Title: Quantum Error Mitigation and Error Correction: a Mathematical Approach

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Abstract: Error-correcting codes were invented to correct errors on noisy communication channels. Quantum error correction (QEC), however, has a wider range of uses, including information transmission, quantum simulation/computation, and fault-tolerance. These invite us to rethink QEC, in particular, the role that quantum physics plays in terms of encoding and decoding. The fact that many quantum algorithms, especially near-term hybrid quantum-classical algorithms, only use limited types of local measurements on quantum states, leads to various new techniques called Quantum Error Mitigation (QEM). We examine the task of QEM from several perspectives. Using some intuitions built upon classical and quantum communication scenarios, we clarify some fundamental distinctions between QEC and QEM. We then discuss the implications of noise invertibility for QEM, and give an explicit construction called Drazin-inverse for non-invertible noise, which is trace-preserving while the commonly-used MoorePenrose pseudoinverse may not be. Finally, we study the consequences of having imperfect knowledge about system noise and derive conditions when noise can be reduced using QEM.

Zoom link: https://pitp.zoom.us/j/91543402893?pwd=b09IS3VWNk5KZi8ya3gzSmRKRFJidz09

Quantum Error Mitigation and Correction: A Mathematical Approach

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December 5th 2022
at Perimeter Institute

arXiv: 2111.02345, joint work with Junan Lin, David Kribs, Yiu-Tung Poon, Bei Zeng, and Raymond Laflamme
Beyond CPTP maps (To be appear on arXiv), joint work with Maxwell Fitzsimmons, Yiu-Tung Poon, Raymond Laflamme

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Outline

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The beginning of the story

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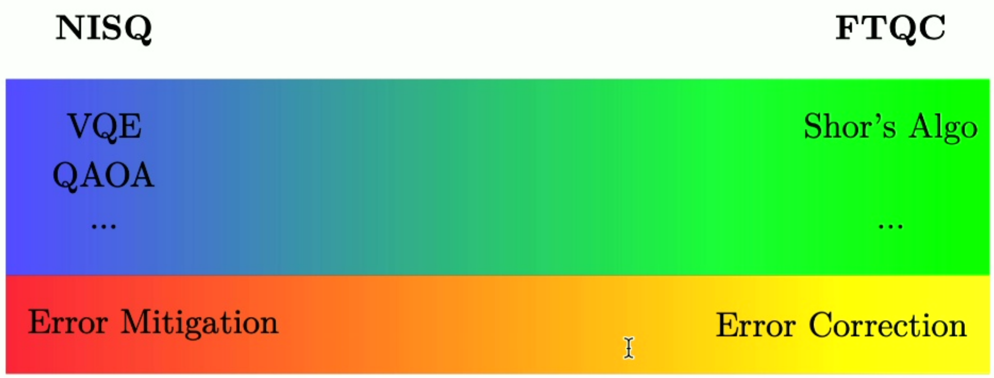
Motivation



- What is Error Mitigation?
- What is its relation with Error Correction?
- Can we develop a theory for EM?

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Error mitigation? Error correction?

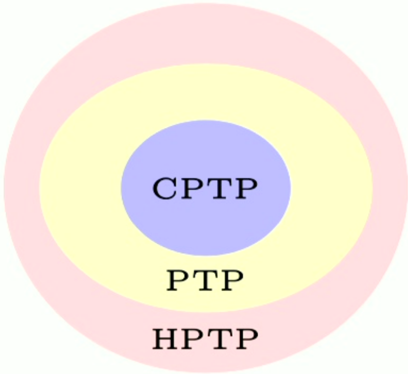
Preliminary

Quantum Channels

A quantum channel is a Completely Positive and Trace Preserving (CPTP) map.

HP

A Hermitian preserving (HP) map Φ is a linear map that takes a Hermitian operator A to another Hermitian operator $\Phi(A)$.



A Schematic Diagram

Inversion of CPTP

For an invertible CPTP map Φ, where $\dim_{in} = \dim_{out}$, the inverse map Φ^{-1} is CPTP iff Φ is unitary. Otherwise, Φ^{-1} is HPTP.

Watrous, John. The theory of quantum information. Cambridge university press, 2018.

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Protocols called Quantum Error Mitigation

"Active" Error Mitigation ($\mathcal{U}, \mathcal{N}, M$)

- Probabilistic error cancellation PRL, 119(18), 180509...
- Zero error extrapolation (Richardson extrapolation) PRX 7, 021050 (2017); PRL, 119(18), 180509...
- Virtual distillation arXiv:2011.07064...
- Measurement Error Mitigation Qiskit, ...
- Neural Networks (Clifford Data Regression) arXiv:2005.10189...
- ...

Utilize the problem structure

- Subspace Expansion Physical Review A 95, 042308 (2017).
- Symmetry Verification Physical Review Letters 122, 180501 (2019).

Preventing Error from Happening

- Compelling-Based Error Mitigation arXiv:2202.07628...
- Quantum Control techniques...

Reviews:

- McArdle, S., Endo, S., Aspuru-Guzik, A., Benjamin, S. C., & Yuan, X. (2020). Reviews of Modern Physics, 92(1), 015003.
- Tilly, J., Chen, H., Cao, S., Picozzi, D., Setia, K., Li, Y., ... & Tennyson, J. (2021). arXiv:2111.05176.
- Cai, Z., Babbush, R., Benjamin, S. C., Endo, S., Huggins, W. J., Li, Y., ... & O'Brien, T. E. (2022). arXiv:2210.00921.

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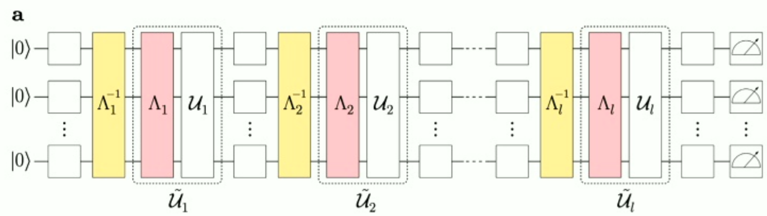
What is QEM?



Berg, E. V. D., Minev, Z. K., Kandala, A., & Temme, K. (2022): arXiv:2201.09866.

With an input state ρ_{in}, the ideal output state ρ_{out}^{ideal} can be written as

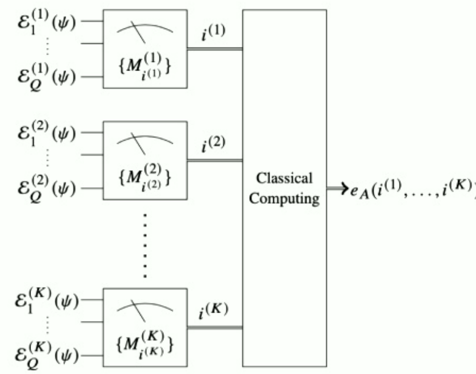
$$\rho_{out}^{ideal} = \mathcal{U}_{N_g} \circ \mathcal{U}_{N_g-1} \cdots \circ \mathcal{U}_1(\rho_{in}) \qquad (97)$$

and the noisy output state ρ_{out} we obtain in practice is

$$\rho_{out} = \mathcal{E}_{N_g} \circ \mathcal{U}_{N_g} \circ \cdots \circ \mathcal{E}_1 \circ \mathcal{U}_1(\rho_{in}). \qquad (98)$$

Here N_g is the number of gates.

Endo, S., Cai, Z., Benjamin, S. C., & Yuan, X. (2021): arXiv:2011.01382



Takagi, R., Endo, S., Minagawa, S., & Gu, M. (2021):arXiv:2109.04457.

Quek, Y., França, D. S., Khatri, S., Meyer, J. J., & Eisert, J. (2022):arXiv:2210.11505.

$$\rho_{\mathrm{EM}} = \mathcal{E}^{-1}(\rho_{\mathrm{out}}) = \mathcal{E}^{-1} \circ \mathcal{E}_{\mathrm{eff}}(\rho_{\mathrm{out}}^{\mathrm{ideal}})$$

Approximating the effect of inverse $\mathcal{E}^{-1}(\rho_{\mathrm{out}})$ under different assumptions.

Error Correction in a Nutshell

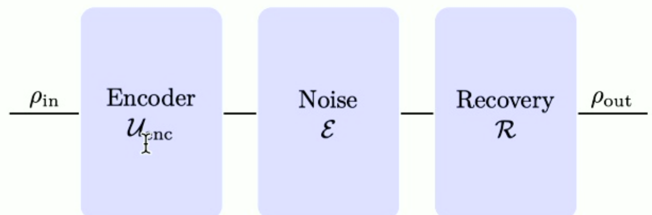
Classical repetition code

$$0 \to 000; 1 \to 111$$

Shor's nine-qubit code

$$|0\rangle \to (|000\rangle + |111\rangle)^{\otimes 3}$$

$$|1\rangle \to (|000\rangle - |111\rangle)^{\otimes 3}$$



$$\mathcal{R} \circ \mathcal{E}(P\rho P) = P\rho P$$

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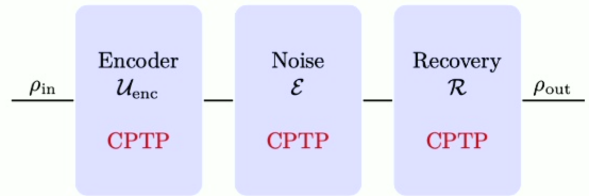
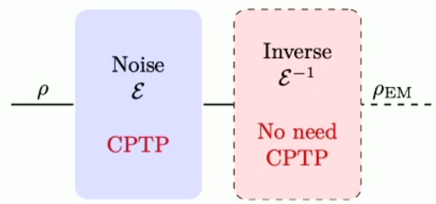
QEM and QEC: a high level perspective

$$\mathcal{R} \circ \mathcal{E}(\sigma) = \sigma,$$



$$\mathcal{E}^{-1} \circ \mathcal{E}(\rho) = \rho,$$



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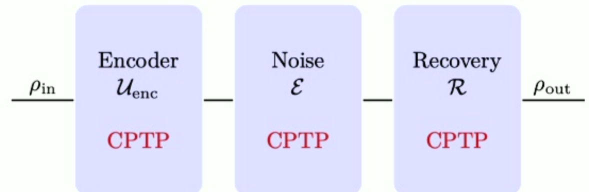
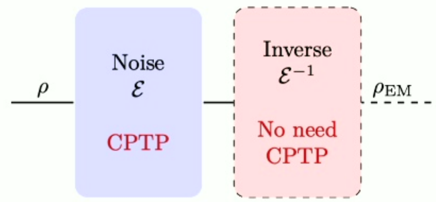
QEM and QEC: a high level perspective

$$\mathcal{R} \circ \mathcal{E}(\sigma) = \sigma,$$



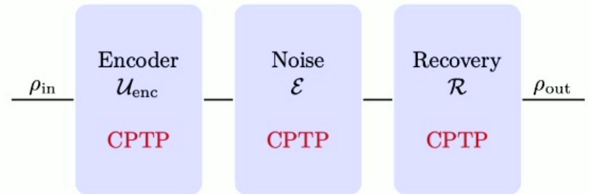
$$\mathcal{E}^{-1} \circ \mathcal{E}(\rho) = \rho,$$



QEM and QEC: a high level perspective

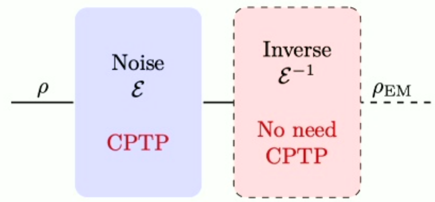
$$\mathcal{R} \circ \mathcal{E}(\sigma) = \sigma,$$

$\sigma \in$ Code Space



$$\mathcal{E}^{-1} \circ \mathcal{E}(\rho) = \rho,$$

$\rho \in$ All Density Matrices



- Encode into a subspace, enabling the recovery $\mathcal{R}$ to be CPTP
- Non-CP maps cannot be dilated to Unitary channels (operational in quantum devices). Non-CP-ness of $\mathcal{E}^{-1}$ has to be dealt with classical processing.
- The less noisy ρ_{EM} does not exist in the quantum device.
  - Many ways to approximate.
  - Classical description of ρ_{EM} stored on paper can be less noisy.
  - Statistical: active QEM cannot invert one-shot experiments.

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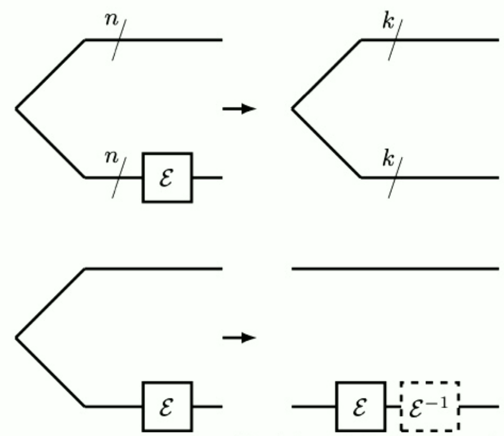
QEM and QEC in communication

QEC = 1-EPP (entanglement purification protocols)

Bennett, C. H., DiVincenzo, D. P., Smolin, J. A., & Wootters, W. K. (1996). Mixed-state entanglement and quantum error correction. Physical Review A, 54(5), 3824.

QEC:

QEM:



QEM can improve the local measurements on Bob's side, but does not preserve important quantum resources.

QEC: Quantum states/information still "alive".
(Active) QEM: Statistical data are less noisy/closer to ideal on paper.

How to invert a quantum channel?

Channel Representations

We will focus on the channels which has **the same input and output dimensions** d.

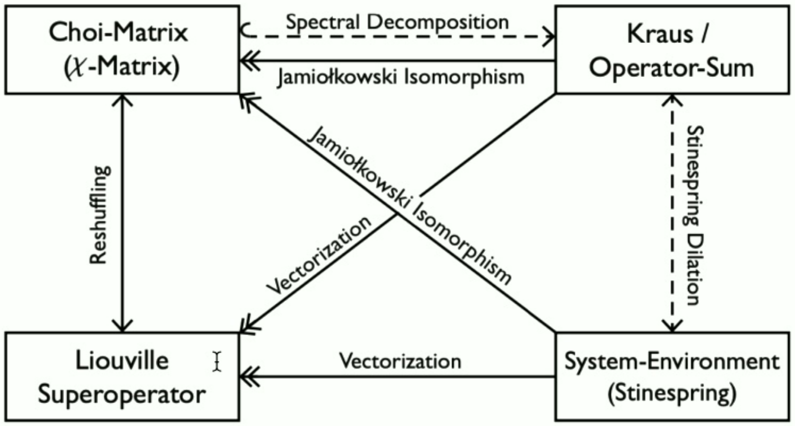


Figure: Quantum Channel Representations

Wood, Christopher J., Jacob D. Biamonte, and David G. Cory. "Tensor networks and graphical calculus for open quantum systems." arXiv:1111.6950 (2011).

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Kraus operators

$$\mathcal{N}(\rho) = \sum_i K_i \rho K_i^\dagger$$

for a set of Kraus operators $\{K_i\}$

Commonly used

Choi representation

$$C(\mathcal{N}) = \sum_{a,b} E_{a,b} \otimes \mathcal{N}(E_{a,b})$$

$C(\mathcal{N})$ is Hermitian, then $\mathcal{N}$ is HP; Partial Trace of $C(\mathcal{N})$ is identity, then $\mathcal{N}$ is TP; If $C(\mathcal{N})$ is positive, then $\mathcal{N}$ is CP. But $C(\mathcal{N}_1 \circ \mathcal{N}_2) \neq C(\mathcal{N}_1) \cdot C(\mathcal{N}_2)$

Superoperator

$$\mathbf{v}(\mathcal{N}(\rho)) = \mathbf{v}(\mathcal{N})\mathbf{v}(\rho)$$

Good for calculation, $\mathbf{v}(\mathcal{N}_1 \circ \mathcal{N}_2) = \mathbf{v}(\mathcal{N}_1) \cdot \mathbf{v}(\mathcal{N}_2)$. $\mathbf{v}(\rho) = |\rho\rangle\rangle$

An example

Let the Choi representation of a quantum channel $\mathcal{N}$ be

$$C(\mathcal{N}) = \left(\begin{array}{cc|cc} \frac{3}{4} & 0 & -\frac{i}{8} & \frac{1}{2}+\frac{i}{8} \\ 0 & \frac{1}{4} & -\frac{i}{8} & \frac{i}{8} \\ \hline \frac{i}{8} & \frac{i}{8} & \frac{1}{4} & 0 \\ \frac{1}{2}-\frac{i}{8} & -\frac{i}{8} & 0 & \frac{3}{4} \end{array}\right).$$

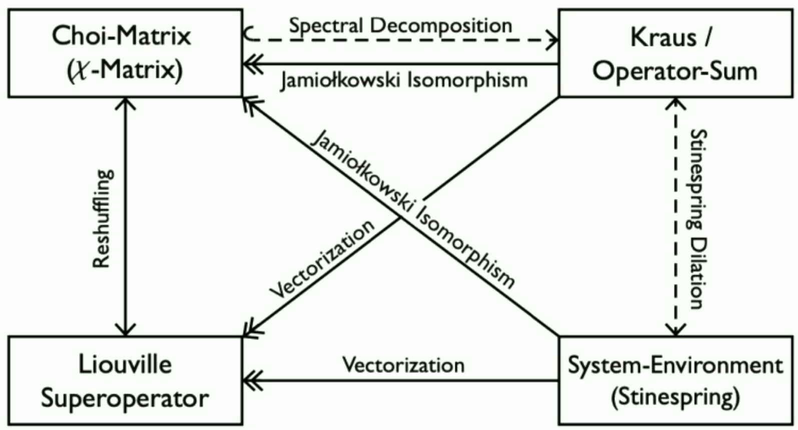
Its superoperator is

$$\mathbf{v}(\mathcal{N}) = \begin{pmatrix} \frac{3}{4} & \frac{i}{8} & -\frac{i}{8} & \frac{1}{4} \\ 0 & \frac{1}{2}-\frac{i}{8} & -\frac{i}{8} & 0 \\ 0 & \frac{i}{8} & \frac{1}{2}+\frac{i}{8} & 0 \\ \frac{1}{4} & -\frac{i}{8} & \frac{i}{8} & \frac{3}{4} \end{pmatrix}.$$

Recall

Calculation X; Properties ✓

Calculation ✓; Properties ✓; Inverse X



Calculation ✓; Properties X

How to invert a quantum channel (CPTP map)

The previous works mainly focused on CPTP inverse.

[Petz recovery]

$$\mathcal{R}^{\mathrm{P}}_{\sigma,\mathcal{N}}(\cdot) \equiv \sigma^{1/2}\mathcal{N}^{+}\left([\mathcal{N}(\sigma)]^{-1/2}(\cdot)[\mathcal{N}(\sigma)]^{-1/2}\right)\sigma^{1/2}$$

Petz, D. (1986). Sufficient subalgebras and the relative entropy of states of a von Neumann algebra. Communications in mathematical physics, 105(1), 123-131.

Petz, D. (1988). Sufficiency of channels over von Neumann algebras. The Quarterly Journal of Mathematics, 39(1), 97-108.

[Other CPTP inverse]

Unitary channels are enough for inverting qubit channels to reach the maximum average fidelity.

Karimipour, V., Benatti, F., & Floreanini, R. (2020). Quasi-inversion of qubit channels. Physical Review A, 101(3), 032109.

For a higher dimensional Φ, exist CPTP maps Ψ s.t. $\Psi \circ \Phi$ maximize the average fidelity.

Shahbeigi, F., Sadri, K., Moradi, M., Życzkowski, K., & Karimipour, V. (2021). Quasi-inversion of quantum and classical channels in finite dimensions. arXiv preprint arXiv:2104.06062.

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How to invert a quantum channel: invertible case

Directly from the representation theory of linear maps.

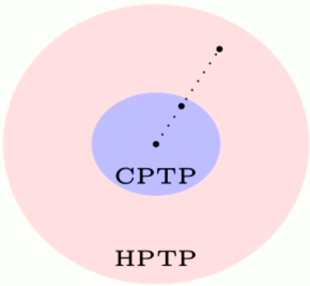
Theorem 1

The quantum channel $\mathcal{N}$ is invertible iff $\mathbf{v}(\mathcal{N})$ is an invertible matrix.

The inverse of $\mathcal{N}$ is unique, $\mathbf{v}(\mathcal{N}^{-1}) = \mathbf{v}(\mathcal{N})^{-1}$

Otherwise it is HPTP. $\Phi_{HPTP} = a_1\Phi_1 - a_2\Phi_2$.

$\dim_{in} = \dim_{out}$, only unitary channels have CPTP inverses.



Nayak, A., & Sen, P. (2006): arXiv preprint quant-ph/0605041.

Jiang, J., Wang, K., & Wang, X. (2020): Quantum 5, 600 (2021)

Regula, B., Takagi, R., & Gu, M. (2021): Quantum, 5, 522.

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Invert a quantum channel

Modulus of the non-zero part of the spectrum of a CPTP map has to be less or equal to 1.

Wolf, M. M., & Perez-Garcia, D. (2010). The inverse eigenvalue problem for quantum channels. arXiv preprint arXiv:1005.4545.

Proposition 1

If a non-zero eigenvalue λ of a quantum channel $\mathcal{N}$ has modulus less than 1 ($|\lambda| < 1$), then the inverse (or quasi-inverse) channel $\mathcal{N}^+$ is not completely positive.

- A CPTP inverse CAN NOT deal the $|\lambda_i| < 1$ parts; "Classical Mix"
  most of the noise channels have $|\lambda_i| < 1$ parts; caused by the "classical mix". e.g. $\{\sqrt{p_1}I, \sqrt{p_2}X, \sqrt{p_3}Y, \sqrt{(1-p_1-p_2-p_3)}Z\}$
- (High Level) QEC: add redundancy; reduce the classical uncertainty; recover by CPTP maps.

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How to invert a quantum channel: non-invertible cases

[Non-invertible?]

- "Zoom out": Unitary on a larger system, always invertible
- "Zoom in": A non-invertible CPTP
- Meaning: Non-local + local effects determine the dynamics of the local density matrix

[Challenges for non-invertible case]

- The generalized inverse of a non-invertible operator is not unique. (The commonly used one is Moore-Penrose Inverse.)
- The superoperator representation is not good for observing properties (CP, TP, HP) of a map.

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Example: an non-invertible channel

The Choi rep of the channel $\mathcal{N}$

$$C(\mathcal{N}) = \frac{1}{20}\left(\begin{array}{cc|cc} 8 & 0 & 1 & 6 \\ 0 & 12 & 2 & -1 \\ \hline 1 & 2 & 8 & 0 \\ 6 & -1 & 0 & 12 \end{array}\right)$$

Its superoperator

$$\mathbf{v}(\mathcal{N}) = \frac{1}{20}\begin{pmatrix} 8 & 1 & 1 & 8 \\ 0 & 6 & 2 & 0 \\ 0 & 2 & 6 & 0 \\ 12 & -1 & -1 & 12 \end{pmatrix}$$

The Moore-Penrose inverse of $\mathbf{v}(\mathcal{N})$ is

$$\mathbf{v}(\mathcal{N}^p) = \begin{pmatrix} \frac{115}{294} & \frac{10}{441} & \frac{10}{441} & \frac{505}{882} \\ \frac{50}{147} & \frac{3245}{882} & -\frac{1165}{882} & -\frac{100}{441} \\ \frac{50}{147} & -\frac{1165}{882} & \frac{3245}{882} & -\frac{100}{441} \\ \frac{115}{294} & \frac{10}{441} & \frac{10}{441} & \frac{505}{882} \end{pmatrix},$$

and its Choi representation is

$$C(\mathcal{N}^p) = \left(\begin{array}{cc|cc} \frac{115}{294} & \frac{50}{147} & \frac{10}{441} & \frac{3245}{882} \\ \frac{50}{147} & \frac{115}{294} & -\frac{1165}{882} & \frac{10}{441} \\ \hline \frac{10}{441} & -\frac{1165}{882} & \frac{505}{882} & -\frac{100}{441} \\ \frac{3245}{882} & \frac{10}{441} & -\frac{100}{441} & \frac{505}{882} \end{array}\right)$$

which is Hermitian preserving but **not trace preserving**.

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The Drazin Inverse

[Intuition] Superoperator's eigen-structure should carry some info; try my best to preserve it! (arXiv:1005.4545)
[A small challenge] Superoperator could be defective or non-normal (doesn't have the spectrum decomposition).

The Jordan decomposition of $\mathbf{v}(\mathcal{N})$:

$$\mathbf{v}(\mathcal{N}) = Q \cdot J \cdot Q^{-1}$$

The Drazin Inverse $\mathcal{N}^+$

$$\mathbf{v}(\mathcal{N}^+) = Q \cdot J' \cdot Q^{-1}.$$

$$J_{\lambda_i} = \begin{pmatrix} \lambda_i & 1 & & \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{pmatrix},$$

let the corresponding block J'_{λ_i} in J' be the inverse of J_{λ_i}

$$J'_{\lambda_i} := J_{\lambda_i}^{-1} = \begin{pmatrix} \frac{1}{\lambda_i} & -\frac{1}{\lambda_i^2} & & \cdots & (-1)^{k+1}\frac{1}{\lambda_i^k} \\ & \frac{1}{\lambda_i} & -\frac{1}{\lambda_i^2} & \cdots & (-1)^k\frac{1}{\lambda_i^{k-1}} \\ & & \ddots & \ddots & \vdots \\ & & & \frac{1}{\lambda_i} & -\frac{1}{\lambda_i^2} \\ & & & & \frac{1}{\lambda_i} \end{pmatrix}.$$

And $J'_0 = 0_{k_0}$.

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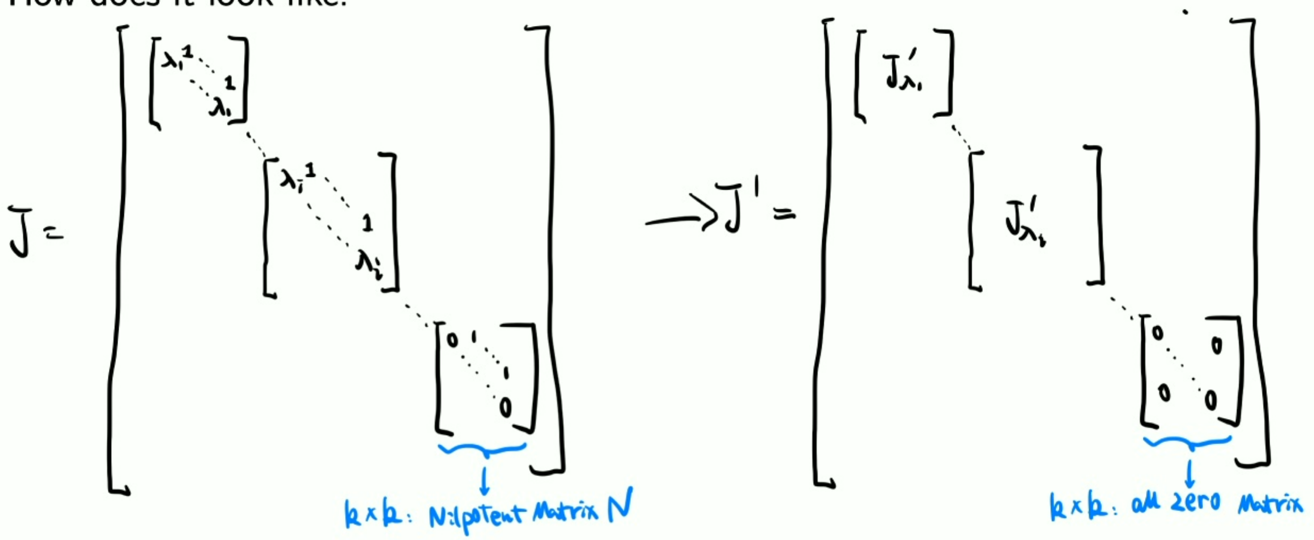
How does it look like:



[Another challenge] It seems helpless for proving any property from the superoperator representation. Because it is reshuffling the entries from the Choi rep, and we always say shuffling the matrix entries completely destroys the structure of a Matrix.

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The Drazin Inverse is TP

Denote the trace operation in the vector representation $\mathbf{v}(A)$ of a d by d matrix A as $\mathrm{sTr}[\cdot]$, where $\mathrm{sTr}[\mathbf{v}(A)] = \mathrm{Tr}(A)$.

Lemma 1

If a linear map $\mathcal{N}: M_d \to M_d$ is trace preserving, the eigenvectors v and generalized eigenvectors v^g of eigenvalue $\lambda \neq 1$ of the superoperator $\mathbf{v}(\mathcal{N})$ is trace zero, i.e. $\mathrm{sTr}[v] = \mathrm{sTr}[v^g] = 0$.

Lemma 2

For a trace persevering linear map $\mathcal{N}: M_d \to M_d$, if there is a k by k $(k > 1)$ defective Jordan Block of eigenvalue $\lambda = 1$ in $\mathbf{v}(\mathcal{N})$, the eigenvector v and first $k-2$ generalized eigenvector v^{g_i} has to be trace zero, i.e. $\mathrm{sTr}[v] = \mathrm{sTr}[v^{g_i}] = 0$ for $i \in \{1, \cdots, k-2\}$.

Theorem 2

The Drazin-inverse $\mathcal{N}^+$ of a trace preserving map $\mathcal{N}$ is also trace preserving.

The non-invertible example

The Choi rep of the channel $\mathcal{N}$

$$C(\mathcal{N}) = \frac{1}{20}\left(\begin{array}{cc|cc} 8 & 0 & 1 & 6 \\ 0 & 12 & 2 & -1 \\ \hline 1 & 2 & 8 & 0 \\ 6 & -1 & 0 & 12 \end{array}\right)$$

Its superoperator

$$\mathbf{v}(\mathcal{N}) = \frac{1}{20}\begin{pmatrix} 8 & 1 & 1 & 8 \\ 0 & 6 & 2 & 0 \\ 0 & 2 & 6 & 0 \\ 12 & -1 & -1 & 12 \end{pmatrix}$$

$J = \mathrm{diag}(0, 1, \frac{2}{5}, \frac{1}{5})$, and $J' = \mathrm{diag}(0, 1, \frac{5}{2}, 5)$.

The superoperator of Drazin Inverse $\mathcal{N}^+$ is

$$\mathbf{v}(\mathcal{N}^+) = \begin{pmatrix} \frac{2}{5} & \frac{5}{16} & \frac{5}{16} & \frac{2}{5} \\ 0 & \frac{15}{4} & -\frac{5}{4} & 0 \\ 0 & -\frac{5}{4} & \frac{15}{4} & 0 \\ \frac{3}{5} & -\frac{5}{16} & -\frac{5}{16} & \frac{3}{5} \end{pmatrix}$$

The Choi representation of $\mathcal{N}^+$ is

$$C(\mathcal{N}^+) = \left(\begin{array}{cc|cc} \frac{2}{5} & 0 & \frac{5}{16} & \frac{15}{4} \\ 0 & \frac{3}{5} & -\frac{5}{4} & -\frac{5}{16} \\ \hline \frac{5}{16} & -\frac{5}{4} & \frac{2}{5} & 0 \\ \frac{15}{4} & -\frac{5}{16} & 0 & \frac{3}{5} \end{array}\right)$$

The Drazin Inverse $\mathcal{N}^+$ is trace preserving, Hermitian preserving, but not completely positive.

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A new perspective of the Superoperator

The superoperator is not a representation only for calculation. Its eigen-structure carries the properties of the channel. In fact, it is the best representation to do matrix decomposition.

Directly working with superoperators is largely unexplored territory.
The new perspective can bridge the study of generalized channel inversions with the well-studied matrix generalized inversion theory.

Moore-Penrose Inverse

Moore-Penrose inverse $\mathcal{N}^p$ of a CPTP map $\mathcal{N}$ is HP but may not be TP.
The composed map $\mathcal{N}^p \circ \mathcal{N}$ is HPTP and unital.

Maximal Rank Inverse

Maximal Rank Inverse $\mathcal{N}^m$, always HPTP.

Beyond CPTP maps, Cao N., Fitzsimmons M., Poon Y.-T., Laflamme R. (To be on arXiv)

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What would happen if we did not know the noise so well?

What if: imperfect noise characterization?

[Setting]

$$\rho_{\text{out}}^{\text{ideal}} = \mathcal{U}_n \circ \cdots \circ \mathcal{U}_1(\rho_{\text{in}}).$$
$$\rho_{\text{out}}^{\text{exp}} = \mathcal{N}_n \circ \mathcal{U}_n \circ \cdots \circ \mathcal{N}_1 \circ \mathcal{U}_1(\rho_{\text{in}})$$
$$\rho_{\text{EM}} = \mathcal{E}_{\text{EM}}^{-1}(\rho_{\text{out}}^{\text{exp}})$$

[Motivation] Many protocols assume that we know the noise perfectly. But **what if we don't?**
How would the imperfect knowledge on noise effect $\mathcal{E}_{\text{EM}}^{-1}$ and then effect EM?

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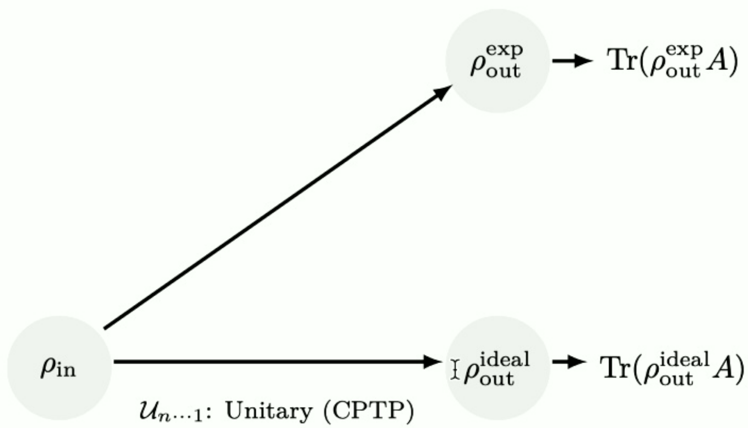
The Inverse Map Structure

$$\rho_{\text{out}}^{\text{exp}} = \mathcal{N}_n \circ \mathcal{U}_n \circ \cdots \circ \mathcal{N}_1 \circ \mathcal{U}_1(\rho_{\text{in}})$$

$$\rho_{\text{EM}} = \mathcal{E}_{\text{EM}}^{-1}(\rho_{\text{out}}^{\text{exp}})$$



where $\mathcal{U}_{1\cdots n} := \mathcal{U}_n \circ \cdots \circ \mathcal{U}_1$.

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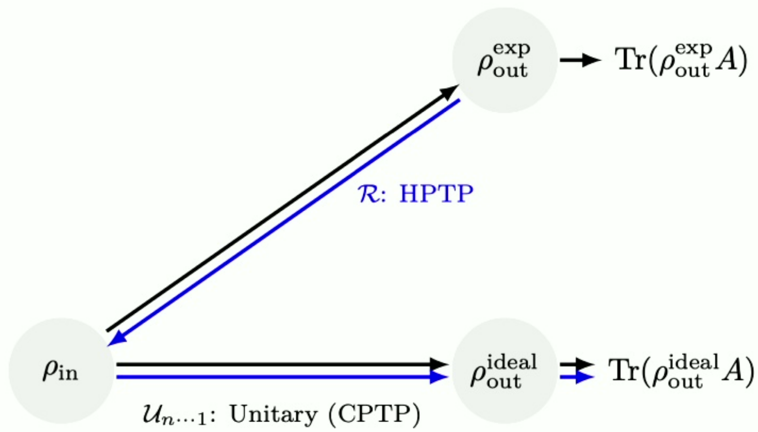
The Inverse Map Structure



where $\mathcal{U}_{1\dots n} := \mathcal{U}_n \circ \cdots \circ \mathcal{U}_1$. $\mathcal{E}_{EM}^{-1} = \mathcal{U} \circ \mathcal{R}$

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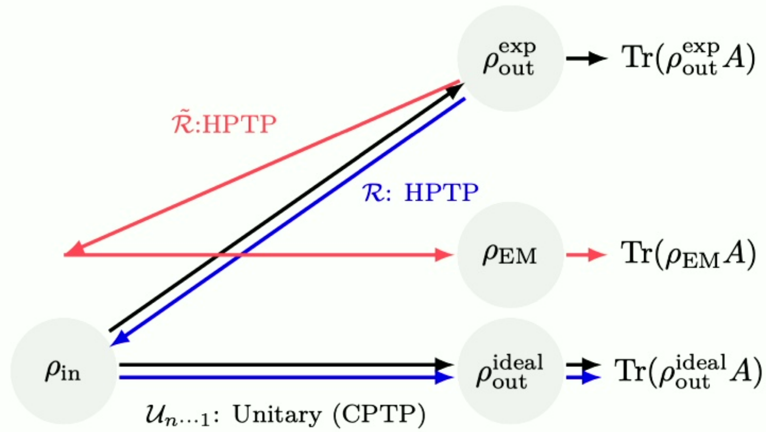
The Inverse Map Structure



where $\mathcal{U}_{1\dots n} := \mathcal{U}_n \circ \cdots \circ \mathcal{U}_1$, $\mathcal{E}_{EM}^{-1} = \mathcal{U} \circ \tilde{\mathcal{R}}$.

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State Fidelity Bounds

Proposition 2

The first order estimation of fidelity between ρ_{EM} and $\rho_{\text{out}}^{\text{ideal}}$ is

$$\left(1-\frac{1}{2}\sqrt{d}C_{\text{exp}}\left\|\mathbf{v}(\Delta\mathcal{N}^{(1)})\right\|\right)^2 \leq F^{(1)}(\rho_{\text{EM}},\rho_{\text{out}}^{\text{ideal}})$$
$$\leq 1-\frac{1}{4}\left(l_U\cdot\left\|\mathbf{v}(\Delta\mathcal{N}^{(1)})\mathbf{v}(\rho_{\text{out}}^{\text{exp}})\right\|\right)^2, \tag{1}$$

where $C_{\text{exp}} := \|\mathbf{v}(\mathcal{U}_{n\cdots 1})\|\cdot\|\mathbf{v}(\rho_{\text{out}}^{\text{exp}})\|$ is an experiment-related constant, and $l_U := \inf_{\|x\|=1}\|\mathbf{v}(\mathcal{U}_{n\cdots 1})x\|$ is the lower Lipschitz constant of the ideal operations $\mathcal{U}_{n\cdots 1}$. The norm $\|\cdot\|$ is 2-norm for vectors and is the induced matrix norm for matrices.

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Expectation Values Bound

Proposition 3

If the following condition eq. (2) is satisfied, EM is guaranteed to improve the expectation value of any observable A for any circuit $\mathcal{U}_{n\cdots 1}$.

$$\|\mathbf{v}(\Delta\mathcal{N})\| \leq l_{\text{ideal-exp}}, \tag{2}$$

where $l_{\text{ideal-exp}} := \inf_{\|x\|=1} \left\|\mathbf{v}((\mathcal{U}^{\dagger}\mathcal{N}^{-1})_{1\cdots n} - \mathcal{U}^{\dagger}_{1\cdots n})x\right\|$ is the lower Lipschitz constant of $\mathbf{v}((\mathcal{U}^{\dagger}\mathcal{N}^{-1})_{1\cdots n} - \mathcal{U}^{\dagger}_{1\cdots n})$.

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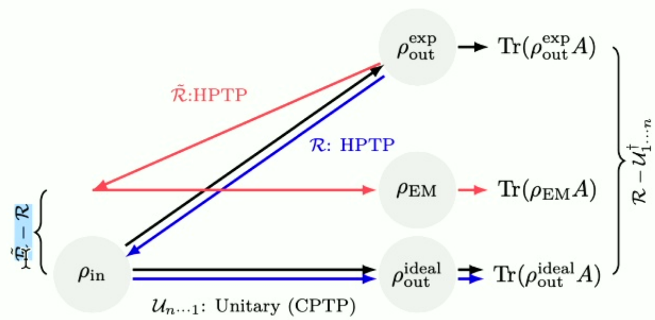

The idea behind Proposition 2

By bounding $\tilde{\mathcal{R}} - \mathcal{R}$, one can bound the distance between ρ_{EM} and $\rho_{\text{out}}^{\text{ideal}}$.

The idea behind Proposition 3

To grantee a improvement on the outcomes of EM, one need to have $\tilde{\mathcal{R}} - \mathcal{R}$ "less than" $\mathcal{R} - \mathcal{U}_{1\cdots n}^{\dagger}$.

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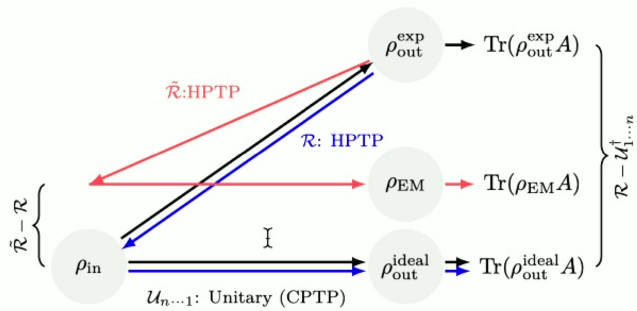

[More]

- It could be harder than directly classically simulate the ideal circuit;
- It is a competition between experimental precision and noise characterization.

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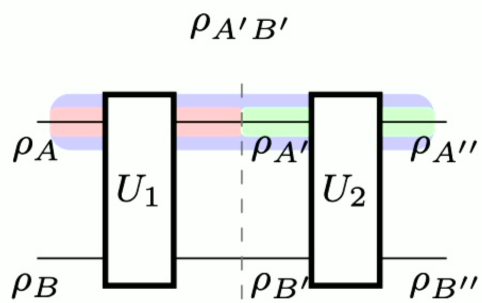
Beyond CPTP maps



If $\rho_{AB} = \rho_A \otimes \rho_B$, Φ_1^A and Φ_{21}^A are CPTP maps.

- If Φ_1^A is invertible,

$$\Phi_2^A := \Phi_{21}^A \circ (\Phi_1^A)^{-1}.$$

  We have $\rho_{A''} = \Phi_2^A(\rho_{A'})$. A local map characterizes the green process. Φ_2^A can be (non-CP) HPTP sometimes since $(\Phi_1^A)^{-1}$ is HPTP.
- An example: let $U_2 = U_1^\dagger$, then $\Phi_{21}^A = \mathbb{I}$, and Φ_2^A is HPTP.
- Non-Markovian (CP-divisible) Phys. Rev. Lett. 105, 050403; Quantum, 5, 522.

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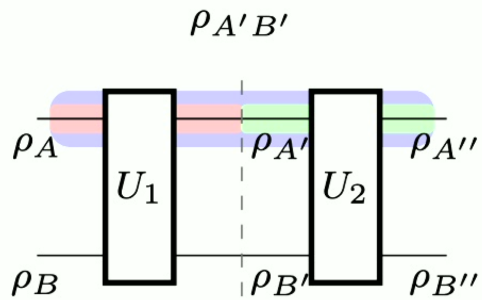
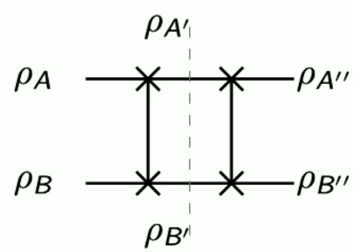


- If Φ_1^A is non-invertible, there is no local linear map can capture the process. The information non-local to system A is "irreplaceable" for $\rho_{A'} \to \rho_{A''}$.
- An simple example:



Wait, any contradiction? No.

- HPTP: information non-local to the subsystem
  Still cannot use this piece of information
- (Non-CP) HPTP is "physical" but not operational

- Now the question is:
  How would this fact changing QEC, QEM, noise characterization and learning etc.?

Beyond CPTP maps, Cao N., Fitzsimmons M., Poon Y.-T., Laflamme R. (To be on arXiv)

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Conclusions and Outlook

Take home messages

- QEC:

$$\rho_{\text{out}} = R_n \circ \mathcal{N}_n \circ \mathcal{U}_n \circ \cdots \circ R_1 \circ \mathcal{N}_1 \circ \mathcal{U}_1 (\rho_{in})$$

  Active QEM:

$$\rho_{\text{EM}} = \mathcal{E}_{\text{EM}}^{-1}(\rho_{\text{out}}^{\text{exp}}) = \widehat{\mathcal{N}}_1^{-1} \circ \widehat{\mathcal{N}}_2^{-1} \circ \cdots \circ \widehat{\mathcal{N}}_n^{-1}(\rho_{\text{out}}^{\text{exp}})$$

- Active QEM can not preserve quantum resources such as entanglement;
- A new (and better) way to built channel inverse;
- Completing the picture of channel representation;
- Upper bound the performance of EM protocols from the knowledge of noise channels;
- (Non-CP) HPTP can be "physical" (non-Markovian)

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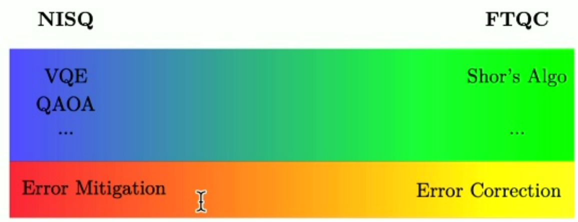
Outlook

A lot more questions can be ask and need to be answered, some examples

- More protocols that merging QEM and QEC



$\Uparrow$

QEC + QEM

(e.g. Phys. Rev. Lett. 127, (2021); PRX QUANTUM 3, 010345 (2022))

- New protocols: quantum operations that not denoise, but make the classical post-processing easy.
- Robustness, Computational efficiency, Sampling cost, etc.
- Non-CP HPTP (a subset of HPTP) can be physical. How would it change QEC, QEM, noise characterization, learning...
- ...

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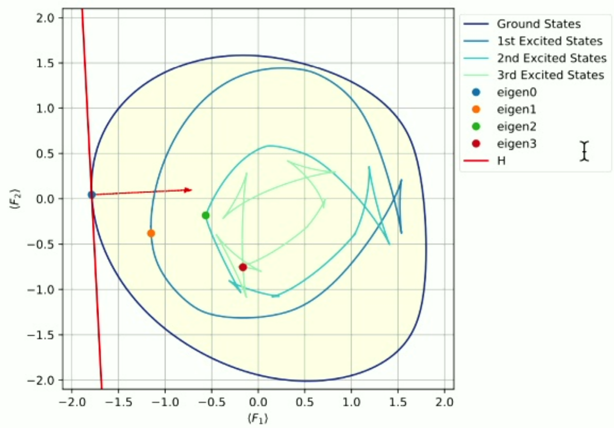
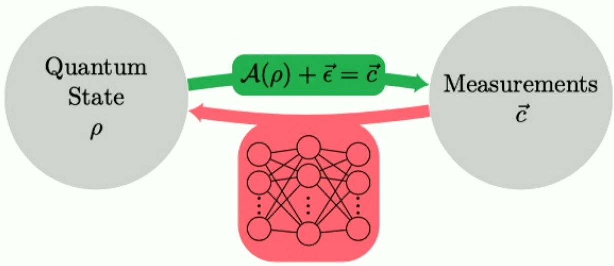
Thank you!



Joint Algebraic Numerical Range and Measurements in Quantum Information



Classical Neural Networks for Quantum Inverse Problems

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