

Title: Cosmological parameters from BOSS and eBOSS data. Theoretical modeling of one-point probability distribution function for cosmological counts in cells.

Speakers: Anton Chudaykin

Series: Cosmology & Gravitation

Date: December 19, 2022 - 12:00 PM

URL: <https://pirsa.org/22120072>

Abstract: In the first part of my talk, I present the effective-field theory (EFT)-based cosmological full-shape analysis of the anisotropic power spectrum of eBOSS quasars. We perform extensive tests of our pipeline on simulations, paying particular attention to the modeling of observational systematics. Assuming the minimal Λ CDM model, we find the Hubble constant $H_0 = (66.7 \pm 3.2)$ km/s/Mpc, the matter density fraction $\Omega_m = 0.32 \pm 0.03$, and the late-time mass fluctuation amplitude $\sigma_8 = 0.95 \pm 0.08$. These measurements are fully consistent with the Planck cosmic microwave background results. Our work paves the way for systematic full-shape analyses of quasar samples from future surveys like DESI. I also present the cosmological constraints from the full-shape BOSS+eBOSS data in various extensions of the Λ CDM model, such as massive neutrinos, dynamical dark energy and spatial curvature.

In the second part, I study the one-point probability distribution function (PDF) for matter density averaged over spherical cells. The leading part to the PDF is defined by the dynamics of the spherical collapse whereas the next-to-leading part comes from the integration over fluctuations around the saddle-point solution. The latter calculation receives sizable contributions from unphysical short modes and must be renormalized. We propose a new approach to renormalization by modeling the effective stress-energy tensor for short perturbations. The model contains three free parameters which can be related to the counterterms in the one-loop matter power spectrum and bispectrum. We demonstrate that this relation can be used to impose priors in fitting the model to the PDF data. We confront the model with the results of high-resolution N-body simulations and find excellent agreement for cell radii $r \geq 10$ Mpc/h at all redshifts up to $z=0$.

Zoom link: <https://pitp.zoom.us/j/92219627192?pwd=eGg4MDUrbGlrR2JqY0xyWHdwQ2lZZz09>

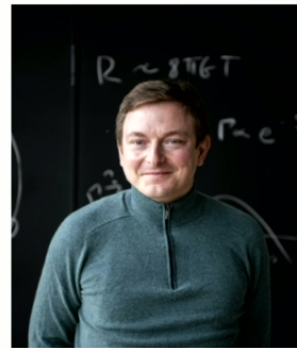
Cosmological parameters from BOSS+eBOSS data.
Theoretical modeling of one-point probability
distribution function for cosmological counts in cells.

Anton Chudaykin

McMaster University & Perimeter Institute



M. M. Ivanov



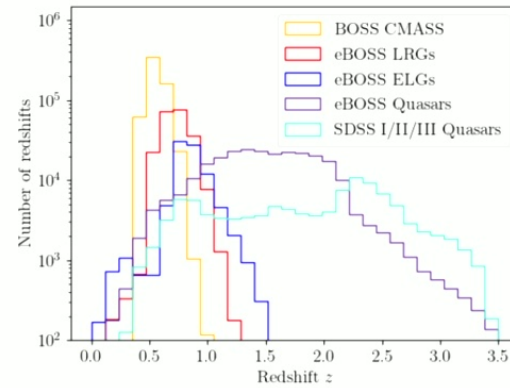
S. Sibiryakov

2210.17044
2212.XXXXX

Other papers
1907.06666
2004.10607
2009.10724
2009.10106
2201.07238



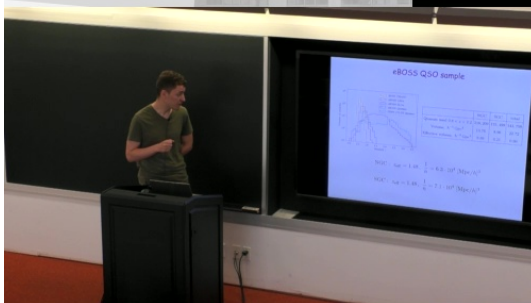
eBOSS QSO sample



	NGC	SGC	total
Quasars used $0.8 < z < 2.2$	218, 209	125, 499	343, 708
Volume, $h^{-3} \text{Gpc}^3$	13.76	8.96	22.72
Effective volume, $h^{-3} \text{Gpc}^3$	0.39	0.21	0.60

$$\text{NGC} : z_{\text{eff}} = 1.48, \quad \frac{1}{\bar{n}} = 6.3 \cdot 10^4 [\text{Mpc}/h]^3$$

$$\text{SGC} : z_{\text{eff}} = 1.48, \quad \frac{1}{\bar{n}} = 7.1 \cdot 10^4 [\text{Mpc}/h]^3$$

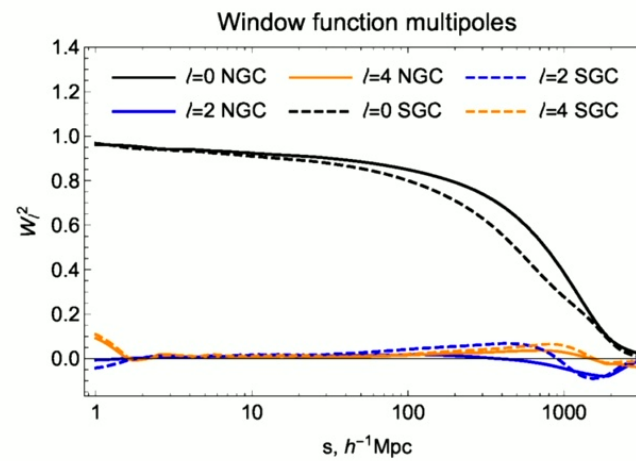


Survey geometry

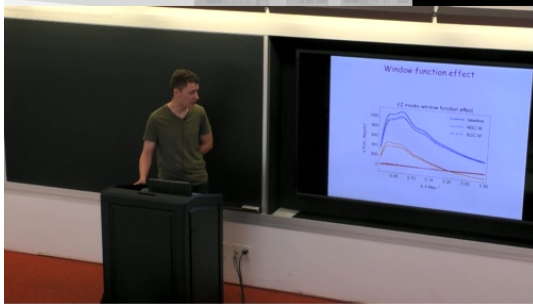
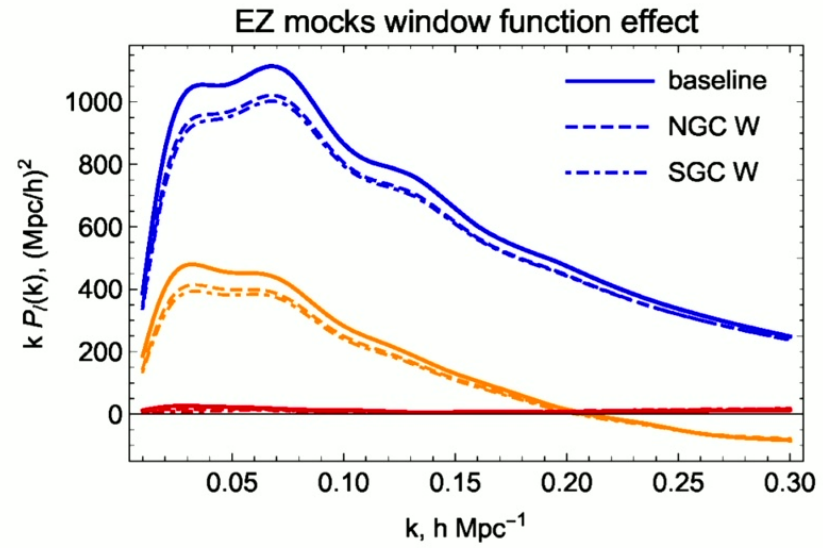
$$\xi_0^{\text{win}} = \xi_0^{\text{true}} W_0^2 + \frac{1}{5} \xi_2^{\text{true}} W_2^2 + \frac{1}{9} \xi_4^{\text{true}} W_4^2,$$

$$\xi_2^{\text{win}} = \xi_0^{\text{true}} W_2^2 + \xi_2^{\text{true}} \left[W_0^2 + \frac{2}{7} W_2^2 + \frac{2}{7} W_4^2 \right] + \xi_4^{\text{true}} \left[\frac{2}{7} W_2^2 + \frac{100}{693} W_4^2 \right]$$

$$\xi_4^{\text{win}} = \xi_0^{\text{true}} W_4^2 + \xi_2^{\text{true}} \left[\frac{18}{35} W_2^2 + \frac{20}{77} W_4^2 \right] + \xi_4^{\text{true}} \left[W_0^2 + \frac{20}{77} W_2^2 + \frac{162}{1001} W_4^2 \right]$$

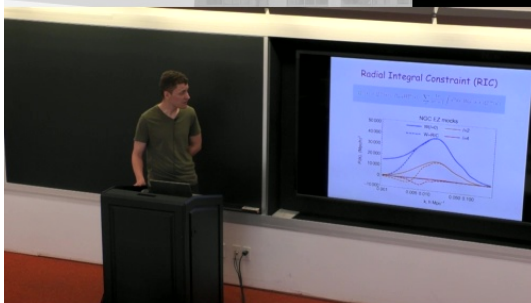
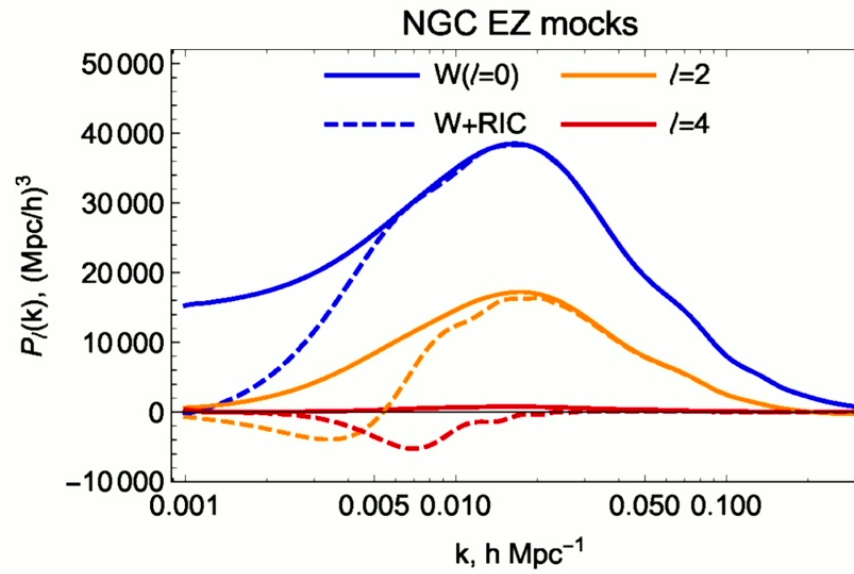


Window function effect



Radial Integral Constraint (RIC)

$$\xi_{\ell}^{\text{icc}}(s) = \xi_{\ell}^{\text{win}}(s) - P_{\text{shot}} \mathcal{W}_{\ell}^{\text{sn}}(s) - \sum_{\ell'} \frac{4\pi}{2\ell' + 1} \int s'^2 ds' \mathcal{W}_{\ell\ell'}(s, s') \xi_{\ell'}^{\text{true}}(s')$$



Fiber collisions

Hahn et al., 2017

$$\Delta P_\ell = \Delta P_\ell^{\text{uncorr}} + \Delta P_\ell^{\text{corr}}$$

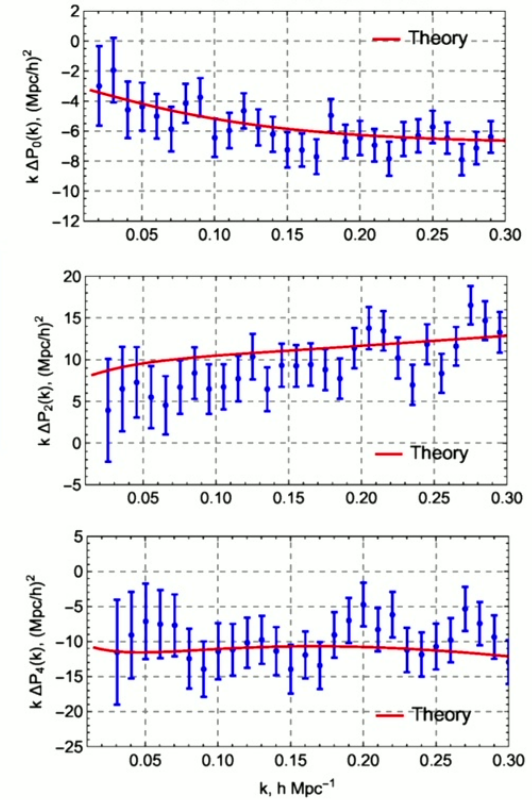
$$\Delta P_\ell^{\text{uncorr}}(k) = -f_s(2\ell + 1)L_\ell(0)\frac{(\pi D_{\text{fc}})^2}{k}W_{2D}(kD_{\text{fc}})$$

$$\Delta P_\ell^{\text{corr}}(k) = -f_s D_{\text{fc}}^2 \int \frac{d^2 q_\perp}{(2\pi)^2} P(k_\parallel, q_\perp) W_{2D}(q_\perp D_{\text{fc}})$$

$$D_{\text{fc}} = 0.9 \text{ hMpc}^{-1}$$

$$f_s(\text{NGC}) = 0.36, \quad f_s(\text{SGC}) = 0.45$$

EZ mocks fibre collision effect

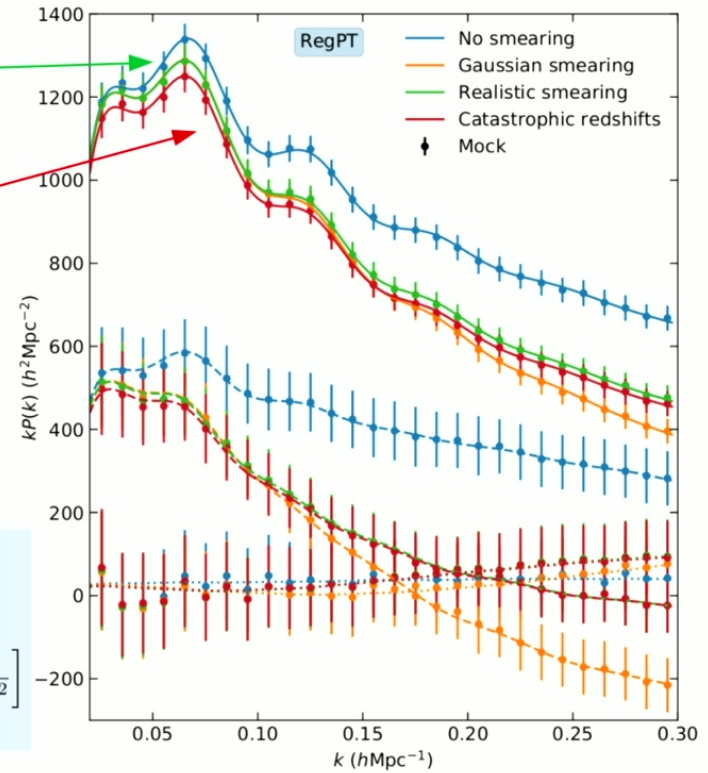


Systematic effects

1. Redshift smearing

2. Catastrophic redshifts

3. Spectroscopic uncertainties



$$P^{\text{FOG}}(k, \mu) = D(k, \mu) P_{\text{SPT}}(k, \mu)$$

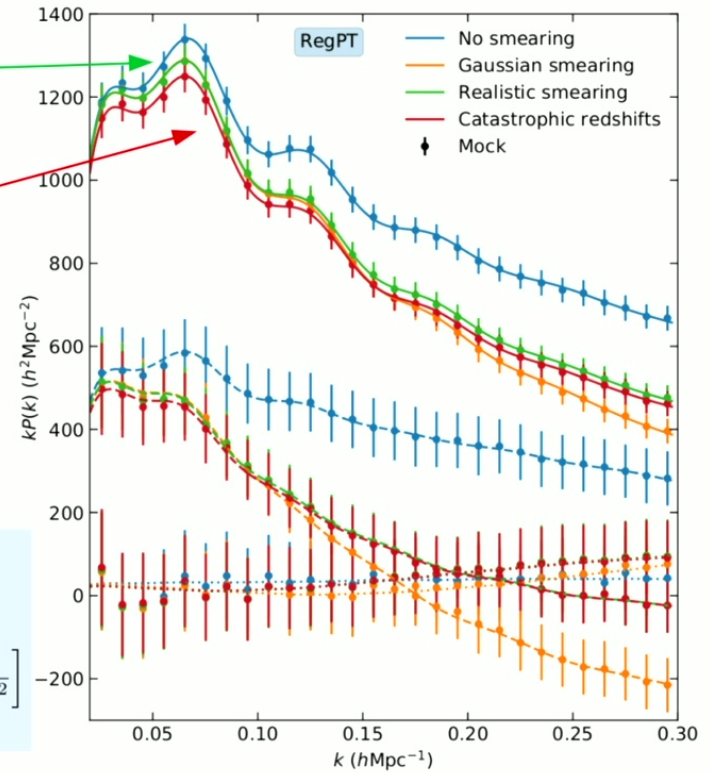
$$D(k, \mu) = \frac{1}{\sqrt{1 + (k\mu a_{\text{vir}})^2}} \exp\left[-\frac{(k\mu\sigma_v)^2}{1 + (k\mu a_{\text{vir}})^2}\right]$$

Systematic effects

1. Redshift smearing

2. Catastrophic redshifts

3. Spectroscopic uncertainties

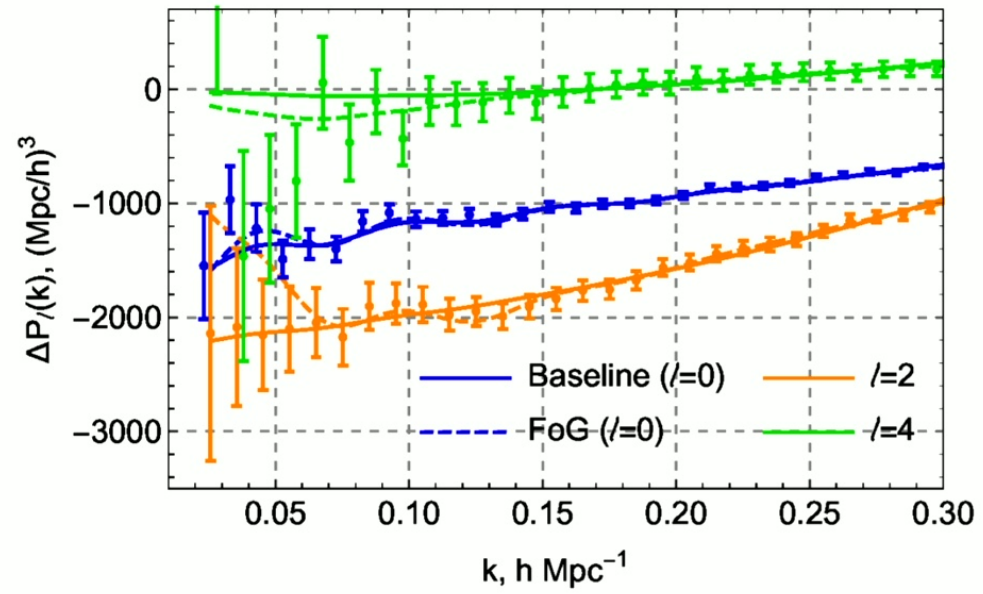


$$P^{\text{FOG}}(k, \mu) = D(k, \mu) P_{\text{SPT}}(k, \mu)$$

$$D(k, \mu) = \frac{1}{\sqrt{1 + (k\mu a_{\text{vir}})^2}} \exp \left[-\frac{(k\mu\sigma_v)^2}{1 + (k\mu a_{\text{vir}})^2} \right]$$

Theoretical model

$$P_2(k) \rightarrow P_2(k) + P_{\text{shot},2}$$

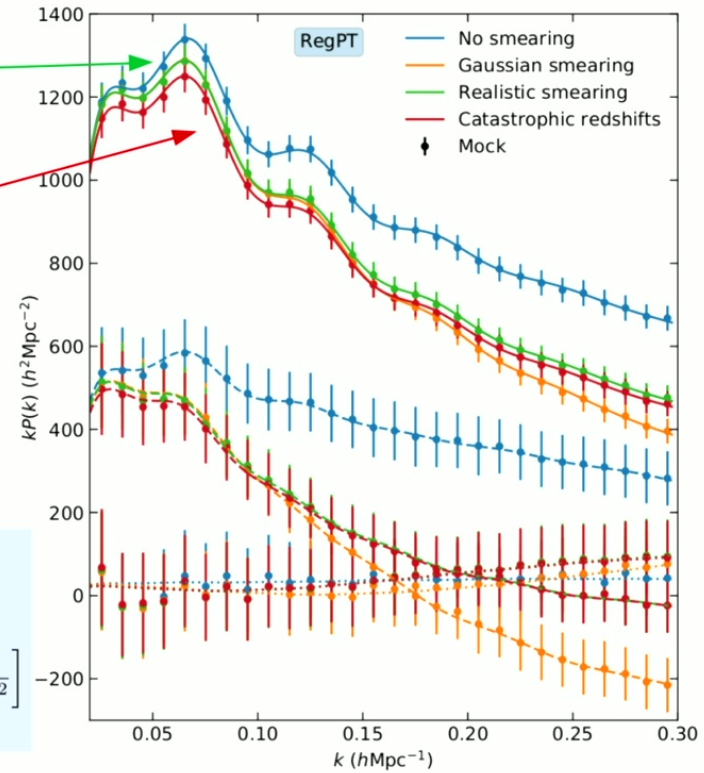


Systematic effects

1. Redshift smearing

2. Catastrophic redshifts

3. Spectroscopic uncertainties

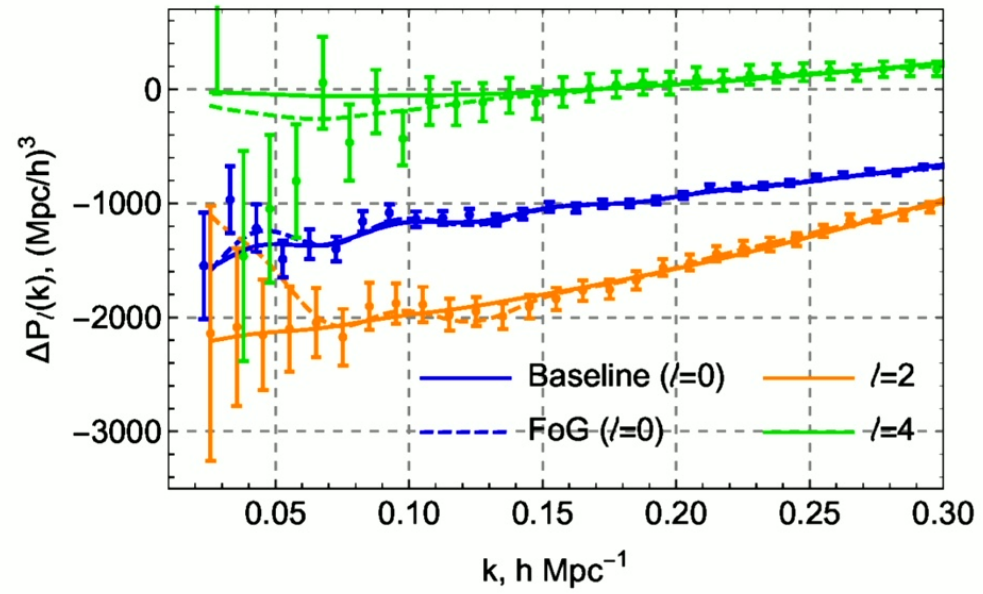


$$P^{\text{FOG}}(k, \mu) = D(k, \mu) P_{\text{SPT}}(k, \mu)$$

$$D(k, \mu) = \frac{1}{\sqrt{1 + (k\mu a_{\text{vir}})^2}} \exp\left[-\frac{(k\mu\sigma_v)^2}{1 + (k\mu a_{\text{vir}})^2}\right]$$

Theoretical model

$$P_2(k) \rightarrow P_2(k) + P_{\text{shot},2}$$



EFTofLSS

Cosmological parameters

ω_b fixed
 n_s fixed

$$h \in \text{flat}[0.4, 1], \quad \omega_{cdm} \in \text{flat}[0.03, 0.7], \\ \ln(10^{10} A_s) \in \text{flat}[0.1, 10]$$

Bias

$$b_1 \in \text{flat}[0, 4], \quad b_2 \sim \mathcal{N}(0, 1^2), \\ b_{\mathcal{G}_2} \sim \mathcal{N}(-0.4, 1^2), \quad b_{\Gamma_3} \sim \mathcal{N}(0.77, 1^2)$$

Higher-derivative terms

$$\frac{c_0}{[\text{Mpc}/h]^2} \sim \mathcal{N}(20, 20^2), \quad \frac{c_2}{[\text{Mpc}/h]^2} \sim \mathcal{N}(30, 20^2) \\ \frac{c_4}{[\text{Mpc}/h]^2} \sim \mathcal{N}(20, 20^2), \quad \frac{\tilde{c}}{[\text{Mpc}/h]^4} \sim \mathcal{N}(0, 200^2)$$

Stochastic bias

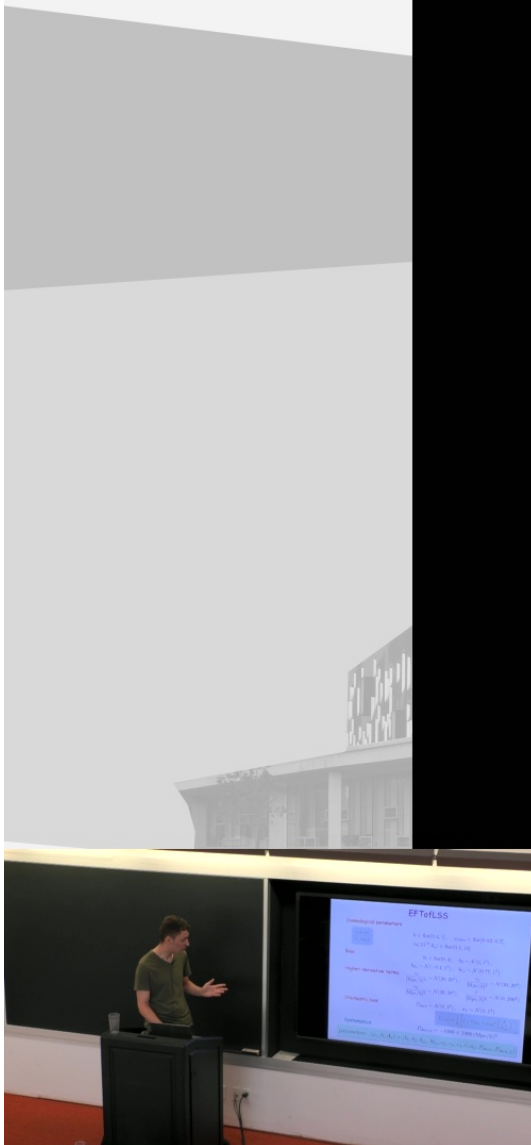
$$P_{\text{shot}} \sim \mathcal{N}(0, 1^2), \quad a_2 \sim \mathcal{N}(0, 1^2)$$

$$P_{\text{stoch}} = \frac{1}{\bar{n}} \left[1 + P_{\text{shot}} + a_2 \mu^2 \left(\frac{k}{k_{\text{NL}}} \right)^2 \right]$$

Systematics

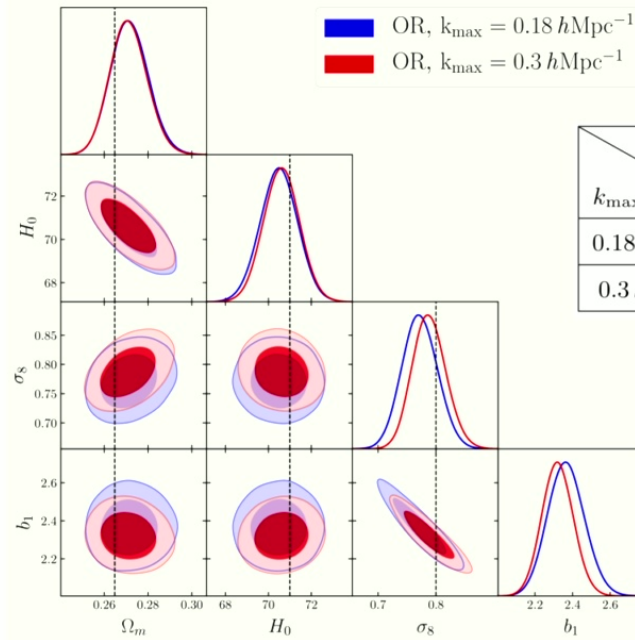
$$P_{\text{shot},2} = -1000 \pm 1000 (\text{Mpc}/h)^3$$

parameters : $(\omega_c, h, A_s) \times (b_1, b_2, b_{\mathcal{G}_2}, b_{\Gamma_3}, c_0, c_2, c_4, \tilde{c}, a_2, P_{\text{shot}}, P_{\text{shot},2})$



Outer Rim mocks: results

parameters : $(\omega_c, h, A_s) \times (b_1, b_2, b_{G_2}, b_{\Gamma_3}, c_0, c_2, c_4, \tilde{c}, a_2, P_{\text{shot}}, P_{\text{shot},2})$



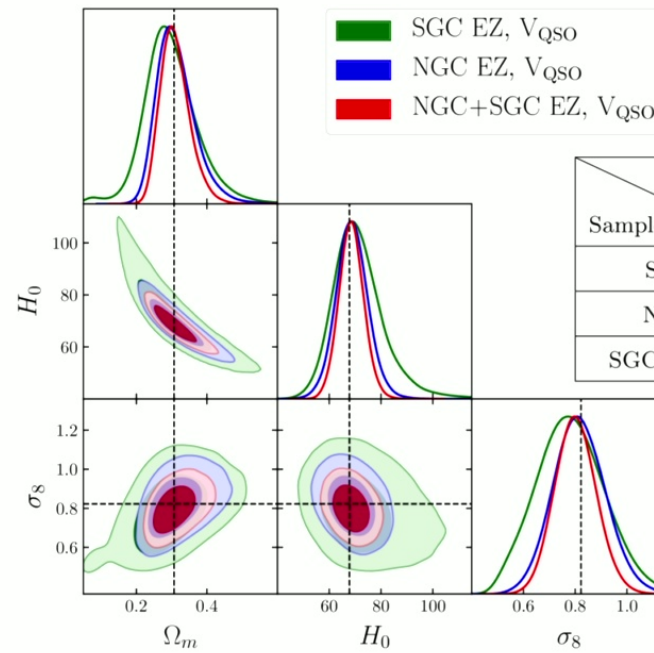
Param \ k_{max}	Ω_m	H_0	σ_8
0.18 hMpc^{-1}	$0.2716^{+0.0084}_{-0.0092}$	$70.48^{+0.9}_{-0.89}$	$0.7722^{+0.031}_{-0.032}$
0.3 hMpc^{-1}	$0.2711^{+0.0083}_{-0.009}$	$70.61^{+0.85}_{-0.86}$	$0.7884^{+0.028}_{-0.031}$

$V = 10 \times V_{\text{QSO}}$



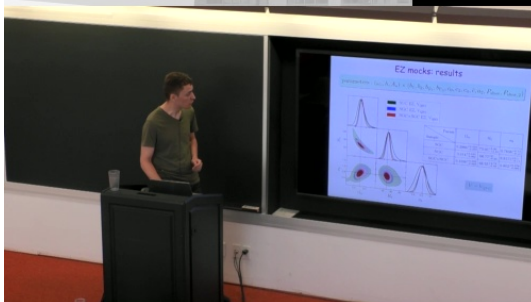
EZ mocks: results

parameters : $(\omega_c, h, A_s) \times (b_1, b_2, b_{G_2}, b_{\Gamma_3}, c_0, c_2, c_4, \tilde{c}, a_2, P_{\text{shot}}, P_{\text{shot},2})$



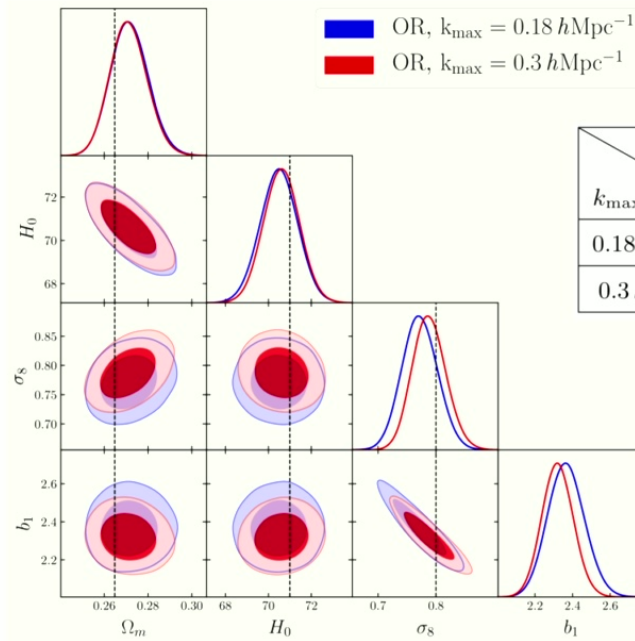
Sample \ Param	Ω_m	H_0	σ_8
SGC	$0.2986^{+0.059}_{-0.079}$	$73.05^{+5.89}_{-12.55}$	$0.7838^{+0.13}_{-0.14}$
NGC	$0.314^{+0.043}_{-0.064}$	$68.77^{+6.}_{-6.56}$	$0.8171^{+0.1}_{-0.11}$
SGC+NGC	$0.3106^{+0.035}_{-0.049}$	$68.55^{+4.73}_{-5.09}$	$0.802^{+0.078}_{-0.085}$

$V = V_{\text{QSO}}$



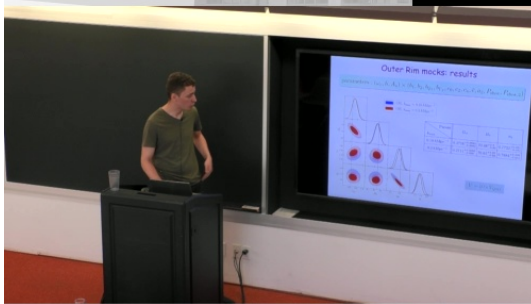
Outer Rim mocks: results

parameters : $(\omega_c, h, A_s) \times (b_1, b_2, b_{G_2}, b_{\Gamma_3}, c_0, c_2, c_4, \tilde{c}, a_2, P_{\text{shot}}, P_{\text{shot},2})$



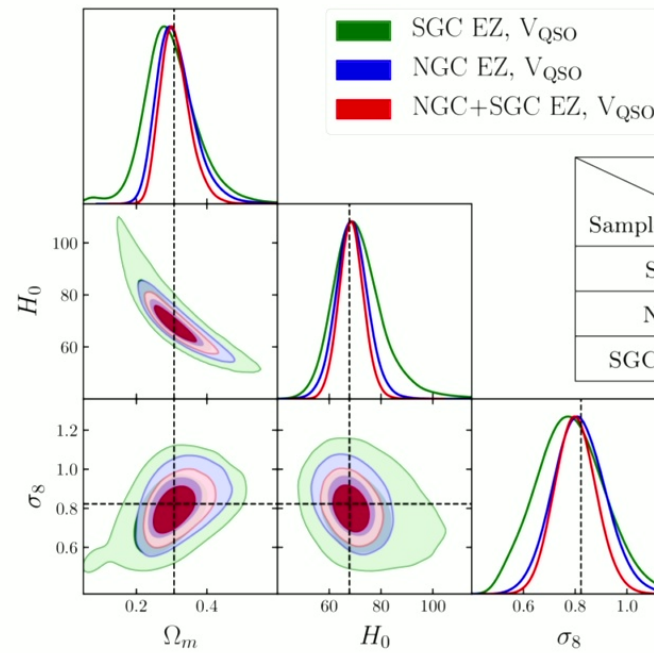
Param \ k_{max}	Ω_m	H_0	σ_8
0.18 hMpc^{-1}	$0.2716^{+0.0084}_{-0.0092}$	$70.48^{+0.9}_{-0.89}$	$0.7722^{+0.031}_{-0.032}$
0.3 hMpc^{-1}	$0.2711^{+0.0083}_{-0.009}$	$70.61^{+0.85}_{-0.86}$	$0.7884^{+0.028}_{-0.031}$

$V = 10 \times V_{\text{QSO}}$

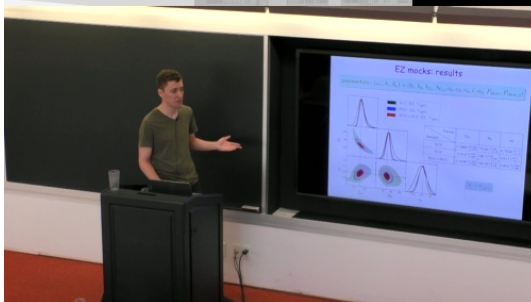


EZ mocks: results

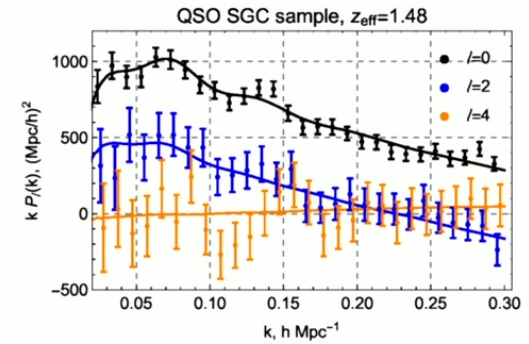
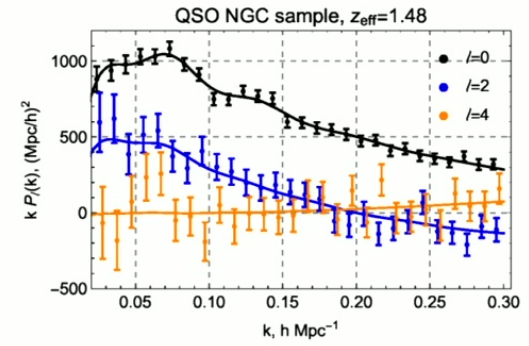
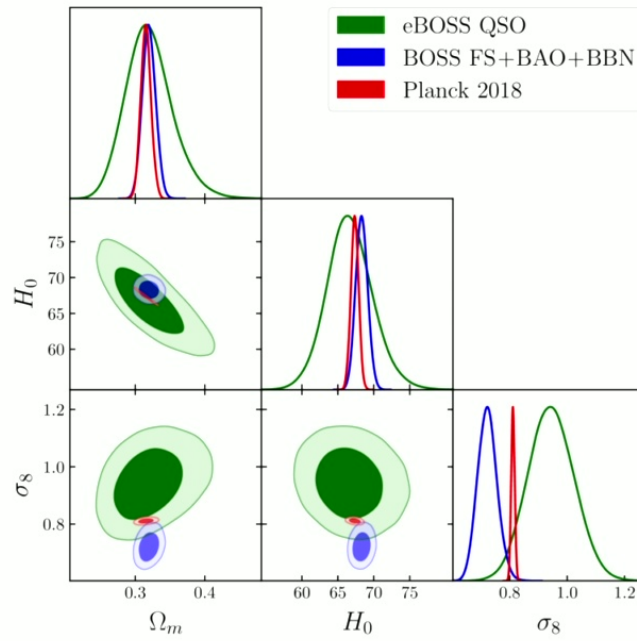
parameters : $(\omega_c, h, A_s) \times (b_1, b_2, b_{G_2}, b_{\Gamma_3}, c_0, c_2, c_4, \tilde{c}, a_2, P_{\text{shot}}, P_{\text{shot},2})$



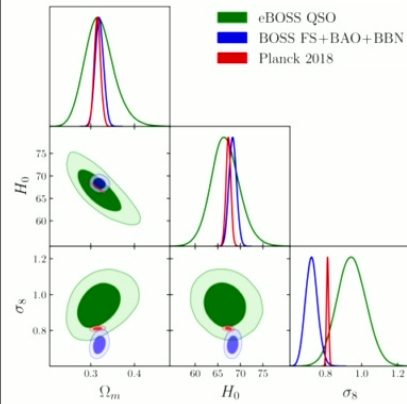
$V = V_{\text{QSO}}$



eBOSS DR14 analysis



eBOSS DR14 analysis



Dataset \ Param.	eBOSS QSO	BOSS FS+BAO+BBN	Planck 2018
ω_{cdm}	$0.1184^{+0.009}_{-0.011}$	$0.1262^{+0.0053}_{-0.0059}$	$0.1200^{+0.0012}_{-0.0012}$
h	$0.6669^{+0.031}_{-0.034}$	$0.6832^{+0.0083}_{-0.0086}$	$0.6736^{+0.0055}_{-0.0053}$
$\ln(10^{10} A_s)$	$3.375^{+0.18}_{-0.18}$	$2.742^{+0.096}_{-0.099}$	$3.044^{+0.014}_{-0.015}$
Ω_m	$0.3205^{+0.03}_{-0.038}$	$0.3196^{+0.01}_{-0.01}$	$0.3153^{+0.0071}_{-0.0077}$
σ_8	$0.9445^{+0.081}_{-0.083}$	$0.7221^{+0.032}_{-0.037}$	$0.8112^{+0.006}_{-0.0062}$
S_8	$0.976^{+0.1}_{-0.12}$	$0.745^{+0.037}_{-0.041}$	$0.832^{+0.013}_{-0.013}$



EFTofLSS / α -analysis

$$\langle \alpha_{\parallel}, \alpha_{\perp}, f\sigma_8 \rangle \Rightarrow D_M, D_H, f\sigma_8$$

$$\alpha_{\perp} = \frac{D_M(z)/r_{\text{drag}}}{D_M^{\text{fid}}(z)/r_{\text{drag}}^{\text{fid}}}$$

$$\alpha_{\parallel} = \frac{D_H(z)/r_{\text{drag}}}{D_H^{\text{fid}}(z)/r_{\text{drag}}^{\text{fid}}}$$

EFTofLSS

$$\begin{aligned} D_H(z = 1.48)/r_{\text{drag}} &= 12.98_{-0.19}^{+0.21}, \\ D_M(z = 1.48)/r_{\text{drag}} &= 30.45 \pm 0.78, \\ f\sigma_8(z = 1.48) &= 0.437_{-0.038}^{+0.036}. \end{aligned}$$

eBOSS analysis

$$\begin{aligned} D_H(z = 1.48)/r_{\text{drag}} &= 13.52 \pm 0.51, \\ D_M(z = 1.48)/r_{\text{drag}} &= 30.68 \pm 0.90, \\ f\sigma_8(z = 1.48) &= 0.476 \pm 0.04. \end{aligned}$$



BOSS+eBOSS(+SN) data

Full-shape measurements

1. BOSS LRG sample ($z=0.38, 0.61$):

- Power spectra (P_0, P_2, P_4)
- Analog to real-space power spectrum (Q_0)
- BAO from post-reconstructed power spectra
- Monopole bispectrum (B_0)

2. eBOSS QSO sample ($z=1.48$):

- Power spectra (P_0, P_2, P_4)

3. eBOSS ELG sample ($z=0.845$):

- Power spectra (P_0, P_2, P_4)
- BAO from post-reconstructed power spectra

BBN

$$\omega_b = 0.02268$$

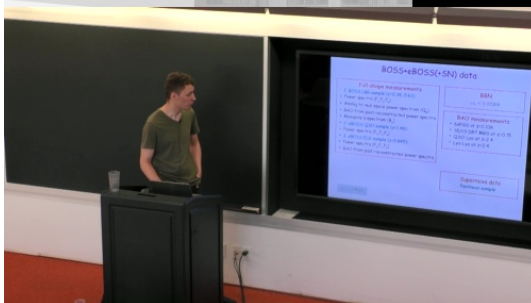
BAO measurements

- 6dFGS at $z=0.106$
- SDSS DR7 MGS at $z=0.15$
- QSO-Lya at $z=2.4$
- Lya-Lya at $z=2.4$

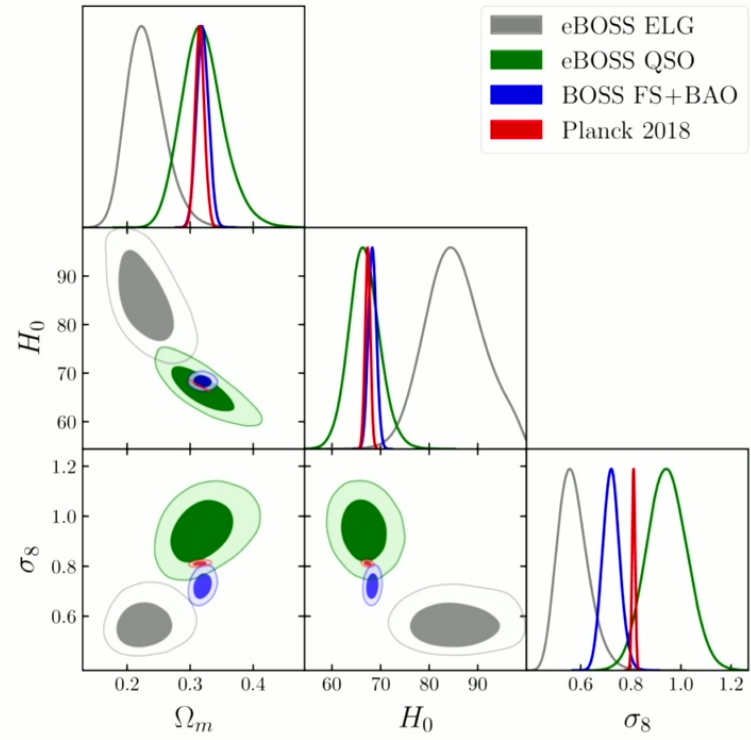
Supernova data

Pantheon sample

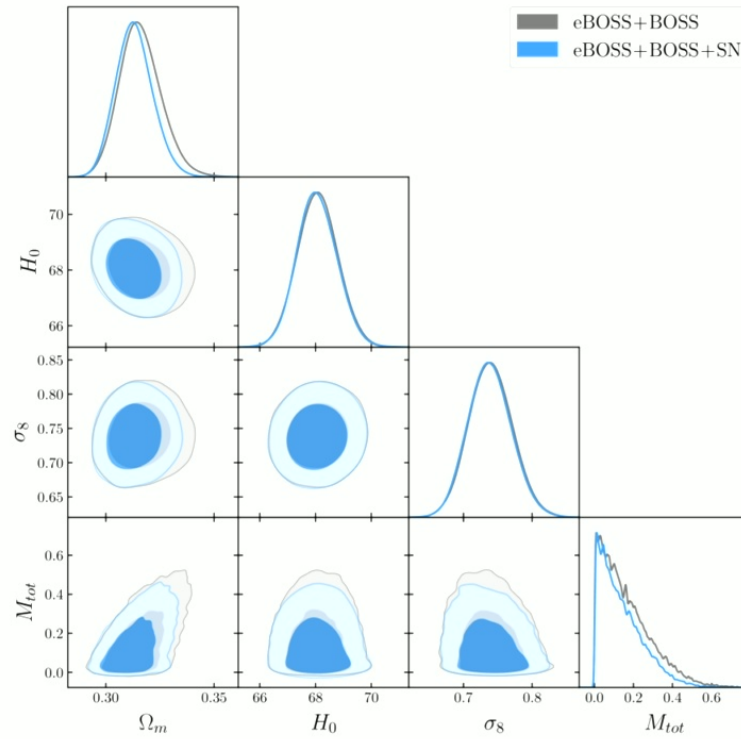
$$n_s = 0.9649$$



Λ CDM

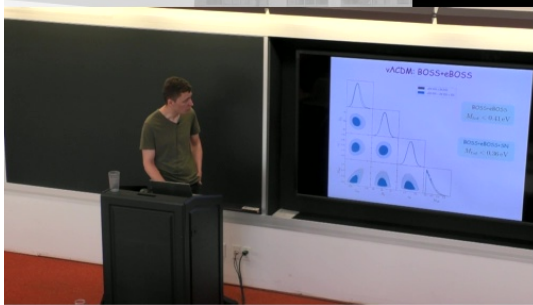


Λ CDM: BOSS+eBOSS

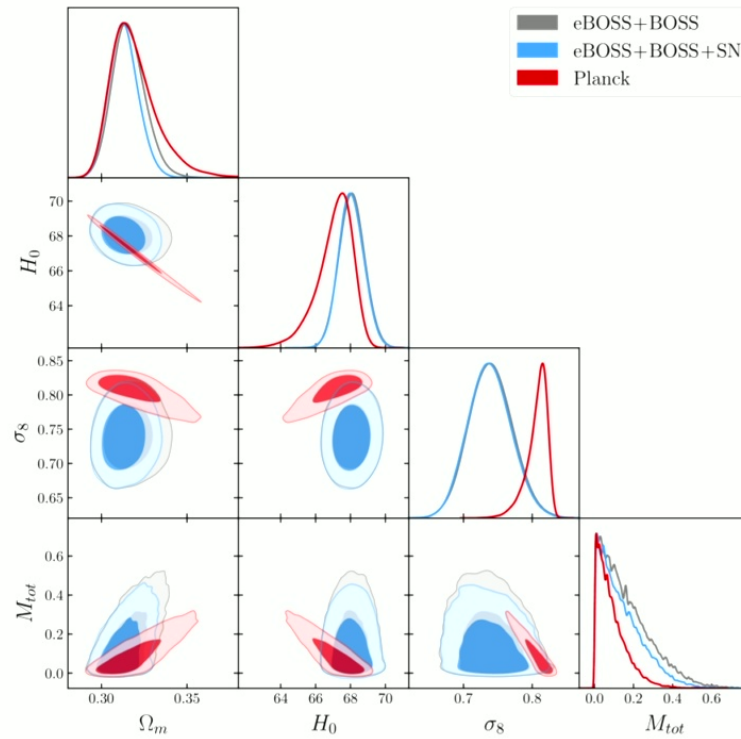


BOSS+eBOSS
 $M_{tot} < 0.41 \text{ eV}$

BOSS+eBOSS+SN
 $M_{tot} < 0.36 \text{ eV}$



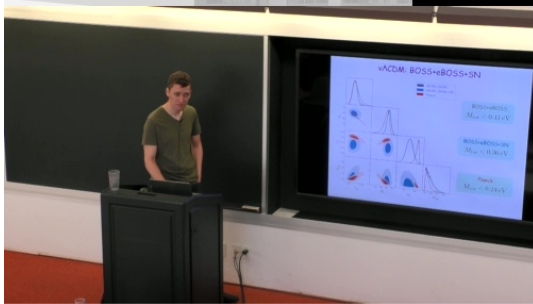
$\nu\Lambda$ CDM: BOSS+eBOSS+SN



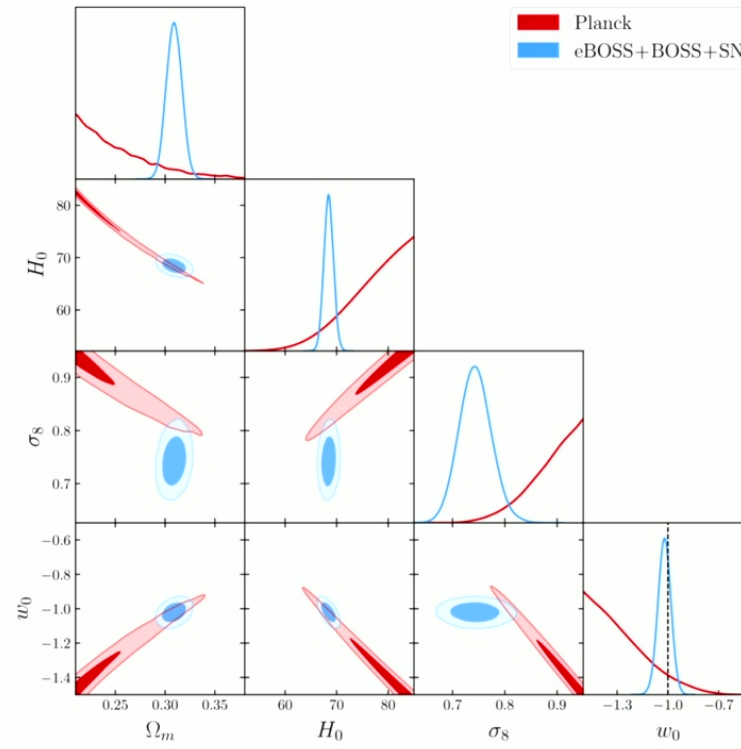
BOSS+eBOSS
 $M_{tot} < 0.41 \text{ eV}$

BOSS+eBOSS+SN
 $M_{tot} < 0.36 \text{ eV}$

Planck
 $M_{tot} < 0.24 \text{ eV}$



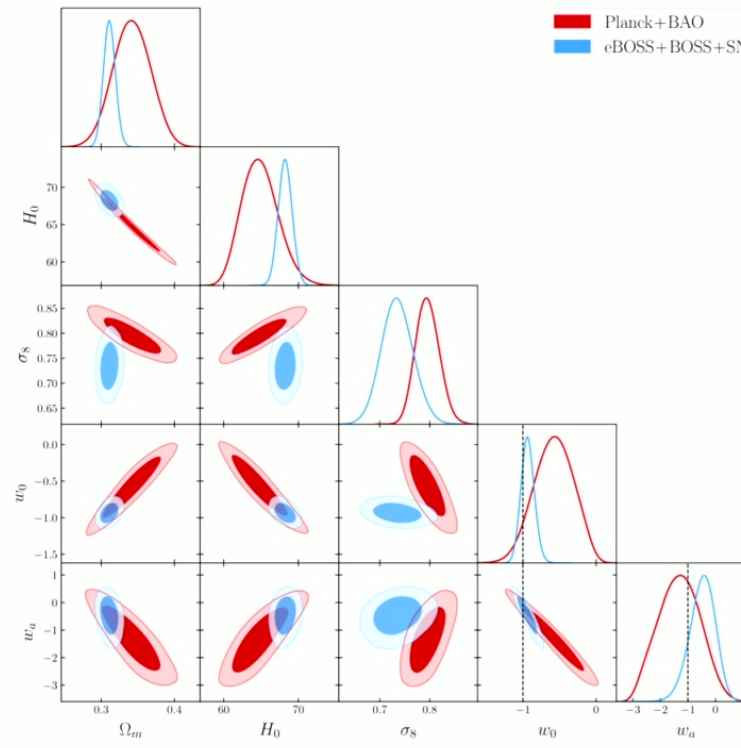
wCDM: eBOSS+BOSS+SN



BOSS+eBOSS+SN
 $w_0 = -1.019 \pm 0.038$



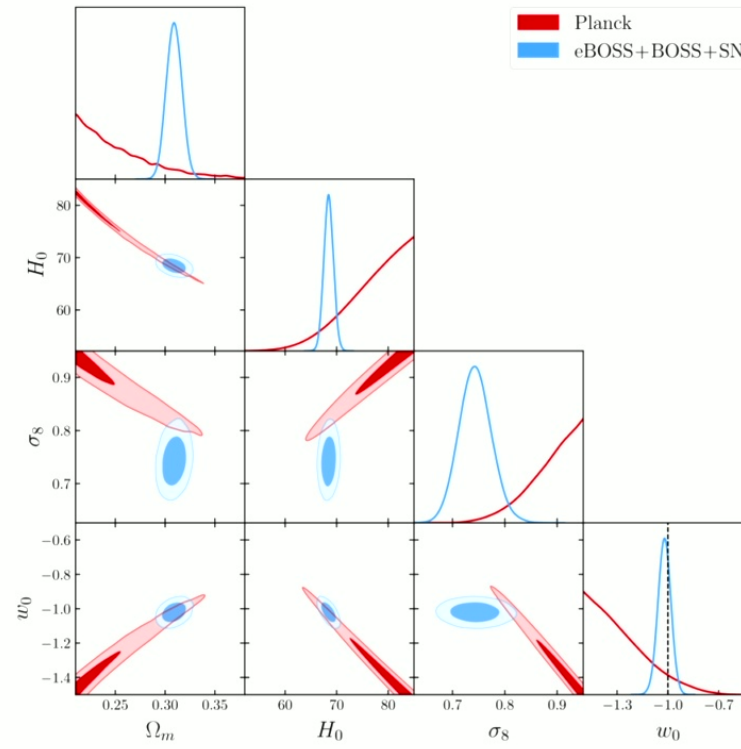
w0waCDM: eBOSS+BOSS+SN



BOSS+eBOSS+SN
 $w_0 = -0.936 \pm 0.093$
 $w_a = -0.473 \pm 0.469$



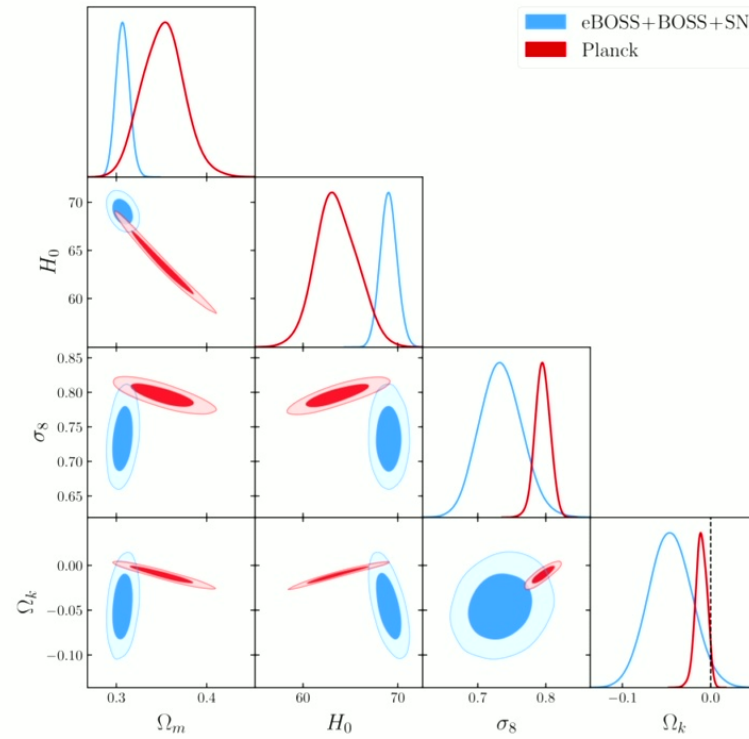
wCDM: eBOSS+BOSS+SN



BOSS+eBOSS+SN
 $w_0 = -1.019 \pm 0.038$



Λ CDM: eBOSS+BOSS+SN

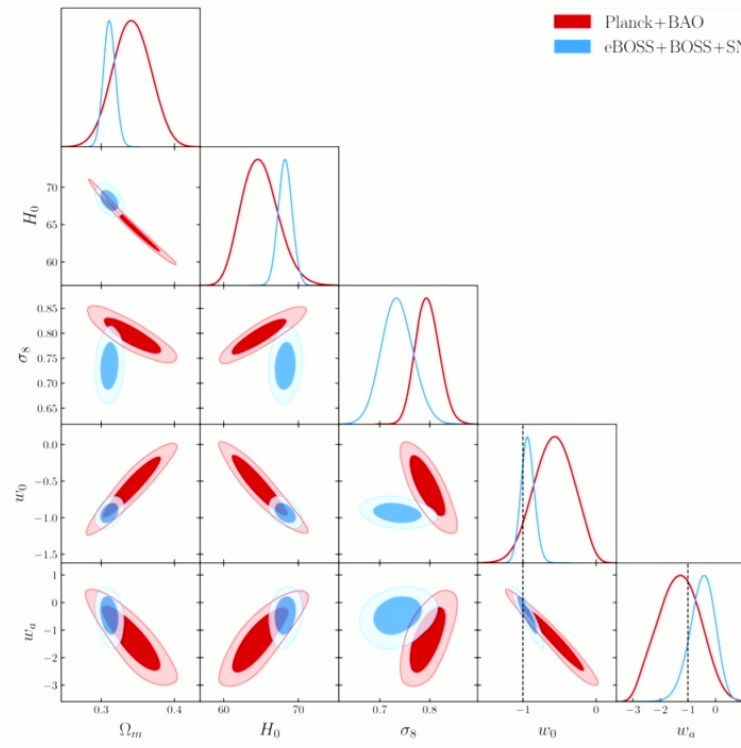


BOSS+eBOSS+SN
 $\Omega_k = -0.044 \pm 0.026$

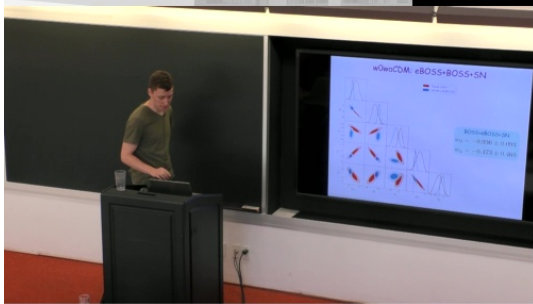
Planck
 $\Omega_k = 0.011 \pm 0.007$



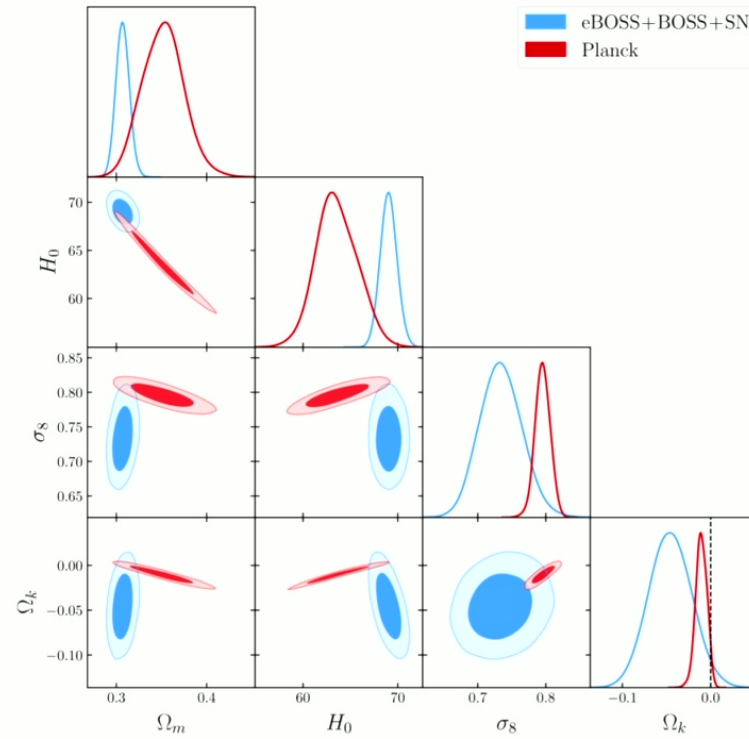
w0waCDM: eBOSS+BOSS+SN



BOSS+eBOSS+SN
 $w_0 = -0.936 \pm 0.093$
 $w_a = -0.473 \pm 0.469$



Λ CDM: eBOSS+BOSS+SN

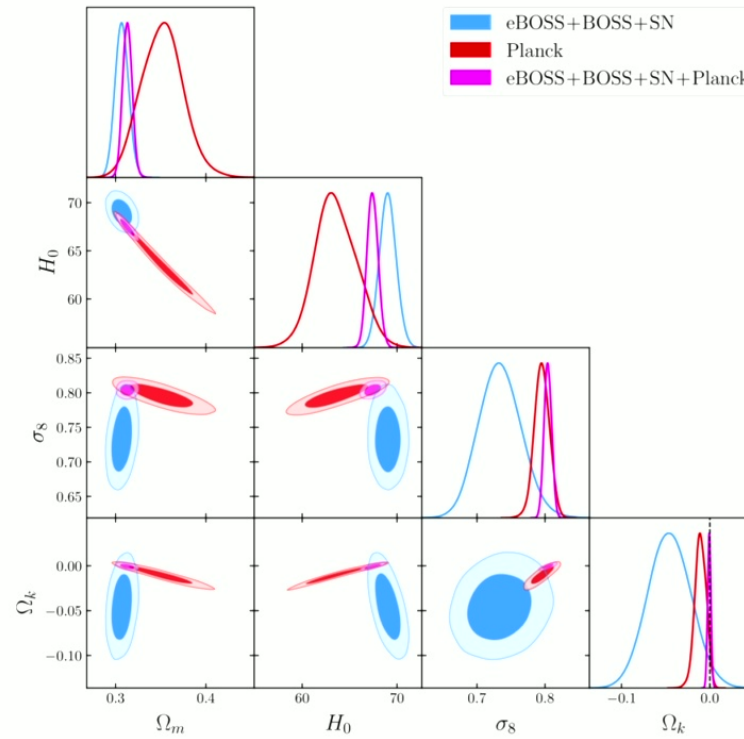


BOSS+eBOSS+SN
 $\Omega_k = -0.044 \pm 0.026$

Planck
 $\Omega_k = 0.011 \pm 0.007$



Λ CDM: eBOSS+BOSS+SN



BOSS+eBOSS+SN

$$\Omega_k = -0.044 \pm 0.026$$

Planck

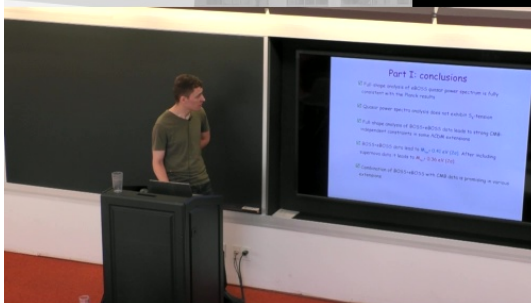
$$\Omega_k = 0.011 \pm 0.007$$

BOSS+eBOSS+SN+Planck

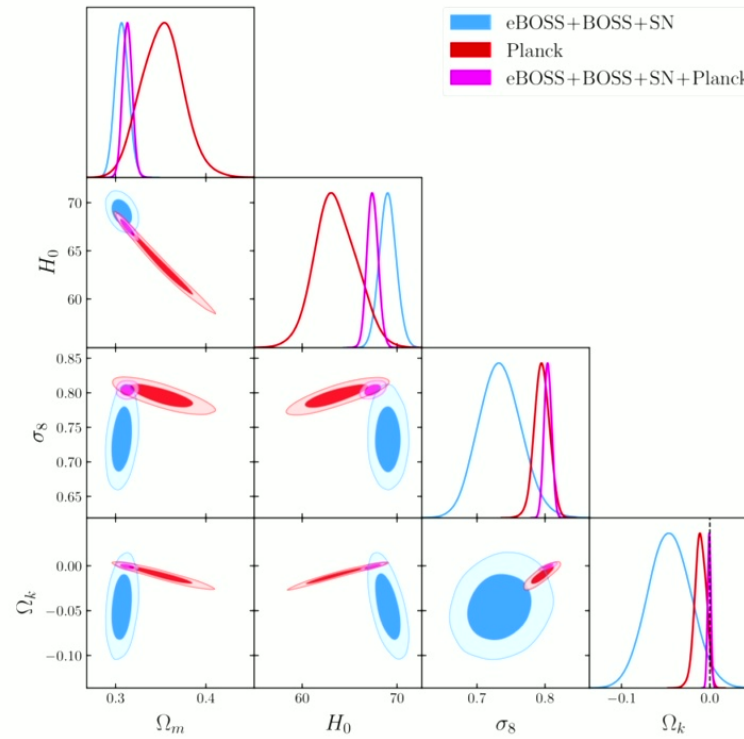
$$\Omega_k = (0.8 \pm 1.9) \cdot 10^{-3}$$

Part I: conclusions

- ✓ Full-shape analysis of eBOSS quasar power spectrum is fully consistent with the Planck results
- ✓ Quasar power spectra analysis does not exhibit S_8 -tension
- ✓ Full-shape analysis of BOSS+eBOSS data leads to strong CMB-independent constraints in some Λ CDM extensions
- ✓ BOSS+eBOSS data lead to $M_{\text{tot}} < 0.41 \text{ eV} (2\sigma)$. After including supernova data it leads to $M_{\text{tot}} < 0.36 \text{ eV} (2\sigma)$.
- ✓ Combination of BOSS+eBOSS with CMB data is promising in various extensions



Λ CDM: eBOSS+BOSS+SN



BOSS+eBOSS+SN

$$\Omega_k = -0.044 \pm 0.026$$

Planck

$$\Omega_k = 0.011 \pm 0.007$$

BOSS+eBOSS+SN+Planck

$$\Omega_k = (0.8 \pm 1.9) \cdot 10^{-3}$$

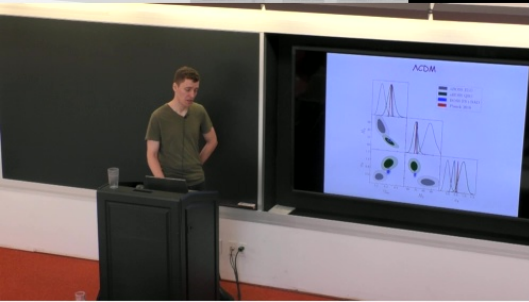
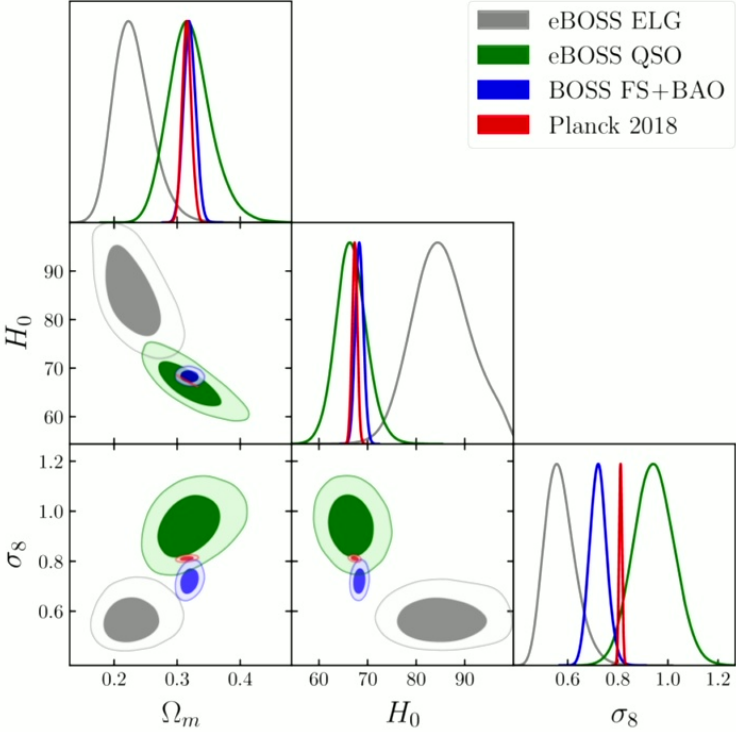


Part I: conclusions

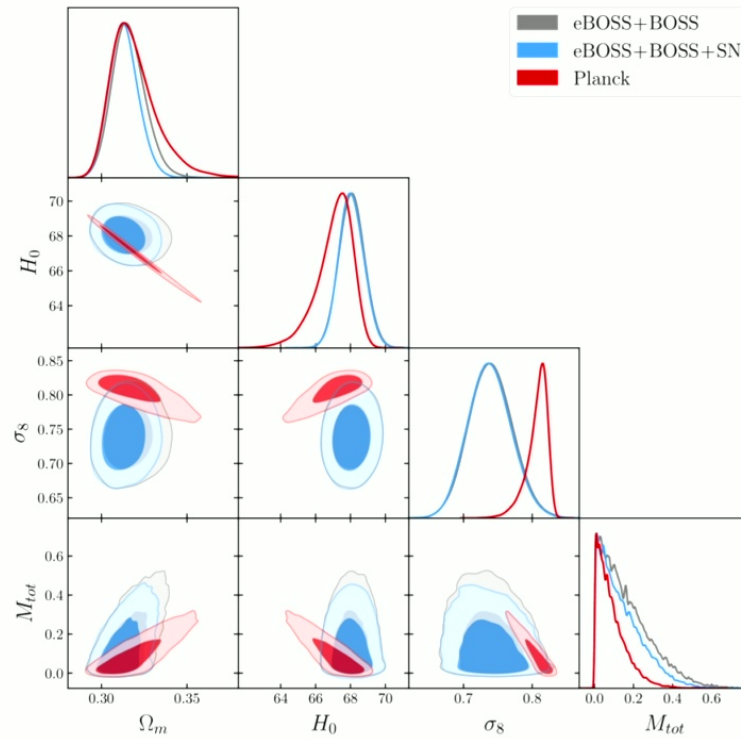
- ✓ Full-shape analysis of eBOSS quasar power spectrum is fully consistent with the Planck results
- ✓ Quasar power spectra analysis does not exhibit S_8 -tension
- ✓ Full-shape analysis of BOSS+eBOSS data leads to strong CMB-independent constraints in some Λ CDM extensions
- ✓ BOSS+eBOSS data lead to $M_{\text{tot}} < 0.41 \text{ eV} (2\sigma)$. After including supernova data it leads to $M_{\text{tot}} < 0.36 \text{ eV} (2\sigma)$.
- ✓ Combination of BOSS+eBOSS with CMB data is promising in various extensions



Λ CDM



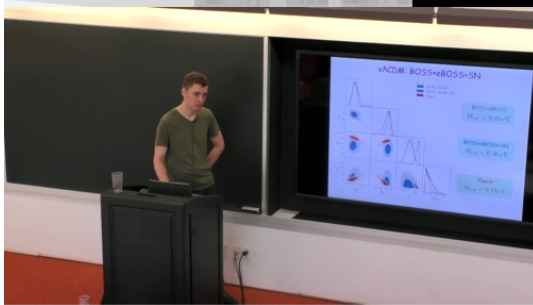
$\nu\Lambda$ CDM: BOSS+eBOSS+SN



BOSS+eBOSS
 $M_{tot} < 0.41 \text{ eV}$

BOSS+eBOSS+SN
 $M_{tot} < 0.36 \text{ eV}$

Planck
 $M_{tot} < 0.24 \text{ eV}$



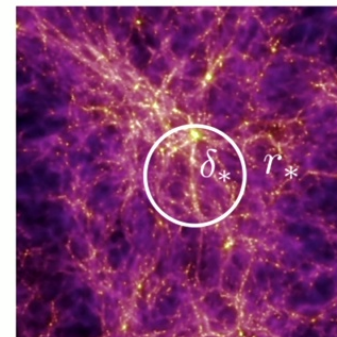
Part II

Theoretical modeling of
one-point probability distribution function
for cosmological counts in cells



Cosmological counts in cells

$\mathcal{P}(\delta_*)$ - probability that a cell of radius r_* has averaged density contrast δ_*



$$\alpha \sim g(z)^2 \sigma_{r_*}^2 \ll 1 \quad \sigma_{r_*}^2 = \langle \delta^2 \rangle_{r_*}$$

$$\mathcal{P}(\delta_*) = \exp \left\{ -\frac{1}{\alpha} (a_0 + \alpha a_1 + \dots) \right\}$$

Saddle point solution
(‘instanton’)
defined by spherical collapse
(Valageas’02)

Prefactor
(‘determinant’)
from perturbations around
the saddle point solution
(M.M. Ivanov’19)



Cosmological counts in cells

$$\mathcal{P}(\delta_*) = \mathcal{N}^{-1} \int \mathcal{D}\delta_L \exp \left\{ - \int_{\mathbf{k}} \frac{|\delta_L(\mathbf{k})|^2}{2g^2 P(k)} \right\} \delta_D^{(1)}(\delta_* - \bar{\delta}_W[\delta_L])$$

Saddle-point solution

$$\mathcal{P}(\delta_*) \propto \exp \left\{ - \frac{F^2(\delta_*)}{2g^2 \sigma_{R_*}^2} \right\}$$

$$\delta_L \equiv F \quad R_* = r_*(1 + \delta_*)^{1/3}$$

$$\bar{\delta}_W = \int \frac{d^3x}{r_*^3} \tilde{W}(r/r_*) \delta(\mathbf{x}) = \int_{\mathbf{k}} W(kr_*) \delta(\mathbf{k})$$

Prefactor from fluctuations

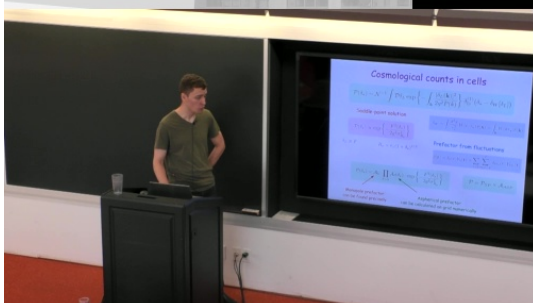
$$\delta(\mathbf{r}) = \delta_0(r) Y_0(\hat{\mathbf{r}}) + \sum_{\ell > 0} \sum_{m=-\ell}^{\ell} \delta_{\ell m}(r) Y_{\ell m}(\hat{\mathbf{r}})$$

$$\mathcal{P}(\delta_*) = \mathcal{A}_0 \cdot \prod_{\ell > 0} \mathcal{A}_{\ell}(\delta_*) \cdot \exp \left\{ - \frac{F^2(\delta_*)}{2g^2 \sigma_{R_*}^2} \right\}$$

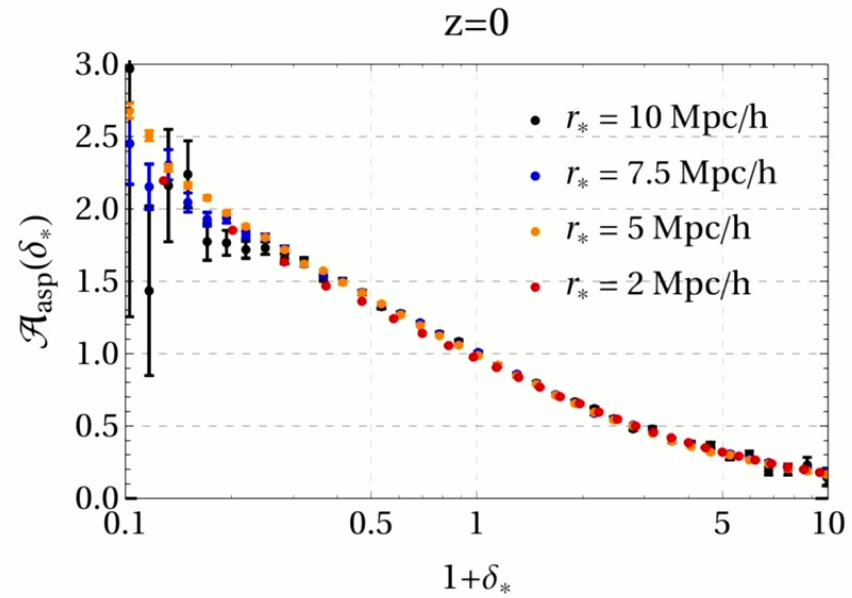
$$\mathcal{P} = \mathcal{P}_{SP} \times \mathcal{A}_{ASP}$$

Monopole prefactor
can be found precisely

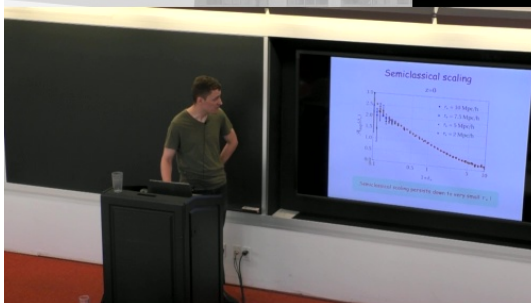
Aspherical prefactor
can be calculated on grid numerically



Semiclassical scaling



Semiclassical scaling persists down to very small r_* !



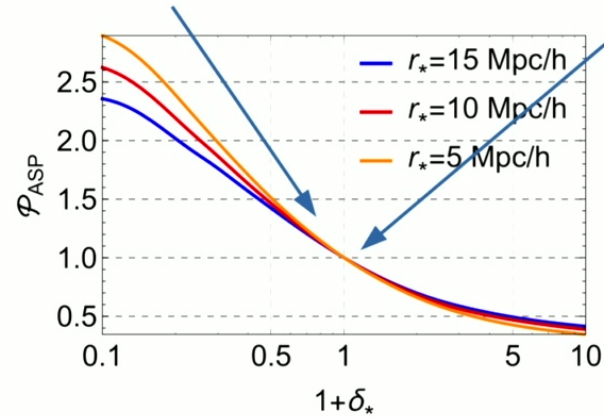
Fluctuation determinant

Value at the origin
controlled by unitarity

$$\int_{-1}^{\infty} d\delta_* \mathcal{P}(\delta_*) = 1$$

Slope at the origin
controlled by
translation invariance

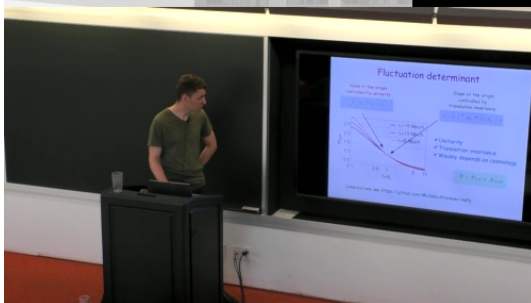
$$\langle \delta_* \rangle \equiv \int_{-1}^{\infty} d\delta_* \mathcal{P}(\delta_*) \delta_* = 0$$



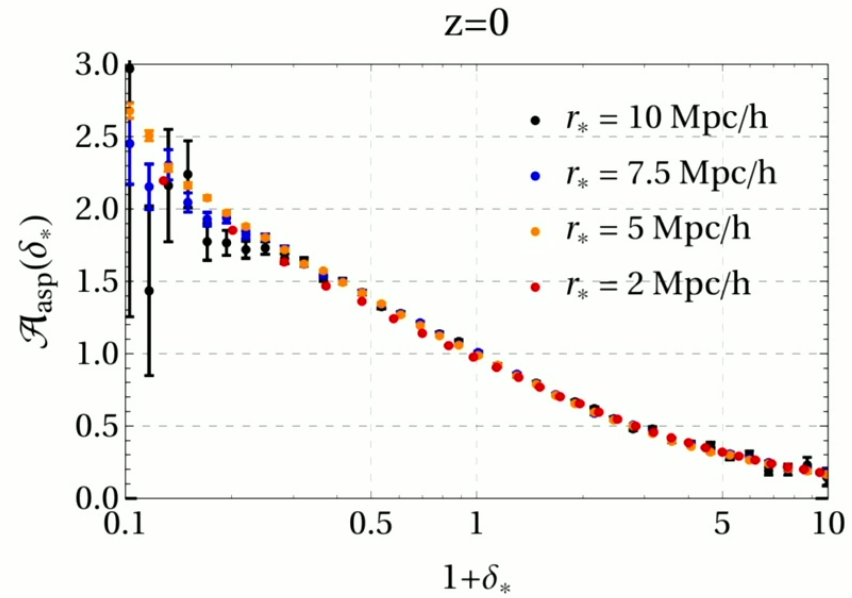
- ✓ Unitarity
- ✓ Translation invariance
- ✓ Weakly depends on cosmology

$$\mathcal{P} = \mathcal{P}_{SP} \times \mathcal{A}_{ASP}$$

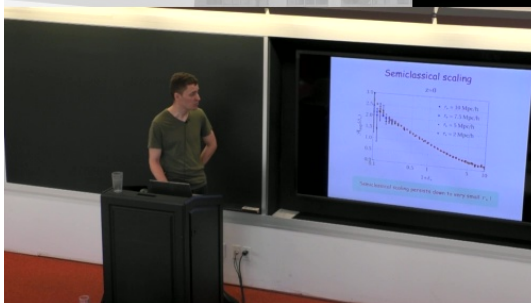
Computations use <https://github.com/Michalychforever/AsPy>



Semiclassical scaling



Semiclassical scaling persists down to very small r_* !



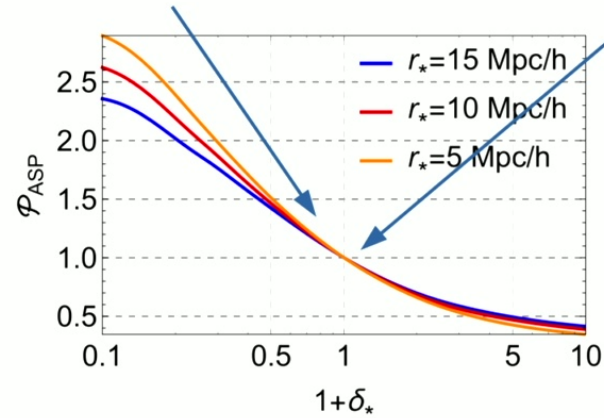
Fluctuation determinant

Value at the origin
controlled by unitarity

$$\int_{-1}^{\infty} d\delta_* \mathcal{P}(\delta_*) = 1$$

Slope at the origin
controlled by
translation invariance

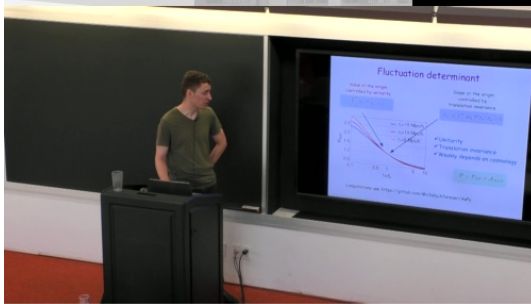
$$\langle \delta_* \rangle \equiv \int_{-1}^{\infty} d\delta_* \mathcal{P}(\delta_*) \delta_* = 0$$



- ✓ Unitarity
- ✓ Translation invariance
- ✓ Weakly depends on cosmology

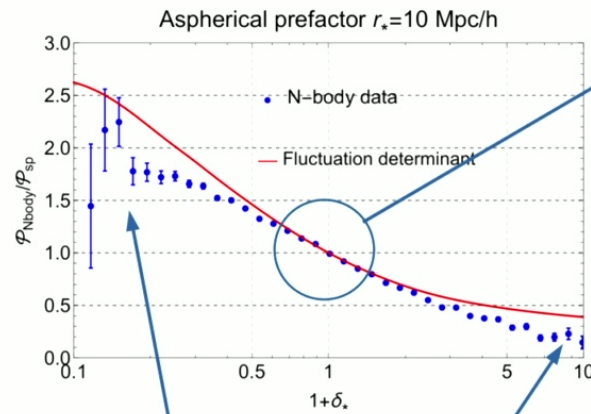
$$\mathcal{P} = \mathcal{P}_{SP} \times \mathcal{A}_{ASP}$$

Computations use <https://github.com/Michalychforever/AsPy>



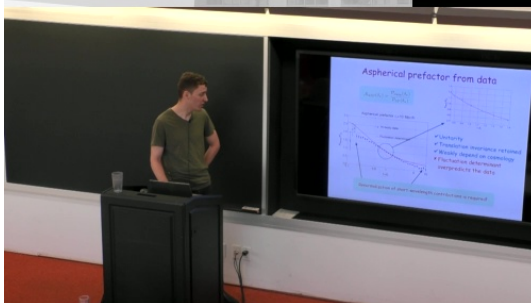
Aspherical prefactor from data

$$\mathcal{A}_{ASP}(\delta_*) = \frac{\mathcal{P}_{\text{data}}(\delta_*)}{\mathcal{P}_{\text{SP}}(\delta_*)}$$



- ✓ Unitarity
- ✓ Translation invariance retained
- ✓ Weakly depend on cosmology
- ✗ Fluctuation determinant overpredicts the data

Renormalization of short-wavelength contributions is required!



Shell crossing scale

$$\frac{dx_i}{d\eta} = -\partial_i \hat{\Psi} - \partial_i \Psi^{(1)}$$

$$\frac{dx_i}{d\eta} = -\partial_i \hat{\Psi}$$

$$\frac{dx_i}{d\eta} = -\partial_i \partial_j \hat{\Psi} x_j^{(1)} - \partial_i \Psi^{(1)}$$

$$\Theta_\ell \equiv \partial_i^2 \Psi_\ell$$

$$\varkappa \equiv \frac{k}{\ell + 1/2}$$

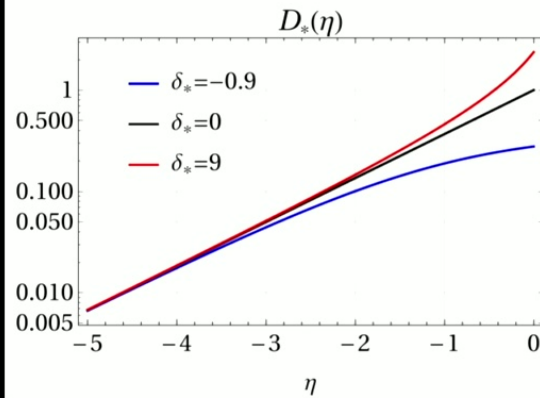
$$\tilde{\Theta}_\ell = \Theta_\ell(\ell + 1/2)$$

$$\partial_j x_i^{(1)} \equiv \frac{\partial x_i^{(1)}}{\partial \hat{x}_j}$$

Consider its trace

$$\frac{d}{d\eta} \partial_i x_i^{(1)} = -\Delta \Psi^{(1)} = -\Theta^{(1)}$$

$$\begin{aligned} \langle (\partial_i x_i^{(1)})^2 \rangle_{k_{\max}} &= \sum_\ell \frac{2\ell + 1}{4\pi} \int^{k_{\max}} \frac{k^2 dk}{(2\pi)^3} P(k) \left| \int_{-\infty}^\eta d\eta' \Theta_\ell^{(1)}(\eta', r(\eta', R); k) \right|^2 \\ &\approx 4\pi \int^{k_{\max}} \frac{k^2 dk}{(2\pi)^3} P(k) \int_{1/R}^\infty \frac{d\varkappa}{(2\pi)^2 \varkappa} \left| \int_{-\infty}^\eta d\eta' \tilde{\Theta}_{\ell 1}(\eta', R; \varkappa) \right|^2 \end{aligned}$$

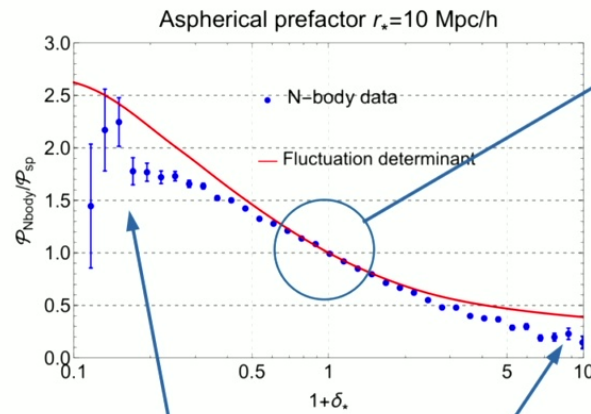


Cummulative growth factor

$$D_*(\eta) = \left(\int_{1/R_*}^\infty \frac{d\varkappa}{(2\pi)^2 \varkappa} \left| \int_{-\infty}^\eta d\eta' \tilde{\Theta}_{\ell 1}(\eta', R_*; \varkappa) \right|^2 \right)^{1/2}$$

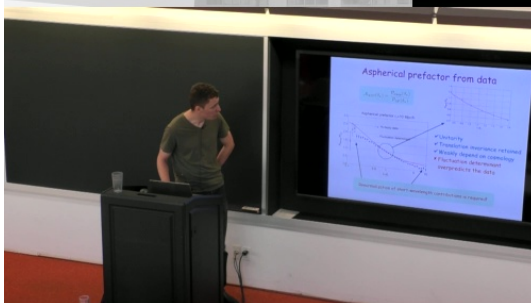
Aspherical prefactor from data

$$\mathcal{A}_{ASP}(\delta_*) = \frac{\mathcal{P}_{\text{data}}(\delta_*)}{\mathcal{P}_{\text{SP}}(\delta_*)}$$



- ✓ Unitarity
- ✓ Translation invariance retained
- ✓ Weakly depend on cosmology
- ✗ Fluctuation determinant overpredicts the data

Renormalization of short-wavelength contributions is required!



Shell crossing scale

$$\frac{dx_i}{d\eta} = -\partial_i \hat{\Psi} - \partial_i \Psi^{(1)}$$

$$\frac{dx_i}{d\eta} = -\partial_i \hat{\Psi}$$

$$\frac{dx_i}{d\eta} = -\partial_i \partial_j \hat{\Psi} x_j^{(1)} - \partial_i \Psi^{(1)}$$

$$\Theta_\ell \equiv \partial_i^2 \Psi_\ell$$

$$\varkappa \equiv \frac{k}{\ell + 1/2}$$

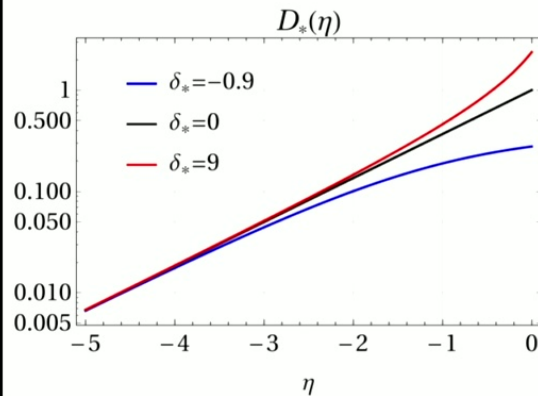
$$\tilde{\Theta}_\ell = \Theta_\ell(\ell + 1/2)$$

$$\partial_j x_i^{(1)} \equiv \frac{\partial x_i^{(1)}}{\partial \hat{x}_j}$$

Consider its trace

$$\frac{d}{d\eta} \partial_i x_i^{(1)} = -\Delta \Psi^{(1)} = -\Theta^{(1)}$$

$$\begin{aligned} \langle (\partial_i x_i^{(1)})^2 \rangle_{k_{\max}} &= \sum_\ell \frac{2\ell + 1}{4\pi} \int^{k_{\max}} \frac{k^2 dk}{(2\pi)^3} P(k) \left| \int_{-\infty}^\eta d\eta' \Theta_\ell^{(1)}(\eta', r(\eta', R); k) \right|^2 \\ &\approx 4\pi \int^{k_{\max}} \frac{k^2 dk}{(2\pi)^3} P(k) \int_{1/R}^\infty \frac{d\varkappa}{(2\pi)^2 \varkappa} \left| \int_{-\infty}^\eta d\eta' \tilde{\Theta}_{\ell 1}(\eta', R; \varkappa) \right|^2 \end{aligned}$$



Cummulative growth factor

$$D_*(\eta) = \left(\int_{1/R_*}^\infty \frac{d\varkappa}{(2\pi)^2 \varkappa} \left| \int_{-\infty}^\eta d\eta' \tilde{\Theta}_{\ell 1}(\eta', R_*; \varkappa) \right|^2 \right)^{1/2}$$

Shell crossing scale

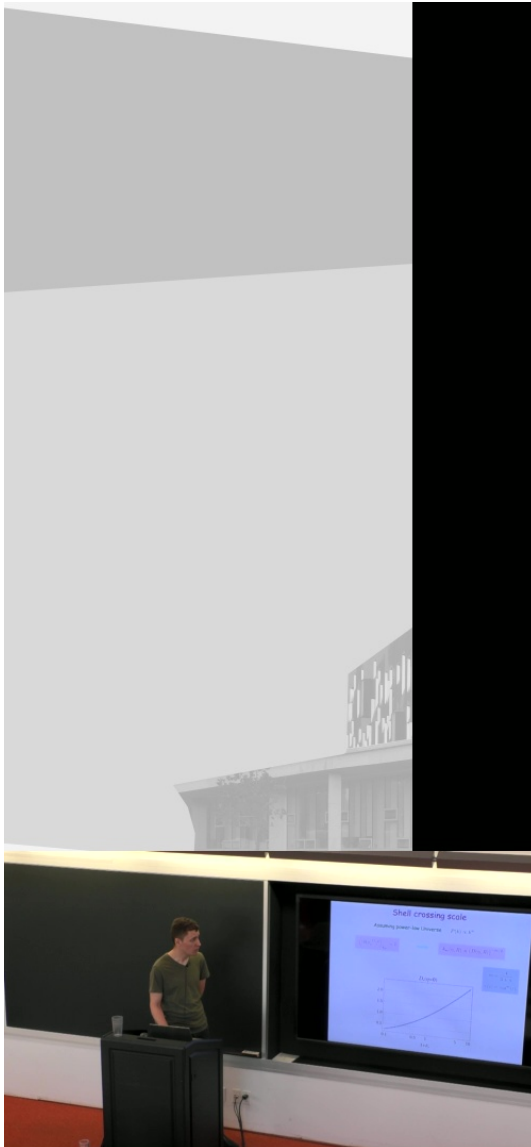
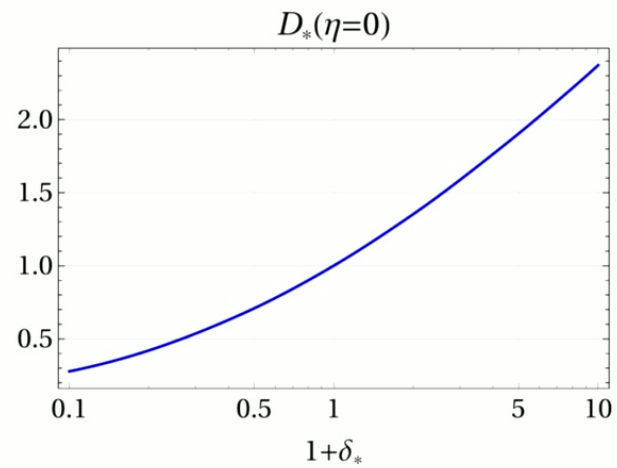
Assuming power-law Universe $P(k) \propto k^n$

$$\langle (\partial_i x_i^{(1)})^2 \rangle_{k_{sc}} \sim 1$$



$$k_{sc}(\eta, R) \propto (D(\eta, R))^{-m/2}$$

$$m = \frac{4}{3+n}$$
$$\gamma(z) = \gamma_0 g^m(z)$$



Effective stress tensor

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{am} \cdot \nabla f - am \nabla \Phi \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

$$\begin{aligned} \delta_{\text{tot}} &= \delta^l + \delta^s \\ u_{\text{tot},i} &= u_i^l + u_i^s \\ \Phi_{\text{tot}} &= \Phi^l + \Phi^s \end{aligned}$$

$$\begin{aligned} \delta^l(\mathbf{x}) &\equiv \int_{x'} W_\Lambda(|\mathbf{x} - \mathbf{x}'|) \delta_{\text{tot}}(\mathbf{x}') \\ \Phi^l(\mathbf{x}) &\equiv \int_{x'} W_\Lambda(|\mathbf{x} - \mathbf{x}'|) \Phi_{\text{tot}}(\mathbf{x}') \\ (1 + \delta^l(\mathbf{x})) u_i^l(\mathbf{x}) &\equiv \int_{x'} W_\Lambda(|\mathbf{x} - \mathbf{x}'|) (1 + \delta(\mathbf{x}')) u_{\text{tot},i}(\mathbf{x}') \end{aligned}$$

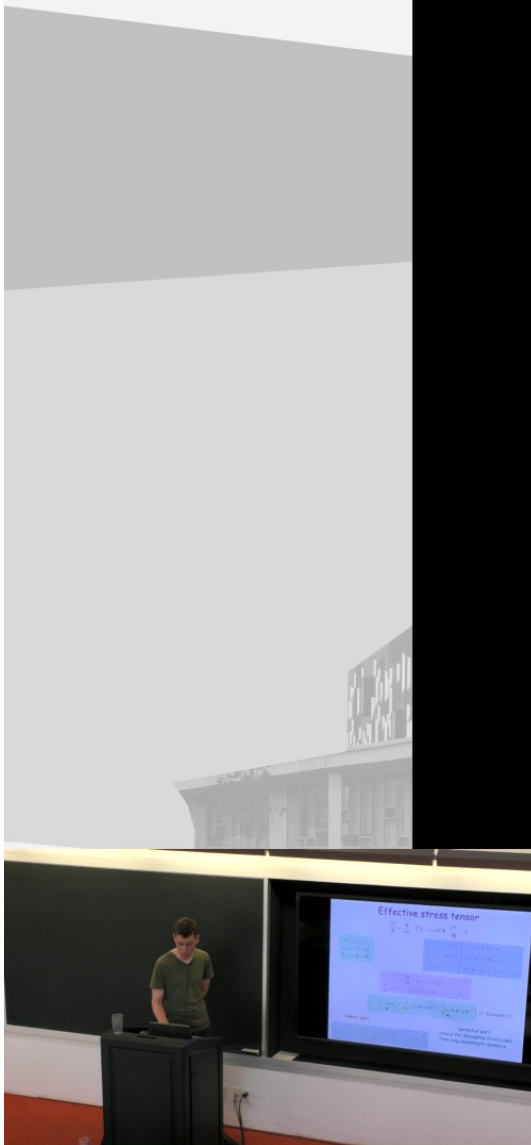
$$\begin{aligned} \frac{\partial \delta^l}{\partial t} + \partial_i ((1 + \delta^l) u_i^l) &= 0 \\ \frac{\partial u_i^l}{\partial t} + \mathcal{H} u_i^l + (u_j^l \partial_j) u_i^l + \partial_i \Phi^l &= -\frac{1}{1 + \delta} \partial_j \tau_{ij} \end{aligned}$$

$$\tau_{ij} = (1 + \delta) \sigma_{ij}^l + \frac{2}{3\mathcal{H}^2} \left([\partial_i \Phi^s \partial_j \Phi^s]^l - \frac{1}{2} \delta_{ij} [\partial_k \Phi^s \partial_k \Phi^s]^l \right) \quad (\text{D. Baumann '12})$$

'kinetic' part

'potential' part
crucial for decoupling virial scales
from long-wavelength dynamics

$$\sigma_{ij}^l = \frac{\int (v_i - u_i^l)(v_j - u_j^l) f^l d^3p}{\int f^l d^3p}, \quad v_i \equiv \frac{p_i}{am}$$



Fluid description

$$1/R_* \ll k_1 < k < k_2 \ll k_{sc}$$

Averaging over initial conditions

$$\sigma_{ij}^l = \langle u_i^{(1)} u_j^{(1)} \rangle = \mathcal{H}^2 \langle \partial_i \Psi^{(1)} \partial_j \Psi^{(1)} \rangle$$

Expanding up to the quadratic order
and averaging over the angles...

$$\begin{aligned} \delta_0 &= \hat{\delta}_0 + \delta_0^{(2)} \\ \Theta_0 &= \hat{\Theta}_0 + \Theta_0^{(2)} \\ \Phi_0 &= \hat{\Phi}_0 + \Phi_0^{(2)} \end{aligned}$$

$$a = \sigma, \Phi$$

$$\Upsilon(\eta) = \frac{1}{\mathcal{H}^2(1 + \hat{\delta})} \partial_j \tau_{ij} \Big|_{i \rightarrow r}$$

$$\dot{\mu}^{(2)} + \dot{r}_\eta^{(2)} \hat{r}_\eta^2 (1 + \hat{\delta}(\hat{r}_\eta)) + r_\eta^{(2)} \frac{d}{d\eta} (\hat{r}_\eta^2 (1 + \hat{\delta}(\hat{r}_\eta))) = 0$$

$$\ddot{r}_\eta^{(2)} + \frac{\dot{r}_\eta^{(2)}}{2} + \left(1 + \frac{3}{2} \hat{\delta}(\hat{r}_\eta) - \frac{R_*^3}{\hat{r}_\eta^3} \right) r_\eta^{(2)} + \frac{3}{2 \hat{r}_\eta^2} \mu^{(2)} = -\Upsilon^a(\hat{r}_\eta)$$

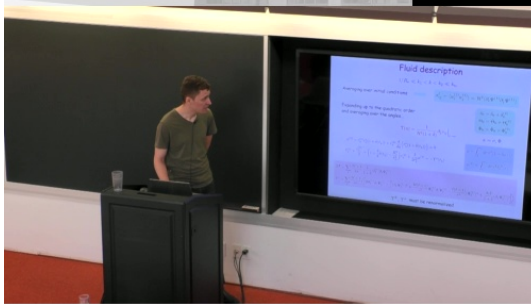
$$\hat{\mu} = \int_0^{r_\eta} dr r^2 (1 + \hat{\delta}_0(r))$$

$$\mu^{(2)} = \int_0^{r_\eta} dr r^2 \delta_0^{(2)}(r)$$

$$\Upsilon^\Phi = \sum_\ell \frac{2\ell + 1}{4\pi} \frac{1}{1 + \hat{\delta}} \langle \delta_\ell^{(1)} \partial_r \Phi_\ell^{(1)} \rangle$$

$$\Upsilon^\sigma = \sum_\ell \frac{2\ell + 1}{4\pi} \left\langle \left[2\Theta_\ell^{(1)} \partial_r \Psi_\ell^{(1)} - \frac{2}{r} (\partial_r \Psi_\ell^{(1)})^2 + \frac{2\ell(\ell + 1)}{r^2} \Psi_\ell^{(1)} \partial_r \Psi_\ell^{(1)} - \frac{\ell(\ell + 1)}{r^3} (\Psi_\ell^{(1)})^2 + \frac{\partial_r \hat{\delta}}{1 + \hat{\delta}} (\partial_r \Psi_\ell^{(1)})^2 \right] \right\rangle$$

$\Upsilon^\Phi, \Upsilon^\sigma$ must be renormalized



Effective stress tensor

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{am} \cdot \nabla f - am \nabla \Phi \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

$$\begin{aligned} \delta_{\text{tot}} &= \delta^l + \delta^s \\ u_{\text{tot},i} &= u_i^l + u_i^s \\ \Phi_{\text{tot}} &= \Phi^l + \Phi^s \end{aligned}$$

$$\begin{aligned} \delta^l(\mathbf{x}) &\equiv \int_{x'} W_\Lambda(|\mathbf{x} - \mathbf{x}'|) \delta_{\text{tot}}(\mathbf{x}') \\ \Phi^l(\mathbf{x}) &\equiv \int_{x'} W_\Lambda(|\mathbf{x} - \mathbf{x}'|) \Phi_{\text{tot}}(\mathbf{x}') \\ (1 + \delta^l(\mathbf{x})) u_i^l(\mathbf{x}) &\equiv \int_{x'} W_\Lambda(|\mathbf{x} - \mathbf{x}'|) (1 + \delta(\mathbf{x}')) u_{\text{tot},i}(\mathbf{x}') \end{aligned}$$

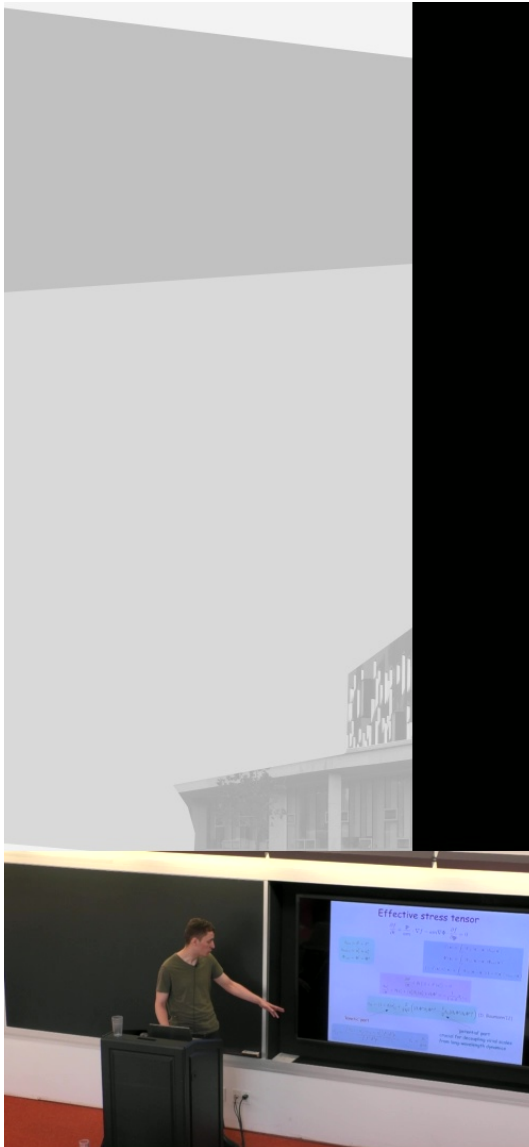
$$\begin{aligned} \frac{\partial \delta^l}{\partial t} + \partial_i ((1 + \delta^l) u_i^l) &= 0 \\ \frac{\partial u_i^l}{\partial t} + \mathcal{H} u_i^l + (u_j^l \partial_j) u_i^l + \partial_i \Phi^l &= -\frac{1}{1 + \delta} \partial_j \tau_{ij} \end{aligned}$$

$$\tau_{ij} = (1 + \delta) \sigma_{ij}^l + \frac{2}{3\mathcal{H}^2} \left([\partial_i \Phi^s \partial_j \Phi^s]^l - \frac{1}{2} \delta_{ij} [\partial_k \Phi^s \partial_k \Phi^s]^l \right) \quad (\text{D. Baumann '12})$$

'kinetic' part

'potential' part
crucial for decoupling virial scales
from long-wavelength dynamics

$$\sigma_{ij}^l = \frac{\int (v_i - u_i^l)(v_j - u_j^l) f^l d^3p}{\int f^l d^3p}, \quad v_i \equiv \frac{p_i}{am}$$



Fluid description

$$1/R_* \ll k_1 < k < k_2 \ll k_{sc}$$

Averaging over initial conditions

$$\sigma_{ij}^l = \langle u_i^{(1)} u_j^{(1)} \rangle = \mathcal{H}^2 \langle \partial_i \Psi^{(1)} \partial_j \Psi^{(1)} \rangle$$

Expanding up to the quadratic order
and averaging over the angles...

$$\begin{aligned} \delta_0 &= \hat{\delta}_0 + \delta_0^{(2)} \\ \Theta_0 &= \hat{\Theta}_0 + \Theta_0^{(2)} \\ \Phi_0 &= \hat{\Phi}_0 + \Phi_0^{(2)} \end{aligned}$$

$$a = \sigma, \Phi$$

$$\Upsilon(\eta) = \frac{1}{\mathcal{H}^2(1 + \hat{\delta})} \partial_j \tau_{ij} \Big|_{i \rightarrow r}$$

$$\dot{\mu}^{(2)} + \dot{r}_\eta^{(2)} \hat{r}_\eta^2 (1 + \hat{\delta}(\hat{r}_\eta)) + r_\eta^{(2)} \frac{d}{d\eta} (\hat{r}_\eta^2 (1 + \hat{\delta}(\hat{r}_\eta))) = 0$$

$$\ddot{r}_\eta^{(2)} + \frac{\dot{r}_\eta^{(2)}}{2} + \left(1 + \frac{3}{2} \hat{\delta}(\hat{r}_\eta) - \frac{R_*^3}{\hat{r}_\eta^3} \right) r_\eta^{(2)} + \frac{3}{2 \hat{r}_\eta^2} \mu^{(2)} = -\Upsilon^a(\hat{r}_\eta)$$

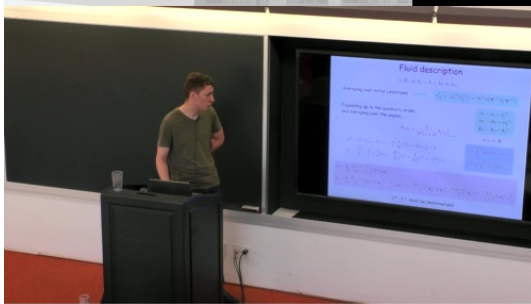
$$\hat{\mu} = \int_0^{r_\eta} dr r^2 (1 + \hat{\delta}_0(r))$$

$$\mu^{(2)} = \int_0^{r_\eta} dr r^2 \delta_0^{(2)}(r)$$

$$\Upsilon^\Phi = \sum_\ell \frac{2\ell + 1}{4\pi} \frac{1}{1 + \hat{\delta}} \langle \delta_\ell^{(1)} \partial_r \Phi_\ell^{(1)} \rangle$$

$$\Upsilon^\sigma = \sum_\ell \frac{2\ell + 1}{4\pi} \left\langle \left[2\Theta_\ell^{(1)} \partial_r \Psi_\ell^{(1)} - \frac{2}{r} (\partial_r \Psi_\ell^{(1)})^2 + \frac{2\ell(\ell + 1)}{r^2} \Psi_\ell^{(1)} \partial_r \Psi_\ell^{(1)} - \frac{\ell(\ell + 1)}{r^3} (\Psi_\ell^{(1)})^2 + \frac{\partial_r \hat{\delta}}{1 + \hat{\delta}} (\partial_r \Psi_\ell^{(1)})^2 \right] \right\rangle$$

$\Upsilon^\Phi, \Upsilon^\sigma$ must be renormalized



Renormalization

$$\partial_j \tau_{ij}^a \Big|_{i \rightarrow r} = \frac{1}{\mathcal{H}^2(1+\hat{\delta})} \left(\partial_r \tau_{\parallel}^a + \frac{2}{r} \tau_{\parallel}^a - \frac{2}{r} \tau_{\perp}^a \right) \Big|_{\eta, \hat{r}(\eta)}$$

Counterterm must fix UV behaviour

$$\tau_{\alpha}^{a,\text{ctr}} \sim 2\mathcal{H}^2 \int_{k_{\text{sc}}}^{\infty} \frac{dk P(k)}{(2\pi)^3} \cdot \int \frac{d\boldsymbol{\varkappa}}{\boldsymbol{\varkappa}} \chi_{\alpha}^a$$

$$\int_{k_{\text{sc}}}^{\infty} dk P(k) \propto \frac{1}{k_{\text{sc}}^2 D^2} \propto D^{m-2}$$

$$\tau_{\alpha}^{a,\text{ctr}}(\eta, R) = \zeta^a \cdot 2\mathcal{H}^2 [D(\eta, R)]^{m-2} \int_{R^{-1}}^{\infty} \frac{d\boldsymbol{\varkappa}}{\boldsymbol{\varkappa}} \chi_{\alpha}^a(\eta, R; \boldsymbol{\varkappa}), \quad a = \sigma, \Phi; \quad \alpha = \parallel, \perp$$

$$\Upsilon^{a,\text{ctr}}(\eta) = 2\zeta^a [D_*(\eta)]^{m-2} \left[\int_{R_*^{-1}}^{\infty} \frac{d\boldsymbol{\varkappa}}{\boldsymbol{\varkappa}} v^a(\eta; \boldsymbol{\varkappa}) + (m-2) \frac{\partial_r [\ln D]}{1+\hat{\delta}} \Big|_{\eta, \hat{r}(\eta)} \int_{R_*^{-1}}^{\infty} \frac{d\boldsymbol{\varkappa}}{\boldsymbol{\varkappa}} \chi_{\parallel}^a(\eta, R_*; \boldsymbol{\varkappa}) \right]$$

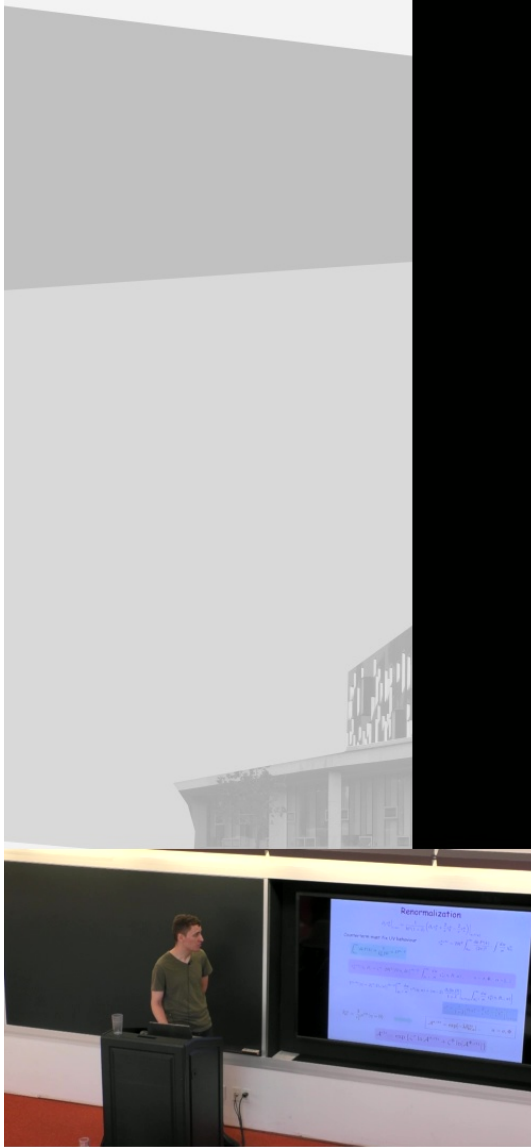
$$v^a = \frac{1}{1+\hat{\delta}} \left[\partial_r \chi_{\parallel}^a + \frac{2}{r} \chi_{\parallel}^a - \frac{1}{r} \chi_{\perp}^a \right] \Big|_{\eta, \hat{r}(\eta)}$$

$$\bar{\delta}_W^{\text{ctr}} = \frac{3}{r_*^3} \mu^{\text{ctr}}(\eta = 0)$$



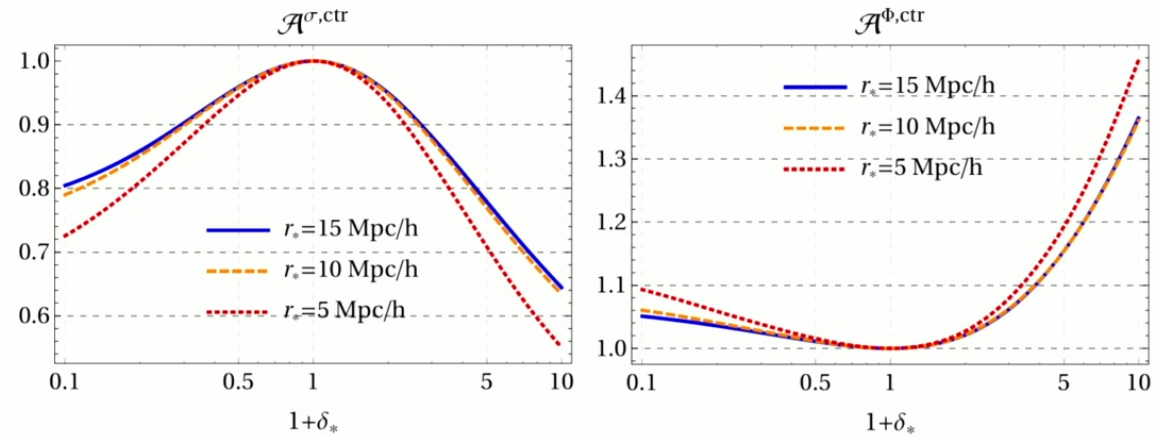
$$\mathcal{A}^{a,\text{ctr}} = \exp[-\hat{\lambda} \bar{\delta}_W^{\text{ctr}}], \quad a = \sigma, \Phi$$

$$\mathcal{A}^{\text{ctr}} = \exp \left\{ \zeta^{\sigma} \ln \mathcal{A}^{\sigma,\text{ctr}} + \zeta^{\Phi} \ln [\mathcal{A}^{\Phi,\text{ctr}}] \right\}$$



Counterterm prefactor

$$\mathcal{A}^{\text{ctr}} = \exp \left\{ \zeta^\sigma \ln \mathcal{A}^{\sigma, \text{ctr}} + \zeta^\Phi \ln [\mathcal{A}^{\Phi, \text{ctr}}] \right\}$$



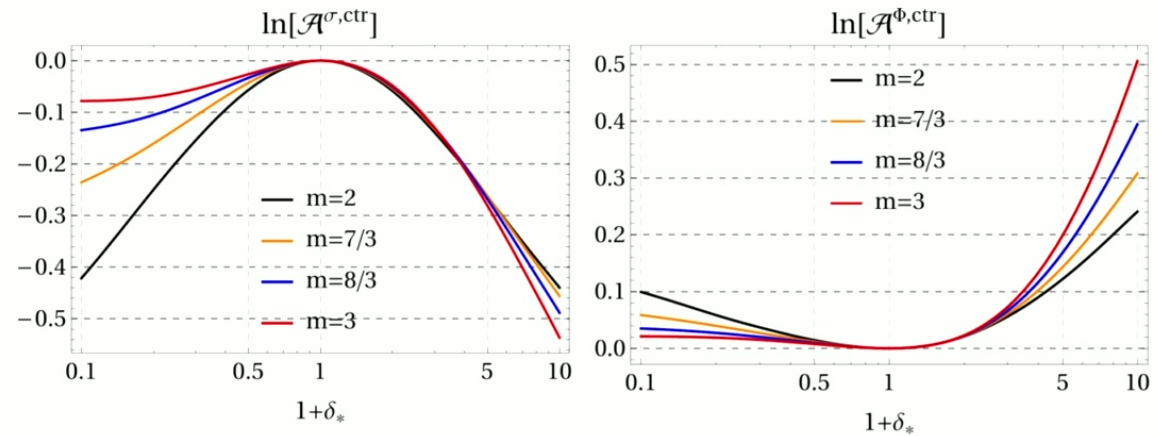
Time dependence

$$\ln[\mathcal{A}^{a, \text{ctr}}] \propto [g(z)]^{m-2}$$



Counterterm prefactor

$$\mathcal{A}^{\text{ctr}} = \exp \left\{ \zeta^\sigma \ln \mathcal{A}^{\sigma, \text{ctr}} + \zeta^\Phi \ln [\mathcal{A}^{\Phi, \text{ctr}}] \right\}$$



$\langle \zeta^\sigma, \zeta^\Phi, m \rangle$ can be related to one-loop EFT parameters!

Consistency conditions

$$\ln[\mathcal{A}^{\text{ctr}}(\delta_*)] = \frac{\delta_*^2}{2\sigma_{r_*}^4} c_{W,2}^{\text{ctr}} - \frac{\delta_*^3}{6\sigma_{r_*}^6} \left(c_{W,3}^{\text{ctr}} - 18 \left(\frac{4}{21} - \frac{\xi_{r_*}}{\sigma_{r_*}^2} \right) c_{W,2} \right) + \mathcal{O}(\delta_*^4)$$

$$c_{W,n}^{\text{ctr}} = g^{-2n} \int_{\mathbf{k}_1} \dots \int_{\mathbf{k}_n} W(k_1 r_*) \dots W(k_n r_*) \langle \delta(\mathbf{k}_1) \dots \delta(\mathbf{k}_n) \rangle^{\text{ctr}}$$

$$\ln \mathcal{A}^{\text{ctr}}(z) = -\delta_*^2 \frac{\gamma(z)}{g^2(z)} \frac{\Sigma_{r_*}^2}{\sigma_{r_*}^4} + \mathcal{O}(\delta_*^2)$$

$$\xi_{r_*} = 4\pi \int [dk] \frac{\sin kr_*}{kr_*} W_{\text{th}}(kr_*) P(k)$$

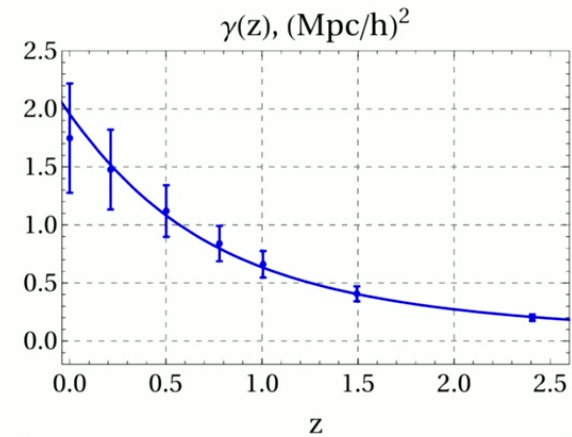
$$\Sigma_{r_*}^2 \equiv 4\pi \int [dk] k^2 |W_{\text{th}}(kr_*)|^2 P(k)$$

$$\gamma(z) = \gamma_0 g^m(z)$$

$$\gamma_0 = 1.95 \pm 0.26 \text{ (Mpc/h)}^2$$

$$m = 2.26 \pm 0.21$$

$$\text{corr}(\gamma_0, m) = 0.85$$



Theoretical model and fitting strategy

$$\mathcal{P}(\delta_*) = \mathcal{P}_{\text{SP}} \cdot \prod_{\ell > 0} \mathcal{A}_\ell(\delta_*) \cdot \mathcal{A}^{\text{ctr}}$$

Spherical PDF
computed precisely

Fluctuation determinant
calculated on grid numerically

Counterterm prefactor
predicted given
model parameters
(α, β, m)

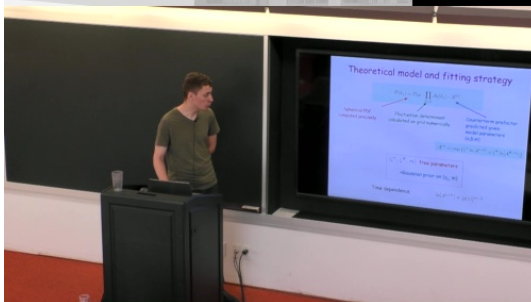
$$\mathcal{A}^{\text{ctr}} = \exp \{ \zeta^\sigma \ln \mathcal{A}^{\sigma, \text{ctr}} + \zeta^\Phi \ln [\mathcal{A}^{\Phi, \text{ctr}}] \}$$

$\langle \zeta^\sigma, \zeta^\Phi, m \rangle$ free parameters

+Gaussian prior on (γ_0, m)

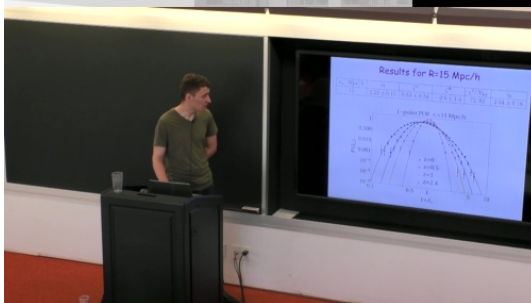
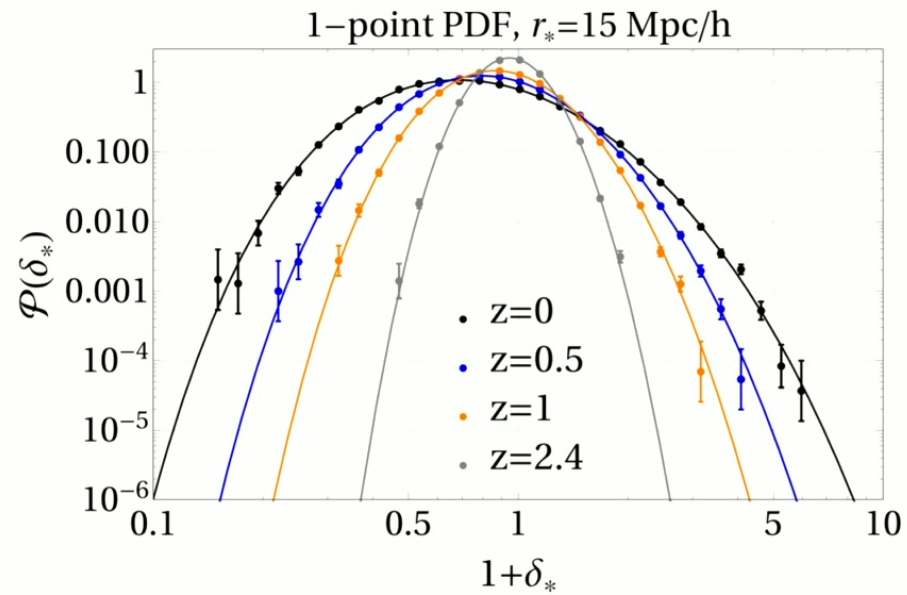
Time dependence

$$\ln[\mathcal{A}^{a, \text{ctr}}] \propto [g(z)]^{m-2}$$



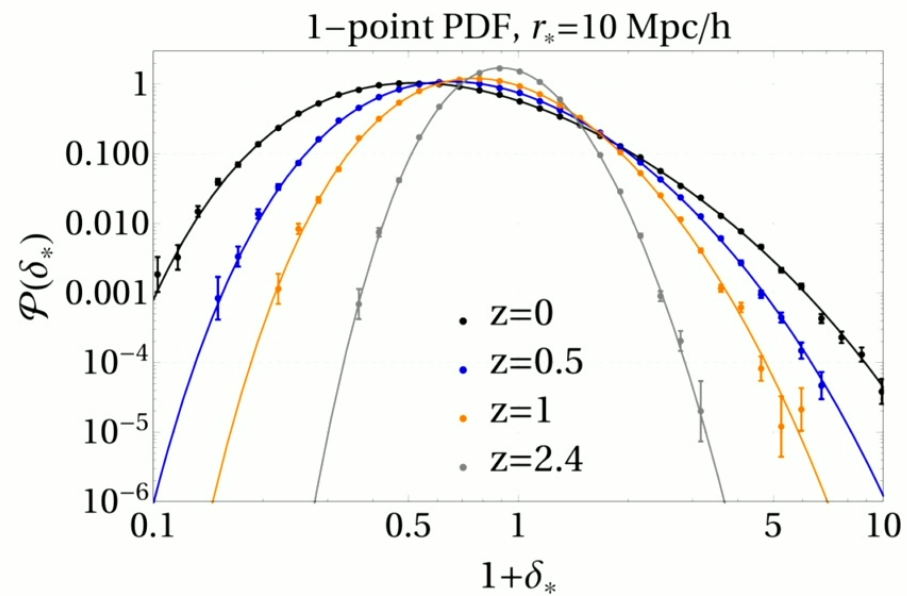
Results for $R=15 \text{ Mpc}/h$

r_* , Mpc/h	m	ζ^σ	ζ^Φ	χ^2/N_{dof}	γ_0
15	2.23 ± 0.15	0.53 ± 0.54	-2.6 ± 1.4	72/82	2.04 ± 0.16

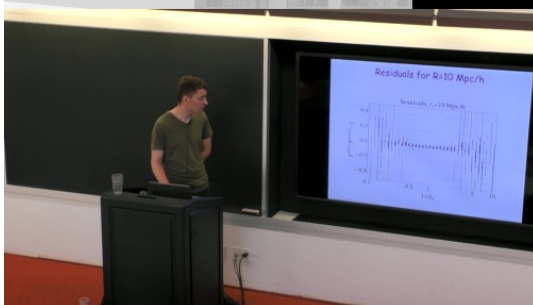
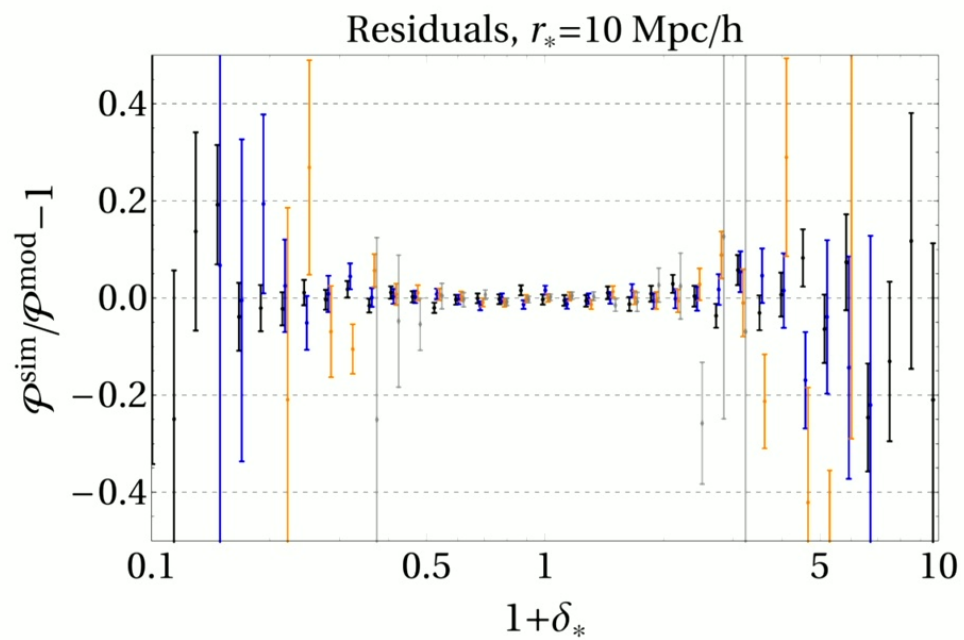


Results for $R=10$ Mpc/h

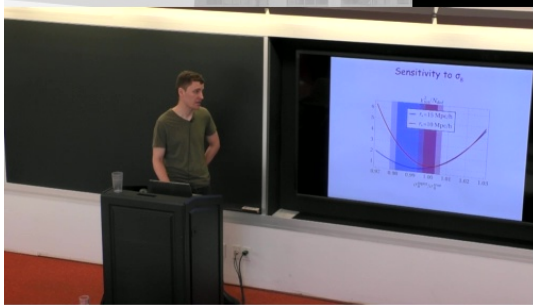
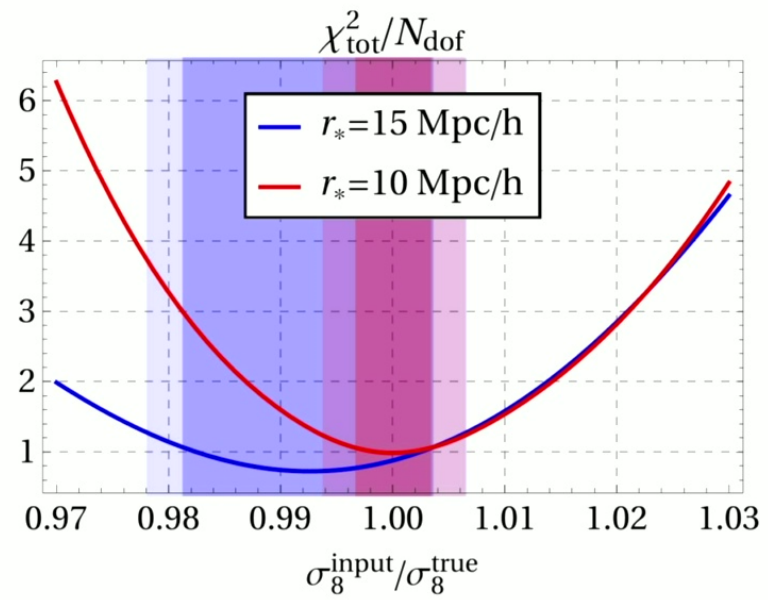
r_* , Mpc/h	m	ζ^σ	ζ^Φ	χ^2/N_{dof}	γ_0
10	2.14 ± 0.10	0.48 ± 0.29	-2.03 ± 0.69	108/110	1.73 ± 0.08



Residuals for R=10 Mpc/h

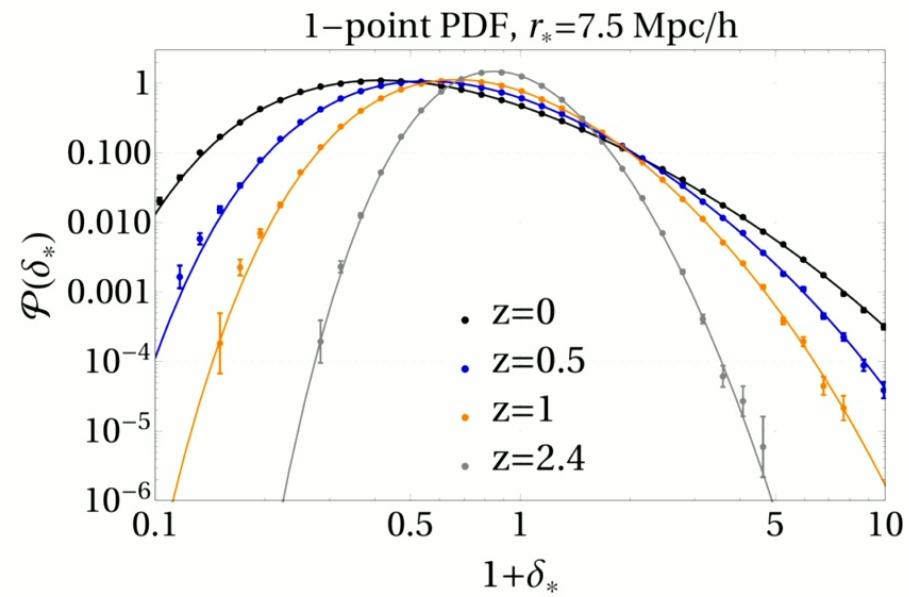


Sensitivity to σ_8



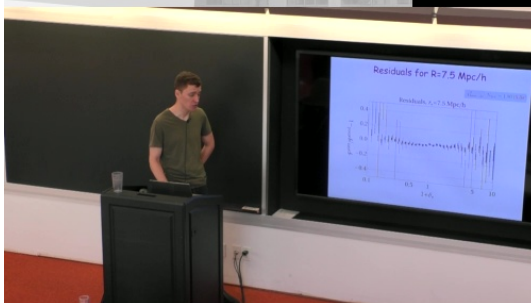
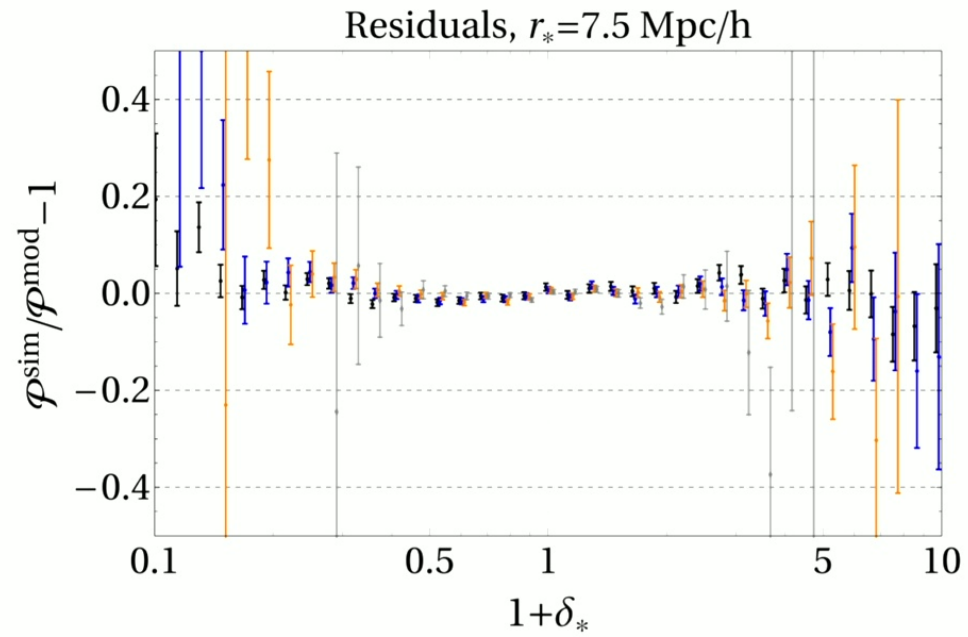
Results for $R=7.5 \text{ Mpc}/h$

r_* , Mpc/h	m	ζ^σ	ζ^Φ	χ^2/N_{dof}	γ_0
7.5	2.37 ± 0.06	1.06 ± 0.18	-0.49 ± 0.36	225/125	1.71 ± 0.05



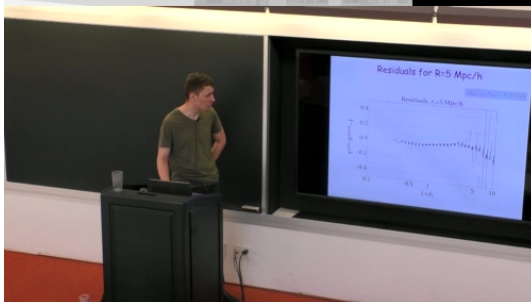
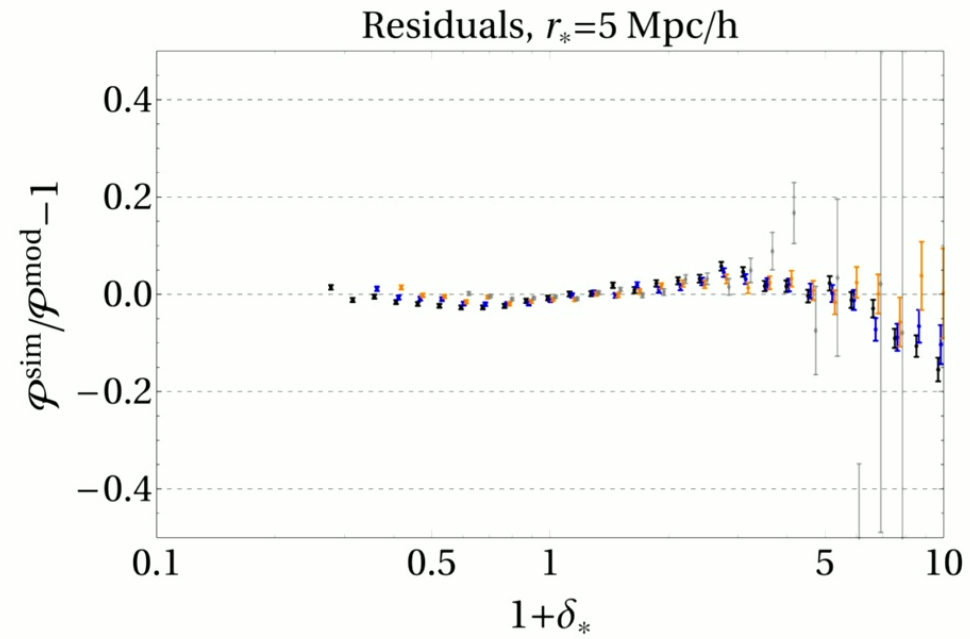
Residuals for $R=7.5 \text{ Mpc/h}$

$$\chi^2_{\text{best-fit}}/N_{\text{dof}} = 1.80 \text{ (5.3}\sigma\text{)}$$

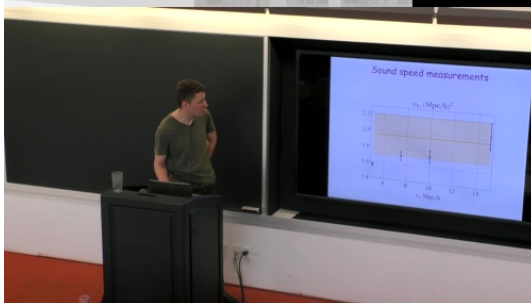
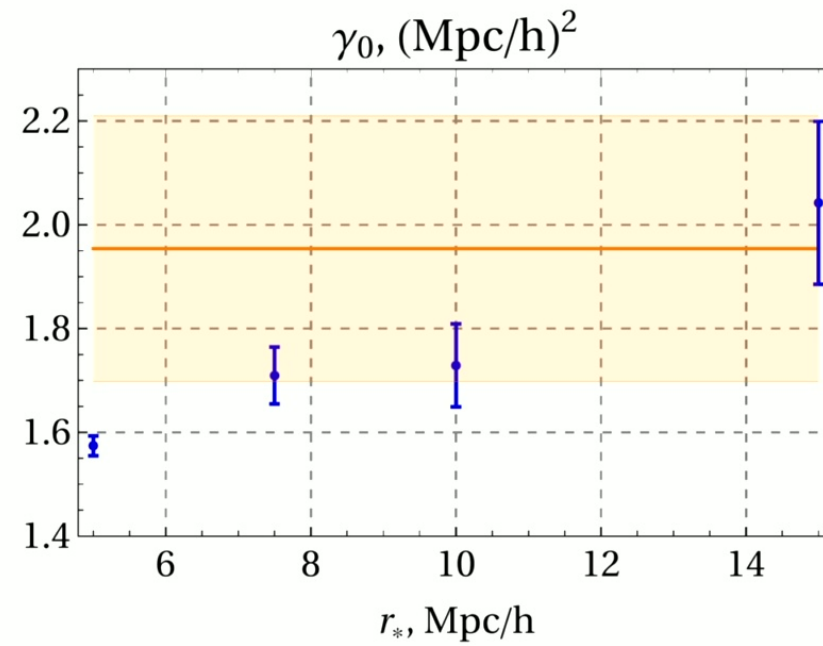


Residuals for R=5 Mpc/h

$$\chi^2_{\text{best-fit}}/N_{\text{dof}} = 9.74 (25\sigma)$$



Sound speed measurements



Part II: conclusions

- ✓ Three-parametric model for counterterm prefactor is in a percent agreement with N-body data for $r_* \geq 10$ Mpc/h
- ✓ PDF can provide better constraints on the one-loop counterterm coefficient than the correlation functions
- ✓ PDF of $r_* < 10$ Mpc/h has a per-mile sensitivity to σ_8
- ✓ Joint analysis of PDF and full-shape power spectra is required
- ✓ For $5 \text{ Mpc/h} < r_* < 10 \text{ Mpc/h}$ there is a few percent deviation between the model and the N-body data due to the 2-loop correction.
- ✓ Theoretical error is needed for robust measurements at $r_* < 10$ Mpc/h

