Title: Phase space extensions at Null infinity

Speakers: Javier Peraza

Series: Quantum Gravity

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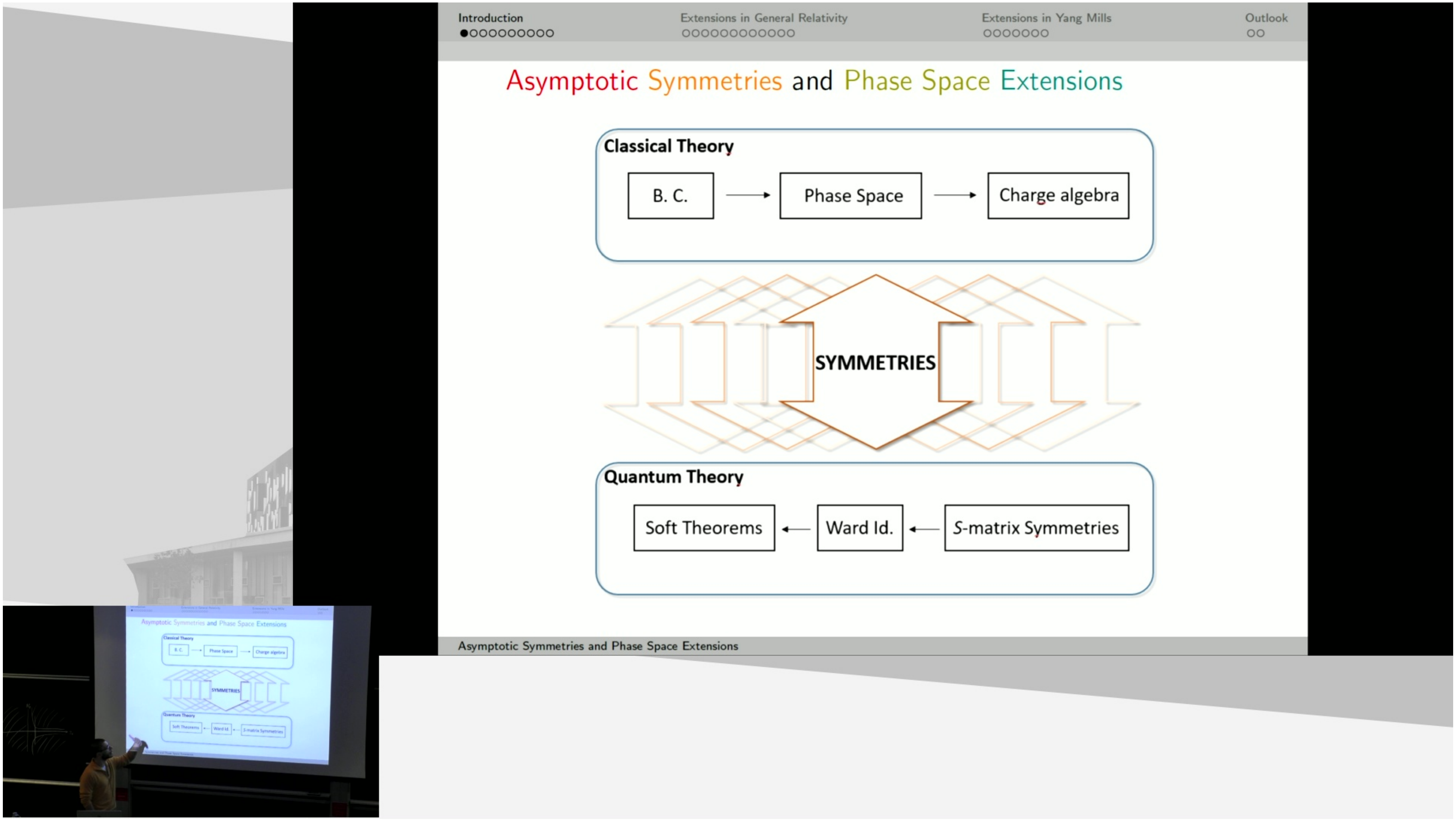
Abstract: Extensions of asymptotic phase spaces and their corresponding asymptotic symmetry groups have become a topic of increasing interest in recent years, due to results that connect them with a wide spectrum of areas, such as symmetries of the S-matrix, soft theorems, corner symmetries, double copy maps and celestial holography. The study of these extensions aims to characterize the degrees of freedom of the physical theories at the classical level, gathering information on how their symmetries can be upgraded to the quantum theories. In this talk, I will describe extensions of phase spaces at null infinity for gravity and gauge theories, such that the charges are consistent with tree-level soft (graviton, gluon or photon) theorems and act canonically. First, as motivation, I will show how the Geroch group for cylindrically symmetric general relativity can be upgraded as a quantum symmetry, exploiting the integrability of the system (arXiv:1906.04856 [gr-qc]). Second, I will review the construction of an extended phase space where the generalized BMS symmetry group acts canonically (arXiv:2002.06691 [gr-qc]). I will show that this construction is consistent with the extended corner symmetry approach (via the embedding maps). Finally, I will show that a similar approach can be done in Yang Mills, by extending the phase space with "Goldstone modes" that transform inhomogeneously under linearized O(r) symmetries (arXiv:2111.00973 [hep-th]). Some preliminary results regarding the extension to higher order O(r^n) will be discussed (arXiv:2211.12991 [hep-th]).

Zoom link: https://pitp.zoom.us/j/95866794894?pwd=UEtQYlBpV0NCM3cyeWpDcTI2WVp6UT09

Asymptotic Symmetries and Phase Space Extensions

Javier Peraza

Perimeter Institute, January 19th 2023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Asymptotic Symmetries and Phase Space Extensions
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B. C.
Phase Space
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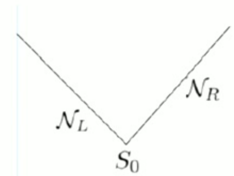
Asymptotic Symmetries and Phase Space Extensions

Non-trivial example: Cylindrically symmetric gravity

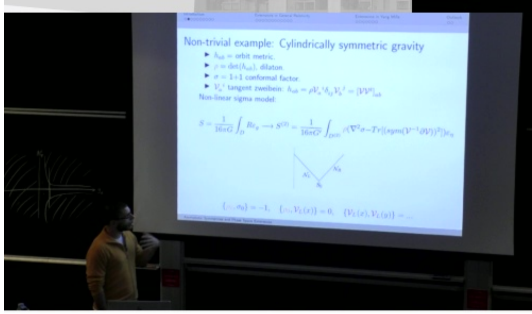
- h_{ab} = orbit metric.
- $\rho = \det(h_{ab})$, dilaton.
- σ = 1+1 conformal factor.
- ${\mathcal{V}_a}^i$ tangent zweibein: $h_{ab} = \rho {\mathcal{V}_a}^i \delta_{ij} {\mathcal{V}_b}^j = [\mathcal{V}\mathcal{V}^t]_{ab}$

Non-linear sigma model:

$$S = \frac{1}{16\pi G}\int_D R\varepsilon_g \longrightarrow S^{(2)} = \frac{1}{16\pi G'}\int_{D^{(2)}} \rho(\nabla^2\sigma - Tr[(sym(\mathcal{V}^{-1}\partial\mathcal{V}))^2])\varepsilon_\eta$$



$$\{\rho_0, \sigma_0\} = -1, \quad \{\rho_0, \mathcal{V}_L(x)\} = 0, \quad \{\mathcal{V}_L(x), \mathcal{V}_L(y)\} = \ldots$$



Via deformation with a (variable) spectral parameter, $u(x) = \sqrt{\frac{\rho^+(x)+w}{\rho^-(x)+w}}$,

$$\hat{\mathcal{V}}(y;u) = \mathcal{V}(0)\mathcal{P}\exp\left(\int_0^y \hat{J}\right)$$

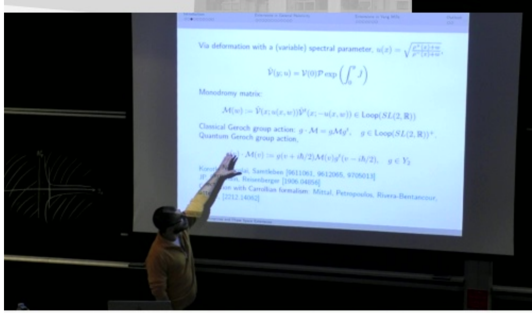
Monodromy matrix:

$$\mathcal{M}(w) := \hat{\mathcal{V}}(x;u(x,w))\hat{\mathcal{V}}^t(x;-u(x,w)) \in \text{Loop}(SL(2,\mathbb{R}))$$

Classical Geroch group action: $g\cdot\mathcal{M} = g\mathcal{M}g^t, \quad g \in \text{Loop}(SL(2,\mathbb{R}))^+$.
Quantum Geroch group action,

$$g(v)\cdot\mathcal{M}(v) := g(v+i\hbar/2)\mathcal{M}(v)g^t(v-i\hbar/2), \quad g \in Y_2$$

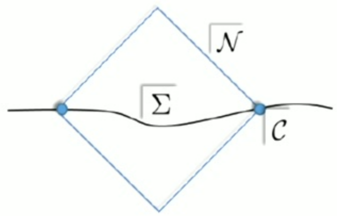
Korotkin, Nicolai, Samtleben [9611061, 9612065, 9705013]
JP, Paternain, Reisenberger [1906.04856]
Connection with Carrollian formalism: Mittal, Petropoulos, Rivera-Bentancour, Vilatte, [2212.14062]



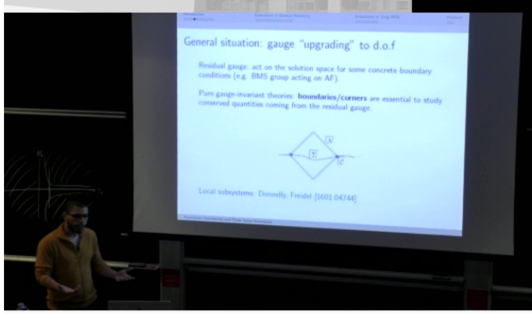
General situation: gauge "upgrading" to d.o.f

Residual gauge: act on the solution space for some concrete boundary conditions (e.g. BMS group acting on AF).

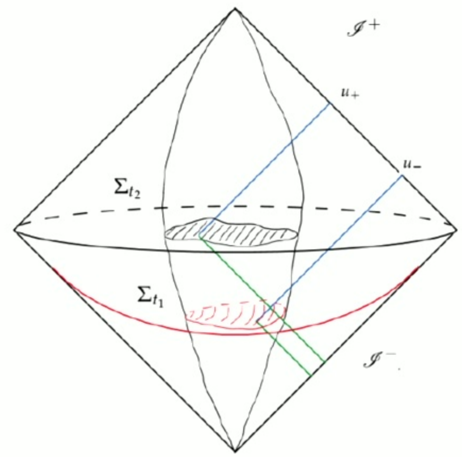
Pure gauge-invariant theories: **boundaries/corners** are essential to study conserved quantities coming from the residual gauge.



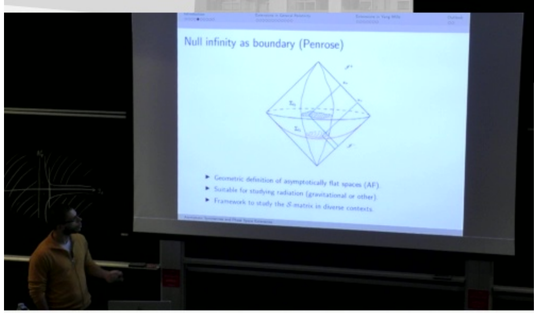
Local subsystems: Donnelly, Freidel [1601.04744]



Null infinity as boundary (Penrose)



- Geometric definition of asymptotically flat spaces (AF).
- Suitable for studying radiation (gravitational or other).
- Framework to study the $\mathcal{S}$-matrix in diverse contexts.



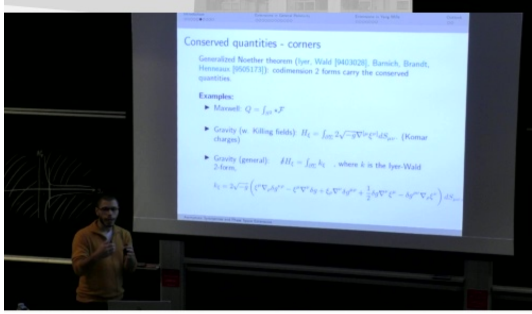
Conserved quantities - corners

Generalized Noether theorem (Iyer, Wald [9403028], Barnich, Brandt, Henneaux [9505173]): codimension 2 forms carry the conserved quantities.

Examples:

- Maxwell: $Q = \int_{S^2} \star\mathcal{F}$
- Gravity (w. Killing fields): $H_\xi = \int_{\partial\Sigma} 2\sqrt{-g}\nabla^{[\mu}\xi^{\nu]} dS_{\mu\nu}$. (Komar charges)
- Gravity (general): $\delta\!\!\!/ H_\xi = \int_{\partial\Sigma} k_\xi$, where k is the Iyer-Wald 2-form,

$$k_\xi = 2\sqrt{-g}\left(\xi^\mu \nabla_\rho \delta g^{\nu\rho} - \xi^\mu \nabla^\nu \delta g + \xi_\rho \nabla^\nu \delta g^{\mu\rho} + \frac{1}{2}\delta g \nabla^\nu \xi^\mu - \delta g^{\rho\nu}\nabla_\rho \xi^\nu\right) dS_{\mu\nu},$$



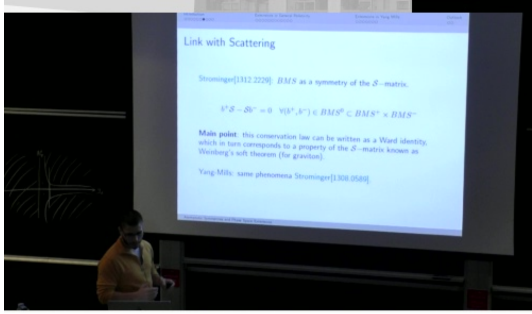
Link with Scattering

Strominger[1312.2229]: BMS as a symmetry of the $\mathcal{S}-$matrix.

$$b^+\mathcal{S} - \mathcal{S}b^- = 0 \quad \forall (b^+, b^-) \in BMS^0 \subset BMS^+ \times BMS^-$$

Main point: this conservation law can be written as a Ward identity, which in turn corresponds to a property of the $\mathcal{S}-$matrix known as Weinberg's soft theorem (for graviton).

Yang-Mills: same phenomena Strominger[1308.0589].



Soft theorems: universal property

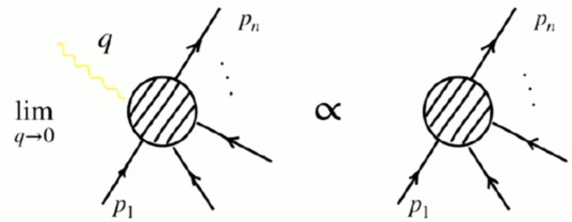
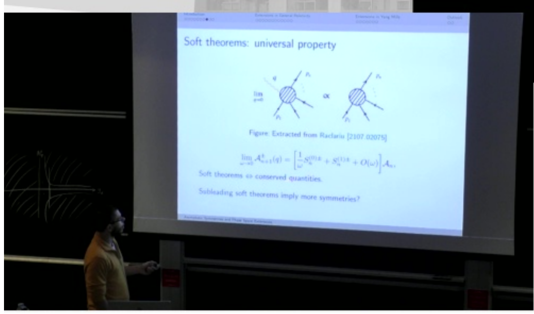


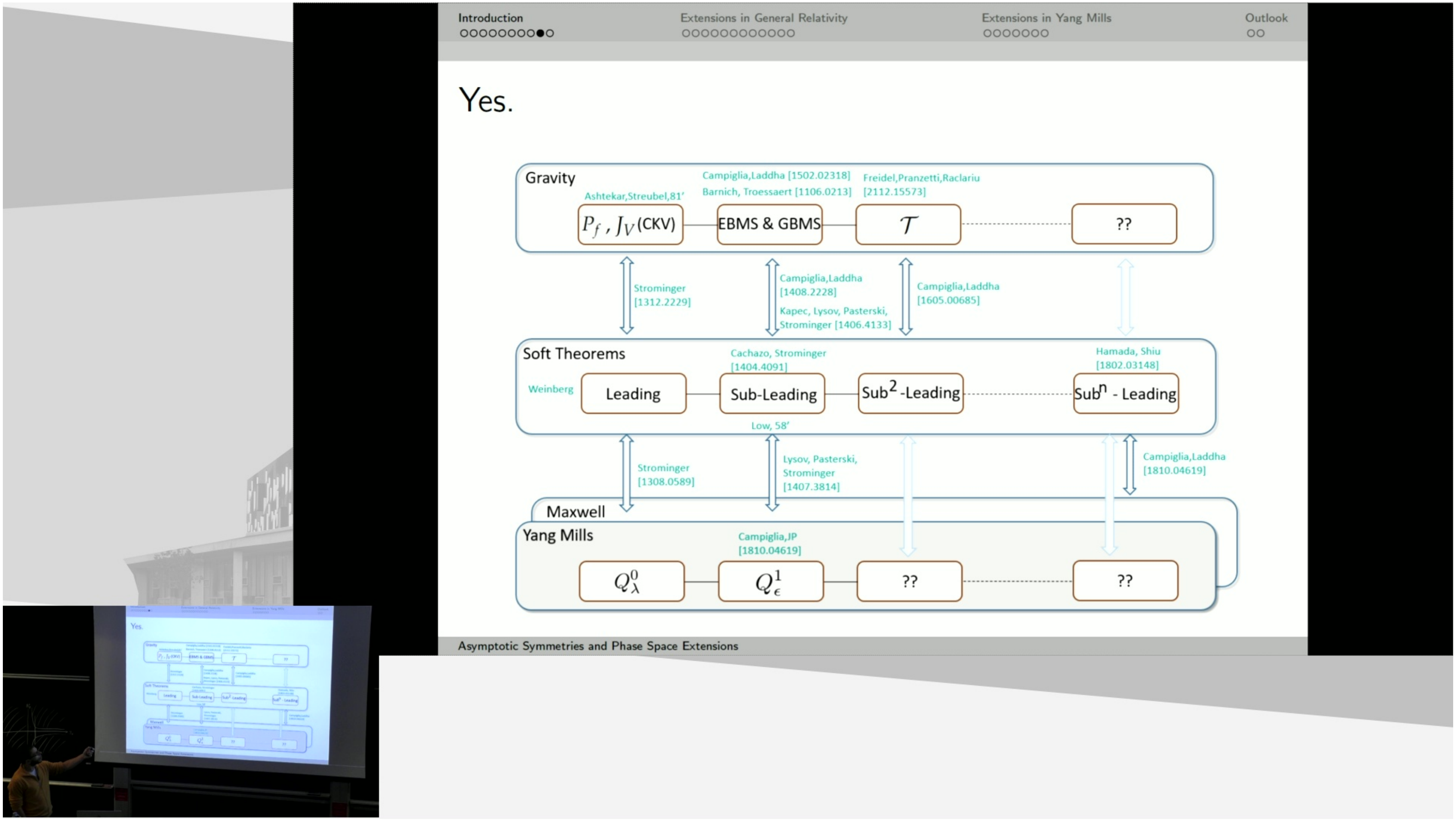
Figure: Extracted from Raclariu [2107.02075]

$$\lim_{\omega\to 0}\mathcal{A}^{\pm}_{n+1}(q)=\left[\frac{1}{\omega}S_n^{(0)\pm}+S_n^{(1)\pm}+O(\omega)\right]\mathcal{A}_n,$$

Soft theorems ⇔ conserved quantities.

Subleading soft theorems imply more symmetries?




Introduction
Extensions in General Relativity
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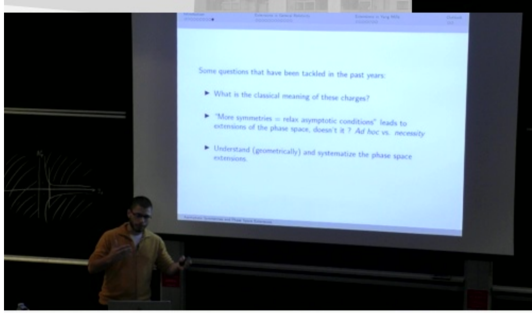
Outlook
Yes.
Gravity
Ashtekar,Streubel,81'
Campiglia,Laddha [1502.02318]
Barnich, Troessaert [1106.0213]
Freidel,Pranzetti,Raclariu
[2112.15573]
P_f , J_V (CKV)
EBMS & GBMS
$\mathcal{T}$
??
Strominger
[1312.2229]
Campiglia,Laddha
[1408.2228]
Kapec, Lysov, Pasterski,
Strominger [1406.4133]
Campiglia,Laddha
[1605.00685]
Soft Theorems
Cachazo, Strominger
[1404.4091]
Hamada, Shiu
[1802.03148]
Weinberg
Leading
Sub-Leading
Sub^2-Leading
Sub^n - Leading
Low, 58'
Strominger
[1308.0589]
Lysov, Pasterski,
Strominger
[1407.3814]
Campiglia,Laddha
[1810.04619]
Maxwell
Yang Mills
Campiglia,JP
[1810.04619]
Q^0_λ
Q^1_ϵ
??
??
Asymptotic Symmetries and Phase Space Extensions

Some questions that have been tackled in the past years:

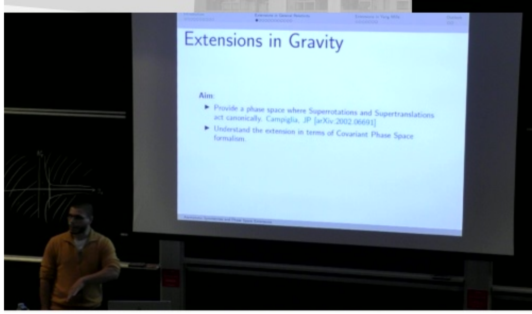
- What is the classical meaning of these charges?
- "More symmetries = relax asymptotic conditions" leads to extensions of the phase space, doesn't it ? *Ad hoc* vs. *necessity*
- Understand (geometrically) and systematize the phase space extensions.



Extensions in Gravity

Aim:

- Provide a phase space where Superrotations and Supertranslations act canonically. Campiglia, JP [arXiv:2002.06691]
- Understand the extension in terms of Covariant Phase Space formalism.



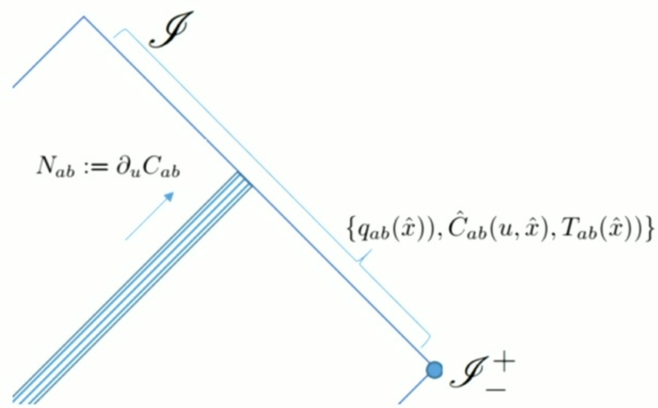
Bondi coordiantes:

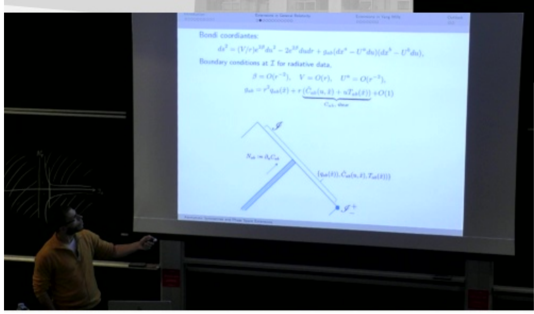
$$ds^2 = (V/r)e^{2\beta}du^2 - 2e^{2\beta}dudr + g_{ab}(dx^a - U^a du)(dx^b - U^b du),$$

Boundary conditions at $\mathcal{I}$ for radiative data,

$$\beta = O(r^{-2}), \quad V = O(r), \quad U^a = O(r^{-2}),$$

$$g_{ab} = r^2 q_{ab}(\hat{x}) + r \underbrace{(\hat{C}_{ab}(u,\hat{x}) + uT_{ab}(\hat{x}))}_{C_{ab}\text{, shear}} + O(1)$$





In the search of more asymptotic symmetries

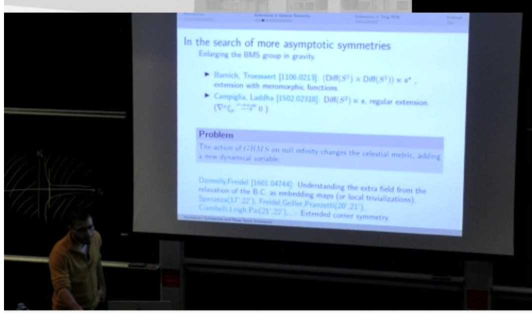
Enlarging the BMS group in gravity.

- Barnich, Troessaert [1106.0213]: $(\text{Diff}(S^1) \times \text{Diff}(S^1)) \ltimes \mathfrak{s}^*$, extension with meromorphic functions.
- Campiglia, Laddha [1502.02318]: $\text{Diff}(S^2) \ltimes \mathfrak{s}$, regular extension. $(\nabla^\mu \xi_\mu \overset{r \to +\infty}{\longrightarrow} 0)$

Problem

The action of $GBMS$ on null infinity changes the celestial metric, adding a new dynamical variable.

Donnelly,Freidel [1601.04744]: Understanding the extra field from the relaxation of the B.C. as embedding maps (or local trivializations).
Speranza(17',22'), Freidel,Geiller,Pranzetti(20',21'), Ciambelli,Leigh,Pai(21',22'),...: Extended corner symmetry.



Diffeomorphisms and Variation algebra

General diffeomorphism preserving the gauge:

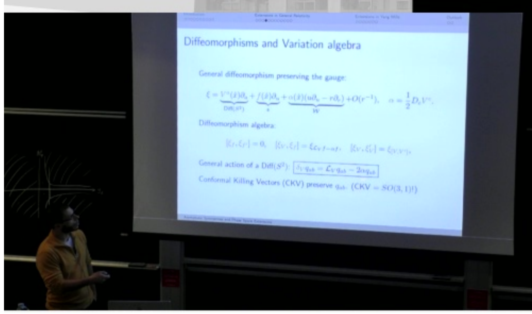
$$\xi = \underbrace{V^a(\hat{x})\partial_a}_{\mathrm{Diff}(S^2)} + \underbrace{f(\hat{x})\partial_u}_{\mathfrak{s}} + \underbrace{\alpha(\hat{x})(u\partial_u - r\partial_r)}_{\mathcal{W}} + O(r^{-1}), \quad \alpha = \frac{1}{2}D_c V^c.$$

Diffeomorphism algebra:

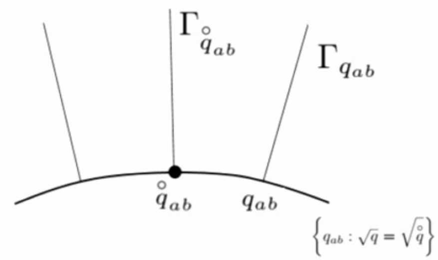
$$[\xi_f, \xi_{f'}] = 0, \quad [\xi_V, \xi_f] = \xi_{\mathcal{L}_V f - \alpha f}, \quad [\xi_V, \xi'_V] = \xi_{[V,V']},$$

General action of a Diff(S^2): $\boxed{\delta_V q_{ab} = \mathcal{L}_V q_{ab} - 2\alpha q_{ab}}$

Conformal Killing Vectors (CKV) preserve q_{ab}. (CKV $= SO(3,1)$!)



Extended Phase Space



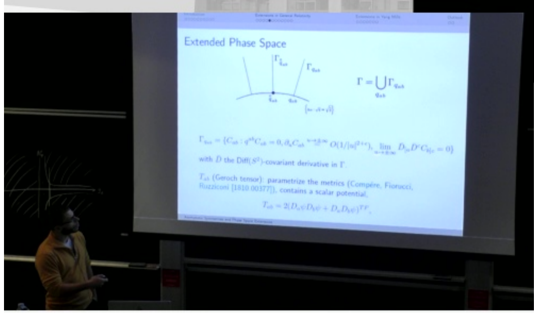
$$\Gamma = \bigcup_{q_{ab}} \Gamma_{q_{ab}}$$

$$\Gamma_{q_{ab}} = \{C_{ab} : q^{ab}C_{ab} = 0, \partial_u C_{ab} \overset{u\to\pm\infty}{=} O(1/|u|^{2+\epsilon}), \lim_{u\to\pm\infty} \bar{D}_{[a}\bar{D}^c C_{b]c} = 0\}$$

with $\bar{D}$ the Diff(S^2)-covariant derivative in Γ.

T_{ab} (Geroch tensor): parametrize the metrics (Compére, Fiorucci, Ruzziconi [1810.00377]), contains a scalar potential,

$$T_{ab} = 2(D_a\psi D_b\psi + D_a D_b\psi)^{TF},$$



Charges from Soft Theorems

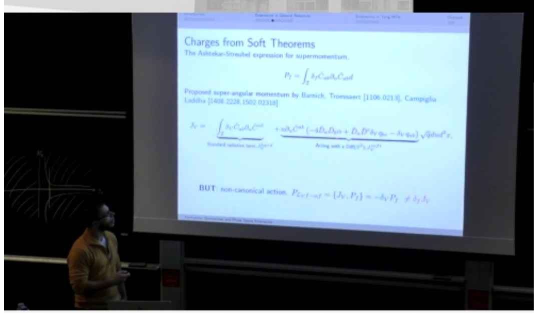
The Ashtekar-Streubel expression for supermomentum,

$$P_f = \int_{\mathcal{I}} \delta_f \hat{C}_{ab} \partial_u \hat{C}_{ab} d$$

Proposed super-angular momentum by Barnich, Troessaert [1106.0213], Campiglia Laddha [1408.2228,1502.02318]

$$J_V = \underbrace{\int_{\mathcal{I}} \delta_V \hat{C}_{ab} \partial_u \hat{C}^{ab}}_{\text{Standard radiative term}, J_V^{hard}} + \underbrace{u \partial_u \hat{C}^{ab} \left(-4 \bar{D}_a \bar{D}_b \alpha + \bar{D}_a \bar{D}^c \delta_V q_{bc} - \delta_V q_{ab} \right)}_{\text{Acting with a Diff}(S^2), J_V^{soft}} \sqrt{q} du d^2 x,$$

BUT: non-canonical action. $P_{\mathcal{L}_V f - \alpha f} = \{J_V, P_f\} = -\delta_V P_f \neq \delta_f J_V$



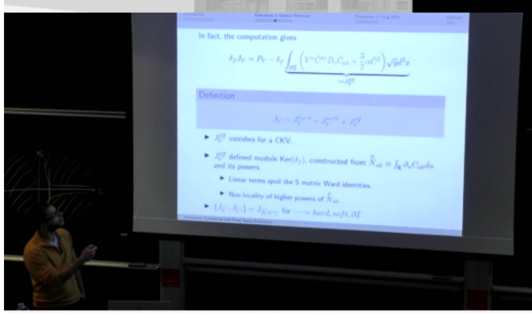
In fact, the computation gives

$$\delta_f J_V = P_V - \delta_f \underbrace{\int_{\partial\mathcal{I}} \left(V^a \hat{C}^{bc} \bar{D}_c \hat{C}_{ab} + \frac{3}{2}\alpha \hat{C}^2 \right) \sqrt{q} d^2x}_{:= J_V^{\partial\mathcal{I}}}$$

Definition

$$J_V = J_V^{hard} + J_V^{soft} + J_V^{\partial\mathcal{I}}$$

- $J_V^{\partial\mathcal{I}}$ vanishes for a CKV.
- $J_V^{\partial\mathcal{I}}$ defined module Ker(δ_f), constructed from $\overset{0}{N}_{ab} \equiv \int_{\mathbb{R}} \partial_u C_{ab} du$ and its powers.
  - Linear terms spoil the S matrix Ward identities.
  - Non-locality of higher powers of $\overset{0}{N}_{ab}$.
- $\{J_V^{\cdots}, J_{V'}^{\cdots}\} = J_{[V,V']}^{\cdots}$, for $\cdots = hard, soft, \partial\mathcal{I}$



Extension of the phase space

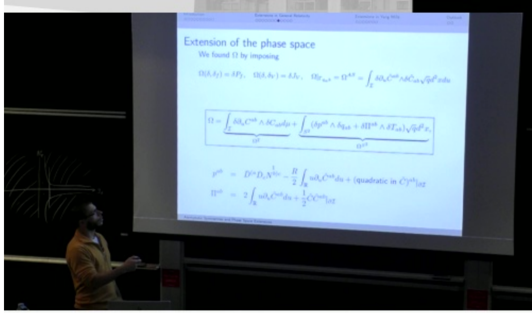
We found Ω by imposing

$$\Omega(\delta,\delta_f)=\delta P_f,\quad \Omega(\delta,\delta_V)=\delta J_V,\quad \Omega|_{\Gamma_{q_{ab}}}=\Omega^{AS}=\int_{\mathcal{I}}\delta\partial_u\hat{C}^{ab}\wedge\delta\hat{C}_{ab}\sqrt{q}\,d^2x\,du$$

$$\Omega=\underbrace{\int_{\mathcal{I}}\delta\partial_u C^{ab}\wedge\delta C_{ab}\,d\mu}_{\Omega^{\mathcal{I}}}+\underbrace{\int_{S^2}(\delta p^{ab}\wedge\delta q_{ab}+\delta\Pi^{ab}\wedge\delta T_{ab})\sqrt{q}\,d^2x}_{\Omega^{S^2}},$$

$$p^{ab} = \bar{D}^{(a}\bar{D}_c\overset{1}{N}{}^{b)c}-\frac{R}{2}\int_{\mathbb{R}}u\partial_u\hat{C}^{ab}du+(\text{quadratic in }\hat{C})^{ab}|_{\partial\mathcal{I}}$$

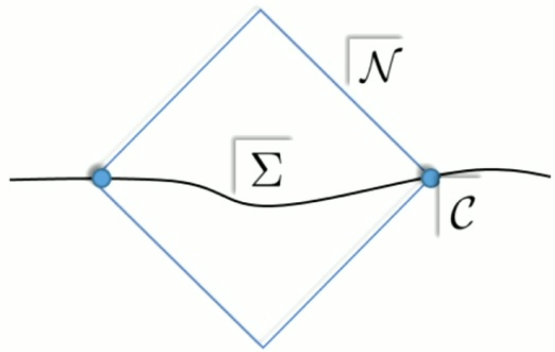
$$\Pi^{ab} = 2\int_{\mathbb{R}}u\partial_u\hat{C}^{ab}du+\frac{1}{2}\hat{C}\hat{C}^{ab}|_{\partial\mathcal{I}}$$



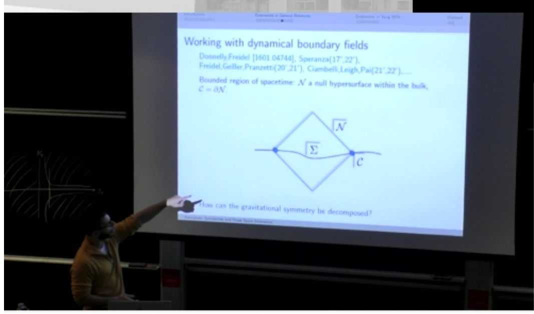
Working with dynamical boundary fields

Donnelly,Freidel [1601.04744], Speranza(17',22'), Freidel,Geiller,Pranzetti(20',21'), Ciambelli,Leigh,Pai(21',22'),....

Bounded region of spacetime: $\mathcal{N}$ a null hypersurface within the bulk, $\mathcal{C} = \partial\mathcal{N}$.



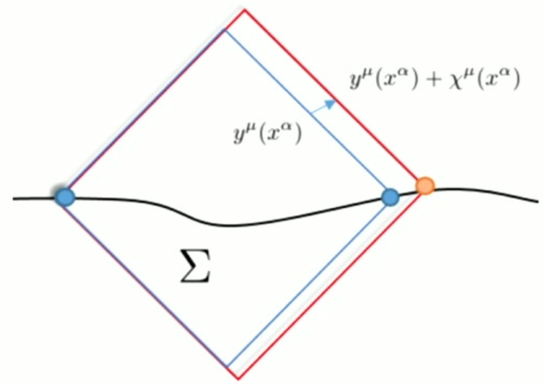
How can the gravitational symmetry be decomposed?

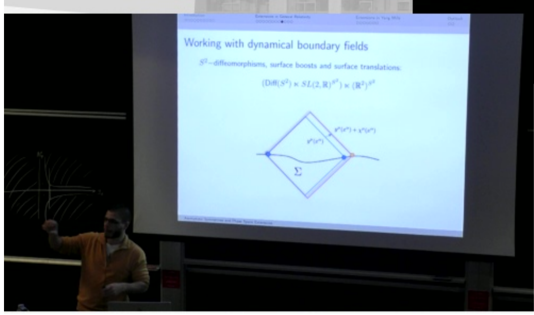


Working with dynamical boundary fields

S^2-diffeomorphisms, surface boosts and surface translations:

$$(\mathrm{Diff}(S^2) \ltimes SL(2,\mathbb{R})^{S^2}) \ltimes (\mathbb{R}^2)^{S^2}$$

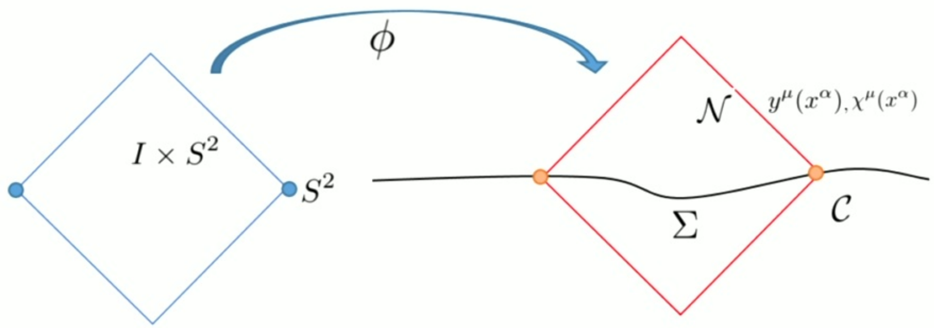




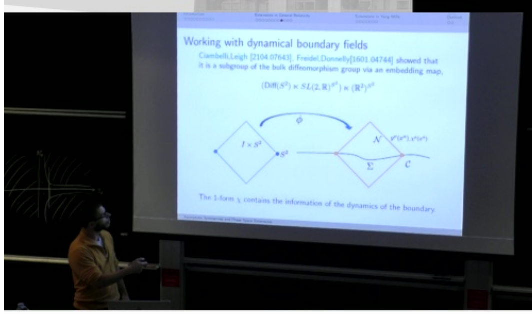
Working with dynamical boundary fields

Ciambelli,Leigh [2104.07643], Freidel,Donnelly[1601.04744] showed that it is a subgroup of the bulk diffeomorphism group via an embedding map,

$$(\mathrm{Diff}(S^2) \ltimes SL(2,\mathbb{R})^{S^2}) \ltimes (\mathbb{R}^2)^{S^2}$$



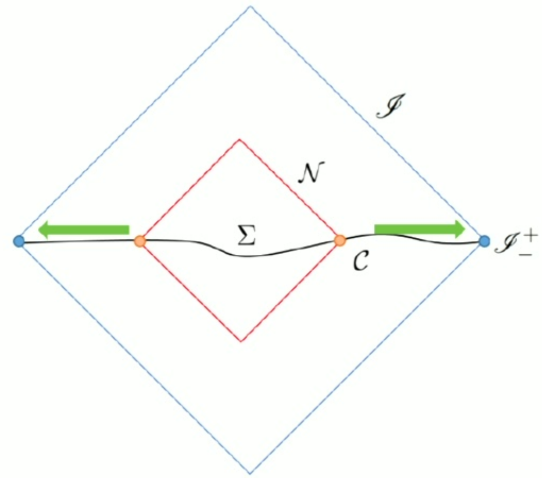
The 1-form χ contains the information of the dynamics of the boundary.

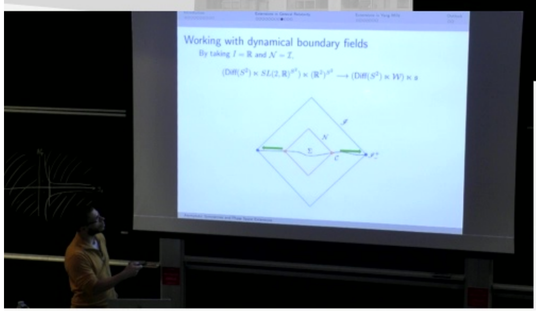


Working with dynamical boundary fields

By taking $I = \mathbb{R}$ and $\mathcal{N} = \mathcal{I}$,

$$(\mathrm{Diff}(S^2) \ltimes SL(2,\mathbb{R})^{S^2}) \ltimes (\mathbb{R}^2)^{S^2} \longrightarrow (\mathrm{Diff}(S^2) \ltimes \mathcal{W}) \ltimes \mathfrak{s}$$





Revealing $J_V^{\partial\mathcal{I}}$ in a more geometric approach

Let $\mathcal{G}$ be the embedding map,

$$\mathcal{G} : Sol \to C^\infty(\mathcal{I} \to \mathcal{M})$$

and $\chi_\mathcal{G}$ its vector field generator, schematically,

$$\chi_\mathcal{G} = \delta\mathcal{G}\mathcal{G}^{-1}, \qquad \chi_\mathcal{G}[\hat{\xi}_V] = -\xi_V$$

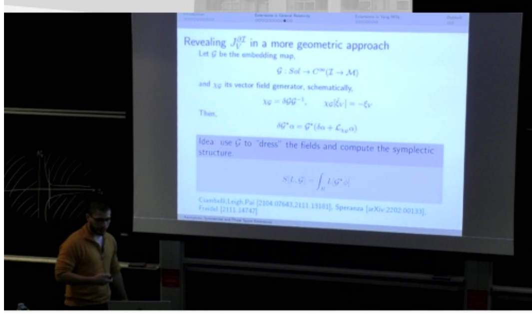
Then,

$$\delta\mathcal{G}^*\alpha = \mathcal{G}^*(\delta\alpha + \mathcal{L}_{\chi_\mathcal{G}}\alpha)$$

Idea: use $\mathcal{G}$ to "dress" the fields and compute the symplectic structure.

$$S[L, \mathcal{G}] = \int_R L[\mathcal{G}^*\phi]$$

Ciambelli,Leigh,Pai [2104.07643,2111.13181], Speranza [arXiv:2202.00133], Freidel [2111.14747]



Symplectic structure

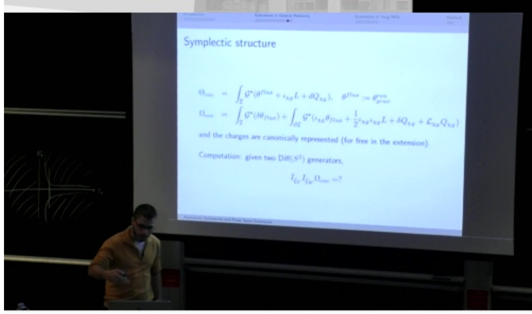
$$\Theta_{cov} = \int_{\mathcal{I}} \mathcal{G}^*(\theta^{flux} + \iota_{\chi_\mathcal{G}} L + dQ_{\chi_\mathcal{G}}), \quad \theta^{flux} := \theta^{ren}_{grav}$$

$$\Omega_{cov} = \int_{\mathcal{I}} \mathcal{G}^*(\delta\theta_{flux}) + \int_{\partial\mathcal{I}} \mathcal{G}^*(\iota_{\chi_\mathcal{G}}\theta_{flux} + \frac{1}{2}\iota_{\chi_\mathcal{G}}\iota_{\chi_\mathcal{G}} L + \delta Q_{\chi_\mathcal{G}} + \mathcal{L}_{\chi_\mathcal{G}} Q_{\chi_\mathcal{G}})$$

and the charges are canonically represented (for free in the extension).

Computation: given two Diff(S^2) generators,

$$I_{\hat{\xi}_V} I_{\hat{\xi}_W} \Omega_{cov} = ?$$



$$I_{\hat{\xi}_V} I_{\hat{\xi}_W} \left(\int_{\partial \mathcal{I}} \mathcal{G}^*(\iota_{\chi_\mathcal{G}} \theta_{flux}) \right) = \int_{\partial \mathcal{I}} \alpha_W u N^{ab} \delta_V C_{ab} - (V \leftrightarrow W) = 0(\text{ by B.C}$$

$$I_{\hat{\xi}_V} I_{\hat{\xi}_W} \left(\delta Q_{\chi_\mathcal{G}} + \mathcal{L}_{\chi_\mathcal{G}} Q_{\chi_\mathcal{G}} \right) = \underbrace{(\mathfrak{L}_{\hat{\xi}_W} - \mathcal{L}_{\xi_W})}_{\text{Anomaly operator}} Q_{\xi_V} - (V \leftrightarrow W)$$

Freidel, Oliveri, Pranzetti, Speziale [2104.12881]

Finally

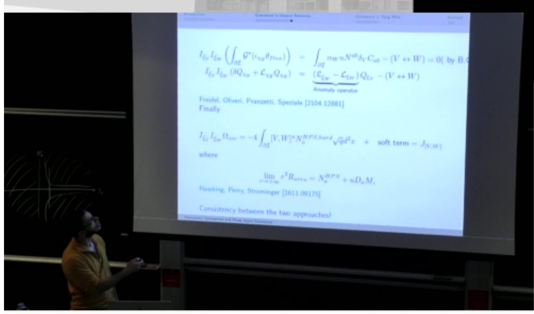
$$I_{\hat{\xi}_V} I_{\hat{\xi}_W} \Omega_{cov} = -4 \int_{\partial \mathcal{I}} [V, W]^a N_a^{HPS,hard} \sqrt{q} d^2 x \quad + \quad \text{soft term} = J_{[V,W]}$$

where

$$\lim_{r \to +\infty} r^3 R_{arru} = N_a^{HPS} + u D_a M,$$

Hawking, Perry, Strominger [1611.09175]

Consistency between the two approaches!



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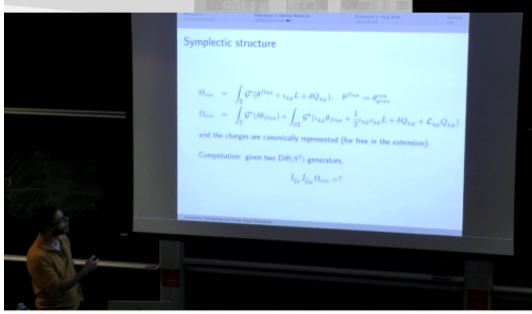

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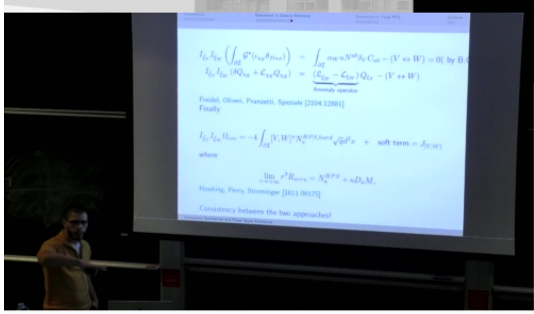


Extensions in Yang-Mills

M. Campiglia, JP [arXiv:2111.00973], S. Nagy, JP [arXiv:2211.00973]

Aim

Understand higher order extensions in non-linear regimes.

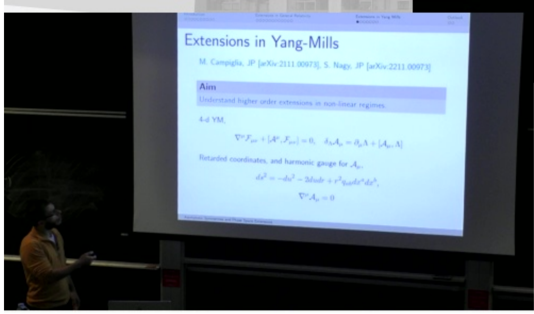
4-d YM,

$$\nabla^\mu \mathcal{F}_{\mu\nu} + [\mathcal{A}^\mu, \mathcal{F}_{\mu\nu}] = 0, \quad \delta_\Lambda \mathcal{A}_\mu = \partial_\mu \Lambda + [\mathcal{A}_\mu, \Lambda]$$

Retarded coordinates, and harmonic gauge for $\mathcal{A}_\mu$,

$$ds^2 = -du^2 - 2dudr + r^2 q_{ab} dx^a dx^b,$$

$$\nabla^\mu \mathcal{A}_\mu = 0$$



Large Gauge transformations

Harmonic gauge condition to the gauge transformation gives,

$$\nabla^\mu \delta_\Lambda \mathcal{A}_\mu = \Delta\Lambda + [\mathcal{A}_\mu, \nabla^\mu \Lambda] = 0 \Rightarrow \boxed{\Lambda = \Lambda^0 + r\Lambda^1 + \ldots}$$

Lysov, Pasterski, Strominger [1407.3814]: $O(r^0)$ LGT $\leftrightarrow$ leading charges, $O(r^1)$ LGT $\leftrightarrow$ subleading charges.

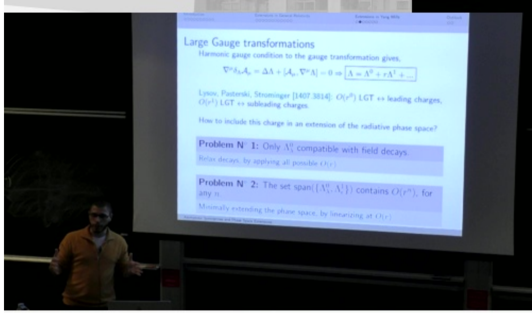
How to include this charge in an extension of the radiative phase space?

Problem N° 1: Only Λ^0_λ compatible with field decays.

Relax decays, by applying all possible $O(r)$

Problem N° 2: The set span$(\{\Lambda^0_\lambda, \Lambda^1_\epsilon\})$ contains $O(r^n)$, for any n.

Minimally extending the phase space, by linearizing at $O(r)$



Charges and Symplectic form

$$\boxed{Q^0_\lambda = Q^{0,\text{rad}}_\lambda + Q^1_{[\phi,\lambda]}}\,,\quad \boxed{Q^1_\epsilon = \int_{S^2} \text{Tr}(\epsilon\pi) d^2x}$$

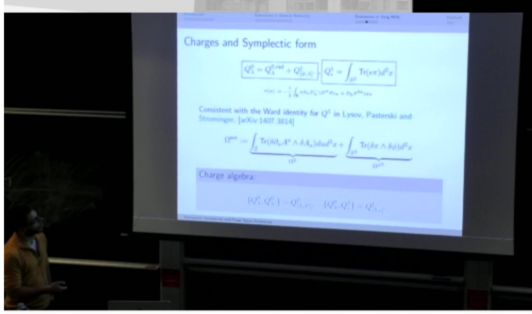
$$\pi(x) := -\frac{1}{2}\int_{\mathbb{R}} u\partial_u D^-_a (D^a F_{ru} + D_b F^{ba}) du$$

Consistent with the Ward identity for Q^1 in Lysov, Pasterski and Strominger, [arXiv:1407.3814]

$$\Omega^{\text{ext}} := \underbrace{\int_{\mathcal{I}} \text{Tr}(\delta\partial_u A^a \wedge \delta A_a) du d^2x}_{\Omega^{\mathcal{I}}} + \underbrace{\int_{S^2} \text{Tr}(\delta\pi \wedge \delta\phi) d^2x}_{\Omega^{S^2}}$$

Charge algebra:

$$\{Q^0_\lambda, Q^0_{\lambda'}\} = Q^0_{[\lambda,\lambda']}, \quad \{Q^0_\lambda, Q^1_\epsilon\} = Q^1_{[\lambda,\epsilon]}.$$



Abelian Theory: infinite tower of charges

Campiglia, Laddha [1810.04619]: Sub^n-leading soft photon theorem $\leftrightarrow$ $O(r^n)$ LGT associated charge, given by parameters $\{\epsilon_k\}$.

Divergences both in r and in u:

$$\delta Q_\Lambda = \Omega(\delta, \delta_\Lambda) \Longrightarrow Q_{\Lambda_\epsilon^N} = \lim_{t\to\infty} \int_{\Sigma_t} (\partial_r - \partial_u)(r^2 \Lambda F_{ru}) dx^2 du,$$

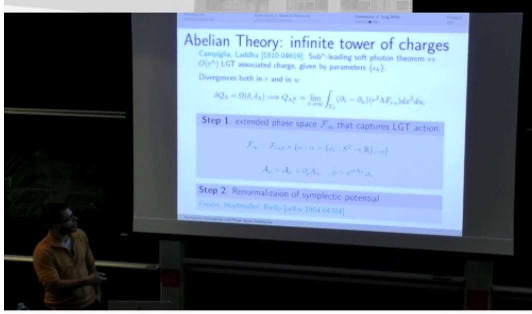
Step 1: extended phase space $\mathcal{F}_\infty$ that captures LGT action.

$$\mathcal{F}_\infty = \mathcal{F}_{rad} \times \{\alpha : \alpha = \{a_i : S^2 \to \mathbb{R}\}_{i>0}\}$$

$$\hat{\mathcal{A}}_\mu = \mathcal{A}_\mu + \partial_\mu \Lambda_\alpha, \quad \hat{\phi} = e^{ie\Lambda_\alpha}\phi,$$

Step 2: Renormalizaion of symplectic potential

Freidel, Hopfmüller, Riello [arXiv:1904.04384]



Step 3: Symplectic form expression

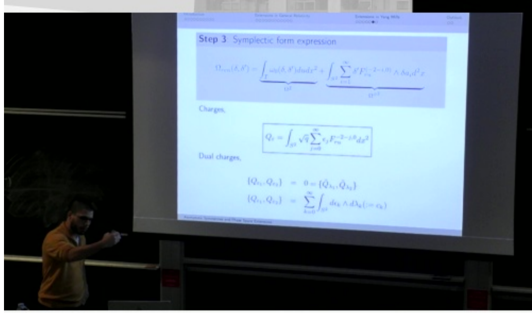
$$\Omega_{ren}(\delta,\delta') = \underbrace{\int_{\mathcal{I}} \omega_0(\delta,\delta') du dx^2}_{\Omega^{\mathcal{I}}} + \underbrace{\int_{S^2} \sum_{i=1}^{\infty} \delta' F_{ru}^{(-2-i,0)} \wedge \delta a_i d^2 x}_{\Omega^{S^2}}$$

Charges,

$$\boxed{Q_\varepsilon = \int_{S^2} \sqrt{q} \sum_{j=0}^{\infty} \epsilon_j F_{ru}^{-2-j,0} dx^2}$$

Dual charges,

$$\{Q_{\varepsilon_1}, Q_{\varepsilon_2}\} = 0 = \{\tilde{Q}_{\lambda_1}, \tilde{Q}_{\lambda_2}\}$$
$$\{Q_{\varepsilon_1}, Q_{\varepsilon_2}\} = \sum_{k=0}^{\infty} \int_{S^2} d\epsilon_k \wedge d\lambda_k (:= c_k)$$



Yang Mills revisited: beyond linear order

S. Nagy, JP [arXiv:2211.00973]

First approach: in Self Dual Yang-Mills, use finite transformations,

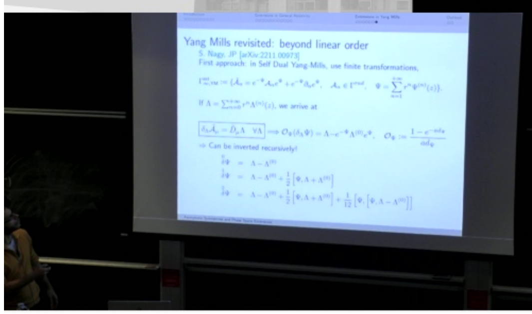
$$\Gamma^{\text{ext}}_{\infty,\text{YM}} := \{\tilde{\mathcal{A}}_\alpha = e^{-\Psi}\mathcal{A}_\alpha e^{\Psi} + e^{-\Psi}\partial_\alpha e^{\Psi}, \quad \mathcal{A}_\alpha \in \Gamma^{rad}, \quad \Psi = \sum_{n=1}^{+\infty} r^n \Psi^{(n)}(z)\}.$$

If $\Lambda = \sum_{n=0}^{+\infty} r^n \Lambda^{(n)}(z)$, we arrive at

$$\boxed{\delta_\Lambda \tilde{\mathcal{A}}_\mu = \tilde{D}_\mu \Lambda \quad \forall \Lambda} \Longrightarrow \mathcal{O}_\Psi(\delta_\Lambda \Psi) = \Lambda - e^{-\Psi}\Lambda^{(0)} e^{\Psi}, \quad \mathcal{O}_\Psi := \frac{1 - e^{-ad_\Psi}}{ad_\Psi}$$

$\Rightarrow$ Can be inverted recursively!

$$\begin{aligned}
\overset{0}{\delta}\Psi &= \Lambda - \Lambda^{(0)} \\
\overset{1}{\delta}\Psi &= \Lambda - \Lambda^{(0)} + \frac{1}{2}\left[\Psi, \Lambda + \Lambda^{(0)}\right] \\
\overset{2}{\delta}\Psi &= \Lambda - \Lambda^{(0)} + \frac{1}{2}\left[\Psi, \Lambda + \Lambda^{(0)}\right] + \frac{1}{12}\left[\Psi, \left[\Psi, \Lambda - \Lambda^{(0)}\right]\right]
\end{aligned}$$



Outlook

Many open questions and avenues to continue...

- Asymptotic Symmetries are intrinsic to holography (e.g., AdS/CFT conjecture, Killing horizons). They could in principle led to new insights of a more general holographic correspondence.
- Study higher order diffeomorphism actions. Sub-sub-leading theorems seem to correspond to $O(r)$ diffeomorphisms. (Campiglia,Laddha [1605.00685] - Freidel, Pranzetti, Raclariu [2111.15607]).
- The geometry of embedding maps for higher order symmetries.
- Does there exists a double copy map between charges in gravity and those in Yang-Mills?
- Black hole physics: universality as thermodynamic objects from near horizon geometry symmetries.