Title: On 2-Categories and 3d A-models

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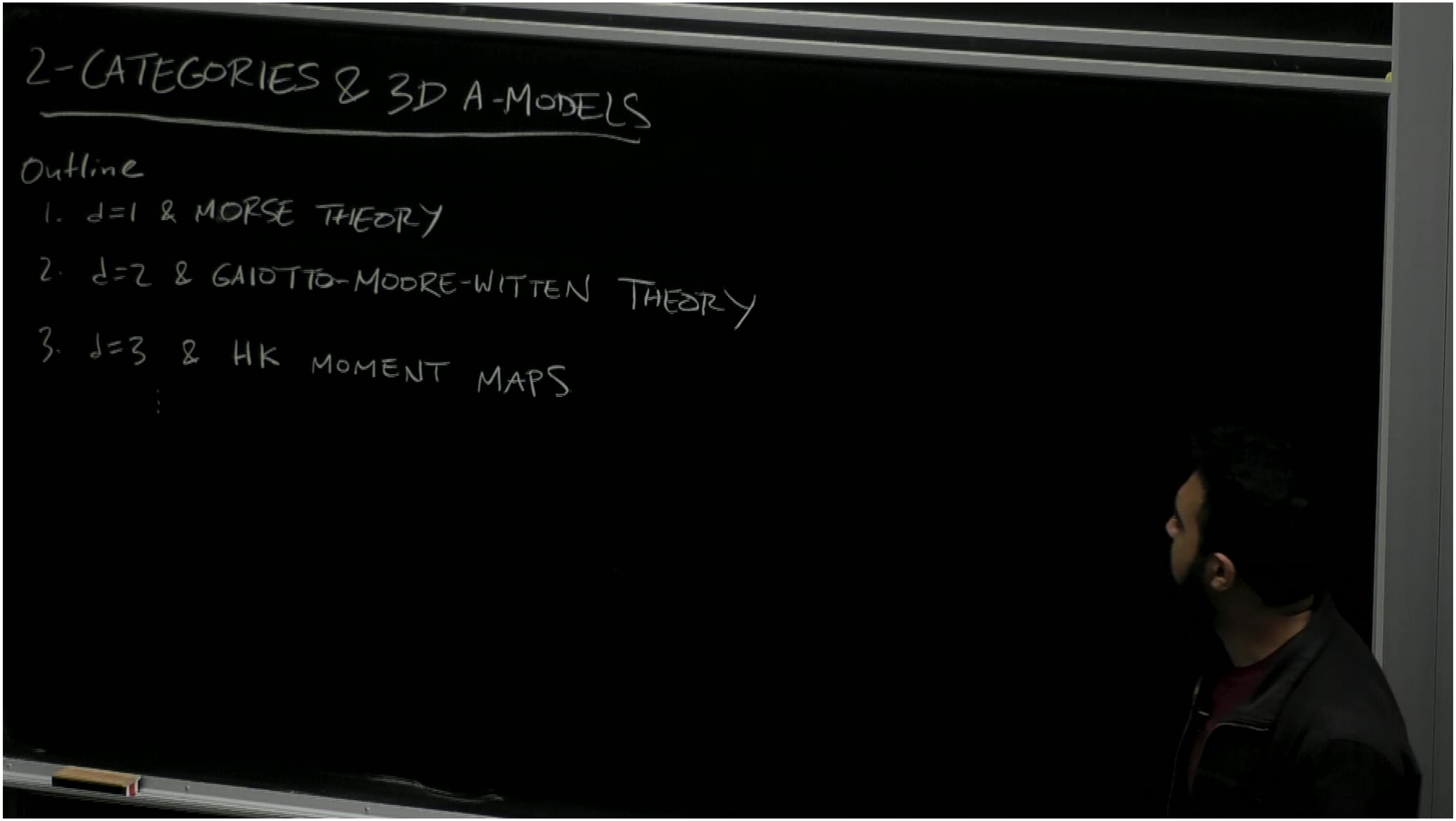
Series: Mathematical Physics

Date: December 09, 2022 - 1:30 PM

URL: https://pirsa.org/22120016

Abstract: The Fukaya category of a symplectic manifold is an excellent toy model for exploring the general structure of boundary conditions in two-dimensional topological field theories. In this talk I will explain an analogue for three dimensional theories. The natural playground for boundary conditions in d=3 is the 3d A-model into a hyperKahler target space. Here the fundamental BPS equation is the Fueter equation. I will explain how studying certain boundary value problems for the Fueter equation, one is lead to something resembling a higher categorical structure. I will then argue how a similar structure also seems to appear after performing a massive deformation by a hyperKahler moment map.

Zoom link: https://pitp.zoom.us/j/96574302292?pwd=dkpIVXN0UmFiRkdkenFzc2o2Nko0dz09


2-CATEGORIES & 3D A-MODELS
Outline
1. d=1 & MORSE THEORY
2. d=2 & GAIOTTO-MOORE-WITTEN THEORY
3. d=3 & HK MOMENT MAPS
⋮

1. $d=1$ & Morse Theory (...)

2. $d=2$ & GAIOTTO-MOORE-WITTEN THEORY (2015)

3. $d=3$ & HK MOMENT MAPS (Newer Stuff)

$\phi : \mathbb{R} \to X$

1. MORSE THEORY (X, g) $h : X \to \mathbb{R}$ $L = g_{ab}\dot{\phi}^a \dot{\phi}^b$ + ferm

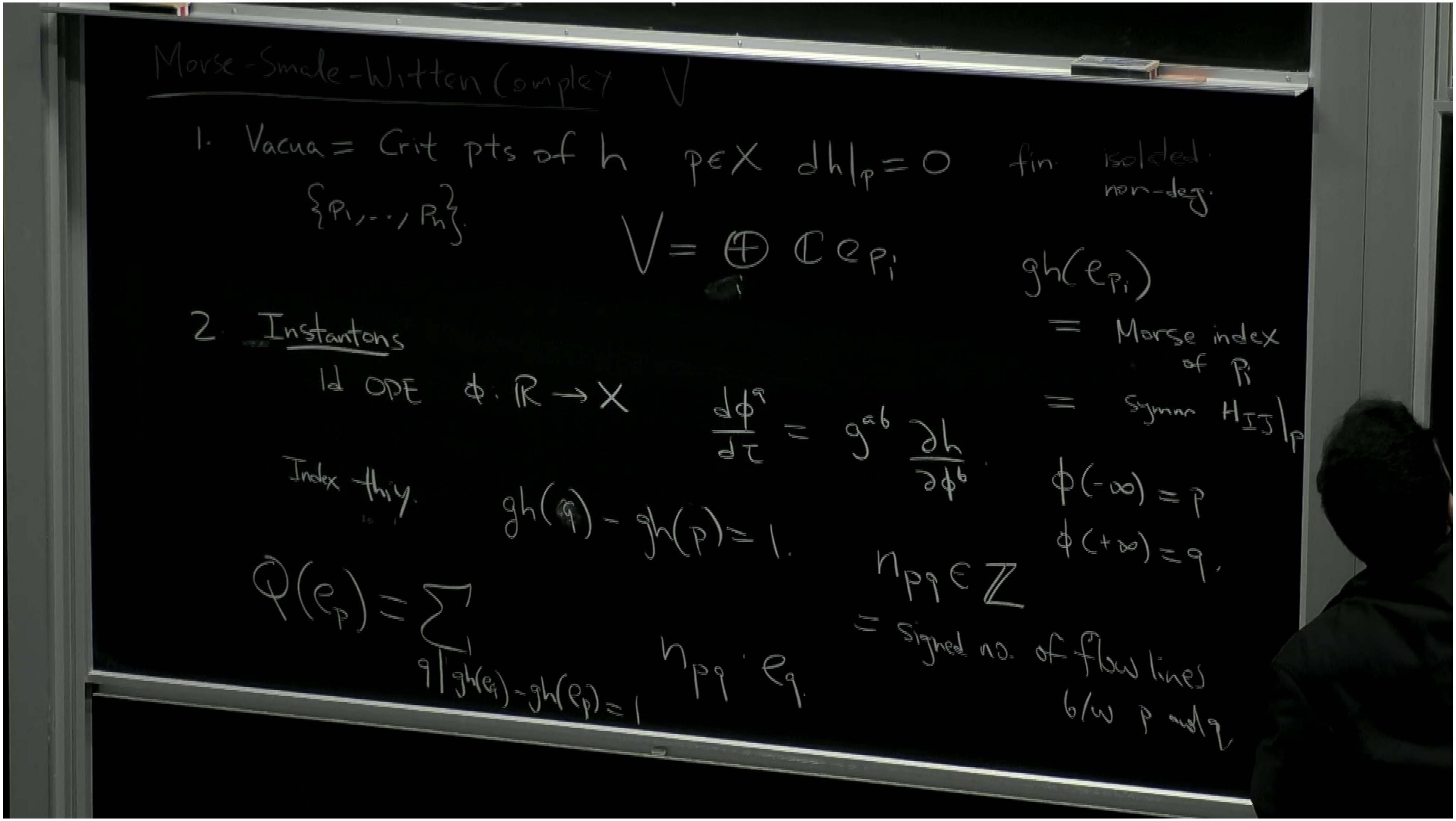
1-dim'l Coh. FT. $N=2$ SQM. $Q, \bar{Q}$ $Q^2 = 0 = \bar{Q}^2$

What does it assign to a pt? $[Q, \bar{Q}] = 2H$.

V. 1. Graded gh. (V, Q) chain complex

2. $Q : V^n \to V^{n+1}$ $Q^2 = 0$.

Morse-Smale-Witten Complex


Morse-Smale-Witten Complex V
1. Vacua = Crit pts of h $p \in X$ $dh|_p = 0$ fin. isolated non-deg.
$\{p_1, \ldots, p_n\}$.
$V = \bigoplus_i \mathbb{C} e_{p_i}$
$gh(e_{p_i})$
= Morse index of p_i
= signtr $H_{IJ}|_p$
2. Instantons
1d ODE $\phi : \mathbb{R} \to X$
$\frac{d\phi^a}{d\tau} = g^{ab} \frac{\partial h}{\partial \phi^b}$.
$\phi(-\infty) = p$
$\phi(+\infty) = q$.
Index thry.
$gh(q) - gh(p) = 1$.
$n_{pq} \in \mathbb{Z}$
= signed no. of flow lines b/w p and q
$Q(e_p) = \sum_{q \mid gh(e_q) - gh(e_p) = 1} n_{pq}\, e_q$.

$Q^2 \overset{?}{=} 0$ holds ?

"Broken flow line" Argument.

Top'l prop. g-independence

indep under "deformation" of h.

(Y, g, I) Kähler $W : Y \longrightarrow \mathbb{C}$ holom

metric, integ. complex str.

$\exists$ a $d=2$ coh. field theory. $N=(2,2)$ Landau-Ginzburg model.

"A-model" $Q_A = \bar{Q}_+ + \zeta Q_-$ $Q_+, \bar{Q}_+, Q_-, \bar{Q}_- \ldots$

twisting sup. $Q_A^2 = 0$

Expect A_∞-category.

$(Y, J, D) + W$ How build A_∞-cat?

GMW recipe for A_∞-category

1. Building blocks (BPS Objects)
2. Comb. Framework combines them into an algebraic str.

<u>GMW recipe for A_∞-category</u>

1. Building blocks (BPS Objects)
2. Comb. Framework combines them into an algebraic str.

1. <u>Vacua</u> $p \in Y$ $\quad dW|_p = 0$

2) $N \geq 1$

2. BPS Solitons

$$X = \mathrm{Map}(\mathbb{R}_x, Y)$$

$\zeta \in S^1$ $\quad h_\zeta[\varphi] = \int \varphi^*(\lambda) - \mathrm{Im}(\zeta^{-1} W)\, dx$

BPS solitons is a crit pt of h

BPS segments $\quad \dfrac{d\phi^a}{dx} = g^{ab} \dfrac{\partial}{\partial \phi^b}\left(\mathrm{Re}(\zeta^{-1} W)\right)$.

pq soliton $\quad -\infty \longrightarrow p$
$\quad\quad\quad\quad +\infty \longrightarrow q.$ $\quad\quad \zeta_{pq} = \dfrac{W_p - W_q}{|W_p - W_q|}$

$\exists$ gr. Vector space $\quad R_{pq} = \bigoplus_{pq\text{-soliton}} \mathbb{C}(\phi_{pq})$.

3.] BPS polygons / BPS Junctions / ζ-instantons.

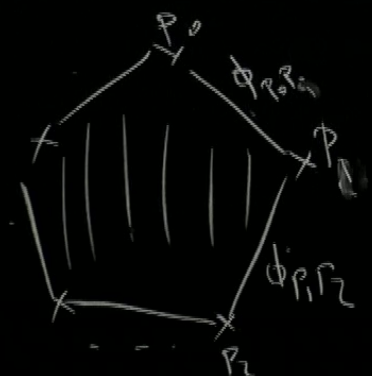
$\phi : \mathbb{R}^2_{\tau,x} \longrightarrow Y$

$$\frac{\partial \phi^a}{\partial x} + I^a{}_b \frac{\partial \phi^b}{\partial \tau} = g^{ab} \frac{\partial}{\partial \phi^b} \mathrm{Re}\, \zeta^{-1} W.$$

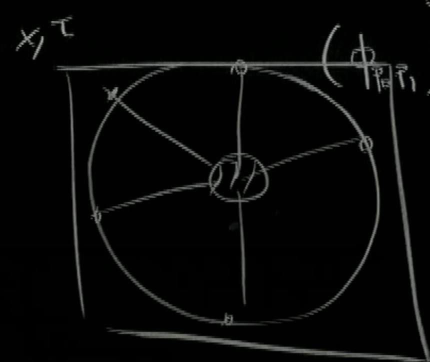
ζ-inst
Witten $\bar\partial$-eqn.

(1) $(p_0, \ldots, p_n)$ coll. of vacua

Crit values convex position.



– label edge by a soliton



$(\phi_{p_0 p_1}, \ldots, \phi_{p_{n-1} p_0})$

Junction of Solitons.

$\omega = d\lambda$

Expect A_∞-category.

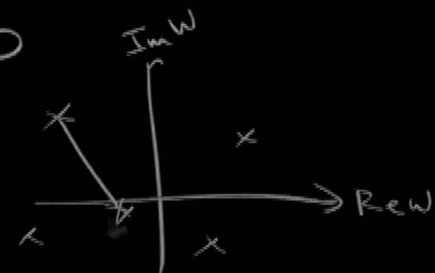
$(Y, J, \mathcal{D}) + W$ How build A_∞-cat?

GMW recipe for A_∞-category

1. Building blocks (BPS Objects)
2. Comb. Framework combines them into an algebraic str.

1. Vacua $p \in Y$ $\quad dW|_p = 0$

Crit values are in general position



Selection rule gradings of $(\phi_{p_0 p_1}, \dots, \phi_{p_n p_0})$

$N_\zeta(\phi_{01}, \dots, \phi_{n0}) \in \mathbb{Z}$ = Signed No of BPS polygons

GMW comb. formalism

1. Objects $\{L_p\}$

A_∞ - assoc. axioms

2. $p, q \to V_{pq}$ $\mathbb{Z}$-gr. v.space

3. $m_k : V_{p_0 p_1} \otimes \cdots \otimes V_{p_{k-1} p_k} \longrightarrow V_{p_0 p_k}$ ---

...dex thry

$gh(q) - gh(p) = 1$

$\phi(+\infty) = q$

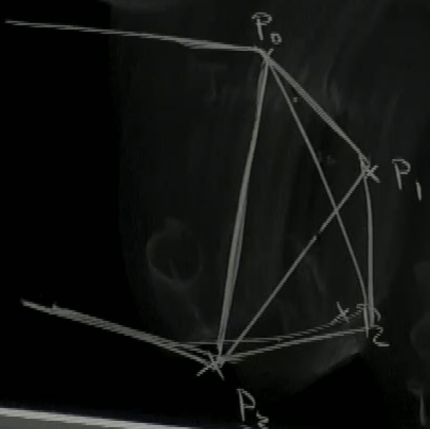
$n_{pq} \in \mathbb{Z}$ = Signed no. of flow lines b/w p and q

$$\ldots(e_p) = \sum_{q \,|\, gh(e_q) - gh(e_p) = 1} n_{pq}\, e_q$$

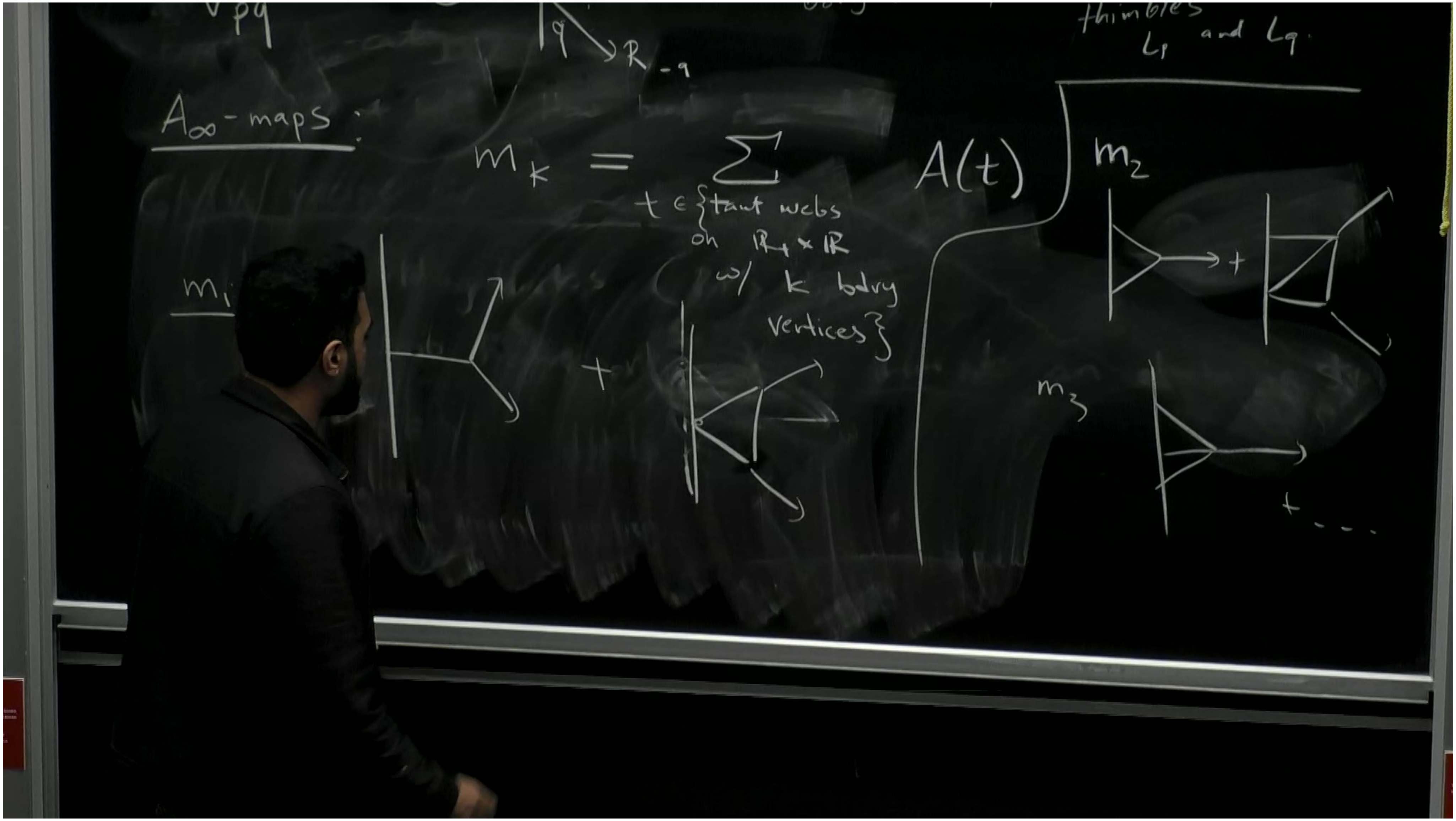
(1) Objects L_p for every vacuum p

Natural ordering

(2) $\zeta =$ $V_{pq} = \bigoplus_{p \in \{pq \text{ semi-int polygons}\}} R_p \sim \otimes\text{-prod of } R's$



$$V_{p_0 p_3} = R_{p_0 p_3} \oplus (R_{p_0 p_1} \otimes R_{p_1 p_3}) \oplus \cdots$$


thimbles
L_p and L_q
A∞-maps:
m_k = Σ A(t)
t ∈ {taut webs on ℝ+ × ℝ w/ k bdry vertices}
m_1
m_2
m_3
+ . . .

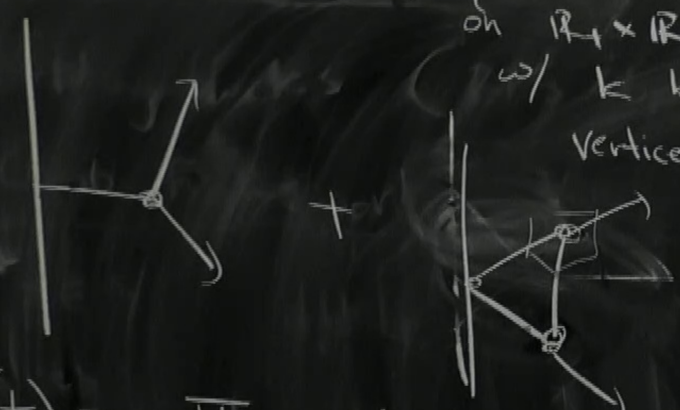
v_{pq} q R_{-q}

thimbles L_1 and L_q.

A_∞-maps:

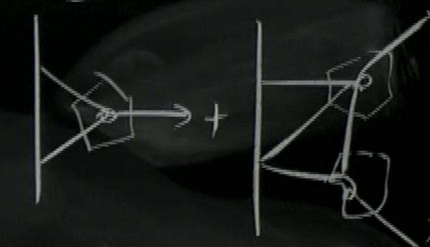
$$m_k = \sum_{t \in \{\text{taut webs on } \mathbb{R}_+ \times \mathbb{R} \text{ w/ } k \text{ bdry vertices}\}} A(t)$$

m_1



$$A(t) = \prod_{v \in V(t)} N(v)$$

m_2



m_3


GMW: Satisfy A∞-axioms...
(physics thm)
Thesis: Third floor
(Z, g, I, J, K) hyperkähler
M: Z → R3
hyper-holo.
M = (MI, MJ, MK)
+ cyclic perms
Z w/ a isometry gen
V
LV I = LV g = 0
M is a HK mom. map.

5. Modularity

$\exists$ 3d $N=4$ σ-model Z $\mathcal{M}$ $\leftarrow$ potentials.

$\bigcup\!|$ sub $\zeta \in \mathbb{CP}^1$.

2d (2,2)

$\bigcup\!|$

Q_A type supercharge

$\wr$

twisting supercharge.

3d coh. FT.

$FT(*) = ? \rightarrow$ 2-category.

(chain level version)

homot. / A_∞ 2-category.

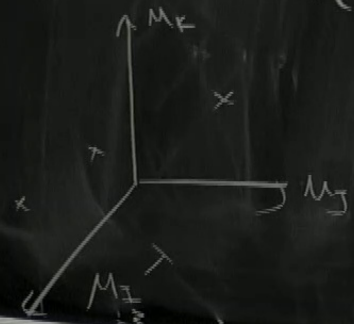
Ω_ζ (2,0) symp I_ζ Z.

$= d\Lambda_\zeta$ $\quad \mu_\zeta$ I_ζ-hol. mom. map.

$$\boxed{W_\zeta[\varphi] = \int \varphi^*(\Lambda_\zeta) + i\mu_\zeta}$$

Four-tier Hierarchy of BPS Objects

1. Vacua = Crit pts of $\vec{\mu}$ = Fixed pts of V



- distinct
- no three lie on a line
- no four lie on a plane

$\zeta \longleftrightarrow \vec{n} \in S^2$

ζ-dom. wall is a crit pt of W_ζ.

$\phi: \mathbb{R} \to Z$ → Grad. flow eqn for $n_1 M_1 + n_2 M_2 + n_3 M_3 = \vec{n} \cdot \vec{M}$

$\vec{n}$-dom.

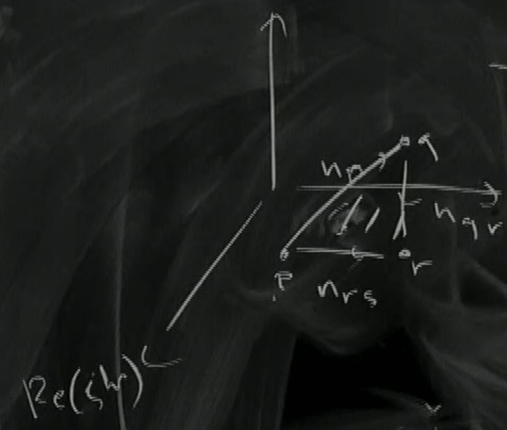
3. Grad. flow for $Re\, W$.

$\phi: \mathbb{R}^2 \to Z$

Witten $\bar{\partial}$-eqn for $\vec{n} \cdot \vec{M}$

$I(\vec{n})$.

$R_{ijk}(\phi_{ij}, \phi_{jk}, \phi_{ki})$.



Intersecting dom. wall junctions   BPS polytopes

$$N_\zeta\left(\quad \right) \in \mathbb{Z}$$

Hopeful Claim: There's an ζ-categorical gadget into which this BPS hierarchy organizes.

$$\times \prod_i \left((\eta\text{-factor})_i \right) \times \langle K \rangle$$

$\uparrow$ $\rho_g \circ \rho_h : a$