

Title: What factorization algebras are (not) good for

Speakers: Owen Gwilliam

Series: Mathematical Physics

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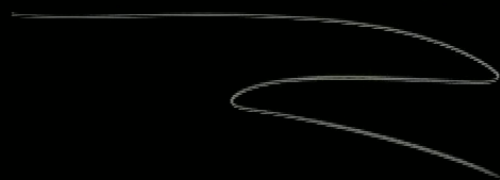
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Abstract: Factorization algebras are local-to-global objects, much like sheaves, and it is natural to ask what kind of topology, geometry, and physics they are sensitive to. We will examine this question with a focus on less-perturbative phenomena, touching on topics like moduli of vacua for 4-dimensional gauge theories and Dijkgraaf-Witten-type TFTs. Apologies hereby issued in advance to the (hopefully) friendly audience (and to my collaborators!) for speaking before achieving complete clarity.

Zoom link: <https://pitp.zoom.us/j/94417858154?pwd=ak54UFpPb3hFbnBwcUlnMnhCdG1odz09>

What factorization algebras

are (not) good for



Owen Gwilliam

General impression

- ① they are the Euclidean field theory cousin of AQFT
- ② they just know about perturbative theories

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② they just know about perturbative theories

perturbative
"Euclidean"
any signature field theory on a manifold M

"observables"

fact algebra
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graph LR; A["perturbative  
'Euclidean'  
any signature field theory on a manifold M"] <-->|Euclidean| B["any signature"]; A -- "observables" --> C["fact algebra on M"]
```

Q: What kind of "nonperturbative" phenomena
might factorization algebras know?

Nonperturbative
instantons, monopoles, ...
Witten effect
defects
dualities

Attempts

$\xrightarrow{?}$ BFN gives E_3 algebra

$\xrightarrow{?}$ DG & KC (Baxter's
 $\mathbb{Q}$ -operator)

KC, DG, TD, NP ...

?

Nonperturbative

instantons, monopoles, ...

Witten effect

defects

dualities

finite & discrete groups

Dijkgraaf-Witten
theory

Attempts

$\xrightarrow{?}$ BFN gives E_3 algebra

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 Q -operator)

KC, DG, TD, NP ...

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DW theory

G finite group

$\text{Bun}_G(M) = (\text{groupoid of}) \text{ } G\text{-bundles over } M$
 $\text{Map}(\pi_1 M, BG)$

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$\text{Map}(\Pi_{\leq 1} M, BG)$

$\leadsto$ functorial TFT

$\text{Bord}_{n-1, n} \xrightarrow{Z_G} \text{Vect}$

$Z_G(M^{n-1}) = "O(\text{Bun}_G(M))"$

$\cong$ v.s. spanned by iso classes of G -bundles

$\Sigma \rightarrow M_{\text{int}}$

$Z_G(\Sigma): Z_G(M_{\text{in}}) \rightarrow Z_G(M_{\text{out}})$

DW theory

← this is a setting where the path integral is well-defined

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DW theory

← this is a setting where the path integral is well-defined

G finite group

$$\lambda \in H^n(BG, U(1))$$

$\text{Bun}_G(M) = (\text{groupoid of})$ G -bundles over M

$$\text{Map}(\Pi_{\leq 1} M, BG)$$

→ functorial TFT

$$\text{Bord}_{n-1, h} \xrightarrow{Z_G} \text{Vect}$$

$$Z_G(M^{n-1}) = "O(\text{Bun}_G(M))"$$

$\cong$ v.s. spanned by iso classes of G -bundles

$$\Pi_n \xrightarrow{\Sigma} M_{n+1}$$

$$Z_G(\Sigma): Z_G(M_n) \rightarrow Z_G(M_{n+1})$$

① Q: Is there a concrete model as a fact.
alg on $\mathbb{C}$ (or Riemann surfaces) for the
lattice vertex algebra?

Rmk: • For the Heisenberg part, we do have a nice model
We're missing the group ring part
• Charge lattices show up in abelian gauge theory

CAUTION

DO NOT TOUCH THE EQUIPMENT
OR THE EQUIPMENT MAY BE DAMAGED
OR THE EQUIPMENT MAY BE DAMAGED
OR THE EQUIPMENT MAY BE DAMAGED

② Discrete & finite group gauge theories "emerge" naturally

Ex Consider YM theory w/ $G=U(1)$
and a complex scalar field ϕ of charge n

Consider a solution of GM where

- the connection is flat
- the scalar is nonzero

Higgs
mechanism
YM theory
 $\simeq$ Proca theory
+ real scalar
field

Ex Consider YMH theory w/ $G=U(1)$
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There is a residual finite gauge symmetry
with group $\mathbb{Z}/n$

Higgs
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② Discrete & finite group gauge theories "emerge" naturally

Ex Consider YM theory w/ $G=U(1)$
and a complex scalar field ϕ of charge n

Consider a solution of GM where

- the connection is flat

- the scalar is nonzero

There is a residual finite gauge symmetry
w/ group $\mathbb{Z}/n$. How to encode it w/ fact alg?

Higgs
mechanism
YM theory
 $\simeq$ Proca theory
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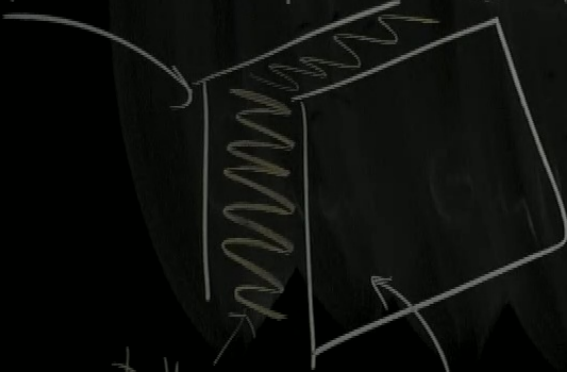
Harvey-Seiberg-Tachikawa

"Reading betw
the lines"

recently Freed-Moore-Teleman
via functorial field theories

gave an approach

4d TFT



bulk 5d DW
theory

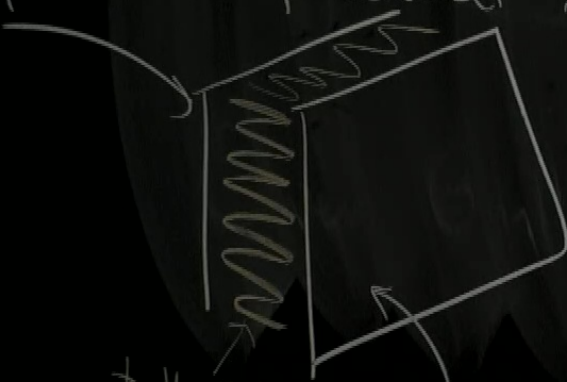
4d-gauge theory
(nontopological)

Hanany-Seiberg-Tachikawa

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Q: Capture RBTL
arguments via
fact algs?

§ Lightning overview of factorization algebras

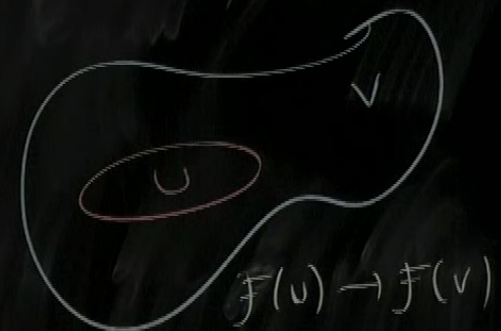
M — "spacetime" (a site)

$\mathcal{P}^\otimes$ — symmetric monoidal $(\infty-)$ category
 $\text{Vect}^\otimes, \text{Ch}^\otimes$

Def A prefact alg F on M with
values in $\mathcal{C}$ is

• a functor $\text{Open}(M) \rightarrow \mathcal{C}$

• for any pairwise disjoint open sets $U_1, \dots, U_k \subset V$

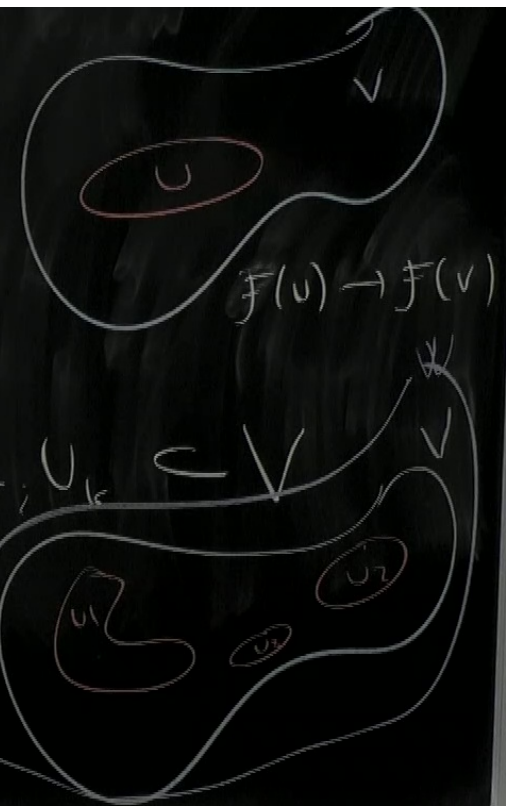


values in $\mathcal{C}$ is

• a functor $\text{Open}(M) \longrightarrow \mathcal{C}$

• for any pairwise disjoint open sets $U_1, \dots, U_k \subset V$

a map $F(U_1) \otimes \dots \otimes F(U_k) \longrightarrow F(V)$



Def A factorization algebra is a prefact alg
that is a cosheaf for the Weiss topology on M .

$\{U_\alpha\}$ opens "covers" V
if any finite set of points in V
is inside some U_α

§ Examples related to DW theory

G discrete group (think S_3 , $\mathbb{Z}$)

$\leadsto$ there are top. spaces associated to G

BG — it classifies G -bundles

$\pi_0 \text{Map}(M, BG) \cong \text{iso classes of } G\text{-bundles}$

G discrete group (think S_3 , $\mathbb{Z}$)

$\leadsto$ there are top. spaces associated to G

• BG — it classifies G -bundles

$\pi_0 \text{Map}(M, BG) \cong$ classes of G -bundles

• if A is an abelian group, there is

$$B^n A = K(A, n) \leadsto \pi_0 \text{Map}(M, B^n A) \cong H^n(M, A)$$

DW theory

"fields" $\text{Bun}_G(M) = \text{Map}(M, BG)$

$\leadsto$ "observables" are $C^*(\text{Map}(M, BG))$
 $\uparrow$ singular cochains

Remark: replace BG by any finite homotopy type X

The functor

$\mathbf{Mfld}_n$

$\longrightarrow \mathbf{Ch}_k$

(fix the commutative ring)

M

$\longrightarrow C^*(\text{Map}(M, X), k)$

is a

prefactor algebra:

$\mathbf{Mfld}_n \xrightarrow{\text{Map}(-, X)} \mathbf{Top} \xrightarrow{C^*(-, k)} \mathbf{Ch}_k$

$M \sqcup M'$

$\longrightarrow \text{Map}(M \sqcup M', X) \cong \text{Map}(M, X) \times \text{Map}(M', X)$

Q. Is it a fact alg?

Theorem (Ayala-Francis; Grosse-Trudler-Zeraviani)

This functor is a factorization algebra if

- ① $\pi_i(X)$ are finite for $i \leq n$ less restrictive than DW
- ②

Theorem (Ayala-Francis; Gmot-Trudler-Zeraviani)

This functor is a factorization algebra if

① $\pi_i(X)$ are finite for $i \leq n$ less restrictive than DW

② X is nilpotent space more restrictive than DW

$\pi_1 X$ is nilpotent group & $\pi_1 X$ acts nilpotently on higher htpy groups

There's an analogous construction to encode "symmetries"

$$Mfld_n \xrightarrow{\text{Map}_c(-, X)} \text{Top}^X \xrightarrow{C_*(-, \ell)} Ch_k^{\otimes}$$

based spaces

(X, τ)

$$\text{Map}_c(M, (X, \tau)) = \left\{ \phi: M \rightarrow X \text{ cts} : \overline{\phi^{-1}(X - \tau)} \text{ has compact closure} \right\}$$

Thus we have a prefactorization algebra

CAUTION

DO NOT TOUCH THE BOARD OR THE BOARDER

IT IS PROHIBITED TO TOUCH THE BOARD OR THE BOARDER

GROUP POLICY BOARD

Nonabelian Poincaré duality (Salvatore, Luné)

This functor is a factorization algebra
if $\pi_i(X, x) = 0$ for $i \leq n-1$.

Ex: $n=1$ G any discrete group $\Rightarrow BG$

$$C_+(\underbrace{\text{Flap}_c(\mathbb{R}, BG)}_{\Omega BG \simeq G}, \ell_c) \simeq \mathbb{C}[G] \quad \text{group ring}$$

factorization algebra
 if $\pi_i(X, x) = 0$ for $i \leq n-1$.

Ex: $n=1$ G any discrete group $\Rightarrow BG$

$$C_+(\underbrace{\text{Map}_c(\mathbb{R}, BG)}_{\Omega BG \simeq G}, \ell_c) \simeq \mathbb{C}[G] \quad \text{group ring}$$

rep $\mathbb{C}[G] \xrightarrow{\text{a symmetry}} \text{End}(V)$

a symmetry of fact alg $\mathcal{F}$

$$C_+(\text{Map}_c(-, BG)) \longrightarrow \mathcal{F}$$