Title: Measurement-induced phase transitions on dynamical quantum trees

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Series: Perimeter Institute Quantum Discussions

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Abstract: Monitored many-body systems fall broadly into two dynamical phases, ``entangling" or ``disentangling", separated by a transition as a function of the rate at which measurements are made on the system. Producing an analytical theory of this measurement-induced transition is an outstanding challenge. Recent work made progress in the context of tree tensor networks, which can be related to all-to-all quantum circuit dynamics with forced (postselected) measurement outcomes. So far, however, there are no exact solutions for dynamics of spin-1/2 degrees of freedom (qubits) with ``real" measurements, whose outcome probabilities are sampled according to the Born rule. Here we define dynamical processes for qubits, with real measurements, that have a tree-like spacetime interaction graph, either collapsing or expanding the system as a function of time. The former case yields an exactly solvable measurement transition. We explore these processes analytically and numerically, exploiting the recursive structure of the tree. We compare the case of ``real" measurements with the case of ``forced" measurements. Both cases show a transition at a nontrivial value of the measurement strength, with the real measurement case exhibiting a smaller entangling phase. Both exhibit exponential scaling of the entanglement near the transition, but they differ in the value of a critical exponent. An intriguing difference between the two cases is that the real measurement case lies at the boundary between two distinct types of critical scaling. On the basis of our results we propose a protocol for realizing a measurement phase transition experimentally via an expansion process.

Measurement induced phase transitions on dynamical quantum trees

Xiaozhou Feng, Brian Skinner and Adam Nahum

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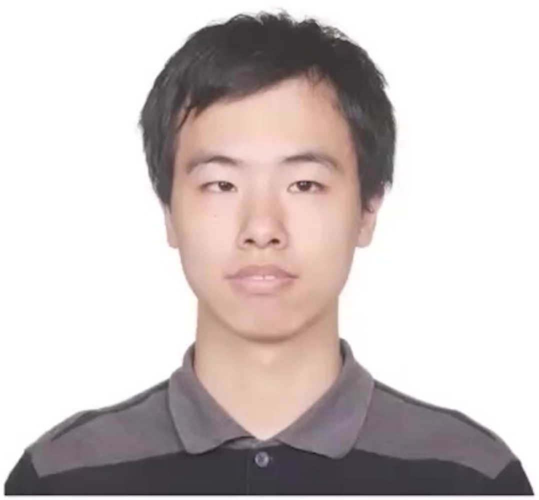

Xiaozhou Feng
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Measurement-induced phase transitions on dynamical quantum trees : arXiv:2210.07264

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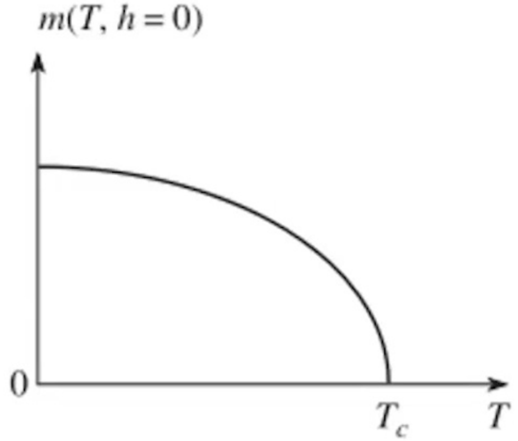
Traditional Phase Transition vs Measurement Induced Phase Transition

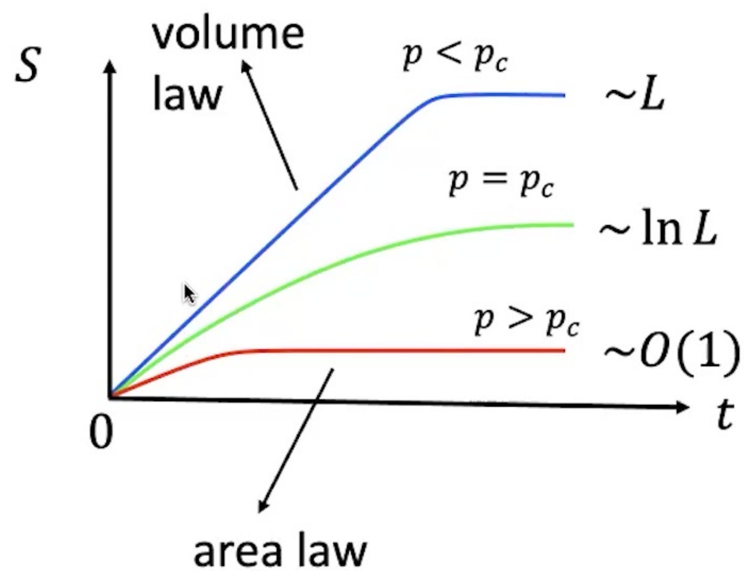
Mean-field ferromagnetic transition of magnetization



M.Kardar, Cambridge 2007

1. equilibrium state or ground state
2. characterized by expectation value of observables

for review: Fisher, Khemani, Nahum and Vijay, arXiv:2207.14280
Potter and Vasseur, arxiv: 2111.08018



1. Dynamics of quantum many body system
2. Characterized by no expectation value of any operator but quantum information dynamics

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Quantum Entanglement

Consider a two spin system

$$\psi_1 = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$ entangled state

→ **Entanglement**: information shared by no subsystem alone but the whole systems together !

With measurement

$$\psi_1 \to |\uparrow\rangle \otimes |\downarrow\rangle \quad \text{or} \quad |\downarrow\rangle \otimes |\uparrow\rangle$$

→ **Measurement turns an entangled state into a product state**

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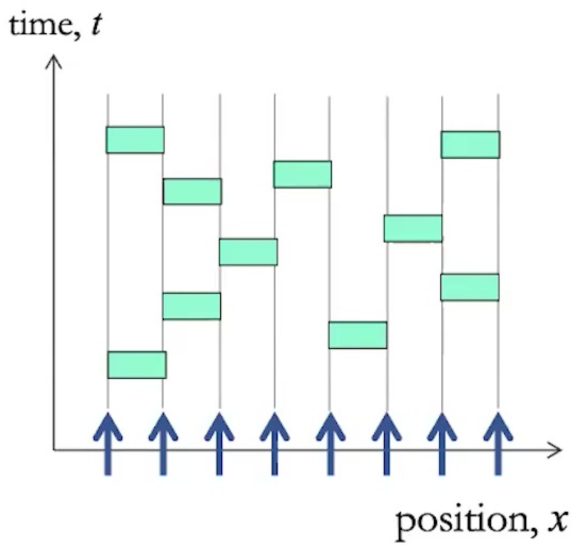
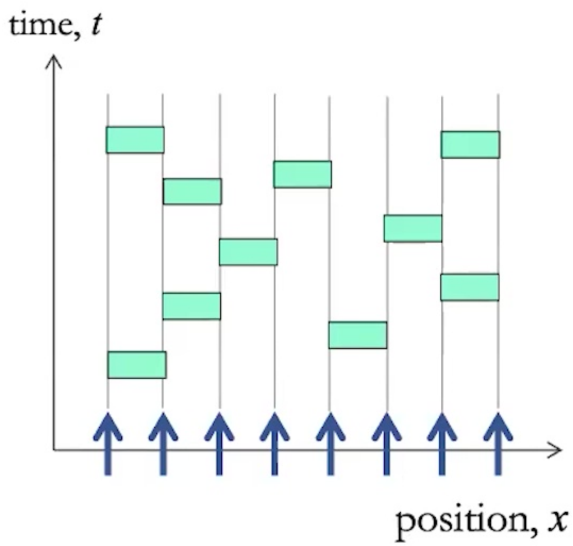
Measurement induced phase transition

A phase transition characterized by the dynamics of quantum entanglement



Measurement induced phase transition

A phase transition characterized by the dynamics of quantum entanglement



Consider a 1D qubit chain with a product state at the beginning and then turn on a random unitary evolution

a quantum circuit without any measurements

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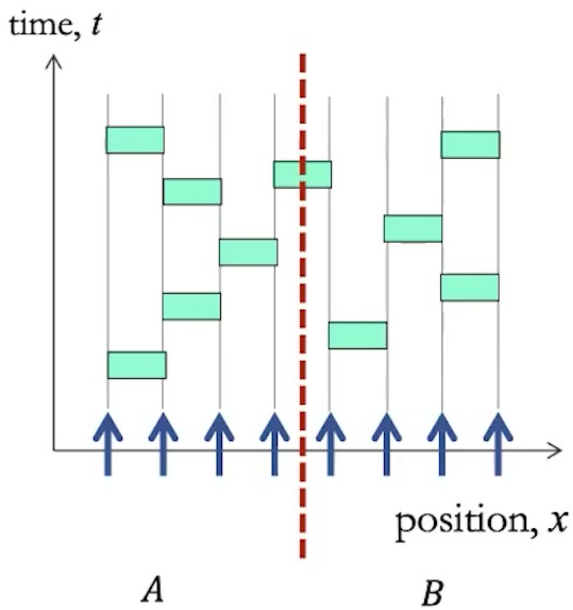
Measurement induced phase transition

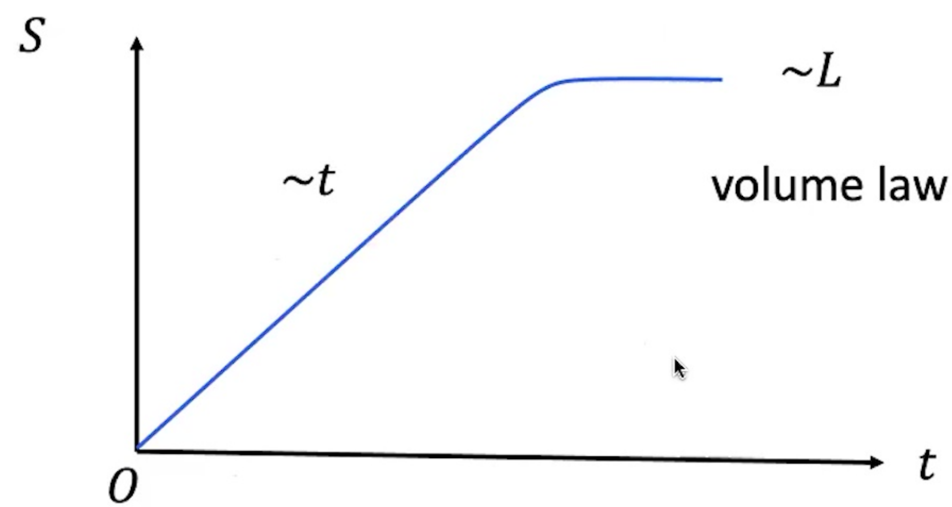
A phase transition characterized by the dynamics of quantum entanglement

$$S = -\mathrm{tr}\rho_A \ln \rho_A$$

entanglement entropy between subsystem A and B



How does the entanglement between the two subsystems evolve ?



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Measurement induced phase transition

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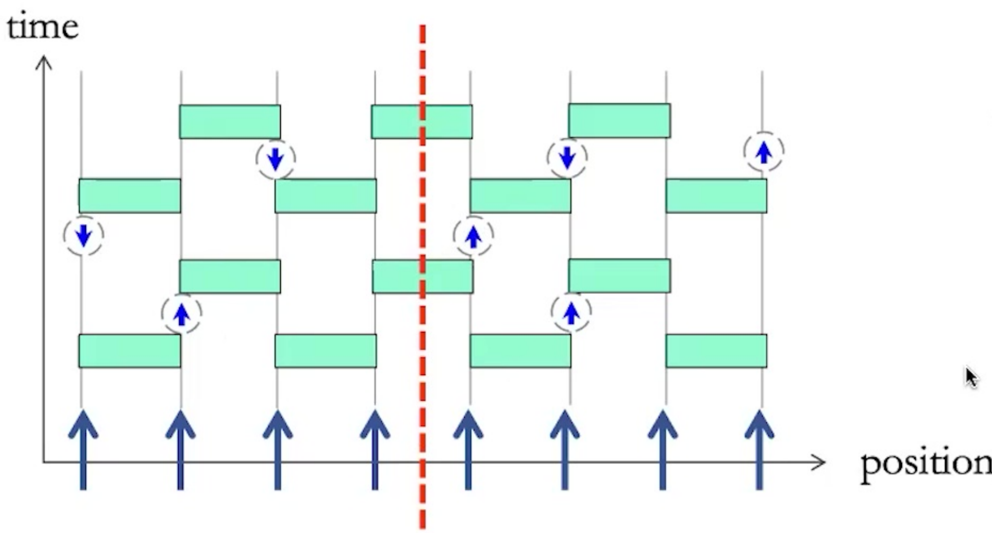
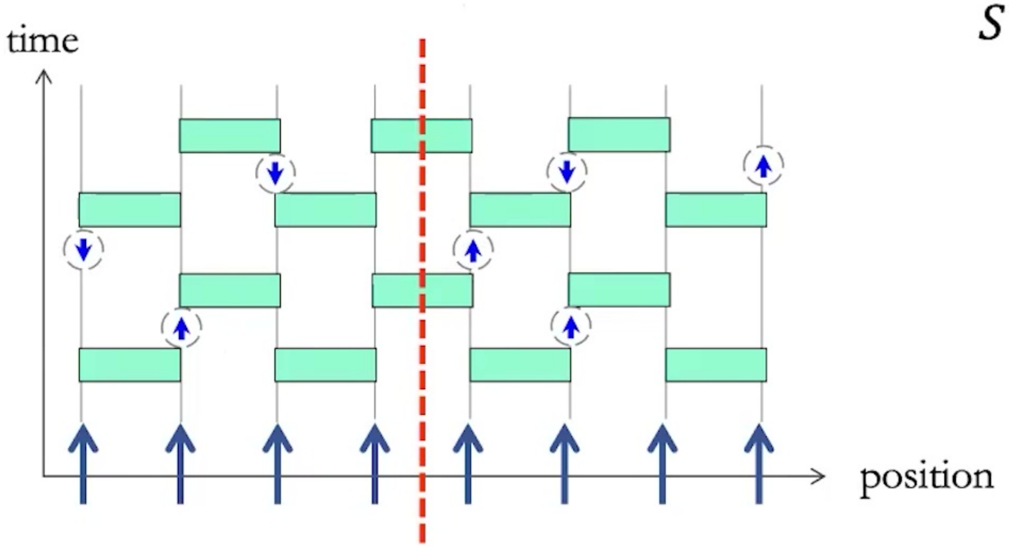


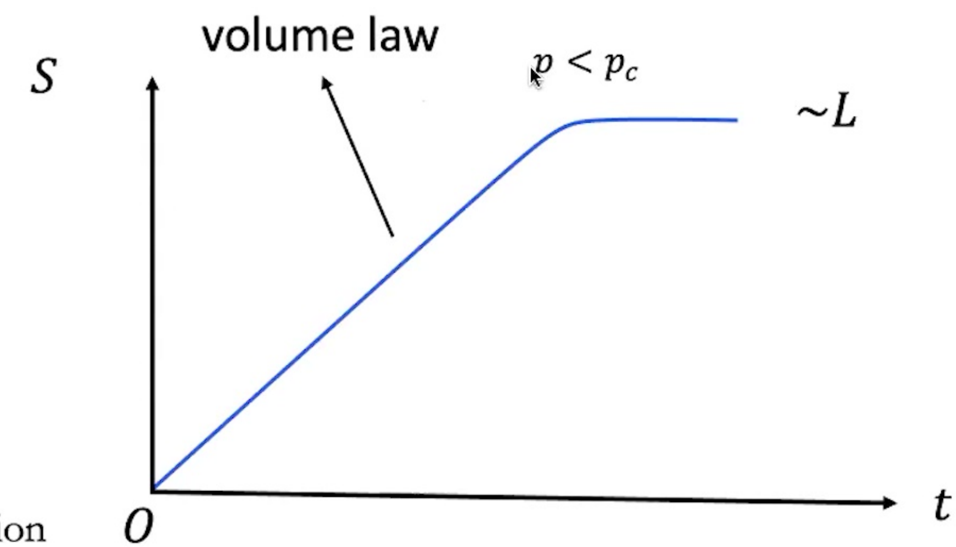
now randomly measure qubits at each time slice with rate p per spin

Measurement induced phase transition

A phase transition characterized by the dynamics of quantum entanglement

Skinner, Ruhman and Nahum, PRX 2019
Chan, Nandkishore,Pretko and Smith, PRB 2019
Li, Chen and Fisher, PRB 2019





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Measurement induced phase transition

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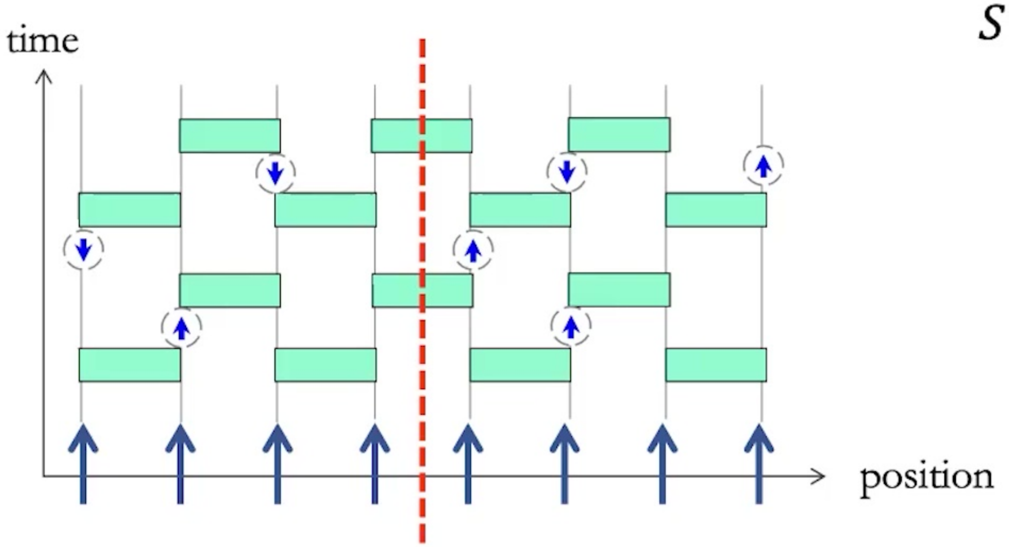

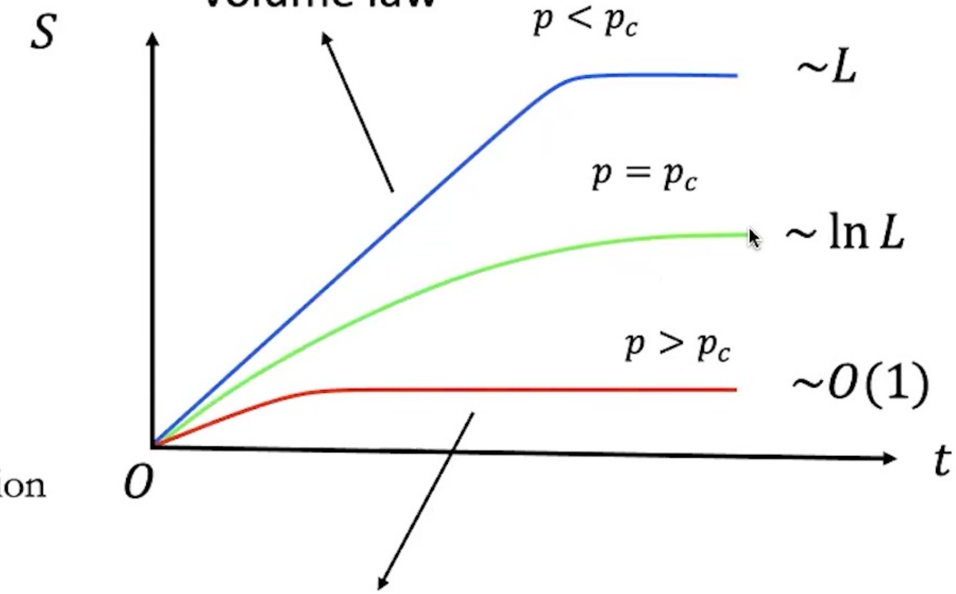

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Measurement induced phase transition

By tuning the measurement rate, the growth of entanglement exhibits different scaling laws !!!

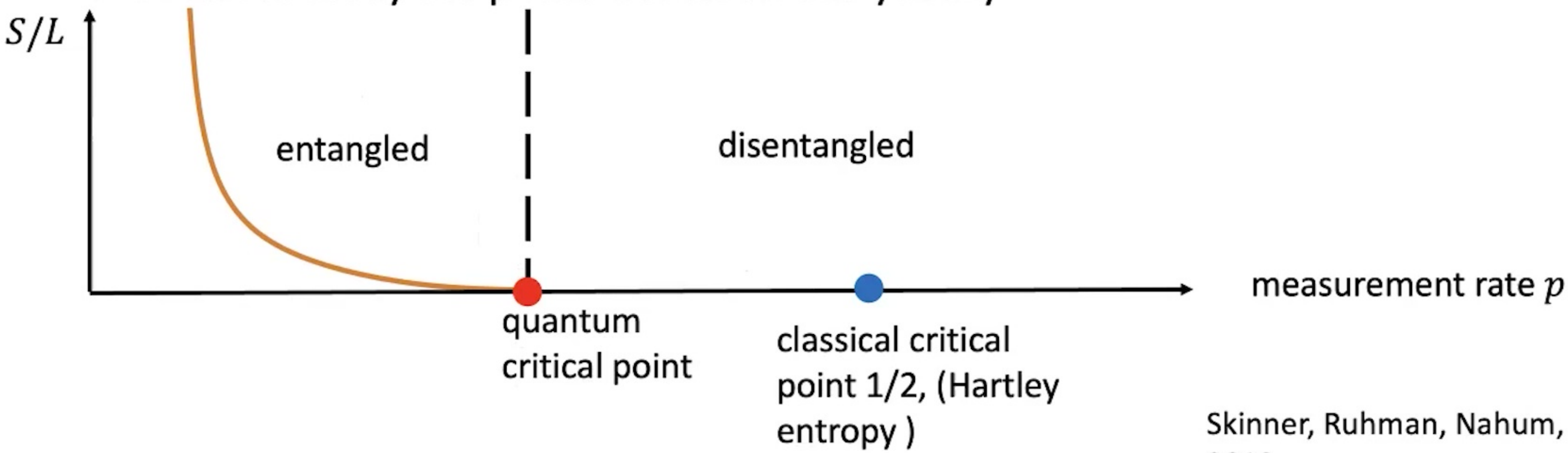
We call this measurement induced phase transition (MPT)

unitary time evolution: increase the entanglement

measurement: cancel the entanglement between the qubit with the rest of the system

Challenges

1. Difficult to study the phase transition analytically



Skinner, Ruhman, Nahum, PRX 2019

No analytical understanding about the quantum critical point and critical exponents !

Challenges

2. Hard to simulate for large system size

Haar random circuit: $\sim$ 24 qubits

hard to get accurate enough critical point/ exponents

3. Hard to achieve in experiment

quantum tomography requires exponential number of detections for the same density operator

post-selection problem requires the exact same measurement outcome

Challenges

2. Hard to simulate for large system size

Haar random circuit: ~ 24 qubits

hard to get accurate enough critical point/ exponents

3. Hard to achieve in experiment

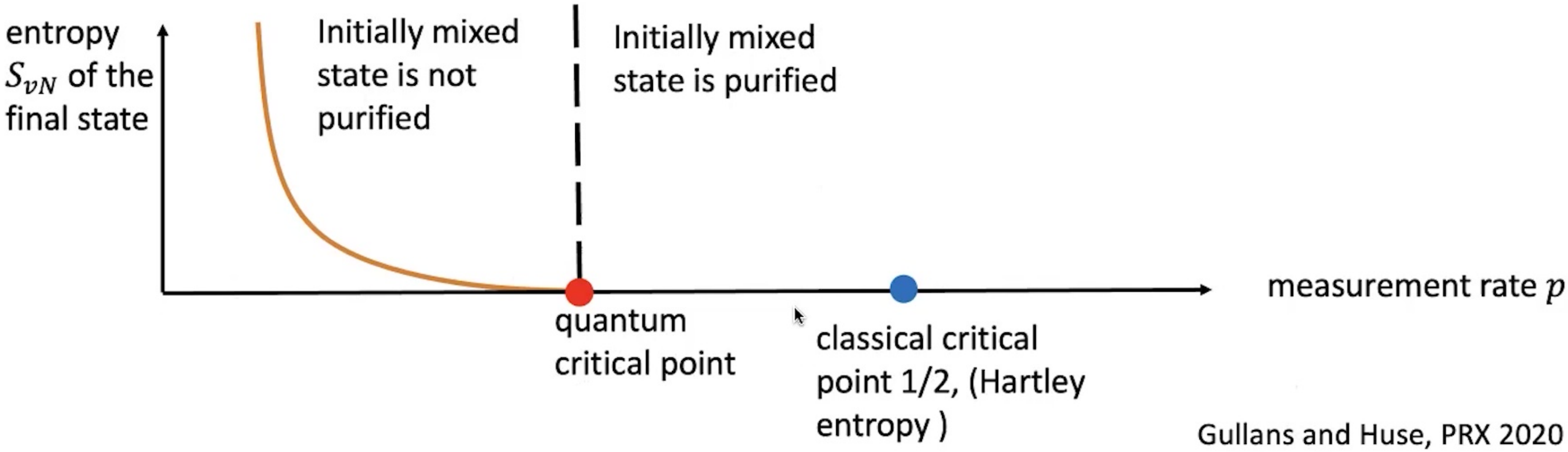
quantum tomography requires exponential number of detections for the same density operator

post-selection problem requires the exact same measurement outcome

superconducting qubits: Koh, Sun, Motta, and Minnich, arXiv:2203.04338
trapped ions: Noel, Niroula, Zhu,Risinger, Egan, Biswas,Cetina, Gorshkov,Gullans,Huse and Monroe Nature Physics 2022

Purification Phase transition

starting with a maximally mixed state



Gullans and Huse, PRX 2020

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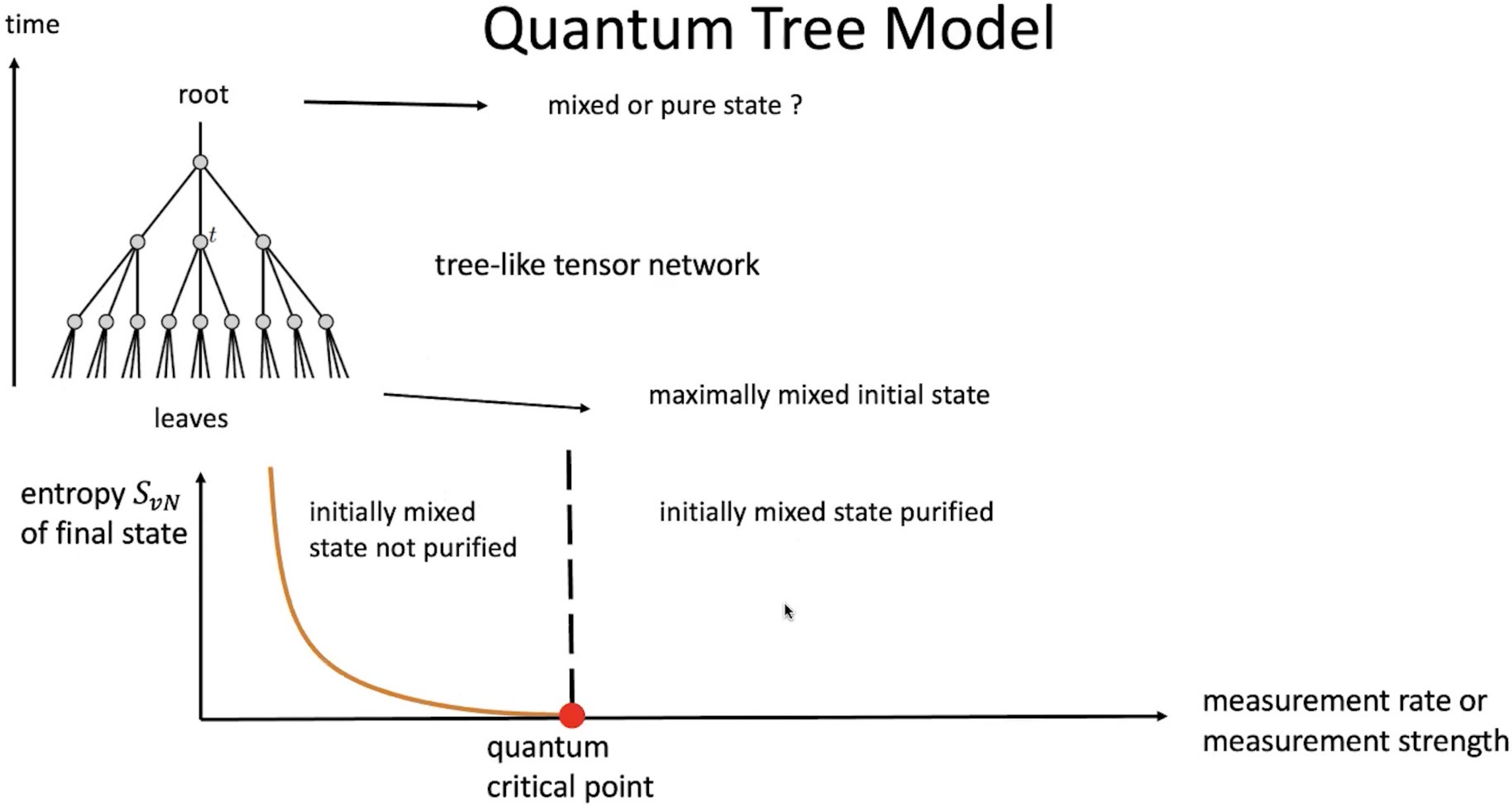
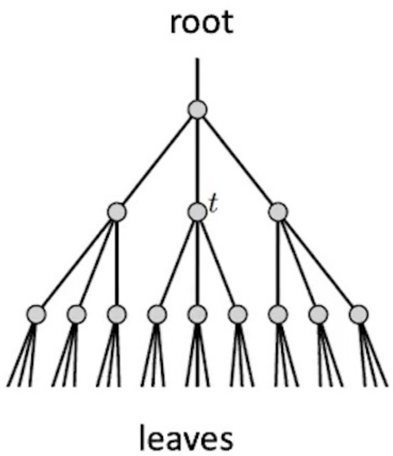
Quantum Tree Model



Quantum Tree Model

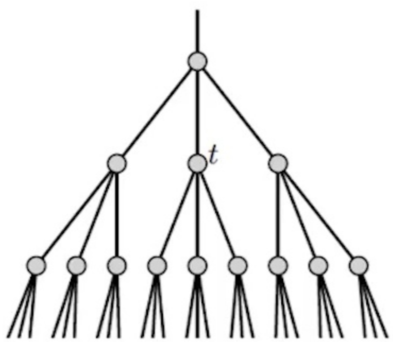


tree-like tensor network

1. The critical point and critical exponents can be exactly solved by mapping to statistical polymer problems

2. The numerical calculation can be done efficiently by a pool method for even the infinite large tree

Quantum Tree Model



tree-like tensor network

Nahum, Roy, Skinner, and Ruhman, PRX Quantum 2, 010352 (2021)

Only works for the forced measurement case

No solution for a tree model with real measurements

Real Measurement vs Forced Measurement

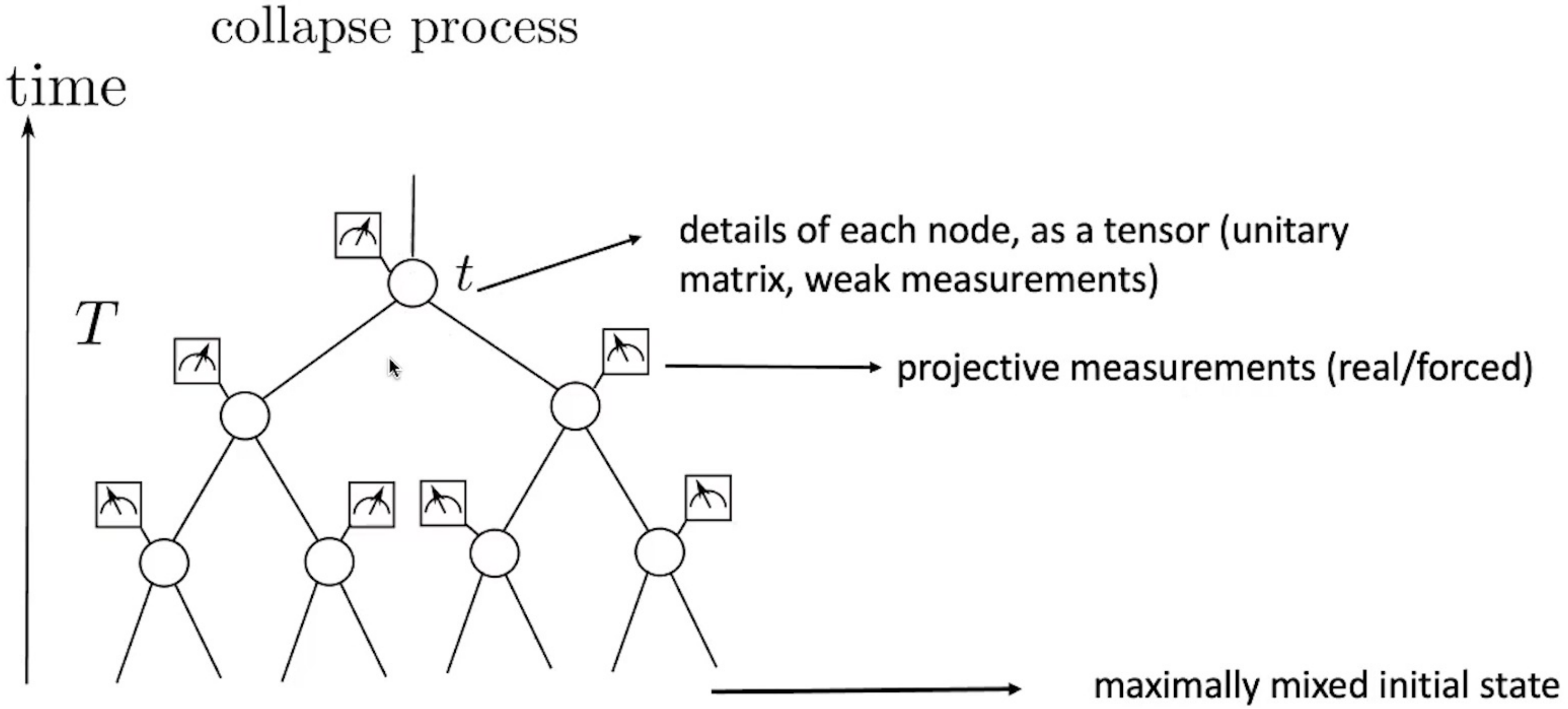
Real measurement: the usual measurement in physics. The measurement outcomes are random with probabilities based on the Born rule

Forced measurement: the measurement outcome is pre-determined with no randomness

To get forced measurement: run the system with real measurements multiple times and discard the realizations with wrong measurement outcomes

Quantum Tree Model

Here we propose a model which has well-defined time order and can have real measurements



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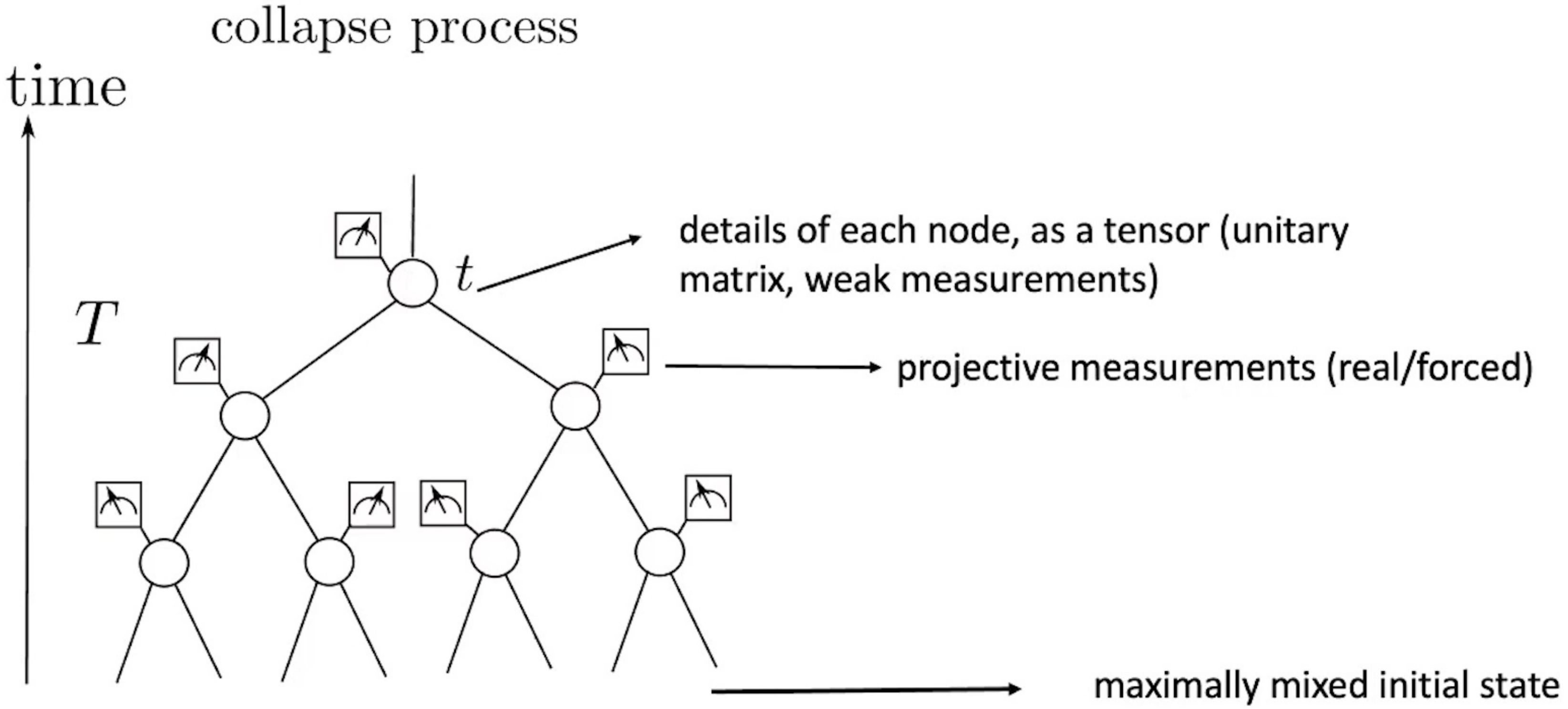
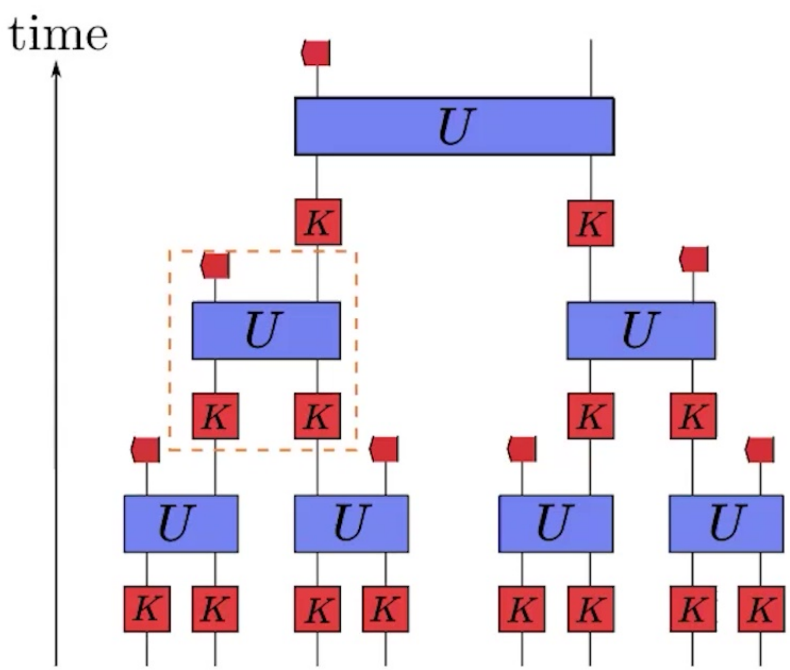
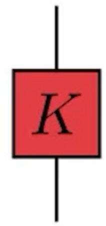


no randomness in locations of measurements

Quantum Tree Model

Here we propose a model which has well-defined time order and can have real measurements





weak measurements with strength θ

$$K_\sigma = \cos\theta \mathbf{1} + (\sin\theta - \cos\theta) u \left|\sigma\right\rangle \left\langle\sigma\right| u^\dagger$$

$$\sum_\sigma K_\sigma^\dagger K_\sigma = \mathbf{1}$$

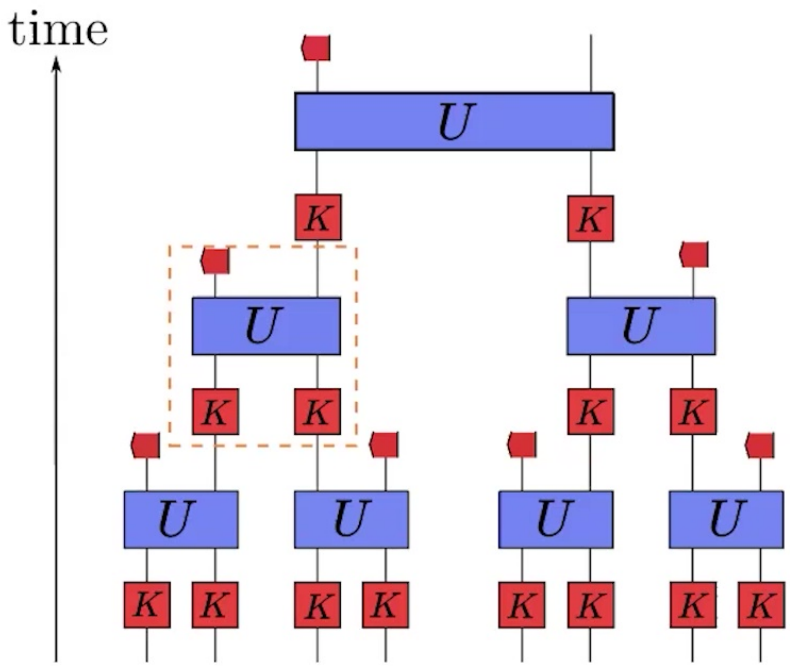
$$\rho \to \frac{K_\sigma^\dagger \rho K_\sigma}{\mathrm{tr} K_\sigma^\dagger \rho K_\sigma}$$

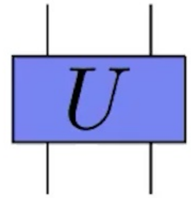
$\theta = \frac{\pi}{4}$, no measurement and K behaves like identity operator, always stay mixed

$\theta = \frac{\pi}{2}$, projective measurement, purified at the first step

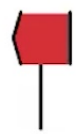
Quantum Tree Model

Here we propose a model which has well-defined time order and can have real measurements

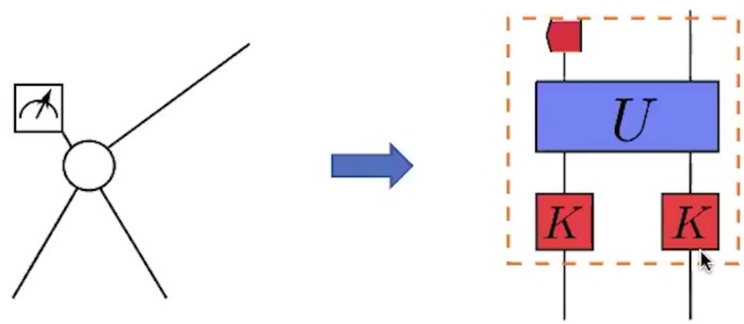




random unitary operator



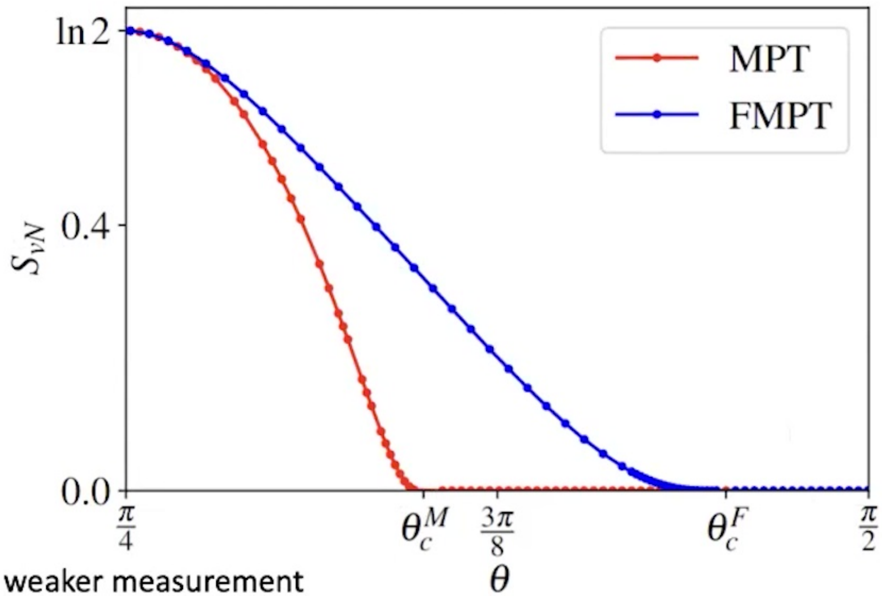
projective measurement
(real/forced)



MPT in our Quantum Tree Model

We are interested in the purity of the top qubit versus the increase of system size (total layers k)

It is found that our tree model exhibits a phase transition by tuning the weak measurement strength



weaker measurement

stronger measurement

$$S_{vN} = -\mathrm{tr}\rho \ln \rho$$

MPT: real measurement induced phase transition

FMPT: forced measurement induced phase transition

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MPT in our Quantum Tree Model

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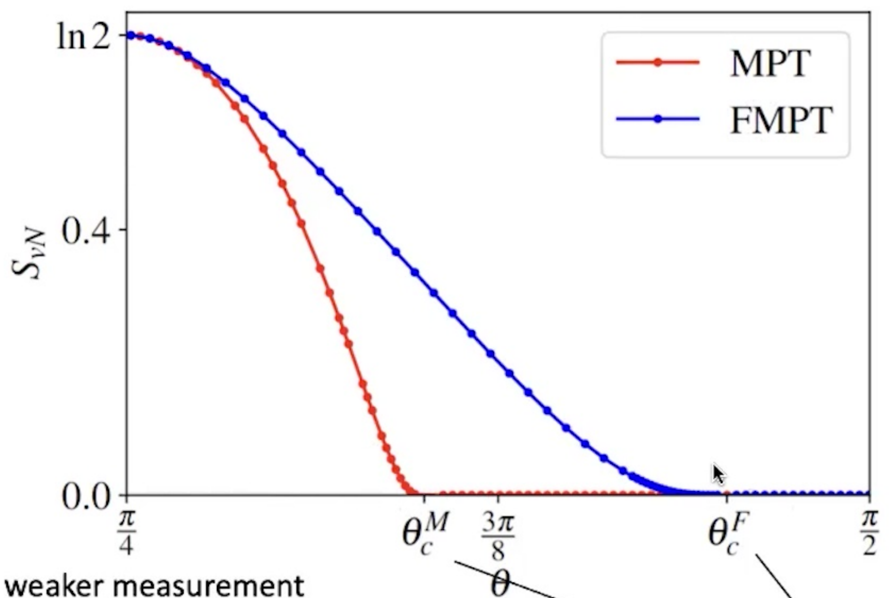


weaker measurement

stronger measurement

critical points

$$S_{vN} = -\mathrm{tr}\rho \ln \rho$$

MPT: real measurement induced phase transition

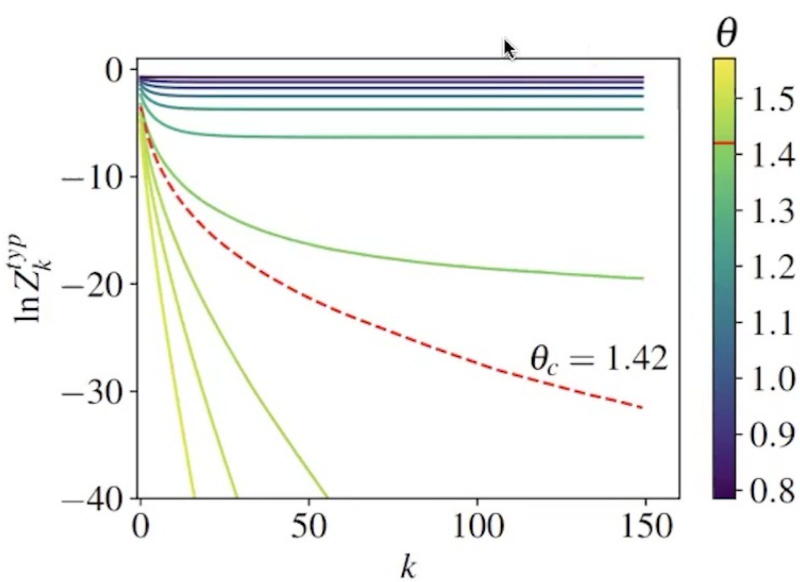
FMPT: forced measurement induced phase transition

MPT in our Quantum Tree Model

$$\rho_f = \begin{pmatrix} 1-Z & 0 \\ 0 & Z \end{pmatrix} \qquad 0 \le Z \le \frac{1}{2}$$

Z encodes all interesting properties of our tree model. $Z = 0$, pure state. $Z = 1/2$, maximally mixed state

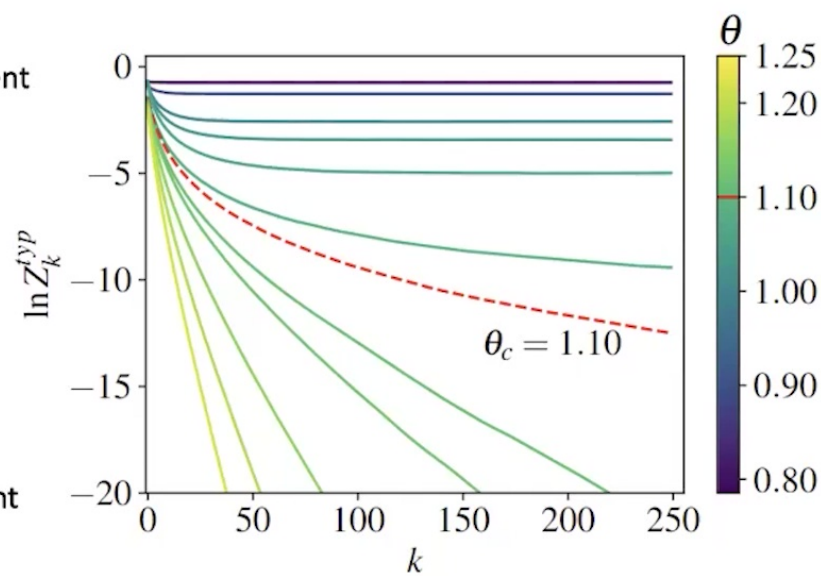
We define a typical purity for each k by $\ln Z^{typ} \equiv \langle \ln Z \rangle$



forced measurement

stronger measurement

weaker measurement



real measurement

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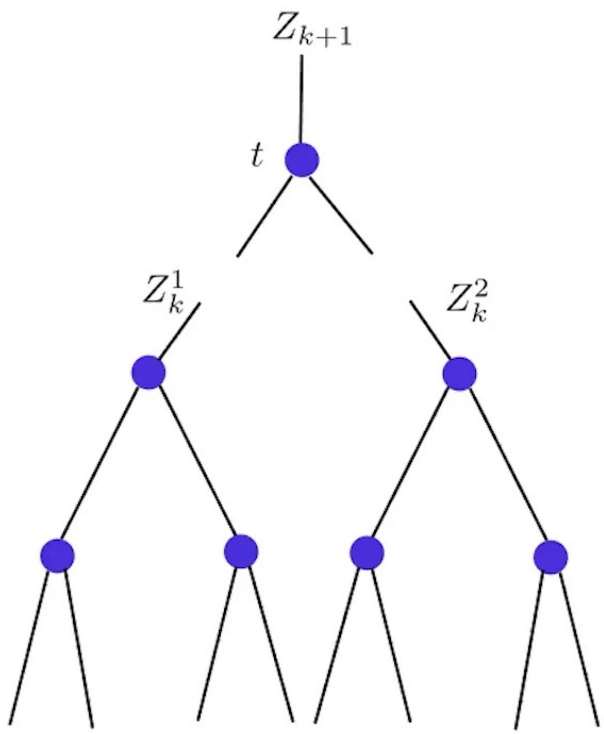
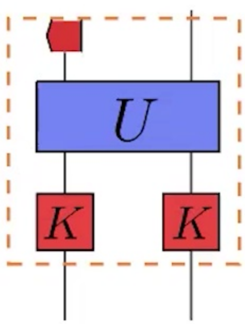
Recursion Relation



$$Z_{k+1} = f(Z_k^1, Z_k^2, t, s)$$

s : the set of measurement outcomes

$$\rho_k^1 = \begin{pmatrix} 1 - Z_k^1 & 0 \\ 0 & Z_k^1 \end{pmatrix} \qquad \rho_k^2 = \begin{pmatrix} 1 - Z_k^2 & 0 \\ 0 & Z_k^2 \end{pmatrix}$$



$$[t(\sigma_1, \sigma_2, \sigma_2')] = [UK_{\sigma_1}^{(1)} \otimes K_{\sigma_2}^{(2)}]_{cd}^{a\sigma_2'}$$

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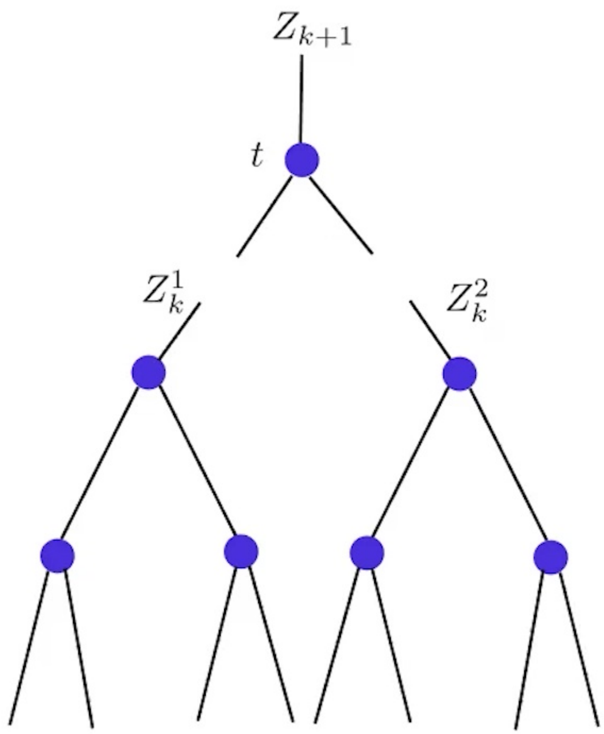
Recursion Relation



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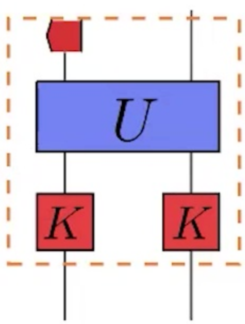

$$[t(\sigma_1, \sigma_2, \sigma_2')] = [UK_{\sigma_1}^{(1)} \otimes K_{\sigma_2}^{(2)}]_{cd}^{a\sigma_2'}$$

$$\rho_{k+1} = \frac{t(\rho_k^1 \otimes \rho_k^2)t^\dagger}{\mathrm{tr}t(\rho_k^1 \otimes \rho_k^2)t^\dagger} \qquad p_s = \mathrm{tr}t(\rho_k^1 \otimes \rho_k^2)t^\dagger$$

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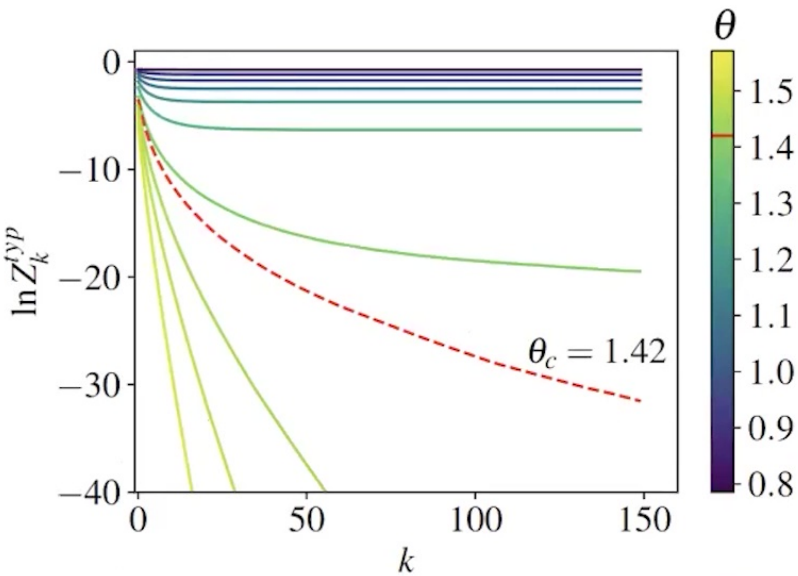
Forced Measurement Phase Transition



In the non purifying phase, non zero Z^{typ} even when system size goes to infinity

In the purifying phase, Z^{typ} decays to zero with system size increasing to infinity

$$\rho_{k+1} = \frac{t(\rho_k^1 \otimes \rho_k^2)t^\dagger}{\mathrm{tr}t(\rho_k^1 \otimes \rho_k^2)t^\dagger}$$

$$s = (\uparrow, \uparrow, \uparrow)$$

$$Z_{k+1} = A_1 Z_k^1 + A_2 Z_k^2 + O((Z_k)^2)$$

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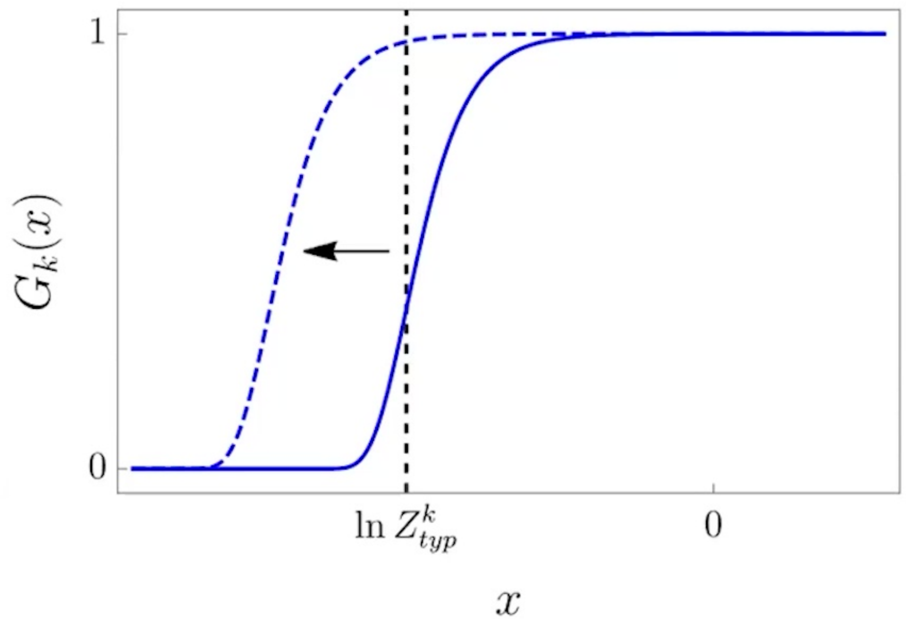
Forced Measurement Phase Transition

$$Z_{k+1} = A_1 Z_k^1 + A_2 Z_k^2$$

$$G_k(x) = \langle \exp(-e^x Z_k) \rangle$$

$G(x)$ can be viewed as a moving wave to the left with k as time and x as position. Its recursion relation can be viewed as the discrete version of Fisher-KPP equation.



Forced/Real Measurement Phase Transition

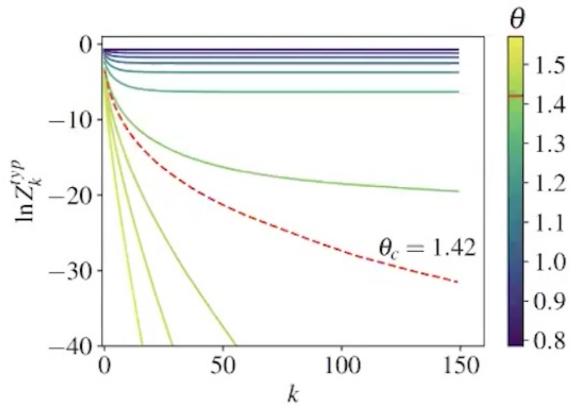
minimal velocity

parameter labeling members of solution

$$v = \frac{1}{\lambda}\ln(\langle A_1^{\lambda}\rangle + \langle A_2^{\lambda}\rangle) \qquad v(\theta_c) = 0, \quad \frac{\partial v}{\partial \lambda} = 0$$

$$\frac{\ln \tan \theta_c}{\tan \theta_c - 1/\tan \theta_c} = \frac{75}{256}$$

$$\theta_c = 1.4201$$

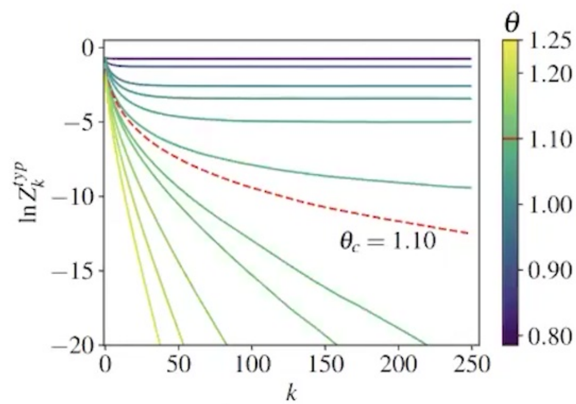


forced measurement

$$v = \frac{1}{\lambda}\ln \sum_s \langle p(s)(A_1^{\lambda}(s) + A_2^{\lambda}(s))\rangle$$

$$\frac{\ln \tan \theta_c}{\tan^2 \theta_c - 1/\tan^2 \theta_c} = \frac{3}{16}$$

$$\theta_c = 1.1001$$



real measurement

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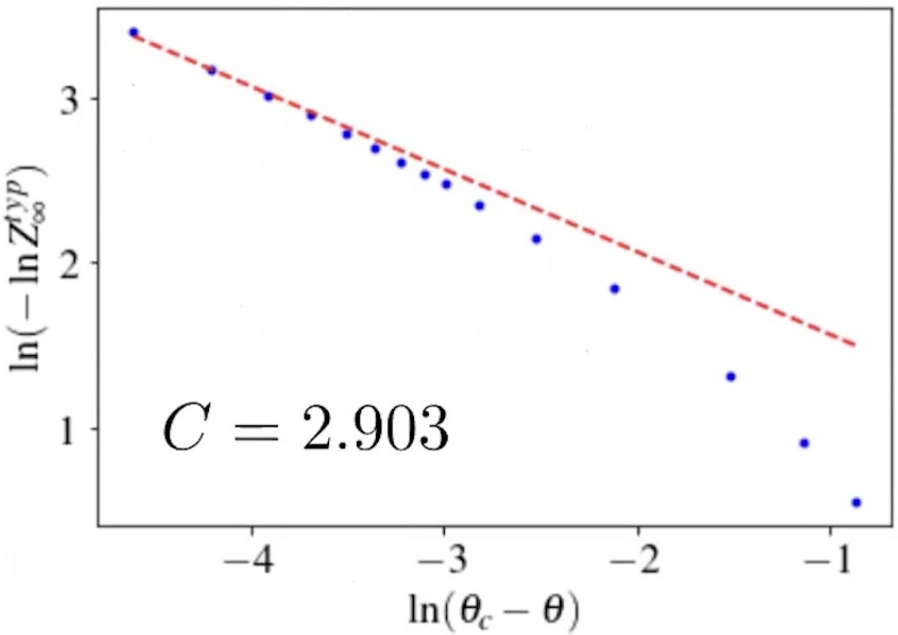
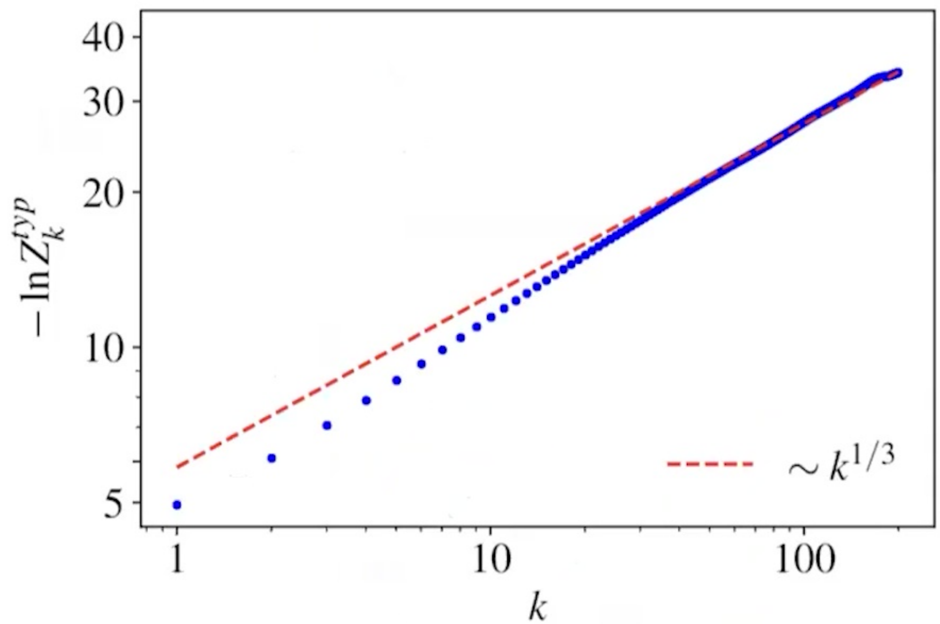
Forced Measurement Phase Transition

$$\ln Z^{typ}_{k\to\infty} = -\frac{C}{\sqrt{\theta_c - \theta}}, \quad \theta_c - \theta \ll 1$$

$$\ln Z^{typ}_{k} \sim -k^{1/3}, \quad \theta = \theta_c$$





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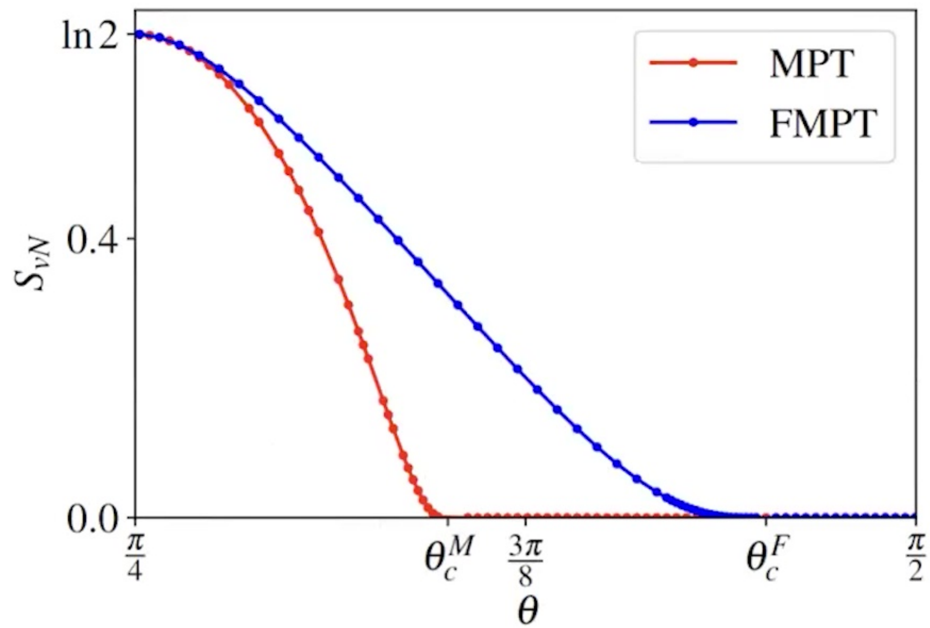
Forced Measurement vs Real Measurement

1. Different critical points

2. Similar scaling exponents



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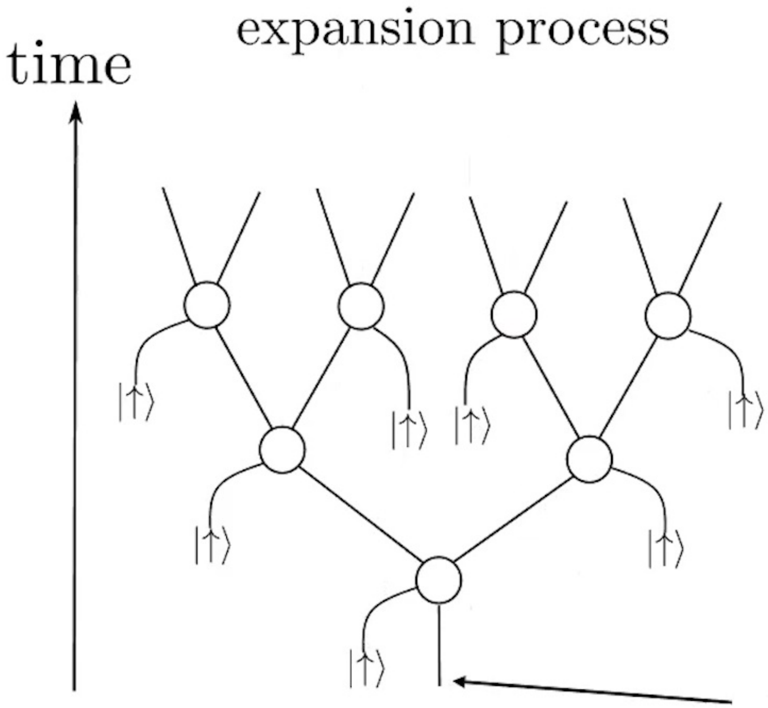
Experimental Protocol

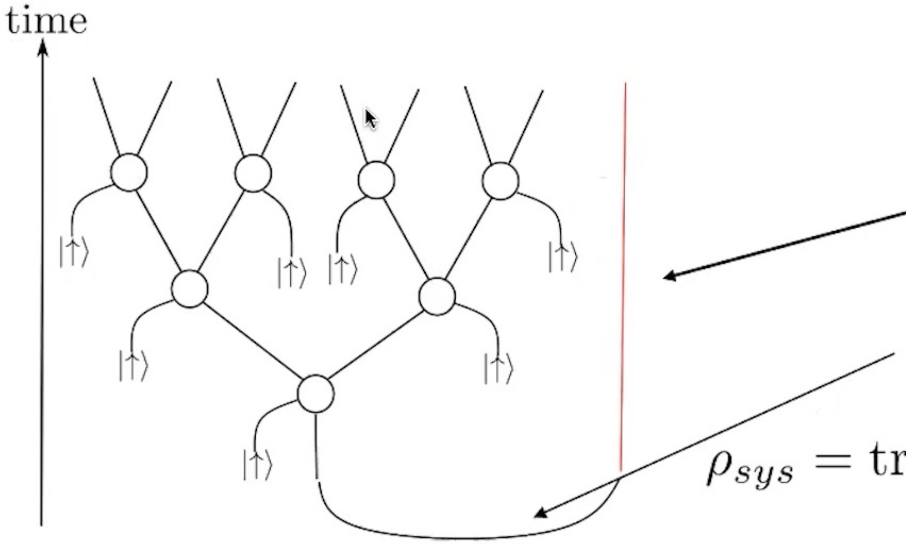


expansion process: inverse of the collapse process

final state of the system exhibits phase transition from a mixed state to a pure state

maximally mixed single qubit state

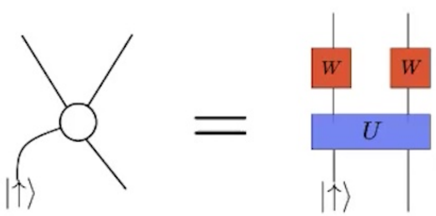
Experimental Protocol



reference qubit

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$\rho_{sys} = \mathrm{tr}_{ref} T\,|\psi_0\rangle\langle\psi_0|\,T^\dagger = \tfrac{1}{2} T(|0\rangle\langle 0| + |1\rangle\langle 1|)T^\dagger$$



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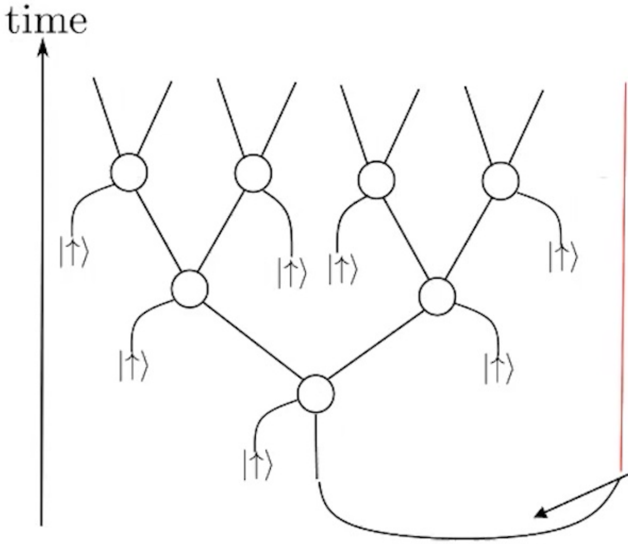

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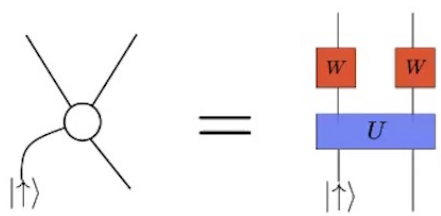
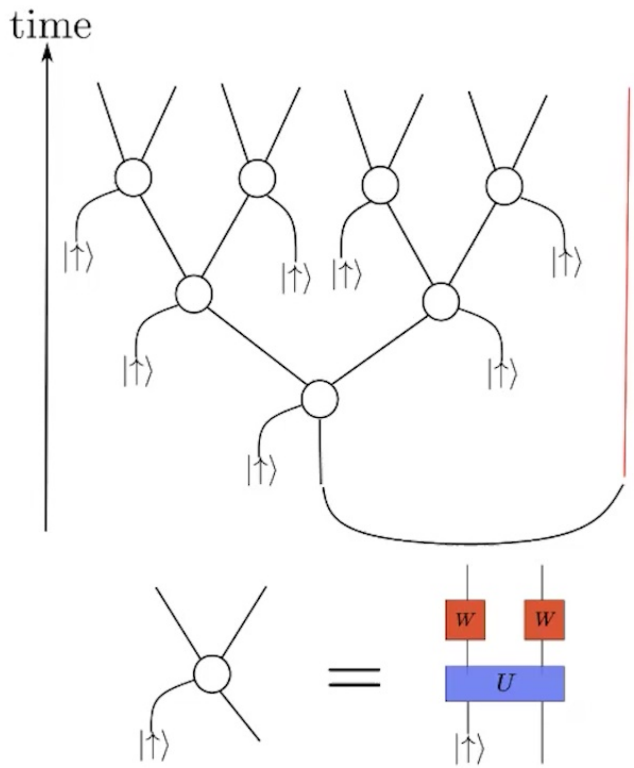
$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\rho_{sys} = \mathrm{tr}_{ref} T |\psi_0\rangle\langle\psi_0| T^\dagger = \frac{1}{2} T(|0\rangle\langle 0| + |1\rangle\langle 1|)T^\dagger$$



purity of the system can be obtained from the density matrix of the reference qubit only

Experimental Protocol



protocol:

1.Prepare the GHZ state as discussed and construct the tree with random unitary operators and measurements

2. Classically calculate the density operator of the reference qubit using the measurement outcomes

$$\rho_R = (1 + \vec{n} \cdot \vec{\sigma})/2, \quad |\vec{n}| = 1 - 2Z$$

3. Measure the reference qubit along the direction $\vec{n}$ and record the outcome τ (± 1)

purifying phase: high accuracy of prediction

Non-purifying phase: low accuracy of prediction

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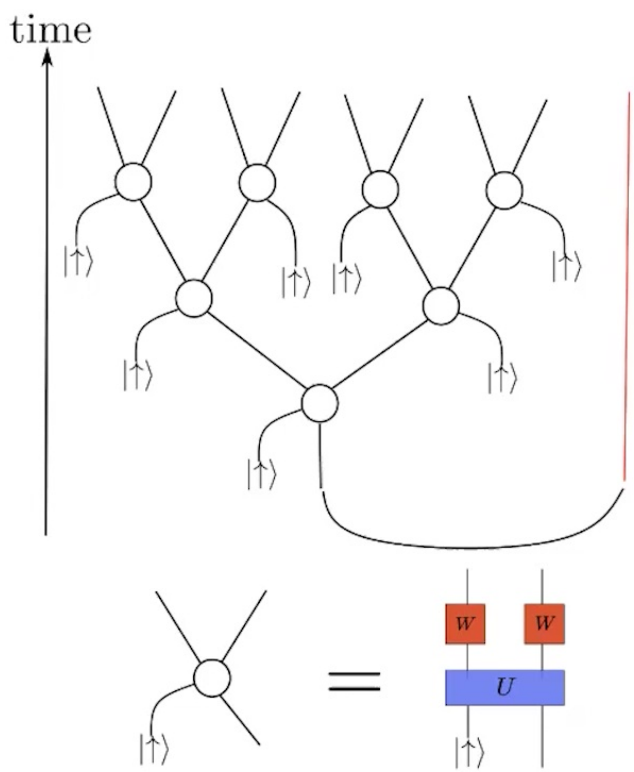
Experimental Protocol



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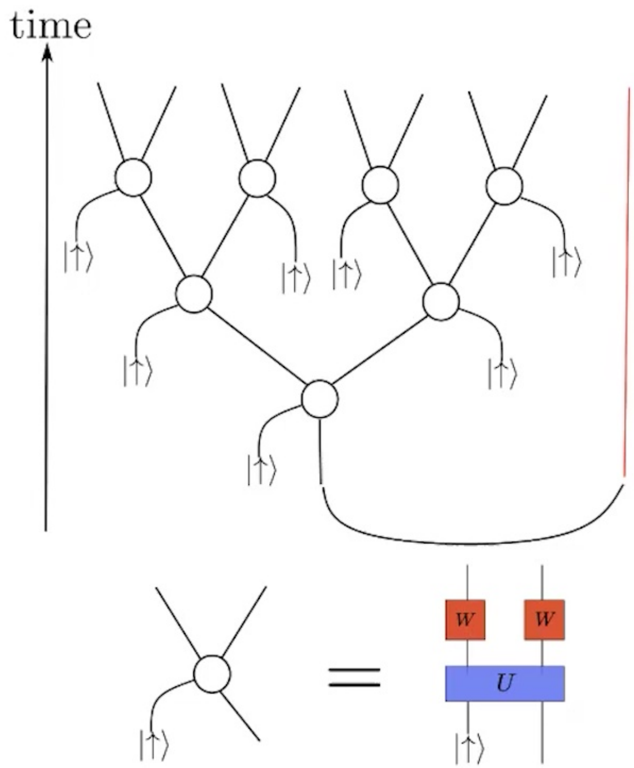
purifying phase: high accuracy of prediction

Non-purifying phase: low accuracy of prediction

4.Repeat above procedure many times

Experimental Protocol



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3. Measure the reference qubit along the direction $\vec{n}$ and record the outcome τ (± 1)

purifying phase: high accuracy of prediction

Non-purifying phase: low accuracy of prediction

4.Repeat above procedure many times

$$\langle Z \rangle_{exp} = \frac{1}{2}(1 - \langle \tau \rangle) = \langle Z \rangle_{th}$$

Summary

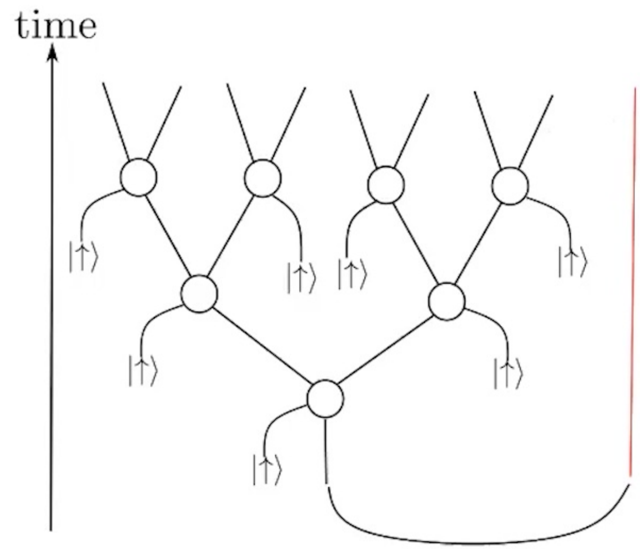
1. Proposed a model which exhibits MPT for both forced and real measurement

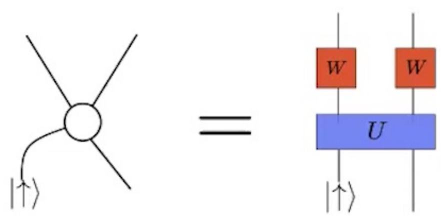
2. Studied the phase diagram and derive a solution for critical points and critical exponents

3.Proposed an experimental protocol to see MPT without post-selection problem

Measurement–induced phase transitions on dynamical quantum trees : arXiv:2210.07264

Experimental Protocol





protocol:

1.Prepare the GHZ state as discussed and construct the tree with random unitary operators and measurements

2. Measure the reference qubit along the z direction and record the outcome τ (± 1)

3. Classically calculate the density operator of the reference qubit using the measurement outcomes

$$\rho_R = (1 + \vec{n}\cdot\vec{\sigma})/2, \quad |\vec{n}| = 1 - 2Z$$

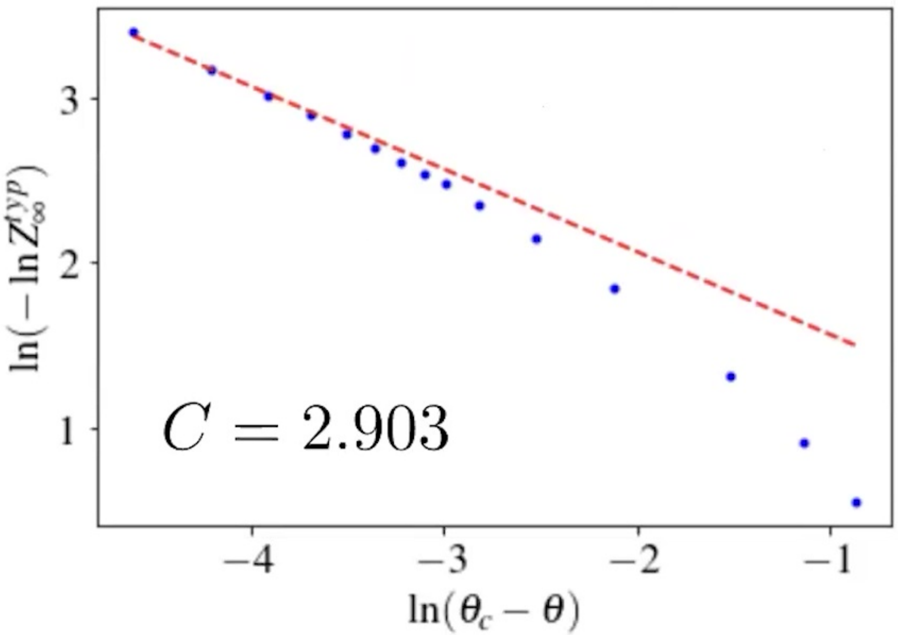
4.Repeat above procedure many times

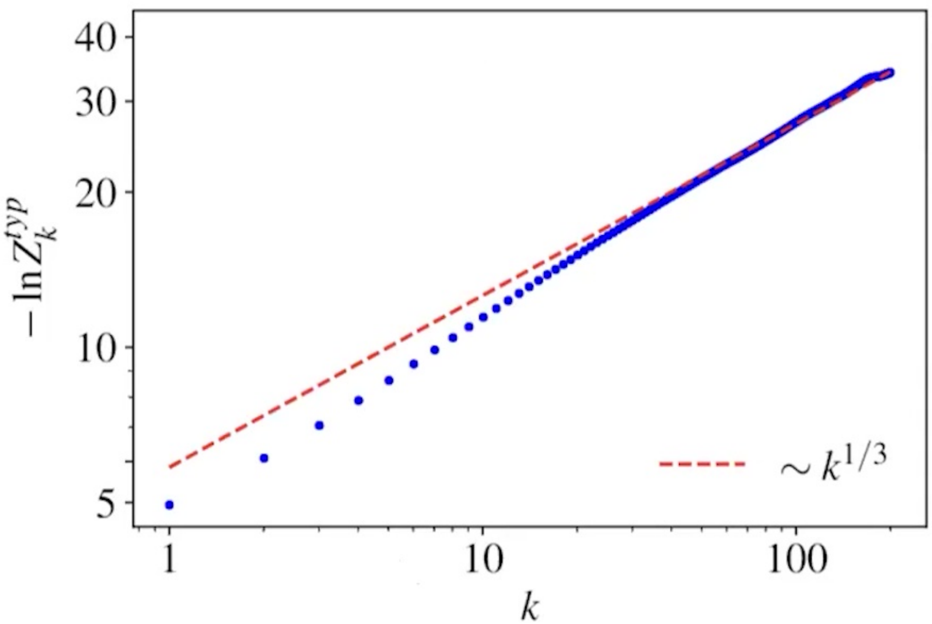
$$\langle Z \rangle = \frac{1}{2}\left(1 - \left\langle \frac{\tau}{\hat{n}_z} \right\rangle\right)$$

Forced Measurement Phase Transition

$$\ln Z_{k\to\infty}^{typ} = -\frac{C}{\sqrt{\theta_c - \theta}}, \quad \theta_c - \theta \ll 1$$

$$\ln Z_k^{typ} \sim -k^{1/3}, \quad \theta = \theta_c$$





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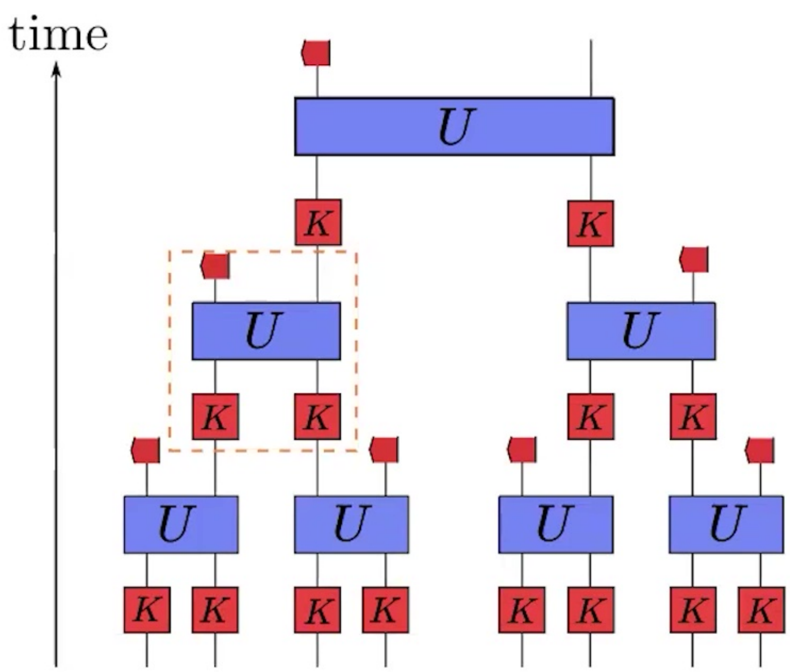
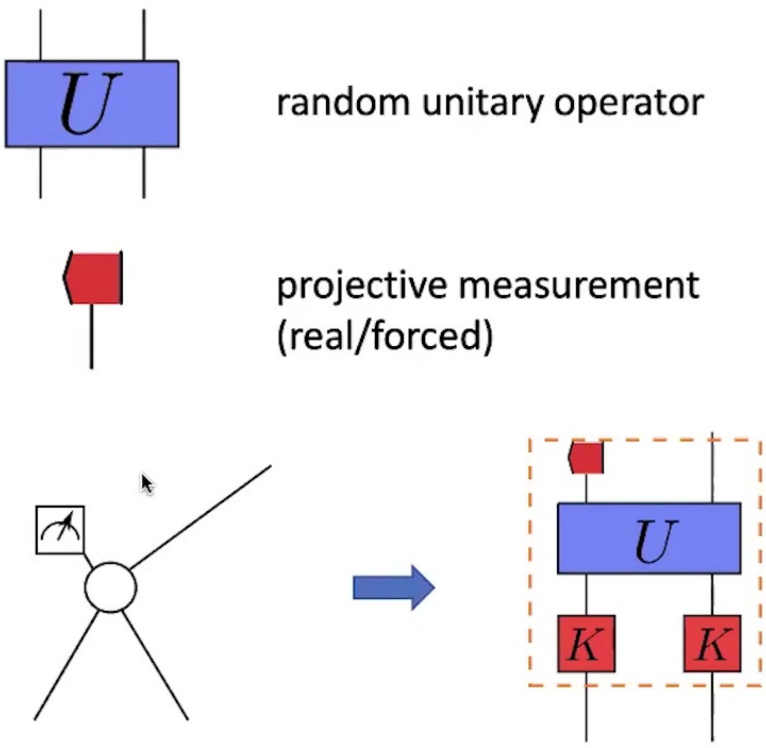
Quantum Tree Model

Here we propose a model which has well-defined time order and can have real measurements





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