Title: QFT2 - Quantum Electrodynamics - Afternoon Lecture

Speakers: Cliff Burgess

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

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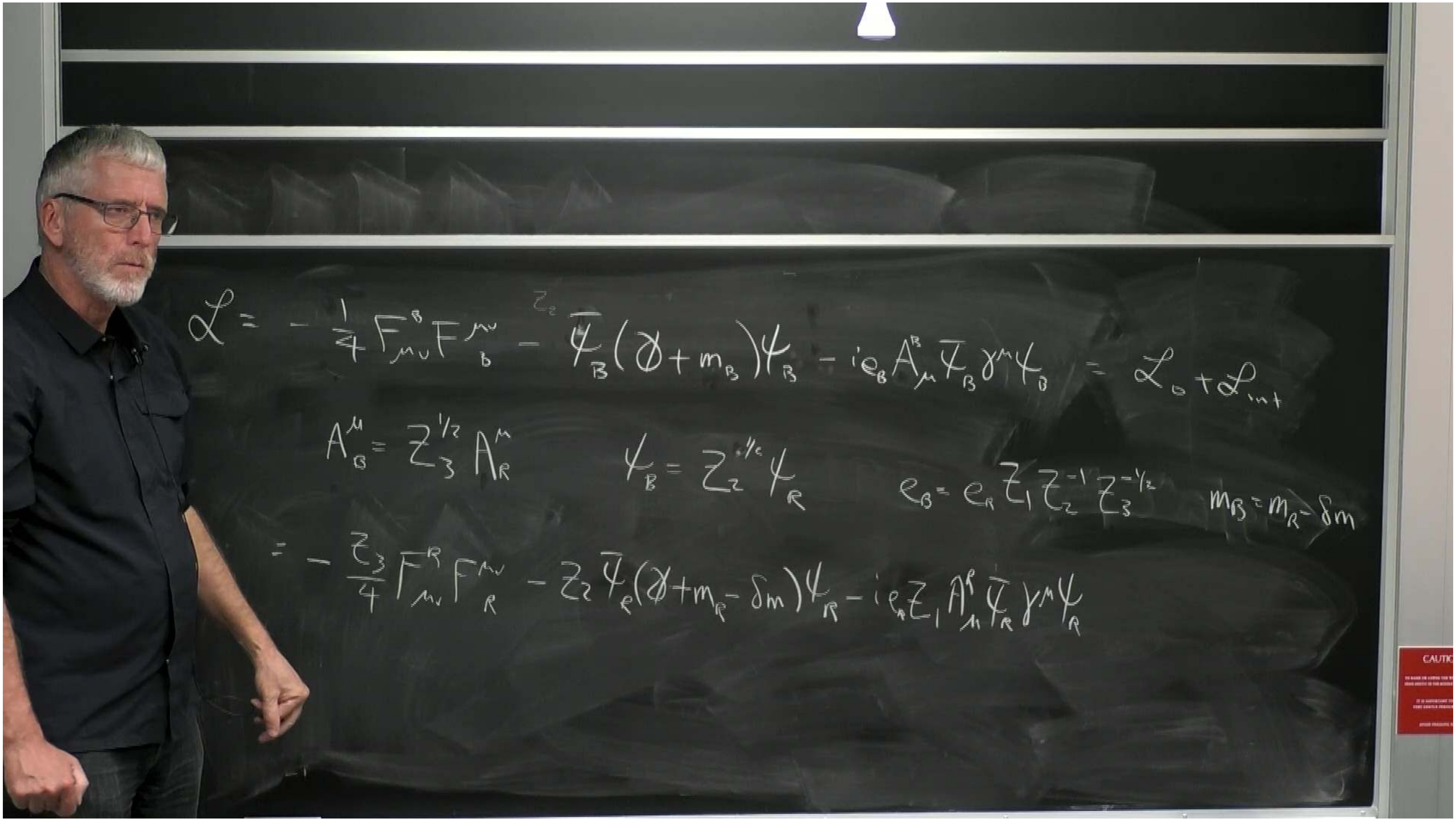
Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


$\mathcal{L} = -\frac{1}{4} F^{B}_{\mu\nu} F^{\mu\nu}_{B} - \bar{\psi}_B (\not{\partial} + m_B)\psi_B - i e_B A^{B}_{\mu} \bar{\psi}_B \gamma^\mu \psi_B = \mathcal{L}_0 + \mathcal{L}_{int}$
$A^\mu_B = Z_3^{1/2} A^\mu_R$ $\psi_B = Z_2^{1/2} \psi_R$ $e_B = e_R Z_1 Z_2^{-1} Z_3^{-1/2}$ $m_B = m_R - \delta m$
$= -\frac{Z_3}{4} F^{R}_{\mu\nu} F^{\mu\nu}_{R} - Z_2 \bar{\psi}_R (\not{\partial} + m_R - \delta m)\psi_R - i e_R Z_1 A^R_\mu \bar{\psi}_R \gamma^\mu \psi_R$

$$Z_2 - 1,\ Z_3 - 1 = O(\alpha)$$

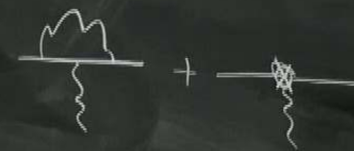
$$\mathcal{L}_0 = -\frac{1}{4} F^{(R)}_{\mu\nu} F^{\mu\nu}_{(R)} - \bar{\psi}_{(R)}(\not\partial + m_R)\psi_{(R)}$$

$$\mathcal{L}_{int} = -ie_R A^R_\mu \bar{\psi}_R \gamma^\mu \psi_R - ie_R (Z_1 - 1) A^R_\mu \bar{\psi}_R \gamma^\mu \psi_R$$

$$- \frac{1}{4}(Z_3 - 1) F^R_{\mu\nu} F^{\mu\nu}_R - (Z_2 - 1)\bar{\psi}_R(\not\partial + m_R)\psi_R + Z_2 \delta m \bar{\psi}_R \psi_R$$

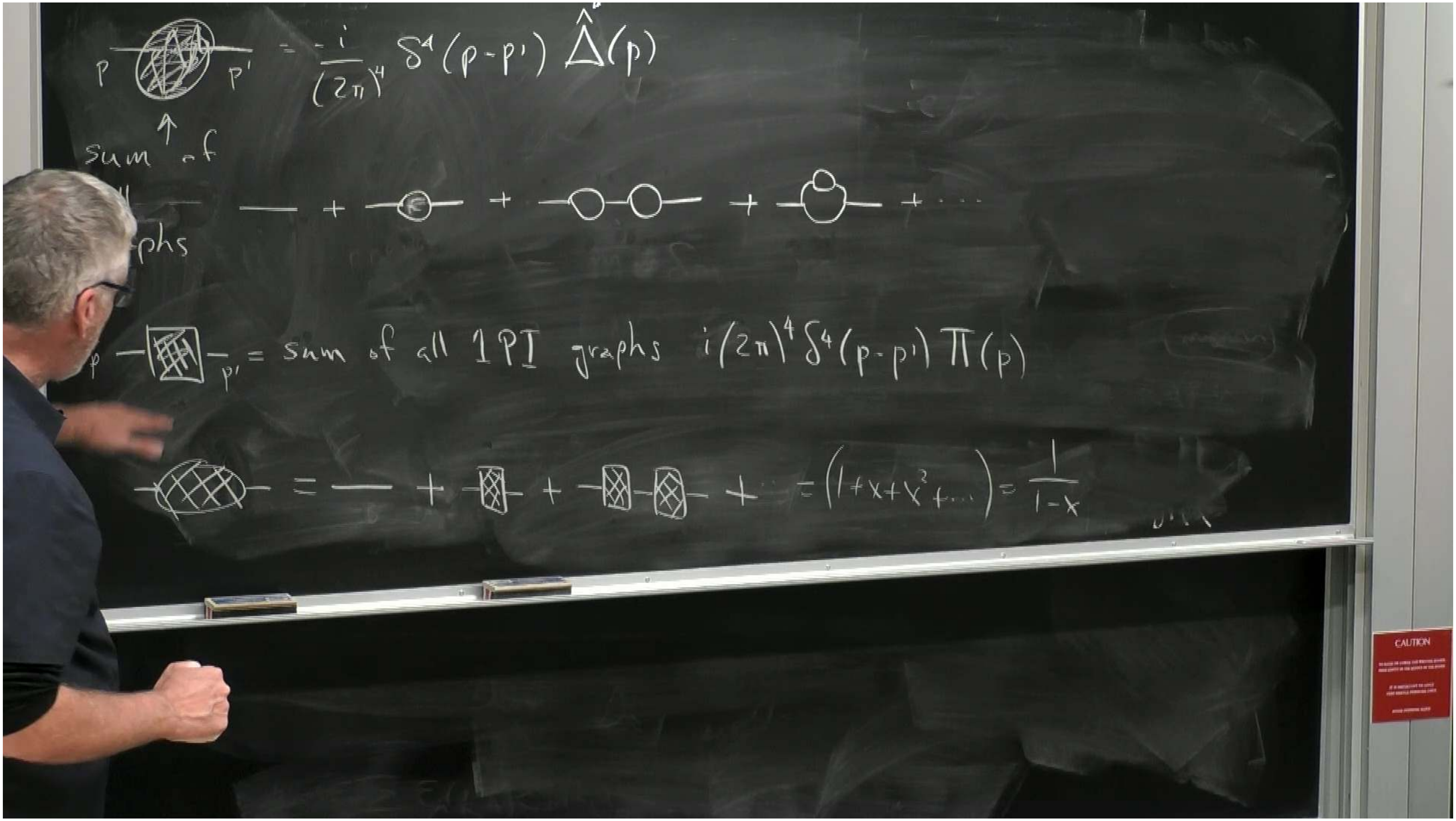
$i(2\pi)^4\delta^4(\ldots)A \qquad A$

 $e_R(\gamma^\mu)_{mn}$  $e_R(Z_1 - 1)(\gamma^\mu)_{mn}$



 $-i(2\pi)^4\delta^4(p-p')(Z_3-1)(p^2\eta_{\mu\nu} - p_\mu p_\nu)$

 $\left[-i(2\pi)^4(Z_2-1)(i\not p + m_R) + i(2\pi)^4 \delta m Z_2\right]\delta^4(p-p')$


p
p'
$= \frac{-i}{(2\pi)^4} \delta^4(p-p') \hat{\Delta}(p)$
sum of all graphs
+ + + + ...
sum of all 1PI graphs $i(2\pi)^4 \delta^4(p-p') \Pi(p)$
$= (1+x+x^2+\ldots) = \frac{1}{1-x}$
CAUTION

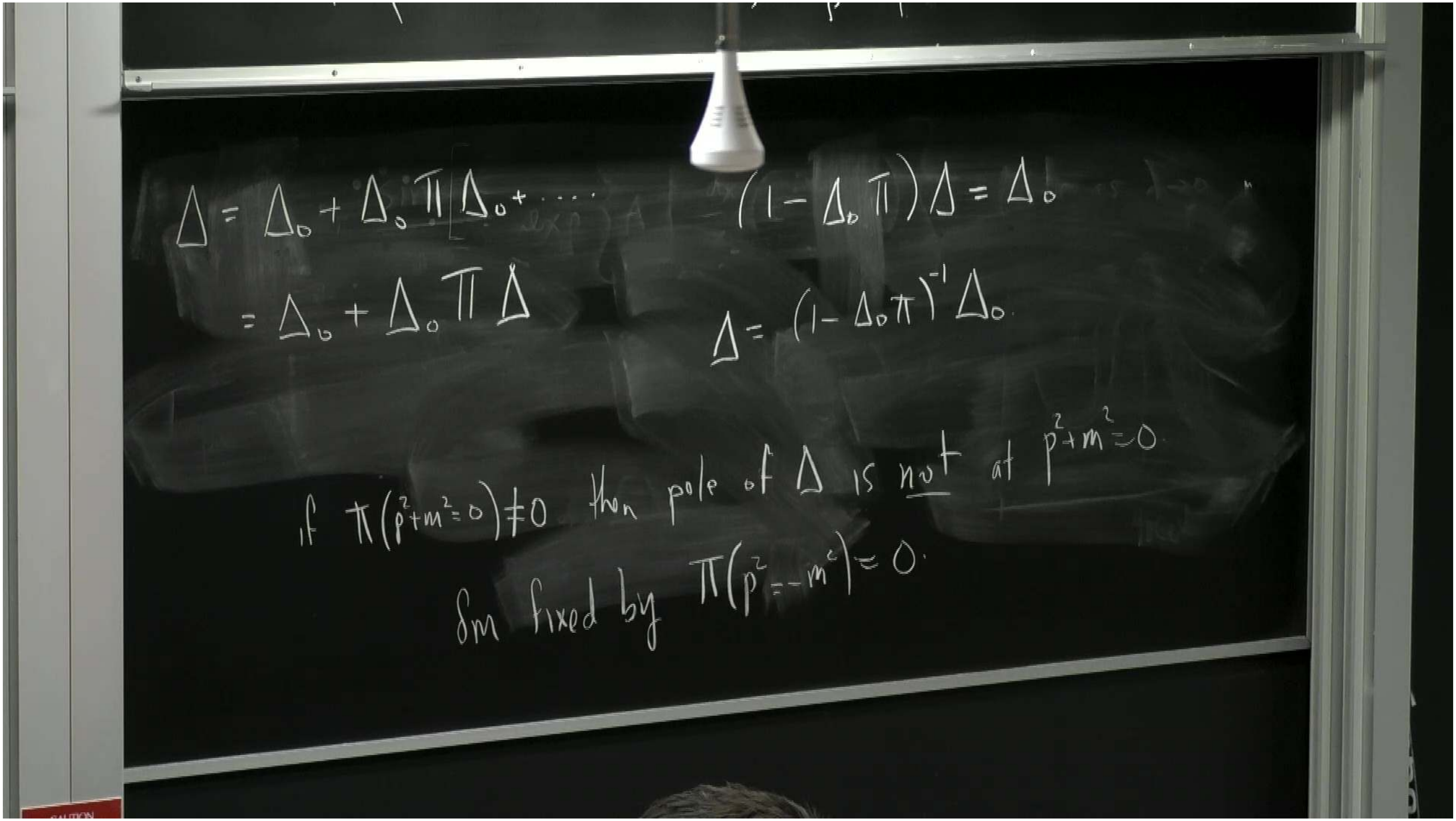
$$\Delta_0 = \frac{1}{p^2+m^2-i\epsilon}$$

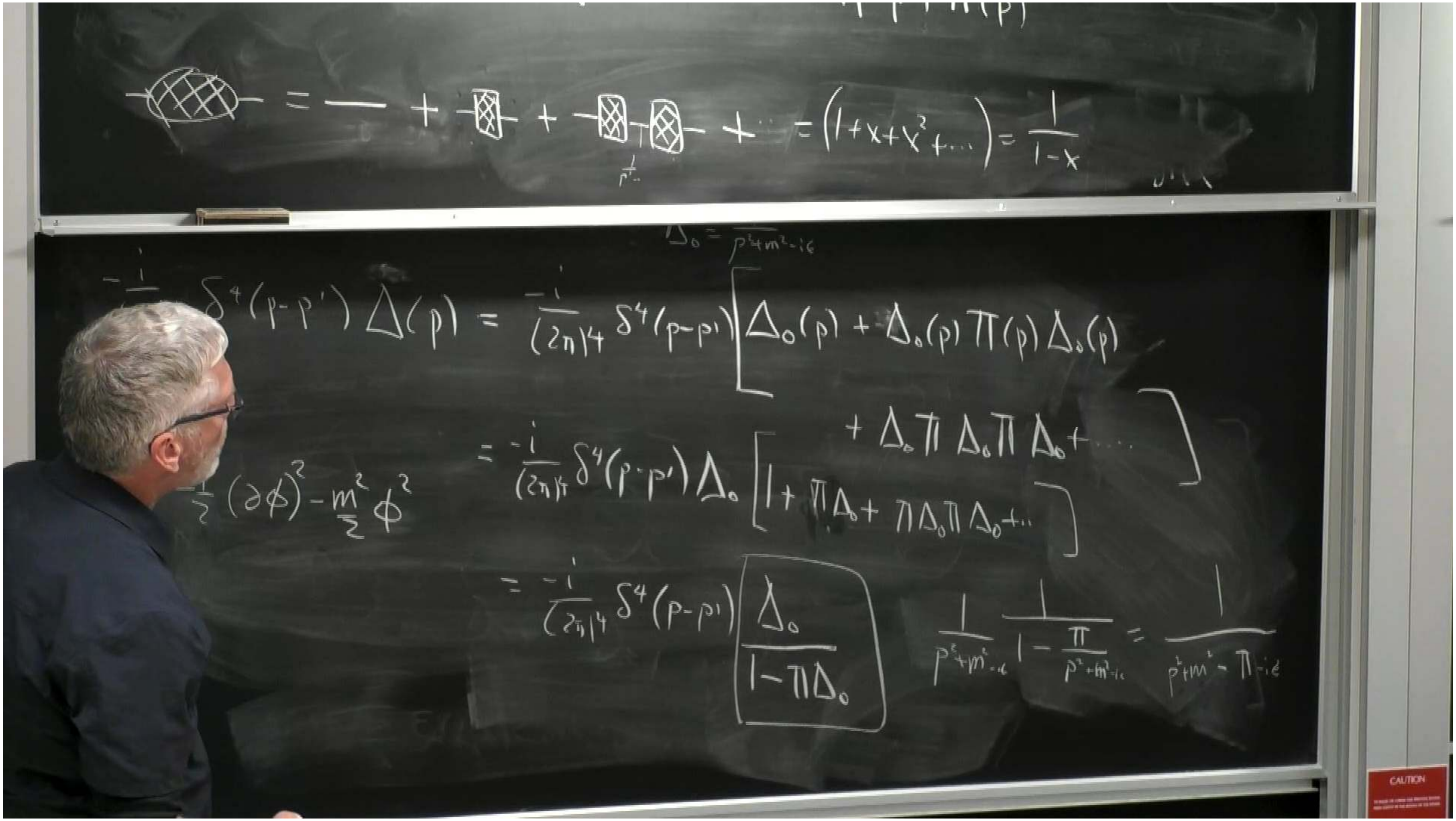
$$\frac{-i}{(2\pi)^4}\delta^4(p-p')\,\Delta(p) = \frac{-i}{(2\pi)^4}\delta^4(p-p')\Big[\Delta_0(p) + \Delta_0(p)\,\Pi(p)\,\Delta_0(p) + \Delta_0\Pi\Delta_0\Pi\Delta_0 + \cdots\Big]$$

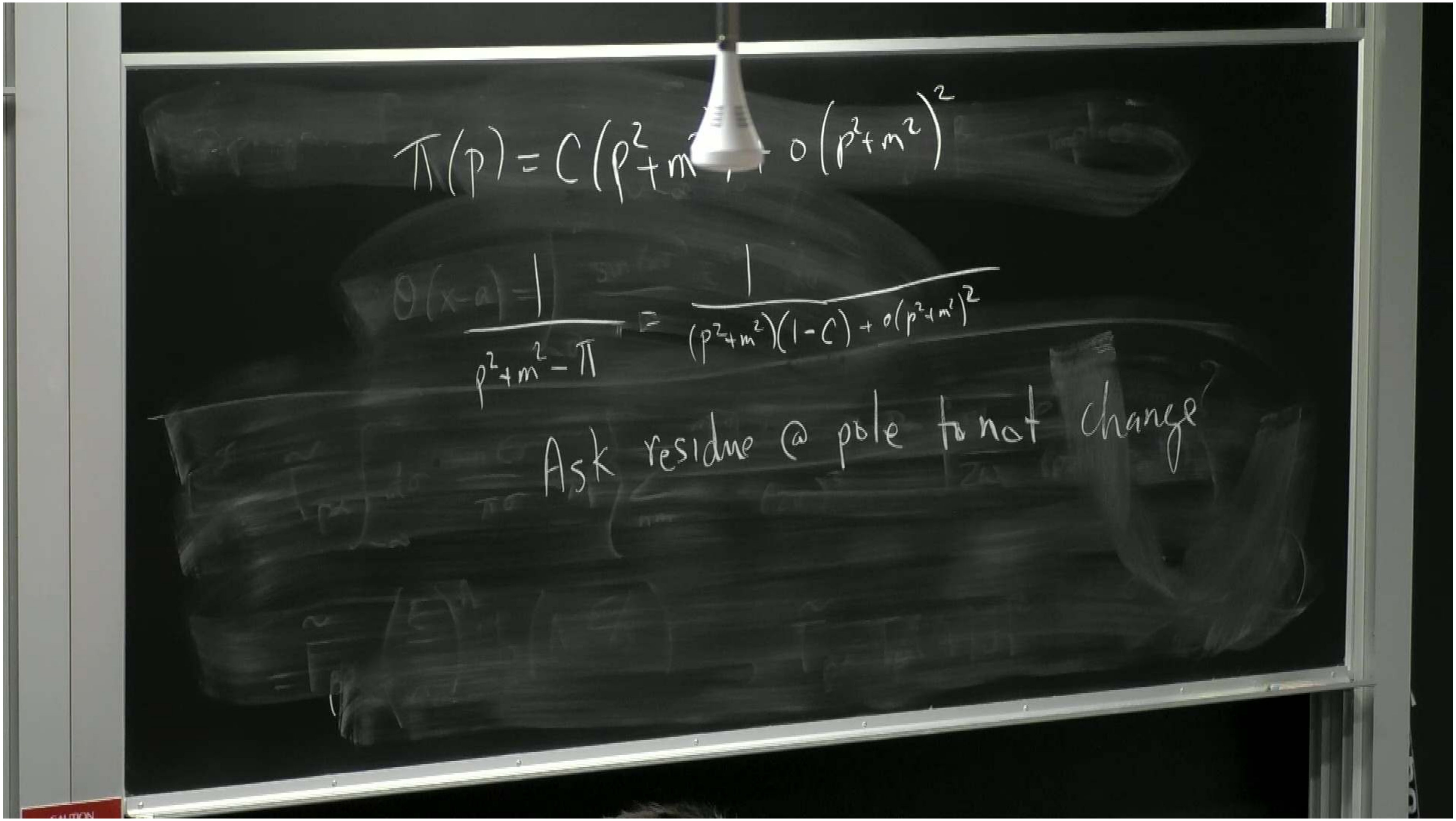
$$= \frac{-i}{(2\pi)^4}\delta^4(p-p')\,\Delta_0\Big[1 + \Pi\Delta_0 + \Pi\Delta_0\Pi\Delta_0 + \cdots\Big]$$

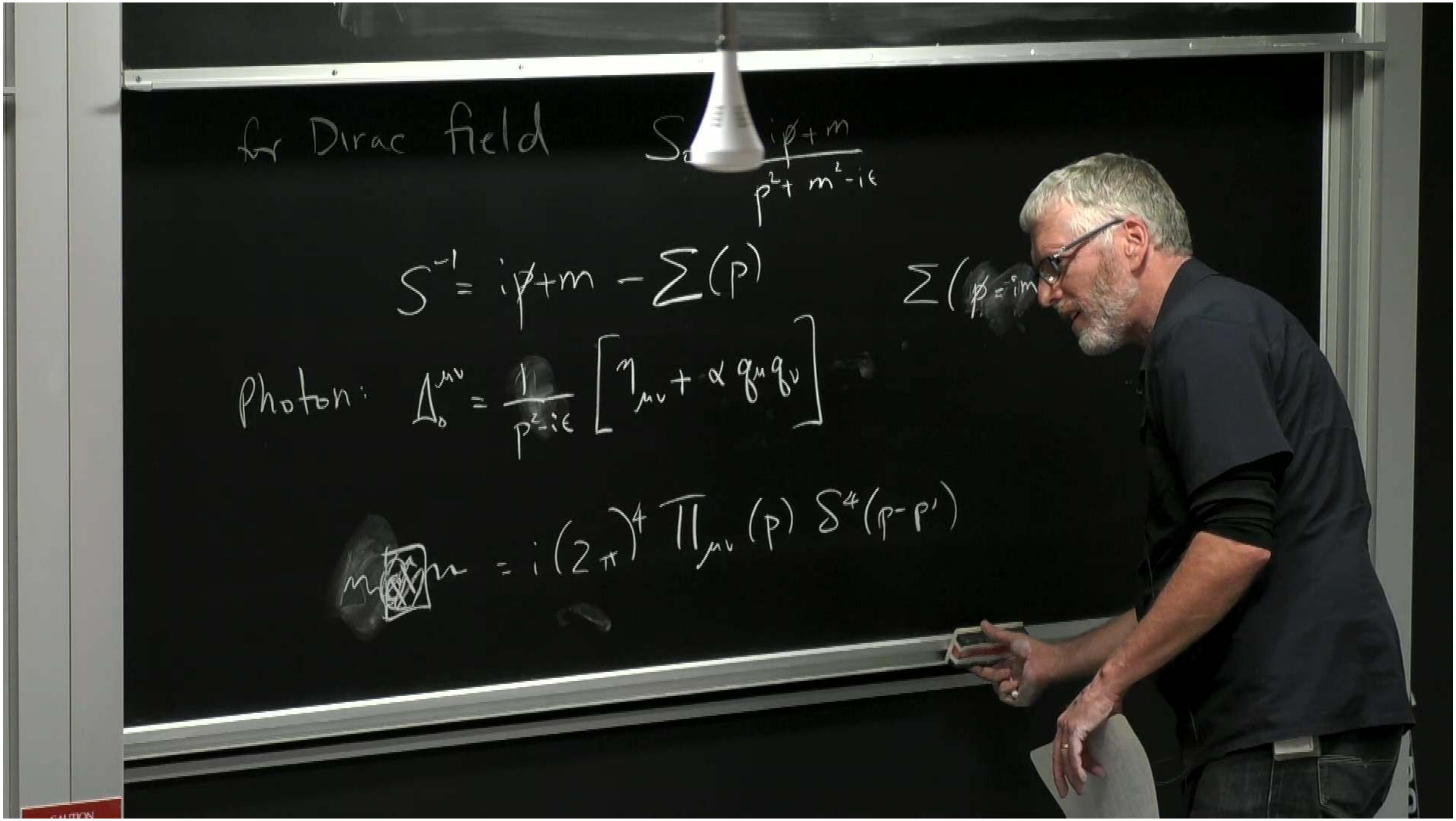
$$= \frac{-i}{(2\pi)^4}\delta^4(p-p')\boxed{\frac{\Delta_0}{1-\Pi\Delta_0}}$$

$$\frac{1}{p^2+m^2-i\epsilon}\,\frac{1}{1-\frac{\Pi}{p^2+m^2-i\epsilon}} = \frac{1}{p^2+m^2-\Pi-i\epsilon}$$


$\Delta = \Delta_0 + \Delta_0 \Pi \Delta_0 + \dots$
$= \Delta_0 + \Delta_0 \Pi \Delta$
$(1 - \Delta_0 \Pi)\Delta = \Delta_0$
$\Delta = (1 - \Delta_0 \pi)^{-1} \Delta_0$
if $\pi(p^2+m^2=0) \neq 0$ then pole of Δ is not at $p^2+m^2=0$
δm fixed by $\Pi(p^2=-m^2)=0$.


$= (1+x+x^2+\dots) = \frac{1}{1-x}$
$\Delta_0 = \frac{1}{p^2+m^2-i\epsilon}$
$\delta^4(p-p')\,\Delta(p) = \frac{-i}{(2\pi)^4}\delta^4(p-p')\left[\Delta_0(p) + \Delta_0(p)\Pi(p)\Delta_0(p) + \Delta_0\Pi\Delta_0\Pi\Delta_0 + \dots\right]$
$= \frac{-i}{(2\pi)^4}\delta^4(p-p')\,\Delta_0\left[1+\Pi\Delta_0+\Pi\Delta_0\Pi\Delta_0+\dots\right]$
$= \frac{-i}{(2\pi)^4}\delta^4(p-p')\boxed{\frac{\Delta_0}{1-\Pi\Delta_0}}$
$\frac{1}{p^2+m^2-i\epsilon}\,\frac{1}{1-\frac{\Pi}{p^2+m^2-i\epsilon}} = \frac{1}{p^2+m^2-\Pi-i\epsilon}$
$-\frac{1}{2}(\partial\phi)^2-\frac{m^2}{2}\phi^2$
CAUTION


$\Pi(p) = C(p^2+m^2) + o(p^2+m^2)^2$
$\frac{1}{p^2+m^2-\Pi} = \frac{1}{(p^2+m^2)(1-C) + o(p^2+m^2)^2}$
Ask residue @ pole to not change?


for Dirac field
$S_0 = \frac{i\not{p}+m}{p^2+m^2-i\epsilon}$
$S^{-1} = i\not{p}+m - \Sigma(p)$
$\Sigma($
Photon: $\Delta_0^{\mu\nu} = \frac{1}{p^2-i\epsilon}\left[\eta_{\mu\nu} + \alpha\, q_\mu q_\nu\right]$
$= i(2\pi)^4\, \Pi_{\mu\nu}(p)\, \delta^4(p-p')$

$$\Delta_{\mu\nu} = \frac{\eta_{\mu\nu}}{p^2 - p^2\Pi - i\epsilon} + p_\mu p_\nu \left[\frac{\alpha}{p^2} - \frac{\Pi(p^2)}{1-\Pi(p^2)} \left(\frac{1}{p^4}\right) \right]$$

$\Pi(p^2)$ is regular at $p^2=0$ pole remains at $p^2=0$.

$\Pi(p^2=0)=0$ keeps residue unity

SSB corresponds to $\Pi(p^2) \simeq \frac{c}{p^2} + O(1)$

$$\Delta_{\mu\nu} = \frac{\eta_{\mu\nu}}{p^2 - p^2\Pi - i\epsilon} + p_\mu p_\nu \left[\frac{\alpha}{p^2} - \frac{\Pi(p^2)}{1-\Pi(p^2)} \left(\frac{1}{p^4} \right) \right]$$

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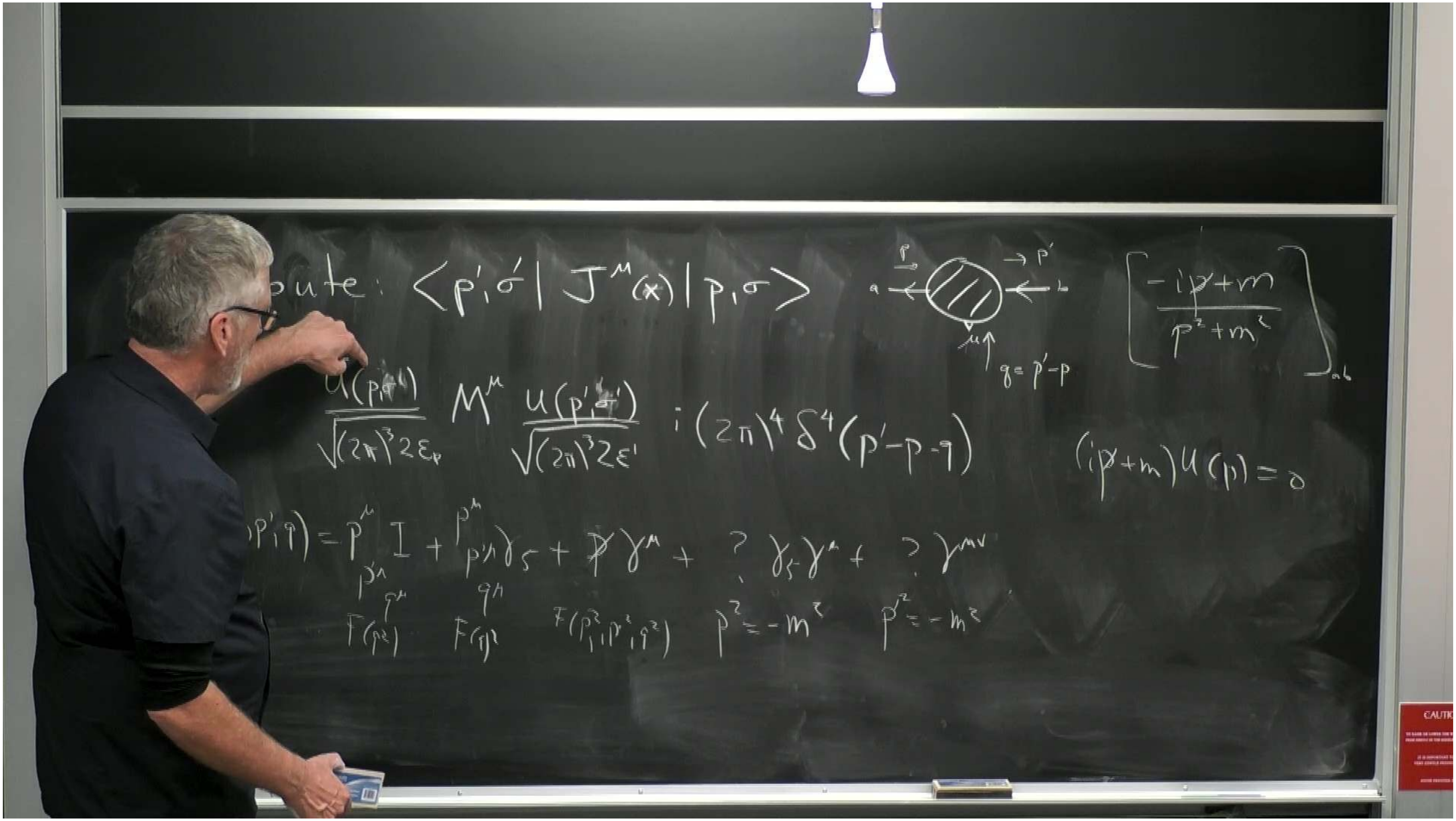
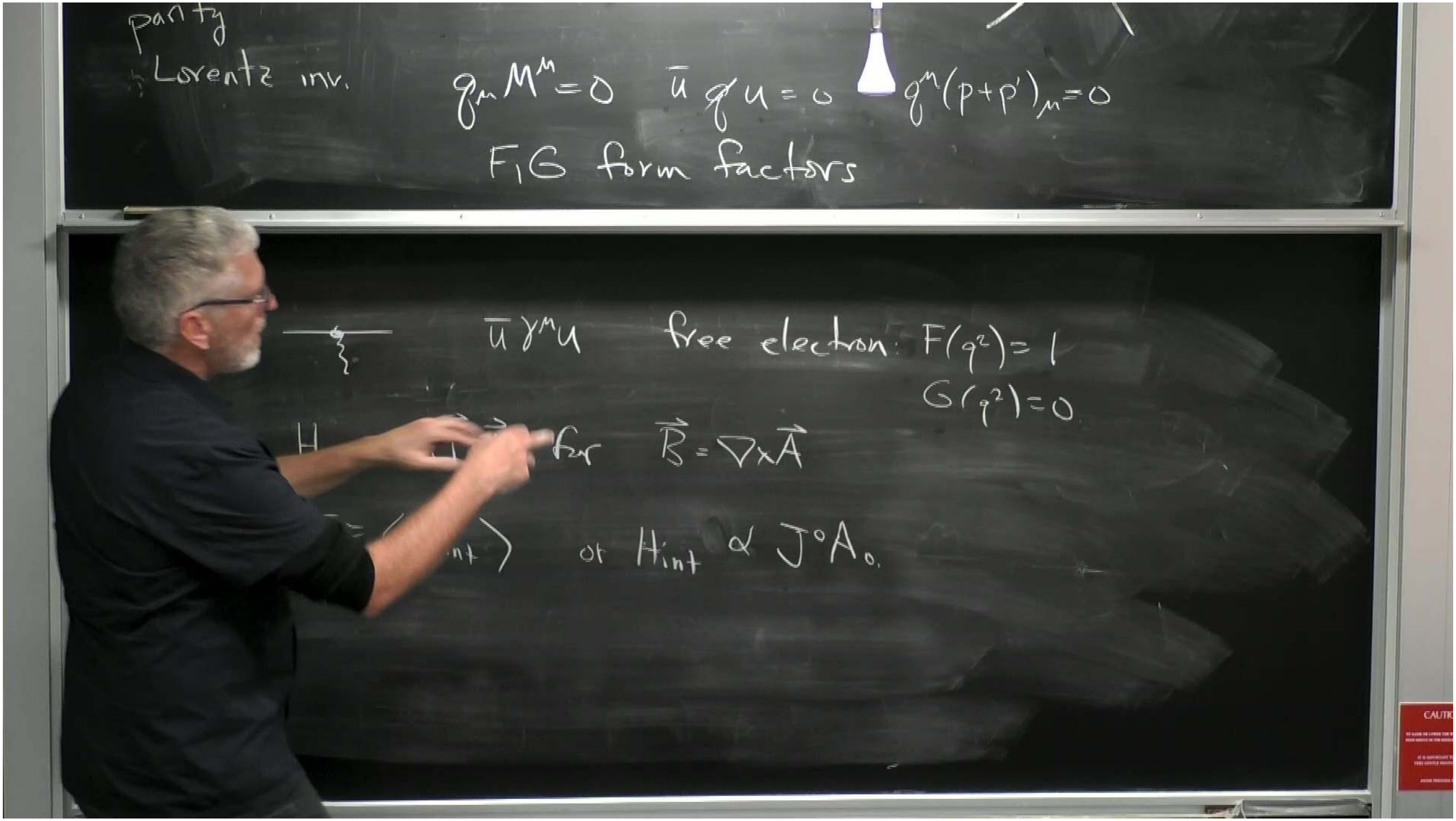


ute: ⟨p', σ'| J^μ(x) |p, σ⟩
q = p' − p
−ip̸ + m
p² + m²
M^μ
i (2π)⁴ δ⁴(p' − p − q)
(ip̸ + m) u(p) = 0
γ₅
CAUTI


parity
Lorentz inv.
q_μ M^μ = 0   ū q̸ u = 0   q^μ (p+p')_μ = 0
F, G form factors
ū γ^μ u   free electron: F(q²) = 1
G(q²) = 0
for B = ∇×A
or H_int ∝ J⁰A₀
CAUTION

parity

Lorentz inv.

$q_\mu M^\mu = 0 \quad \bar{u}\, \not{q}\, u = 0 \quad q^\mu (p+p')_\mu = 0$

F, G form factors

$G(q^2) = 0$

$H_{int} = -\vec{J}\cdot\vec{A}$ for $\vec{B} = \nabla\times\vec{A}$

$\delta E = \langle H_{int} \rangle$ or $H_{int} \propto J^0 A_0$

anomalous magnetic moment

Upshot: $Q = -e\left[F(0) + G(0)\right]$ $\quad \mu = \frac{Q}{2m} F(0) = \frac{Q}{2m}\left[1 - G(0)\right]$

$F(0) + G(0) = 1$

$$D = \left[(p-k)^2 + m^2 - i\epsilon\right]\left[(p'-k)^2 + m^2 - i\epsilon\right](k^2 - i\epsilon)$$

$$N^\mu = \gamma^\rho\left[-i(\not{p}'-\not{k})+m\right]\gamma^\mu\left[-i(\not{p}-\not{k})+m\right]\gamma_\rho$$

$$= -4(p'-k)\cdot(p-k)\gamma^\mu + 4imk^\mu - 4(p+p')^\mu\not{k} + 2\gamma^\mu k^2 + 4k^\mu\not{k}$$

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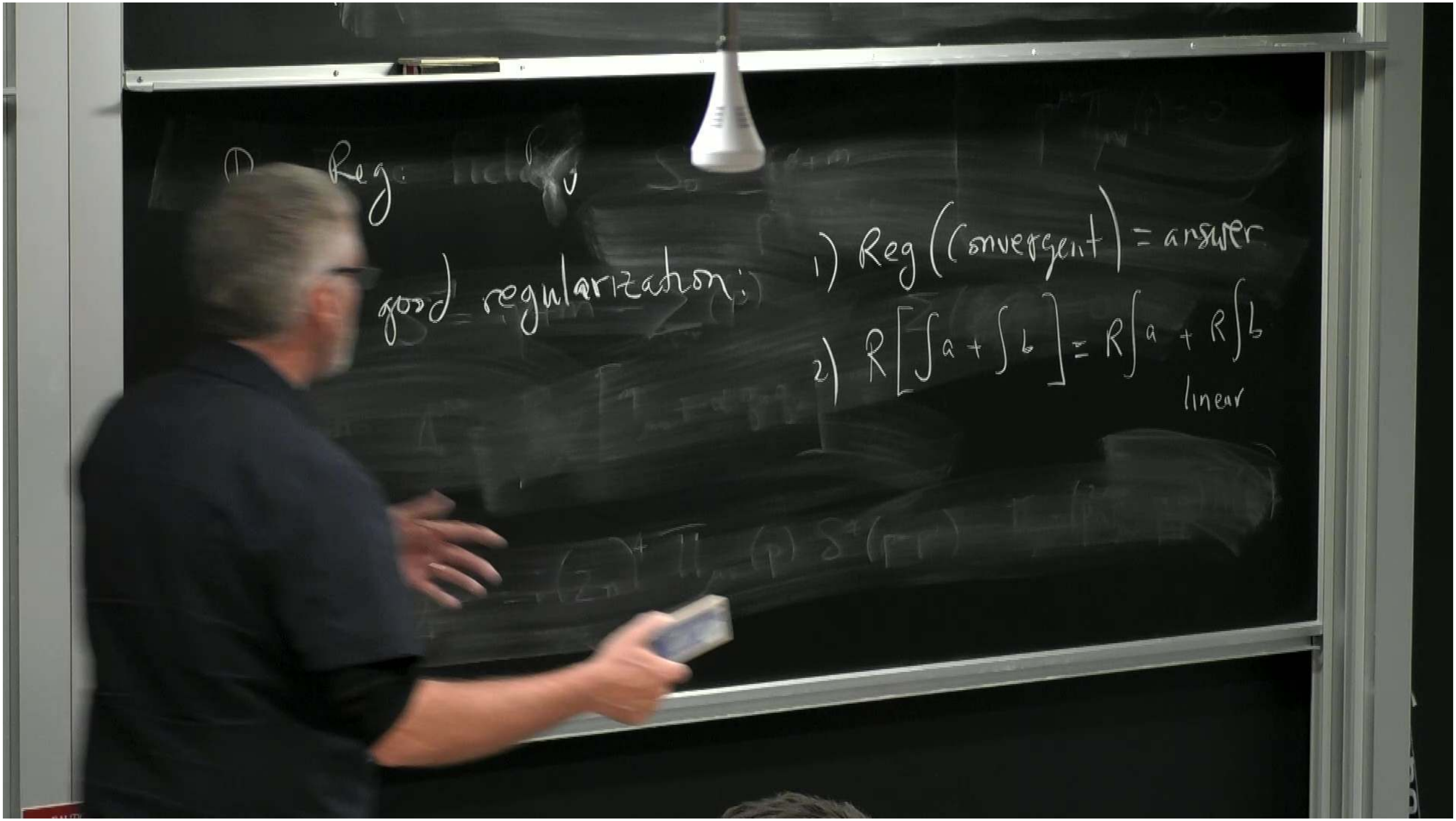

Reg:
good regularization:
1) Reg(convergent) = answer
2) R[∫a + ∫b] = R∫a + R∫b
linear

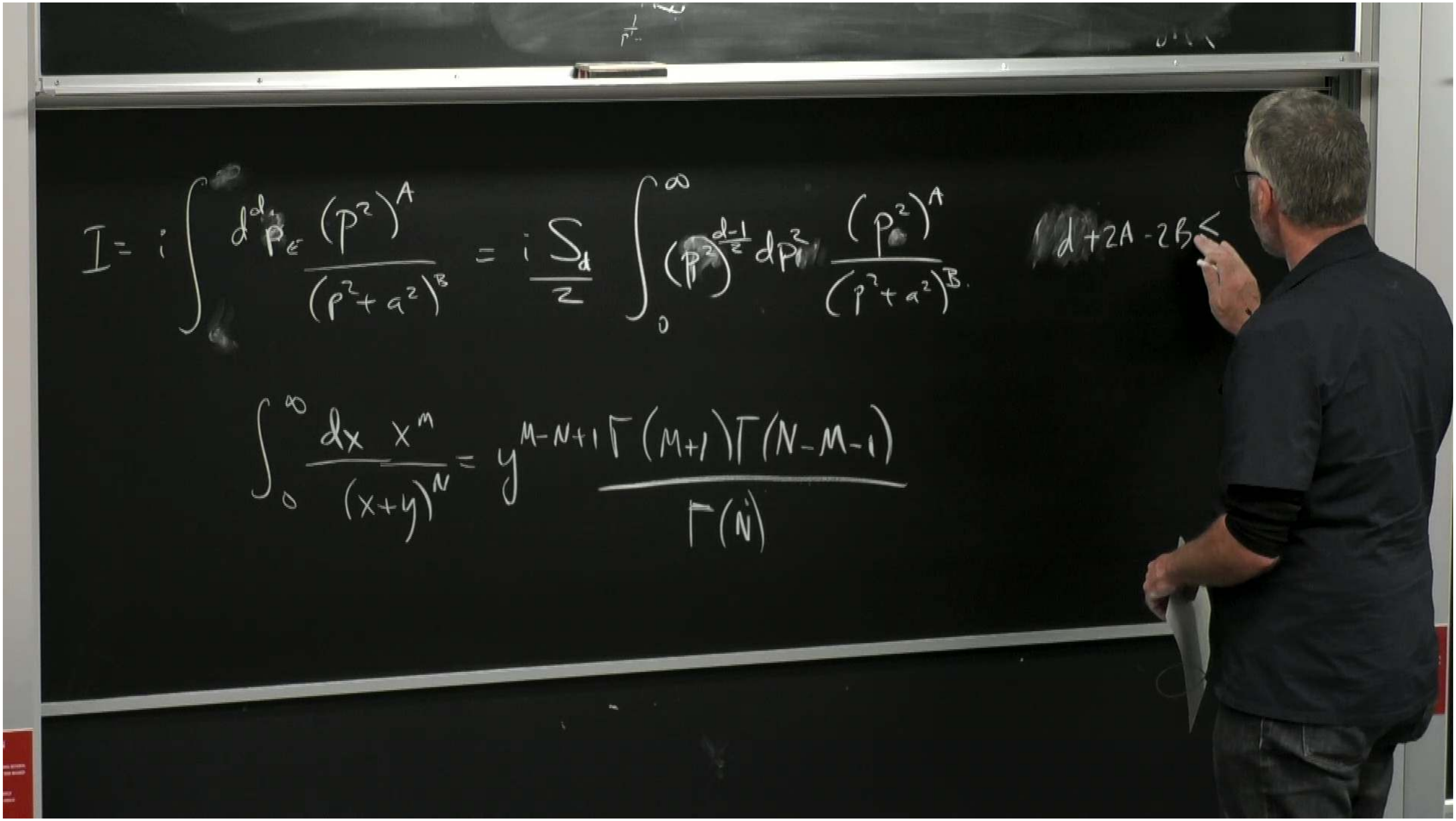
$$I = i \int d^4 p_E \frac{(p_E^2)^A}{(p_E^2 + a^2)^B}$$

diverges in UV if $4 + 2A \geq 2B$

2) $R\left[\int a + \int b\right] = R\int a + R\int b$

linear

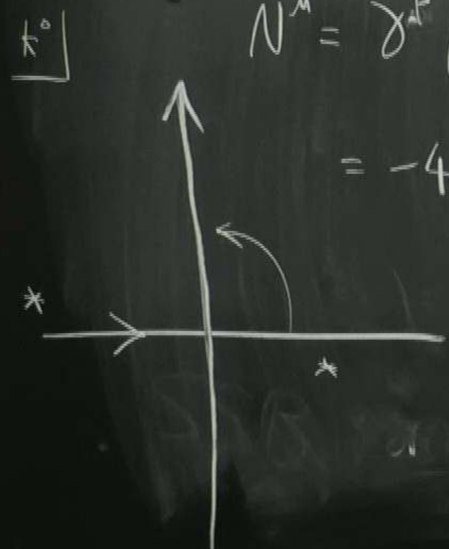
δm fixed by $\Pi(p^2 = -m^2) = 0$. physical ren. scheme


$I = i \int d^d p_E \frac{(p^2)^A}{(p^2+a^2)^B} = i \frac{S_d}{2} \int_0^\infty (p^2)^{\frac{d-1}{2}} dp^2 \frac{(p^2)^A}{(p^2+a^2)^B}.$
$d + 2A - 2B <$
$\int_0^\infty \frac{dx \; x^M}{(x+y)^N} = y^{M-N+1} \frac{\Gamma(M+1)\Gamma(N-M-1)}{\Gamma(N)}$

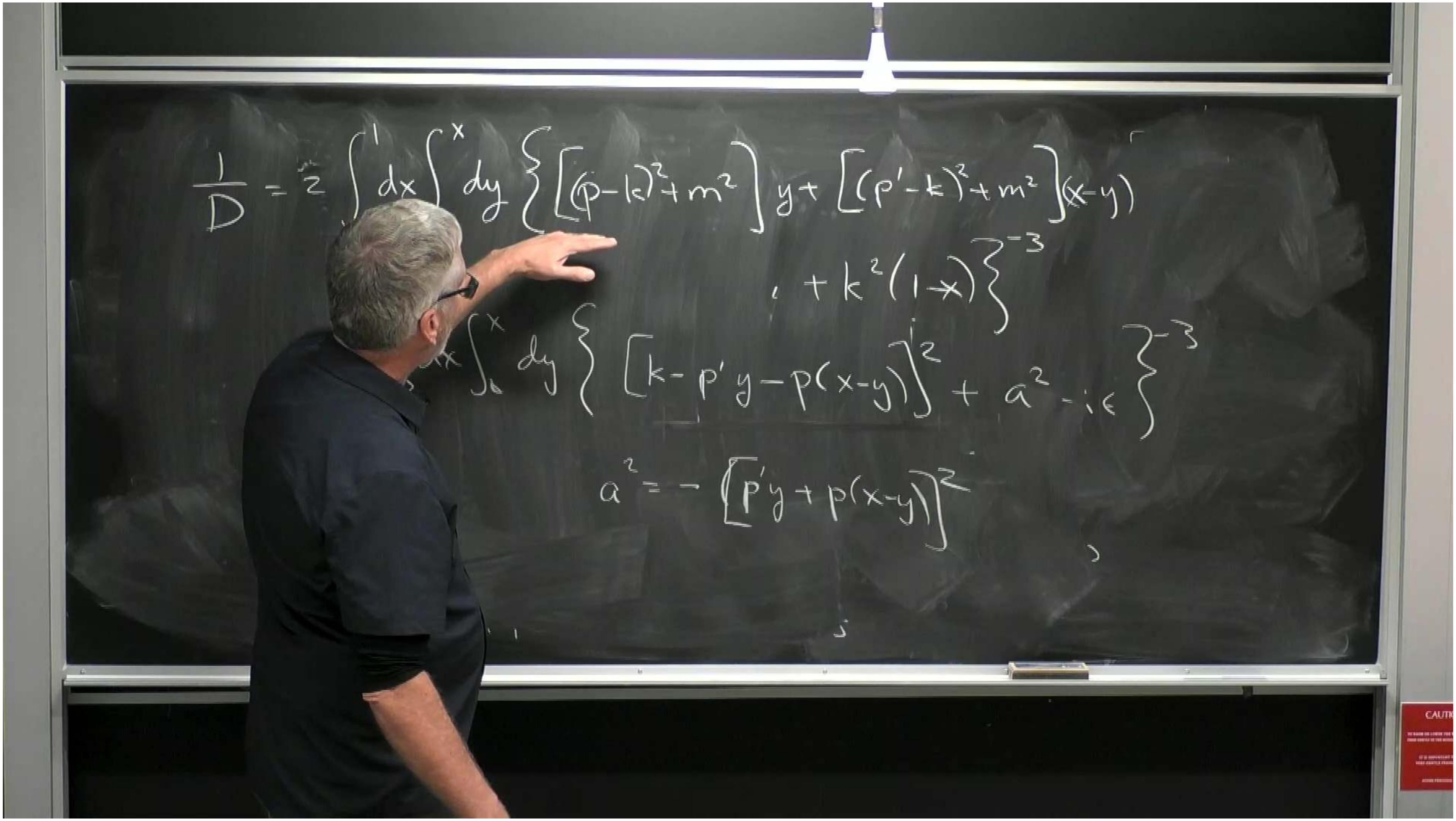
$$D = \left[(p'-k)^2+m^2\right]\left[(p-k)^2+m^2\right]k^2$$

$$N^\mu = \gamma^\rho\left[-i(\not{p}'-\not{k})+m\right]\gamma^\mu\left[-i(\not{p}-\not{k})+m\right]\gamma_\rho$$

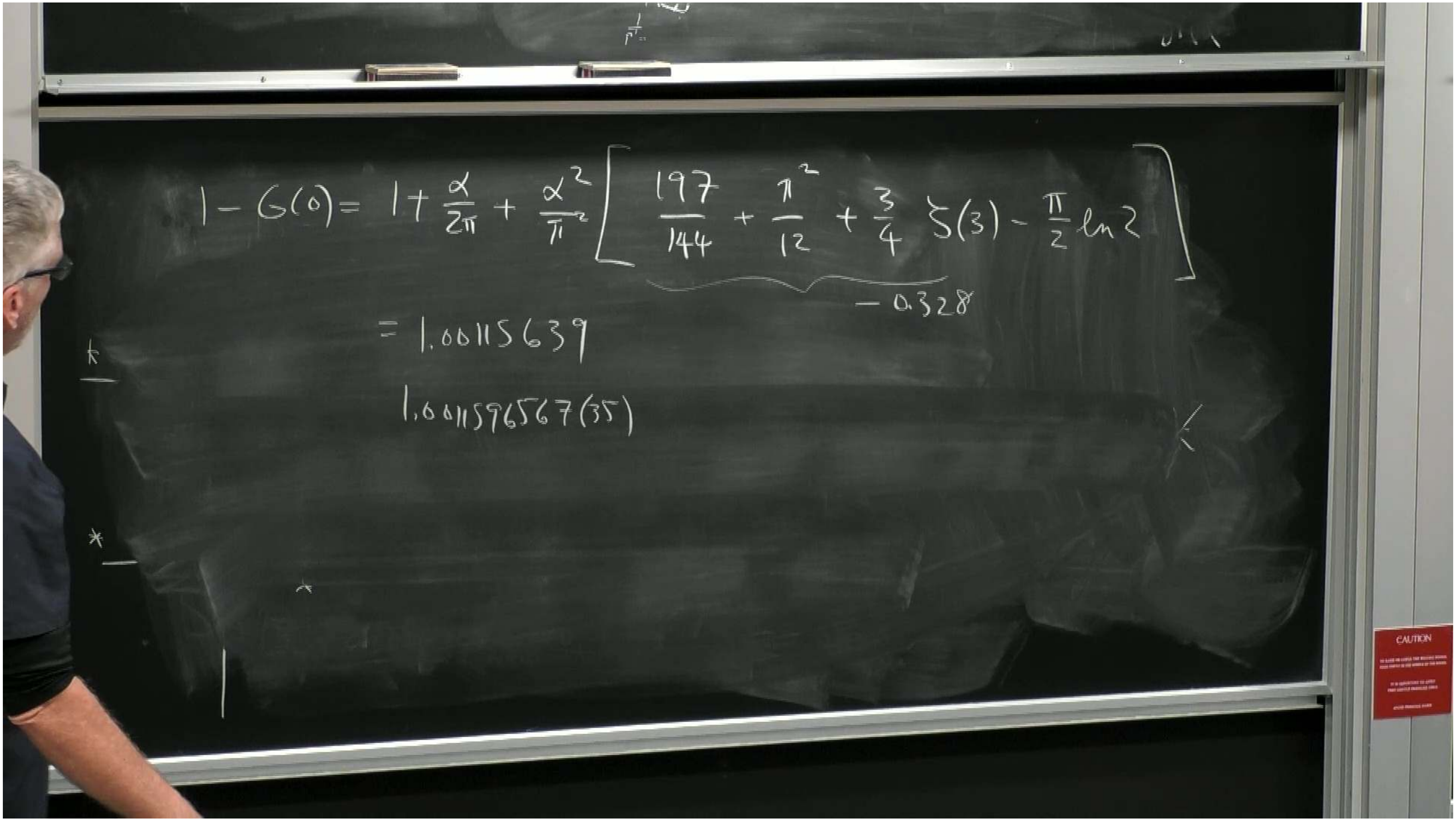
$$= -4(p'-k)\cdot(p-k)\gamma^\mu + 4im\,k^\mu - 4(p+p')^\mu\not{k} + 2\gamma^\mu k^2 + 4k^\mu\not{k}$$

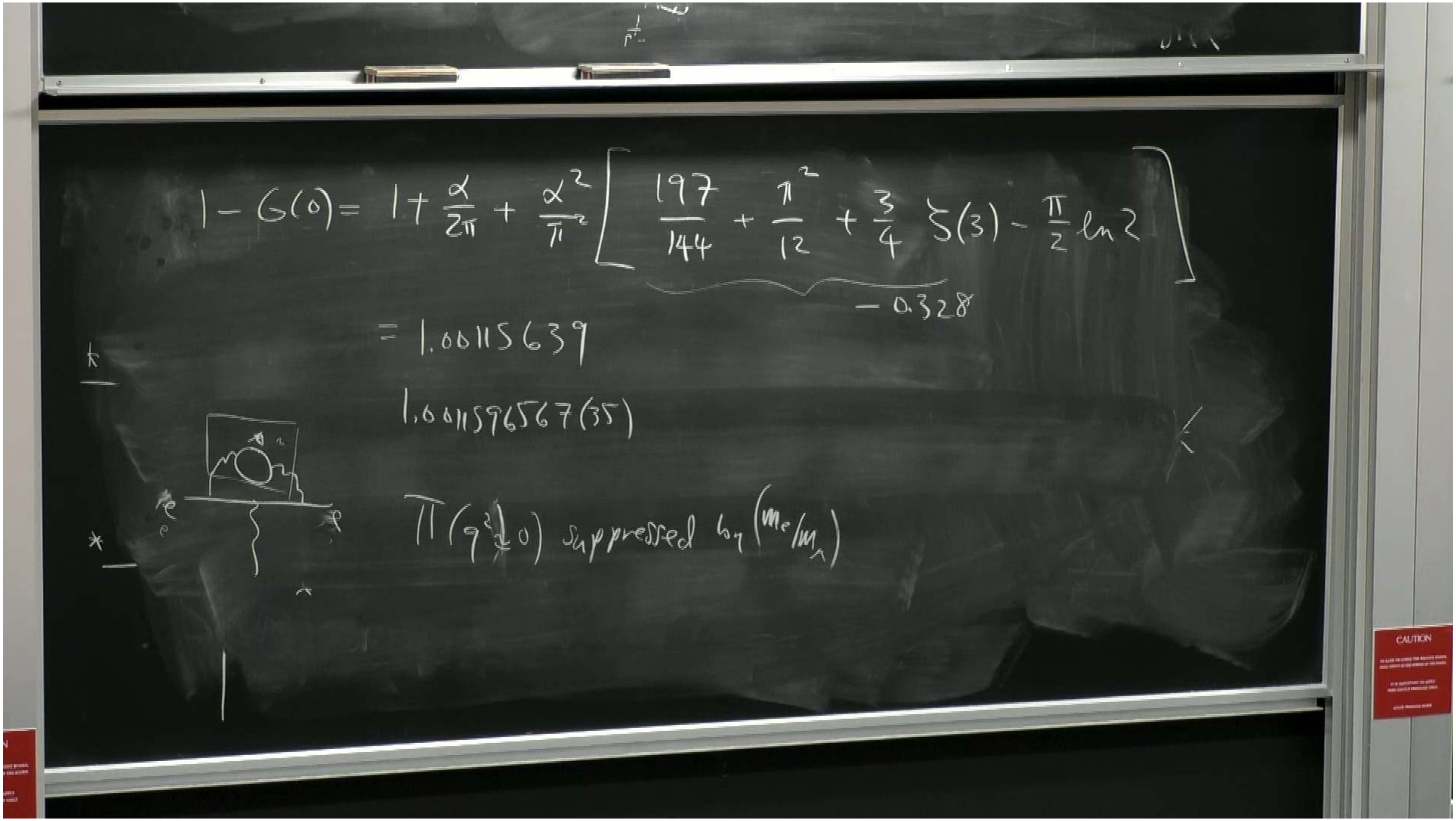


$$k^0 = i k^4 \qquad d^4k = i\,d^4k_E$$



$\frac{1}{D} = 2 \int^1 dx \int^x dy \{ [(p-k)^2 + m^2] y + [(p'-k)^2 + m^2](x-y)$
$+ k^2(1-x) \}^{-3}$
$\int_0^x dy \{ [k - p'y - p(x-y)]^2 + a^2 - i\epsilon \}^{-3}$
$a^2 = -[p'y + p(x-y)]^2$


$1 - G(0) = 1 + \frac{\alpha}{2\pi} + \frac{\alpha^2}{\pi^2}\left[\underbrace{\frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4}\zeta(3)}_{-0.328} - \frac{\pi}{2}\ln 2\right]$
$= 1.00115639$
$1.0011596567(35)$
CAUTION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-0.328
$= 1.00115639$
$1.0011596567(35)$
$\Pi(q^2 \neq 0)$ suppressed by (m_e/m_Λ)