 4-element Vector{Int64}:
 2
 2
 2
 2

[7]: x = rand(4)

[7]: 4-element Vector{Float64}:
 0.48294720767286936
 0.4059337103377747
 0.7751103621957153
 0.2923902544909047

[ ]:
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 454.36 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

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Mem:733 MB

Untitled6.ipynb

Markdown git Julia 1.8.5

```
[10]: 3×2 Matrix{Int64}:
       1  2
       3  4
       5  6

[11]: A = zeros(4, 4)

[11]: 4×4 Matrix{Float64}:
       0.0  0.0  0.0  0.0
       0.0  0.0  0.0  0.0
       0.0  0.0  0.0  0.0
       0.0  0.0  0.0  0.0

[13]: using LinearAlgebra

[14]: A + I

[14]: 4×4 Matrix{Float64}:
       1.0  0.0  0.0  0.0
       0.0  1.0  0.0  0.0
       0.0  0.0  1.0  0.0
       0.0  0.0  0.0  1.0

`I
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 732.94 MB Saving completed Mode: Edit Ln 1, Col 3 Untitled6.ipynb

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File Edit View Run Kernel Git Tabs Settings Help

Mem:462 MB

Untitled6.ipynb

Code git Julia 1.8.5

[14]:
```
A + I
```

[14]:
```
4×4 Matrix{Float64}:
 1.0  0.0  0.0  0.0
 0.0  1.0  0.0  0.0
 0.0  0.0  1.0  0.0
 0.0  0.0  0.0  1.0
```

`I` is the identity matrix

[15]:
```
typeof(I)
```

[15]:
```
UniformScaling{Bool}
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 462.14 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

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Mem:539 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
 -1.23421    0.192585   0.0726337      -0.161421  -0.454981   -0.655838
 -0.248926  -1.38171   -0.850878        0.368427  -0.485841   -1.13174
 -0.393313   0.110753   0.0775899      -0.105413   1.50259     0.540897
  0.708857  -0.250389  -0.970402        0.490682   0.0857331   0.849621
  0.381393   1.15025   -1.04459     …  -0.31922    0.374296    1.15026
 -0.351643   0.467111  -1.17917         0.796063   1.008       0.405301
  1.38006   -1.7385     0.607196       -0.256575   2.2319     -0.255111
 -0.438223   0.18321    0.0133778      -0.344515   0.483224   -0.411225
 -1.49231   -1.15968   -0.681522       -1.63951   -0.83442    -2.20031
```

[17]:
```
A * x
```

[17]:
```
10-element Vector{Float64}:
 -2.2061665484685413
 -1.3625462041697145
  1.0821298822044938
 -2.793372741226245
  1.4856317713434266
  0.9424865771821518
 -0.988370044961673
  3.116386898144311
 -3.583547001508562
  1.4758111550053108
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 538.71 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

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Mem:578 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
 -0.9883700449616 73
  3.116386898144311
 -3.583547001508562
  1.4758111550053108
```

[18]:

```julia
x * A
```

```
DimensionMismatch: A has dimensions (10,1) but B has dimensions (10,10)

Stacktrace:
 [1] gemm_wrapper!(C::Matrix{Float64}, tA::Char, tB::Char, A::Matrix{Float64}, B::Matrix{Float64}, _add::LinearAlg
ebra.MulAddMul{true, true, Bool, Bool})
   @ LinearAlgebra ~/.julia/juliaup/julia-1.8.5+0.x64.linux.gnu/share/julia/stdlib/v1.8/LinearAlgebra/src/matmul.j
l:646
 [2] mul!
   @ ~/.julia/juliaup/julia-1.8.5+0.x64.linux.gnu/share/julia/stdlib/v1.8/LinearAlgebra/src/matmul.jl:161 [inlined
]
 [3] mul!
   @ ~/.julia/juliaup/julia-1.8.5+0.x64.linux.gnu/share/julia/stdlib/v1.8/LinearAlgebra/src/matmul.jl:276 [inlined
]
 [4] *
   @ ~/.julia/juliaup/julia-1.8.5+0.x64.linux.gnu/share/julia/stdlib/v1.8/LinearAlgebra/src/matmul.jl:148 [inlined
]
 [5] *(a::Vector{Float64}, B::Matrix{Float64})
   @ LinearAlgebra ~/.julia/juliaup/julia-1.8.5+0.x64.linux.gnu/share/julia/stdlib/v1.8/LinearAlgebra/src/matmul.j
l:62
 [6] top-level scope
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 577.97 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

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Mem:456 MB

Untitled6.ipynb

Code git Julia 1.8.5

[19]:
```
x' * A
```

[19]:
```
1×10 adjoint(::Vector{Float64}) with eltype Float64:
 2.5766  -1.5098  0.0773024  0.223512  …  3.00766  2.55386  1.96617  3.9474
```

`'` is the transpose

[21]:
```
transpose(A)
```

[21]:
```
10×10 transpose(::Matrix{Float64}) with eltype Float64:
 -0.832528  -1.23421    -0.248926  …   1.38006   -0.438223   -1.49231
 -0.299984   0.192585   -1.38171      -1.7385     0.18321    -1.15968
  1.4285     0.0726337  -0.850878      0.607196   0.0133778  -0.681522
  0.608733  -0.164068    1.7296        0.585388  -1.98229    -0.975746
  0.221681  -0.521551   -0.572459      0.715231  -0.707857    0.944254
  0.335325  -0.051091   -0.609413  …   0.942417   1.74462     0.893738
 -0.983688   0.660729    1.01052      -0.788336  -0.960347   -1.16343
 -1.07821   -0.161421    0.368427     -0.256575  -0.344515   -1.63951
  0.632342  -0.454981   -0.485841      2.2319     0.483224   -0.83442
  1.43361   -0.655838   -1.13174      -0.255111  -0.411225   -2.20031
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 456.26 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

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Mem:470 MB

Untitled6.ipynb

Code

git

Julia 1.8.5

```
 -0.983688   0.660729   1.01052    -0.788336  -0.960347  -1.16343
 -1.07821   -0.161421   0.368427   -0.256575  -0.344515  -1.63951
  0.632342  -0.454981  -0.485841    2.2319     0.483224  -0.83442
  1.43361   -0.655838  -1.13174    -0.255111  -0.411225  -2.20031
```

[22]:

```julia
y = randn(10)
```

[22]:

```
10-element Vector{Float64}:
 -1.0508822790221664
  0.29220327162202425
 -1.717972337645649
 -0.939954419742296
 -0.48657081100579574
  1.0533297563233868
 -1.035099816164604
  1.320557789457694
 -0.9552778143059417
 -0.17146363772917292
```

[ ]:

```julia
y'
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 469.91 MB Saving completed Mode: Edit Ln 1, Col 4 Untitled6.ipynb

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help Mem:571 MB

Untitled6.ipynb

Code git Julia 1.8.5

[22]: y = randn(10)

[22]:
```
10-element Vector{Float64}:
 -1.0508822790221664
  0.29220327162202425
 -1.717972337645649
 -0.939954419742296
 -0.48657081100579574
  1.0533297563233868
 -1.035099816164604
  1.320557789457694
 -0.9552778143059417
 -0.17146363772917292
```

[23]:
```
y' * A * x
```

[23]: 11.26540446631752

[24]:
```
y[1] * A[2, 3] * x[4]
```

[24]: 0.022050592496814198

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 571.23 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help Mem:604 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
  1.0533297563233868
 -1.035099816164604
  1.320557789457694
 -0.9552778143059417
 -0.17146363772917292
```

[23]:
```julia
y' * A * x
```

[23]:
```
11.26540446631752
```

[24]:
```julia
y[1] * A[2, 3] * x[4]
```

[24]:
```
0.022050592496814198
```

[25]:
```julia
x'
```

[25]:
```
1×10 adjoint(::Vector{Float64}) with eltype Float64:
 0.876756  -0.693656  0.231876  -0.288887  …  0.176596  0.237286  -0.912327
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 604.37 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:458 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
  1.0353297563233868
 -1.035099816164604
  1.320557789457694
 -0.9552778143059417
 -0.17146363772917292
```

[23]:
```julia
y' * A * x
```

[23]:
```
11.26540446631752
```

[24]:
```julia
y[1] * A[2, 3] * x[4]
```

[24]:
```
0.022050592496814198
```

[26]:
```julia
typeof(x)
```

[26]:
```
Vector{Float64} (alias for Array{Float64, 1})
```

[27]:
```julia
typeof(x')
```

[27]:
```
Adjoint{Float64, Vector{Float64}}
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 457.66 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help Mem:568 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
 1.0333297363233868
-1.035099816164604
 1.320557789457694
-0.9552778143059417
-0.17146363772917292
```

```
[23]: y' * A * x
```

```
[23]: 11.26540446631752
```

```
[24]: y[1] * A[2, 3] * x[4]
```

```
[24]: 0.022050592496814198
```

```
[26]: typeof(x)
```

```
[26]: Vector{Float64} (alias for Array{Float64, 1})
```

```
[27]: typeof(x')
```

```
[27]: Adjoint{Float64, Vector{Float64}}
```

```
[28]: typeof(x'')
```

```
[28]: Vector{Float64} (alias for Array{Float64, 1})
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 568.09 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help Mem:495 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
[23]: 11.26540446631752
```

```
[24]: y[1] * A[2, 3] * x[4]
```

```
[24]: 0.022050592496814198
```

```
[26]: typeof(x)
```

```
[26]: Vector{Float64} (alias for Array{Float64, 1})
```

```
[27]: typeof(x')
```

```
[27]: Adjoint{Float64, Vector{Float64}}
```

```
[28]: typeof(x'')
```

```
[28]: Vector{Float64} (alias for Array{Float64, 1})
```

```
[30]: M = [1;2;; 3;4;;; 5;6;; 7;8]
```

```
[30]: 2×2×2 Array{Int64, 3}:
      [:, :, 1] =
       1  3
       2  4
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 495.50 MB Saving completed Mode: Command Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help Mem:495 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
[26]: typeof(x)

[26]: Vector{Float64} (alias for Array{Float64, 1})

[27]: typeof(x')

[27]: Adjoint{Float64, Vector{Float64}}

[28]: typeof(x'')

[28]: Vector{Float64} (alias for Array{Float64, 1})

[30]: M = [1;2;; 3;4;;; 5;6;; 7;8]

[30]: 2×2×2 Array{Int64, 3}:
      [:, :, 1] =
       1  3
       2  4

      [:, :, 2] =
       5  7
       6  8

[ ]:
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 495.50 MB Saving completed Mode: Command Ln 1, Col 1 Untitled6.ipynb

[28]: Vector{Float64} (alias for Array{Float64, 1})

```julia
[30]: M = [1;2;; 3;4;;; 5;6;; 7;8]
```

```
[30]: 2×2×2 Array{Int64, 3}:
[:, :, 1] =
 1  3
 2  4

[:, :, 2] =
 5  7
 6  8
```

```julia
[31]: reshape(M, (4,2))
```

```
[31]: 4×2 Matrix{Int64}:
 1  5
 2  6
 3  7
 4  8
```

```
[ ]:
```

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help Mem:513 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
 1  3  5  7
 2  4  6  8
```

[33]:
```julia
reshape(A, 100)
```

[33]:
```
100-element Vector{Float64}:
 -0.8325277244809408
 -1.234209625462004
 -0.2489256040114567
 -0.3933129986237735
  0.7088566948403249
  0.3813934698118687
 -0.35164282677444353
  1.3800625880413442
 -0.4382229724131867
 -1.4923126986562871
 -0.299984368946636
  0.19258528594011065
 -1.3817055315942097
  ⋮
  0.4832244608223171
 -0.8344202157533473
  1.4336133647236358
 -0.6558378431558194
 -1.1317353294895178
  0.5408971634847218
  0.8496208853420525
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 512.67 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help

Mem:564 MB

Untitled6.ipynb

Code

git

Julia 1.8.5

```
 -1.4923126986562871
 -0.299984368946636
  0.19258528594011065
 -1.3817055315942097
  ⋮
  0.4832244608223171
 -0.8344202157533473
  1.4336133647236358
 -0.6558378431558194
 -1.1317353294895178
  0.5408971634847218
  0.8496208853420525
  1.1502630688586468
  0.40530143360961496
 -0.2551108380524427
 -0.41122509187345907
 -2.2003104344197575
```

[36]:
```
reshape(Avec, (10, 10)) == A
```

[36]:
```
true
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 564.87 MB Saving completed Mode: Command Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:559 MB

Untitled6.ipynb

Markdown git Julia 1.8.5

```
  0.8496208853420525
  1.1502630688586468
  0.40530143360961496
 -0.2551108380524427
 -0.41122509187345907
 -2.2003104344197575
```

[36]:
```julia
reshape(Avec, (10, 10)) == A
```

[36]:
```
true
```

Matrix types

```
- dense (full)
- symmetric
- diagonal
- bi-, tri-diagonal
- spar
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 559.24 MB Saving completed Mode: Edit Ln 5, Col 7 Untitled6.ipynb

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help

Mem:600 MB

Untitled6.ipynb

Code git Julia 1.8.5

[36]:
```
reshape(Avec, (10, 10)) == A
```

[36]:
```
true
```

Matrix types

- dense (full)
- symmetric
- diagonal
- bi-, tri-diagonal
- sparse

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 599.63 MB Saving completed Mode: Command Ln 1, Col 1 Untitled6.ipynb

- dense (full)
- symmetric
- diagonal
- bi-, tri-diagonal
- sparse

[38]:
```julia
Symmetric(A)
```

[38]:
```
10×10 Symmetric{Float64, Matrix{Float64}}:
 -0.832528  -0.299984   1.4285     …  -1.07821    0.632342   1.43361
 -0.299984   0.192585   0.0726337     -0.161421  -0.454981  -0.655838
  1.4285     0.0726337 -0.850878       0.368427  -0.485841  -1.13174
  0.608733  -0.164068   1.7296        -0.105413   1.50259    0.540897
  0.221681  -0.521551  -0.572459       0.490682   0.0857331  0.849621
  0.335325  -0.051091  -0.609413   …  -0.31922    0.374296   1.15026
 -0.983688   0.660729   1.01052        0.796063   1.008      0.405301
 -1.07821   -0.161421   0.368427      -0.256575   2.2319    -0.255111
  0.632342  -0.454981  -0.485841       2.2319     0.483224  -0.411225
  1.43361   -0.655838  -1.13174       -0.255111  -0.411225  -2.20031
```

[ ]:

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help Mem:501 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
[38]: 10×10 Symmetric{Float64, Matrix{Float64}}:
 -0.832528  -0.299984   1.4285     …  -1.07821    0.632342    1.43361
 -0.299984   0.192585   0.0726337     -0.161421  -0.454981   -0.655838
  1.4285     0.0726337 -0.850878       0.368427  -0.485841   -1.13174
  0.608733  -0.164068   1.7296        -0.105413   1.50259     0.540897
  0.221681  -0.521551  -0.572459       0.490682   0.0857331   0.849621
  0.335325  -0.051091  -0.609413   …  -0.31922    0.374296    1.15026
 -0.983688   0.660729   1.01052        0.796063   1.008       0.405301
 -1.07821   -0.161421   0.368427      -0.256575   2.2319     -0.255111
  0.632342  -0.454981  -0.485841       2.2319     0.483224   -0.411225
  1.43361   -0.655838  -1.13174       -0.255111  -0.411225   -2.20031
```

```
[39]: Symmetric(A) - A
```

```
[39]: 10×10 Matrix{Float64}:
  0.0         0.0        0.0       …  0.0      0.0      0.0       0.0
  0.934225    0.0        0.0          0.0      0.0      0.0       0.0
  1.67742     1.45434    0.0          0.0      0.0      0.0       0.0
  1.00205    -0.274821   1.65201      0.0      0.0      0.0       0.0
 -0.487176   -0.271162   0.397943     0.0      0.0      0.0       0.0
 -0.0460683  -1.20134    0.43518   …  0.0      0.0      0.0       0.0
 -0.632045    0.193618   2.18969      0.0      0.0      0.0       0.0
 -2.45828     1.57708   -0.238769     1.5844   0.0      0.0       0.0
  1.07056    -0.638191  -0.499219     1.96834  2.57641  0.0       0.0
  2.92593     0.503846  -0.450213     1.56874  1.3844   0.423195  0.0
```

```
[ ]: 
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 501.07 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

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Mem:501 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
  1.4285     0.0726337  -0.850878      0.368427  -0.485841   -1.13174
  0.608733  -0.164068    1.7296       -0.105413   1.50259     0.540897
  0.221681  -0.521551   -0.572459      0.490682   0.0857331   0.849621
  0.335325  -0.051091   -0.609413  …  -0.31922    0.374296    1.15026
 -0.983688   0.660729    1.01052       0.796063   1.008       0.405301
 -1.07821   -0.161421    0.368427     -0.256575   2.2319     -0.255111
  0.632342  -0.454981   -0.485841      2.2319     0.483224   -0.411225
  1.43361   -0.655838   -1.13174      -0.255111  -0.411225   -2.20031
```

[39]:
```
Symmetric(A) - A
```

[39]:
```
10×10 Matrix{Float64}:
  0.0         0.0        0.0       …  0.0      0.0      0.0       0.0
  0.934225    0.0        0.0          0.0      0.0      0.0       0.0
  1.67742     1.45434    0.0          0.0      0.0      0.0       0.0
  1.00205    -0.274821   1.65201      0.0      0.0      0.0       0.0
 -0.487176   -0.271162   0.397943     0.0      0.0      0.0       0.0
 -0.0460683  -1.20134    0.43518   …  0.0      0.0      0.0       0.0
 -0.632045    0.193618   2.18969      0.0      0.0      0.0       0.0
 -2.45828     1.57708   -0.238769     1.5844   0.0      0.0       0.0
  1.07056    -0.638191  -0.499219     1.96834  2.57641  0.0       0.0
  2.92593     0.503846  -0.450213     1.56874  1.3844   0.423195  0.0
```

[ ]:
```
Symmetric(A + A')
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 501.11 MB Saving completed Mode: Edit Ln 1, Col 17 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:591 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
 -0.0460683  -1.20134    0.43518   …  0.0      0.0      0.0       0.0
 -0.632045    0.193618   2.18969      0.0      0.0      0.0       0.0
 -2.45828     1.57708   -0.238769     1.5844   0.0      0.0       0.0
  1.07056    -0.638191  -0.499219     1.96834  2.57641  0.0       0.0
  2.92593     0.503846  -0.450213     1.56874  1.3844   0.423195  0.0
```

[40]:

```julia
Symmetric(A + A')
```

[40]:

```
10×10 Symmetric{Float64, Matrix{Float64}}:
 -1.66506    -1.53419     1.17957   …   0.301849     0.194119   -0.0586993
 -1.53419     0.385171   -1.30907      -1.89992     -0.27177    -1.81552
  1.17957    -1.30907    -1.70176       0.975623    -0.472463   -1.81326
  0.21542    -0.0533157   1.80719       0.479975    -0.4797     -0.434849
  0.930537   -0.77194    -1.54286       1.20591     -0.622124    1.79388
  0.716719    1.09916    -1.654     …   0.623197     2.11891     2.044
 -1.33533     1.12784    -0.168648      0.00772748   0.0476504  -0.758133
  0.301849   -1.89992     0.975623     -0.513149     1.88738    -1.89462
  0.194119   -0.27177    -0.472463      1.88738      0.966449   -1.24565
 -0.0586993  -1.81552    -1.81326      -1.89462     -1.24565    -4.40062
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 591.03 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help   Mem:605 MB

Untitled6.ipynb

Code   git   Julia 1.8.5

```
  0.21542    -0.0533157   1.80719       0.479975    -0.4797     -0.434849
  0.930537   -0.77194    -1.54286       1.20591     -0.622124    1.79388
  0.716719    1.09916    -1.654     …   0.623197     2.11891     2.044
 -1.33533     1.12784    -0.168648      0.00772748   0.0476504  -0.758133
  0.301849   -1.89992     0.975623     -0.513149     1.88738    -1.89462
  0.194119   -0.27177    -0.472463      1.88738      0.966449   -1.24565
 -0.0586993  -1.81552    -1.81326      -1.89462     -1.24565    -4.40062
```

[41]:
```
Diagonal(randn(10))
```

[41]:
```
10×10 Diagonal{Float64, Vector{Float64}}:
 0.567709    ⋅          ⋅          ⋅         …    ⋅          ⋅          ⋅
  ⋅        -0.774464    ⋅          ⋅              ⋅          ⋅          ⋅
  ⋅          ⋅        -0.482402    ⋅              ⋅          ⋅          ⋅
  ⋅          ⋅          ⋅        -0.448824        ⋅          ⋅          ⋅
  ⋅          ⋅          ⋅          ⋅              ⋅          ⋅          ⋅
  ⋅          ⋅          ⋅          ⋅         …    ⋅          ⋅          ⋅
  ⋅          ⋅          ⋅          ⋅              ⋅          ⋅          ⋅
  ⋅          ⋅          ⋅          ⋅            -0.568468    ⋅          ⋅
  ⋅          ⋅          ⋅          ⋅              ⋅         0.637312    ⋅
  ⋅          ⋅          ⋅          ⋅              ⋅          ⋅        -0.728199
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 604.74 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help   Mem:736 MB

Untitled6.ipynb

Code   git   Julia 1.8.5

```
 ⋅          ⋅          ⋅          ⋅        …   ⋅          ⋅          ⋅
 ⋅          ⋅          ⋅          ⋅            ⋅          ⋅          ⋅
 ⋅          ⋅          ⋅          ⋅          -0.568468    ⋅          ⋅
 ⋅          ⋅          ⋅          ⋅            ⋅         0.637312    ⋅
 ⋅          ⋅          ⋅          ⋅            ⋅          ⋅        -0.728199
```

Operations on matrices

```
[42]: inv(A)
```

```
[42]: 10×10 Matrix{Float64}:
  0.130278    1.7048     0.280147   …   0.915988   1.51103    -0.750954
 -0.0243551   1.57268   -0.0785341      0.534775   0.902169   -0.532399
 -0.03324    -0.524719  -0.340812      -0.152099  -0.661166    0.106677
  0.226124    0.56677    0.367328       0.277119   0.547419   -0.328856
 -0.227858   -1.05724   -0.586824      -0.349975  -1.25985     0.589493
  0.303004    0.756102   0.259343   …   0.355829   1.03577    -0.381055
  0.206381    2.5551     0.055039       0.933829   1.04996    -0.638287
 -0.71551    -4.59555   -0.528365      -1.83528   -2.93262     1.2273
 -0.174087    0.519366  -0.191103       0.321094  -0.190178   -0.00862159
  0.349828   -0.344257   0.0846334     -0.232566   0.0438897  -0.0272964
```

```
[ ]: inv(A) *
```

Simple 0 $_ 1 Julia 1.8.5 | Idle Mem: 736.02 MB Saving completed Mode: Edit Ln 1, Col 9 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:507 MB

Untitled6.ipynb

Code git Julia 1.8.5

Operations on matrices

[42]:
```julia
inv(A)
```

[42]:
```
10×10 Matrix{Float64}:
  0.130278    1.7048     0.280147   …   0.915988   1.51103    -0.750954
 -0.0243551   1.57268   -0.0785341      0.534775   0.902169   -0.532399
 -0.03324    -0.524719  -0.340812      -0.152099  -0.661166    0.106677
  0.226124    0.56677    0.367328       0.277119   0.547419   -0.328856
 -0.227858   -1.05724   -0.586824      -0.349975  -1.25985     0.589493
  0.303004    0.756102   0.259343   …   0.355829   1.03577    -0.381055
  0.206381    2.5551     0.055039       0.933829   1.04996    -0.638287
 -0.71551    -4.59555   -0.528365      -1.83528   -2.93262     1.2273
 -0.174087    0.519366  -0.191103       0.321094  -0.190178   -0.00862159
  0.349828   -0.344257   0.0846334     -0.232566   0.0438897  -0.0272964
```

[43]:
```julia
inv(A) * A == I
```

[43]:
```
false
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 507.02 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help Mem:654 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
  3.19627e-16   1.0           1.89735e-16      7.69694e-17   7.09145e-17
 -8.40632e-17  -9.24309e-17   1.0             -8.93017e-17  -3.54948e-17
  7.83634e-19   1.57446e-16  -2.74425e-17      3.76598e-17   1.3855e-16
 -6.7869e-17   -1.63028e-16  -2.56539e-16     -6.87892e-17  -4.2522e-16
 -1.3759e-16    1.38515e-16  -2.14349e-18  …   3.59727e-16  -1.50524e-16
 -1.33949e-16  -1.18129e-16  -1.60627e-16      4.52372e-16  -7.74971e-16
 -1.55093e-16  -6.39193e-16  -4.52137e-16     -1.06987e-15  -5.94712e-16
  7.27855e-17  -1.6318e-16   -3.70697e-17      1.0          -1.26759e-16
  4.78803e-17  -2.23129e-18   7.53905e-17     -8.72726e-17   1.0
```

[46]:
```
B = rand(-100:100, 10, 10)
```

[46]:
```
10×10 Matrix{Int64}:
 -11   65  -32  -99  -99  -76  -42  -53   55    76
  50  -58  -76  -44   34  -36   -7   78   62   -28
  12   71   57  -87  -13  -40   56  -45   31    65
  43  -44  -46   14  -97   31  -35  -10  -70   -72
  51  -45  -42   67   89  -97  -62  -31   79    25
 -41   16    9   77   -2   99  -88   13  -90   -73
  88   26  -15   28   12  -95  -40   35  -19    74
 -77  -54  -41  -41   30   -1   -8  -22  -85  -100
  -1   86  -43   76   -4  -54  -10  -75  -62    68
  64   60   64   11   42  -69   58   51   27   -28
```

```
## Floating point numbers
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 653.91 MB Saving completed Mode: Edit Ln 1, Col 5 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help Mem:798 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
  3.19627e-16   1.0            1.89735e-16      7.69694e-17   7.09145e-17
 -8.40632e-17  -9.24309e-17    1.0             -8.93017e-17  -3.54948e-17
  7.83634e-19   1.57446e-16   -2.74425e-17      3.76598e-17   1.3855e-16
 -6.7869e-17   -1.63028e-16   -2.56539e-16     -6.87892e-17  -4.2522e-16
 -1.3759e-16    1.38515e-16   -2.14349e-18  …   3.59727e-16  -1.50524e-16
 -1.33949e-16  -1.18129e-16   -1.60627e-16      4.52372e-16  -7.74971e-16
 -1.55093e-16  -6.39193e-16   -4.52137e-16     -1.06987e-15  -5.94712e-16
  7.27855e-17  -1.6318e-16    -3.70697e-17      1.0          -1.26759e-16
  4.78803e-17  -2.23129e-18    7.53905e-17     -8.72726e-17   1.0
```

[47]:
```
B = big.(rand(-100:100, 10, 10))
```

[47]:
```
10×10 Matrix{BigInt}:
 -54   75  -94  -58   37   45  -75  -31    0  -52
  90   91  -64   11   88   42  -17  -67  -73   37
  67  -73   83  -73   97   14  -30   29  -35   15
 -12   10  -83   63   68  -16   -8   48  -16  -88
 -58    8   -1   85   57  -91    7  -71  -36    7
  -1   -4   44  -73  -27  -67   46  -94  -13  -99
 -21   68   16  -46   14  -88   82   57   42    7
 -90   46   52   58  -29   82   32   15   33  -42
 -10  -67  -51  -42  -55  -73  -10  -45  -83   93
 -10  -11   35   58  -54   27  -46  -42  -23  -19
```

```
## Floating point numbers
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 798.28 MB Saving completed Mode: Edit Ln 1, Col 5 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help Mem:509 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
 -6.7869e-17   -1.63028e-16  -2.56539e-16     -6.87892e-17  -4.2522e-16
 -1.3759e-16    1.38515e-16  -2.14349e-18  …   3.59727e-16  -1.50524e-16
 -1.33949e-16  -1.18129e-16  -1.60627e-16      4.52372e-16  -7.74971e-16
 -1.55093e-16  -6.39193e-16  -4.52137e-16     -1.06987e-15  -5.94712e-16
  7.27855e-17  -1.6318e-16   -3.70697e-17      1.0          -1.26759e-16
  4.78803e-17  -2.23129e-18   7.53905e-17     -8.72726e-17   1.0
```

[48]:
```julia
B = big.(rand(-100:100, 10, 10)) // 100
```

[48]:
```
10×10 Matrix{Rational{BigInt}}:
  -9//20     11//25    -43//100    -9//100  …   73//100    49//50   -39//100
 -37//100   -63//100     1//2     -91//100     -13//50     79//100  -43//50
   3//50    -23//100    31//100   -53//100      -3//5      -7//25     1//2
 -29//50    -71//100   -17//20    -27//50      -11//20     -3//25    -7//25
 -14//25     57//100    -1//20    -19//50       89//100   -31//50    -1//2
  41//100     9//25     -7//25    -19//100  …  -31//50    -57//100   71//100
 -81//100   -83//100   -49//50     -6//25        1//100    23//50    87//100
 -11//20    -49//100   -51//100   -61//100     -39//50     27//50    87//100
 -51//100   -39//50     -4//5      49//50       29//50     79//100   59//100
   9//20    -11//50      3//4      77//100      13//100    -9//50    47//100
```

[ ]:

```
## Floating point numbers
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 508.66 MB Saving completed Mode: Command Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help | Mem:551 MB

Untitled6.ipynb

Markdown git | Julia 1.8.5

```
 -37//100  -63//100    1//2    -91//100      -13//50    79//100  -43//50
   3//50   -23//100   31//100  -53//100       -3//5     -7//25     1//2
 -29//50   -71//100  -17//20   -27//50       -11//20    -3//25    -7//25
 -14//25    57//100   -1//20   -19//50        89//100  -31//50    -1//2
  41//100    9//25    -7//25   -19//100  …   -31//50   -57//100   71//100
 -81//100  -83//100  -49//50    -6//25         1//100   23//50    87//100
 -11//20   -49//100  -51//100  -61//100      -39//50    27//50    87//100
 -51//100  -39//50    -4//5     49//50        29//50    79//100   59//100
   9//20   -11//50     3//4     77//100       13//100   -9//50    47//100
```

```
[49]: invB = inv(B)
```

```
[49]: 10×10 Matrix{Rational{BigInt}}:
  4185321811449106160//7301769964863648269  …  -3197990405178368555//7301769964863648269
  -6707658691221757300//7301769964863648269      1575220561316549292 5//2190530989459094480 7
 -14060843445096022600//7301769964863648269     12857655717073854150//7301769964863648269
 -31454235454931936300//7301769964863648269     20565336038471464100//7301769964863648269
  22230949934185172500//7301769964863648269    -40977415448680267900//21905309894590944807
 -10597036561349064 00//7301769964863648269  …  14317035254824832975//21905309894590944807
   9432994612030080300//7301769964863648269    -21621014859107194075//21905309894590944807
  29681940064641174600//7301769964863648269    -16231259257795961975//7301769964863648269
  23672778373208715300//7301769964863648269    -14767434873845602300//7301769964863648269
  15528126102563742700//7301769964863648269    -17094047475720082600//21905309894590944807
```

```
## Floating point numbers
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 550.73 MB Saving completed Mode: Command Ln 3, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:596 MB

Untitled6.ipynb

Markdown git Julia 1.8.5

```
 -1406084344509602260//7301769964863648269      1285765571707385415//7301769964863648269
 -3145423545493193630//7301769964863648269      2056533603847146410//7301769964863648269
  2223094993418517250//7301769964863648269     -4097741544868026790//21905309894590944807
 -1059703656134906400//7301769964863648269  …   1431703525482483297//21905309894590944807
   943299461203008030//7301769964863648269     -21621014859107194075//21905309894590944807
  2968194006464117460//7301769964863648269     -1623125925779596197//7301769964863648269
  2367277837320871530//7301769964863648269     -1476743487384560230//7301769964863648269
  1552812610256374270//7301769964863648269     -17094047475720082600//21905309894590944807
```

[50]:
```
invB * B
```

[50]:
```
10×10 Matrix{Rational{BigInt}}:
 1//1  0//1  0//1  0//1  0//1  0//1  0//1  0//1  0//1  0//1
 0//1  1//1  0//1  0//1  0//1  0//1  0//1  0//1  0//1  0//1
 0//1  0//1  1//1  0//1  0//1  0//1  0//1  0//1  0//1  0//1
 0//1  0//1  0//1  1//1  0//1  0//1  0//1  0//1  0//1  0//1
 0//1  0//1  0//1  0//1  1//1  0//1  0//1  0//1  0//1  0//1
 0//1  0//1  0//1  0//1  0//1  1//1  0//1  0//1  0//1  0//1
 0//1  0//1  0//1  0//1  0//1  0//1  1//1  0//1  0//1  0//1
 0//1  0//1  0//1  0//1  0//1  0//1  0//1  1//1  0//1  0//1
 0//1  0//1  0//1  0//1  0//1  0//1  0//1  0//1  1//1  0//1
 0//1  0//1  0//1  0//1  0//1  0//1  0//1  0//1  0//1  1//1
```

```
## Floating point numbers
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 595.89 MB Saving completed Mode: Command Ln 3, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help | Mem:632 MB

Untitled6.ipynb

Code | git | Julia 1.8.5

```
## Floating point numbers
```

```
[52]: isapprox(inv(A) * A, I)
```

```
[52]: true
```

```
[53]: inv(A) * A ≈ I
```

```
[53]: true
```

\approx

```
[ ]:
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 631.93 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:529 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
[55]: 42

[56]: # ẋ x′

[58]: Rational(0.1)

[58]: 3602879701896397//36028797018963968

[60]: float(Rational(0.1) - 1//10)

[60]: 5.551115123125783e-18

[61]: π

[61]: π = 3.1415926535897...

[ ]:
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 529.08 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help Mem:588 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
[58]: Rational(0.1)

[58]: 3602879701896397//36028797018963968

[60]: float(Rational(0.1) - 1//10)

[60]: 5.551115123125783e-18

[61]: π

[61]: π = 3.1415926535897...

[62]: Float64(π)

[62]: 3.141592653589793

[63]: BigFloat(π)

[63]: 3.141592653589793238462643383279502884197169399375105820974944592307816406286198

[ ]:
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 588.18 MB Saving completed Mode: Edit Ln 1, Col 1 Untitled6.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help Mem:667 MB

Untitled6.ipynb

Code git Julia 1.8.5

```
[58]: Rational(0.1)

[58]: 3602879701896397//36028797018963968

[60]: float(Rational(0.1) - 1//10)

[60]: 5.551115123125783e-18

[61]: π

[61]: π = 3.1415926535897...

[62]: Float64(π)

[62]: 3.141592653589793

[63]: BigFloat(π)

[63]: 3.141592653589793238462643383279502884197169399375105820974944592307816406286198

[ ]: isapprox()
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 666.70 MB Saving completed Mode: Edit Ln 1, Col 10 Untitled6.ipynb

approximate $\sin(x)$ via polynomials on $I = [0, \frac{\pi}{2}]$

$$f(x) = \sum_{p=0}^{N} c_p x^p$$

choose R points on I

$$x_i = \frac{\pi}{2} \frac{i}{R-1} \quad \text{Lin Range}$$

goal $$G = \sum_{i=0}^{R-1} \left(\sin x_i - \sum_{p=0}^{N} c_p x^p \right)^2$$

minimize G

$$\left(-\langle c|A^T + \langle b|\right)\left(|b\rangle - A|c\rangle\right)$$

$$G = \langle b|b\rangle - \langle c|A^T|b\rangle - \langle b|A|c\rangle + \langle c|A^TA|c\rangle$$

$$-2\langle b|A + 2\langle c|A^TA = 0$$

$$\boxed{A^TA|c\rangle = A^T|b\rangle}$$

symmetry.pi.local

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File Edit View Run Kernel Git Tabs Settings Help

Mem:517 MB

Linear Algebra 2023.ipynb

Markdown git Julia 1.8.5

[52]: myisapprox (generic function with 1 method)

```
[53]: myisapprox(1, 1.00001)
```

[53]: true

```
# L
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 516.56 MB Saving completed Mode: Edit Ln 1, Col 4 Linear Algebra 2023.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:570 MB

Linear Algebra 2023.ipynb

Code git Julia 1.8.5

[53]:
```julia
myisapprox(1, 1.00001)
```

[53]:
```
true
```

Least Squares

[54]:
```julia
# expansion coefficients
pmax = 4
```

[54]:
```
4
```

[57]:
```julia
# evaluation points
npoints = 11
x = LinRange(0, π/2, npoints)
```

[57]:
```
11-element LinRange{Float64, Int64}:
 0.0,0.15708,0.314159,0.471239,0.628319,…,1.09956,1.25664,1.41372,1.5708
```

[ ]:

Simple 0 1 Julia 1.8.5 | Idle Mem: 569.75 MB Saving completed Mode: Command Ln 1, Col 1 Linear Algebra 2023.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:522 MB

Linear Algebra 2023.ipynb

Code git Julia 1.8.5

```
[57]: 11-element LinRange{Float64, Int64}:
 0.0,0.15708,0.314159,0.471239,0.628319,…,1.09956,1.25664,1.41372,1.5708
```

```
[58]: # target function
f(x) = sin(x)
```

```
[58]: f (generic function with 1 method)
```

```
[59]: # matrix
A = [x[i]^p for i in 1:npoints, p in 0:pmax]
```

```
[59]: 11×5 Matrix{Float64}:
 1.0  0.0       0.0       0.0         0.0
 1.0  0.15708   0.024674  0.00387578  0.000608807
 1.0  0.314159  0.098696  0.0310063   0.00974091
 1.0  0.471239  0.222066  0.104646    0.0493134
 1.0  0.628319  0.394784  0.24805     0.155855
 1.0  0.785398  0.61685   0.484473    0.380504
 1.0  0.942478  0.888264  0.837169    0.789014
 1.0  1.09956   1.20903   1.32939     1.46175
 1.0  1.25664   1.57914   1.9844      2.49367
 1.0  1.41372   1.99859   2.82545     3.99438
 1.0  1.5708    2.4674    3.87578     6.08807
```

```
[ ]:
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 522.44 MB Saving completed Mode: Edit Ln 1, Col 1 Linear Algebra 2023.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:619 MB

Linear Algebra 2023.ipynb

Code git Julia 1.8.5

[61]:
```
10-element Vector{Float64}:
 -1.0384011702665148
  0.5849557288759659
 -0.6944263228609986
 -0.6838666812665662
 -0.33453006696954807
 -1.048578179494093
  0.41134951216906357
  1.042736044039715
 -1.5564326152880863
 -0.43795555286709137
```

[62]:
```
2 / 3
```

[62]:
```
0.6666666666666666
```

[63]:
```
2 \ 3
```

[63]:
```
1.5
```

[ ]:
```
* x = A \ b
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 619.21 MB Saving completed Mode: Edit Ln 1, Col 1 Linear Algebra 2023.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help Mem:508 MB

Linear Algebra 2023.ipynb

Code git Julia 1.8.5

[62]:
```
2 / 3
```

[62]:
```
0.6666666666666666
```

[63]:
```
2 \ 3
```

[63]:
```
1.5
```

[65]:
```
A * x = b
```

[65]:
```
10-element Vector{Float64}:
 -0.9063723567849704
 -0.930448325898977
 -1.6857217510127993
 -0.3235371956379028
 -1.509754594119294
  0.2849739657226849
 -1.0920555313836793
  1.5372840342353487
 -0.004295349465374819
  0.911999627253853
```

[66]:
```
A * x \app b
```

[66]:
```
false
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 507.68 MB Saving completed Mode: Edit Ln 1, Col 9 Linear Algebra 2023.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help Mem:573 MB

Linear Algebra 2023.ipynb

Code git Julia 1.8.5

[60]:
```
# target function
fstar(x) = sin(x)
```

[60]: fstar (generic function with 1 method)

[59]:
```
# matrix
A = [x[i]^p for i in 1:npoints, p in 0:pmax]
```

[59]:
```
11×5 Matrix{Float64}:
 1.0  0.0       0.0       0.0         0.0
 1.0  0.15708   0.024674  0.00387578  0.000608807
 1.0  0.314159  0.098696  0.0310063   0.00974091
 1.0  0.471239  0.222066  0.104646    0.0493134
 1.0  0.628319  0.394784  0.24805     0.155855
 1.0  0.785398  0.61685   0.484473    0.380504
 1.0  0.942478  0.888264  0.837169    0.789014
 1.0  1.09956   1.20903   1.32939     1.46175
 1.0  1.25664   1.57914   1.9844      2.49367
 1.0  1.41372   1.99859   2.82545     3.99438
 1.0  1.5708    2.4674    3.87578     6.08807
```

[ ]:

Solving linear systems

Simple 0 1 Julia 1.8.5 | Idle Mem: 571.97 MB Saving completed Mode: Edit Ln 1, Col 1 Linear Algebra 2023.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help Mem:629 MB

Linear Algebra 2023.ipynb

Markdown git Julia 1.8.5

```
 1.0  0.628319  0.394784  0.24805    0.155855
 1.0  0.785398  0.61685   0.484473   0.380504
 1.0  0.942478  0.888264  0.837169   0.789014
 1.0  1.09956   1.20903   1.32939    1.46175
 1.0  1.25664   1.57914   1.9844     2.49367
 1.0  1.41372   1.99859   2.82545    3.99438
 1.0  1.5708    2.4674    3.87578    6.08807
```

[70]:
```
c = (A'*A) \ (A'*b)
```

[70]:
```
10-element Vector{Float64}:
 -0.06099791561759082
 -3.56311781844371
 -0.28299302279520777
  8.446950631598714
 -0.5035651073066433
 -5.433917594114555
 -1.3983261684328165
 -3.963733076297944
 -1.142244136318507
  2.3977209043388754
```

Solving linear systems

[69]:
```
A = randn(10,10)
b = randn(10)
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 628.68 MB Saving completed Mode: Command Ln 1, Col 26 Linear Algebra 2023.ipynb

symmetry.pi.local

JupyterLab

File Edit View Run Kernel Git Tabs Settings Help

Mem:800 MB

Linear Algebra 2023.ipynb

Code git Julia 1.8.5

[73]: fstar (generic function with 1 method)

[74]:
```julia
# matrix
A = [x[i]^p for i in 1:npoints, p in 0:pmax]
```

[74]: 11×5 Matrix{Float64}:

```
 1.0  0.0       0.0       0.0         0.0
 1.0  0.15708   0.024674  0.00387578  0.000608807
 1.0  0.314159  0.098696  0.0310063   0.00974091
 1.0  0.471239  0.222066  0.104646    0.0493134
 1.0  0.628319  0.394784  0.24805     0.155855
 1.0  0.785398  0.61685   0.484473    0.380504
 1.0  0.942478  0.888264  0.837169    0.789014
 1.0  1.09956   1.20903   1.32939     1.46175
 1.0  1.25664   1.57914   1.9844      2.49367
 1.0  1.41372   1.99859   2.82545     3.99438
 1.0  1.5708    2.4674    3.87578     6.08807
```

[75]:
```julia
c = (A'*A) \ (A'*b)
```

```
DimensionMismatch: second dimension of A, 11, does not match length of x, 10

Stacktrace:
 [1] gemv!(y::Vector{Float64}, tA::Char, A::Matrix{Float64}, x::Vector{Float64}, α::Bool, β::Bool)
   @ LinearAlgebra ~/.julia/juliaup/julia-1.8.5+0.x64.linux.gnu/share/julia/stdlib/v1.8/LinearAlgebra/src/matmul.j
l:493
```

Simple 0 1 Julia 1.8.5 | Idle Mem: 799.72 MB Saving completed Mode: Command Ln 1, Col 1 Linear Algebra 2023.ipynb