

Title: Surrogate model for gravitational wave signals from black hole binaries built on black hole perturbation theory waveforms calibrated to numerical relativity : one model to rule both comparable and extreme mass ratio regime

Speakers: Tousif Islam

Series: Strong Gravity

Date: December 01, 2022 - 1:00 PM

URL: <https://pirsa.org/22120052>

Abstract: We present a reduced-order surrogate model of gravitational waveforms from non-spinning binary black hole systems with comparable to large mass-ratio configurations. This surrogate model, BHPTNRSur1dq1e4, is trained on waveform data generated by point-particle black hole perturbation theory (ppBHPT) with mass ratios varying from 2.5 to 10,000. BHPTNRSur1dq1e4 can generate waveforms up to  $30,500 m_1$  (where  $m_1$  is the mass of the primary black hole), includes several more spherical harmonic modes up to  $\ell=10$ , and calibrates both dominant and subdominant modes to numerical relativity (NR) data. In the comparable mass-ratio regime, including mass ratios as low as 2.5, the gravitational waveforms generated through ppBHPT agree surprisingly well with those from NR after this simple calibration step. We argue that this scaling essentially captures higher order self-force corrections in a much simpler way. We also compare our model to recent SXS and RIT NR simulations at mass ratios ranging from 15 to 32, and find the dominant quadrupolar modes agree to better than  $10^{-3}$ . We expect our model to be useful to study intermediate-mass-ratio binary systems in current and future gravitational-wave detectors. Finally, we discuss avenues for improving the model by extending its region of validity.

Zoom link: <https://pitp.zoom.us/j/99971588372?pwd=ZVUveUINeTI1SE5iMzNnVDh0L2xkQT09>

---

**Surrogate model for gravitational wave signals from  
black hole binaries built on black hole perturbation  
theory waveforms calibrated to numerical relativity:**

**one model to rule both comparable and extreme mass  
ratio regime**

**Tousif Islam**

**Center for Scientific Computing and Data Science Research  
University of Massachusetts Dartmouth**

[ [tislam@umassd.edu](mailto:tislam@umassd.edu) ]

[ <https://tousifislam.com> ]

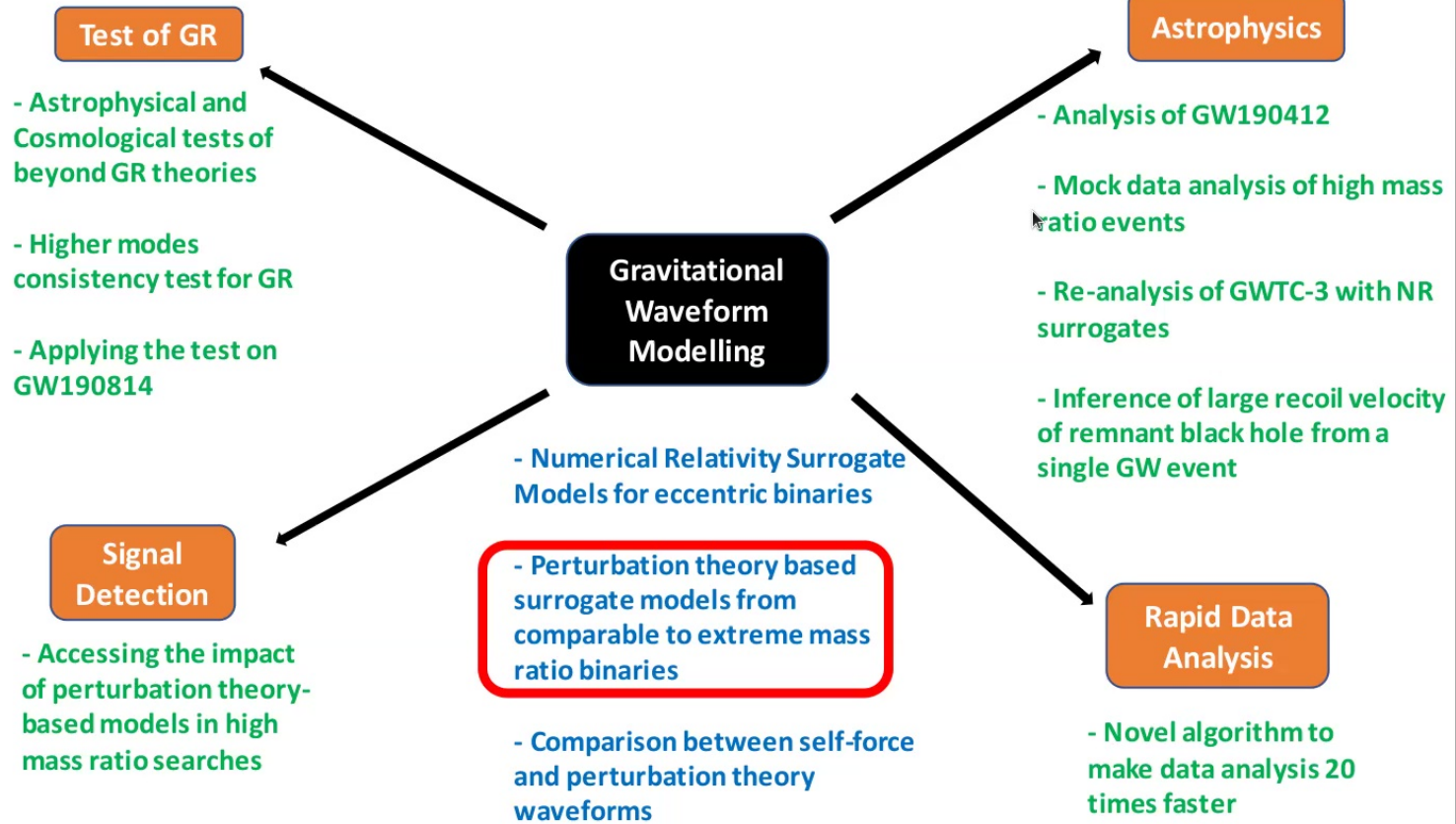


**lisa**



**Dec 1, 2022  
Strong Gravity Seminar  
Perimeter Institute**

# Things I do



Tousif Islam | UMass Dartmouth

## Collaborators

2

### Perturbation Surrogate

Scott Field (UMassD), Scott Hughes (MIT), Gaurav Khanna (URI), Vijay Varma (AEI), Matthew Giesler (Cornell), Mark Scheel (Caltech)  
[ Islam +, arXiv.2204.01972 ]

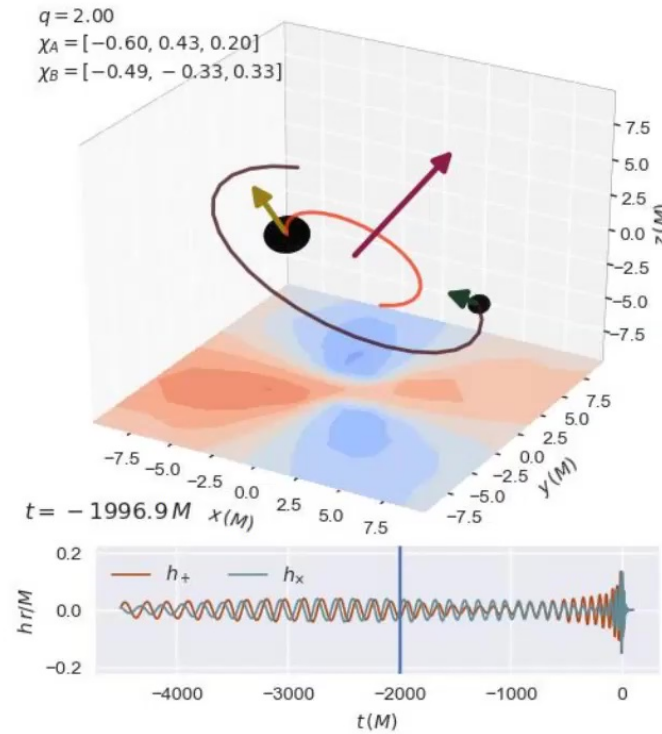
### Comparison with Self-force

Barry Wardell (UCD), Adam Pound (Southampton), Niels Warburton (UCD), Scott Field (UMassD), Gaurav Khanna (URI)  
[ Islam +, In preparation ]

Tousif Islam | UMass Dartmouth

# Gravitational Waves from Binary Black Holes

3



[ Courtesy: Vijay Varma ]

Tousif Islam | UMass Dartmouth

# Gravitational Waves from Binary Black Holes

4

Gravitational radiation can be written as a superposition of  $-2$  spin-weighted spherical harmonics

$$\begin{aligned} \hat{h}(t; \mathbf{n}, \lambda) &:= h_+(t; \mathbf{n}, \lambda) - i h_\times(t; \mathbf{n}, \lambda) \\ &= \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} Y_{\ell m}^{-2}(\mathbf{n}) \hat{h}_{\ell m}(t; \lambda), \end{aligned}$$

Described by at-least 17 parameters

- **Intrinsic parameters:**  
masses and spin components of the binary, {eccentricity, mean anomaly} (2+6+2)
- **Extrinsic parameters:**  
inclination, luminosity distance, right ascension and declination, polarization, phase and merger time (7)

Tousif Islam | UMass Dartmouth

# Gravitational Waveform Models

5

Numerical Relativity (NR) waveforms

Phenomenological Methods

Post Newtonian Approximations

[ Blanchet+, Iyer+, Will+, .. ]

Phenomenological waveform Models

[ Ajith+, London+, Khan+, Hannam+, .. ]

Effective-one-body (EOB) waveform models

[ Bohe+, Cotesta+, Pan+, Babak+, .. ]

NR Surrogate waveform models

[ Field+, Blackman+, Varma+, Islam+ ]

Black Hole Perturbation Theory

[ Rifat+, Islam+ ]



Tousif Islam | UMass Dartmouth

---

## Point-Particle Black Hole Perturbation Theory (ppBHPT)

6

The smaller black hole is modeled as a point-particle with no internal structure.

The framework was originally developed for extreme mass ratio inspirals and/or solving ringdown regime.

First, we compute the trajectory taken by the point-particle.

We use that trajectory to compute the gravitational wave emission by solving Teukolsky equation.

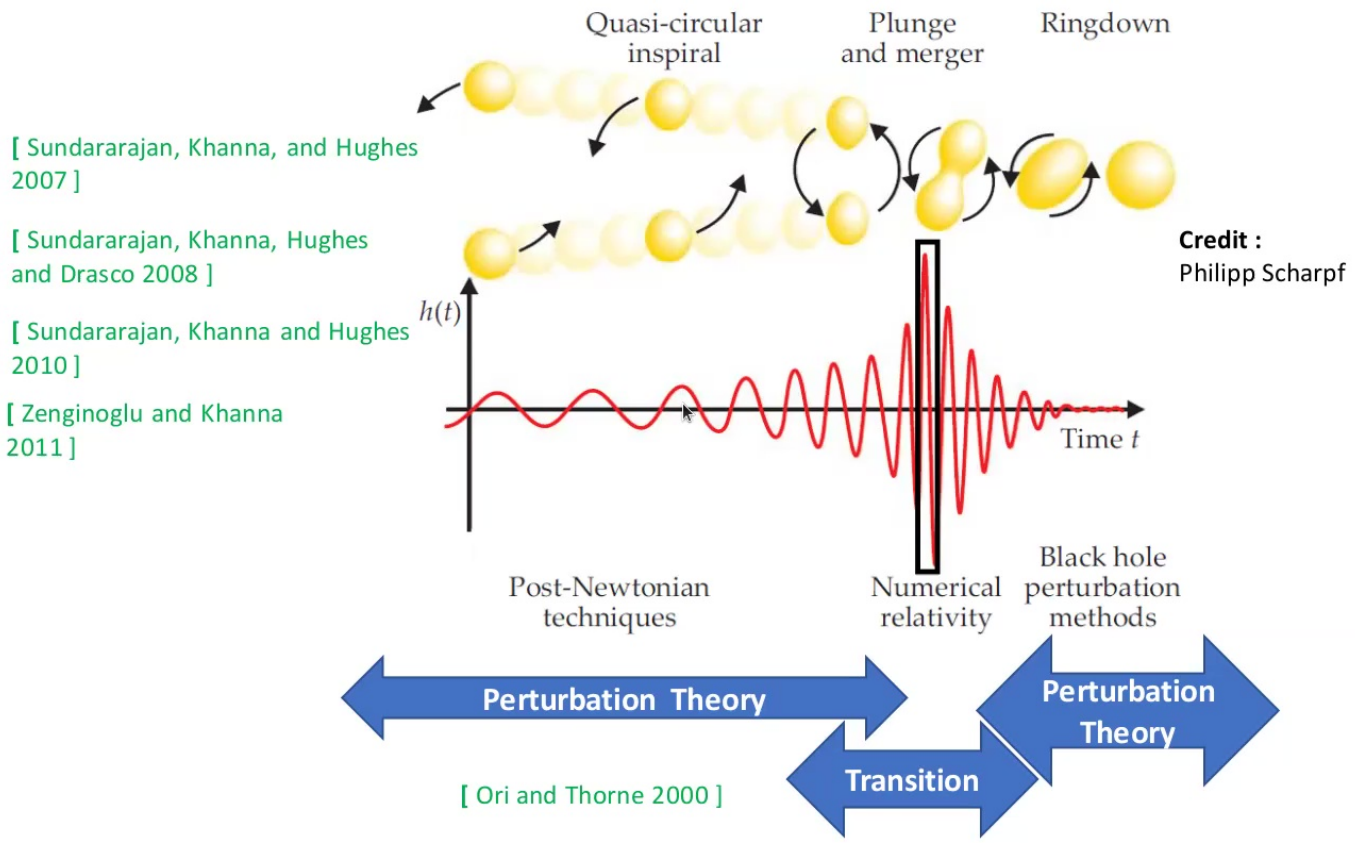
Best Way to generate accurate Waveform for extreme mass ratio binaries

Tousif Islam | UMass Dartmouth



# ppBHPT Inspiral-Merger-Ringdown Waveform

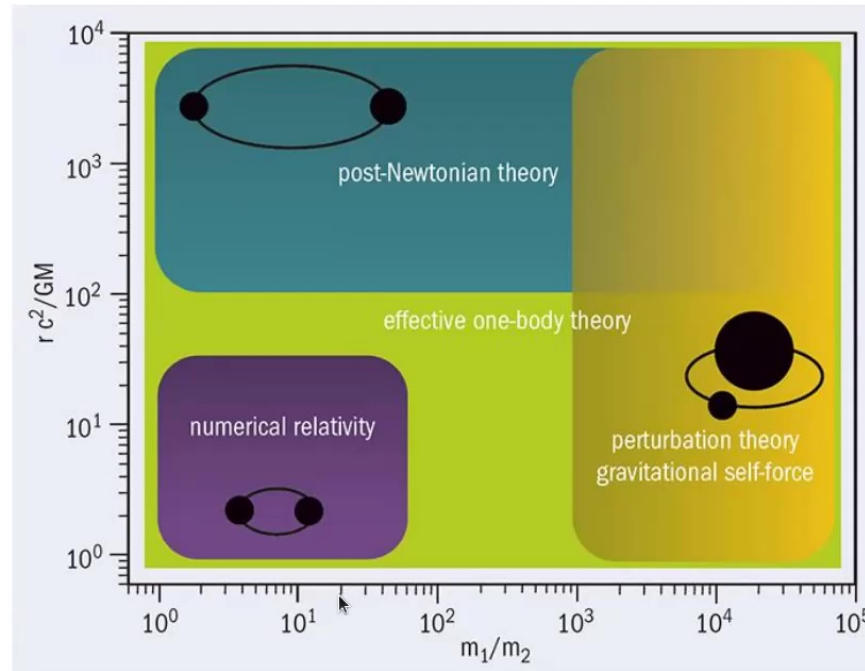
7



Tousif Islam | UMass Dartmouth

## Where should we trust ppBHPT ?

8



[ Image credit: arXiv:1410.7832 ]

Tousif Islam | UMass Dartmouth

## Why Another Model ?

9

None of the current NR-Surrogate / EOB / Phenom models are accurate in high mass ratio regime

Intermediate Mass Ratio Binaries : GW190814 (mass ratio  $q \sim 10$ )

ppBHPT gives most accurate waveform for higher mass ratio systems; However, it is computationally expensive

"Kludge" Models

[ Barack+, Babak+, Gair+, Chua+]

Second Order Self Force

[ Wardell+]

Tousif Islam | UMass Dartmouth

## BHPTNRSur1dq1e4 : an overview

10

Available via



Black-hole Perturbation Toolkit

gw-surrogate

Non-Spinning Binaries : BHPTNRSur1dq1e4

- **Covers comparable to large Mass Ratio** :  $q=2.5$  to  $q=10^4$
- **Trained on ppBHPT waveforms from time-domain Teukolsky Solver** :
  - OPA waveforms, updated plunge model in the ppBHPT framework
- **Many modes** :
  - 25 modes up to  $\ell=10$
- **Longer waveforms** : [ Relevant for LIGO, Cosmic Explorer, Einstein Telescope ]
  - 35000M
- **Calibrated to NR in the small mass ratio regime** :
  - modes are calibrated up to  $\ell=5$

Tousif Islam | UMass Dartmouth

---

## Tuning ppBHPT to NR in small mass ratio regime

11

- Can we build a single model from comparable to large/extreme mass ratio binaries?
- Can we extend perturbation theory framework in small mass ratio regime?
- Do we need to calibrate ppBHPT to NR in the small mass ratio regime ?
- Will the calibration work for all modes?


Tousif Islam | UMass Dartmouth

## Tuning ppBHPT to NR in small mass ratio regime

12

- Rescaling ppBHPT waveforms:
  - up to  $\ell=5$

$$\mathbf{h}_{\text{NR}} := h_{\text{S},\alpha^\ell,\beta}^{\ell,m}(t; q) = \alpha^\ell h_{\text{S}}^{\ell,m}(t\beta; q)$$

rescaled ppBHPT  raw ppBHPT

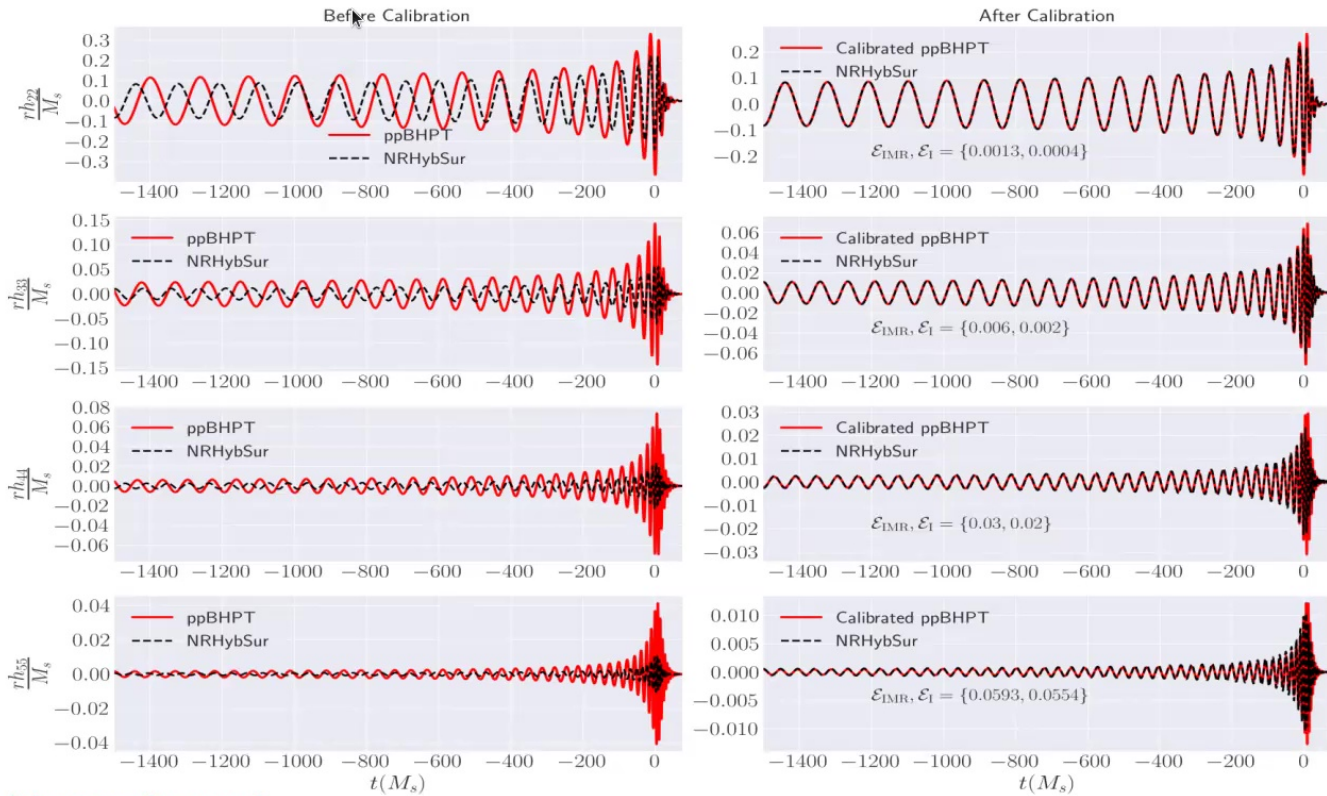
- Obtain the scaling parameters by optimizing the error between scaled ppBHPT and NR (NRHybSur3dq8) :

$$\min_{\alpha^\ell,\beta} \frac{\int_{t=-5000M}^{t=115M} \left| h_{\text{S},\alpha^\ell,\beta}^{\ell,m}(t; q) - h_{\text{NRHyb}}^{\ell,m}(t; q) \right|^2 dt}{\int_{t=-5000M}^{t=115M} \left| h_{\text{NRHyb}}^{\ell,m}(t; q) \right|^2 dt}$$

Tousif Islam | UMass Dartmouth

# Small Mass Ratio Regime : Comparison to NRHybSur3dq8 / example waveforms

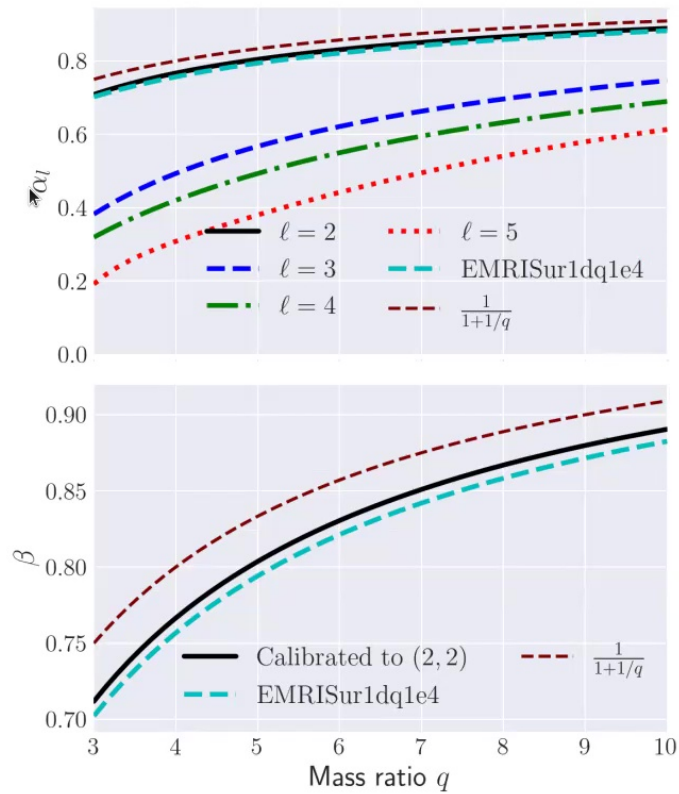
13



Tousif Islam | UMass Dartmouth



## Small Mass Ratio Regime : Scaling ppBHPT waveforms to match NR



Fourth Order polynomial formula used for the scaling parameters as a function of small mass ratio ( $1/q$ )

Tousif Islam | UMass Dartmouth



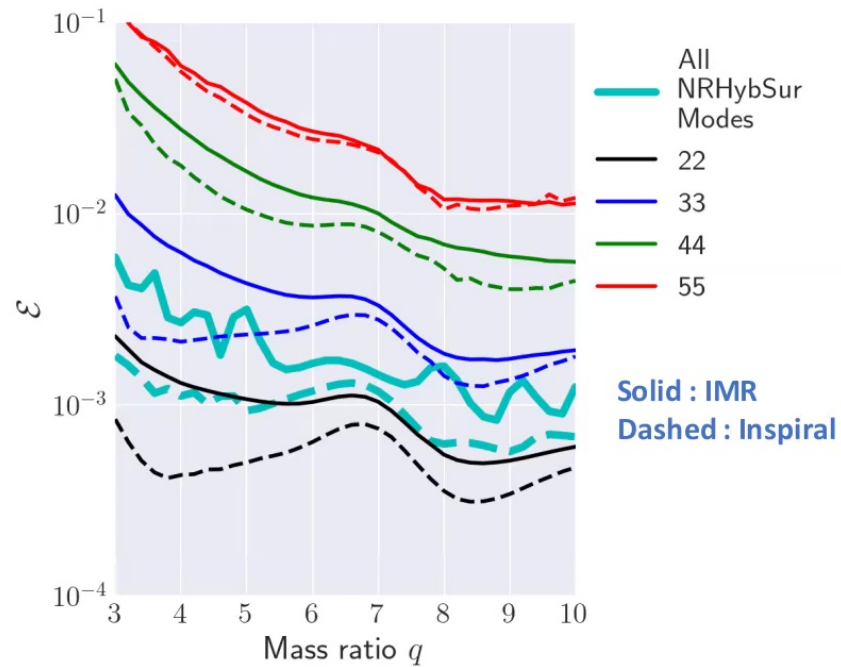
## Small Mass Ratio Regime : Comparison to NRHybSur3dq8 / Time Domain Error

15

(2,2) mode error  $\sim 10^{-3}$

(3,3) and (4,4) modes  
error  $\sim 10^{-2}$

Errors drop further when only  
INSPIRAL waveform is  
considered



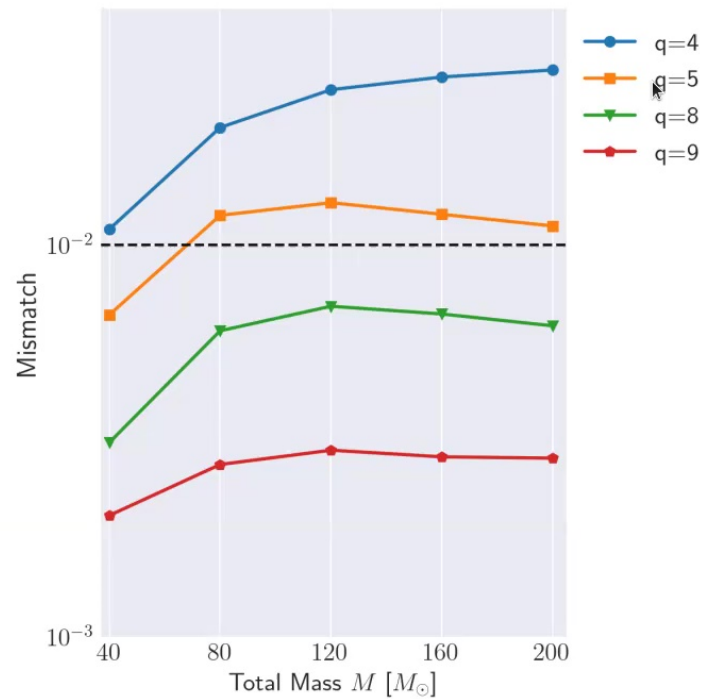
Tousif Islam | UMass Dartmouth

## Small Mass Ratio Regime : Comparison to NRHybSur3dq8 / Frequency Domain Mismatch

Mismatches are computed  
assuming advanced LIGO  
detector

Mismatches decrease as  
mass ratio increases

For  $q \geq 5$ , mismatches are  
always below  $\sim 0.01$

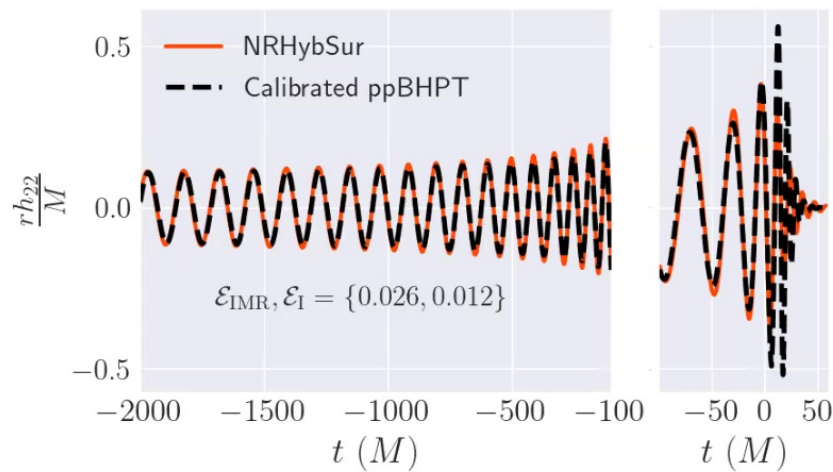


Tousif Islam | UMass Dartmouth

## Small Mass Ratio Regime :

How small can we go in mass ratio and still get a good match?

17



We have been able to obtain reasonable scaling until  $q=1.2$

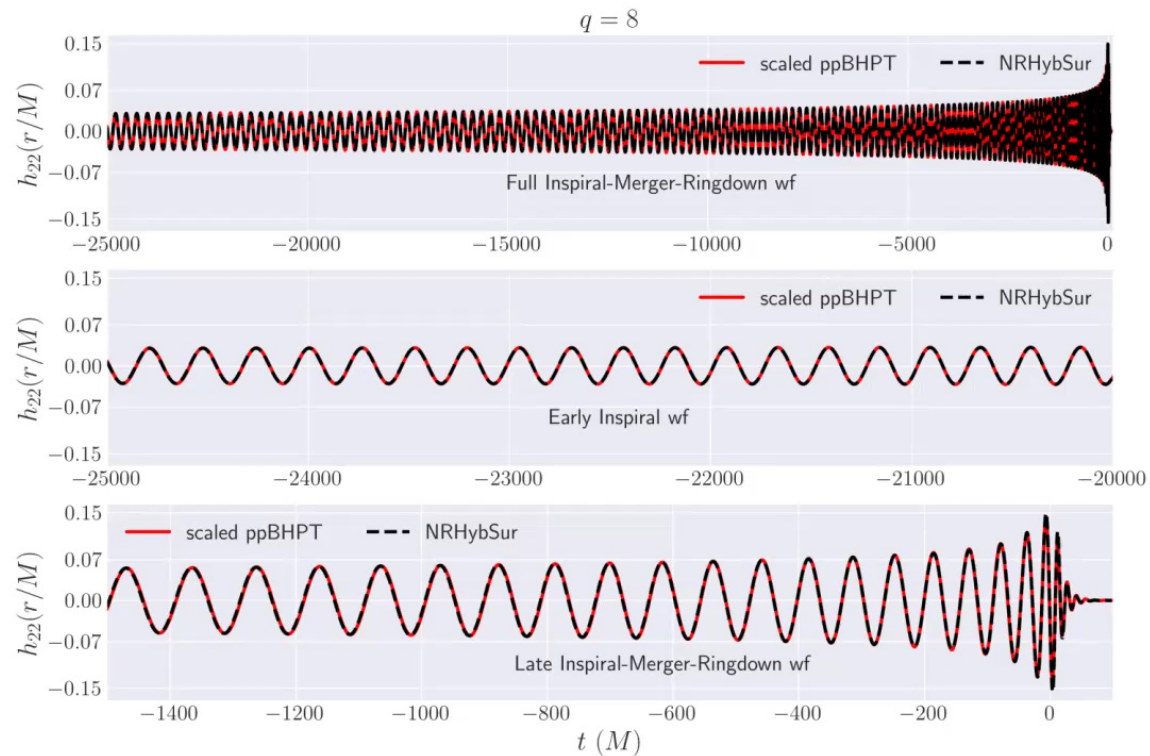
Surrogate provides an alternative way to generate wfs because ppBHPT code breaks there

Higher modes errors are not as good

Tousif Islam | UMass Dartmouth

## Small Mass Ratio Regime : [Validation] Testing Scaling over a longer time window

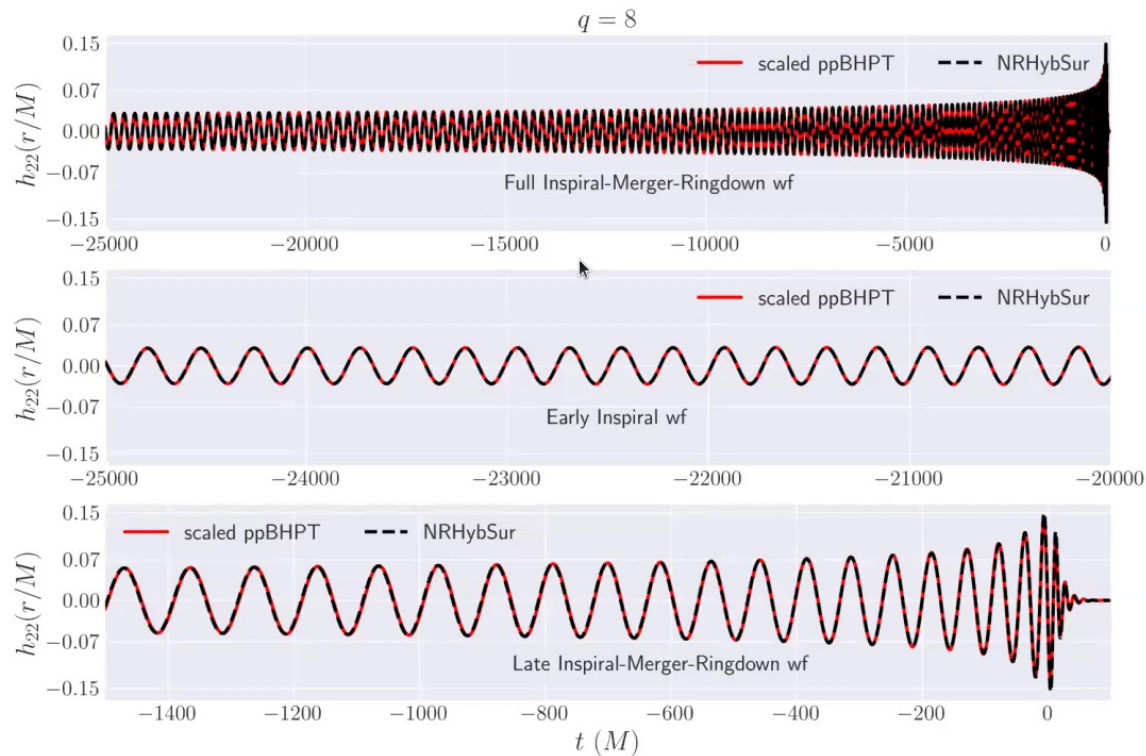
18



Tousif Islam | UMass Dartmouth

# Small Mass Ratio Regime : [Validation] Testing Scaling over a longer time window

18

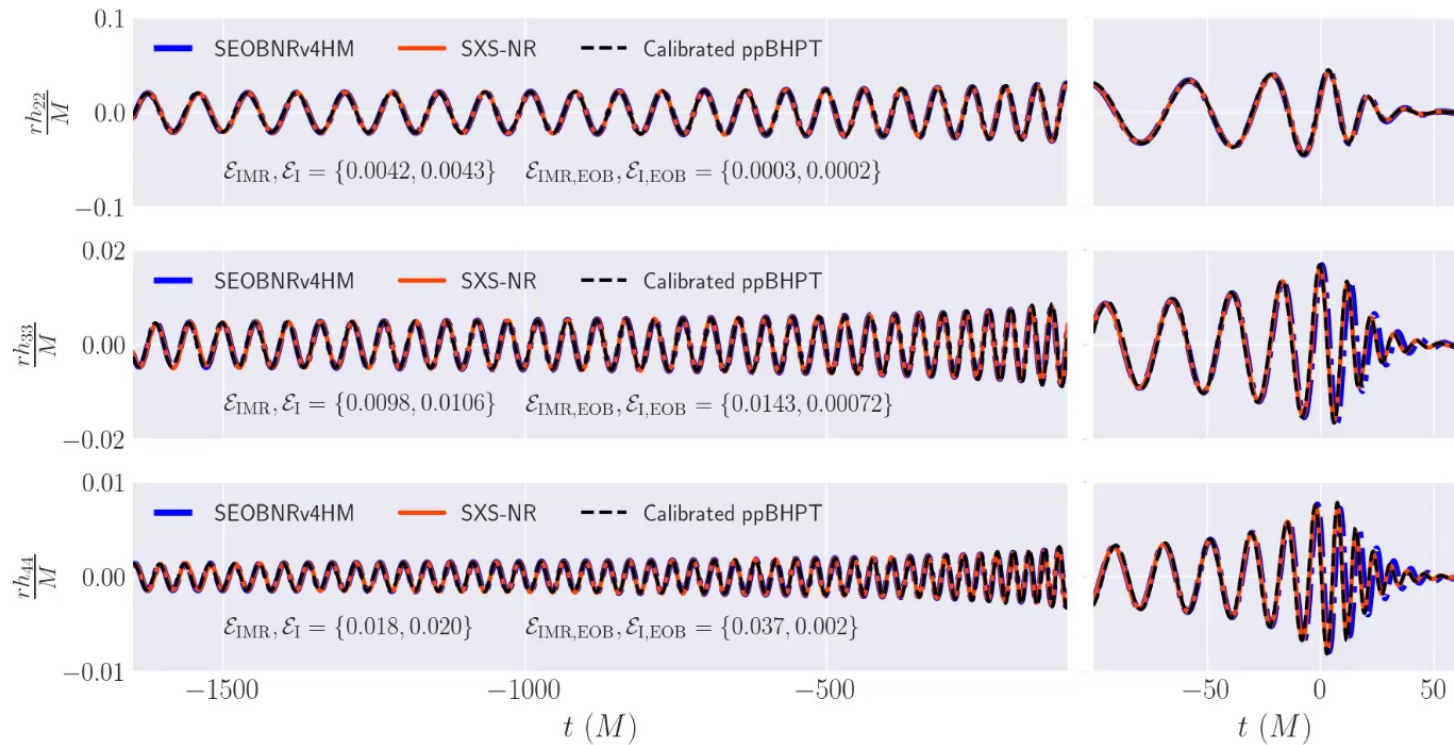


Tousif Islam | UMass Dartmouth

# Testing Scaling in Intermediate Mass Ratio Regime : [Validation] Comparison to SXS NR at q=30

19

NR Simulation : **Matthew Giesler, Mark Scheel *et al***





Tousif Islam | UMass Dartmouth



## Explaining Alpha-Beta Scaling: Comparison to Higher Order Self-force Calculation


20

$$\mathbf{h}_{\text{NR}} := h_{\text{S},\alpha^\ell,\beta}^{\ell,m}(t;q) = \alpha^\ell h_{\text{S}}^{\ell,m}(t\beta;q)$$

rescaled ppBHPT  raw ppBHPT 

### Second Order Self-force Calculation

$$\mathbf{h}_{\text{NR}} := \mathbf{h}_{\text{OPA}} + \mathbf{h}_{\text{1PA}}$$

 raw ppBHPT

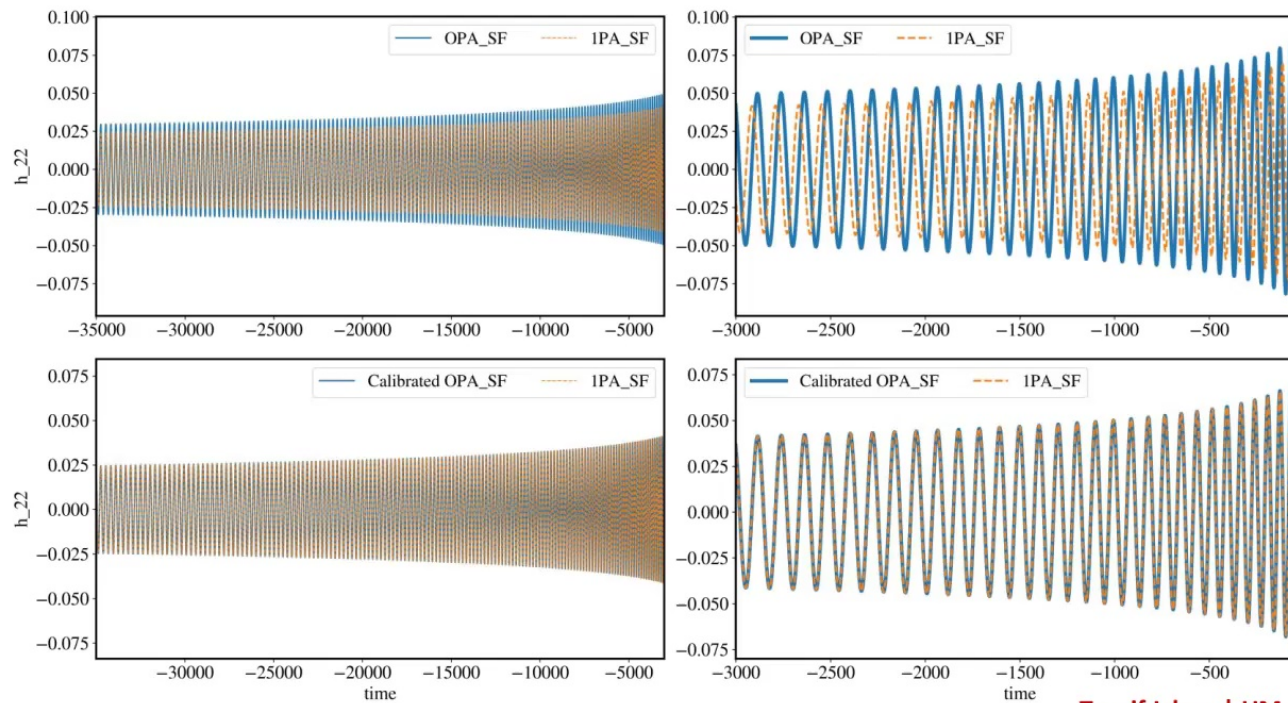
[ Wardell, Pound, Warburton et al, 2021 ]

Tousif Islam | UMass Dartmouth

## Explaining Alpha-Beta Scaling: Comparison to Higher Order Self-force Calculation

21

1PA self-force waveform can be obtained using OPA self-force waveform using an alpha-beta rescaling !!




Tousif Islam | UMass Dartmouth



## Explaining Alpha-Beta Scaling: Comparison to Higher Order Self-force Calculation


20

$$\mathbf{h}_{\text{NR}} := h_{\text{S},\alpha^\ell,\beta}^{\ell,m}(t;q) = \alpha^\ell h_{\text{S}}^{\ell,m}(t\beta;q)$$

rescaled ppBHPT  raw ppBHPT

### Second Order Self-force Calculation

$$\mathbf{h}_{\text{NR}} := \mathbf{h}_{\text{OPA}} + \mathbf{h}_{\text{1PA}}$$

 raw ppBHPT

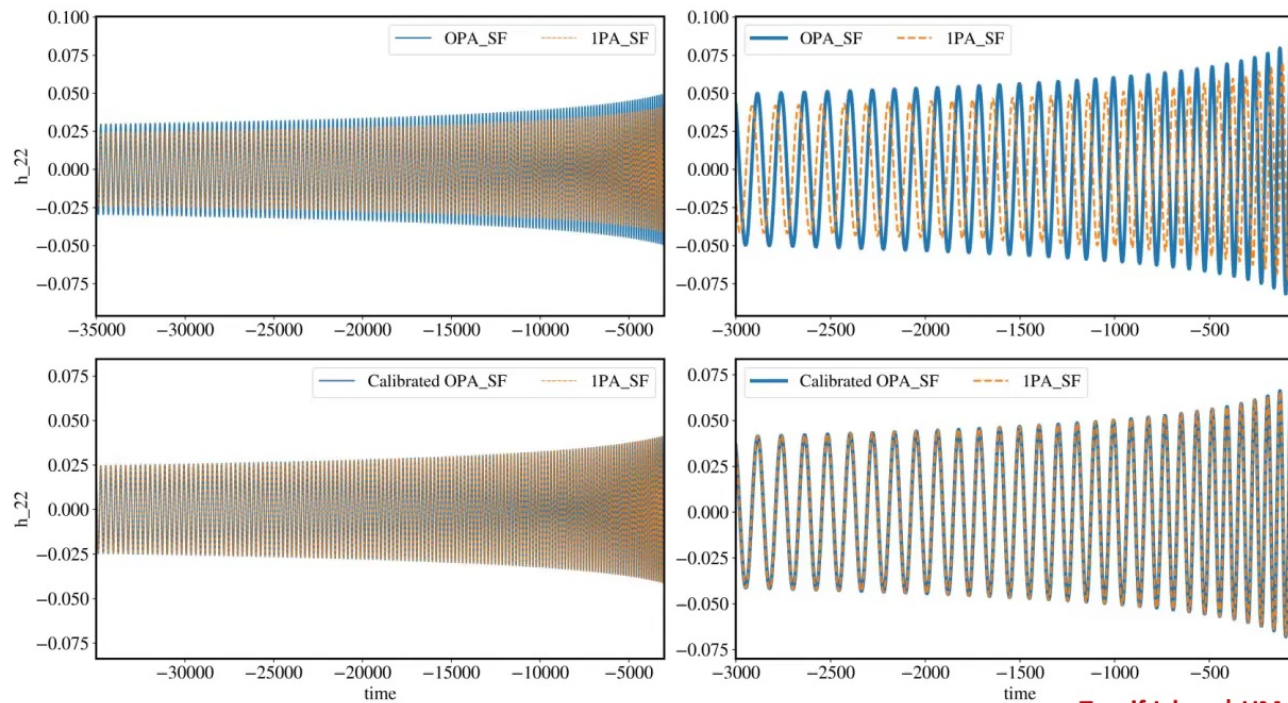
[ Wardell, Pound, Warburton et al, 2021 ]

Tousif Islam | UMass Dartmouth

## Explaining Alpha-Beta Scaling: Comparison to Higher Order Self-force Calculation

21

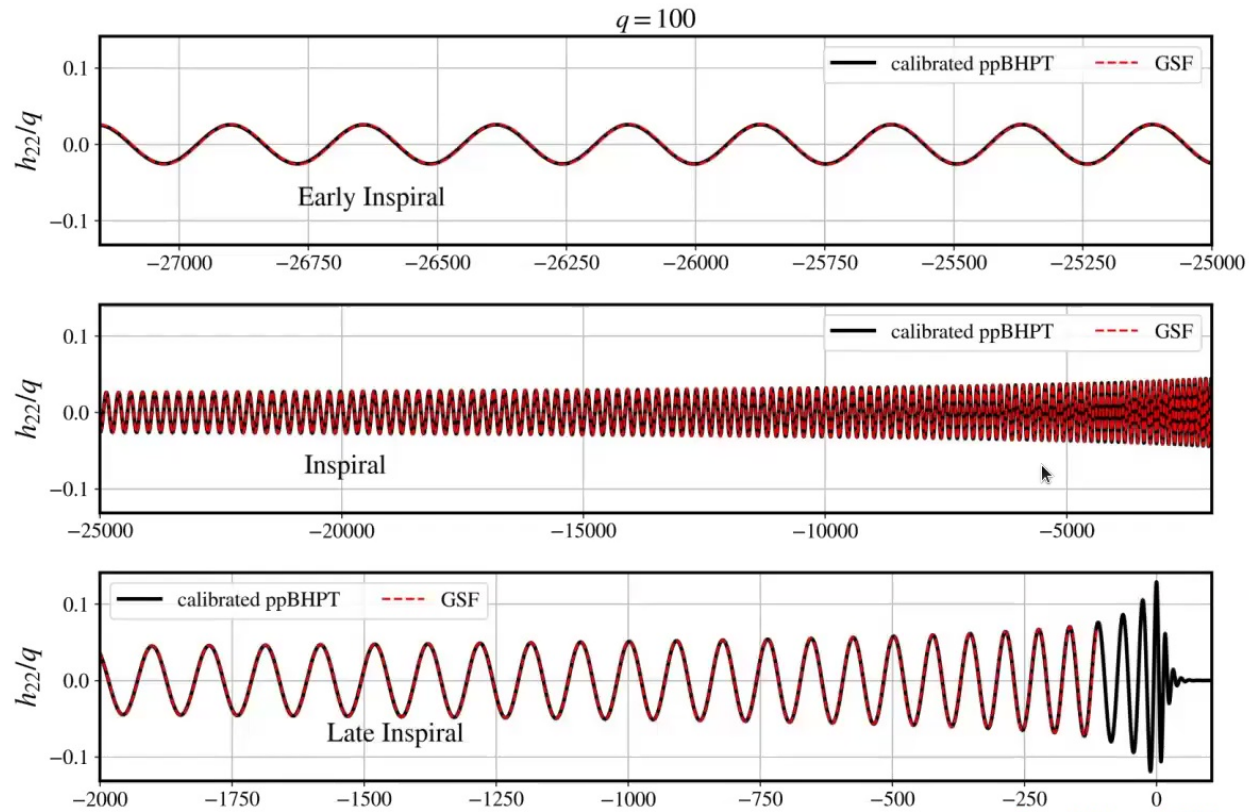
1PA self-force waveform can be obtained using OPA self-force waveform using an alpha-beta rescaling !!



Tousif Islam | UMass Dartmouth

# Explaining Alpha-Beta Scaling: Comparison to Higher Order Self-force Calculation

22



Tousif Islam | UMass Dartmouth

## What's Next in BHPTNRSur?

23

### Aligned spinning Model

[ $3 \leq q \leq 10000$ ;  $-0.6 \leq a \leq 0.6$ ]

Katie Rink, Kevin González-Quesada, Scott Field, *Tousif Islam*, Gaurav Khanna, Vijay Varma

### Eccentric Model

[ $3 \leq q \leq 100$ ;  $0.0 \leq ecc \leq 0.2$ ]

*Tousif Islam*, Scott Field, Gaurav Khanna, Niels Warburton

### Precessing Spin Model

[ $3 \leq q \leq 10000$ ; slightly misaligned system]

Ritesh Bacchar, *Tousif Islam*, Scott Field, Gaurav Khanna

```
git clone https://github.com/BlackHolePerturbationToolkit/BHPTNRSurrogate.git
```

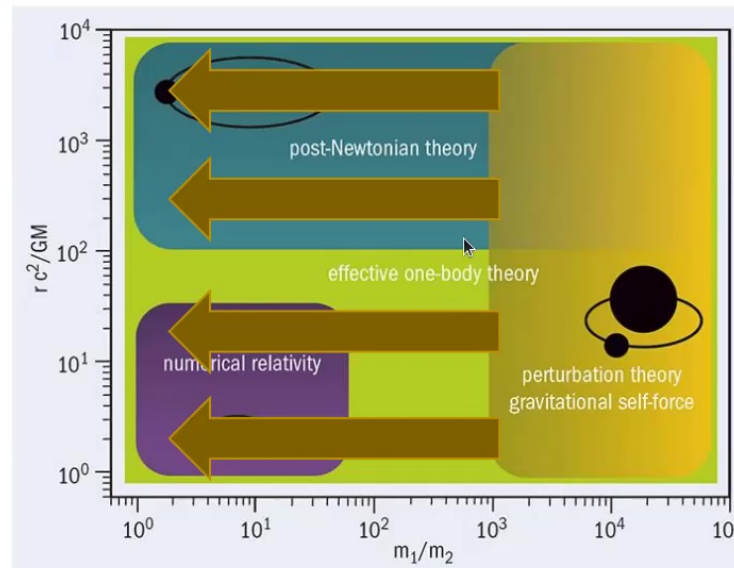
```
pip install gwsurrogate
```

Tousif Islam | UMass Dartmouth

## Summary

24

ppBHPT waveform model from comparable to extreme mass ratios **including higher order modes** ( $1.2 \leq q \leq 10,000$ )



Thank You

Tousif Islam | UMass Dartmouth