Title: Emergent classical gauge symmetry from quantum entanglement

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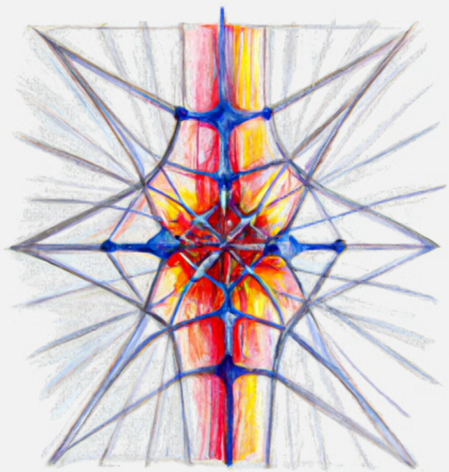
Abstract: Inspired by the emergence of bulk diffeomorphism invariance in holography, I will give an explicit description how entanglement between quantum subsystems can lead to emergent gauge symmetry in a classical limit. Along the way, I will provide a precise characterisation of when it is consistent to treat a quantum subsystem classically in such a limit, and show that this gives strong constraints on the entanglement structure of classical states. I will explain how this generically leads to emergent fundamentally non-local classical degrees of freedom, which may nevertheless be accounted for in a kinematically local way if one employs an appropriately redundant description. The mechanism I describe is general and elementary, but for concreteness I will exhibit a toy example involving three entangled spins at high angular momentum, and I will also describe a significant generalisation of this toy example based on coadjoint orbits. If there is time, I will discuss evidence for the role this phenomenon plays in gravity. This talk is based on arXiv:2209.03979.

Zoom link: https://pitp.zoom.us/j/92066956880?pwd=OTRySTlOVGgvM3RCRmkzWHFVSUF3Zz09

Emergent classical gauge symmetry from quantum entanglement



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Seminar at Perimeter Institute, 15th December 2022

Any theory of quantum gravity must in some way account for *bulk diffeomorphism invariance*. There are basically two options:

1. Diffeomorphism invariance is *fundamentally* part of the quantum theory.
2. Diffeomorphism invariance is only *emergent* in the semiclassical regime.

Evidence to take the latter seriously: AdS/CFT. The bulk spacetime itself is emergent at large N. So diffeomorphisms of that bulk are also emergent.

Diffeomorphism invariance is a kind of gauge symmetry. What does it mean for a gauge symmetry to be emergent?

Gauge symmetries are *redundancies* in our description of reality, whose purpose is to allow for a better conceptual and computational grasp of the underlying physics.

In differing regimes, we may use different descriptions of reality, which are redundant in different ways.

A gauge symmetry is *emergent* if our description of physics in a more fundamental regime is less redundant than our description in a less fundamental regime.

quantum theory —*classical limit*→ classical theory

less redundant description → more redundant description (*emergent gauge symmetry*)

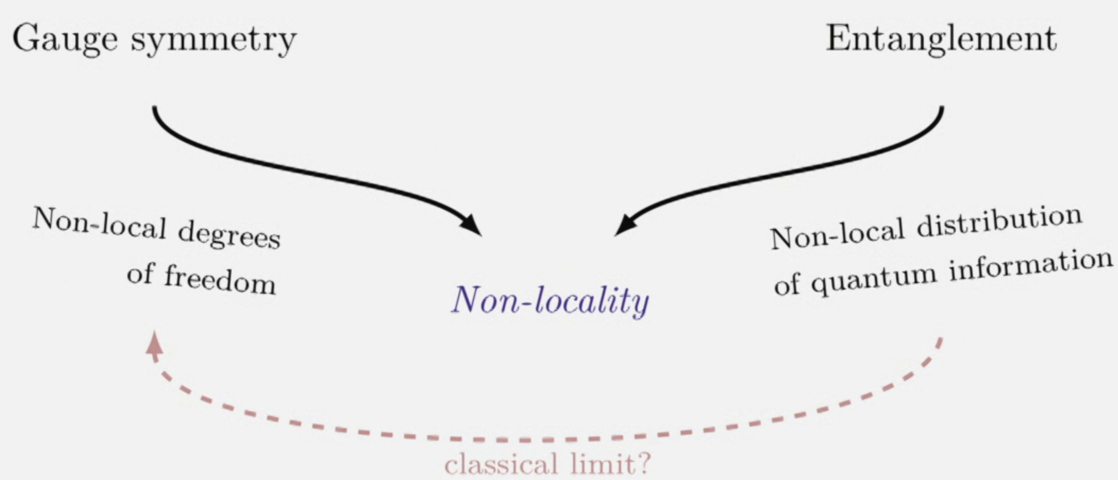
In field theory and gravity, gauge symmetries allow us to use mathematically local structures to describe non-local degrees of freedom.

So, instead of *emergent gauge symmetry*, you may also think of this talk as being about *emergent non-local degrees of freedom*.

[Rovelli, 2013, *"Why gauge?"*] [Witten, 2016, *"Symmetry and Emergence"*]

In holography, the emergence of the bulk appears to be largely quantum information theoretic in nature.

I will focus on one aspect of this story: the link between *gauge symmetry* and *entanglement*. Motivation comes from gravity, but I won't restrict to the gravitational setting.



Quantum entanglement would give rise to emergent classical gauge symmetry.

This talk: Identify the mechanism for this to happen.

Precise and elementary, but also very general (not model-specific).

Takeaways:

- You don't need something like traditional constraint quantisation to quantise a theory with gauge symmetry — you can use entanglement instead.
- The structure of multipartite entanglement simplifies significantly in the classical limit — you can describe it with gauge symmetry.

Toy example: three entangled spins

Consider a spin j with Hilbert space $\mathcal{H}$. Let $\hat{\mathbf{J}}$ be the angular momentum operator, and let $|j, m\rangle$ be usual eigenbasis.

Spin coherent states are an overcomplete basis of $\mathcal{H}$:

$$|\mathbf{n}\rangle = \hat{D}(\mathbf{n})\,|j, -j\rangle\,, \quad \text{where} \quad \hat{D}(\mathbf{n}) = \exp\!\left(i\frac{\theta}{\sin(\theta)}(\mathbf{n}_i \times \mathbf{e}^3)\cdot\hat{\mathbf{J}}\right),$$

where $\mathbf{n}$ is a unit 3-vector of angle θ from $\mathbf{e}^3 = (0, 0, 1)$.

These states provide us with a notion of a classical limit at large j.

$$\lim_{j\to\infty} |\langle\mathbf{n}|\mathbf{n}'\rangle|^2 = \begin{cases} 1 & \text{if } \mathbf{n} = \mathbf{n}', \\ 0 & \text{otherwise.} \end{cases}$$

The classical state space is S^2, and any classical observable $A : S^2 \to \mathbb{C}$ has an operator representation

$$\hat{A} = \int_{S^2} \frac{\mathrm{d}^2\mathbf{n}}{4\pi}\,(2j+1)\,|\mathbf{n}\rangle\langle\mathbf{n}|\,A(\mathbf{n}),$$

such that

$$\hat{A}\,|\mathbf{n}\rangle \approx A(\mathbf{n})\,|\mathbf{n}\rangle\,.$$

Classical limit of unentangled spins

Suppose we have three spins j_1, j_2, j_3.

We can get a classical limit by defining

$$|\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3\rangle = |\mathbf{n}_1\rangle \otimes |\mathbf{n}_2\rangle \otimes |\mathbf{n}_2\rangle$$

These states satisfy

$$\lim_{j_1, j_2, j_3 \to \infty} |\langle \mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3 | \mathbf{n}'_1, \mathbf{n}'_2, \mathbf{n}'_3\rangle|^2 = \begin{cases} 1 & \text{if } \mathbf{n}_i = \mathbf{n}'_i, \quad i = 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$

The classical state space is $S^2 \times S^2 \times S^2$.

Classical limit of entangled spins

Let $|0,0\rangle \in \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ be the unique state of zero angular momentum.

This is a highly entangled state:

$$|0,0\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_1=-j_2}^{j_2} \sum_{m_3=-j_3}^{j_3} \underbrace{\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}}_{\text{Wigner } 3j\text{-symbol}} |j_1, m_1\rangle \otimes |j_2, m_2\rangle \otimes |j_3, m_3\rangle \,.$$

Act with $\mathrm{SU}(2)$ representations of the first two spins:

$$|\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2\rangle = \big(U_1(\psi_1, \mathbf{m}_1) \otimes U_2(\psi_2, \mathbf{m}_2) \otimes \mathbb{1}_3\big) \, |0,0\rangle \,.$$

where $U_i(\psi_i, \mathbf{m}_i) = \exp\!\big(2i\psi_i \, \mathbf{m} \cdot \hat{\mathbf{J}}^i\big)$.

Here $\psi_1, \psi_2 \in [0, \pi)$ and $\mathbf{m}_1, \mathbf{m}_2$ are unit 3-vectors. Each of $(\psi_1, \mathbf{m}_1)$ and $(\psi_2, \mathbf{m}_2)$ parametrise points on S^3, the group manifold of $\mathrm{SU}(2)$.

Thus, we have an $S^3 \times S^3$ of such states.

Classical limit of entangled spins

These states form an overcomplete basis for $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$. Moreover:

$$\lim_{j_1,j_2,j_3\to\infty} \left|\langle \psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2 | \psi_1', \psi_2'; \mathbf{m}_1', \mathbf{m}_2' \rangle\right|^2 = \begin{cases} 1 & \text{if } \psi_i = \psi_i' \text{ and } \mathbf{m}_i = \mathbf{m}_i', \\ 0 & \text{otherwise.} \end{cases}$$

Thus, these states give a classical limit, with classical state space $S^3 \times S^3$.

Classical observables $A : S^3 \times S^3 \to \mathbb{C}$ may be represented as operators:

$$\hat{A} = \int_{S^3 \times S^3} \mathrm{d}\mu \; A(\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2) \; |\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2\rangle \, \langle \psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2| \, ,$$

(for some measure μ) which obey

$$\hat{A} \, |\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2\rangle \approx A(\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2) \, |\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2\rangle \, .$$

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Thus, we have an $S^3 \times S^3$ of such states.

Two classical limits of the same quantum system:

states:	$\|\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3\rangle$	$\|\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2\rangle$
regime:	$j_1, j_2, j_3 \to \infty$	$j_1, j_2, j_3 \to \infty$
entanglement:	*separable*	*highly entangled*
Hilbert space:	$\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$	$\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$
classical state space:	$S^2 \times S^2 \times S^2$	$S^3 \times S^3$
local structure:	*preserved*	*?*

What happened to the local structure (i.e. decomposition into three spins)?

To answer this, consider

$$\rho_i(\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2) = \mathrm{tr}_{\bar{i}} \left|\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2\right\rangle \left\langle\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2\right|,$$
$$\rho_{ij}(\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2) = \mathrm{tr}_{\overline{ij}} \left|\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2\right\rangle \left\langle\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2\right|,$$

i.e. the reduced states of each spin i and pair of spins ij.

One may show:

$$\rho_i(\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2) = \frac{\mathbb{1}_i}{2j_i + 1},$$

so there is actually *only one* possible classical state for each individual spin.

Similarly (w.l.o.g. consider $ij = 23$):

$$\rho_{23}(\psi_1, \psi_2; \mathbf{m}_1, \mathbf{m}_2) \approx \frac{\hat{\pi}_{23}(\psi_1, \mathbf{m}_1)}{2j_1 + 1},$$

where

$$\hat{\pi}_{23}(\psi, \mathbf{m})\hat{\pi}_{23}(\psi', \mathbf{m}') \approx \delta_{\psi\psi'}\delta_{\mathbf{m}\mathbf{m}'}\hat{\pi}_{23}(\psi, \mathbf{m})$$

are a set of *approximately mutually orthogonal projection operators*, labelled by points in S^3. Thus, there are S^3 possible classical states for each pair of spins.

Aside: 'classically resolvable'. See later.

To recover the local structure we can use gauge symmetry.

For each spin define a space of *kinematical* states $\mathcal{N}_i^{\text{kin.}} = \text{SU}(2) = S^3$, and introduce a gauge group $G = \text{SU}(2)$ that acts simultaneously (i.e. diagonally) from the right.

Then the space of *physical* states for the full system is

$$\frac{\mathcal{N}_1^{\text{kin.}} \times \mathcal{N}_2^{\text{kin.}} \times \mathcal{N}_3^{\text{kin.}}}{G} = \frac{\text{SU}(2) \times \text{SU}(2) \times \text{SU}(2)}{\text{SU}(2)} = \text{SU}(2) \times \text{SU}(2) = S^3 \times S^3.$$

Similarly, the space of *physical* states of each individual spin is a singleton:

$$\frac{\mathcal{N}_i^{\text{kin.}}}{G} = \frac{\text{SU}(2)}{\text{SU}(2)},$$

while the space of *physical* states for each pair of spins is an S^3:

$$\frac{\mathcal{N}_i^{\text{kin.}} \times \mathcal{N}_j^{\text{kin.}}}{G} = \frac{\text{SU}(2) \times \text{SU}(2)}{\text{SU}(2)} = \text{SU}(2) = S^3,$$

as required. This S^3 parametrises bilocal degrees of freedom: the kinematical state of one spin relative to the kinematical state of another.

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entanglement:	*separable*	*highly entangled*	
Hilbert space:	$\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$	$\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$	$\leftarrow$ no constraints
classical state space:	$S^2 \times S^2 \times S^2$	$S^3 \times S^3$	
local structure:	*preserved*	*preserved*	
gauge symmetry:	*none*	SU(2)	

Thus, *quantum entanglement* has led to *emergent classical gauge symmetry*.

This was a toy example. For most of the rest of the talk, I will describe the general mechanism underlying this phenomenon.

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Classically resolvable subsystems

Locality and the classical limit

Physical systems have 'local structure' if they are *composite*, i.e. divisible into subsystems (e.g. the spins, subregion in QFT, ...).

Each subsystem s has a set of observables $\mathcal{O}_s$ 'local to' that subsystem.

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If $A \in \mathcal{O}_{s \cup s'}$ cannot be formed as a combination of observables in $\mathcal{O}_s$ and $\mathcal{O}_{s'}$, then A measures *non-local* degrees of freedom (e.g. A is a Wilson line).

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To understand *emergent gauge symmetry*, we need to know what happens to *quantum* subsystems in a *classical limit*. Actually not all quantum subsystems will be well-behaved in this limit (e.g. very small subregion in QFT).

What is a quantum subsystem?

A quantum subsystem is a *von Neumann algebra* $\mathcal{A}_s \subset L(\mathcal{H})$, where $\mathcal{H}$ is the Hilbert space of the full system.

In this talk: assume $\dim(\mathcal{H})$ finite. Will also assume no pre-existing quantum gauge symmetry. This implies $\mathcal{A}_s$ is a *Type I factor*. Can then write:

$$\mathcal{A}_s = \mathcal{B}(\mathcal{H}_s) \otimes \mathbb{1}_{\bar{s}}, \quad \text{where} \quad \mathcal{H} = \overbrace{\mathcal{H}_s}^{\text{Hilbert space of } s} \otimes \underbrace{\mathcal{H}_{\bar{s}}}_{\substack{\text{Hilbert space of} \\ \text{complement of } s}}$$

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More generally:

$$\mathcal{A}_i = \mathbb{1}_1 \otimes \cdots \otimes \mathbb{1}_{i-1} \otimes \mathcal{B}(\mathcal{H}_i) \otimes \mathbb{1}_{i+1} \otimes \cdots \otimes \mathbb{1}_n,$$

where

$$\mathcal{H} = \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_i \otimes \cdots \otimes \mathcal{H}_n.$$

is a tensor factorisation into subsystem Hilbert spaces $\mathcal{H}_i$.

What is a classical limit?

Let $\mathcal{N}$ be space of classical states, $C(\mathcal{N})$ be set of functions $\mathcal{N} \to \mathbb{C}$, and pick a set of operators $\mathcal{C} \subset L(\mathcal{H})$ that is 'approximately isomorphic' to $C(\mathcal{N})$:

$$C(\mathcal{N}) \to \mathcal{C}, \quad A \mapsto \hat{A},$$

i.e.

$$\widehat{A^*} \approx \hat{A}^\dagger, \quad \widehat{AB} \approx \hat{A}\hat{B}, \quad \widehat{(\alpha A + \beta B)} \approx \alpha^* \hat{A}^\dagger + \beta^* \hat{B}^\dagger, \quad \text{etc} \ldots.$$

("$\approx$" denotes equality in the *classical limit* $\chi \to 0$ where $\chi = \hbar, G, 1/N^2, \ldots$.)

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("$\approx$" denotes equality in the *classical limit* $\chi \to 0$ where $\chi = \hbar, G, 1/N^2, \ldots$.)

At leading order in χ, the map $A(x) \mapsto \hat{A}$ is implemented by

$$\hat{A} \approx \int_{\mathcal{N}} \mathrm{d}\mu(x)\, \frac{N}{N(x)} \hat{\pi}(x)\, A(x).$$

for some measure μ on $\mathcal{N}$ and projection operators $\hat{\pi}(x)$ obeying

$$\hat{\pi}(x)\hat{\pi}(y) \approx \delta_{xy}\hat{\pi}(x), \quad \text{for all} \quad x, y \in \mathcal{N},$$

Here, $N = \dim(\mathcal{H})$ and $N(x) = \mathrm{rank}(\hat{\pi}(x))$.

‘Complete’ classical limit

By measuring $\hat{\pi}(x)$ we can determine with high precision if the state of the classical degrees of freedom is x.

If $N(x) = \text{rank}(\hat{\pi}(x)) > 1$, then there is more than one quantum state consistent with a given classical state x. This indicates that there are still some ‘left over’ quantum degrees of freedom in the classical limit.

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If $N(x) = \text{rank}(\hat{\pi}(x)) > 1$, then there is more than one quantum state consistent with a given classical state x. This indicates that there are still some ‘left over’ quantum degrees of freedom in the classical limit.

A ‘complete’ classical limit is one for which $N(x) = 1$, so that classical degrees of freedom suffice to determine the full state. Then we may write

$$\hat{\pi}(x) = |x\rangle \langle x| \, ,$$

and

$$\hat{A} \approx \int_{\mathcal{N}} \mathrm{d}\mu(x) \, N \, |x\rangle \langle x| \, A(x),$$

and we have

$$\hat{A} \, |x\rangle \approx A(x) \, |x\rangle \text{ for all } \hat{A} \in \mathcal{C}, \, x \in \mathcal{N}.$$

What is the classical limit of a subsystem?

Two sets of operators:

- $\mathcal{A}_s = \mathcal{B}(\mathcal{H}_s) \otimes \mathbb{1}_{\bar{s}}$ defining quantum subsystem s.
- $\mathcal{C}$ defining (complete) classical limit.

To understand the classical limit of the subsystem, consider the intersection:

$$\mathcal{C}_s = \mathcal{A}_s \cap \mathcal{C}.$$

This consists of operators measuring classical degrees of freedom in s.

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Gelfand-Naimark theorem implies $\mathcal{C}_s$ is approximately isomorphic to an algebra $C(\mathcal{N}_s)$ of functions on a space $\mathcal{N}_s$ of classical subsystem states.

Explicitly:

$$\mathcal{N}_s = \left\{ x_s \mid x_s : \hat{A} \mapsto \langle x|\hat{A}|x\rangle, x \in \mathcal{N} \right\} \subset \mathcal{C}_s^*.$$

The classical observable $A_s \in C(\mathcal{N}_s)$ corresponding to $\hat{A} \in \mathcal{C}_s$ is defined via $A_s(x_s) = x_s(\hat{A})$.

This construction of $\mathcal{N}_s$ works for any quantum subsystem. But in general $\mathcal{N}_s$ doesn't fully account for physics in s.

It is not guaranteed that knowledge of *classical* degrees of freedom in s suffices to determine *complete state* of s.

So a complete classical limit for the full system does not necessarily imply its quantum subsystems behave in a completely classical way.

Consider classical subsystem operators:

$$\mathcal{C}_s = \left\{ \hat{A} \mid \hat{A} = \hat{A}_s \otimes \mathbb{1}_{\bar{s}} \approx \int_{\mathcal{N}} \mathrm{d}\mu(x)\, N \, |x\rangle \langle x| \, A(x) \right\}$$

In an extreme case: the only $\hat{A}_s$ satisfying this condition is $\hat{A}_s \propto \mathbb{1}_s$. Then there is only one element in $\mathcal{N}_s$, i.e. no classical degrees of freedom.

We would need to describe the subsystem in a completely quantum way.

Classical resolvability

We are interested in the opposite case, where classical degrees of freedom suffice to describe subsystem.

Suppose we know full system is in some classical state in $\mathcal{N}$ (but we don't know which one).

We will say subsystem s is *classically resolvable* if knowledge of x_s suffices to determine its quantum state ρ_s (to a high degree of accuracy in the classical limit).

More precisely, suppose $x, y \in \mathcal{N}$ correspond to $x_s, y_s \in \mathcal{N}_s$. We already know:

$$\rho_s(x) = \rho_s(y) \implies x_s = y_s$$

(by definition of x_s, y_s). Subsystem s is *classically resolvable* if reverse is true:

$$x_s = y_s \implies \rho_s(x) = \rho_s(y).$$

States of classically resolvable subsystem

Suppose s classically resolvable, $\hat{A} \in \mathcal{C}_s$.

$$\hat{A} = \hat{A}_s \otimes \mathbb{1}_{\bar{s}} \approx \int_{\mathcal{N}} \mathrm{d}\mu(x)\, N\, |x\rangle \langle x|\, A(x). \qquad (*)$$

$\frac{1}{N_{\bar{s}}} \operatorname{tr}_{\bar{s}}$ both sides, where $N_{\bar{s}} = \dim(\mathcal{H}_{\bar{s}})$:

$$\hat{A}_s \approx \int_{\mathcal{N}} \mathrm{d}\mu(x)\, N_s\, \rho_s(x) A_s(x_s),$$

where $N_s = \dim(\mathcal{H}_s)$, and $A_s(x_s) = x_s(\hat{A}_s) = \langle x| \big(\hat{A}_s \otimes \mathbb{1}_{\bar{s}}\big) |x\rangle = A(x)$.

Classical resolvability means we can set

$$A_s(x_s) \propto \begin{cases} 1 & \text{if } \rho_s(x) \approx \rho_s(y), \\ 0 & \text{otherwise.} \end{cases}$$

Then $\hat{A}_s$ appproximately proportional to $\rho_s(x)$.

But $(*)$ implies $\hat{A}_s$ is approximately proportional to a projection operator.

States of classically resolvable subsystem

Thus, in a *classically resolvable* subsystem s, the reduced density matrix $\rho_s(x)$ is always approximately proportional to a projection operator, for any $x \in \mathcal{N}$:

$$\rho_s(x) \approx \frac{\hat{\pi}_s(x_s)}{N_s(x_s)} \qquad \text{where} \qquad N_s(x_s) = \text{rank}(\hat{\pi}_s(x_s)).$$

(c.f. toy model)

The projection operator $\hat{\pi}_s(x_s)$ is a classical operator. It measures whether x_s is the classical state of subsystem s.

The classical degrees of freedom in subsystem s can't be in more than one state:

$$\hat{\pi}_s(x_s)\hat{\pi}_s(y_s) \approx \delta_{x_s y_s}\hat{\pi}_s(x_s).$$

$\rho_s(x)$ determines the way in which s is entangled with other subsystems. So these are strong constraints on entanglement.

Classically resolvable subsystem $\not\Rightarrow$ complete classical limit for subsystem

General classical subsystem operator for classically resolvable subsystem:

$$\hat{A}_s \approx \int_{\mathcal{N}_s} \mathrm{d}\mu_s(x_s)\, \frac{N_s}{N_s(x_s)}\, \hat{\pi}_s(x_s) A_s(x_s),$$

μ_s the pushforward of μ to $\mathcal{N}$.

So subsystem s may be treated with a self-contained classical limit of the kind previously described.

Note: in general $\hat{\pi}_s(x_s)$ has rank greater than 1, so this is not a complete classical limit.

But a classically resolvable system can be described using only classical degrees of freedom...

This is consistent because we are assuming s is part of a larger completely classical system. This is extra information compared to before, where we only considered classical limits of isolated systems.

Gauge symmetry from entanglement

Gauge symmetry from entanglement

Take a completely classical limit of a quantum system. Classical states $x \in \mathcal{N}$ corresponding to quantum states $|x\rangle \in \mathcal{H}$.

Assume the quantum system has a classically resolvable 'local structure', i.e. division into classically resolvable subsystems (and unions of subsystems also classically resolvable). Then we decompose

$$\mathcal{H} = \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_i \otimes \cdots \otimes \mathcal{H}_n.$$

For each subsystem s_i there is a classical space $\mathcal{N}_i$ of subsystem states, and a map from $x \in \mathcal{N}$ to corresponding $x_i \in \mathcal{N}_i$. Reduced states are approximately proportional to projection operators acting on $\mathcal{H}_i$:

$$\rho_i(x) \approx \frac{\hat{\pi}_i(x_i)}{N_i(x_i)} \qquad \text{where} \qquad N_i(x_i) = \mathrm{rank}(\hat{\pi}_i(x_i)).$$

Classical subsystem operators may be written

$$\hat{A}_i \approx \int_{\mathcal{N}_i} \mathrm{d}\mu_i(x_i)\, \frac{N_i}{N_i(x_i)}\, \hat{\pi}_i(x_i) A_i(x_i) \qquad \text{where} \qquad N_i = \dim(\mathcal{H}_i).$$

Projection operators are mutually approximately orthogonal:

$$\hat{\pi}_i(x_i)\hat{\pi}_i(y_i) \approx \delta_{x_i y_i} \hat{\pi}_i(x_i).$$

Gauge symmetry from entanglement

$\rho_i(x)$ tells us how s_i is entangled with the other subsystems in the state x.

In this case, $\rho_i(x) \propto \hat{\pi}_i(x_i)$ means some part of s_i is maximally entangled with some part of its complement.

Roughly speaking: s_i and its complement share $\log_2(N_i(x_i))$ maximally entangled qubits / Bell pairs.

The unentangled case

Sanity check: *separable subsystems*. Should be no gauge symmetry. Not hard to confirm this.

Unentangled subsystems implies $\rho_i(x)$ is rank 1, so $\rho_i(x) = |x_i\rangle \langle x_i|$ for some $|x_i\rangle \in \mathcal{H}_i$. We have $\langle x_i|y_i\rangle \approx \delta_{x_i y_i}$. Classical subsystem operators may be written

$$\hat{A}_i \approx \int_{\mathcal{N}_i} \mathrm{d}\mu_i(x_i)\, N_i\, |x_i\rangle \langle x_i|\, A_i(x_i).$$

So separable subsystems undergo a *complete classical limit*, unlike entangled case.

Set of subsystem states x_i determines full system state via

$$|x\rangle \langle x| = |x_1\rangle \langle x_1| \otimes \cdots \otimes |x_i\rangle \langle x_i| \otimes \cdots \otimes |x_n\rangle \langle x_n|\,.$$

x also determines $x_1, \ldots, x_n$, so we have a bijection

$$\mathcal{N} \longleftrightarrow \mathcal{N}_1 \times \cdots \times \mathcal{N}_i \times \cdots \times \mathcal{N}_n.$$

No non-local degrees of freedom, so no gauge symmetry.

The entangled case

Return to the entangled case. Classical subsystem operators:

$$\mathcal{C}_i = \left\{ \hat{A}_i \approx \int_{\mathcal{N}_i} \mathrm{d}\mu_i(x) \, \frac{N_i}{N_i(x_i)} \hat{\pi}_i(x_i) A_i(x_i) \mid A_i : \mathcal{N}_i \to \mathbb{C} \right\}.$$

Define

$$\mathcal{C}_{\text{local}} = \mathcal{C}_1 \otimes \cdots \otimes \mathcal{C}_i \otimes \cdots \otimes \mathcal{C}_n.$$

This is the set of operators which only depend on local degrees of freedom.

Most general $\hat{A}_{\text{local}} \in \mathcal{C}_{\text{local}}$:

$$\hat{A}_{\text{local}} \approx \int_{\mathcal{N}_1} \mathrm{d}\mu_1(x_1) \cdots \int_{\mathcal{N}_n} \mathrm{d}\mu_n(x_n) \; N \, \hat{\pi}_{\text{local}}(x_1, \ldots, x_n) \, A_{\text{local}}(x_1, \ldots, x_n),$$

where

$$\hat{\pi}_{\text{local}}(x_1, \ldots, x_n) = \hat{\pi}_1(x_1) \otimes \cdots \otimes \hat{\pi}_n(x_n)$$

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Non-local degrees of freedom

Define $\mathcal{C}_{\text{non-local}} = \mathcal{C} \setminus \mathcal{C}_{\text{local}}$.

If $\mathcal{C}_{\text{non-local}}$ is non-empty, then there are non-local classical degrees of freedom.

Suppose $|x\rangle$ is entangled, and $\hat{A}_{\text{local}} \in \mathcal{C}_{\text{local}}$ satisfies $\hat{A}_{\text{local}} |x\rangle = 1$. Then

$$\text{rank}(\hat{A}_{\text{local}}) \geq \text{rank}(\hat{\pi}_{\text{local}}(x_1, \ldots, x_n)) > 1.$$

On the other hand, $|x\rangle \langle x|$ is rank 1 and $|x\rangle \langle x| \in \mathcal{C}$.

Therefore $\mathcal{C}_{\text{non-local}}$ is non-empty: it contains $|x\rangle \langle x|$.

$|x\rangle \langle x|$ measures if the classical state is x. Thus, to know if the classical state is x, we have to measure non-local degrees of freedom.

Non-local degrees of freedom

N.B. presence of non-local degrees of freedom depends on structure of entanglement between subsystems in the state $|x\rangle$.

Separable implies no non-local degrees of freedom.

Subsystems could be *entangled* in some states, but *separable* in others. Moreover, when entangled, can be entangled in different ways.

One can show that the number of non-local degrees of freedom shared by two subsystems s_i, s_j is counted by their mutual information

$$I_{i:j}(x) = \mathrm{tr}_{ij}(\rho_{ij}(x)\log\rho_{ij}(x)) - \mathrm{tr}_i(\rho_i(x)\log\rho_i(x)) - \mathrm{tr}_j(\rho_j(x)\log\rho_j(x)).$$

They share non-local degrees of freedom if and only if the mutual information is non-vanishing (in the classical limit).

Can be interpreted as a *variable 'bulk topology'*.

Kinematical states

Have shown that entanglement between classically resolvable subsystems leads to *classically emergent non-local degrees of freedom*.

Now: account for these non-local degrees of freedom using *gauge symmetry*.

First step is to construct a space of kinematical states for each subsystem. A kinematical state will include a *purification* of $\rho_i(x)$:

$$|\psi_i\rangle \in \mathcal{H}_i \otimes \widetilde{\mathcal{H}}_i(x_i) \quad \text{such that} \quad \widetilde{\mathrm{tr}}_i\big(|\psi_i\rangle\langle\psi_i|\big) = \rho_i(x),$$

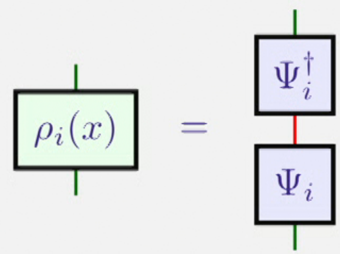
where $\widetilde{\mathcal{H}}_i(x_i)$ is an auxiliary Hilbert space of sufficiently high dimension.

For simplicity we can set $\widetilde{\mathcal{H}}_i(x_i) = \mathbb{C}^{N_i(x_i)}$.

We are adding $\log_2 N_i(x_i)$ qubits to the subsystem — these are *edge modes*.

More notationally convenient to view the purification as a map $\Psi : \mathcal{H}_i \to \widetilde{\mathcal{H}}_i(x_i)^*$.

The purification condition may then be written $\rho_i(x) = \Psi_i^\dagger \Psi_i$.



Kinematical states

Kinematical subsystem state space:

$$\mathcal{N}_i^{\text{kin.}} = \{(x_i, \Psi_i) \mid x_i \in \mathcal{N}_i,\ \Psi_i \in \mathcal{P}_i(x_i)\},$$

where $\mathcal{P}_i(x_i)$ is the space of purifications of $\rho_i(x)$.

Full kinematical state space:

$$\mathcal{N}^{\text{kin.}} = \mathcal{N}_1^{\text{kin.}} \times \cdots \times \mathcal{N}_i^{\text{kin.}} \times \cdots \times \mathcal{N}_n^{\text{kin.}}.$$

A general kinematical state may be written

$$\big((x_1, \Psi_1), \ldots, (x_i, \Psi_i), \ldots, (x_n, \Psi_n)\big) \in \mathcal{N}^{\text{kin.}}.$$

How do we project to physical states? Two steps: impose constraints, then gauge reduce.

Constraints, gluing and gauge reduction

Let $X : x \mapsto (x_1, \ldots, x_n)$ map full system states $x \in \mathcal{N}$ to corresponding subsystem states $x_i \in \mathcal{N}_i$, and let

$$\text{image}(X) = \overline{\mathcal{N}_1 \times \cdots \times \mathcal{N}_n}.$$

First constraint is almost trivial:

$$(x_1, \ldots, x_n) \in \overline{\mathcal{N}_1 \times \cdots \times \mathcal{N}_n}.$$

So $(x_1, \ldots, x_n)$ can come from at least one state of the full system.

Constraints, gluing and gauge reduction

Let us introduce some additional *reference structures*. Analogous to e.g. coordinate systems in gravity.

1. A section Y of X, i.e. a map

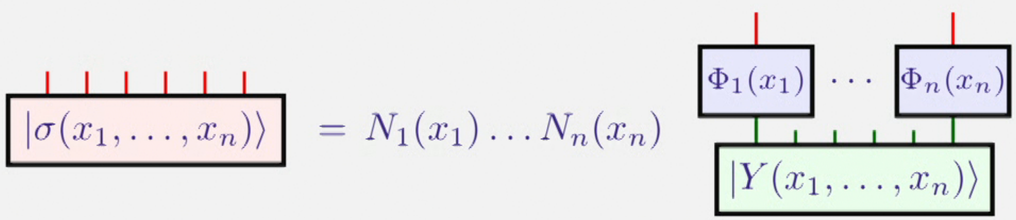
$$Y : \overline{\mathcal{N}_1 \times \cdots \times \mathcal{N}_1} \to \mathcal{N}$$

   such that $X \circ Y$ is the identity.

2. A choice of purification $\Phi_i(x_i) \in \mathcal{P}_i(x_i)$ for each subsystem s_i and each $x_i \in \mathcal{N}_i$.

From this, define *gluing states* for $(x_1, \ldots, x_n) \in \overline{\mathcal{N}_1 \times \cdots \times \mathcal{N}_n}$:

$$|\sigma(x_1, \ldots, x_n)\rangle = N_1(x_1) \ldots N_n(x_n) \left(\Phi_1(x_1) \otimes \cdots \otimes \Phi_n(x_n)\right) |Y(x_1, \ldots, x_n)\rangle \,.$$

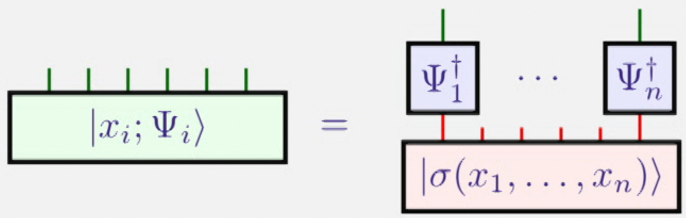


Constraints, gluing and gauge reduction

We can use these states to glue together the kinematical subsystem states.

If $(x_1, \ldots, x_n) \in \overline{\mathcal{N}_1 \times \cdots \times \mathcal{N}_n}$, we define $|x_i; \Psi_i\rangle \in \mathcal{H}$, by

$$|x_i; \Psi_i\rangle = \left(\Psi_1^\dagger \otimes \cdots \otimes \Psi_n^\dagger\right) |\sigma(x_1, \ldots, x_n)\rangle \, .$$



Essentially, take tensor product of purifications for each subsystem, then project auxiliary degrees of freedom onto $|\sigma(x_1, \ldots, x_n)\rangle$.

Constraints, gluing and gauge reduction

One can show: we can obtain any physical state by gluing in this way.

But not all kinematical states when glued together will yield sensible classical states. Restricting to those which do:

$$\overline{\mathcal{N}^{\text{kin.}}} = \Big\{\big((x_1, \Psi_1), \ldots, (x_n, \Psi_n)\big) \in \mathcal{N}^{\text{kin.}} \mid (x_1, \ldots, x_n) \in \overline{\mathcal{N}_1 \times \cdots \times \mathcal{N}_n} \text{ and } |x_i; \Psi_i\rangle = |y\rangle \text{ for some } y \in \mathcal{N}\Big\}.$$

This is the 'constraint surface' $\overline{\mathcal{N}^{\text{kin.}}} \subset \mathcal{N}^{\text{kin.}}$.

Gauge reduction map is

$$R : \overline{\mathcal{N}^{\text{kin.}}} \to \mathcal{N}, \quad \big((x_1, \Psi_1), \ldots, (x_n, \Psi_n)\big) \mapsto |x_i; \Psi_i\rangle .$$

Any physical observable may be written as a function of the kinematical subsystem states. In particular, *non-local* physical observables can be decomposed into *local* kinematical observables.

Constraints, gluing and gauge reduction

We can also glue together any proper subset of the full set of subsystems.

E.g. the physical states of a union of subsystems $s_i \cup s_j$ are in one-to-one correspondence with the reduced states $\rho_{ij}(x)$, which can be obtained by gluing the kinematical states (x_i, Ψ_i) and (x_j, Ψ_j):

$$\rho_{ij}(x) = (\Psi_i \otimes \Psi_j)\sigma_{ij}(x_i, x_j)(\Psi_i \otimes \Psi_j)^\dagger,$$

where

$$\sigma_{ij}(x_i, x_j) = \widetilde{\mathrm{tr}}_{\overline{ij}} \left|\sigma(x_1, \ldots, x_n)\right\rangle \left\langle\sigma(x_1, \ldots, x_n)\right|.$$

Classical resolvability of s_i, s_j and $s_i \cup s_j$ ensure: σ_{ij} only depends on x_i, x_j.

Gauge transformations

A local gauge transformation is a change of purifications $\Psi_i \to U_i^\dagger \Psi_i$, with $U_i \in U(N_i(x_i))$. This changes the kinematical state of the subsystem, but not its physical state.

Thus, the *local* gauge group in subsystem s_i is $U(N_i(x_i))$.

In order to leave the physical state $|x_i; \Psi_i\rangle$ of the *full system* invariant, U_i must obey

$$|\sigma(x_1, \dots, x_n)\rangle = \big(U_1 \otimes \dots \otimes U_n\big) \, |\sigma(x_1, \dots, x_n)\rangle \, .$$

This defines the *global* gauge group:

$$G(x) = \mathrm{Stab}_{U(N_1(x_1)) \times \dots \times U(N_n(x_n))} \big(\, |\sigma(x_1, \dots, x_n)\rangle \, \big).$$

N.B. these are state dependent gauge groups. (Same is true in gravity.)

Summary

- For subsystems to have good classical limits, they must be *classically resolvable*, meaning the subsystem state can be determined by classical measurements alone.
- Classical resolvability implies that the reduced state in any subsystem is approximately proportional to one of a set of mutually orthogonal projection operators, $\rho_i(x) \propto \hat{\pi}_i(x_i)$.
- If $\mathrm{rank}(\hat{\pi}_i(x_i)) > 1$, then there are emergent non-local degrees of freedom.
- These can be accounted for by introducing a gauge symmetry where the kinematical states are *purifications* of $\rho_i(x)$. In other words, $\log_2 N_i(x_i)$ auxiliary qubits as 'edge modes'.
- We can glue together arbitrary collections of subsystems by projecting auxiliary qubits onto entangled gluing states $|\sigma(x_1, \ldots, x_n)\rangle$.
- Any classical physical observable may be decomposed into a combination of local kinematical observables.

This provides a precise general picture of how entanglement leads to classically emergent gauge symmetry.

Entangled group coherent states

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Entangled group coherent states

The toy model of three spins is a special case of a larger family based on unitary representations of Lie groups.

Let $\mathcal{H} = \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n$, and let G_i act unitarily and irreducibly on $\mathcal{H}_i$.

In separable case, there is a well-known construction of classical limits, with classical space of states $\mathcal{N}_i$ for each subsystem being a *coadjoint orbit* of G.

Construct coherent states for full system by taking the tensor product of coherent states for the subsystems.

Then space of states for full system is $\mathcal{N} = \mathcal{N}_1 \times \cdots \times \mathcal{N}_n$, which is a coadjoint orbit of $G = G_1 \times \cdots \times G_n$.

There is no emergent gauge symmetry (consistent with no entanglement).

Entangled group coherent states

In the paper: a modification of this construction.

Involves the choice of a subgroup

$$H \subseteq G = G_1 \times \cdots \times G_n$$

with special properties. I explain how to construct a set of coherent states for full system, with entanglement determined by H.

I show: this yields a *good classical limit*, and *all subsystems are classically resolvable*. Also: the entanglement leads to an *emergent gauge symmetry*, with a certain gauge group K satisfying

$$H \subseteq K \subseteq G.$$

For the three spin toy model: $G_i = \mathrm{SU}(2)$, H is the diagonal subgroup, and $K = H$.

This gives a very large and varied class of classical limits with gauge symmetry emerging from entanglement.

‘Gravitational’ aspects

Gravity from entanglement

There is a general expectation that spacetime can emerge from structure of quantum entanglement.

So diffeomorphism invariance of that spacetime should also emerge from entanglement.

The mechanism described here is a very generic (model-independent) way for this to happen.

Not too much of a stretch to suggest that it is general enough to include the gravitational case.

Classical resolvability of subregions in gravity

Reduced density matrix of spacetime subregion:

$$\rho = \frac{\exp\left(-\hat{A}/4G + \dots\right)}{\mathrm{tr}\left(\exp\left(-\hat{A}/4G + \dots\right)\right)},$$

where $\hat{A}$ is an area operator. [Jafferis, Lewkowycz, Maldecena, Suh, 2015]

In classical $G \to 0$ limit, this is proportional to a projection operator onto minimal area states.

Can show fidelity of reduced density matrices ρ, ρ' for two different subsystem states obeys [Kirklin, 2019]

$$\mathrm{tr}\left(\sqrt{\sqrt{\rho}\rho'\sqrt{\rho}}\right) = \exp\left(-\mathcal{O}(1/G)\right) \to 0,$$

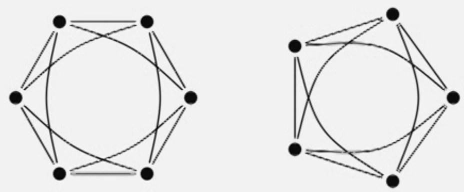
which implies projection operators for ρ, ρ' are approximately mutually orthogonal.

Thus, gravitational subsystems are *classically resolvable*, consistent with present work.

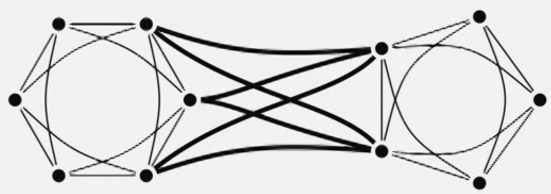
Variable 'bulk topology'

Different states have different kinds of entanglement, so different kinds of non-local degrees of freedom. Interpret this as different *'bulk topologies'*.

For example:



vs.



Represents entanglement structure in two different states.

Dots are subsystems, lines are present when mutual information is non-vanishing.

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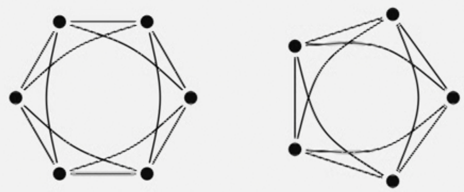

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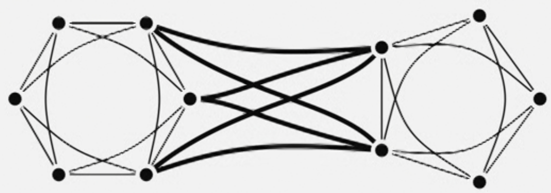

Represents entanglement structure in two different states.

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Modular symmetries

The emergent gauge transformations I have described are approximate *modular symmetries* of each subsystem — i.e. transformations which do not change the reduced density matrix.

In gravity, subregion modular symmetries are transformations of geometric edge modes. [Czech, de Boer, Ge, Lamprou, 2019]

So this is consistent. (Suggests a direct entanglement interpretation of geometric edge modes...)

Quantum error correction

QEC has had important conceptual implications in gravity.

QEC also plays a role here. Suppose we take a classical limit of a system with Hilbert space $\mathcal{H}$ and obtain a classical state space $\mathcal{N}$ with emergent gauge symmetry. So

$$\mathcal{N}^{\text{kin.}} \supset \overline{\mathcal{N}^{\text{kin.}}} \to \mathcal{N}.$$

Suppose we do constrained quantisation of $\mathcal{N}$. So quantise $\mathcal{N}^{\text{kin.}}$ to $\mathcal{H}^{\text{kin.}}$, and identify physical states

$$\mathcal{H}^{\text{phys.}} \subset \mathcal{H}^{\text{kin.}}$$

by imposing some operator constraints.

There is then a sense in which original Hilbert space $\mathcal{H}$ is embedded as a code subspace of $\mathcal{H}^{\text{kin.}}$.

Conclusion

- A general mechanism for the *emergence of classical gauge symmetry from quantum entanglement*.
- This proceeded from an understanding of *classical resolvability*.
- Toy model of three entangled spins, and a group-theoretic generalisation.
- Evidence that the mechanism could be responsible (in part) for diffeomorphism invariance in gravity.

Future directions

- Phase space structure (Berry curvature $\to$ symplectic form).
- *Semi*-classical limit (some degrees of freedom remain quantum... to what extent do they respect emergent gauge symmery?).
- $\dim(\mathcal{H}) = \infty$, Type II and Type III subsystems, compatibility with *pre-existing* quantum gauge symmetry.
- Specific gravitational applications: coadjoint orbits of the *corner symmetry group*, relationship with *spin network states*, etc.
- Other implications of classical resolvability (e.g. *entropy cone*...).
- Dynamics... *chaos*, *decoherence*, etc.

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Thank you for listening!