Title: Hidden symmetries in cosmology and black holes

Speakers: Francesco Sartini

Series: Quantum Gravity

Date: December 01, 2022 - 2:30 PM

URL: https://pirsa.org/22120053

Abstract: Cosmological models and black holes belong to classes of space-time metrics defined in terms of a finite number of degrees of freedom, for which the Einstein-Hilbert action reduces to a one-dimensional mechanical model. We investigate their classical symmetries and the algebra of the corresponding Noether charges. These dynamical symmetries have a geometric interpretation, not in terms of spacetime geometry, but in terms of motion on the field space. Moreover, they interplay with the fiducial scales, introduced to regulate the homogenous model, suggesting a relationship with the boundary symmetries of the full theory.

Finally, the existence of these symmetries unravels new aspects of the physics of black holes and cosmology. It opens the way towards a rigorous group quantization of the reduced model and to the study of their holographic properties. It might have significant consequences on the propagation of test fields and the corresponding perturbation theory.

Zoom link: https://pitp.zoom.us/j/92846533238?pwd=cERGUjd6OXB5S0ZaSzVIdVJyMHZxUT09

Hidden symmetries in cosmology and black holes

and other minisuperspaces

Francesco Sartini

based on

Geiller, Livine, FS:
arXiv:gr-qc/2010.07059 [SciPost]
arXiv:gr-qc/2107.03878 [CQG]
arXiv:gr-qc/2205.02615 [PRD]

1/12/2022
Perimeter Institute

Motivations

What is quantum gravity?

Reduced models

- Lower dimensional theories
  - Two and three-dimensional gravity have a *simpler* dynamics
  - Interesting for boundary structures
  - Entropy and holography
- **Symmetry reduced models**
  - Simple but physically relevant models
  - Insights on phenomenology

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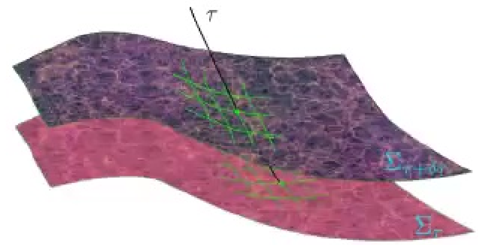
Symmetries

- Gauge vs physical
- Control the structure of the classical solutions
- Non-perturbative handle on quantization
- **Gravity:** gauge (diffeos) + ? $\left\{ \begin{array}{l} \text{boundary/asymptotic symmetries} \\ \text{NJ algorithm, symmetries of qnm, love numbers...} \end{array} \right.$

Minisuperspaces

Simple, but physically relevant subsets of solutions in GR: *e.g.* black holes and cosmology [Ashtekar, Ellis, Hartle, Hawking, Horowitz, Misner, Page, Unruh, Vilenkin ...]

- We fix the degrees of freedom on the slices
- Finite number of degrees of freedom
- We study the dynamics in the orthogonal direction
  τ = evolution parameter
  $\vec{x}$ = homogeneous coordinates



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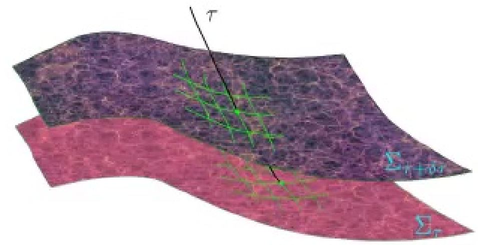

Cosmology and Bianchi models

- Homogeneous slices, with a three-dimensional group of isometries
  $\rightarrow$ Bianchi classification, Bianchi [1897]

Black holes

- Spherically-symmetric slice, with topology $\mathbb{R} \times S^2$
- I will focus on this illustrative example

An example: black hole minisuperspace

Line element and action

- Metric with homogeneous fields $V_1(\tau)$, $V_2(\tau)$

$$\mathrm{d}s^2 = 2N\mathrm{d}\tau\mathrm{d}x + \frac{V_1}{2V_2}\mathrm{d}x + L_s^2\, V_1\,\mathrm{d}\Omega^2$$

- L_s is a constant, to have dimensionless fields
- Reduced Einstein-Hilbert action = mechanical model

$$\mathcal{S}_{\mathrm{BH}} = \int \mathrm{d}\tau \left[\frac{\mathcal{V}_0}{G}\left(\frac{\dot{V}_1(V_2\dot{V}_1 - 2V_1\dot{V}_2)}{2NV_1^2} + \frac{N}{L_s^2}\right)\right]$$

- **Regulator** in the null x direction $x \in [0, L_0]$

$$\int_\Sigma \mathrm{d}x\mathrm{d}\Omega \;\rightarrow\; \mathcal{V}_0 := \frac{L_s^2 L_0}{4}$$

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On-shell metric

- The solutions represent Schwarzschild black holes (interior and exterior)
- τ is the radius, x is the incoming null direction ☺
- The mass is one of the initial conditions (family of solutions)

Field space approach

General method to find the symmetries of minisuperspaces

- Lagrangian: describe the motion of a particle on curved space [Christodoulakis et al.]

$$\mathcal{L}_{\text{minis.}} = \frac{1}{2N} G_{\alpha\beta} \dot{q}^\alpha \dot{q}^\beta - NU(q)$$

$$\mathcal{L}_{\text{BH}} = \frac{1}{N} \frac{\mathcal{V}_0 (V_2 \dot{V}_1^2 - 2 V_1 \dot{V}_1 \dot{V}_2)}{2 G V_1^2} + N \frac{\mathcal{V}_0}{G L_s^2}$$

- Hamiltonian formalism: phase space $\{q^\mu, p_\nu\} = \delta^\mu_\nu$

$$H_{\text{ADM}} = N \frac{2\pi_{ab}\pi^{ab} - \pi^2}{2\sqrt{q}} - N\sqrt{q} R \qquad \rightarrow \qquad H_{\text{minis.}} = \frac{N}{2} G^{\alpha\beta} p_\alpha p_\beta + NU$$

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⚠ $G_{\alpha\beta}$ is not related to the spacetime metric, we study **the geometry of the geometry!**

⚠ Mind the lapse: we need to choose a clock

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Evolution of the spacetime metric ⇔ Geodesic motion on field space

Conformal properties of the supermetric and symmetries

- We can build conserved quantities from Killing vectors ξ

$$\pounds_\xi G_{\alpha\beta} = 0 \qquad \Rightarrow \qquad \xi^\alpha p_\alpha = const.$$

- Generalization to conformal Killing vectors, such that

$$\pounds_\xi G_{\alpha\beta} = \lambda G_{\alpha\beta} \qquad \pounds_\xi U = -\lambda U \qquad \lambda = const.$$

$$\mathcal{Q}_0 = H \qquad \mathcal{Q}_1 = \xi^\alpha p_\alpha - \lambda\tau H$$

- Whenever the supermetric is flat, from $\xi^\alpha \xi_\alpha$ we can build charges quadratic in time

$$\mathcal{Q}_2 = \xi^2 + \tau\xi^\alpha p_\alpha + \tau^2 H$$

These are conserved $\dot{\mathcal{Q}}_i = \partial_\tau \mathcal{Q}_i + \{\mathcal{Q}, H\} = 0$

- In general there might be many vectors ξ_i

$$\mathcal{Q}_0 = H \qquad \mathcal{Q}_1^i = \xi^\alpha p_\alpha - \lambda_i \tau H \qquad \mathcal{Q}_2^{ij} = \xi_i^{\ \alpha} \xi_{j\,\alpha} + \tau a_{ij}^k \xi_k^\alpha p_\alpha + b_{ij}\tau^2 H$$

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Schrödinger algebra

Applying this procedure to

- Black holes
- FLRW cosmology with a scalar field

gives a eight-dimensional algebra of observables
[Geiller, Livine, FS: 2205.02615, Ben Achour, Livine, Oriti, Piani: 2207.07312]

$$\mathfrak{sh}(2) = (\mathfrak{sl}(2,\mathbb{R}) \times \mathfrak{so}(2)) \ltimes \mathfrak{h}_2$$

The Schrödinger algebra generates:

- The dynamical symmetry of the compressible Navier-Stokes equation
- The group of symmetry of a free particle in quantum mechanics

CVH subalgebra in cosmology

Cosmology

- In flat FLRW cosmology the $\mathrm{SL}(2,\mathbb{R})$ symmetry was already known!
  [Pioline, Waldron 2002], [Bergeron, Gazeau 2013], [Ben Achour, Livine 2019]
- This is due to the conformal scaling properties of the volume and energy
- Is this specific to FLRW?

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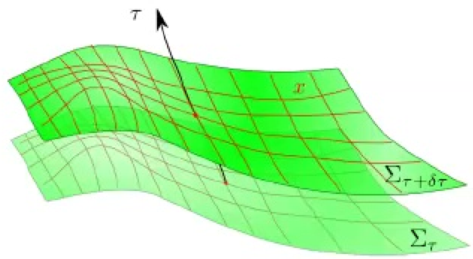
Full GR

- Consider Hamiltonian description
- Induced metric $q_{ab}(x)$, extrinsic curvature $\pi^{ab}(x)$



$$\mathcal{H} = \frac{2\pi_{ab}\pi^{ab} - \pi^2}{2\sqrt{q}} - \sqrt{q}R = \mathcal{H}_{\text{kin}} - \sqrt{q}R$$

- The time derivative of the volume ($V = \int_\Sigma \sqrt{q}$) is proportional to the Gibbons-Hawking term, i.e. extrinsic curvature or *complexifier* ($C = \int_\Sigma \pi^{ab} q_{ab}$)
- If the hypersurface is flat $R = 0 \Rightarrow \{C, V, H\}$ generate an $\mathrm{SL}(2,\mathbb{R})$ symmetry

Möbius transformation for black holes

Exponentiating the action of the $\mathfrak{sl}(2,\mathbb{R})$ charges:

$$\mathcal{L}_n = n(n-1)V_2 - n\frac{\tau^{n-1}}{\kappa \ell_{\mathsf{Pl}}} C - \tau^n \tilde{H} \qquad n = \pm 1, 0$$

$$\kappa = \frac{\mathcal{V}_0}{\ell_{\mathsf{Pl}}^3} \qquad C = -P_i V_i \qquad \tilde{H} = G^{\alpha\beta} p_\alpha p_\beta$$

- $\mathrm{SL}(2,\mathbb{R})$: Möbius transformation on τ, and conformal on V_i

$$\tau \mapsto \tilde{\tau} = \frac{a\tau + b}{c\tau + d} \qquad \text{with} \qquad ad - bc = 1$$

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What is the interpretation?

Physical interpretation of the symmetry

- We act with the symmetry on the initial conditions:
  singularity and horizon location (τ_0, τ_1), the mass M, and the fiducial scale L_0

Möbius $\tau \mapsto f(\tau)$

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\begin{aligned}
\tau_0 &\mapsto f(\tau_0) \\
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- The mass changes $\Rightarrow$ Not residual diffeos!
- The fiducial length (boundary) interplays with the symmetry

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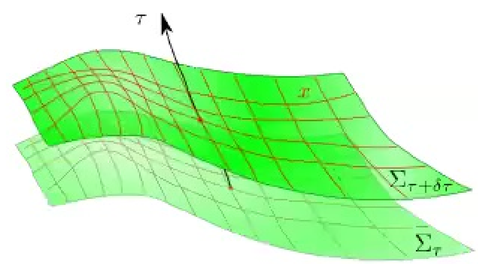

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What happens if there is a curvature?

An example: black hole minisuperspace

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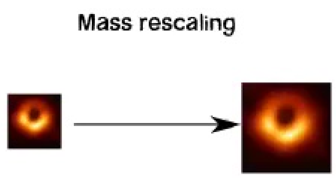

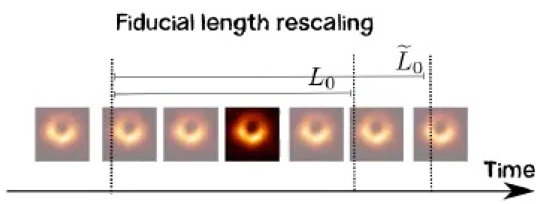

A note on the central charge

Heisenberg subalgebra

- The Schrödinger algebra contains an Heisenberg subalgebra

$$\mathfrak{sh}(2) = (\mathfrak{sl}(2,\mathbb{R}) \times \mathfrak{so}(2)) \ltimes \mathfrak{h}_2$$

- whose central charge is κ

$$\mathfrak{h}_2 : \{X_i, \Pi_j\} = \kappa \delta_{ij}$$

- $\kappa = \mathcal{V}_0/\ell_{\mathrm{Pl}}^3$ ratio between IR and UV scales
  - This happens for both cosmology and BH
  - central charge = number of Planckian cells

Virasoro extension

- Considering a generalisation of the Möbius transformation: $\tau \mapsto f(\tau)$
- Equation of motion of V_i transform as in the coadjoint representation of Virasoro, with central charge $\kappa \ell_{\mathrm{Pl}}$

Virasoro extension

- Let us extend $\mathrm{SL}(2,\mathbb{R})$ to Virasoro
- For this, consider generic function f

$$\mathrm{SL}(2,\mathbb{R}) : \left| \begin{array}{lll} \tau & \mapsto & \tilde{\tau} = \frac{a\tau+b}{c\tau+d} \\ V_i & \mapsto & \widetilde{V}_i(\tilde{\tau}) = \frac{V_i(\tau)}{(c\tau+d)^2} \end{array} \right. \quad \mathrm{Diff}(\tau) \left| \begin{array}{lll} \tau & \mapsto & \tilde{\tau} = f(\tau) \\ V_i & \mapsto & \widetilde{V}_i(\tilde{\tau}) = \dot{f}(\tau)\, V_i(\tau) \end{array} \right.$$

They transform the action functional as

$$\Delta_f \mathcal{S} = \frac{\mathcal{V}_0}{G} \int \mathrm{d}\tau \left[\mathrm{Sch}[f] V_2 - \frac{\mathrm{d}}{\mathrm{d}\tau} (\dots) \right]$$

$$f = \frac{a\tau + b}{c\tau + d} \qquad \Leftrightarrow \qquad \mathrm{Sch}[f] = \frac{f^{(3)}}{\dot{f}} - \frac{3}{2} \left(\frac{\ddot{f}}{\dot{f}} \right)^2 = 0$$

Coadjoint representation

- Equivalent rewriting of our action

$$\mathcal{S} = \int \mathrm{d}\tau [\mathcal{J} V_1 + \mathcal{P} V_2] \qquad \delta\mathcal{S} = \int \mathrm{d}\tau [\mathcal{J}\delta V_1 + \mathcal{P}\delta V_2 + \mathrm{d}_\tau \theta]$$

$$\mathcal{P} = \kappa \ell_{\mathsf{Pl}} \left(\frac{\ddot{V}_1}{V_1} - \frac{\dot{V}_1^{\,2}}{2V_1^2} \right)$$

$$\mathcal{J} = \kappa \ell_{\mathsf{Pl}} \left(-\frac{\dot{V}_2 \dot{V}_1}{V_1^2} + \frac{\ddot{V}_2}{V_1} + \frac{V_2}{V_1} \left(\frac{\ddot{V}_1}{V_1} - \frac{\dot{V}_1^{\,2}}{V_1^2} \right) \right)$$

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- EOMs $\mathcal{J}, \mathcal{P}$ transform as in the centrally-extended coadjoint representation of Virasoro, with central charge $c = \kappa\ell_{\mathsf{Pl}}$

$$\mathcal{P} \to \dot{f}^2(\mathcal{P}\circ f) - \kappa\ell_{\mathsf{Pl}}\mathrm{Sch}[f]$$
$$\mathcal{J} \to \dot{f}^2(\mathcal{J}\circ f)$$

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Perturbations and inhomogeneities

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Some hints:

- Quasinormal modes from an $\mathrm{SL}(2,\mathbb{R})$ symmetry in the near horizon region [Chen, Long '10]
- Vanishing of love number for static perturbation from $\mathrm{SL}(2,\mathbb{R})$ symmetry [Ben Achour, Livine, Mukohyama, Uzan '22]

The latter comes from a special conformal transformation of the radius

$$r \to \frac{\lambda r\, r_s}{(\lambda - 1) r + r_s}$$

Perturbations and inhomogeneities

Do the hidden symmetry play a role in perturbation theory?

- Background symmetries $\leftrightarrow$ conserved quantities for test fields?
- Consequences on QNM or Hawking radiation?

Some hints:

- Quasinormal modes from an $\mathrm{SL}(2,\mathbb{R})$ symmetry in the near horizon region [Chen, Long '10]
- Vanishing of love number for static perturbation from $\mathrm{SL}(2,\mathbb{R})$ symmetry [Ben Achour, Livine, Mukohyama, Uzan '22]

The latter comes from a special conformal transformation of the radius

$$r \rightarrow \frac{\lambda r \, r_s}{(\lambda - 1) r + r_s}$$

Is possible to link these to the minisuperspace symmetry?

Conclusions

Despite their apparent simplicity, **minisuperspaces contain a rich symmetry structure**

Physical symmetries

- General method to find minisuperspace symmetries (w/o matter)
- "Second geometrization"
- Hints about interesting role of the fiducial cell

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Beyond minisuperspace

- IR/UV scales from ∞-dimensional extension
- Relationship with scalar perturbation and QNM

What comes next?

Physical consequences of the $\mathfrak{sh}(2)$ symmetry

- Cosmological perturbation and quantum gravity condensates
  [Gielen, Marchetti, Oriti]
- Analog gravity models and hydrodynamical regime

Explore the role of the fiducial cell

- Central charge and degrees of freedom
- Gluing multiple cells and edge modes
- Relationship with coarse graining
  [Bodendorfer, Han, Haneder, Liu, Wuhrer]

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Generalisation

- Midisuperspace (rotating black holes)
- Inhomogeneity in cosmology
- Understanding the connection with the full theory