Title: Catalysed Vacuum Decay

Speakers: Michael Nee

Series: Particle Physics

Date: December 16, 2022 - 11:00 AM

URL: https://pirsa.org/22120048

Abstract: Phase transitions in everyday systems are often catalysed by the presence of impurities, but in cosmology we typically assume the initial state is a homogeneous vacuum. In this talk I will discuss how topological defects can seed first order phase transitions in the early universe, causing them to proceed much more rapidly than in the usual case. The field profiles describing the decay do not have the typically assumed O(3)/O(4) symmetry, requiring an extension of the usual decay rate calculation. To numerically determine the saddle point solutions which describe the decay we use a new algorithm based on the mountain pass theorem. I will present results showing the significance of this effect for catalysis by magnetic monopoles in a simplified model, then discuss the same effect for domain walls catalysing the electroweak phase transition. The presence of domain walls can significantly modify the predictions for gravitational wave signal which may be observed with LISA.

Zoom link: https://pitp.zoom.us/j/96182819088?pwd=cXhnVjFlT0tkc1VsRld0Yk43bFROUT09

Catalysed Vacuum Decay

Michael Nee

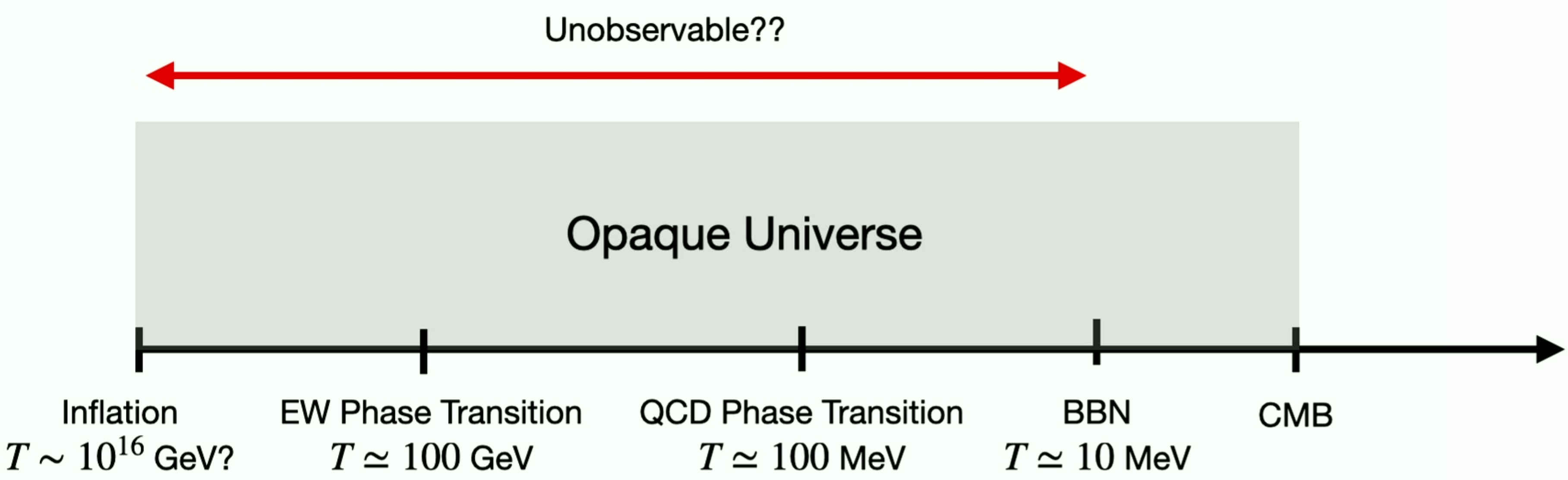
with Prateek Agrawal, Simone Blasi
& Alberto Mariotti

UNIVERSITY OF OXFORD

Outline

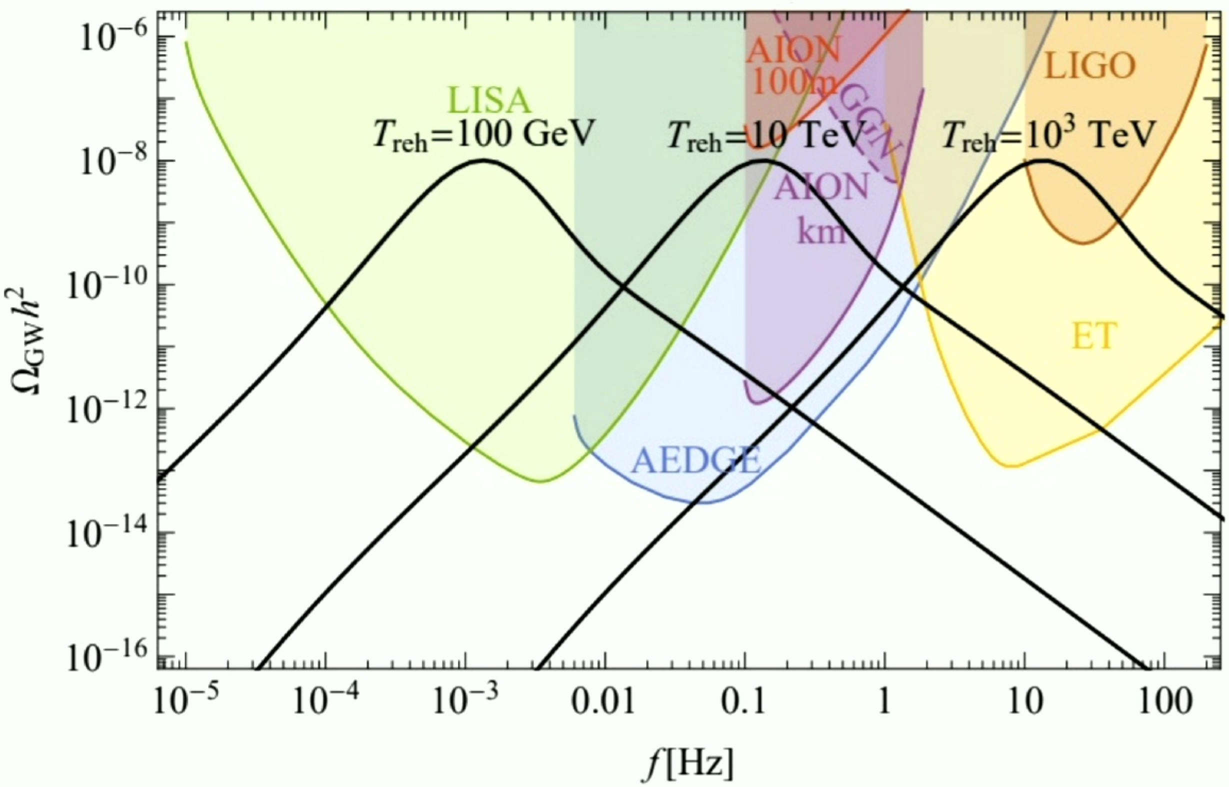
- Phase transitions and Gravitational waves
- Impurity-driven phase transitions
- Monopoles as impurities
  - Mountain pass theorem and algorithm
- Domain walls and the electroweak phase transition
  - Gravitational wave signals

Pre-BBN cosmology



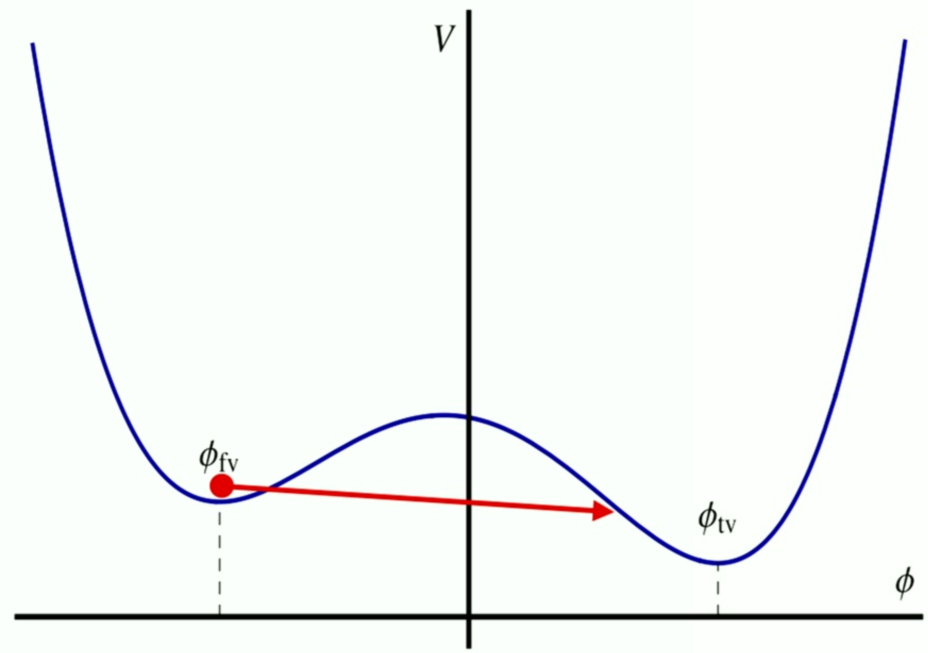
Gravitational Waves

Figure from: Badurina et. al.: hep-ph/1911.11755



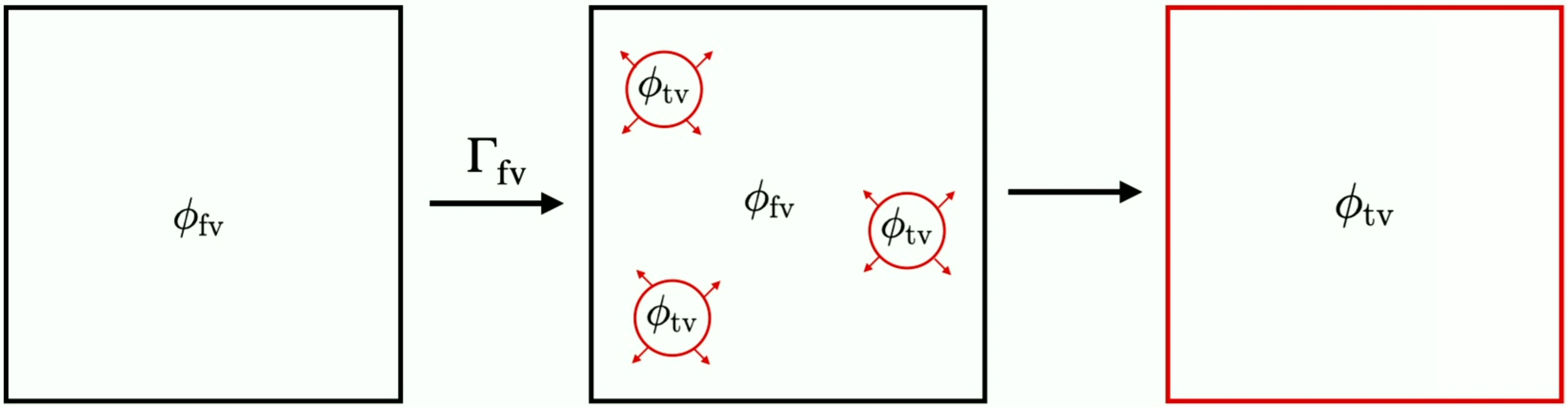
Cosmological Phase transitions

- Start in metastable state (false vacuum)
- Tunnel to ground state (true vacuum) by nucleating bubbles



Cosmological Phase transitions

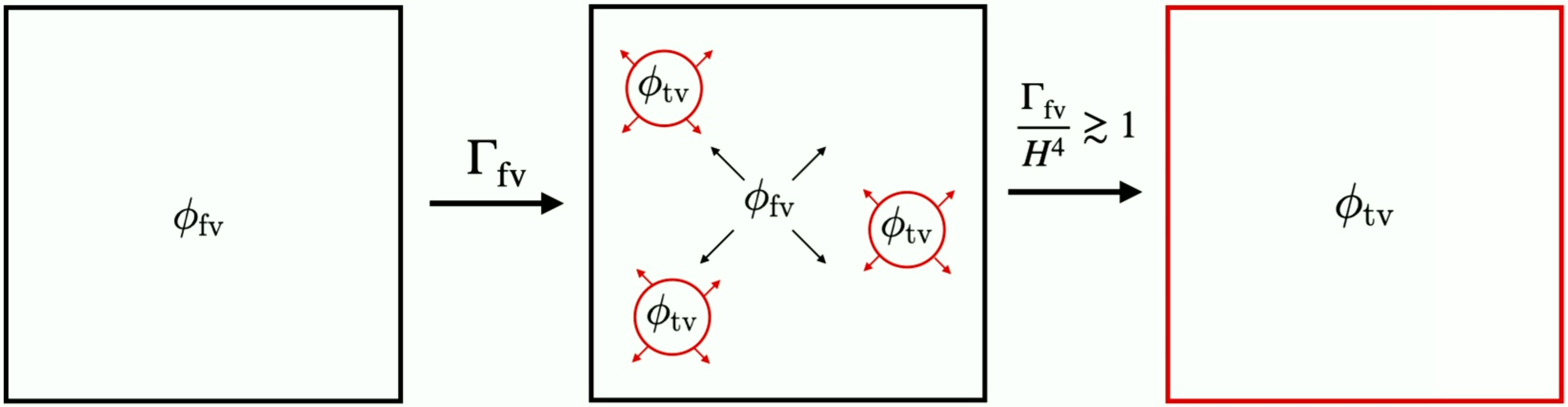
Coleman: *Phys. Rev. D* **15**, 2929, 1977



$$\Gamma_{\rm fv} = \frac{1}{\rm Vol.}\frac{dN_{\rm bubbles}}{dt} \sim \phi_{\rm fv}^4 e^{-B_{\rm fv}}$$

Cosmological Phase transitions

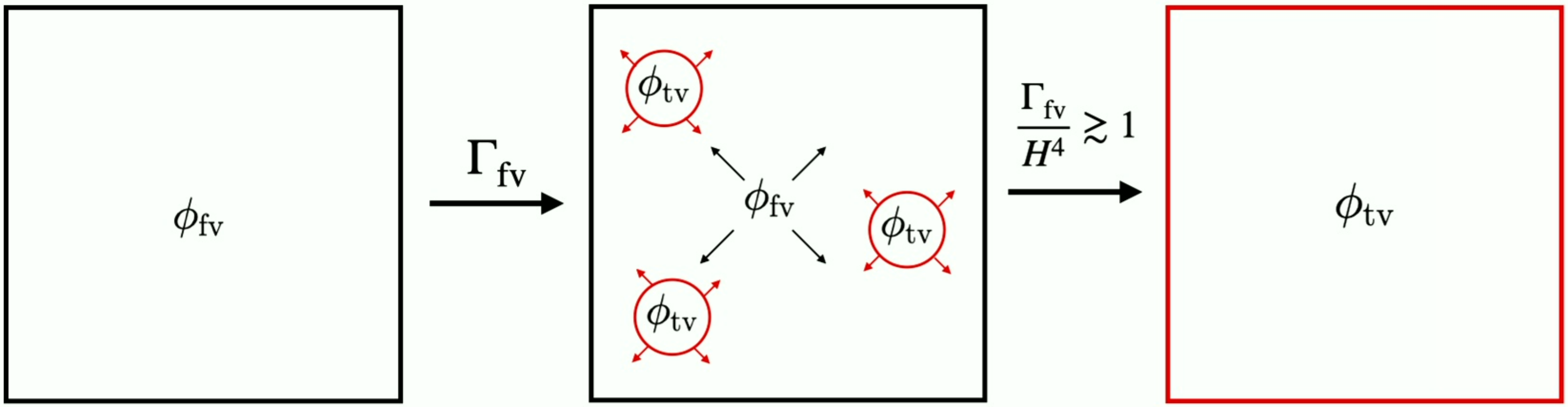
Guth & Weinberg: *Nucl. Phys. B.* B212, 1983



$$\Gamma_{\rm fv} = \frac{1}{\rm Vol.} \frac{dN_{\rm bubbles}}{dt} \sim \phi_{\rm fv}^4 e^{-B_{\rm fv}} \qquad H^2 = \frac{8\pi G_N}{3} \left[\frac{\pi^2 g_*}{30} T^4 + V\left(\phi_{\rm fv}\right) \right]$$

Cosmological Phase transitions

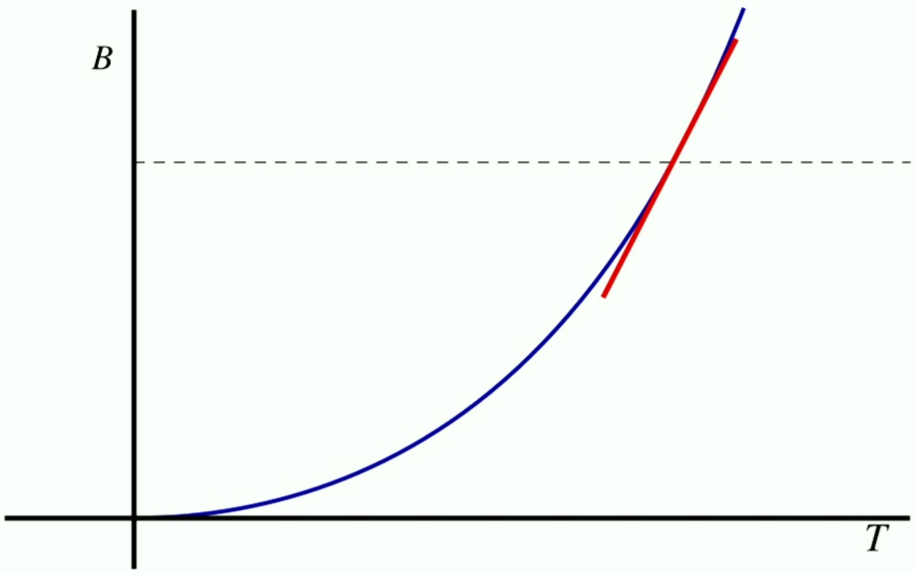
Guth & Weinberg: *Nucl. Phys. B.* B212, 1983

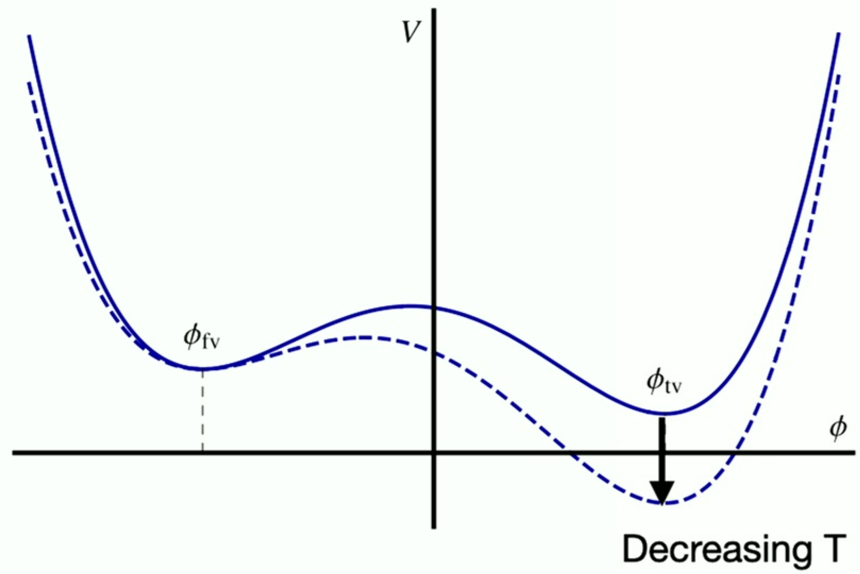


$$\frac{\Gamma_{\rm fv}}{H^4} \gtrsim 1 \quad \Rightarrow \quad B_{\rm fv} \lesssim 140$$ (for EW scale PT)

Gravitational Waves from Phase Transitions

Kowosky et. al. Phys. Rev. D 45 (1992), Phys. Rev. Lett. 69 (1992), Phys. Rev. D 47 (1993), Phys. Rev. D 49 (1994)





$$\frac{\beta}{H_*} = T_* \frac{dB_{\text{fv}}}{dT}$$

$$\alpha = \frac{\rho_{\text{vac}}}{\rho_{\text{rad}}} = \frac{V(\phi_{\text{fv}}) - V(\phi_{\text{tv}})}{\rho_{\text{rad}}}$$

Gravitational Waves from Phase Transitions

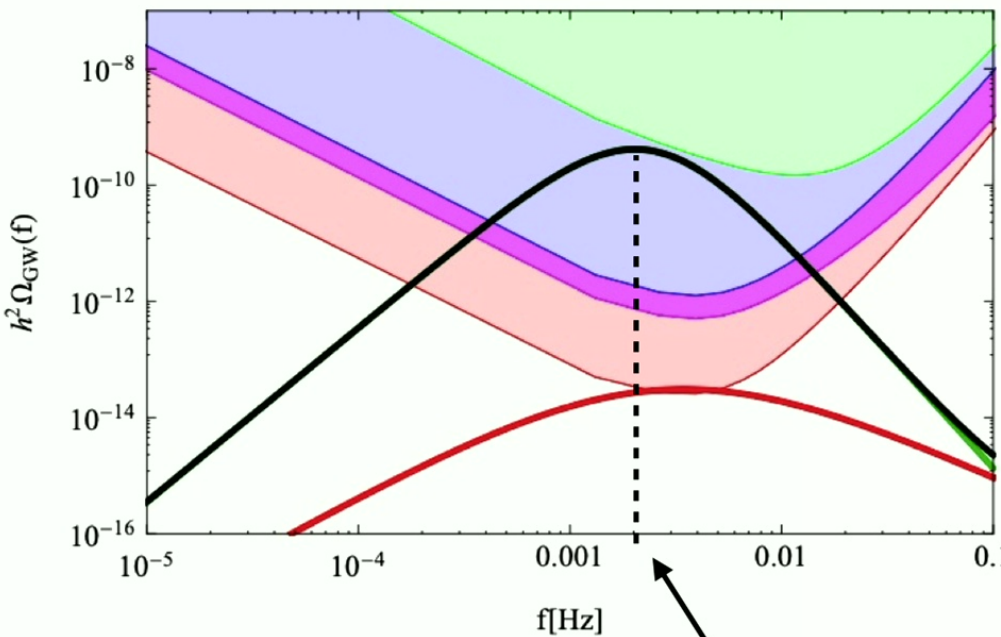
Huber & Konstandin, hep-ph/0806.1828

- GW Signal:

$$\Omega_{GW} \propto \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\alpha}{\alpha+1}\right)^2$$

- β^{-1} = average time for bubbles to collide

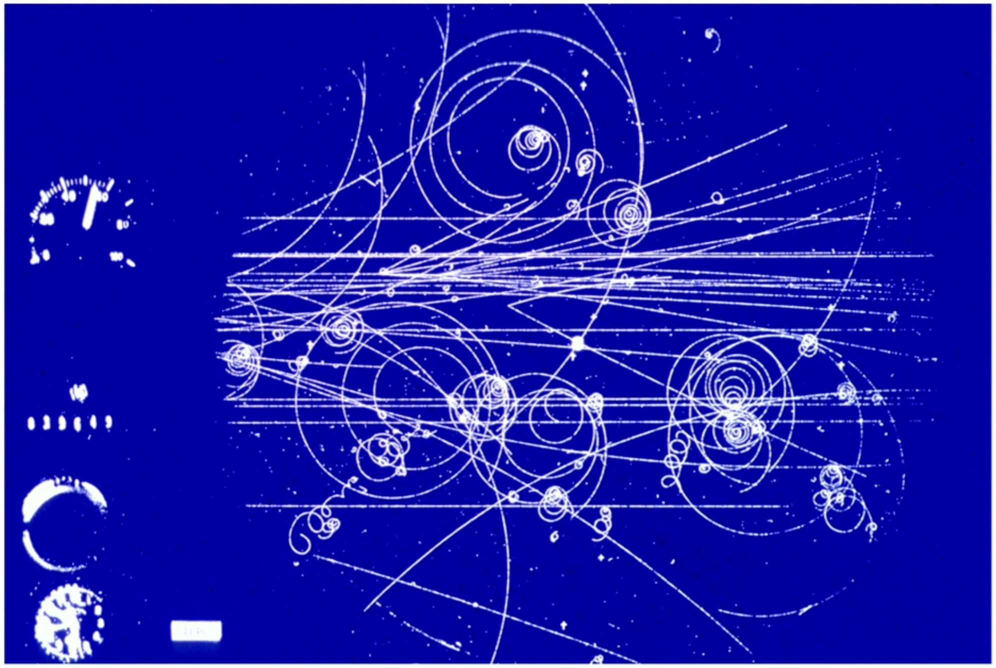
Figure from Caprinia et.al., hep-ph/1512.06239



$$f_{max} \simeq \beta \frac{T_0}{T_*}$$

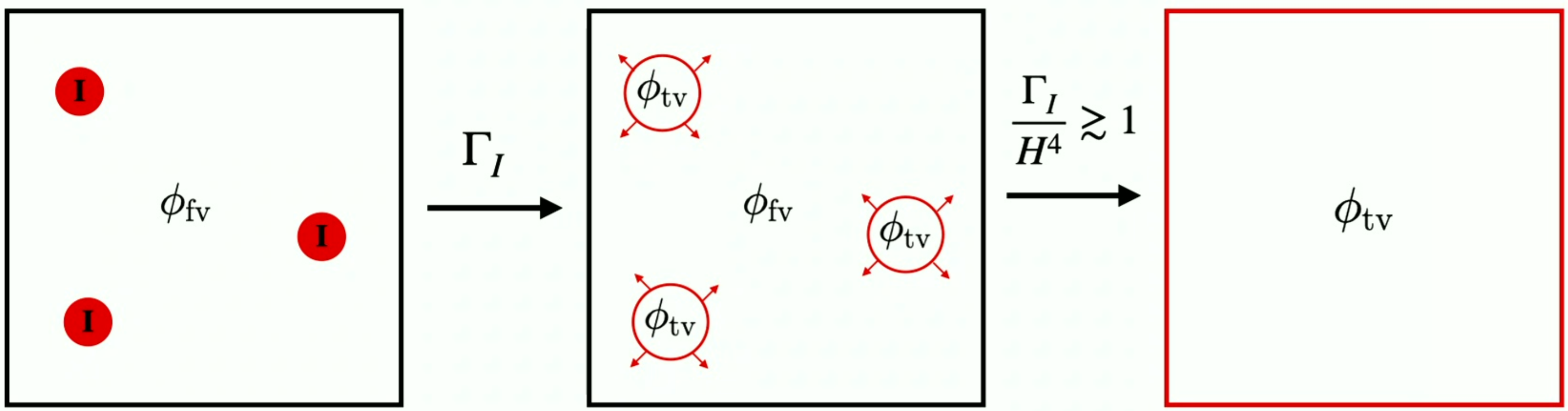
Catalysed phase transitions

- Bubble chambers
- Condensation/rain
- Water freezing
- …



https://cds.cern.ch/record/39474

Catalysed phase transitions



$$\Gamma_I \sim n_I \phi_{\rm fv} e^{-B_I} \qquad B_I < B_{\rm fv} \implies \Gamma_I \ll \Gamma_{\rm fv} \qquad \beta_2 = \frac{v_w}{r_I}$$

Monopole Tunnelling

hep-ph/2202.11102 w/ Prateek Agrawal

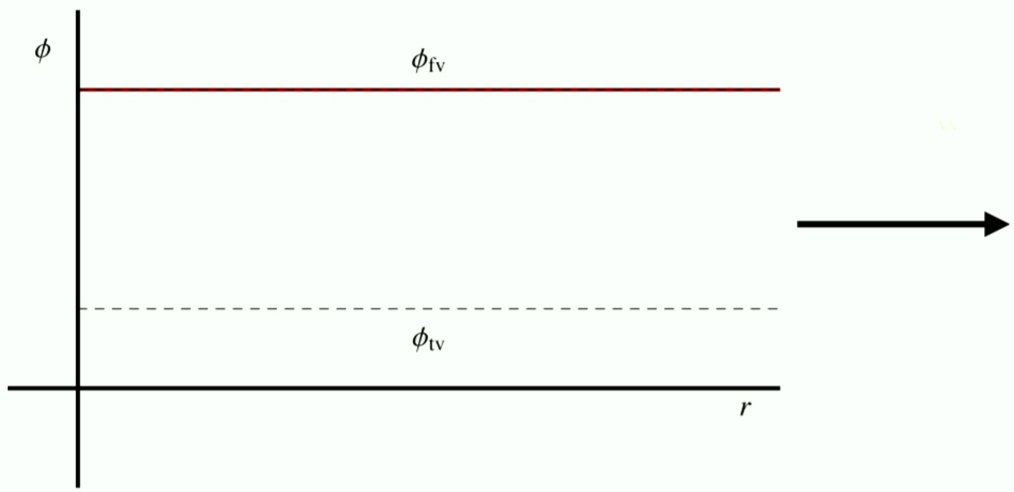
False Vacuum Tunnelling

Coleman: *Phys.Rev.D* 15 (1977), Linde: *Nucl.Phys.B* 216 (1983)

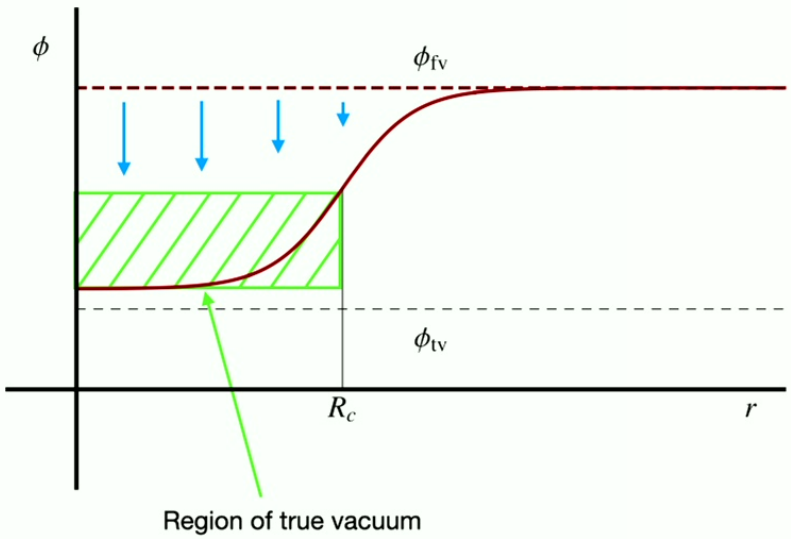
- How to calculate B_{fv} ?

$$B_{\text{fv}} = S_E\left[\phi_{\text{fvb}}\right] - S_E\left[\phi_{\text{fv}}\right]$$

$$\phi_{\text{fvb}}\left(|\tau| \to \infty\right)$$



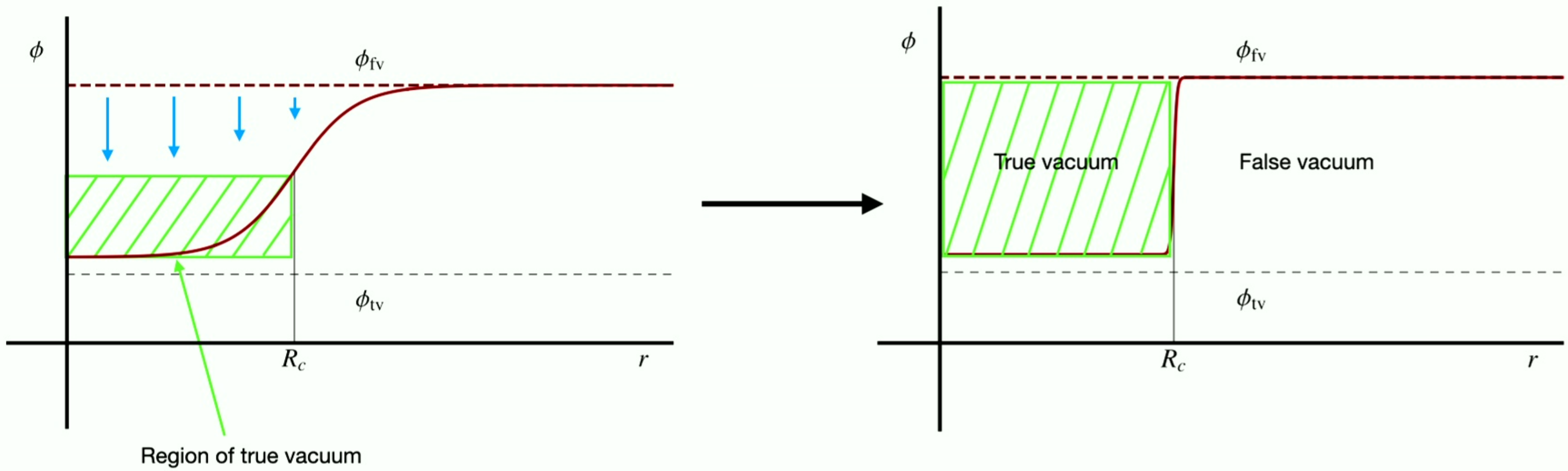
$$\phi_{\text{fvb}}(\tau = 0)$$



Thin Wall Approximation

Coleman: *Phys.Rev.D* 15 (1977)

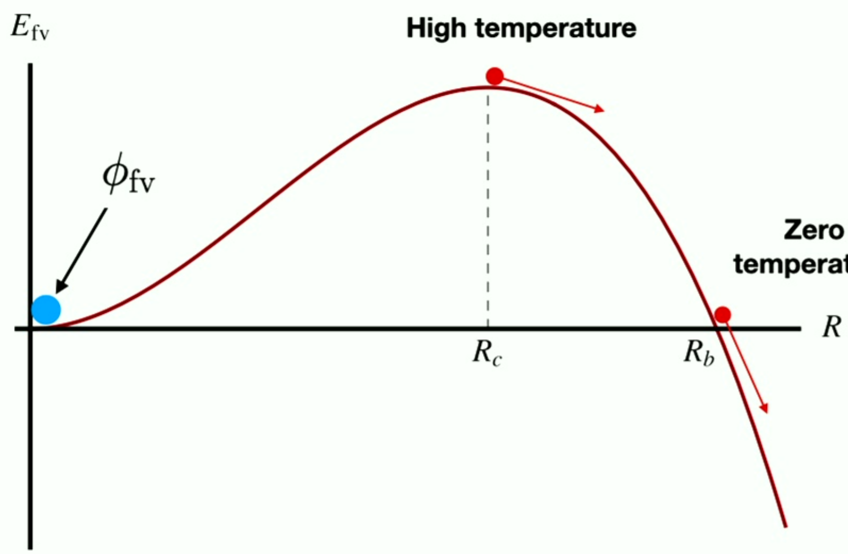
- Radius is only degree of freedom



False Vacuum Tunnelling

Coleman: *Phys.Rev.D* 15 (1977), Linde: *Nucl.Phys.B* 216 (1983)

- Solution is saddle point of action - unstable against expansion



Magnetic Monopoles

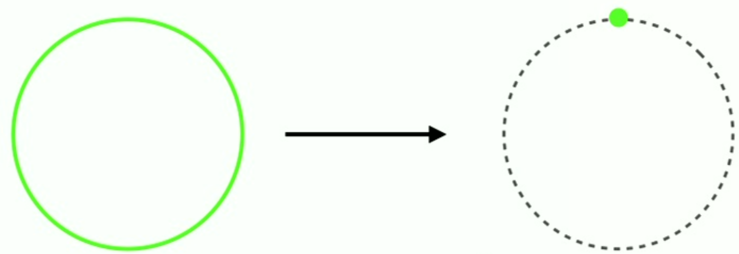
't Hooft: Nucl.Phys.B 79 (1974), Polyakov: JETP Lett. 20 (1974)

- Consider SU(2) gauge group broken to U(1) subgroup by triplet scalar

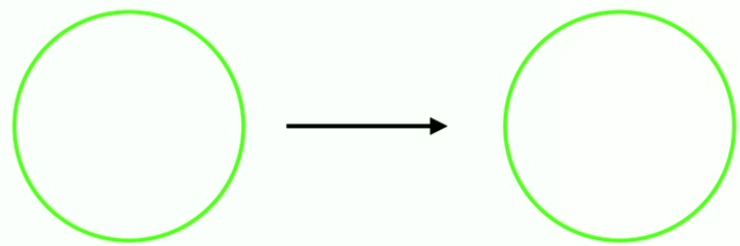
- Vacuum is a two-sphere:

$$\sum_{i=1}^{3} \phi^a \phi^a = v^2$$

- Have topologically distinct maps from spatial two-sphere to vacuum



$\phi^a_\infty(\theta, \varphi) = v\delta^{a3}$



$\phi^a_\infty(\theta, \varphi) = v\hat{r}^a$

Monopoles as Impurities

Steinhardt: Phys. Rev. D **24**, 842 (1981), *Nucl.Phys.B* 190 (1981); Kumar, Paranjape & Yajnik: hep-ph/0908.3949, hep-ph/1006.0693

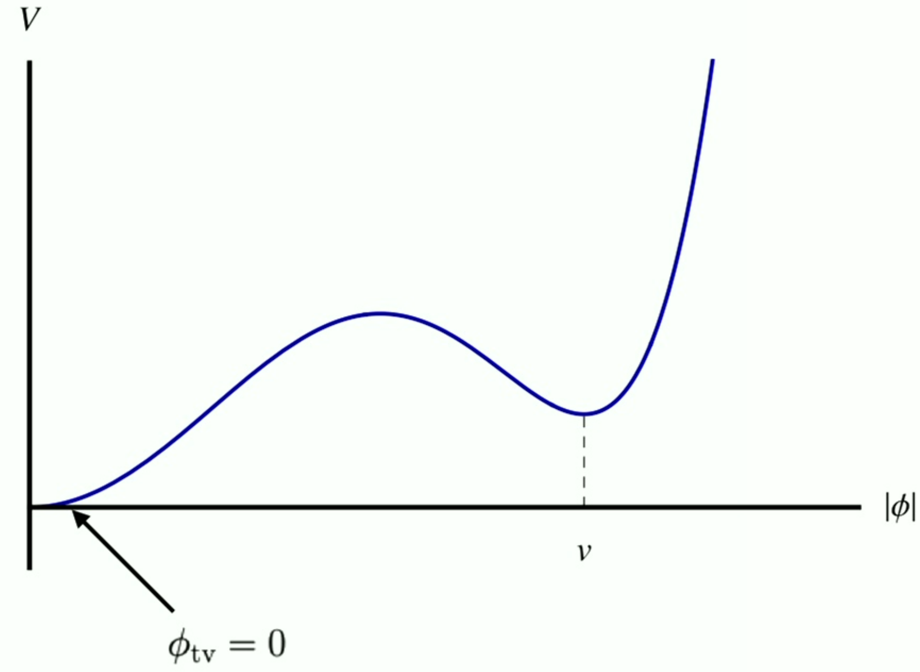
- Monopole solutions:

$$\phi^a = v\hat{r}^a h_{\mathrm{m}}(r)$$

$$A_i^a = \epsilon^{iam}\hat{r}^m \left(\frac{1 - u_{\mathrm{m}}(r)}{gr}\right)$$

- Continuity requires

$$h_m(0) = 0, u_m(0) = 1$$



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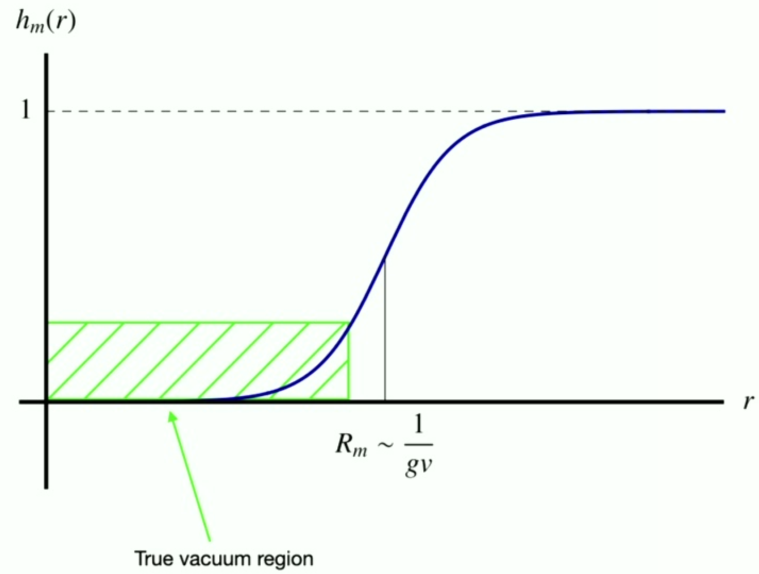

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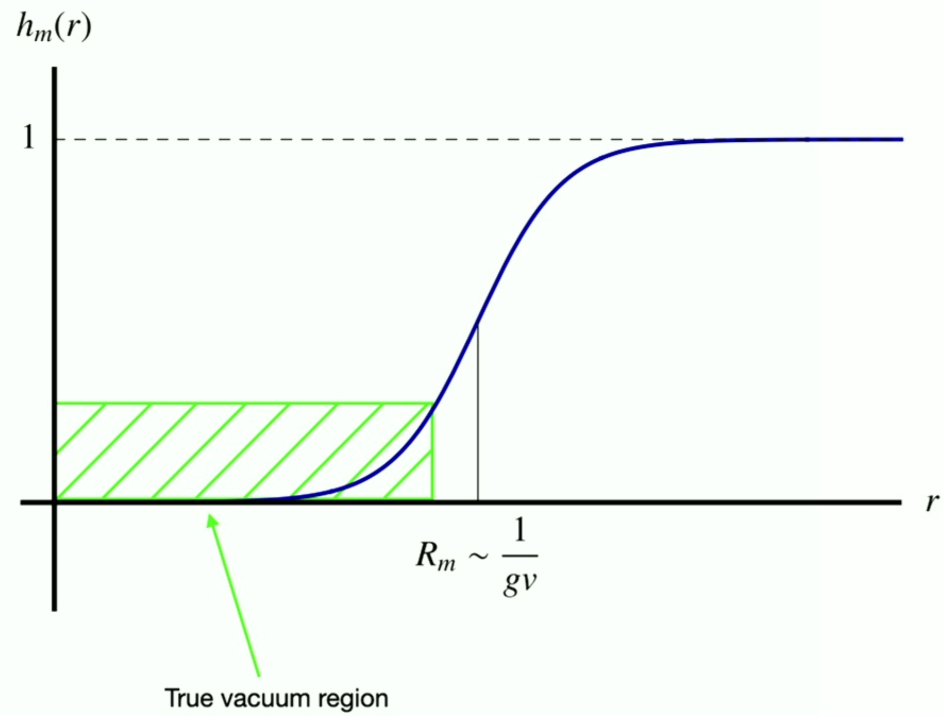
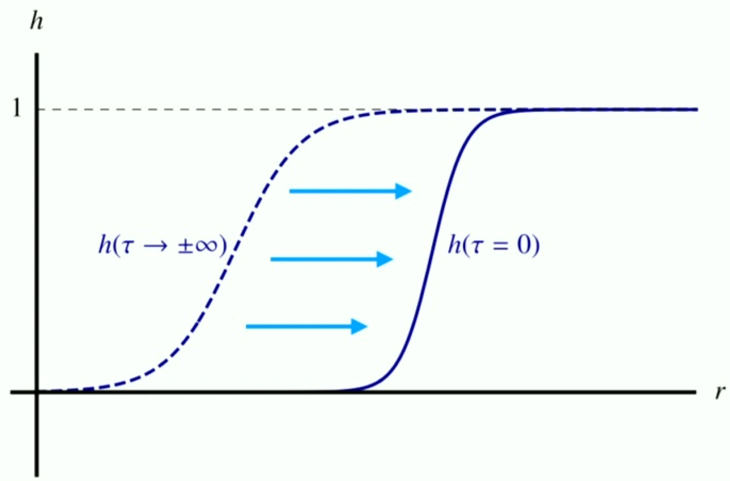


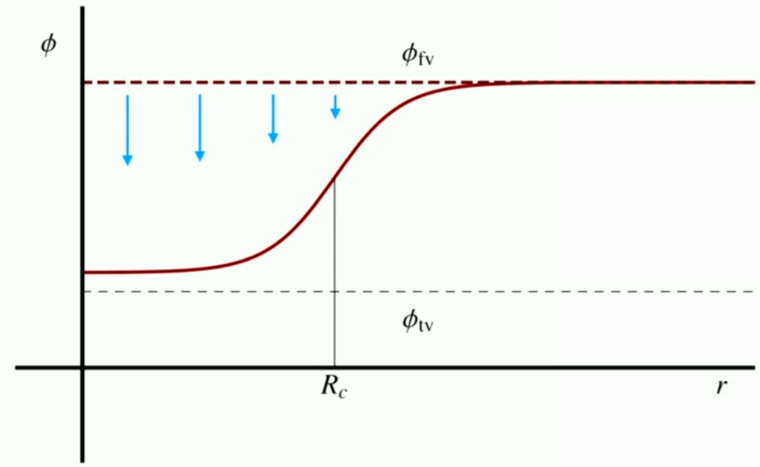
Monopole Bounce

Monopole Tunnelling

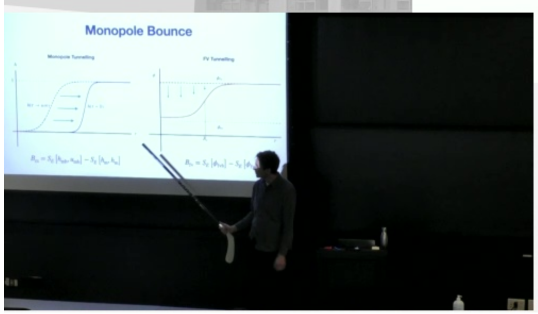


$$B_{\rm m} = S_E\left[h_{\rm mb}, u_{\rm mb}\right] - S_E\left[h_{\rm m}, h_{\rm m}\right]$$

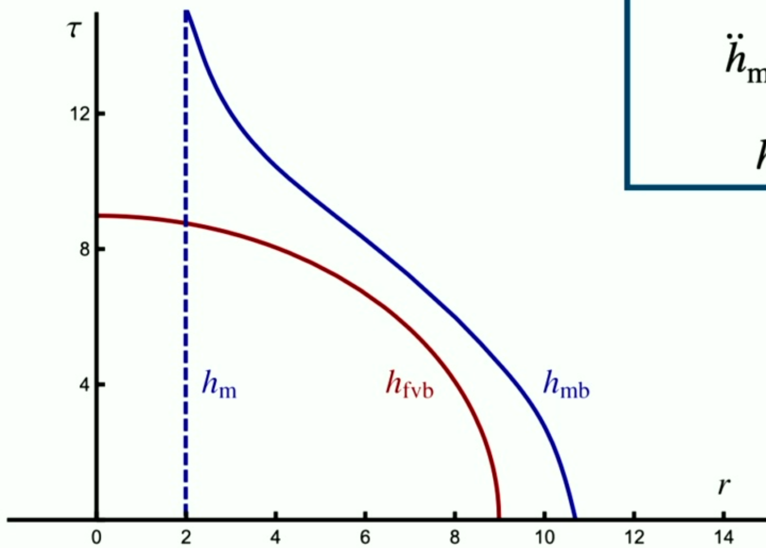
FV Tunnelling



$$B_{\rm fv} = S_E\left[\phi_{\rm fvb}\right] - S_E\left[\phi_{\rm fv}\right]$$



$O(4)$ symmetry



Monopole Tunnelling

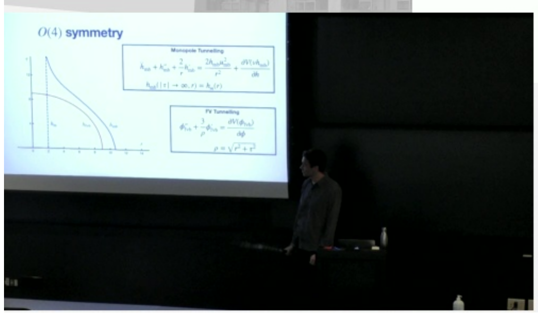
$$\ddot{h}_{\rm mb} + h''_{\rm mb} + \frac{2}{r}h'_{\rm mb} = \frac{2h_{\rm mb}u^2_{\rm mb}}{r^2} + \frac{\partial V(v h_{\rm mb})}{\partial h}$$

$$h_{\rm mb}(|\tau| \to \infty, r) = h_m(r)$$

FV Tunnelling

$$\phi''_{\rm fvb} + \frac{3}{\rho}\phi'_{\rm fvb} = \frac{\partial V(\phi_{\rm fvb})}{\partial \phi}$$

$$\rho = \sqrt{r^2 + \tau^2}$$



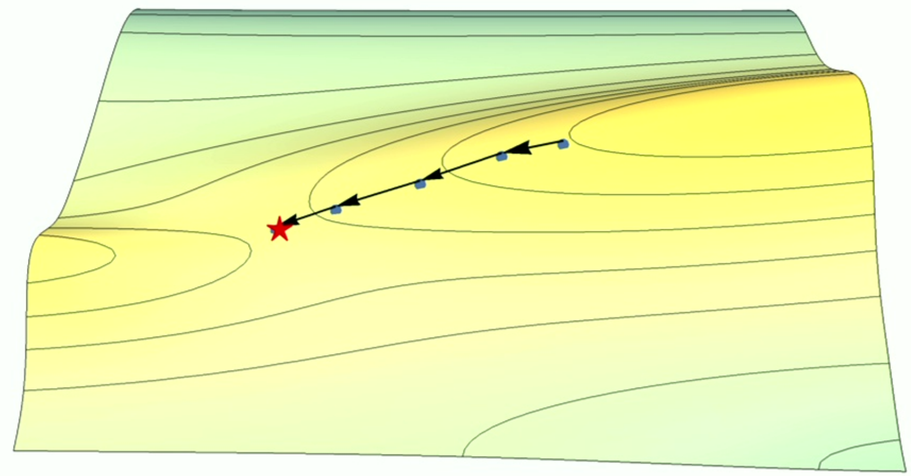
Numerical Procedure

Aim:

- Start with initial guess and improve by minimising action

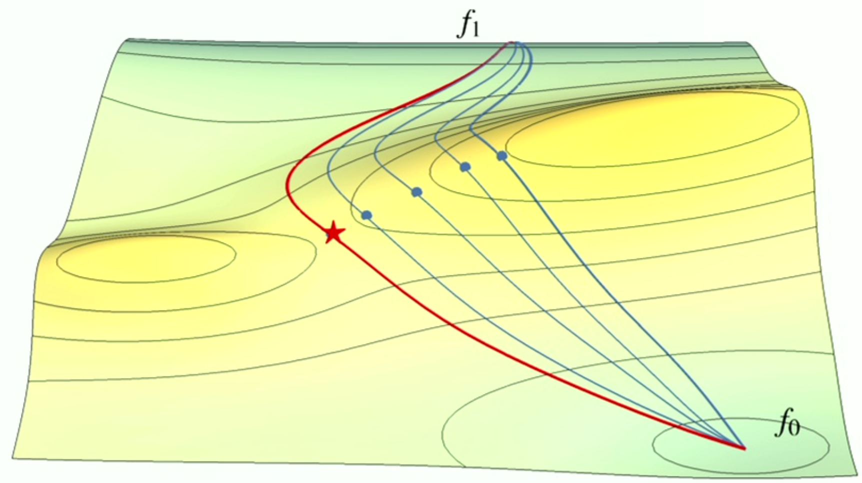
$$h^{n+1} = h^n - \beta_n \frac{\delta S_E}{\delta h^n},$$

$$u^{n+1} = u^n - \beta_n \frac{\delta S_E}{\delta u^n},$$



Mountain Pass Theorem

- $I[f]$ a functional
- f_0 a local minimum, f_1 such that $I[f_1] < I[f_0]$
- $\gamma \in$ paths from f_0 to f_1
- $\min_{\gamma} \max_{\alpha \in [0,1]} I[\gamma(\alpha)]$ is a saddle point

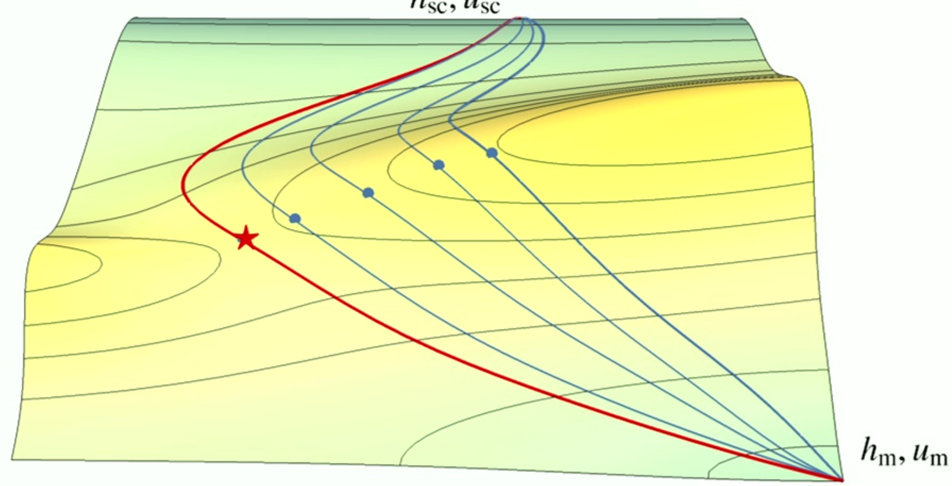


Mountain Pass Theorem

$I[h,u] = S_E[h,u] - S_E[h_\mathrm{m}, u_\mathrm{m}]$

satisfies all of the above conditions:

- $h_\mathrm{m}, u_\mathrm{m}$ is minimum
- need to find supercritical bubble where $I[h_\mathrm{sc}, u_\mathrm{sc}] < 0$
- Saddle point is the monopole bounce

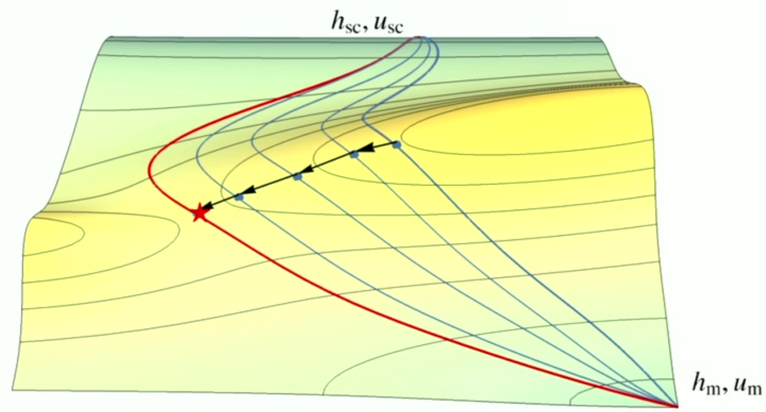


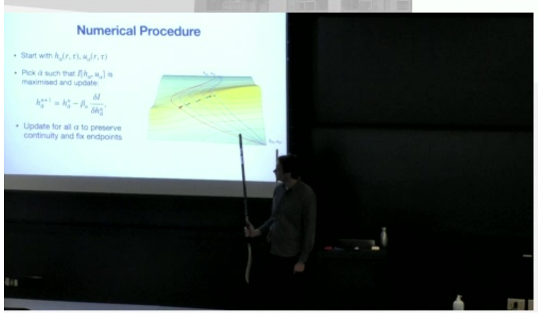
Numerical Procedure

- Start with $h_\alpha(r,\tau), u_\alpha(r,\tau)$
- Pick $\bar{\alpha}$ such that $I[h_\alpha, u_\alpha]$ is maximised and update:

$$h_{\bar{\alpha}}^{n+1} = h_{\bar{\alpha}}^{n} - \beta_n \frac{\delta I}{\delta h_{\bar{\alpha}}^{n}},$$

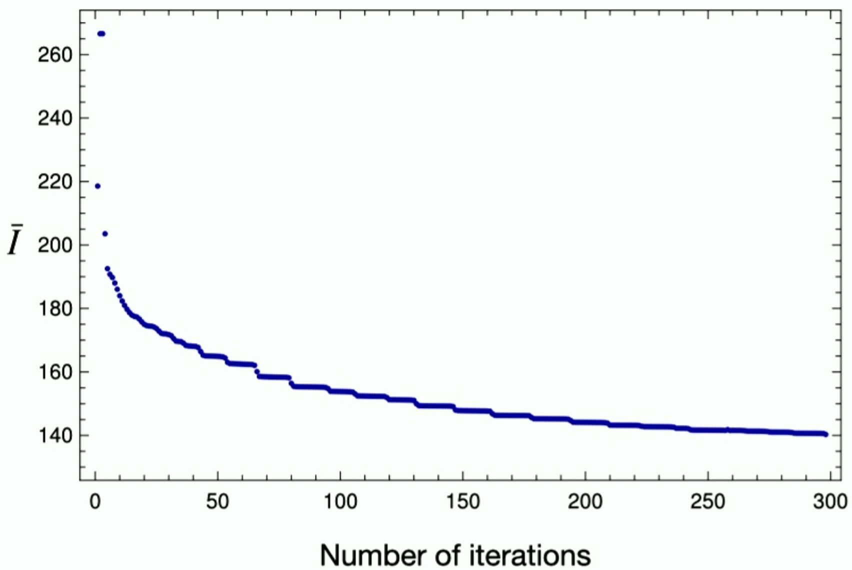
- Update for all α to preserve continuity and fix endpoints

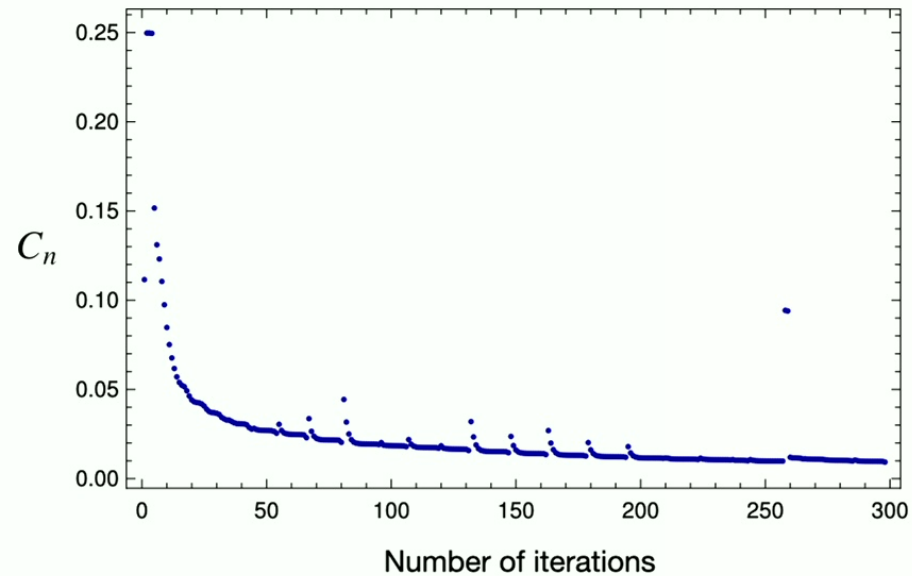




Performance

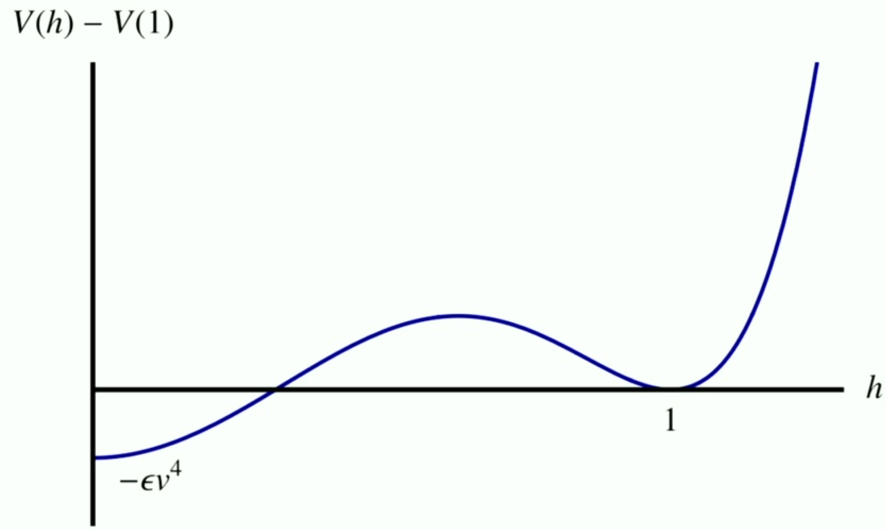
$$C_n = \frac{1}{N_r N_\tau}\left[\sum\left(\left(\frac{\delta I}{\delta h_{\bar{\alpha}}}\right)^2 + \left(\frac{\delta I}{\delta u_{\bar{\alpha}}}\right)^2\right)\right]^{1/2}$$

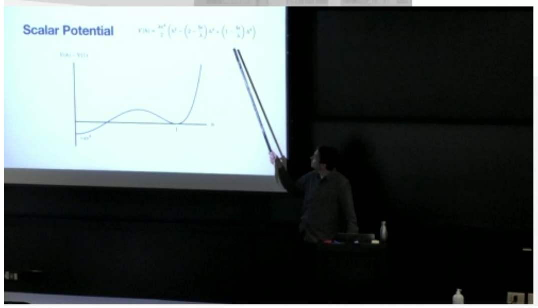




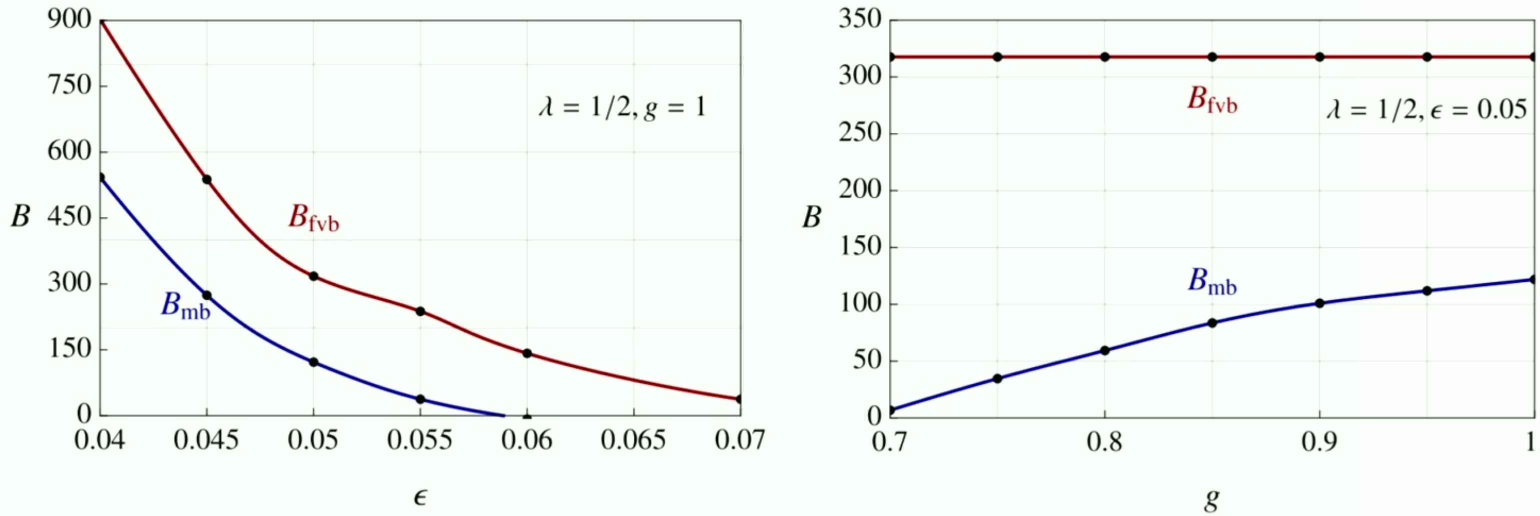
Scalar Potential

$$V(h) = \frac{\lambda v^4}{2}\left(h^2 - \left(2 - \frac{6\epsilon}{\lambda}\right)h^4 + \left(1 - \frac{4\epsilon}{\lambda}\right)h^6\right)$$

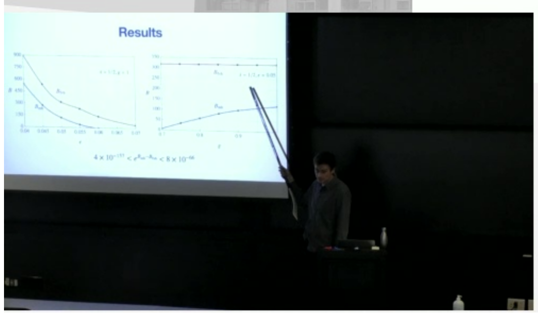




Results



$$4 \times 10^{-157} < e^{B_{\text{mb}} - B_{\text{fvb}}} < 8 \times 10^{-66}$$



Results - early universe PTs

- Condition for monopoles to dominate:

$$n_m \gtrsim v^3 e^{B_{\rm mb}-B_{\rm fv}}.$$

- Kibble mechanism produces monopoles

$$n_{m,\rm kibble}(T) \simeq H^3 \left(\frac{v}{T}\right)^3$$

- Kibble produced monopoles dominate for:

$$\frac{T}{M_P} \gtrsim 1.7 \times 10^{-23} \left(\frac{e^{B_{\rm mb}-B_{\rm fv}}}{10^{-66}}\right)^{1/3}$$

Domain Walls and Electroweak Phase Transition

with Prateek Agrawal, Simone Blasi & Alberto Mariotti

SM + Scalar Singlet

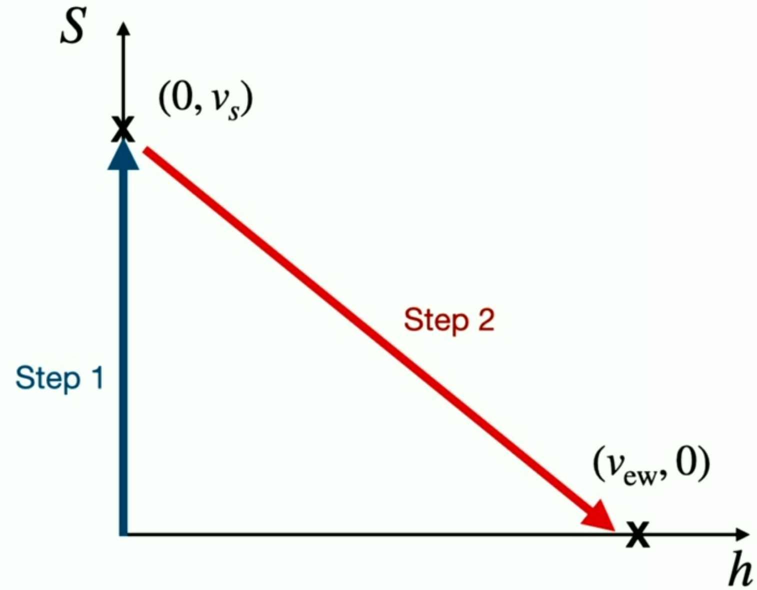
- Tree-level potential:

$$V_{\text{tree}}(h,S) = -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{\kappa}{2}h^2S^2 - \frac{m^2}{2}S^2 + \frac{\eta}{4}S^4$$

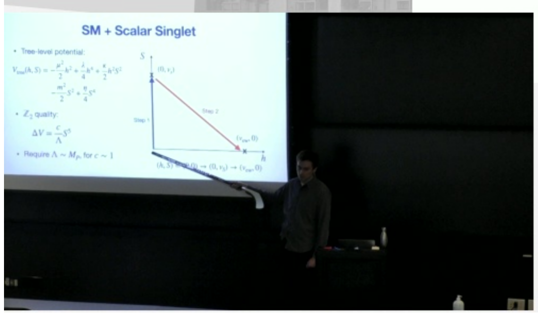
- $\mathbb{Z}_2$ quality:

$$\Delta V = \frac{c}{\Lambda}S^5$$

- Require $\Lambda \sim M_P$, for $c \sim 1$

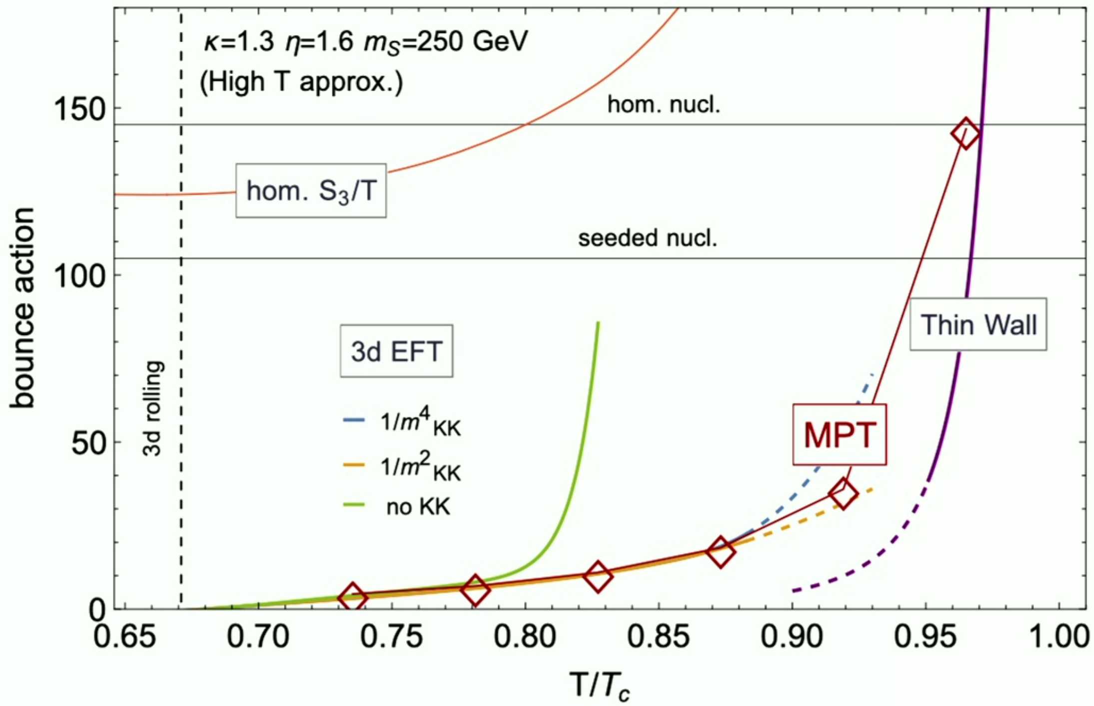


$$(h,S) = (0,0) \to (0,v_S) \to (v_{\text{ew}},0)$$



High Temperature Limit

- Can use mode expansion in z direction
  - Only tractable when keeping T^2 corrections to V
  - Breaks down near where transition completes



Comparison to EFT results of hep-ph/2203.16450

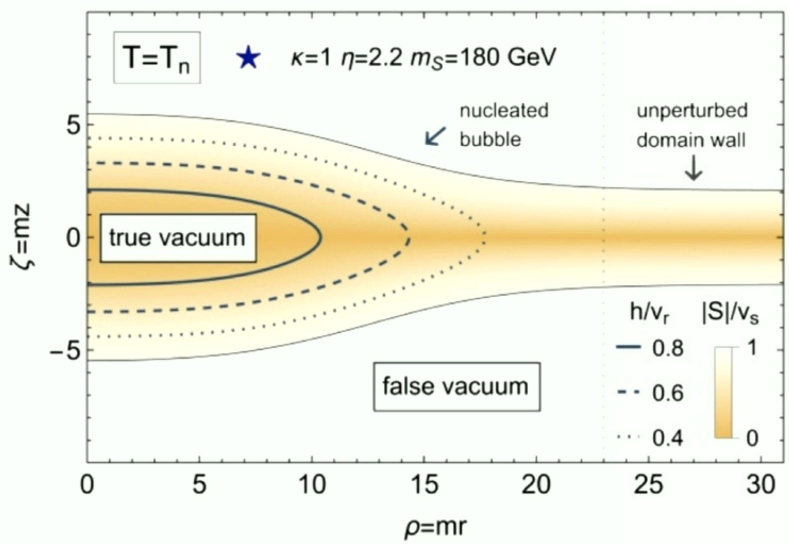
Bounce Calculation

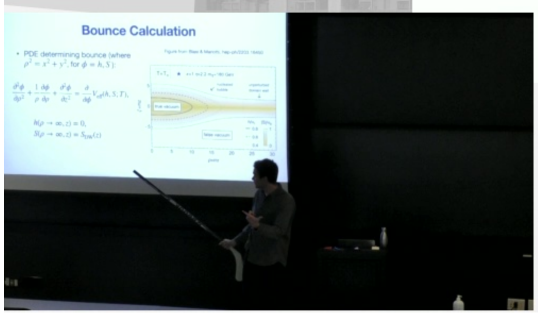
- PDE determining bounce (where $\rho^2 = x^2 + y^2$, for $\phi = h, S$):

$$\frac{\partial^2 \phi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \phi}{\partial \rho} + \frac{\partial^2 \phi}{\partial z^2} = \frac{\partial}{\partial \phi} V_{\text{eff}}(h, S; T),$$

$$h(\rho \to \infty, z) = 0,$$
$$S(\rho \to \infty, z) = S_{\text{DW}}(z)$$

Figure from Blasi & Mariotti, hep-ph/2203.16450





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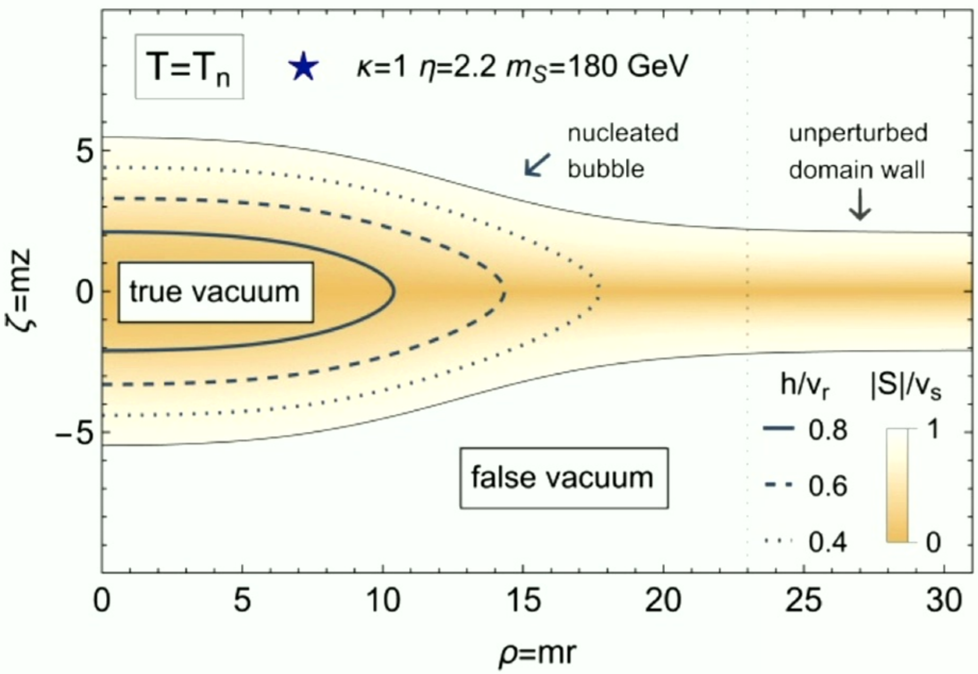

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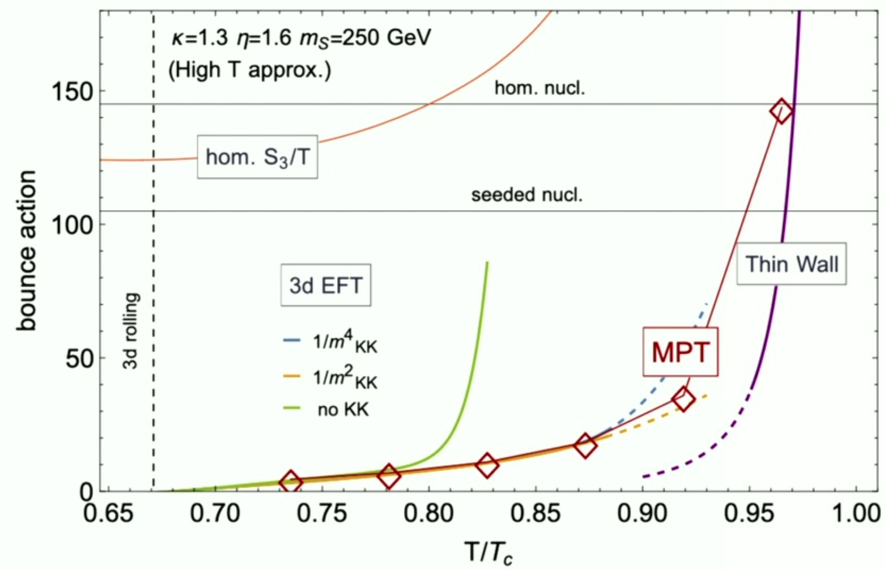
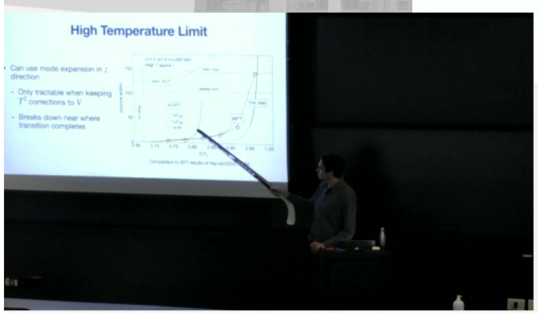
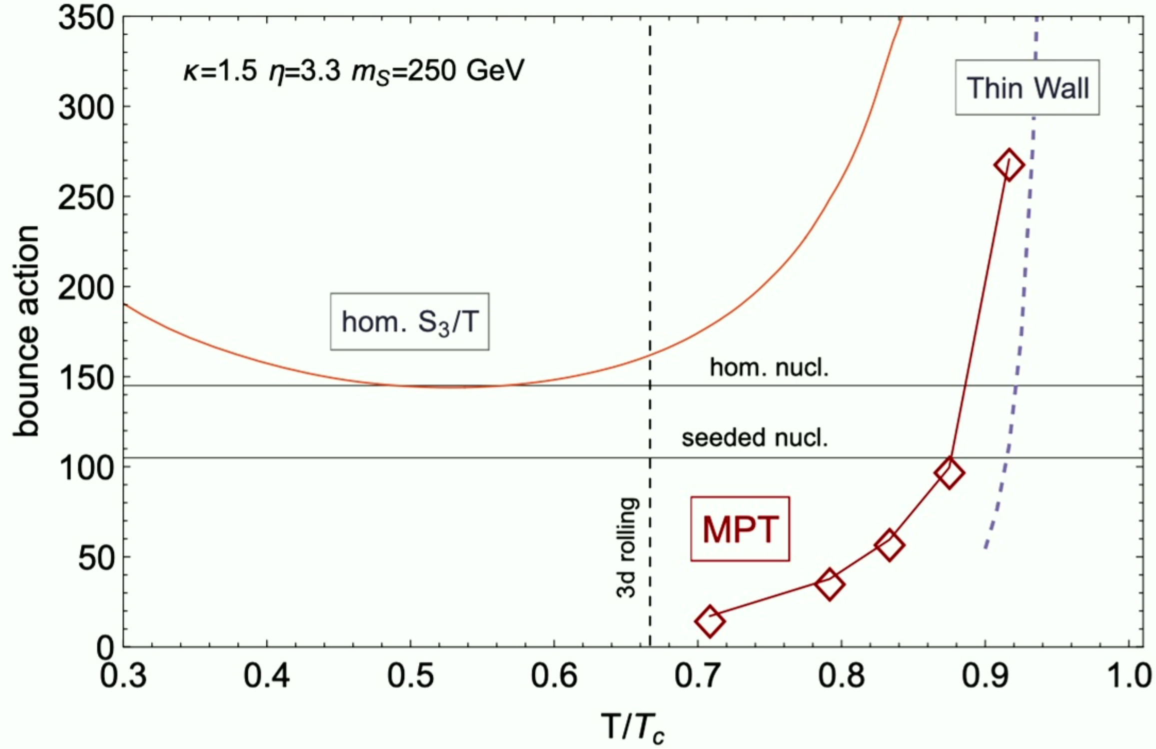


Comparison to EFT results of hep-ph/2203.16450

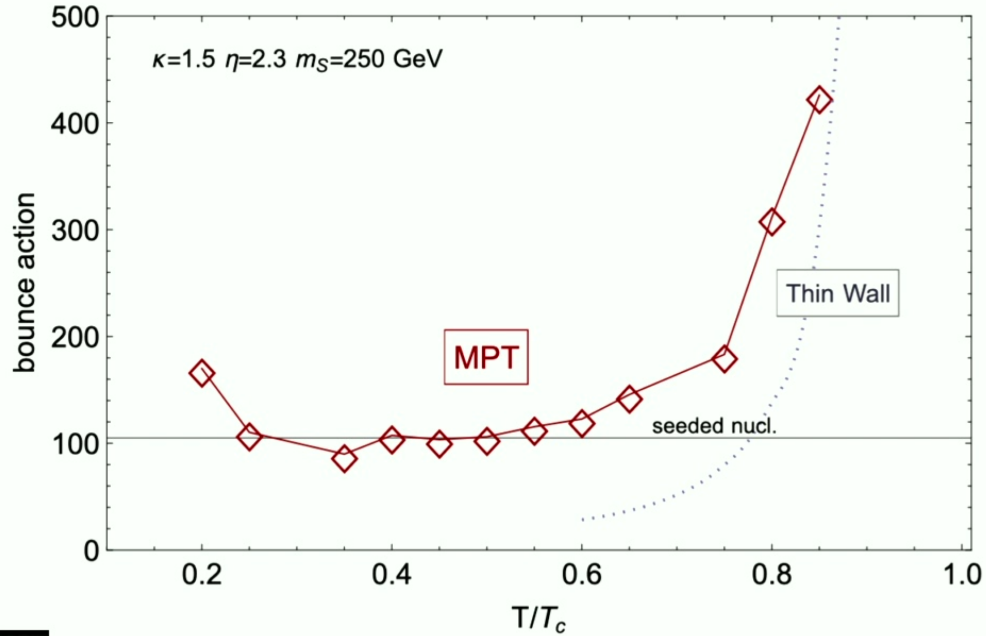


Full Calculation - LISA Benchmark



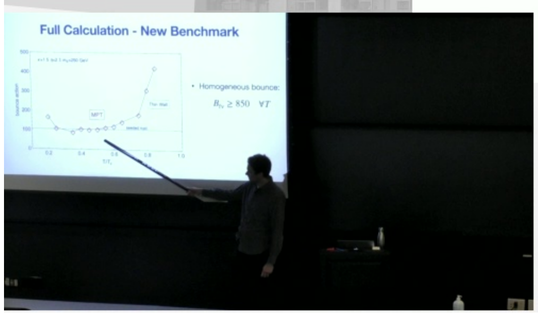
Comparison to Benchmark C of hep-ph/1512.06239

Full Calculation - New Benchmark

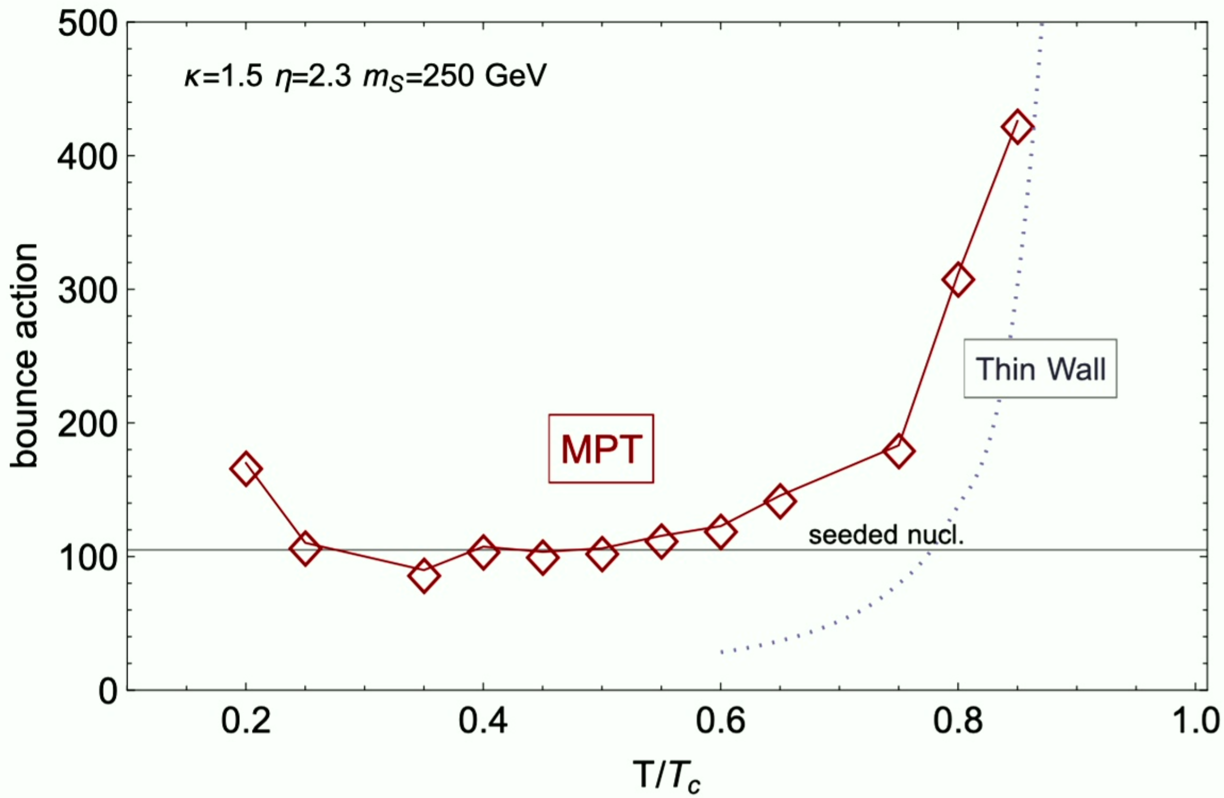


- Homogeneous bounce:

$$B_{\rm fv} \geq 850 \quad \forall T$$



Full Calculation - New Benchmark



- Homogeneous bounce:

$$B_{\rm fv} \geq 850 \quad \forall T$$

Gravitational Waves