Title: On 2-Categories and 3d A-models

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Series: Mathematical Physics

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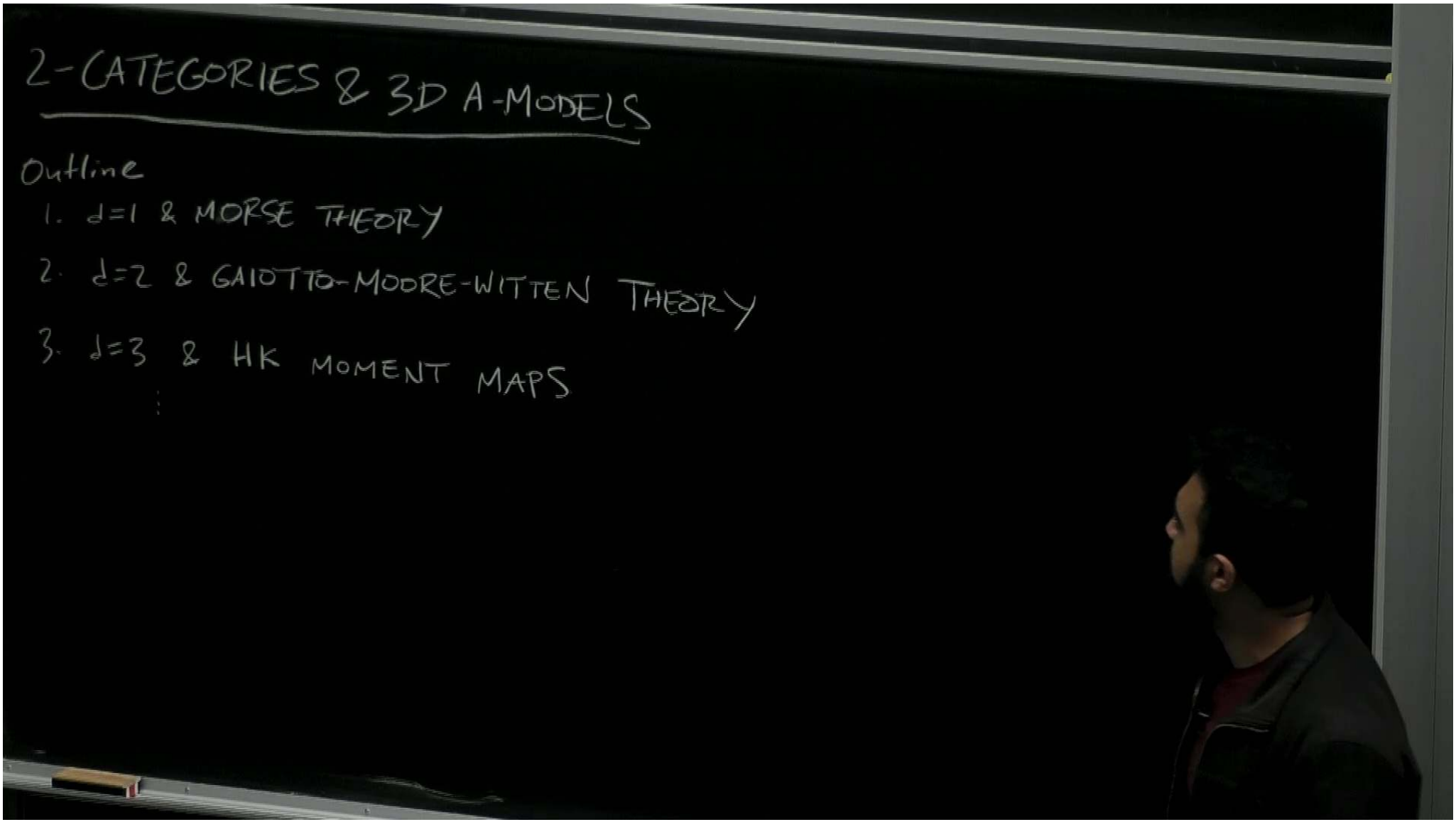
Abstract: The Fukaya category of a symplectic manifold is an excellent toy model for exploring the general structure of boundary conditions in two-dimensional topological field theories. In this talk I will explain an analogue for three dimensional theories. The natural playground for boundary conditions in d=3 is the 3d A-model into a hyperKahler target space. Here the fundamental BPS equation is the Fueter equation. I will explain how studying certain boundary value problems for the Fueter equation, one is lead to something resembling a higher categorical structure. I will then argue how a similar structure also seems to appear after performing a massive deformation by a hyperKahler moment map.

Zoom link: https://pitp.zoom.us/j/96574302292?pwd=dkpIVXN0UmFiRkdkenFzc2o2Nko0dz09


2-CATEGORIES & 3D A-MODELS
Outline
1. d=1 & MORSE THEORY
2. d=2 & GAIOTTO-MOORE-WITTEN THEORY
3. d=3 & HK MOMENT MAPS

2. $d=2$ & GAIOTTO-MOORE-WITTEN THEORY (2015)

3. $d=3$ & HK MOMENT MAPS (Newer Stuff)

$\phi : \mathbb{R} \to X$

1. MORSE THEORY (X, g) $h : X \to \mathbb{R}$ $L = g_{ab} \dot{\phi}^a \dot{\phi}^b$ + ferm

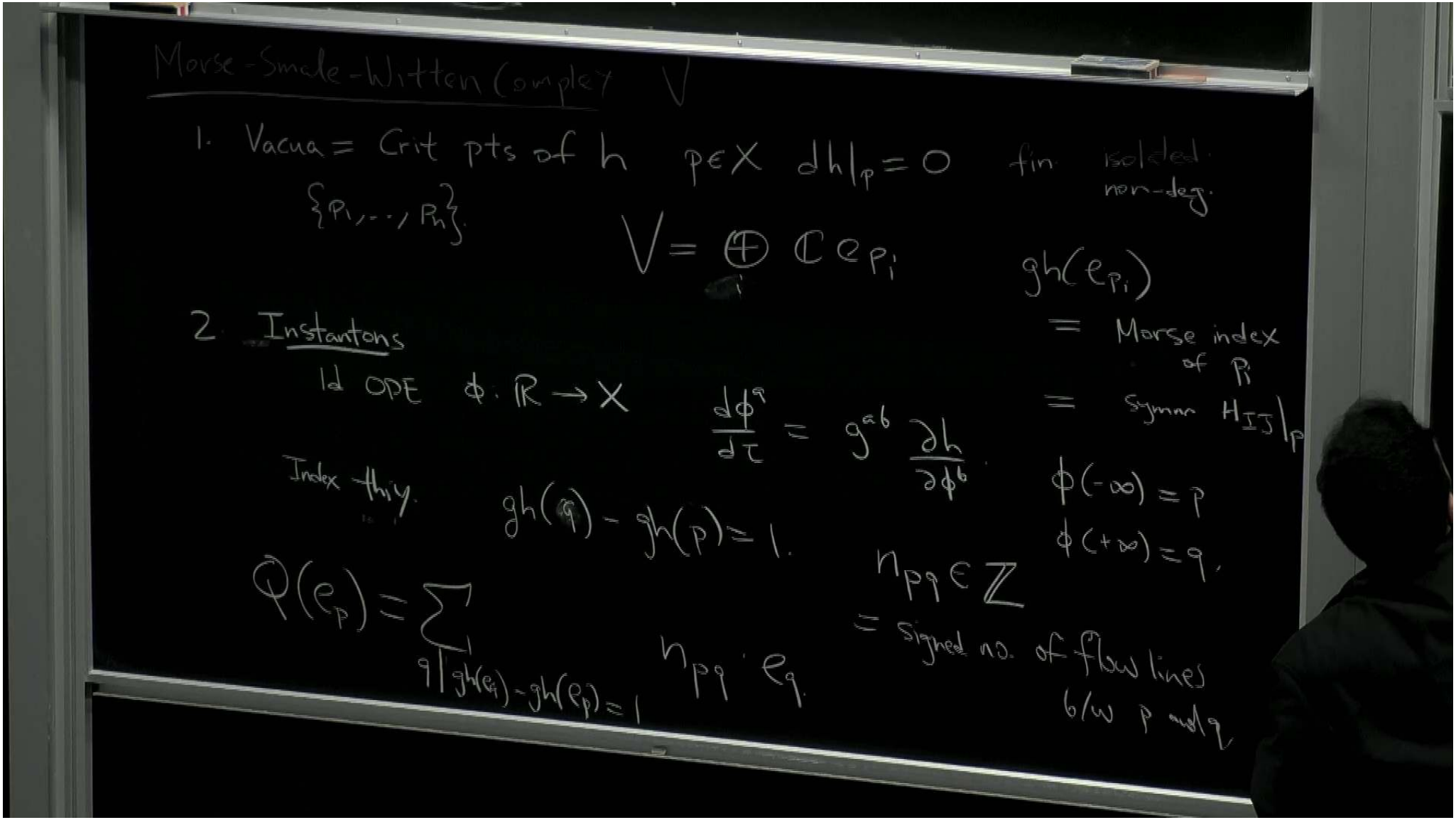
1-dim'l Goh. FT. $N=2$ SQM. $Q, \bar{Q}$ $Q^2 = 0 = \bar{Q}^2$

What does it assign to a pt? $[Q, \bar{Q}] = 2H$.

V 1. Graded gh. (V, Q) chain complex

2. $Q : V^\bullet \to V^{\bullet+1}$ $Q^2 = 0$.

Morse-Smale-Witten Complex


Morse-Smale-Witten Complex
1. Vacua = Crit pts of h
$\{p_1, \dots, p_n\}$.
$p \in X \quad dh|_p = 0$
fin isolated non-deg.
$V = \bigoplus_i \mathbb{C} e_{p_i}$
$gh(e_{p_i})$ = Morse index of p_i = Symm $H_{IJ}|_p$
2. Instantons
1d QFT $\phi : \mathbb{R} \to X$
$\frac{d\phi^a}{d\tau} = g^{ab} \frac{\partial h}{\partial \phi^b}$
$\phi(-\infty) = p$
$\phi(+\infty) = q$.
Index thy.
$gh(q) - gh(p) = 1$.
$n_{pq} \in \mathbb{Z}$
= Signed no. of flow lines b/w p and q
$Q(e_p) = \sum_{q \mid gh(e_q) - gh(e_p) = 1} n_{pq} \cdot e_q$.

$Q^2 = 0$ holds ?

"Broken flow line" Argument.

Top'l prop: g-independence

indep under "deform" of h.

(Y, g, I) Kähler $W : Y \longrightarrow \mathbb{C}$ holom

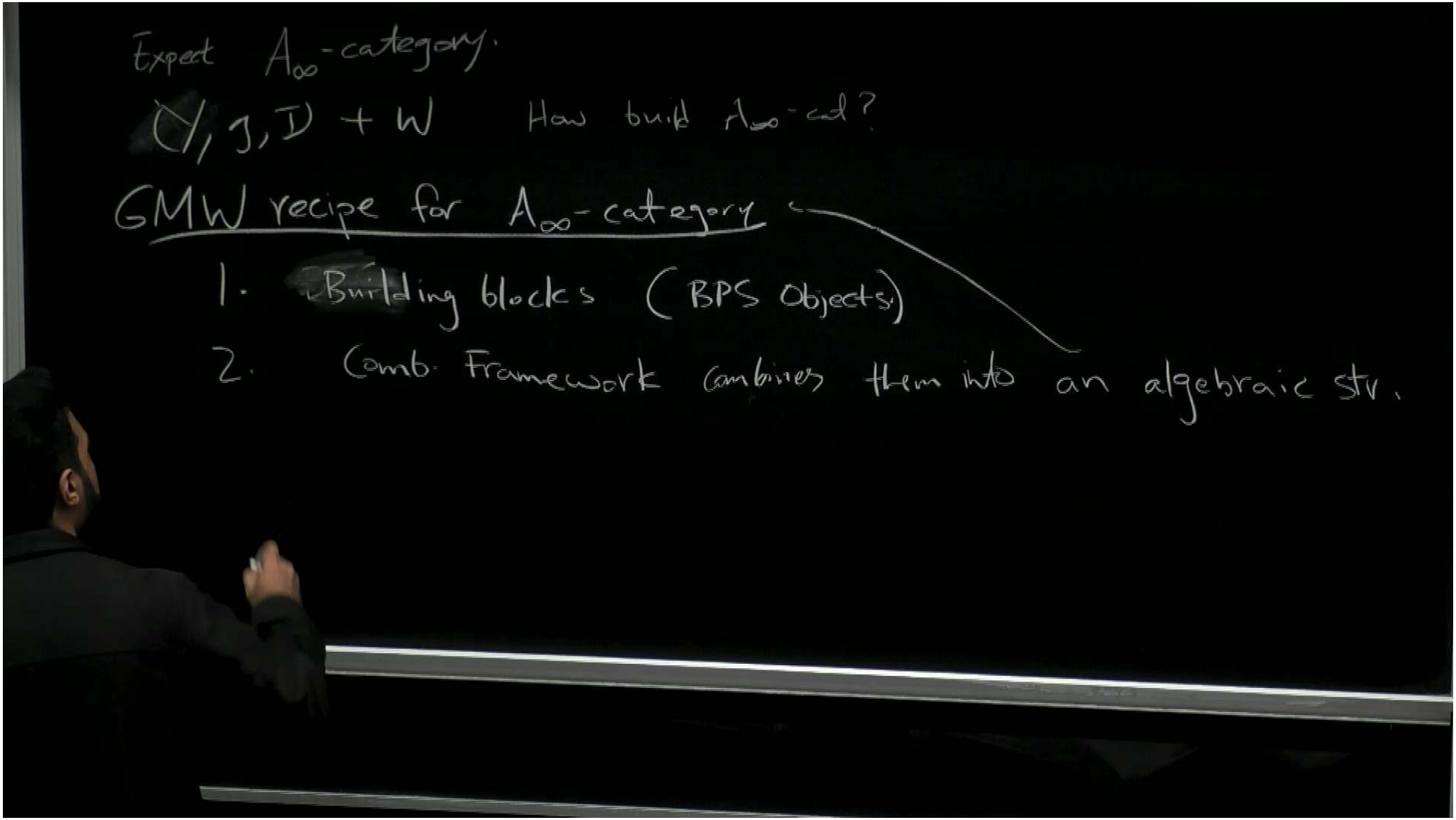
metric integ. complex str.

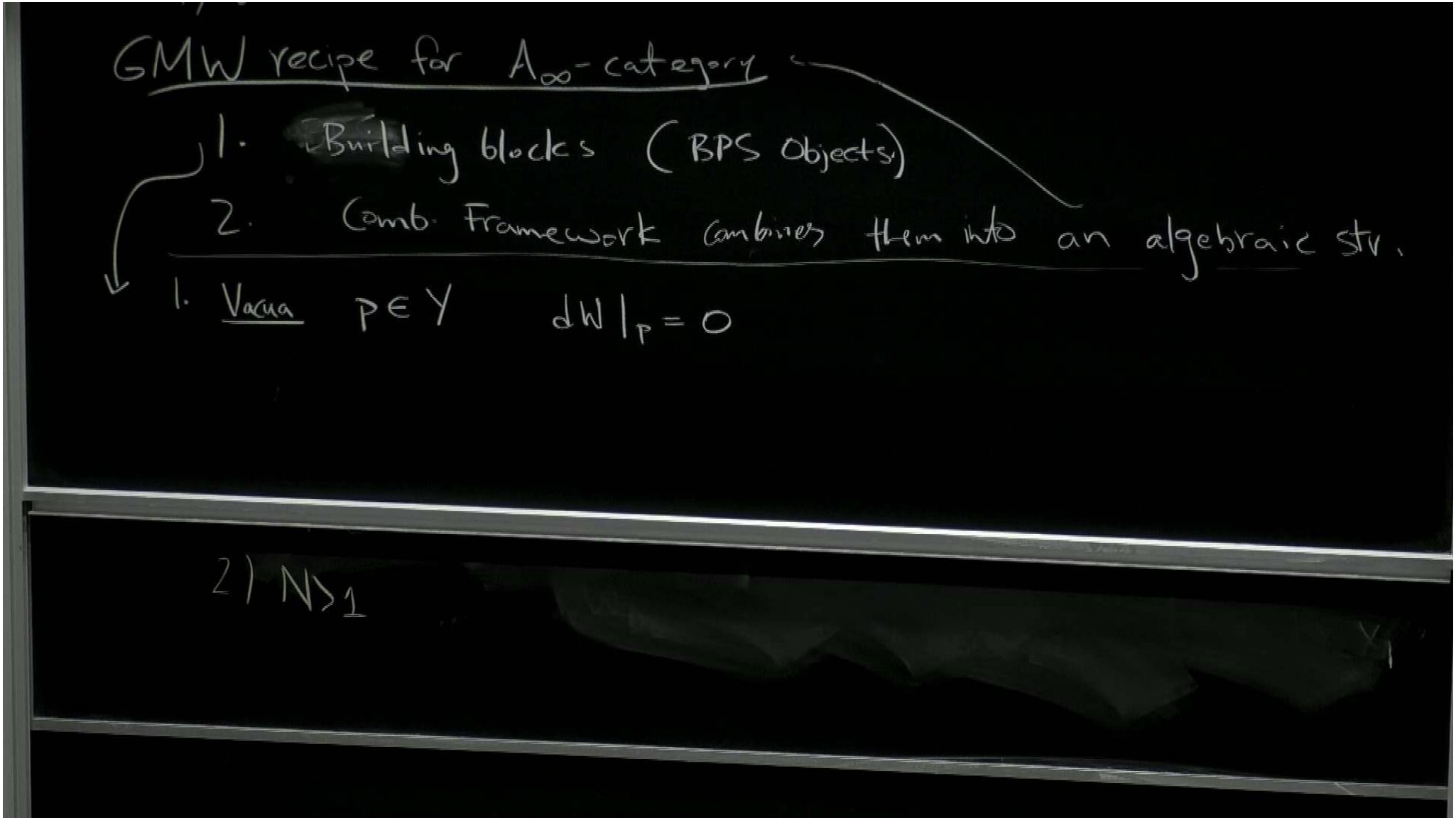
$\exists$ a d=2 coh. field theory. $N=(2,2)$ Landau-Ginzburg mod.l.

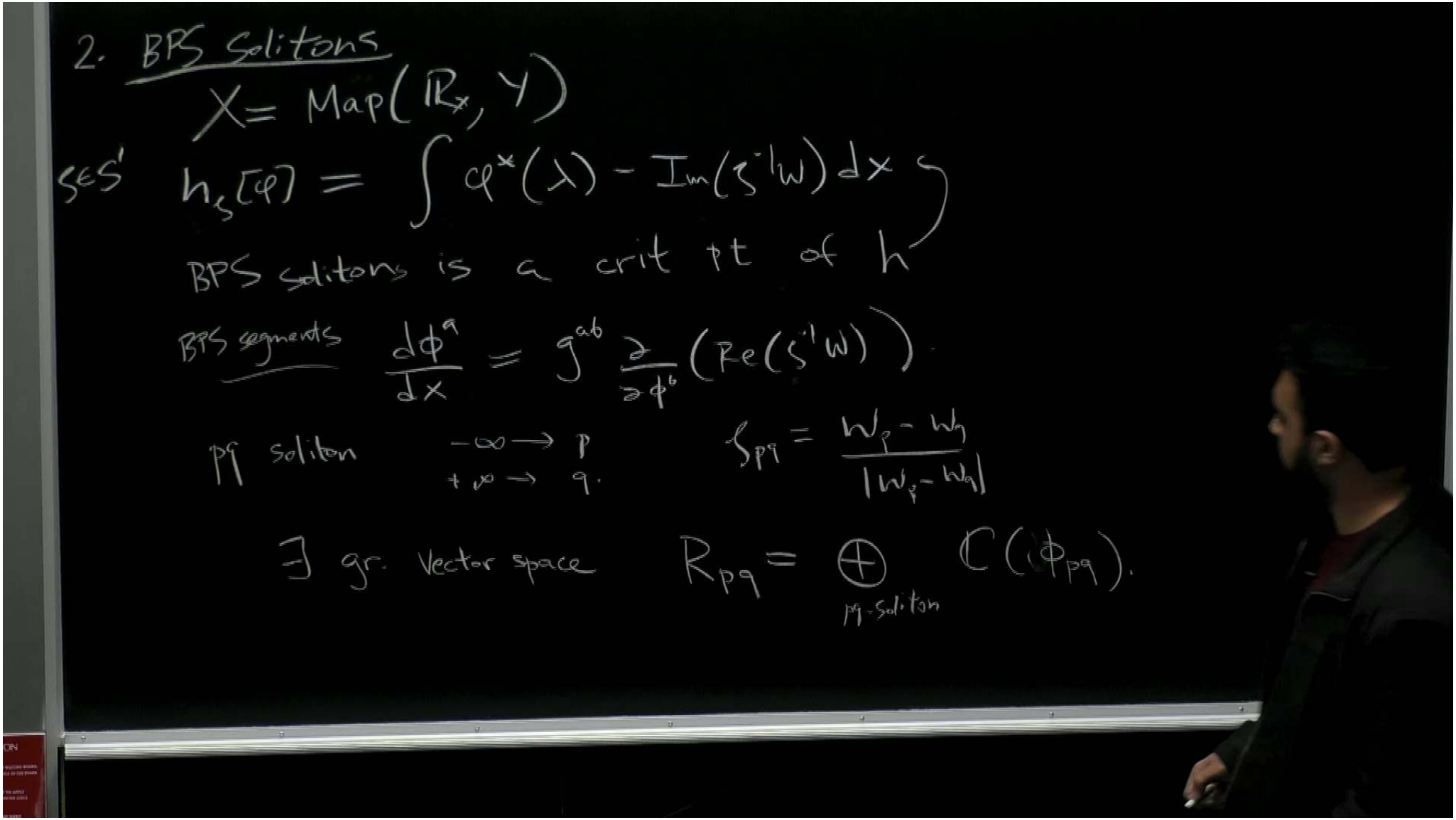
"A-model" $Q_+, \bar{Q}_+, Q_-, \bar{Q}_-$ ...

$Q_A = Q_+ + J Q_-$ $Q_A^2 = 0$

twisting sup.


Expect A_∞-category.
$C, J, D + W$ How build A_∞-cat?
GMW recipe for A_∞-category
1. Building blocks (BPS Objects)
2. Comb. Framework combines them into an algebraic str.


GMW recipe for A_∞-category
1. Building blocks (BPS Objects)
2. Comb. Framework combines them into an algebraic str.
1. Vacua $p \in Y$ $dW|_p = 0$
2) $N > 1$


2. BPS Solitons
$X = \mathrm{Map}(\mathbb{R}, Y)$
SES' $h_\zeta[\varphi] = \int \varphi^*(\lambda) - \mathrm{Im}(\zeta^{-1} W)\, dx$
BPS solitons is a crit pt of h
BPS segments $\frac{d\phi^a}{dx} = g^{ab} \frac{\partial}{\partial \phi^b} (\mathrm{Re}(\zeta^{-1} W))$
pq soliton $-\infty \to p$, $+\infty \to q$. $\zeta_{pq} = \frac{W_p - W_q}{|W_p - W_q|}$
$\exists$ gr. vector space $R_{pq} = \bigoplus_{pq\text{-soliton}} \mathbb{C}(\psi_{pq})$.

3.1 BPS polygons / BPS Junctions / 5-instantons.

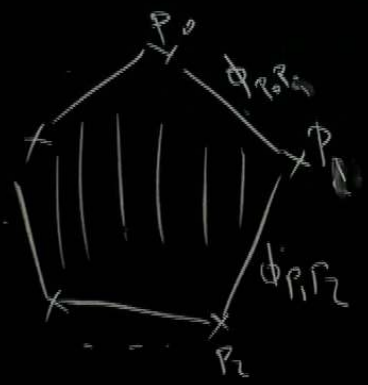
$\phi : \mathbb{R}^2_{\tau,x} \longrightarrow Y$

$$\frac{\partial \phi^a}{\partial x} + I^a{}_b \frac{\partial \phi^b}{\partial \tau} = g^{ab} \frac{\partial}{\partial \phi^b} \mathrm{Re}\, \zeta^{-1} W.$$

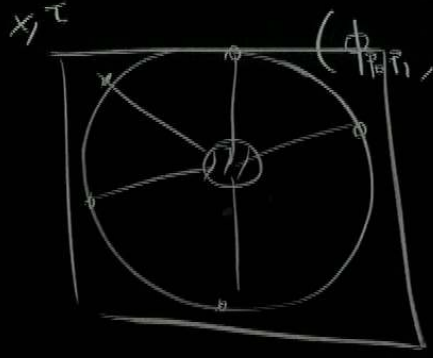
(1) $(p_0, \dots, p_n)$ coll. of vacua

Crit values convex position.

5-inst
Witten 5-eqn.



– label edge by a Soliton



Junction of Solitons.

Expect A_∞-category.

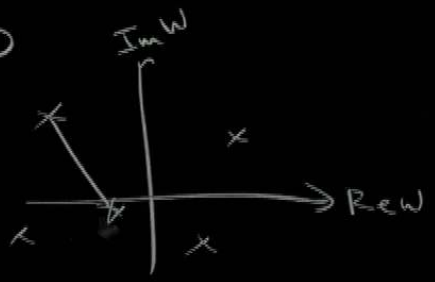
$(C, J, D) + W$ $\omega = d\lambda$ How build A_∞-cat?

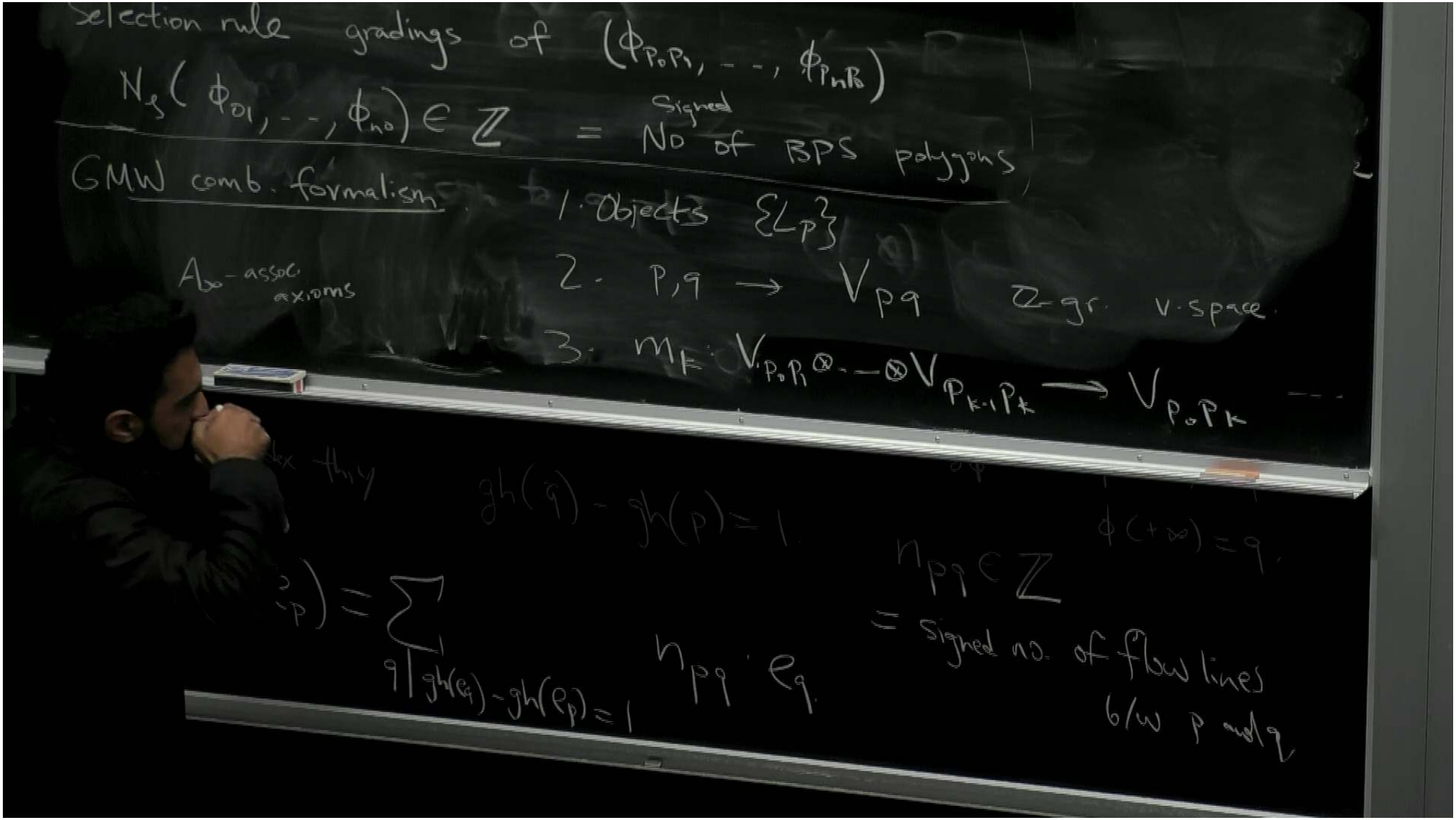
GMW recipe for A_∞-category

1. Building blocks (BPS Objects)
2. Comb. Framework combines them into an algebraic str.

1. Vacua $p \in Y$ $dW|_p = 0$

Crit values are in general position



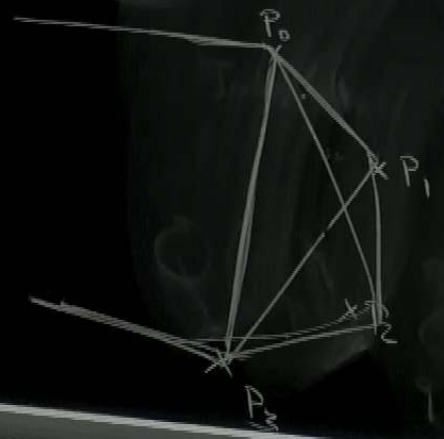

selection rule gradings of $(\phi_{P_0P_1}, \dots, \phi_{P_{n-1}P_n})$
$N_\gamma(\phi_{01}, \dots, \phi_{n0}) \in \mathbb{Z}$ = Signed No of BPS polygons
GMW comb. formalism
1. Objects $\{L_p\}$
A_∞ - assoc. axioms
2. $P, Q \to V_{PQ}$ $\mathbb{Z}$-gr. v. space
3. $m_k : V_{P_0P_1} \otimes \cdots \otimes V_{P_{k-1}P_k} \to V_{P_0P_k}$
$gh(q) - gh(p) = 1$
$n_{pq} \in \mathbb{Z}$
= Signed no. of flow lines b/w p and q
$\phi(+\infty) = q$.
$n_{pq} \cdot e_q$

(1) Objects L_P for every vacuum P.

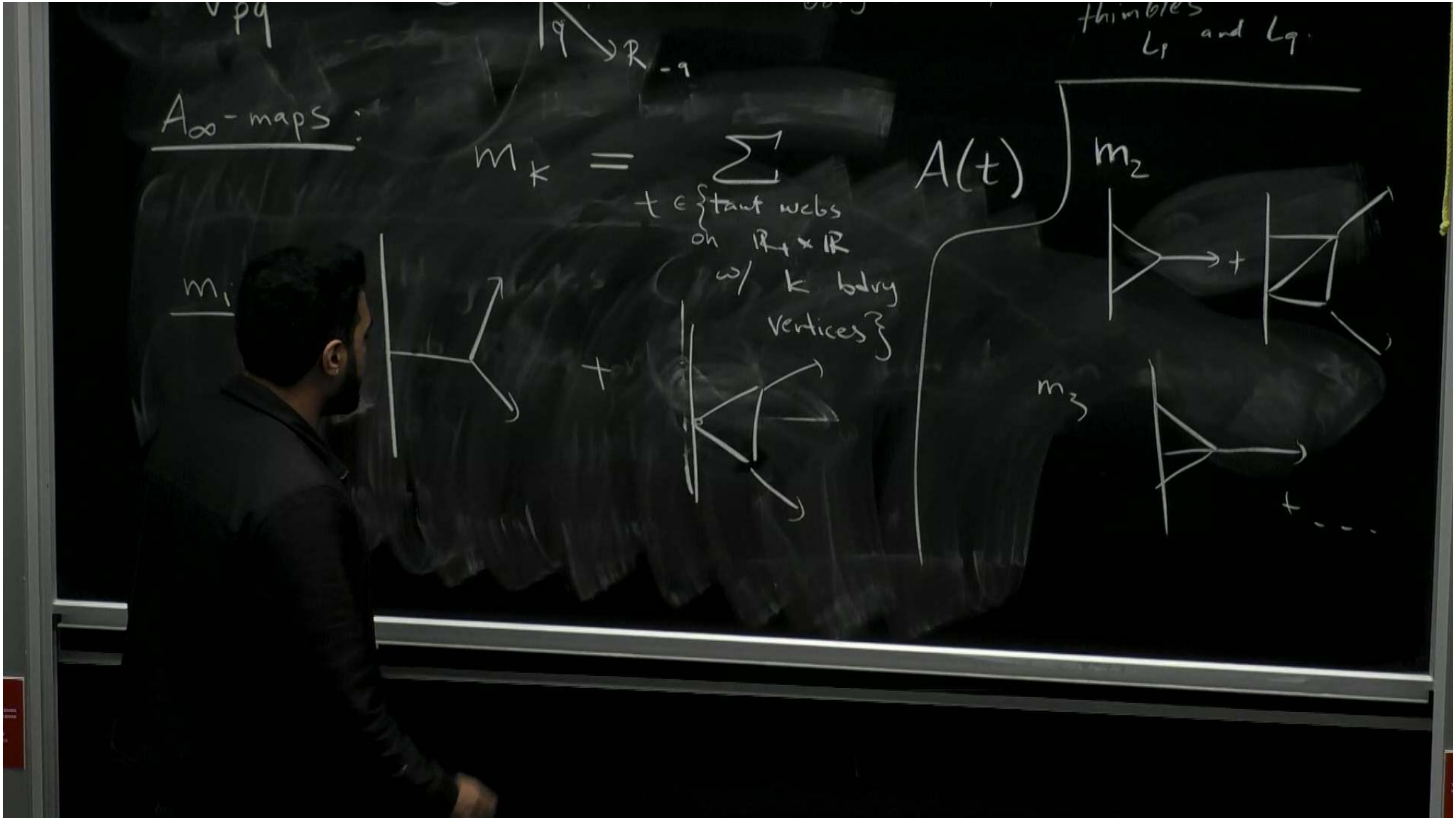
Natural ordering

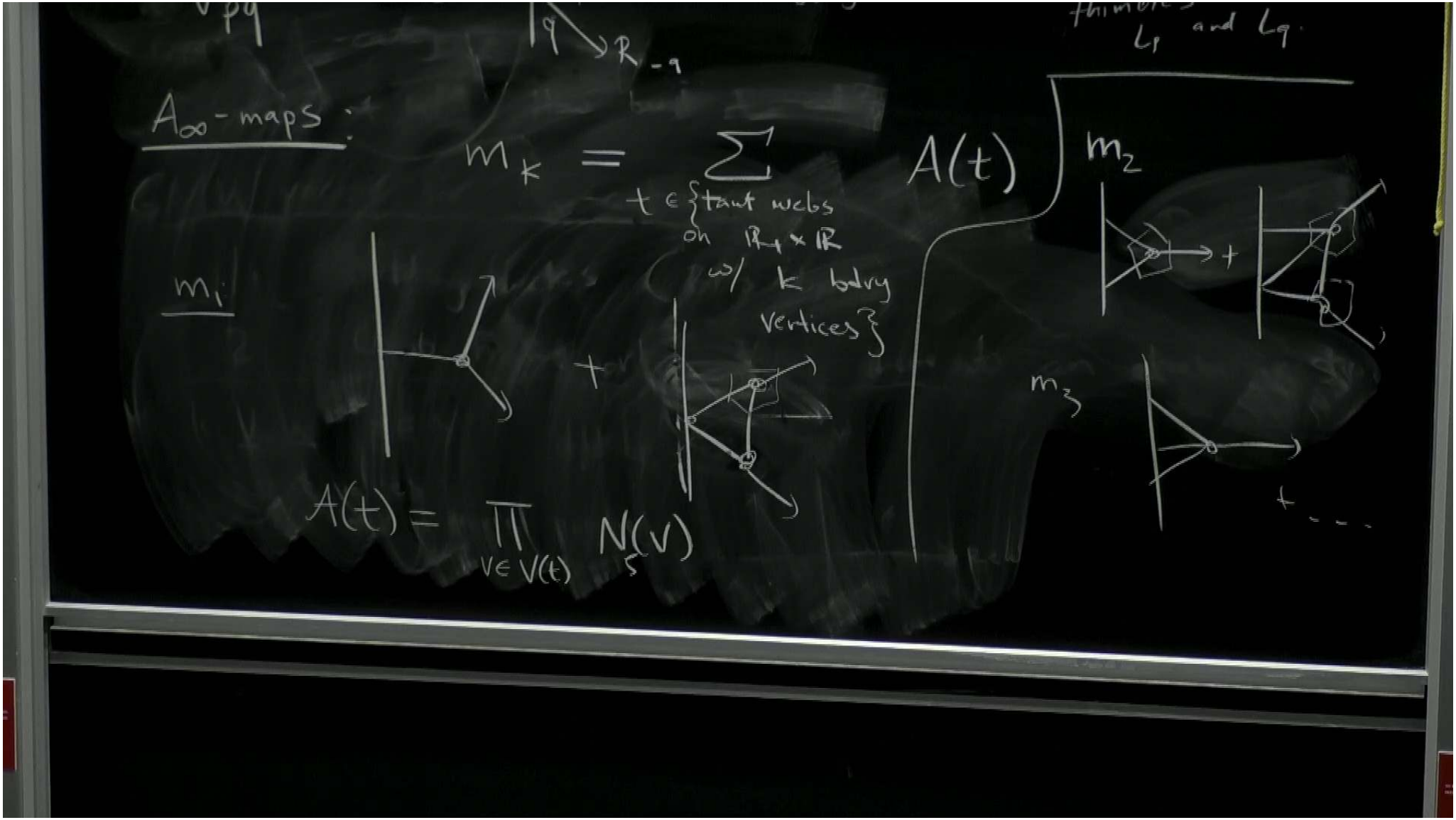
(2) $V_{P,Q} = \bigoplus_{P \in \{Q \text{ semi-int polygons}\}} R_P$

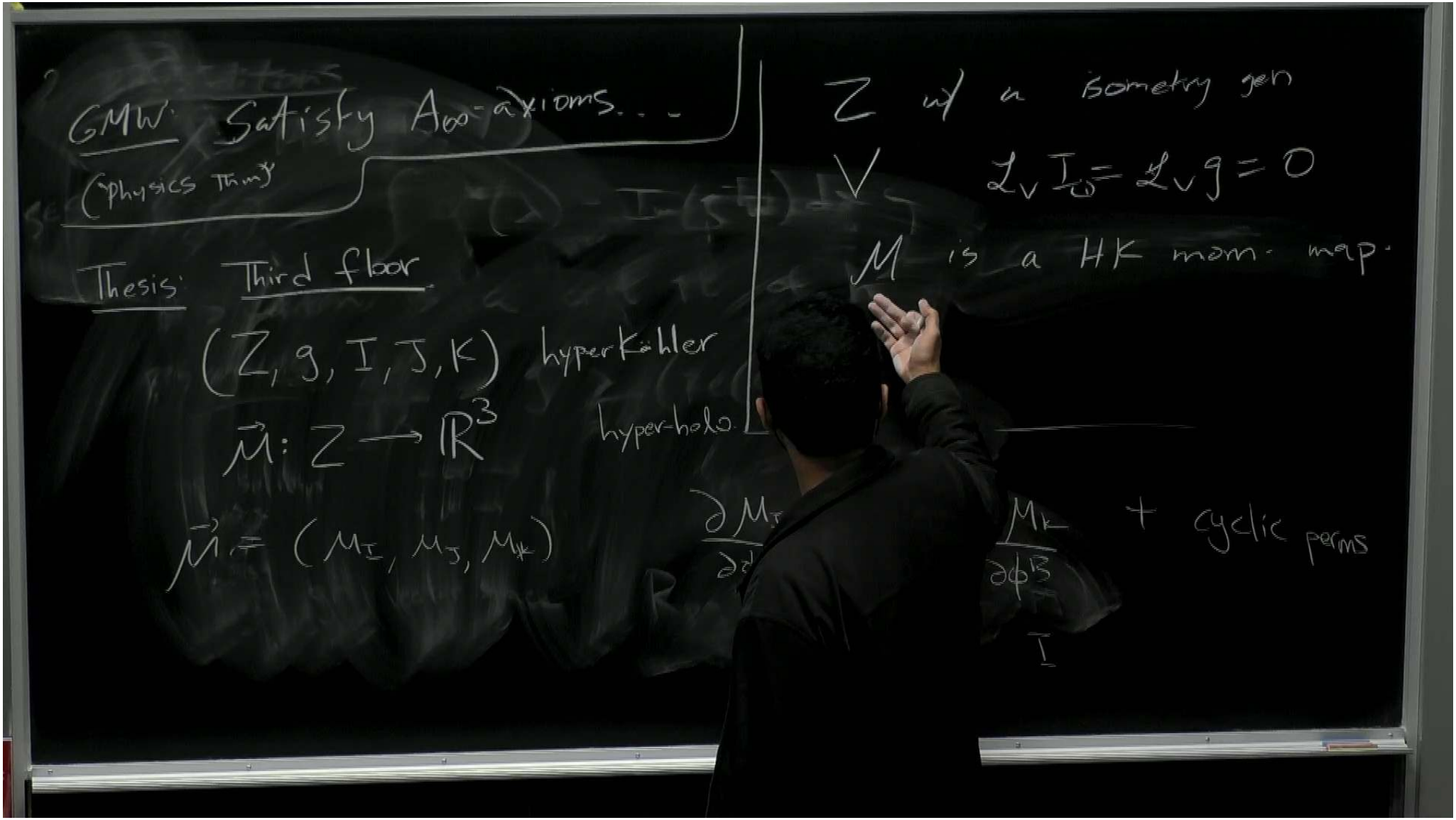
$\otimes$ - prod. of R's

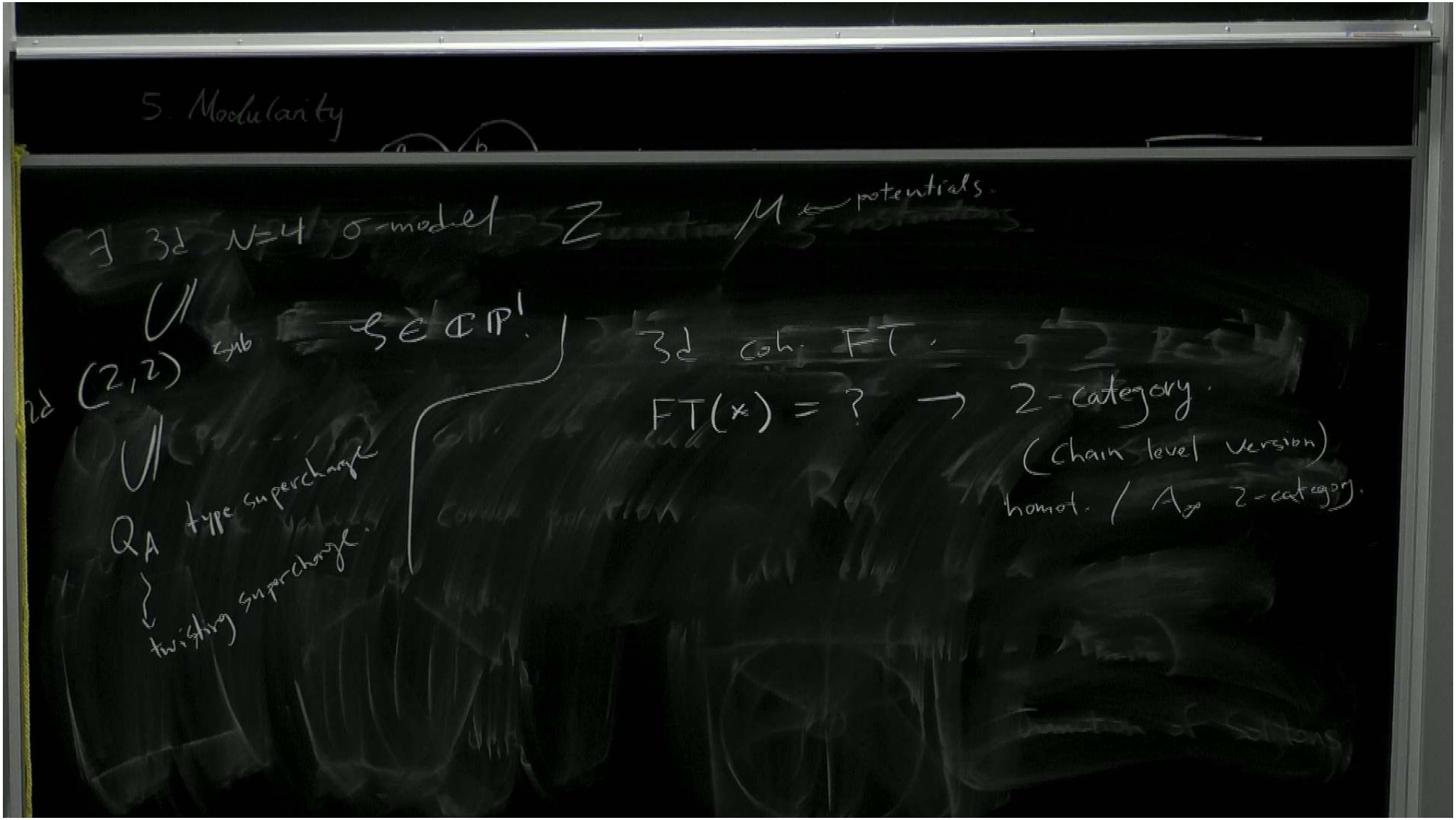


$$V_{P_0P_3} = R_{P_0P_3} \oplus (R_{P_0P_1} \otimes R_{P_1P_3}) \otimes \cdots$$


thimbles
L_p and L_q.
R_{-q}
A_∞ - maps:
$m_k = \sum_{t \in \{\text{tant webs on } \mathbb{R}_+ \times \mathbb{R} \text{ w/ } k \text{ bdry vertices}\}} A(t)$
m_1
m_2
m_3


L_p and L_q.
A_∞-maps:
$m_k = \sum_{t \in \{\text{taut webs on } \mathbb{R}_{\geq 0} \times \mathbb{R} \text{ w/ } k \text{ bdry vertices}\}} A(t)$
m_1
+
$A(t) = \prod_{v \in V(t)} N(v)$
m_2
+
m_3
+ ...


GMW: Satisfy A_∞-axioms...
(physics Thm)
Thesis: Third floor
(Z, g, I, J, K) hyperKähler
$\vec{\mu}: Z \to \mathbb{R}^3$
hyper-holo.
$\vec{\mu} = (\mu_I, \mu_J, \mu_K)$
Z w/ a isometry gen
V $\mathcal{L}_V I_\omega = \mathcal{L}_V g = 0$
M is a HK mom. map.
$\frac{\partial M_I}{\partial}$
$\frac{M_K}{\partial \phi_3}$ + cyclic perms
I


5. Modularity
3d N=4 σ-model
Z
M ← potentials.
3d (2,2) susy
ξ ∈ ℂℙ¹
3d coh. FT.
FT(*) = ? → 2-category.
(chain level version)
homot. / A∞ 2-category.
Q_A type supercharge
twisting supercharge.

Ω_ζ (2,0) symp I_ζ Z

$= d\Lambda_\zeta$ M_ζ I_ζ-hol. mom. map.

$$W_\zeta[\varphi] = \int \varphi^*(\Lambda_\zeta) + i M_\zeta$$

Four-tier Hierarchy of BPS objects

1. Vacua = Crit pts of $\vec{\mu}$ = Fixed pts of V

- distinct
- no three lie on a line
- no four lie on a plane

$\zeta \longleftrightarrow \vec{n} \in S^2$

ζ-dom. wall is a crit pt of W_ζ.

$\phi : \mathbb{R} \to Z$ → Grad. flow eqn for $n_1 M_1 + n_2 M_2 + n_3 M_3 = \vec{n} \cdot \vec{M}$.

$\vec{n}$-dom.

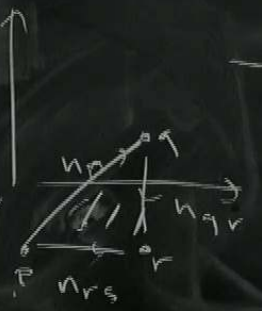
3. Grad. flow for $\mathrm{Re}\, W$.

$\phi : \mathbb{R}^2 \to Z$

Witten $\bar{\partial}$-eqn for $\vec{n} \cdot \vec{M}$

$I(\vec{n})$.

$R_{ijk}(\phi_{ij}, \phi_{jk}, \phi_{ki})$.



$\mathrm{Re}(\zeta h)$

Intersecting dom. wall junctions ↔ BPS polytopes

$N_{\mathcal{S}}(\quad) \in \mathbb{Z}$

Hopeful Claim: There's an ∞-categorical gadget into which [illegible] this BPS hierarchy organizes.

$\times \prod_i \underbrace{(\text{m-factor})_i}_{\uparrow \text{PgoPh}} \times \langle K \rangle$: a