

Title: Non-locality in Quantum Gravity

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Series: Quantum Gravity

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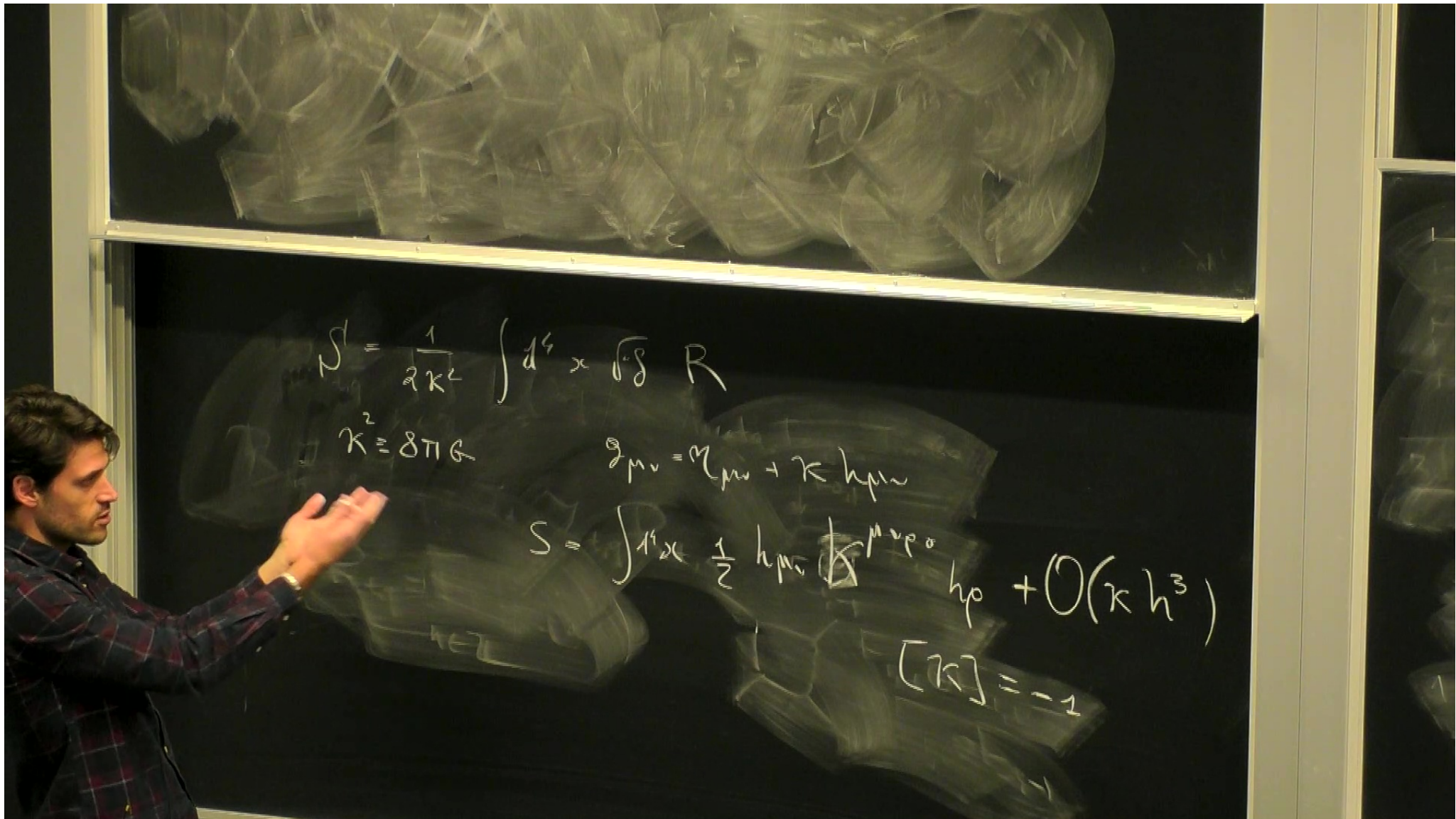
Abstract: A natural way to extend Einstein's General Relativity to the high-energy regime is to introduce quadratic curvature operators, which give rise to a renormalizable gravitational Lagrangian in four dimensions. However, this theory turns out to be pathological due to an additional massive spin-2 ghost that causes classical instabilities and breaks perturbative unitarity at the quantum level. In this talk, we investigate which principles of QFT are usually affected when higher-order derivative operators are present in a Lagrangian, and show that one possibility to avoid ghosts is giving up locality. In fact, the emergence of non-local physics at short distances and high energies is a common prediction for various quantum gravity approaches. We propose a model-independent approach to constrain the non-local Lagrangian in quantum gravity, thus providing a novel scenario in which different quantum gravity programs can be fruitfully contrasted in terms of foundational concepts and computational methods.

Zoom link: <https://pitp.zoom.us/j/96167652193?pwd=OWFDV1JXVGxLdE1qMHo4N3ZKcDByQT09>

Non Locality in QG

$$\begin{array}{l} - + + + \\ \hline \hbar = 1 = \mathbb{G} \\ \Lambda = 0 \end{array}$$

- LOCALITY
- UNITARITY : $S^\dagger S = \mathbb{1}$
- STRICT - RENORM.
- SYMMET.



$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R$$

$$\kappa^2 = 8\pi G$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$S = \int d^4x \frac{1}{2} h_{\mu\nu} \Box^{\mu\nu\rho\sigma} h_{\rho\sigma} + O(\kappa h^3)$$

$$[\kappa] = -1$$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [\gamma R + \alpha R^2 + \beta \epsilon_{\mu\nu\rho\sigma} \epsilon^{\mu\nu\rho\sigma}]$$

$$\Pi_{\mu\nu\rho\sigma}(p^2) = \Pi_{\mu\nu\rho\sigma}^{GR} + \frac{1}{2} \frac{P_{\mu\nu\rho\sigma}^{(0)}}{p^2 + m_0^2} - \frac{P_{\mu\nu\rho\sigma}^{(2)}}{p^2 + m_2^2}$$

$$m_0^2 = \frac{\gamma}{6\alpha}$$

$$m_2^2 = -\frac{\gamma}{2\beta}$$

$$\alpha > 0, \beta < 0$$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[\gamma R + \alpha R^2 + \beta \epsilon_{\mu\nu\rho\sigma} \epsilon^{\mu\nu\rho\sigma} \right]$$

$$\Pi_{\mu\nu\rho\sigma}(p^2) = \Pi_{\mu\nu\rho\sigma}^{\text{GR}} + \frac{1}{2} \frac{P_{\mu\nu\rho\sigma}^{(0)}}{p^2 + m_0^2} \ominus \frac{P_{\mu\nu\rho\sigma}^{(2)}}{p^2 + m_2^2}$$

$$= R_{\mu\nu} R^{\mu\nu}$$

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

$$\Box R$$

$$m_0^2 = \frac{\gamma}{6\alpha}$$

$$\alpha > 0, \beta < 0$$

$$m_2^2 = -\frac{\gamma}{2\beta}$$

$$\mathcal{L} = \frac{1}{2} \phi \square \left(1 - \frac{\square}{m^2} \right) \phi - V(\phi)$$

$$\Pi(p^2) = \frac{m^2}{p^2(p^2 + m^2)} = \frac{1}{p^2} - \frac{1}{p^2 + m^2}$$

$$S^\dagger S = -\mathbb{1}$$

$$S = \mathbb{1} + iT \rightarrow 0$$

$$2 \operatorname{Im} T = T + T^* \geq 0$$

$$\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - \frac{m^2}{2} \phi^2 - V(\phi)$$

$$\Pi(p^2) = \frac{m^2}{p^2(p^2 + m^2)} = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m^2 - i\epsilon}$$

$$S^\dagger S \neq \mathbb{1} \quad ; \quad S = \mathbb{1} + iT \quad \rightarrow \quad 2 \operatorname{Im} T = T + T^\dagger \quad " \geq 0 "$$

$$\operatorname{Im} [\Pi(p^2)] = \pi [\delta(p^2) - \delta(p^2 + m^2)]$$

$$\mathcal{L} = \frac{1}{2} \phi F(\square) \phi - V(\phi)$$

$$\frac{1}{p^2 (p^2 + m^2 + \Lambda p^4 + \dots)}$$

$$\mathcal{L} = \frac{1}{2} \phi F(\square) \phi - V(\phi)$$

F : entire function

$$F(\square) = 2 \gamma(\square) \prod_{i=1}^N (\square - m_i^2)^{\alpha_i}$$

$$1) \quad \begin{matrix} N=1, \gamma=0 \\ \alpha_1=0 \end{matrix} \Rightarrow F(\square) = \square - m^2$$

$$2) \quad N > 1$$

$$\mathcal{L} = \frac{1}{2} \phi F(\square) \phi - V(\phi)$$

F : entire function

$$e^{\gamma(\square)} = \sum_{m=0}^{\infty} \frac{\gamma_m}{m!} \square^m$$

$$F(\square) = e^{\gamma(\square)} \prod_{i=1}^N (\square - m_i^2)^{n_i}$$

$$1) \begin{matrix} N=1, \\ n_1=0 \end{matrix} \gamma=0 \Rightarrow F(\square) = \square - m^2$$

$$2) N > 1$$

$$\mathcal{L} = \frac{1}{2} \phi F(\square) \phi - V(\phi)$$

F : entire function

$$F(\square) = e^{\gamma(\square)} \prod_{i=1}^N (\square - m_i^2)^{n_i}$$

$$1) \quad \begin{matrix} N=1, \gamma=0 \\ n_1=0 \end{matrix} \Rightarrow F(\square) = \square - m^2$$

$$2) \quad N > 1$$

$$\gamma(\square) = \alpha \square$$

$$e^{\alpha \square} =$$

$$e^{\gamma(\square)} = \sum_{n=0}^{\infty} \frac{\gamma^n}{n!} \square^n$$

$$\mathcal{L} = \frac{1}{2} \phi \cdot e^{\gamma(0)} (\square - m^2) \phi - V(\phi)$$

$$\Pi(p^2) = \frac{1}{e^{\gamma(p^2)} (p^2 + m^2)}$$

$\nwarrow$ $p^2 = -m^2$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[R + R F_0(0) R + e_{\mu\nu\sigma} F_2(0) e^{\mu\nu\sigma} \right]$$

$$\Pi_{\mu\nu\sigma} = \frac{P^{(2)}}{\kappa^2 (1 - 2\bar{F}_2(-k^2)k^2)} + \frac{P^{(0)}_{\mu\nu\sigma}}{2\kappa^2 (1 + 6\bar{F}_0(-k^2)k^2)}$$

$$1 - 2\bar{F}_2(-k^2)k^2 = e^{\gamma_2(k^2)}$$

$$1 - 2\bar{F}_0(-k^2)k^2 = e^{\gamma_0(k^2)}$$

11 p u p s =

$$k^2 (1 - 2 \bar{F}_2(-k^2) k^2) + 2 k^2 (1 + 6 \bar{F}_0(k^2) k^2)$$

$$1 - 2 \bar{F}_2(-k^2) k^2 = e^{\gamma_2(k^2)}$$

$$1 - 2 \bar{F}_0(-k^2) k^2 = e^{\gamma_0(k^2)}$$

$$2 \text{Im} \left[\text{Diagram} \right] = \int d\pi_f | \text{Diagram} |^2$$

The diagram is a circle with a cross inside, and a vertical line passing through the center. The top part of the vertical line is labeled $\frac{1}{k^4}$ and the bottom part is labeled $\frac{1}{k^4}$.

1) $N=1, \gamma=0 \Rightarrow F(Q) = Q - m^2$
 $q_1=0$

2) $N>1$

$$\pi_{\mu\nu\rho\sigma} = e^{\gamma(0)} \pi_{GR, \mu\nu\rho\sigma} \rightarrow \int \frac{1}{2\kappa^2} d^4x \sqrt{-g} [R$$

$$\gamma(0) = -\frac{\square}{M^2} + G_{\mu\nu} \frac{e^{-\square/M^2} - 1}{\square} R^{\mu\nu}]$$

$$T_{00} = M \delta^{(3)}(\vec{r})$$

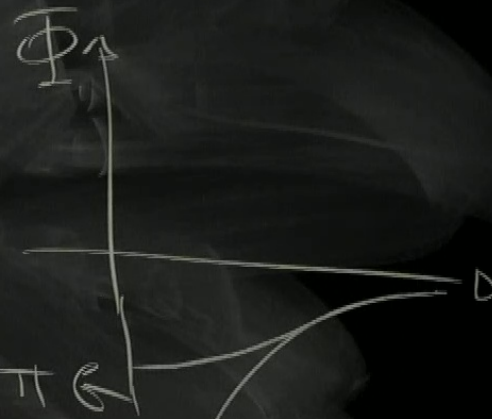
IN QG

$\frac{+}{G}$ • LOCALITY

$$\gamma(0) = -\frac{\square}{M^2}$$

$$+ G_{\mu\nu} \frac{e^{-\square/M^2} - 1}{\square} R^{\mu\nu}$$

$$T_{00} = m \delta^{(3)}(\vec{r})$$



$$e^{-\nabla^2/M^2} \nabla^2 \Phi = m \delta^{(3)}(\vec{r}) \quad 4\pi \epsilon$$

$$\Rightarrow \Phi(r) = -\frac{Gm}{r} \mathcal{E}_2\left(\frac{Mr}{2}\right)$$

NONLOCALITY IN QG

+++

$\hbar = 1 = G$

• LOCALITY

$$\gamma(0) = -\frac{\square}{M^2}$$

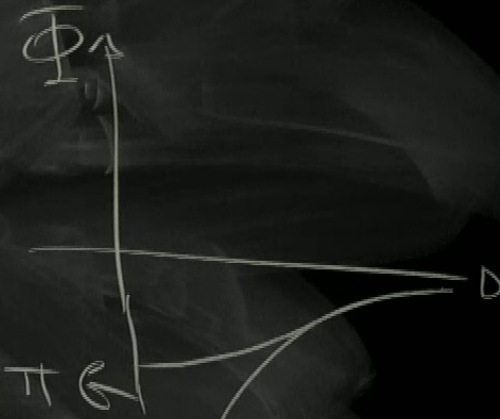
$$+ G_{\mu\nu} \frac{e^{-\sqrt{M^2} r} - 1}{\square} R^{\mu\nu}$$

$$T_{00} = m \delta^{(3)}(\vec{r})$$

$$GMm \ll 1$$

$$e^{-\sqrt{M^2} r} \nabla^2 \Phi = m \delta^{(3)}(\vec{r}) \quad 4\pi G$$

$$\Rightarrow \Phi(r) = -\frac{Gm}{r} \mathcal{E}_2\left(\frac{Mr}{2}\right)$$



NONLOCALITY IN QM

$\begin{array}{c} - + + + \\ \hline \end{array} \quad \bullet \text{ LOCALITY}$

$$e^{\gamma(\Box)}$$

$$V(\phi)$$

γ : entire

$$\gamma(\Box) = \Box^{m+1}$$

$$\Box \Rightarrow -p^2$$

space-like $p^2 > 0$

time-like $p^2 < 0$

$$\Pi_{\mu\nu\rho\sigma} = e^{-\frac{p^2}{M^2}} \Pi_{\mu\nu\rho\sigma}$$

$$\gamma(\Box) = \Box^{2m} \rightarrow (k^2)^{2m}$$

NONLOCALITY IN QG

$$\frac{-++}{h=1=\mathbb{C}} \mid \bullet \text{ LOCALITY}$$

$$e^{\gamma(\Box)}$$

$$V(\phi)$$

γ : entire

$$\gamma(\Box) = \Box^{m+1}$$

$$\Box \Rightarrow -p^2$$

space-like $p^2 > 0$

time-like $p^2 < 0$

$$I_m \left[\text{Feynman diagram with 3 external lines and a loop} \right]$$

$$\Pi_{\mu\nu\rho\sigma} = e^{-\frac{p^2}{M^2}} \Pi_{\mu\nu\rho\sigma}^{\text{free}}$$

$$= \int d\pi_\phi \left| \text{Feynman diagram} \right|^2 e^{-p^2}$$

$$\gamma(\Box) = \Box^{2m}$$

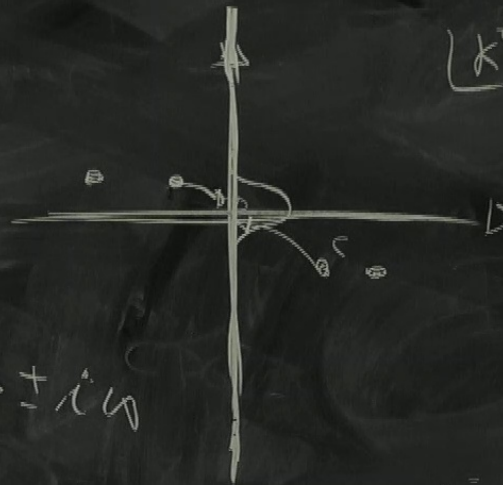
$$\rightarrow (k^2)^{2m}$$

NON LOCALITY IN QG

$$\frac{-\hbar + \hbar + \hbar}{\hbar = 1 = \mathbb{C}} \mid \bullet \text{ LOCALITY}$$

$$e^{\gamma(0)}$$

$$\text{---} \bigcirc \text{---} = \int d^4k \frac{e^{-k^2}}{k^2 + m^2} \frac{e^{-(k-r)^2}}{(k-r)^2 + m^2}$$



$$k^0 \rightarrow \pm i\omega$$

$$[k^0]$$

NONLOCALITY IN QG

$$\frac{-++}{h=1=\infty} \quad \bullet \quad \text{LOCALITY}$$

1)

$R + \text{Group Haleruo}$

$$k^2 + m^2 \quad (k - \gamma)^2 + m^2$$

2)

Non Locality IN QG

$- + + +$

$\hbar = 1 = \mathbb{C}$

• LOCALITY

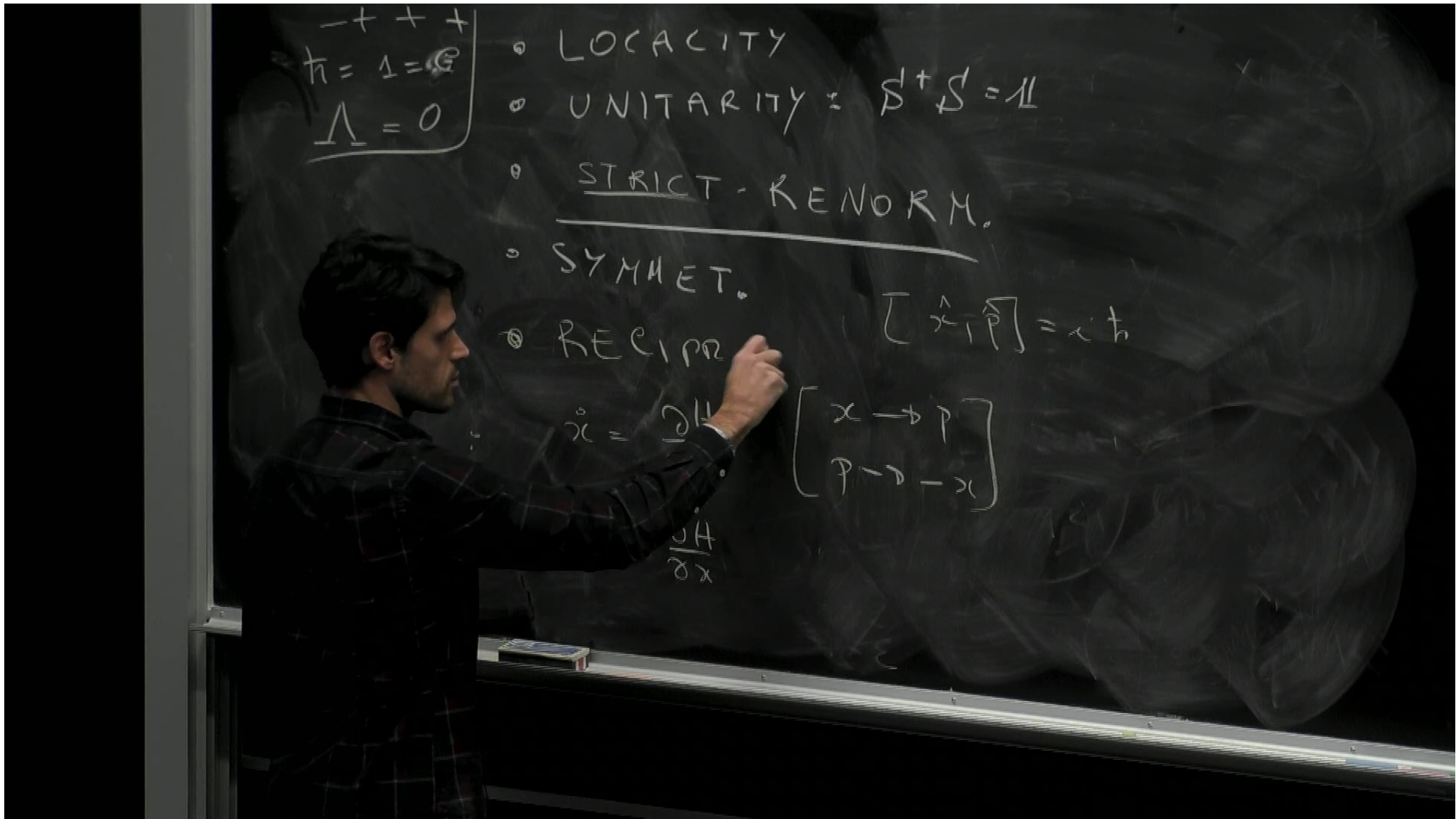
$$\begin{array}{l} - + + + \\ \hbar = 1 = \mathbb{C} \\ \Lambda = 0 \end{array}$$

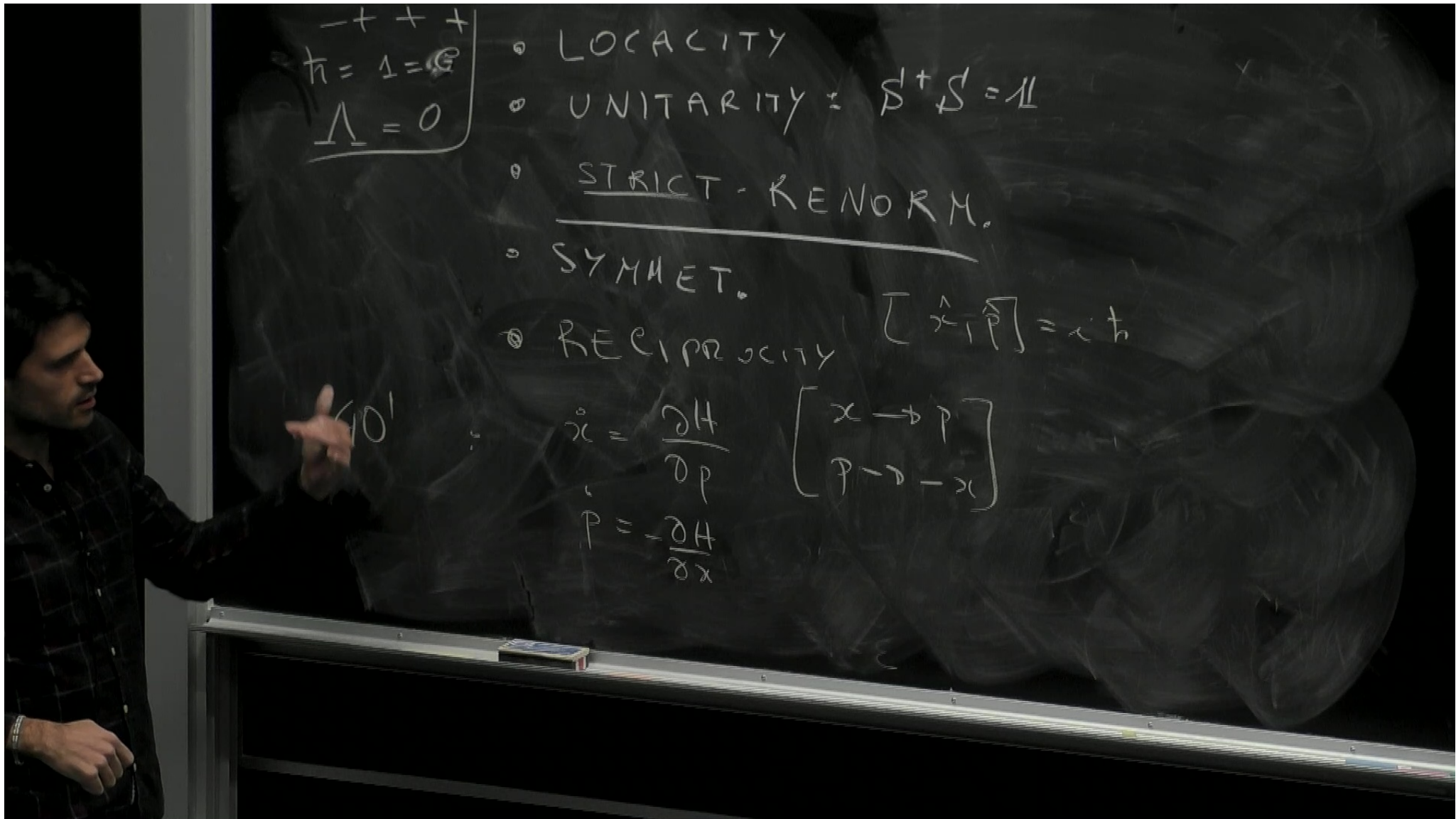
- LOCALITY
- UNITARITY: $S^\dagger S = 11$
- STRICT - RENORM.
- SYMMET.
-

40'

$$\begin{aligned} x &= \frac{\partial H}{\partial p} \\ p &= -\frac{\partial H}{\partial x} \end{aligned}$$

$$\begin{bmatrix} x \rightarrow p \\ p \rightarrow -x \end{bmatrix}$$





$$40' \quad \begin{aligned} \dot{x} &= \frac{\partial H}{\partial p} \\ p &= -\frac{\partial H}{\partial x} \end{aligned} \quad \left[\begin{array}{l} x \rightarrow p \\ p \rightarrow -x \end{array} \right]$$

$$e^{-\alpha\phi} \square A_\mu = 4\pi \partial_\mu$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \tilde{g}_{\mu\nu} dp^\mu dp^\nu$$