Title: Average Symmetry-Protected Topological Phases: Construction and Detection

Speakers: Jianhao Zhang

Series: Quantum Matter

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Abstract: Symmetry-protected topological (SPT) phases are short-range entanglement (SRE) quantum states which cannot be adiabatically connected to trivial product states in the presence of symmetries. Recently, it is shown that symmetry-protected short-range entanglement can still prevail even if part of the protecting symmetry is broken by quenched disorder locally but restored upon disorder averaging, dubbed as the average symmetry-protected topological (ASPT) phases. In this talk, I will systematically construct the ASPT phases as a mixed ensemble or density matrix, which may not be realized in a clean system without any disorder. I will also design the strange correlator of the ASPT phases via a strange density matrix to detect the nontrivial ASPT state. Moreover, it is amazing that the strange correlator of ASPT can be precisely mapped to the loop correlation functions of some proper statistical loop models, with power-law behaviors.

Zoom link: https://pitp.zoom.us/j/91672345456?pwd=N0dNQXNmVVoybnNxYXJuWnVRME8rUT09

Average Symmetry-Protected Topological Phases: Construction and Detection

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Perimeter Institute, December 7, 2022

JHZ, Y. Qi, Z. Bi, arXiv: 2210.17485

R. Ma, JHZ, Z. Bi, M. Cheng, C. Wang, To appear

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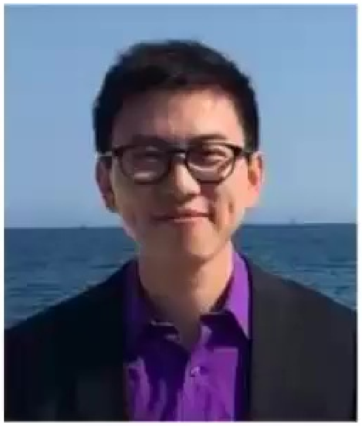
Ruochen Ma, Perimeter


Zhen Bi, Penn State


Yang Qi, Fudan


Meng Cheng, Yale


Chong Wang, Perimeter

Outline

Topological phases: Beyond Symmetry Breaking

Definition: Gapped quantum phases without symmetry breaking, but cannot be connected to a trivial disorder phase without phase transition.

Two basic classes of topological phases:

Topological ordered phase Long-range entangled, cannot be connected to a trivial phase.

Symmetry protected topological (SPT) phases Short-range entangled, cannot be connected to a trivial phase without breaking symmetry.

$$\text{SPT phase} \xrightarrow{\text{symmetry breaking unitaries}} \text{trivial disorder phase}$$

Ref: Z.-C. Gu & X.-G. Wen, PRB (2009)

1D SPT State

1D cluster state:

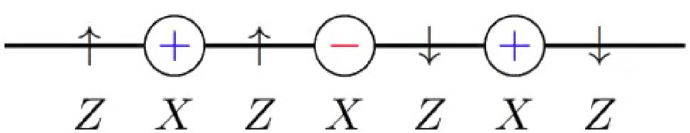
$$H = -\sum_j Z_{j-1} X_j Z_{j+1}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry (odd and even sites)

$$K = \prod_j X_{2j+1}, \; G = \prod_j X_{2j}$$

If $Z_{j-1}Z_{j+1} = \pm 1 \Rightarrow X_j = \pm 1$.



$$|\psi\rangle = \prod_j CZ_{j,j+1} |X = 1\rangle^{\oplus 2N}$$

Domain wall decoration configuration:

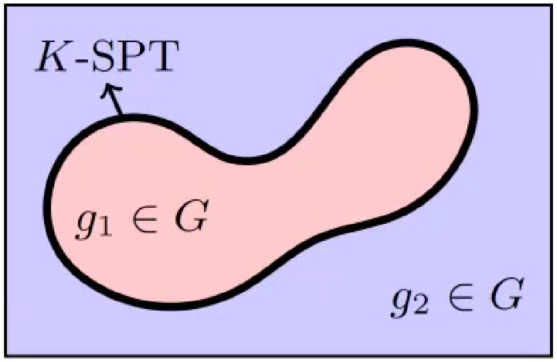
$X = -1$ on the domain wall of Z!

Domain Wall Decoration

Generic symmetry group $\tilde{G}$ with a normal subgroup K and $G = \tilde{G}/K$

$$1 \to K \to \tilde{G} \to G \to 1$$

$(d+1)$D G-SPT phases: decorate a pD K-SPT phase on the $(d+1-p)$D domain wall of G, then proliferate G symmetry.



$$|\Psi_{\text{SPT}}\rangle = \sum_{\mathcal{D}} \sqrt{p_{\mathcal{D}}} |\Psi_{\mathcal{D}}\rangle |a_{\mathcal{D}}\rangle, \ |\Psi_{\mathcal{D}}\rangle \in \mathcal{H}, \ |a_{\mathcal{D}}\rangle \in \mathcal{D}$$

1. $\mathcal{H}$: K-symmetric Hilbert space;
2. $\mathcal{D}$: Hilbert space of all domain wall configurations with codimension-p.

G-symmetry proliferation: quantum superpose all $|\Psi_{\mathcal{D}}\rangle|a_{\mathcal{D}}\rangle$ toward $|\Psi_{\text{SPT}}\rangle$.

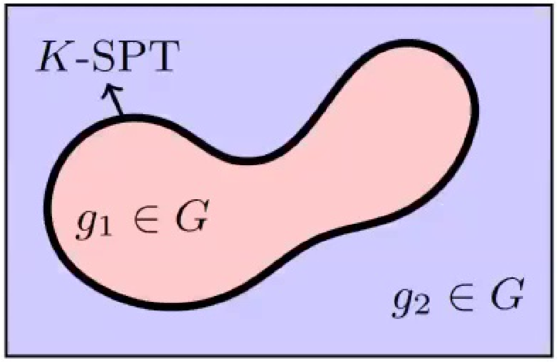
Ref: X. Chen, Y.-M. Lu, and A. Vishwanath, arXiv: 1303.4301

MATHEMATICS OF DOMAIN WALL DECORATION

Simplest case: $\tilde{G} = K \times G$, the $(d+1)$D $\tilde{G}$-SPT phases are classified by group $(d+1)$-cohomology and Künneth formula

$$\mathcal{H}^{d+1}[K \times G, U(1)] = \sum_{p=0}^{D+1} \mathcal{H}^{d+1-p}(G, \mathcal{H}^p[K, U(1)])$$

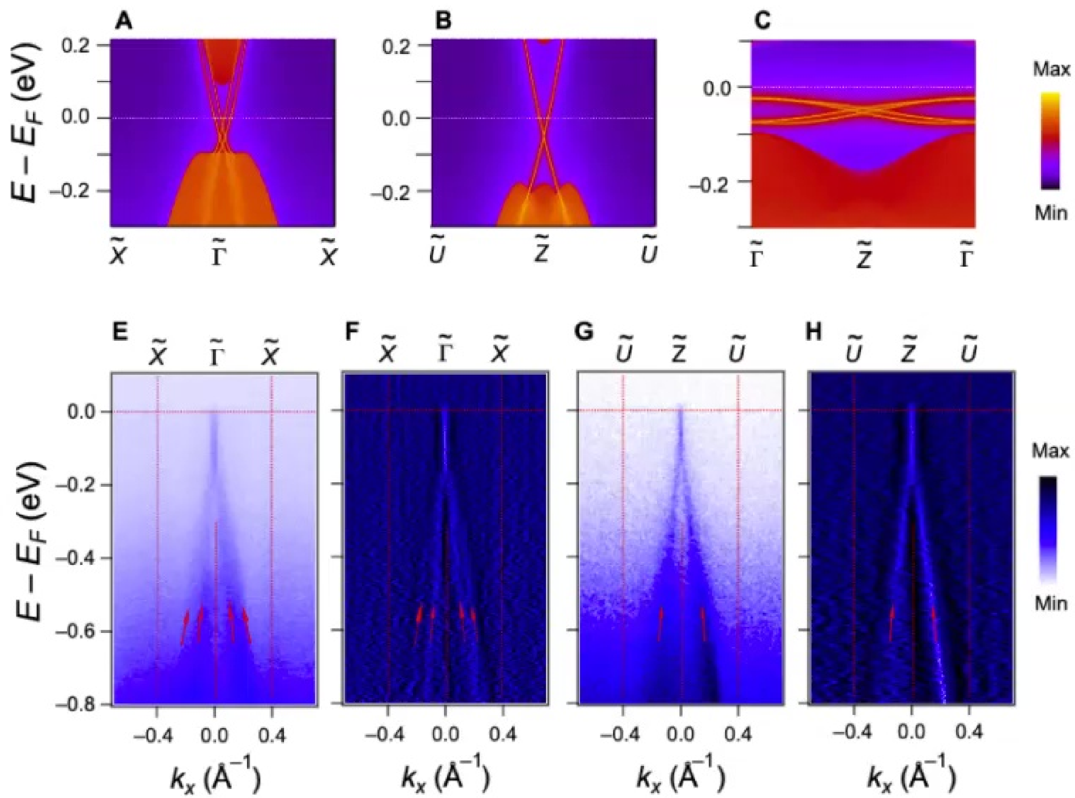
Physical meaning of Künneth formula: Decorate a pD K-SPT phase classified by $\mathcal{H}^p[K, U(1)]$, on the codimension-p domain walls of G symmetry.



$$|\Psi_{\mathrm{SPT}}\rangle = \sum_{\mathcal{D}} \sqrt{p_{\mathcal{D}}} |\Psi_{\mathcal{D}}\rangle |a_{\mathcal{D}}\rangle, \; |\Psi_{\mathcal{D}}\rangle \in \mathcal{H}, \; |a_{\mathcal{D}}\rangle \in \mathcal{D}$$

Ref: arXiv: 1106.4772

TOPOLOGICAL CRYSTALLINE PHASES



KHgSb: $P6_3/mmc$ No. 194 nonsymmorphic space group symmetry.

Gapless fermion on the (010) surface is discovered.

HARD to growth a perfect single crystal!

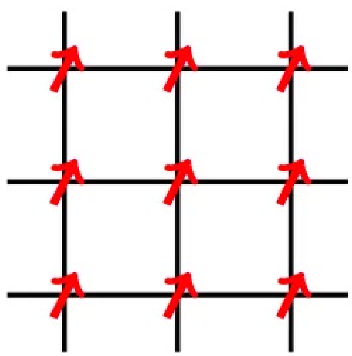
Is the gapless surface is ROBUST even for symmetry on average level?

Ref: Sci. Adv. 3, e1602415 (2017).

Average Symmetry

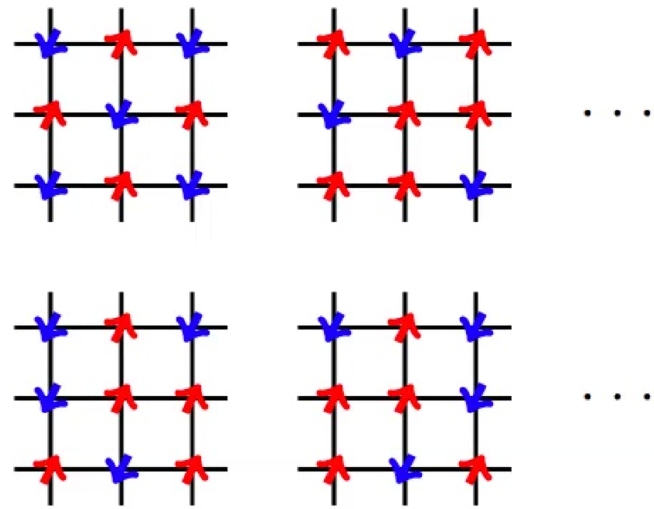
Classical Ising model:

$$H = J \sum_{\langle i,j \rangle} \sigma_i \sigma_j, \ \ \sigma_{i,j} = \pm 1$$



$\mathbb{Z}_2$ Spontaneous symmetry breaking

$\langle \sigma_z \rangle \neq 0$



$\mathbb{Z}_2$ average symmetry, $\langle \sigma_z \rangle = 0$

Average Symmetry and Symmetry SRE Ensemble

Def. (Average Symmetry): An ensemble of local Hamiltonian $\{H_\mathcal{D}\}$ and their ground state $|\Psi_\mathcal{D}\rangle$ with (classical) probability distribution $\{p_\mathcal{D}\}$, where

$$H_\mathcal{D} = H_0 + \sum_i \left(v_i^\mathcal{D} O_i + h.c.\right),\ [G, H_0] = 0,\ [v_i^\mathcal{D} O_i, G] \neq 0$$

The probability distribution $P[v] = \{p_\mathcal{D}\}$ is invariant under G. $\Rightarrow$ Entire statistical ensemble keeps invariant.

Def. (Symmetric SRE ensemble): A K-symmetric SRE ensemble only contains K-symmetric Hamiltonians with SRE ground states $\{H_\mathcal{D}, |\Psi_\mathcal{D}\rangle, p_\mathcal{D}\}$, with any pair of states being adiabatically connected while preserving K symmetry.

$$|\Psi_\mathcal{D}\rangle \xrightarrow{\text{adiabatic path}} |\Psi_{\mathcal{D}'}\rangle,\ \ |\Psi_\mathcal{D}\rangle, |\Psi_{\mathcal{D}'}\rangle \in \{H_\mathcal{D}, |\Psi_\mathcal{D}\rangle, p_\mathcal{D}\}$$

Ref: R. Ma and C. Wang, arXiv: 2209.02723

Domain Wall Decoration

Exact SPT (K and G are exact): the SPT wavefunction is

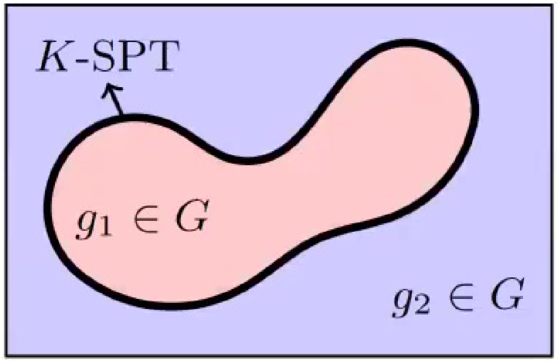
$$|\Psi_{\text{SPT}}\rangle = \sum_{\mathcal{D}} \sqrt{p_{\mathcal{D}}} |\Psi_{\mathcal{D}}\rangle |a_{\mathcal{D}}\rangle$$

1. $|\Psi_{\mathcal{D}}\rangle \in \mathcal{H}$: decorated K-SPT state;
2. $|a_{\mathcal{D}}\rangle \in \mathcal{D}$: domain wall of G symmetry;
3. $\sum_{\mathcal{D}}$: sum over all domain wall configurations – Symmetry proliferation.

ASPT (K is exact and G is average): Assemble all domain wall configurations towards a mixed ensemble $\{H_{\mathcal{D}}, |\Psi_{\mathcal{D}}\rangle, p_{\mathcal{D}}\}$.

$\Leftrightarrow$ Reduced density matrix of $|\Psi_{\text{SPT}}\rangle$:

$$\rho_{\text{ASPT}} = \text{Tr}_{\mathcal{D}} \left(|\Psi_{\text{SPT}}\rangle\langle\Psi_{\text{SPT}}|\right) = \sum_{\mathcal{D}} p_{\mathcal{D}} |\Psi_{\mathcal{D}}\rangle\langle\Psi_{\mathcal{D}}|$$



Example: 1D Average Cluster State

Recall: exact 1D cluster state

$$H_{\text{cluster}} = -\sum_j Z_{j-1} X_j Z_{j+1}$$

Disorder: break $G = \mathbb{Z}_2$ on even sites

$$H^{\mathcal{D}}_{\text{disorder}} = -\sum_j h^{\mathcal{D}}_{2j} Z_{2j}, \ \{h^{\mathcal{D}}_{2j} = \pm 1\}$$

Mixed ensemble $\{H_{\mathcal{D}}, |\Psi_{\mathcal{D}}\rangle, p_{\mathcal{D}} = 1/2^N\}$, where

$$H_{\mathcal{D}} = -\sum_j Z_{2j} X_{2j+1} Z_{2j+2} + h^{\mathcal{D}}_{2j} Z_{2j}$$

$$|\Psi_{\mathcal{D}}\rangle = \sum_j |Z_{2j} = h^{\mathcal{D}}_{2j}\rangle \otimes |X_{2j+1} = h^{\mathcal{D}}_{2j} h^{\mathcal{D}}_{2j+2}\rangle$$

$\Leftrightarrow$ Reduced density matrix

$$\rho = \sum_{\mathcal{D}} \frac{1}{2^N} |\Psi_{\mathcal{D}}\rangle\langle\Psi_{\mathcal{D}}|$$

Gapped edge, with sample-to-sample fluctuation – Average anomaly!

$h_{\text{edge}} > 0$ and $h_{\text{edge}} < 0$

Average anomaly is NOT so obvious!

Example: Haldane Chain Decoration

$K = SO(3)$, $G = \mathbb{Z}_2^A$; Haldane chain decoration phase

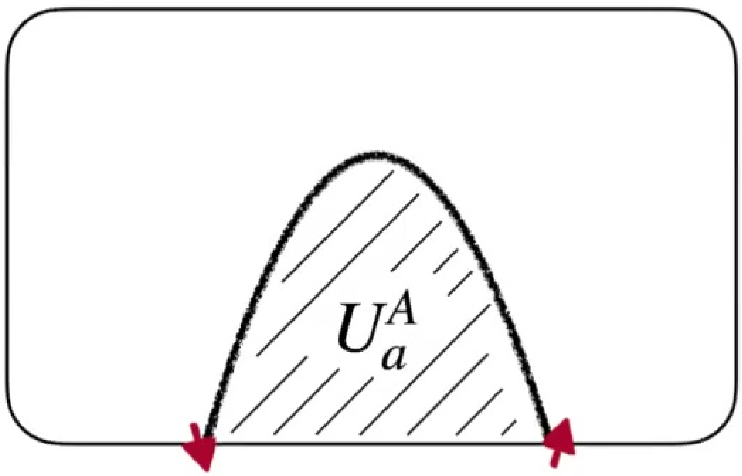
$$\mathcal{H}^1\left(\mathbb{Z}_2, \mathcal{H}^2\left[SO(3), U(1)\right]\right) = \mathbb{Z}_2$$

a: background gauge field of $\mathbb{Z}_2$: $\mathbb{Z}_2$ symmetry domain wall;

Topological action

$$S_{\text{top}} = \pi \int_X a \cup w_2^{SO(3)}$$

$w_2^{SO(3)}$: topological action of the Haldane chain.



Edge: Nonlocal spin-singlet pairs $\Rightarrow$ Long-range entanglement on the edge (probability $p \to 1$) – Average anomaly!

Q: How to detect the nontrivial feature of ASPT from the bulk?

Summary of ASPT with $K \times G$

1. Classification: Künneth formula

$$\sum_{p=0}^{d} \mathcal{H}^{d+1-p}\left(G, \mathcal{H}^p[K, U(1)]\right)$$

   compared with clean SPT, the term $\mathcal{H}^{d+1}[G, U(1)]$ is missing.

2. Can be purified to clean SPT wavefunction

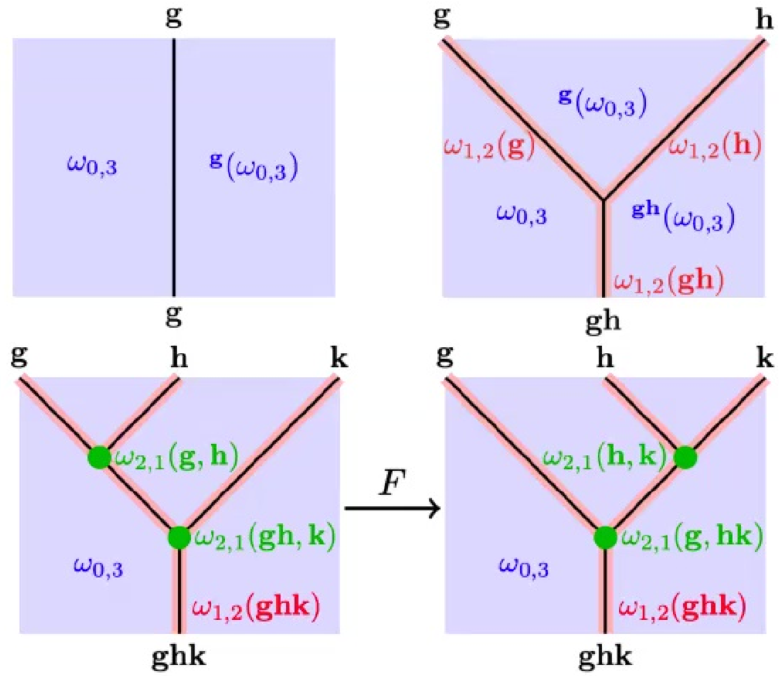
$$\rho_{\mathrm{ASPT}} = \sum_{\mathcal{D}} p_{\mathcal{D}} |\Psi_{\mathcal{D}}\rangle\langle\Psi_{\mathcal{D}}| \xrightarrow{\text{purification}} |\Psi_{\mathrm{SPT}}\rangle$$

Q1: For generic group structure $1 \to K \to \tilde{G} \to G \to 1$, how to classify ASPT in this kind of symmetry class?

Q2: Is there any ASPT ρ_{ASPT} that cannot be purified to clean SPT?

Consistency Conditions of SPT and ASPT

- d_1: The decorated G-defects can be gapped without breaking K;
- d_2: K is preserved during a continuous deformation of the G-defect network;
- d_3: There is no Berry phase accumulated after a closed path of continuous F-move deformations.



ASPT phases: classical ensemble of $|\Psi_{\mathcal{D}}\rangle$. $\Rightarrow$ d_3 is released!

There might be ASPT phases that do not have clean limit!

Generic Symmetry Group

Generic symmetry group $\tilde{G}$

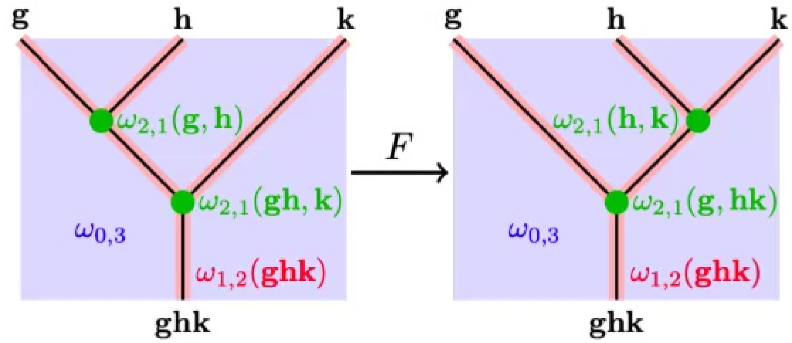
$$1 \to K \to \tilde{G} \to G \to 1,\ \omega_2 \in \mathcal{H}^2(G, K)$$

E_2-page of spectral sequence:

$$\bigoplus_{p+q=d+1} E_2^{p,q} = \bigoplus_{p+q=d+1} \mathcal{H}^p\left[G, h^q(K)\right]$$

Obstruction condition: differentials of spectral sequence:

$$\mathrm{d}_r:\ E_2^{p,q} \to E_2^{p+r,q-r+1}$$



Last layer of differentials: $\mathrm{d}_q:\ E_2^{p,q} \to E_2^{p+q+1,0}$: from F-move of the G-defects, which is no longer important for ASPT!

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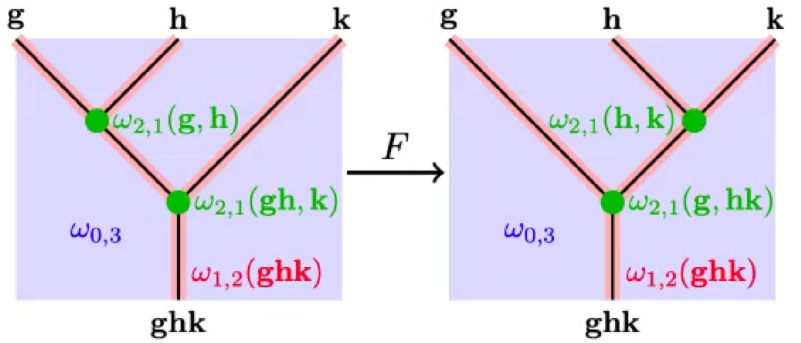

Last layer of differentials: $\mathrm{d}_q :\ E_2^{p,q} \to E_2^{p+q+1,0}$: from F-move of the G-defects, which is no longer important for ASPT!

Intrinsically ASPT Phases

Q: What is the purified state of the intrinsically ASPT density matrix?

$$\rho = \sum_I p_{\mathcal{D}} |\Psi_{\mathcal{D}}\rangle\langle\Psi_{\mathcal{D}}| \xrightarrow{\text{purification}} |\Psi_{\text{pure}}\rangle = ?$$

$$\rho = \sum_{\mathcal{D}} p_{\mathcal{D}} |\Psi_{\mathcal{D}}\rangle\langle\Psi_{\mathcal{D}}| \xleftarrow{\text{reduced density matrix}} |\Psi_{\text{pure}}\rangle = ?$$

Conjecture of $|\Psi_{\text{pure}}\rangle$: Wavefunction of an intrinsically gapless SPT phase?

For intrinsically gapless SPT phases, see arXiv: 2008.06638.

Generic Symmetry Group

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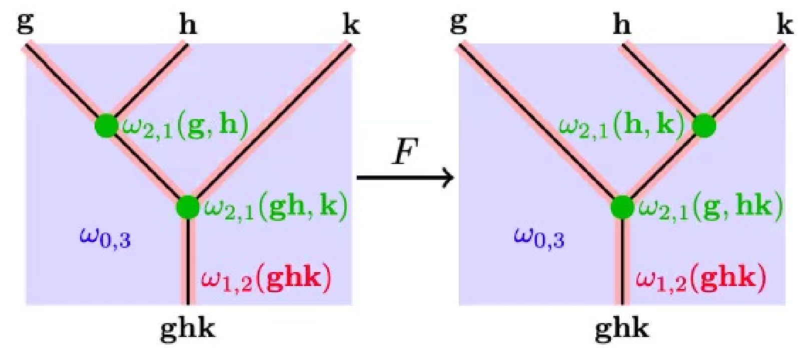

Obstruction condition: differentials of spectral sequence:

$$\mathrm{d}_r : \ E_2^{p,q} \to E_2^{p+r,q-r+1}$$

Last layer of differentials: $\mathrm{d}_q : \ E_2^{p,q} \to E_2^{p+q+1,0}$: from F-move of the G-defects, which is no longer important for ASPT!

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Conjecture of $|\Psi_{\text{pure}}\rangle$: Wavefunction of an intrinsically gapless SPT phase?

For intrinsically gapless SPT phases, see arXiv: 2008.06638.

1D INTRINSICALLY ASPT FROM 0D DECORATION

$K = \mathbb{Z}_2$, $G = \mathbb{Z}_2$, $\tilde{G} = \mathbb{Z}_4$: No clean SPT phase: $H^2[\mathbb{Z}_4, U(1)] = \mathbb{Z}_1$.

Clean system: 1D $\mathbb{Z}_4$ intrinsically gapless SPT

$$U_g = \prod_j \sigma_j^x e^{i\frac{\pi}{4}\sum_j 1-\tau^x_{j+1/2}},\ U_a = \prod_j \tau^x_{j+1/2},\ U_g^2 = U_a$$

Hamiltonian of 1D $\mathbb{Z}_4$ gapless SPT phase

$$H = -\sum_j \sigma_j^x(1-\sigma^z_{j-1}\sigma^z_{j+1})P + \frac{1}{2}\sum_j(1-\sigma_j^z\tau^x_{j+1/2}\sigma^z_{j+1}),\ P = \prod_j \frac{1+\sigma_j^z\tau^x_{j+1/2}\sigma^z_{j+1}}{2}$$

Under (strong) random disorder $-\sum_j h_j^{\mathcal{D}}\sigma_j^z$:

$$H_{\mathcal{D}} = \frac{1}{2}\sum_j\left(1-\sigma_j^z\tau^x_{j+1/2}\sigma^z_{j+1}\right) + h_j^{\mathcal{D}}\sigma_j^z,\ \{h_j^{\mathcal{D}} = \pm 1\}$$

1D $\mathbb{Z}_4$ gapless SPT + disorder $\Rightarrow$ 1D Average cluster state AGAIN!

Higher-Dimensional Decoration

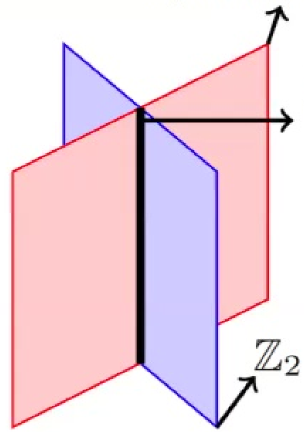
(3+1)D $\mathbb{Z}_2^f \times \mathbb{Z}_4 \times \mathbb{Z}_2$ SPT

- Majorana chain decoration phase: $n_2 \in \mathcal{H}^2[\mathbb{Z}_4 \times \mathbb{Z}_2, \mathbb{Z}_2]$ $(a, b \in \mathbb{Z}_4 \times \mathbb{Z}_2)$

$$n_2(a,b) = a_1 b_2 (\text{mod } 2),\ a = (a_1, a_2),\ b = (b_1, b_2)$$

Two differentials:

1. $\mathrm{d}_2 :\ E_2^{2,2} \to E_2^{4,1}$: $\mathrm{d}_2 n_2 = n_2 \cup n_2 = \mathrm{d}\nu_3$
   – Obstruction-free (fermion parity conservation);
2. $\mathrm{d}_3 :\ E_2^{2,2} \to E_2^{5,0}$:
   $\mathrm{d}_3 n_2 = O_5[n_2] \in \mathcal{H}^5[\mathbb{Z}_4 \times \mathbb{Z}_2, U(1)]$
   – Obstructed (Berry phase inconsistency).



Majorana chain decoration phase n_2 is an intrinsically ASPT phase!

Classification of Intrinsically ASPT Phases

Generic symmetry group $\tilde{G}$

$$1 \to K \to \tilde{G} \to G \to 1$$

E_2-page of spectral sequence and obstruction:

$$\bigoplus_{p+q=d+1} E_2^{p,q} = \bigoplus_{p+q=d+1} \mathcal{H}^p\left[G, h^q(K)\right], \quad \mathrm{d}_r: \ E_2^{p,q} \to E_2^{p+r,q-r+1}$$

Intrinsically ASPT: Elements in $E_2^{p,q}$ which are obstructed by

$$d_{d+2-p}: \ E_2^{p,q} \to E_2^{d+2,0} \simeq H^{d+2}[G, U(1)]$$

and other differentials are obstruction-free

$$d_r : E_2^{p,q} \to E_2^{p+r,q-r+1}, \ 1 \le r \le d+1-p$$

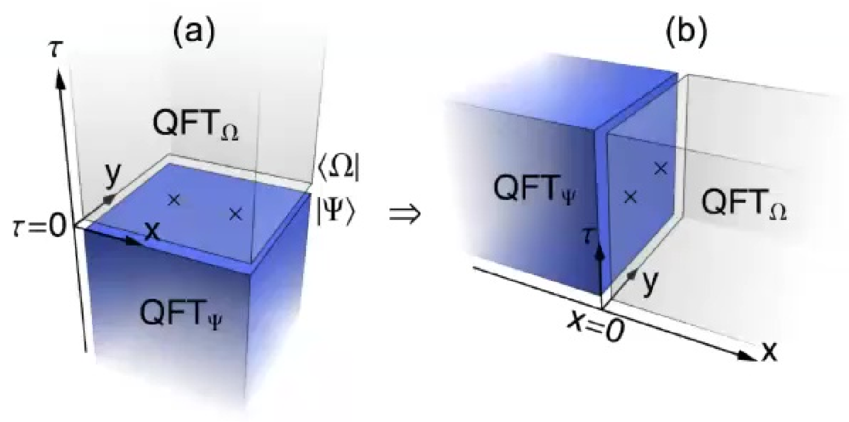
Bulk Detection of SPT

Motivation: Bulk of SPT is SRE $\Rightarrow$ Detection of SPT

1. Edge detection: 't Hooft anomaly;
2. Bulk detection: Gauging the symmetry group (arXiv: 1202.3120);
3. Bulk detection: Strange correlator (arXiv: 1312.0616).

$$C(r,r') = \frac{\langle\Omega|\psi(r)\psi(r')|\Psi\rangle}{\langle\Omega|\Psi\rangle}$$

1. $|\Psi\rangle$: Wavefunction under detection;
2. $|\Omega\rangle$: Direct product state;
3. $\phi(r)$: local operator at r.



$C(r,r')$ is either **long-range** or **power-law decay** $\Rightarrow$ $|\Psi\rangle$ is nontrivial!

Bulk Detection of ASPT

ASPT phase: Average anomaly of $\rho_{\text{ASPT}} = \sum_{\mathcal{D}} p_{\mathcal{D}} |\Psi_{\mathcal{D}}\rangle\langle\Psi_{\mathcal{D}}|$ is not as obvious as the 't Hooft anomaly for clean SPT.

Reference density matrix: trivial ASPT state $\rho_0 = \sum_{\mathcal{D}} p_{\mathcal{D}} |\Phi_{\mathcal{D}}\rangle\langle\Phi_{\mathcal{D}}|$.

Def. (Strange density matrix): For an ASPT state ρ_{ASPT} and reference trivial ASPT state ρ_0, the strange density matrix is defined as

$$\rho_s = \sum_{\mathcal{D}} p_{\mathcal{D}} |\Psi_{\mathcal{D}}\rangle\langle\Phi_{\mathcal{D}}|$$

Def. (Strange correlator): For an ASPT density matrix ρ_{ASPT}, the strange correlator of some local operator $\phi(r)$ is defined as

$$C(r-r') = \frac{\text{Tr}[\rho_s \phi(r)\phi(r')]}{\text{Tr}(\rho_s)}$$

Strange correlator $C(r,r')$ will be either **long-range** or **power-law decay** if ρ_{ASPT} is nontrivial.

Warm Up: 1D Average Cluster State

Mixed ensemble of 1D average cluster state $\{H_{\mathcal{D}}, |\Psi_{\mathcal{D}}\rangle, p_{\mathcal{D}}\}$ and trivial ASPT ensemble $\{H^0_{\mathcal{D}}, |\Phi_{\mathcal{D}}\rangle, p_{\mathcal{D}}\}$ ($\{h^{\mathcal{D}}_{2j} = \pm 1\}$ and $p_{\mathcal{D}} = 1/2^N$)

$$H_{\mathcal{D}} = -\sum_j Z_{2j} X_{2j+1} Z_{2j+2} + h^{\mathcal{D}}_{2j} Z_{2j} \qquad H^0_{\mathcal{D}} = -\sum_j X_{2j+1} + h^{\mathcal{D}}_{2j} Z_{2j}$$

$$|\Psi_{\mathcal{D}}\rangle = \sum_j |Z_{2j} = h_{2j}\rangle \otimes |X_{2j+1} = h_{2j} h_{2j+2}\rangle, \quad |\Phi_{\mathcal{D}}\rangle = \sum_j |Z_{2j} = h_{2j}\rangle \otimes |X_{2j+1} = 1\rangle$$

Strange density matrix and strange correlator

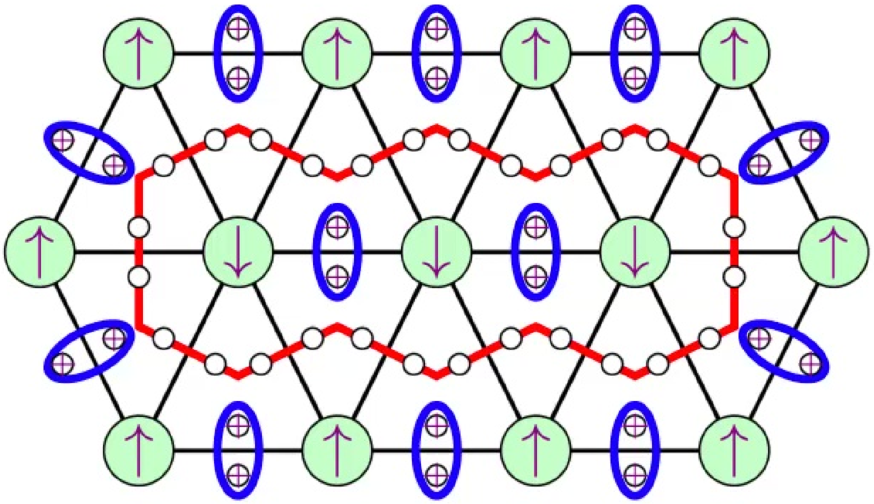
$$\rho_s = \sum_{\mathcal{D}} \frac{1}{2^N} |\Psi_{\mathcal{D}}\rangle\langle\Phi_{\mathcal{D}}|, \; C(Z_i, Z_j) = \frac{\mathrm{Tr}(\rho_s Z_i Z_j)}{\mathrm{Tr}(\rho_s)} = 1$$

Ref: arXiv: 2210.17485

Remark ASPT from 0D decoration has subtle nontrivial effects on the boundary (sample-to-sample fluctuation);

Nontrivial feature: Reflected by the strange correlator in the bulk!

2D $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2^A$ ASPT



1. $\mathbb{Z}_2^A$: Paramagnetic phase of 2D classical Ising model;
2. Each bond: Two spin-1/2 degrees of freedom;
3. Away from domain wall: spins are polarized to $|+\rangle$ state.

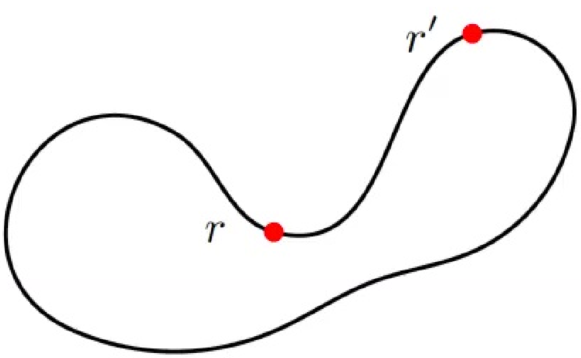
On a domain wall (red): spins form a 1D cluster state.

Mapping to a Loop Model

The strange correlator of K degree of freedom

$$C_K(r,r') = \frac{\mathrm{Tr}\left(\rho_s Z_r Z_{r'}\right)}{\mathrm{Tr}\,\rho_s} = \frac{\sum_{\mathcal{D}} p_{\mathcal{D}} \langle \Psi_{\mathcal{D}} | Z_r Z_{r'} | \Phi_{\mathcal{D}} \rangle}{\sum_{\mathcal{D}} p_{\mathcal{D}} \langle \Psi_{\mathcal{D}} | \Phi_{\mathcal{D}} \rangle}$$

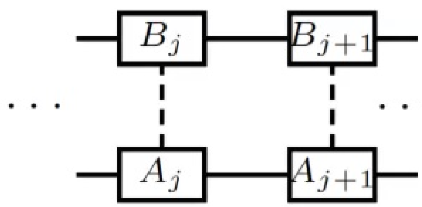
1. $p_{\mathcal{D}}$: Boltzmann weight of a classical Ising model in the paramagnetic phase, $p_{\mathcal{D}} \sim \prod_{l\in\mathcal{D}} x^{-l}$, $x = e^{-2\beta}$ (domain wall tension);
2. $\langle \Psi_{\mathcal{D}} | \Phi_{\mathcal{D}} \rangle \sim e^{-l/l_c} \Rightarrow$ Renormalize domain wall tension;
3. $\langle \Psi_{\mathcal{D}} | Z_r Z_{r'} | \Phi_{\mathcal{D}} \rangle$: Nonzero only when r and r' are connected by the same domain wall.
   – Strange correlator of 1D K-SPT!



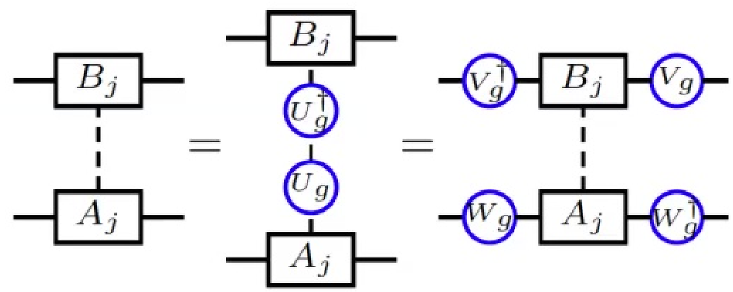
Quantum Correction of Loop Fugacity

$$\langle \Psi_{\mathcal{D}} | \Phi_{\mathcal{D}} \rangle = \prod_{l \in \mathcal{D}} \langle \psi_{\text{cluster}} | \psi_0 \rangle (l) = \prod_{l \in \mathcal{D}} \mathbf{2} \times 2^{-l}$$

Matrix product state representation:



$$\cdots = \mathrm{Tr}(E_1 E_2 \cdots)$$



The factor 2 is universal!

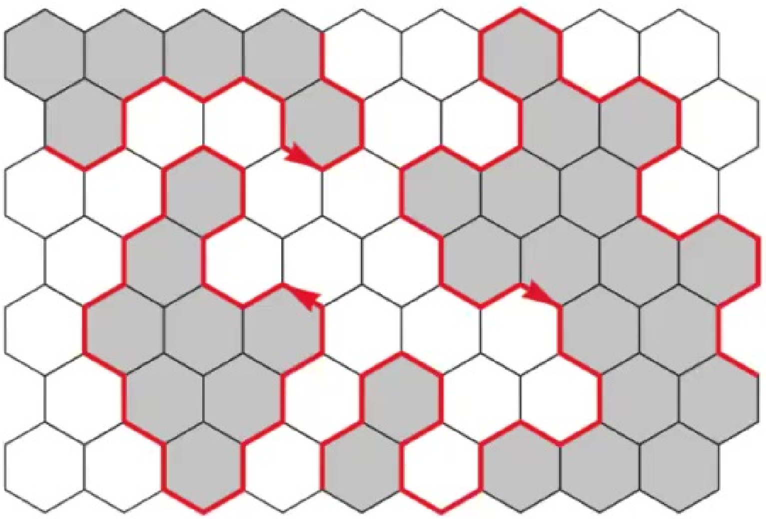
$$\mathrm{Tr}(\rho_s) = 2 \sum_{\mathcal{D}} \tilde{x}^{-L(\mathcal{D})} 2^{n(\mathcal{D})}$$

$O(n)$ Loop Model

$\mathrm{Tr}(\rho_s)$: $O(n)$ loop model with $n = 2$

$$Z_{O(2)} = \mathrm{Tr}(\rho_s) = 2 \sum_{\mathcal{D}} \tilde{x}^{-L(\mathcal{D})} 2^{n(\mathcal{D})}$$

1. $L(\mathcal{D})$: Total length of domain wall;
2. $n(\mathcal{D})$: Number of domain walls in $\mathcal{D}$.

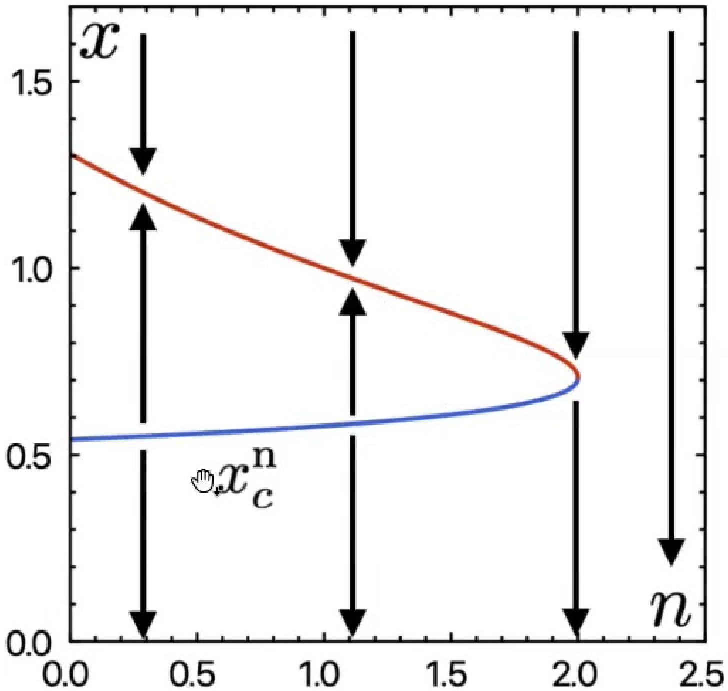


Two kinds of loops with orientations.

$$C_K(r, r') = \frac{\sum_{\mathcal{D}'} \langle Z_r Z_{r'} \rangle_S \tilde{x}^{-L(\mathcal{D}')} 2^{n(\mathcal{D}')}}{\sum_{\mathcal{D}} \tilde{x}^{-L(\mathcal{D})} 2^{n(\mathcal{D})}}$$

Loop correlation function: Probability that r and r' are connected by a single domain wall.

$O(n)$ Loop Model and Correlation Function



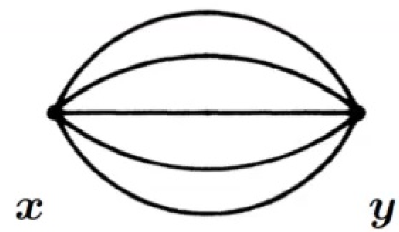
Ref: Duplantier, Phys. Rep. (1989).

For $n \in [-2, 2]$, critical point (blue line) $x_c = [2 + \sqrt{2-n}]^{-1/2}$, and

1. $x > / < x_c$: dense/dilute loop phase;

L-leg watermelon correlation function:

$$C_L(\boldsymbol{x} - \boldsymbol{y}) = |\boldsymbol{x} - \boldsymbol{y}|^{-2\Delta_L}$$



$$\begin{cases} n = -2\cos(\pi g) \\ \Delta_L = \frac{g}{8} L^2 - \frac{(1-g)^2}{2g} \end{cases}$$

2D $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2^A$ ASPT

$x > \sqrt{2}$: $\tilde{x}$ in the dense loop phase

$\Rightarrow K = \mathbb{Z}_2 \times \mathbb{Z}_2$ strange correlator

$$C_K(r, r') \sim \frac{1}{|r - r'|^{2\Delta_2}}$$

Δ_2: 2-leg critical exponent for $O(2)$ model

$$\Delta_2 = 1/2$$

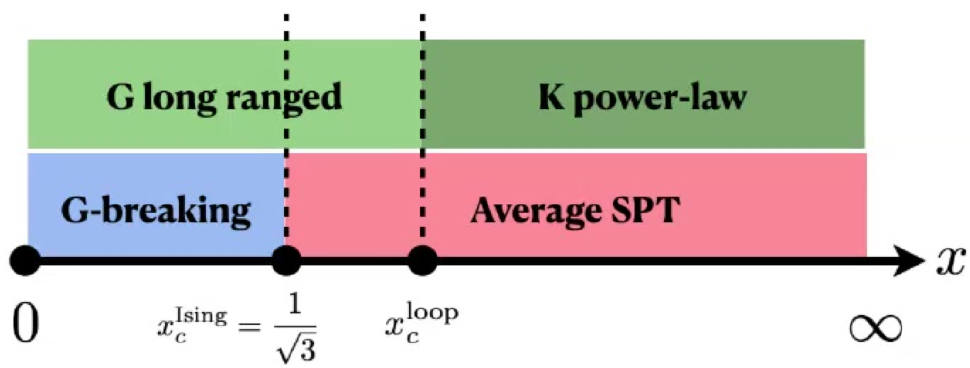
$1/\sqrt{3} < x < \sqrt{2}$: $\tilde{x}$ in the dilute loop phase

$G = \mathbb{Z}_2^A$-strange correlator

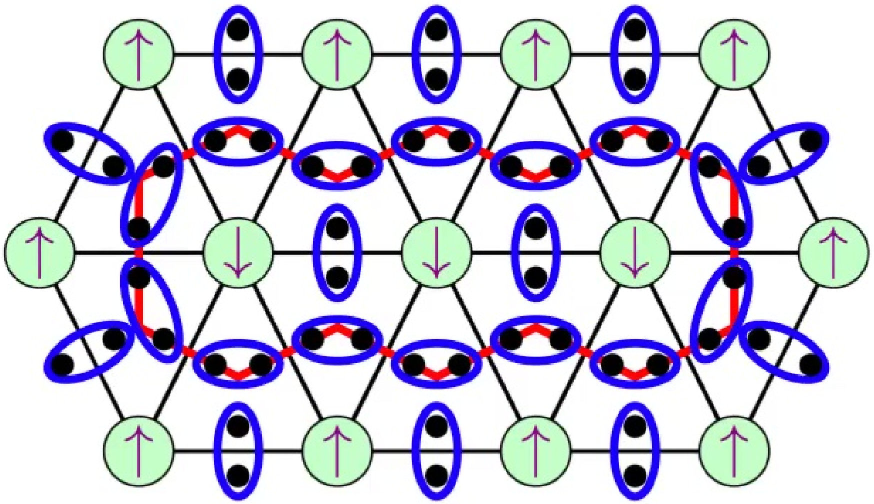
$$C_G(r, r') = \frac{\mathrm{Tr}\,(\rho_s \sigma_r \sigma_{r'})}{\mathrm{Tr}\rho_s}$$

In the dilute loop phase

$$C_G(r, r') \sim \text{const for } \tilde{x} < 1/\sqrt{2}$$



2D $\mathbb{Z}_2^f \times \mathbb{Z}_2^A$ ASPT



1. $\mathbb{Z}_2^A$: Paramagnetic phase of 2D classical Ising model;
2. Each bond: Two Majorana zero modes;
3. Away from domain wall: Local Majorana entanglement pair.

On a domain wall (red): 1D Majorana chain decoration.

QUANTUM CORRECTION OF LOOP FUGACITY

$K = \mathbb{Z}_2^f$-strange correlator

$$C_K(r, r') = \frac{\mathrm{Tr}(\rho_s c(r) c(r'))}{\mathrm{Tr}(\rho_s)}$$

$$\mathrm{Tr}(\rho_s) = \sum_{\mathcal{D}} p_{\mathcal{D}} \langle \Psi_{\mathcal{D}} | \Phi_{\mathcal{D}} \rangle$$

$$\langle \Psi_{\mathcal{D}} | \Phi_{\mathcal{D}} \rangle = \prod_{l \in \mathcal{D}} \langle \psi_{\mathrm{Majorana}} | \psi_0 \rangle (l)$$

$$= \prod_{l \in \mathcal{D}} \sqrt{2} \times \sqrt{2}^{-l}$$

$$Z_{O(\sqrt{2})} = \mathrm{Tr}(\rho_s) = \sqrt{2} \sum_{\mathcal{D}} \tilde{x}^{-L(\mathcal{D})} \sqrt{2}^{n(\mathcal{D})}$$

$\Rightarrow O(n = \sqrt{2})$ loop model!

2D $\mathbb{Z}_2^f \times \mathbb{Z}_2^A$ ASPT

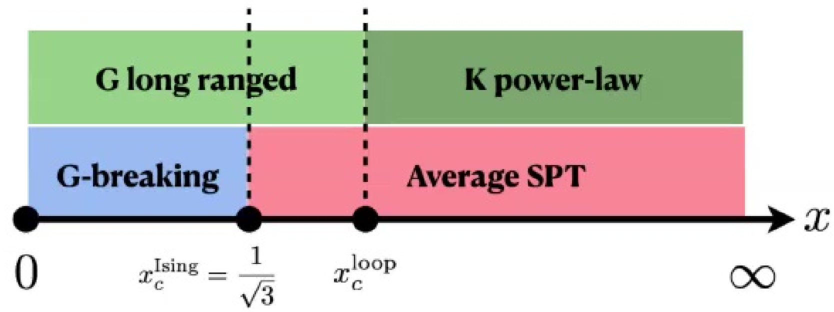
Dilute fixed point $x_c^{n=\sqrt{2}} = 1/\sqrt{2-\sqrt{2-\sqrt{2}}} \simeq 0.601$, and $\tilde{x} = x/\sqrt{2}$

$$C_K(r,r') = \frac{\sum_{\mathcal{D}'} \langle c_r c_{r'} \rangle_S \tilde{x}^{-L(\mathcal{D}')} \sqrt{2}^{n(\mathcal{D}')}}{\sum_{\mathcal{D}} \tilde{x}^{-L(\mathcal{D})} \sqrt{2}^{n(\mathcal{D})}} \sim \frac{1}{|r-r'|^{2\Delta_2}}, \quad x \geq x_c^{n=\sqrt{2}}$$

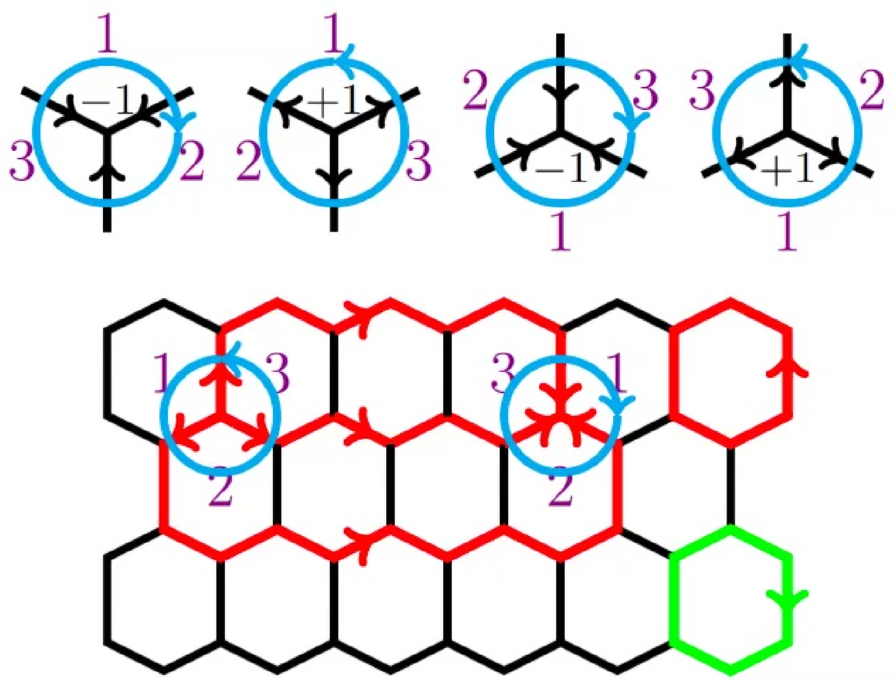
$\tilde{x} > x_c^{n=\sqrt{2}}$: $\Delta_2 = 1/3$; $\tilde{x} = x_c^{n=\sqrt{2}}$: $\Delta_2 = 3/5$.

$\tilde{x} < x_c^{n=\sqrt{2}}$: $G = \mathbb{Z}_2^A$-strange correlator in the dilute loop phase

$$C_G(r,r') = \frac{\mathrm{Tr}(\rho_s \sigma_r \sigma_{r'})}{\mathrm{Tr}\rho_s} \sim \text{const}$$



2D $\mathbb{Z}_3 \times \mathbb{Z}_3^A$ ASPT



1. A nontrivial $\mathbb{Z}_3$ charge is attached on each vortex of the $\mathbb{Z}_3$ – 2D SPT from 0D decoration;
2. Two types of domain walls: with orientations.

$K = \mathbb{Z}_3$-strange correlator

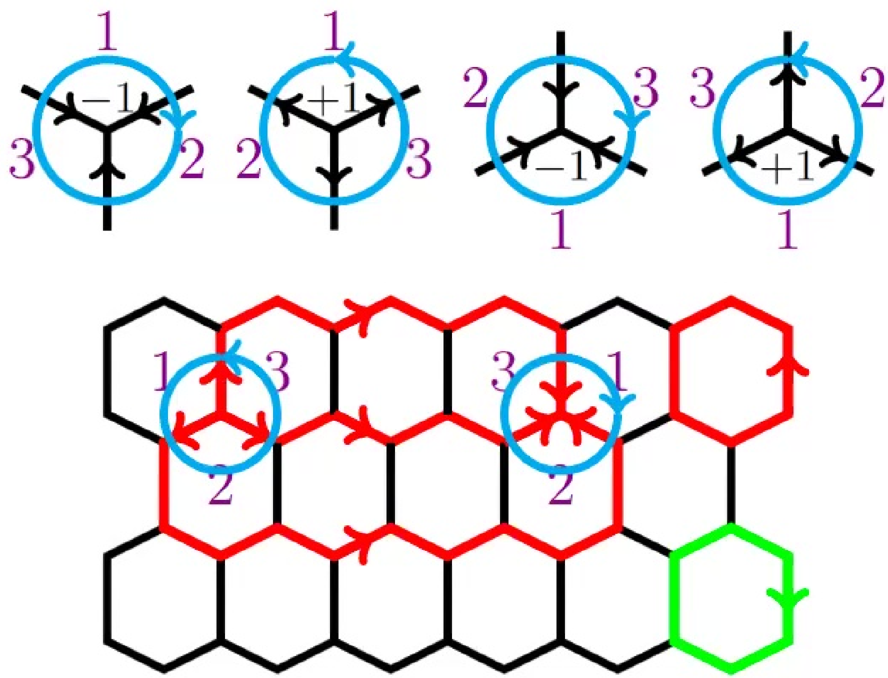
$$C_K(r, r') = \frac{\mathrm{Tr}\left(\rho_s a^\dagger(r) a(r')\right)}{\mathrm{Tr}\rho_s}$$

Denominator:

$$\mathrm{Tr}(\rho_s) = 3 \sum_{\mathcal{D}'} x^{-L(\mathcal{D}')} 2^{n(\mathcal{D}')}$$

$O(2)$ loop model!

2D $\mathbb{Z}_3 \times \mathbb{Z}_3^A$ ASPT



$x \geq x_c = 1/\sqrt{2}$: dense loop phase, with 3-leg watermelon correlation function

$$C_K(r, r') \sim \frac{1}{|r - r'|^{2\Delta_3}}$$

Critical exponent:

$$\Delta_3 = 9/8$$

$x < x_c$: dilute loop phase, measure the $G = \mathbb{Z}_3^A$-strange correlator

$$C_G(r, r') = \frac{\mathrm{Tr}\left(\rho_s b^\dagger(r) b(r')\right)}{\mathrm{Tr}\rho_s} \sim \text{const}$$

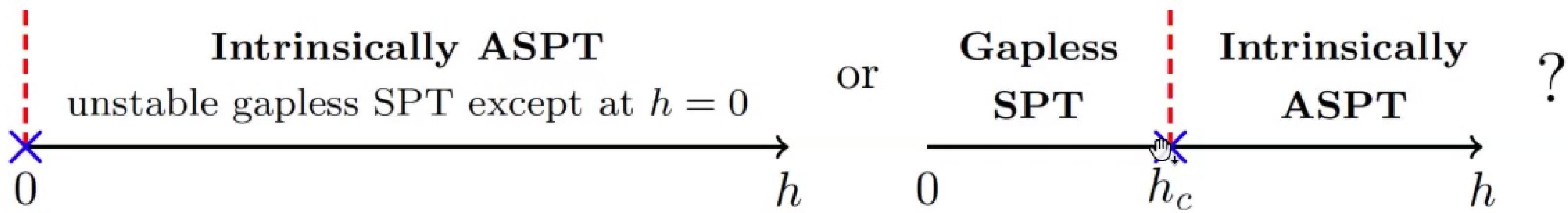
Summary and Outlook

Summary

- Define the ASPT phases in mixed ensembles;
- Intrinsically ASPT phases that do not have a clean limit: from obstructed/intrinsically gapless SPT phases;
- Detecting nontrivial ASPT phases from bulk: Strange correlation functions;
- Mapping the strange correlators of ASPT to the watermelon correlation function of $O(n)$ loop models in 2D.

Outlook

1. Transition between gapless SPT and intrinsically ASPT controlled by disorder intensity;



2. Preparations and phase transitions of ASPT with measurements;

3. Average symmetry-enriched topological (SET) phases, including surface SET of average SPT phases.

Thanks for your attention!