

Title: Geometry in Scattering Amplitudes

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Series: Particle Physics

Date: December 09, 2022 - 1:00 PM

URL: <https://pirsa.org/22120056>

Abstract: One central property of the S-matrix is its invariance under field redefinitions. I will discuss how the geometry of field space makes this invariance manifest. This geometric formulation also has practical consequences. Scattering amplitudes and the renormalization group equations for a theory of scalars and gauge bosons only depend on geometric quantities. Also, the scattering amplitudes satisfy a geometric soft theorem.

Zoom link: <https://pitp.zoom.us/j/98973018515?pwd=MFJFSGM4RS9KRHFOREFBc1gvWXYwQT09>

Geometry in Amplitudes

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J$$

Field Redef

coord

$$\phi^I \rightarrow \tilde{\phi}^I(\phi) = \phi^I + \delta \phi^I + \dots$$

$$\partial_\mu \phi^I \rightarrow \frac{\partial \phi^I}{\partial \phi^J} \partial_\mu \phi^J \leftarrow \text{vector}$$

g_{IJ} is metric

Cheung
Parr-Martinez
Jenkins
Manohar

Geometry in Amplitudes

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J$$

Field Redef

$$\partial_\mu \phi^I \rightarrow \frac{\partial \phi^I}{\partial \phi^J} \partial_\mu \phi^J \leftarrow \text{vector}$$

g_{IJ} as metric

$$g_{IJ} \rightarrow \Gamma^I_{JK} \rightarrow \nabla_I$$

$$g_{IJ} = \delta_{IJ} + R_{IKJL} \phi^K \phi^L + \dots$$

Cheung
Parrera-Martinez
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Manohar

Amplitudes

$$A_3 = 0$$

$$A_4 = R_{IKJL}$$

Geometry in Amplitudes

Sem
min

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J$$

coord
Field Redef

$$\phi^I \rightarrow \tilde{\phi}^I(\phi) = \phi^I + \phi^{I'}$$

$$\partial_\mu \phi^I \rightarrow \frac{\partial \phi^I}{\partial \phi^J} \partial_\mu \phi^J \leftarrow \text{vector}$$

g_{IJ} as metric

$$g_{IJ} \rightarrow \Gamma^I_{JK} \rightarrow \nabla_I$$

$$g_{IJ} = \delta_{IJ} + R_{IKJL} \phi^K \phi^L + \dots$$

$$X + \text{Y} + \dots$$

Cheung
Parrera-Martinez
Jenkins
Manohar

Amplitudes

$$A_3 = 0$$

$$A_4 = R_{IK}$$

Amplitudes

$$A_3 = 0$$

$$A_4 = R_{IKJL} S_{34} + R_{IJKL} S_{24}$$

pp_s

$$S_{15} = (p_1 + p_5)^2$$

Amplitudes

$$S_{15} = (P_1 + P_5)^2$$

$$A_3 = 0$$

$$A_4 = R_{IKJL} S_{34} + R_{IJKL} S_{24}$$

$$A_5 = \nabla_K R_{ILJH} S_{45} + \nabla_L R_{IKJM} S_{35} + \nabla_L R_{IJKM} S_{25} \\ + \nabla_M R_{IKJL} S_{34} + \nabla_M R_{IJKL} (S_{24} + S_{45})$$

Amplitudes

$$S_{12} = (p_1 + p_2)^2$$

$$A_3 = 0$$

$$A_4 = R_{IKJL} S_{34} + R_{ISJKL} S_{24}$$

$$A_5 = \nabla_K R_{IKJL} S_{45} + \nabla_L R_{IKJM} S_{35} + \nabla_L R_{ISJKM} S_{25} \\ + \nabla_M R_{IKJL} S_{34} + \nabla_M R_{ISJKL} (S_{24} + S_{45})$$

Soft Theorem

$$\lim_{q \rightarrow 0} A_{n+1} = \nabla_I A_n$$

Geometry in Amplitudes

sem
min

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J$$

coord Field Redef

$$\phi^I \rightarrow \phi^{\tilde{I}}(\phi) = \phi^I + \phi^{I_2} + \dots$$

$$\partial_\mu \phi^I \rightarrow \frac{\delta \phi^I}{\delta \phi^{\tilde{J}}} \partial_\mu \phi^{\tilde{J}} \leftarrow \text{vector}$$

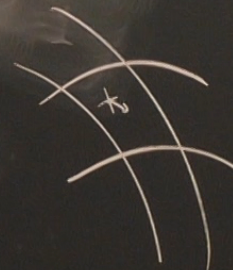
g_{IJ} as metric

$$g_{IJ} \rightarrow \Gamma^I_{JK} \rightarrow \nabla_I$$

$$g_{IJ} = \delta_{IJ} + R_{IKJL} \phi^K \phi^L + \dots$$

$$X + \text{diagram} + \dots$$

Cheung
Parr-Martinez
Jenkins
Manohar



Amplit

$$A_3 = 0$$

$$A_4 = R$$

$$A_5 = \nabla$$

$$+ \nabla$$

Amplitudes

$$A_3 = 0$$

$$A_4 = R_{IKJL} S_{34} + R_{IJKL} S_{24} + \lambda_{IJKL} S_{12} S_{34} + \dots$$

$$A_5 = \nabla_K R_{IKJL} S_{45} + \nabla_L R_{IKJM} S_{35} + \nabla_L R_{IJKM} S_{25} \\ + \nabla_M R_{IKJL} S_{34} + \nabla_M R_{IJKL} (S_{24} + S_{45})$$

$$S_{12} = (p_1 + p_2)^2 \\ = 2p_1 \cdot p_2$$

Soft Theorem

$$\lim_{q \rightarrow 0} A_{n+1} = \nabla_I A_n$$

Amplitudes

$$A_3 = 0$$

$$A_4 = R_{IKJL} S_{34} + R_{IJKL} S_{24} + \lambda_{IJKL} S_{12} S_{34} + \dots$$

$$A_5 = \nabla_K R_{IKJL} S_{45} + \nabla_L R_{IKJM} S_{35} + \nabla_L R_{IJKM} S_{25} + \nabla_M \lambda_{IJKL} S_{12} S_{34} + \dots$$

$$+ \nabla_M R_{IKJL} S_{34} + \nabla_M R_{IJKL} (S_{24} + S_{45})$$

$$S_{I\bar{J}} = (p_I + p_{\bar{J}})^2 \\ = 2p_I \cdot p_{\bar{J}}$$

Soft Theorem

$$\lim_{q \rightarrow 0} A_{n+1} = \nabla_I A_n$$

Geometry in Amplitudes

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J + \lambda_{IJK}(\phi) \partial_\mu \phi^I \partial_\nu \phi^J \partial^\mu \phi^K + \dots$$

Symmetry

$$\phi^I \rightarrow \phi^I + K^I(\phi)$$

Amplitudes

$$A_3 = 0$$

$$A_4 = R$$

$$A_5 = \nabla_K$$

Geometry in Amplitudes

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J + \lambda_{IJKL}(\phi) \partial_\mu \phi^I \partial_\nu \phi^J \partial_\rho \phi^K \partial_\sigma \phi^L + \dots$$

Symmetry

$$\phi^I \rightarrow \phi^I + K^I_{(U)}$$

$$\lim_{\epsilon \rightarrow 0} K^I_{\epsilon} A^{I_1 \dots I_n I}_{n+1} = K^I_{\epsilon} \nabla^I A^{I_1 \dots I_n}_{n+1}$$

$$K^I_{(U)} \neq 0$$

Amplitudes

$$A_3 = 0$$

$$A_4 = R$$

$$A_5 = \nabla_K$$

Geometry in Amplitudes

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu}(\phi) \partial_\mu \phi^I \partial^\nu \phi^I + \lambda_{IJKL}(\phi) \partial_\mu \phi^I \partial_\nu \phi^J \partial^\mu \phi^K \partial^\nu \phi^L + \dots$$

Symmetry

$$\phi^I \rightarrow \phi^I + K^I_{(\psi)}$$

$$K^I_{(\psi)} \neq 0$$

$$\lim_{\epsilon \rightarrow 0} K^I_{\epsilon} A^{I_1 \dots I_n I}_{n+1} = K^I_{\epsilon} \nabla^I A^{I_1 \dots I_n}_n$$

$$= \sum_a (\nabla K^I_a) A^{I_1 \dots I_n I}_{a, n}$$

Amplit

$$A_3 = 0$$

$$A_4 = R$$

$$A_5 = \nabla K$$

Geometry in Amplitudes

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J + \lambda_{IJKL}(\phi) \partial_\mu \phi^I \partial_\nu \phi^J \partial_\rho \phi^K \partial_\sigma \phi^L + \dots$$

Symmetry

$$\phi^I \rightarrow \phi^I + K^I_{(U)}$$

$$K^I_{(U)} \neq 0$$

$$\begin{aligned} \lim_{\hbar \rightarrow 0} K^I_{(U)} A^{I_1 \dots I_n I}_{n+1} &= K^I_{(U)} \nabla^I A^{I_1 \dots I_n}_{n+1} \\ &= \sum_n (\nabla K^I_{(U)}) A^{I_1 \dots I_n I}_{n+1} \\ &= 0 \end{aligned}$$

Amplitudes

$$A_3 = 0$$

$$A_4 = R$$

$$A_5 = \nabla K$$

Amplitudes

$$A_3 = 0$$

$$A_4 = R_{IKJL} S_{34} + R_{ISJKL} S_{24} + \lambda_{ISJKL} S_{12} S_{34} + \dots$$

$$A_5 = \nabla_K R_{ILJM} S_{45} + \nabla_L R_{IKJM} S_{35} + \nabla_L R_{IJKM} S_{25} + \nabla_M \lambda_{ISJKL} S_{12} S_{34} + \dots$$

$$+ \nabla_M R_{IKJL} S_{34} + \nabla_M R_{IJLK} (S_{24} + S_{45})$$

$$S_{IJ} = (P_I + P_J)^2 = 2P_I \cdot P_J$$

Soft Theorem

$$\lim_{q \rightarrow 0} A_{n+1} = \nabla_I A_n + O(q)$$

Geometry in Amplitudes

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu}(\phi) \partial_\mu \phi^I \partial^\nu \phi^{\bar{I}} + \lambda_{IJKL}(\phi) (\partial_\mu \phi^I) (\partial_\nu \phi^{\bar{J}}) (\partial_\rho \phi^K) (\partial^\rho \phi^{\bar{L}}) + \dots$$

$$\phi^{\bar{I}} \rightarrow \bar{\varphi}^{\bar{I}}(\phi, \partial\phi)$$

Amplitudes

$$A_3 = 0$$

$$A_4 = R_I$$

$$A_5 = \nabla_K$$

$$+ \nabla_M$$

Amplitudes

$$A_3 = 0$$

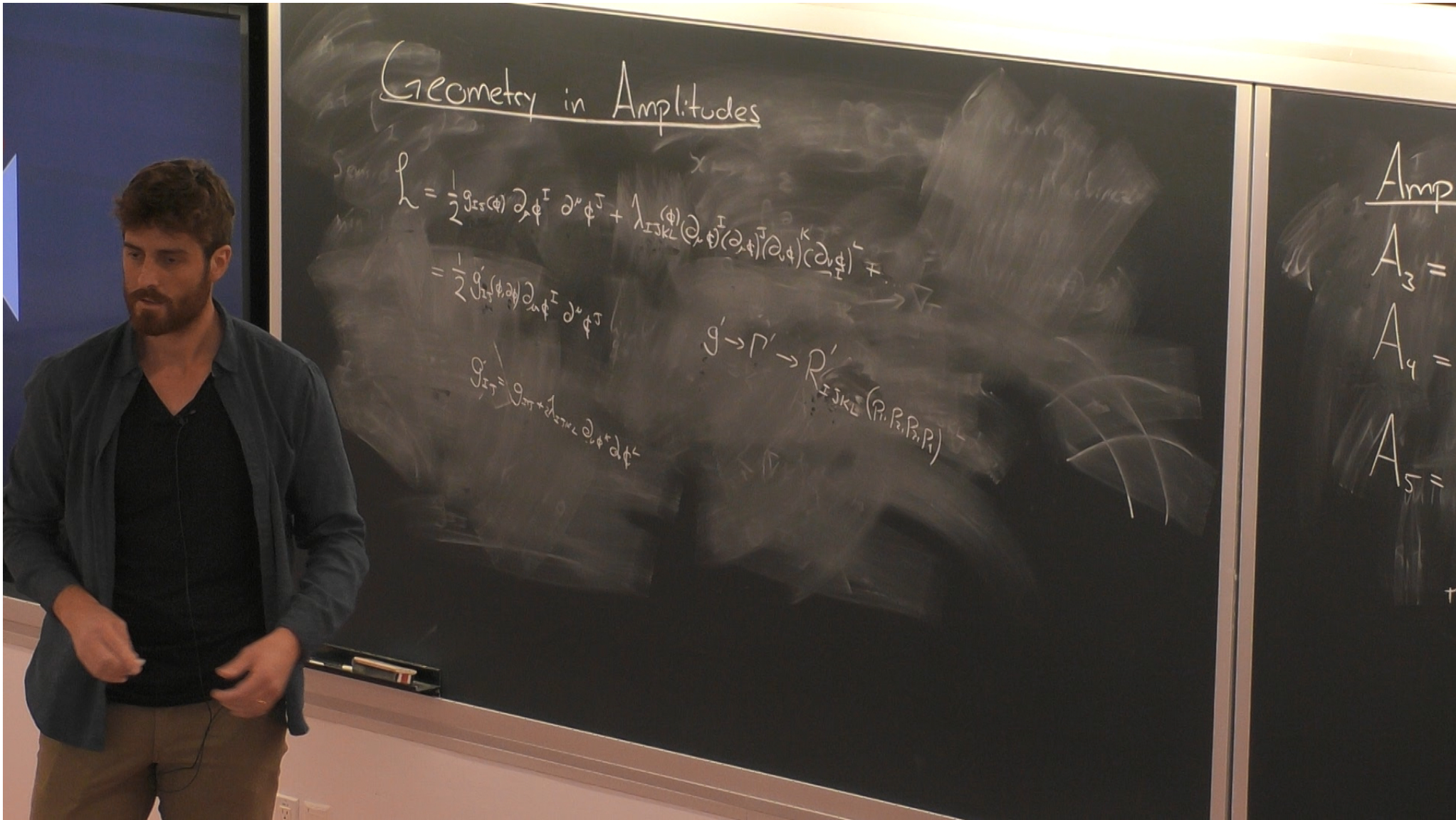
$$A_4 = R'_{IKJL} S_{34} + R'_{ISKJL} S_{24}$$

$$A_5 = \nabla'_K R'_{IKJL} S_{45} + \nabla'_L R'_{IKJM} S_{35} + \nabla'_L R'_{IKJM} S_{25} + \nabla'_M R'_{IKJL} S_{34} + \nabla'_M R'_{IKJL} (S_{24} + S_{15})$$

$$S_{15} = (p_1 + p_5)^2 = 2p_1 \cdot p_5$$

Soft Theorem

$$\lim_{q \rightarrow 0} A_{n+1} = \nabla_I A_n + O(q)$$



Geometry in Amplitudes

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu}(\phi) \partial_\mu \phi^I \partial^\nu \phi^J + \frac{1}{4} \lambda_{IJKL}(\phi) (\partial_\mu \phi^I)(\partial_\nu \phi^J)(\partial^\mu \phi^K)(\partial^\nu \phi^L) + \dots - V(\phi) - \frac{1}{4} F^2$$

$$\frac{1}{P^2 P^2}$$

$$g_{IJ} = g_{IJ} + \lambda_{IJKL} \partial_\mu \phi^K \partial_\nu \phi^L$$

$$g' \rightarrow P' \rightarrow R'_{IJKL}(P_1, P_2, P_3, P_4)$$

$$\frac{\partial}{\partial x^2} (P_{x^2}) = P_2$$

Geometry in Amplitudes

$$D_\mu \phi^I = \partial_\mu \phi^I + \epsilon_{IJ}^K A_\mu^J \phi^K \quad \text{Killing vec} \quad \mathcal{L}_{\text{SMEFT}} \sim \frac{0}{\Lambda^4}$$

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J + \lambda_{IJKL}(\phi) (\partial_\mu \phi^I) (\partial_\nu \phi^J) (\partial_\rho \phi^K) (\partial^\rho \phi^L) + \dots - V(\phi) - \frac{1}{4} F^2$$

$$\frac{1}{R^2 R}$$

$$g_{IJ} = g_{IJ} + \lambda_{IJKL} \partial_\mu \phi^K \partial_\nu \phi^L$$

$$\frac{V_{IJ}}{R^2 R}$$

$$\frac{d}{d\mu} R_{IJKL} \sim$$

$$R_{IJKL} + R_{IJK}^M \nabla_M \nabla_N \nabla_N \nabla^L + R(\nabla \epsilon)(\nabla \epsilon)$$

Geometry in Amplitudes

$$D_\mu \phi^I = \partial_\mu \phi^I + \epsilon_{IJ}^K A_\mu^J \phi^K$$

$$L_{\text{SMEFT}} \sim \frac{O}{\Lambda^4}$$

$$L = \frac{1}{2} g_{IJ}(\phi) D_\mu \phi^I D^\mu \phi^J + \lambda_{IJKL}(\phi) (\partial_\mu \phi^I)(\partial_\nu \phi^J)(\partial_\rho \phi^K)(\partial^\sigma \phi^L) + \dots - V(\phi) - \frac{1}{4} F^2$$

$$\frac{1}{P^2 R}$$

$$g_{IJ} = g_{IJ} + \lambda_{IJKL} \partial_\mu \phi^K \partial_\nu \phi^L$$

$$\frac{d}{ds} R_{IJKL} \sim \phi^I, \epsilon \in I, A_{\mu, \nu}$$

$$R_{IJKL} + R_{IJK}{}^\mu{}_\nu \nabla_\mu \nabla_\nu \phi^I \phi^J \phi^K \phi^L + R(\nabla \epsilon)(\nabla \epsilon)$$

Geometry in Amplitudes

$$D_\mu \phi^I = \partial_\mu \phi^I + \epsilon_{\alpha\beta}^I A_\mu^\alpha \phi^\beta \quad \text{Killing vec} \quad \mathcal{L}_{\text{SMEFT}} \sim \frac{0}{14}$$

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) D_\mu \phi^I D^\mu \phi^J + \lambda_{IJKL}(\phi) (\partial_\mu \phi^I) (\partial_\nu \phi^J) (\partial_\rho \phi^K) (\partial^\rho \phi^L) + \dots - V(\phi) - \frac{1}{4} F^2$$

$$\frac{1}{P_R}$$

$$g_{IJ} = g_{IJ} + \lambda_{IJKL} \partial_\mu \phi^K \partial_\nu \phi^L$$

$$\lim_{q_1, q_2 \rightarrow 0} A_{n+2} = \sum_{i,j} \frac{s_{ij} - s_{2j}}{s_{1j} + s_{2j}} R_{i,j} A_{n-j} + \nabla_{(I} \nabla_{J} A_n$$

Geometry in Amplitudes

$$D_\mu \phi^I = \partial_\mu \phi^I + \epsilon_{\mu}^{I} \overset{\text{Killing vec}}{A_\mu^B} \phi^B \quad \mathcal{L}_{\text{SMEFT}} \sim \frac{G}{\Lambda^4}$$

$$\mathcal{L} = \frac{1}{2} g_{IJ}(\phi) D_\mu \phi^I D^\mu \phi^J + \lambda_{IJKL}(\phi) (\partial_\mu \phi^I) (\partial_\nu \phi^J) (\partial_\rho \phi^K) (\partial_\sigma \phi^L) \epsilon^{\mu\nu\rho\sigma} + \dots - V(\phi) - \frac{1}{4} F^2$$

$$= \frac{1}{2} g_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J + \dots$$

$$\frac{1}{P_R}$$

$$g_{IJ} = g_{IJ} + \lambda_{IJKL} \partial_\mu \phi^K \partial_\nu \phi^L$$

$$\frac{V_{IJ}}{P_R}$$

$$\lim_{q_1, q_2 \rightarrow 0} A_{n+2} = \sum_j \frac{s_{1j} - s_{2j}}{s_{1j} + s_{2j}} R_{+I_2 J}^\dagger A_{n+1, j'} + \nabla_{(I} \nabla_{J)} A_n$$

$$\lim A_{n+1} = \frac{s}{\omega} \nabla R A + \frac{s}{\omega} R \nabla A + \nabla^2 A$$

$$\frac{A_m}{A_3 = A_4 = A_5 = \dots}$$

Amplitudes

$$A_3 = 0$$

$$A_4 = R'_{IKJL} S_{34} + R'_{IJKL} S_{24}$$

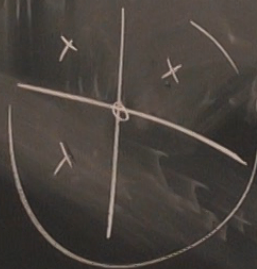
$$A_5 = \nabla'_K R'_{IKJL} S_{45} + \nabla'_L R'_{IKJM} S_{35} + \nabla'_L R'_{IJKM} S_{25} + \nabla'_M R'_{IKJL} S_{34} + \nabla'_M R'_{IJKL} (S_{24} + S_{45})$$

$$S_{15} = (p_i + p_j)^2 = 2p_i \cdot p_j$$

Soft Theorem

$$\lim_{q \rightarrow 0} A_{n+1} = \nabla'_I A_n + O(q)$$

$$\oint_{\Sigma} \frac{dz}{z} A_n(z) = \sum \frac{A_L A_R}{p^2} + A_{CT}$$



Amplitudes

$$A_3 = 0$$

$$A_4 = R'_{IKJL} S_{34} + R'_{IJKL} S_{24}$$

$$A_5 = \nabla'_K R'_{IKJL} S_{45} + \nabla'_L R'_{IKJM} S_{35} + \nabla'_L R'_{IJKM} S_{25} + \nabla'_M R'_{IKJL} S_{34} + \nabla'_M R'_{IJKL} (S_{24} + S_{45})$$

$$S_{IJ} = (p_I + p_J)^2 = 2p_I \cdot p_J$$

Soft Theorem

$$\lim_{q \rightarrow 0} A_{n+1} = \nabla'_I A_n + O(q)$$

$$\oint_{\Sigma} \frac{dz}{z} A_n(z) = \sum \frac{A_L A_R}{F p^2} + A_{CT} + \sum \nabla'_I A$$

