Title: QFT2 - Quantum Electrodynamics - Morning Lecture

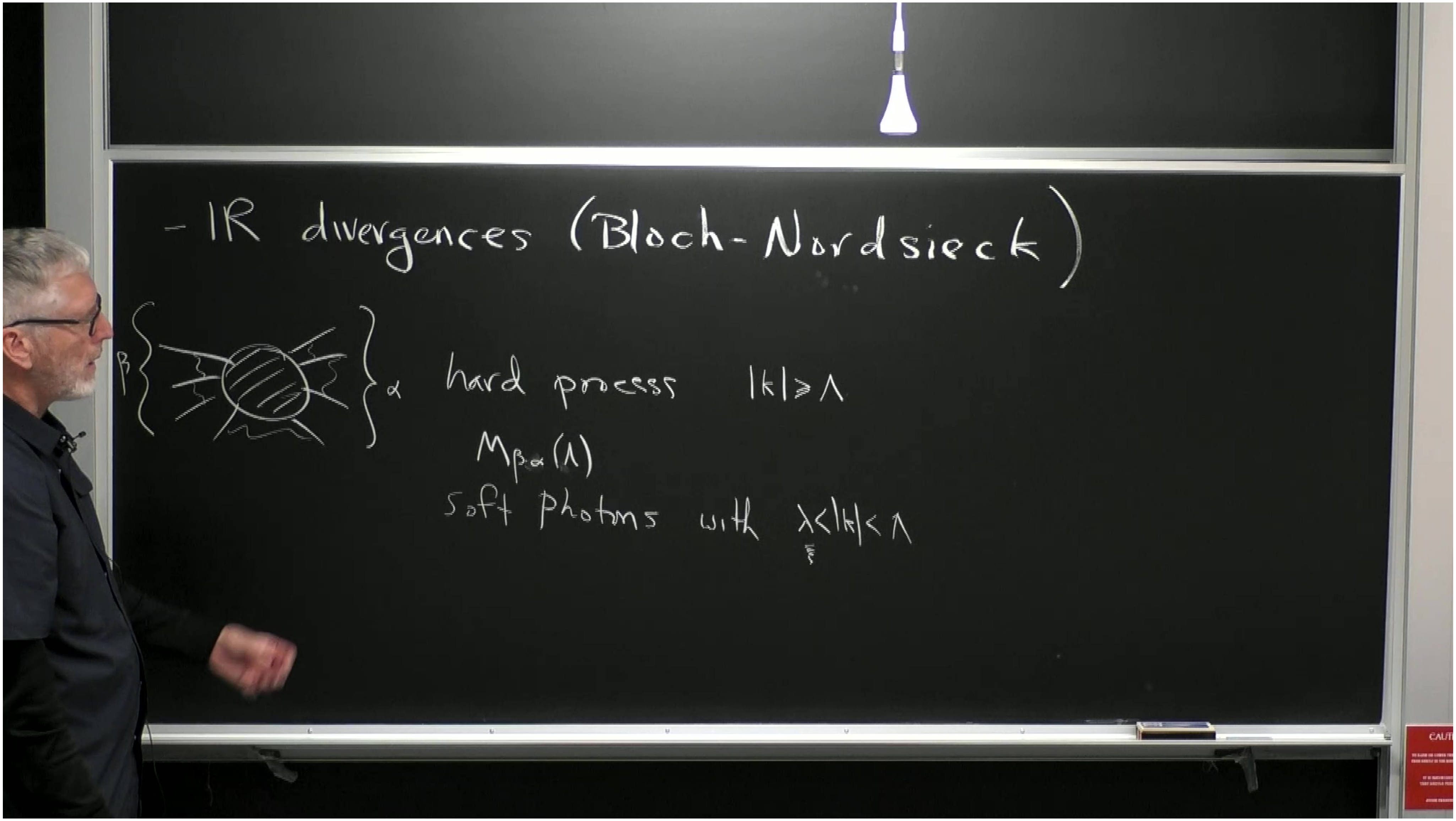
Speakers:

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

Date: December 06, 2022 - 10:00 AM

URL: https://pirsa.org/22120014

Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.

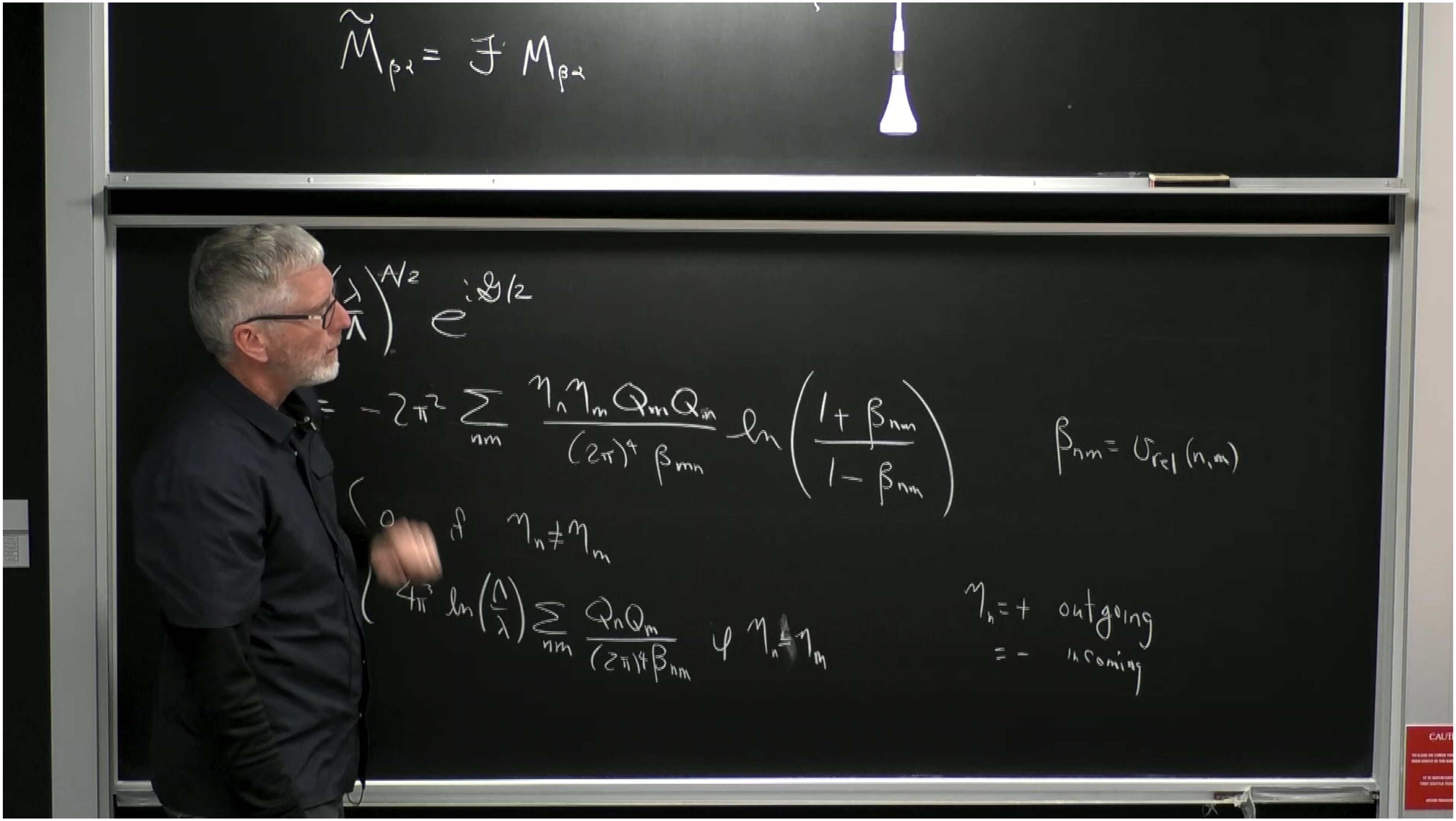

- IR divergences (Bloch-Nordsieck)
β
α
hard process |k| ≥ Λ
M_{βα}(Λ)
soft photons with λ < |k| < Λ

$$\tilde{M}_{\beta\alpha} = \mathcal{F}\, M_{\beta\alpha}$$

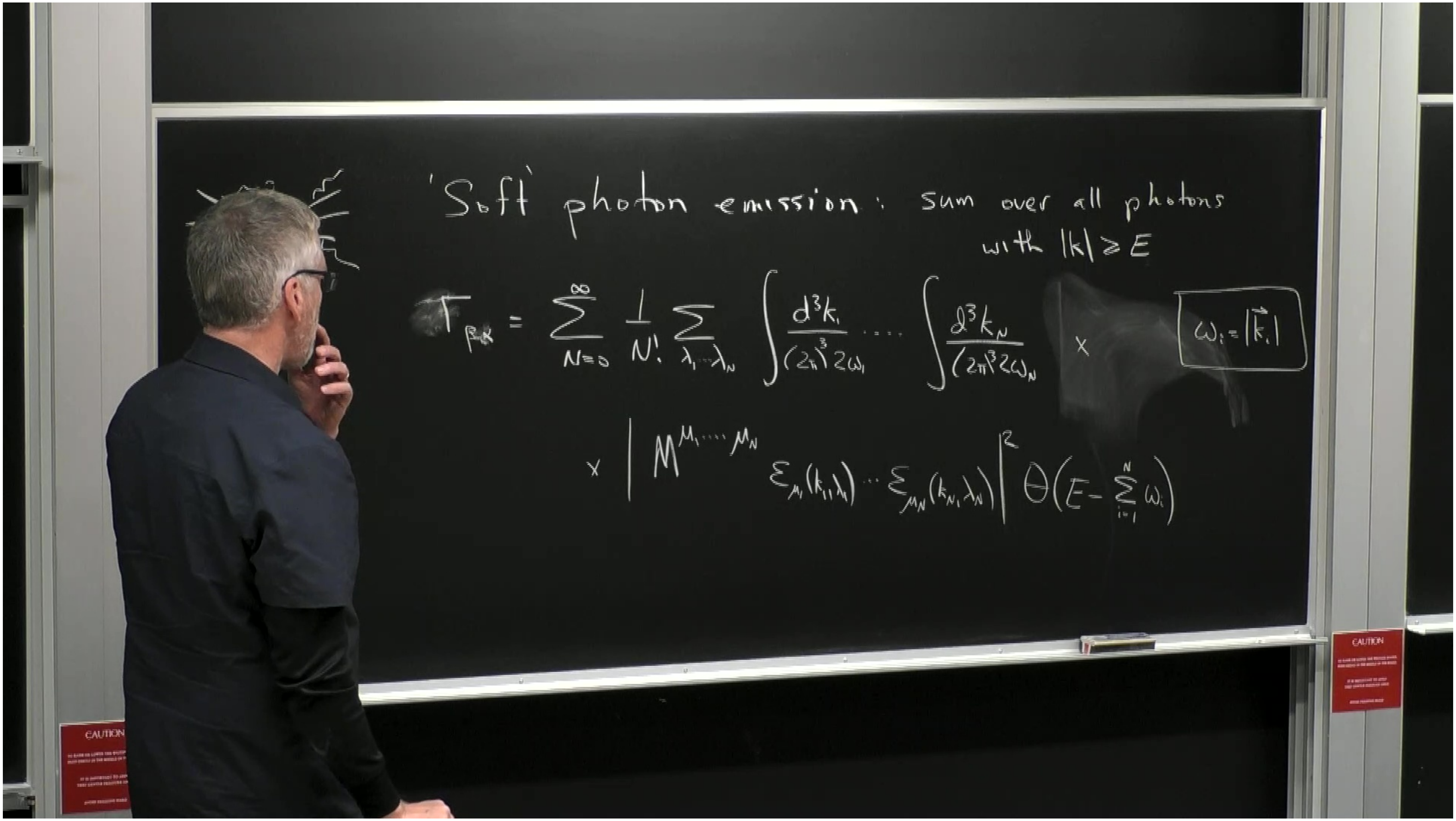
$$\mathcal{F} = \left(\frac{\lambda}{\Lambda}\right)^{A/2} e^{i\mathcal{B}/2}$$

$$A = -2\pi^2 \sum_{nm} \frac{\eta_n \eta_m Q_m Q_m}{(2\pi)^4 \beta_{mn}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right) \qquad \beta_{nm} = v_{rel}(n,m)$$

$$\mathcal{B} = \begin{cases} 0 & \text{if } \eta_n \neq \eta_m \\ 4\pi^3 \ln\left(\frac{\Lambda}{\lambda}\right) \sum_{nm} \frac{Q_n Q_m}{(2\pi)^4 \beta_{nm}} & \text{if } \eta_n \neq \eta_m \end{cases}$$



$\tilde{M}_{\beta\alpha} = \mathcal{F}\, M_{\beta\alpha}$
$\left(\frac{\lambda}{\Lambda}\right)^{A/2} e^{i\mathcal{B}/2}$
$= -2\pi^2 \sum_{nm} \frac{\eta_n \eta_m Q_n Q_m}{(2\pi)^4 \beta_{mn}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right)$
$\beta_{nm} = v_{rel}(n,m)$
0 if $\eta_n \neq \eta_m$
$4\pi^3 \ln\left(\frac{\Lambda}{\lambda}\right) \sum_{nm} \frac{Q_n Q_m}{(2\pi)^4 \beta_{nm}}$ if $\eta_n = \eta_m$
$\eta_n = +$ outgoing
$= -$ incoming



'Soft' photon emission: sum over all photons
with $|k| \geq E$
$\Gamma_{p,k} = \sum_{N=0}^{\infty} \frac{1}{N!} \sum_{\lambda_1 \cdots \lambda_N} \int \frac{d^3k_1}{(2\pi)^3 2\omega_1} \cdots \int \frac{d^3k_N}{(2\pi)^3 2\omega_N} \times$
$\boxed{\omega_i = |\vec{k}_i|}$
$\times \left| M^{\mu_1 \cdots \mu_N} \varepsilon_{\mu_1}(k_1,\lambda_1) \cdots \varepsilon_{\mu_N}(k_N,\lambda_N) \right|^2 \Theta\left(E - \sum_{i=1}^{N} \omega_i\right)$
CAUTION
CAUTION

$$\left|\tilde{M}^{\mu_1\cdots\mu_N}\varepsilon_{\mu_1}\cdots\varepsilon_{\mu_N}\right|^2 = \left|\tilde{M}_{\beta\alpha}\right|^2 \sum_{\substack{m_1\cdots m_N\\ n_1\cdots n_N}} \left[\frac{\eta_{n_1}Q_{n_1}p_{n_1}^{\mu_1}}{p_{n_1}\cdot k_1 - i\eta_{n_1}\varepsilon}\right]\varepsilon_{\mu_1}(k_1,\lambda_1)\cdots$$

$$\cdots\left[\frac{\eta_{m_1}Q_{m_1}p_{m_1}^{\mu_1}}{p_{m_1}\cdot k_1 - i\eta_{m_1}\varepsilon}\right]\varepsilon^*_{\mu_1}(k_1,\lambda_1)\cdots\Big]$$

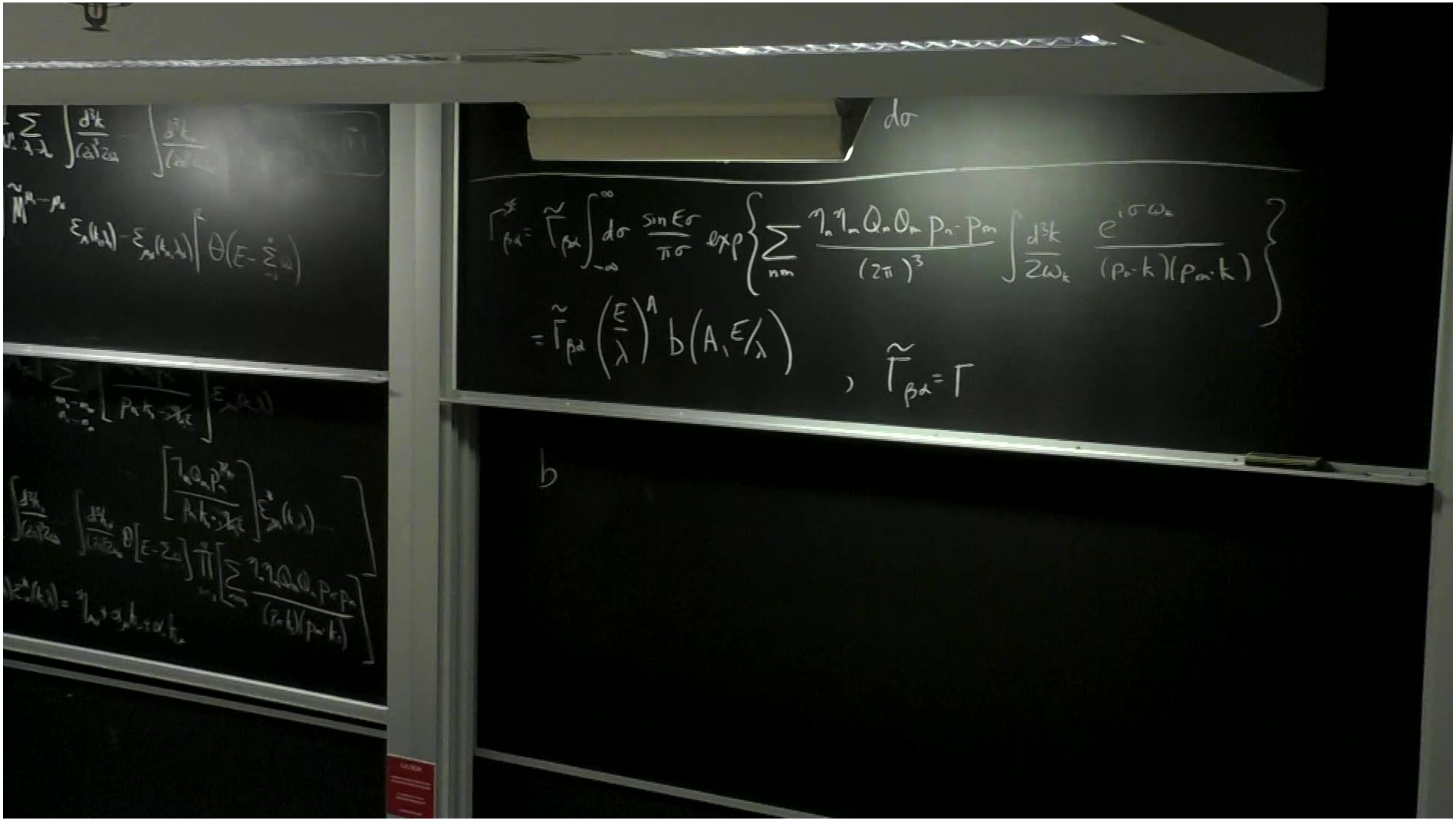
$$\times \left| \tilde{M}^{\mu_1 \cdots \mu_N} \varepsilon_{\mu_1}(k_1,\lambda_1)\cdots\varepsilon_{\mu_N}(k_N,\lambda_N)\right|^2 \Theta\left(E-\sum_{i=1}^{N} \omega_i\right)$$

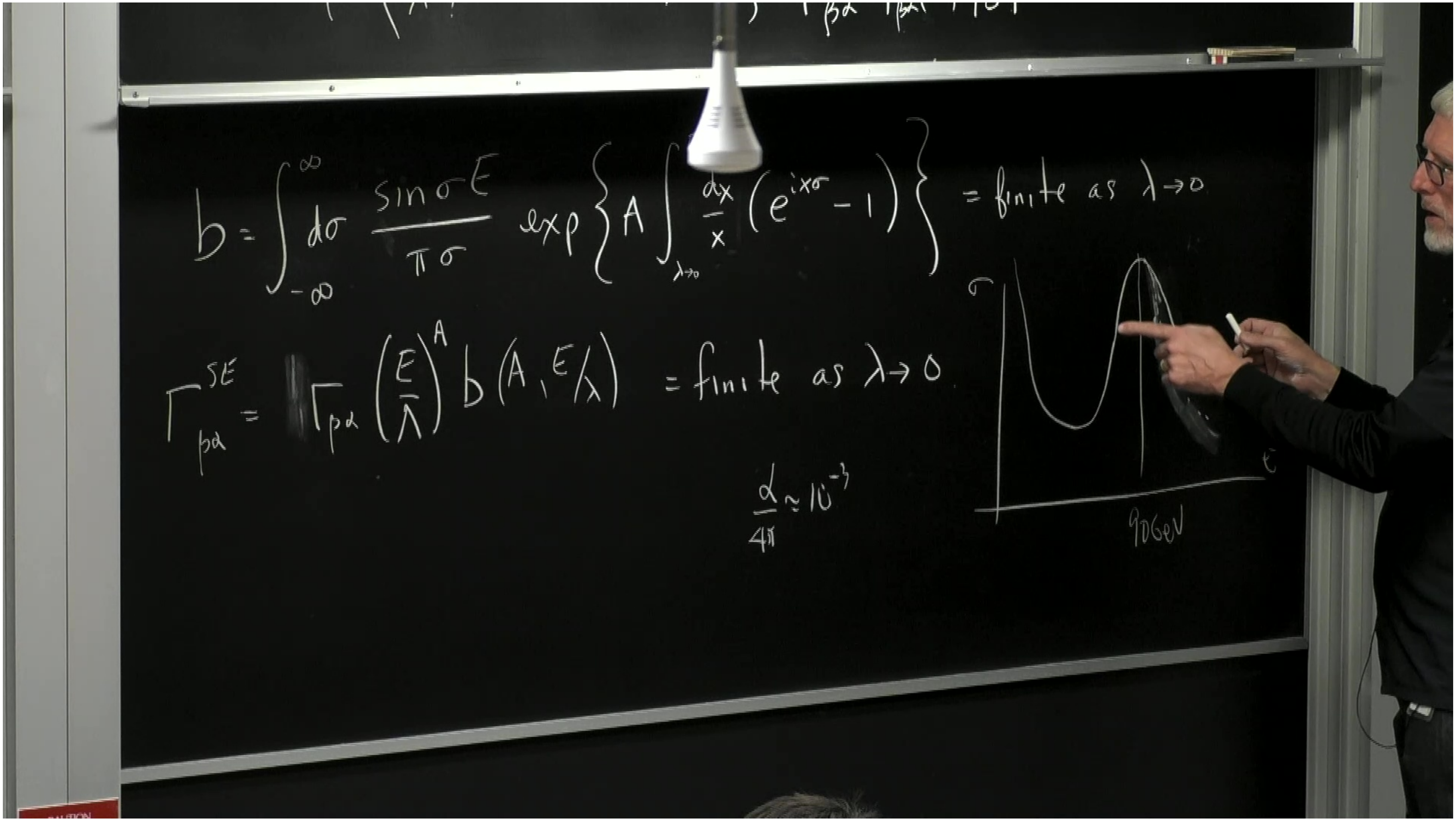
$$\left| M \quad \varepsilon_{\mu_1}\cdots\varepsilon_{\mu_N}\right|^2 = \left|M_{\beta\alpha}\right|^2 \sum_{\substack{m_1\cdots m_N\\ n_1\cdots n_N}} \left[\frac{\eta_{n_1} Q_{n_1} p_{n_1}}{p_{n_1}\cdot k_1 - \cancel{i\eta_{n_1}\epsilon}}\right]\varepsilon_{\mu_1}(k_1,\lambda_1)\cdots$$

$$\cdots\left[\frac{\eta_{m_1}Q_{m_1}p_{m_1}^{\nu_1}}{p_{m_1}\cdot k_1 + \cancel{i\eta_{m_1}\epsilon}}\right]\varepsilon^*_{\nu_1}(k_1,\lambda_1)\cdots\Bigg]$$

$$\Gamma_{\beta\alpha}(E) = \tilde{\Gamma}_{\beta\alpha}\sum_{N=0}^{\infty} \frac{1}{N!}\int\frac{d^3k_1}{(2\pi)^3 2\omega_1}\cdots\int\frac{d^3k_N}{(2\pi)^3 2\omega_N}\,\Theta\left[E-\sum_i\omega_i\right]\prod_{s=1}^{N}\left[\sum_{nm}\frac{\eta_n\eta_m Q_m Q_n\, p_n\cdot p_m}{(p_n\cdot k_s)(p_m\cdot k_s)}\right]$$

uses $\sum_\lambda \varepsilon_\mu(k,\lambda)\varepsilon^*_\nu(k,\lambda) = \eta_{\mu\nu} + \alpha_\mu k_\nu + \alpha_\nu k_\mu$


$d\sigma$
$\Gamma^{\epsilon}_{\beta\alpha} = \tilde{\Gamma}_{\beta\alpha} \int_{-\infty}^{\infty} d\sigma \, \frac{\sin E\sigma}{\pi\sigma} \exp\left\{ \sum_{nm} \frac{\eta_n \eta_m Q_n Q_m \, p_n \cdot p_m}{(2\pi)^3} \int \frac{d^3k}{2\omega_k} \frac{e^{i\sigma\omega_k}}{(p_n \cdot k)(p_m \cdot k)} \right\}$
$= \tilde{\Gamma}_{\beta\alpha} \left(\frac{E}{\lambda}\right)^A b(A, E/\lambda) \quad , \quad \tilde{\Gamma}_{\beta\alpha} = \Gamma$
b


$b = \int_{-\infty}^{\infty} d\sigma \, \frac{\sin \sigma E}{\pi \sigma} \exp\left\{ A \int_{\lambda \to 0} \frac{dx}{x} \left(e^{ix\sigma} - 1 \right) \right\}$ = finite as $\lambda \to 0$
$\Gamma^{SE}_{\beta\alpha} = \Gamma_{\beta\alpha} \left(\frac{E}{\Lambda} \right)^{A} b\left(A, E/\lambda\right)$ = finite as $\lambda \to 0$
$\frac{\alpha}{4\pi} \simeq 10^{-3}$
σ
E
90GeV

$$b = \int_{-\infty}^{\infty} d\sigma \, \frac{\sin \sigma E}{\pi \sigma} \exp\left\{ A \int_{\lambda \to 0} \frac{dx}{x} \left(e^{ix\sigma} - 1 \right) \right\} = \text{finite as } \lambda \to 0$$

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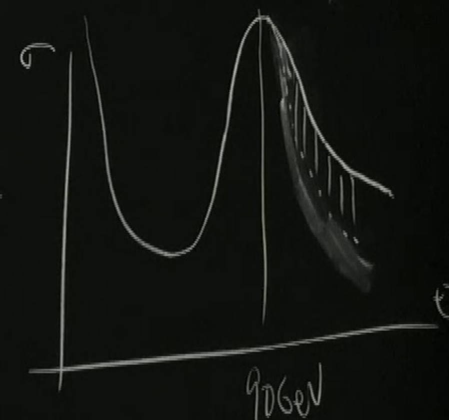

$$\frac{\alpha}{4\pi} \ln\left(\frac{E_1}{E_2}\right) \qquad \frac{\alpha}{4\pi} \simeq 10^{-3}$$

$$= \tilde{\Gamma}_{\beta\alpha} \left(\frac{E}{\lambda}\right)^{A} b\left(A, E/\lambda\right) \quad , \quad \tilde{\Gamma}_{\beta\alpha} = \Gamma_{\beta\alpha}(\lambda)\, |\mathcal{F}|^2 \qquad d\Gamma \propto$$

$$b = \int_{-\infty}^{\infty} d\sigma \, \frac{\sin \sigma E}{\pi \sigma} \exp\left\{ A \int_{\lambda \to 0} \frac{dx}{x} \left(e^{ix\sigma} - 1\right) \right\} = \text{finite as } \lambda \to 0$$

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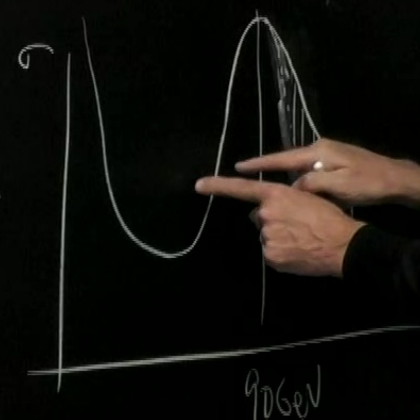
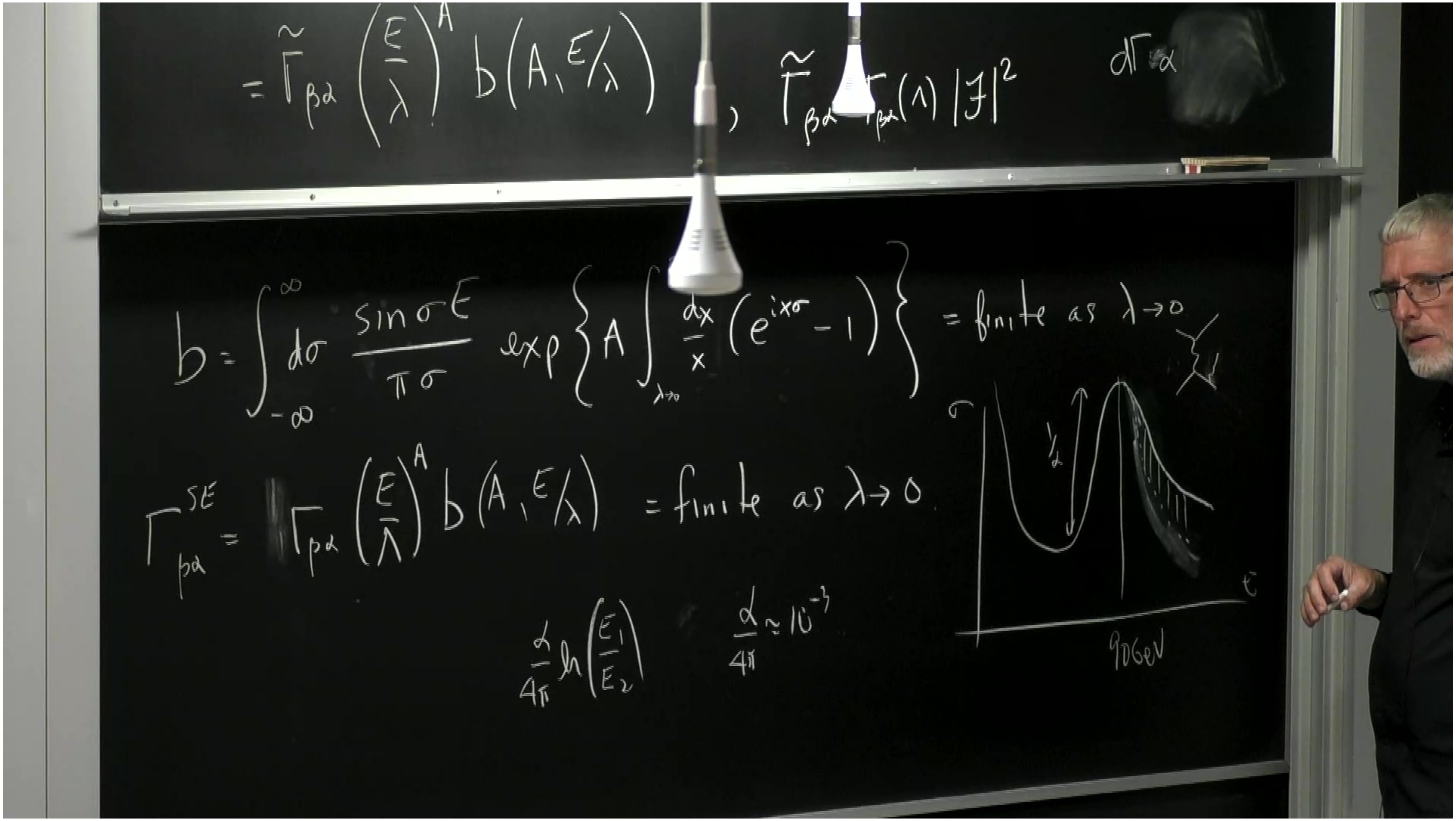
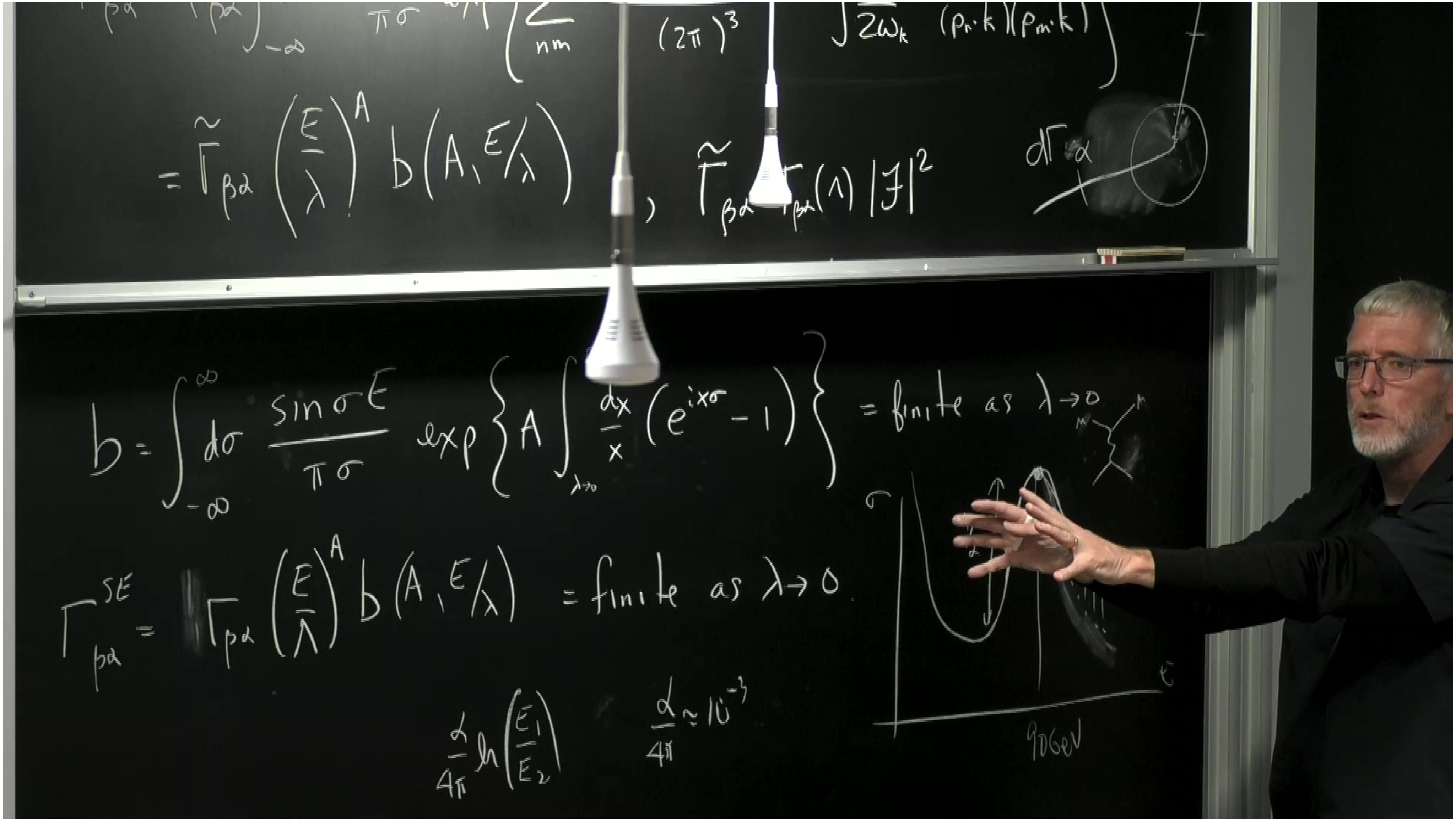



$= \tilde{\Gamma}_{\beta\alpha} \left(\frac{E}{\lambda}\right)^A b(A, E/\lambda)$, $\tilde{\Gamma}_{\beta\alpha} = \Gamma_{\beta\alpha}(\lambda)\, |\mathcal{F}|^2$
$d\Gamma \propto$
$b = \int_{-\infty}^{\infty} d\sigma \, \frac{\sin \sigma E}{\pi \sigma} \exp\left\{ A \int_{\lambda \to 0} \frac{dx}{x} \left(e^{ix\sigma} - 1\right) \right\}$ = finite as $\lambda \to 0$
$\Gamma^{SE}_{\beta\alpha} = \Gamma_{\beta\alpha} \left(\frac{E}{\Lambda}\right)^A b(A, E/\lambda)$ = finite as $\lambda \to 0$
$\frac{\alpha}{4\pi} \ln\left(\frac{E_1}{E_2}\right)$
$\frac{\alpha}{4\pi} \simeq 10^{-3}$
σ
$1/\alpha$
E
90GeV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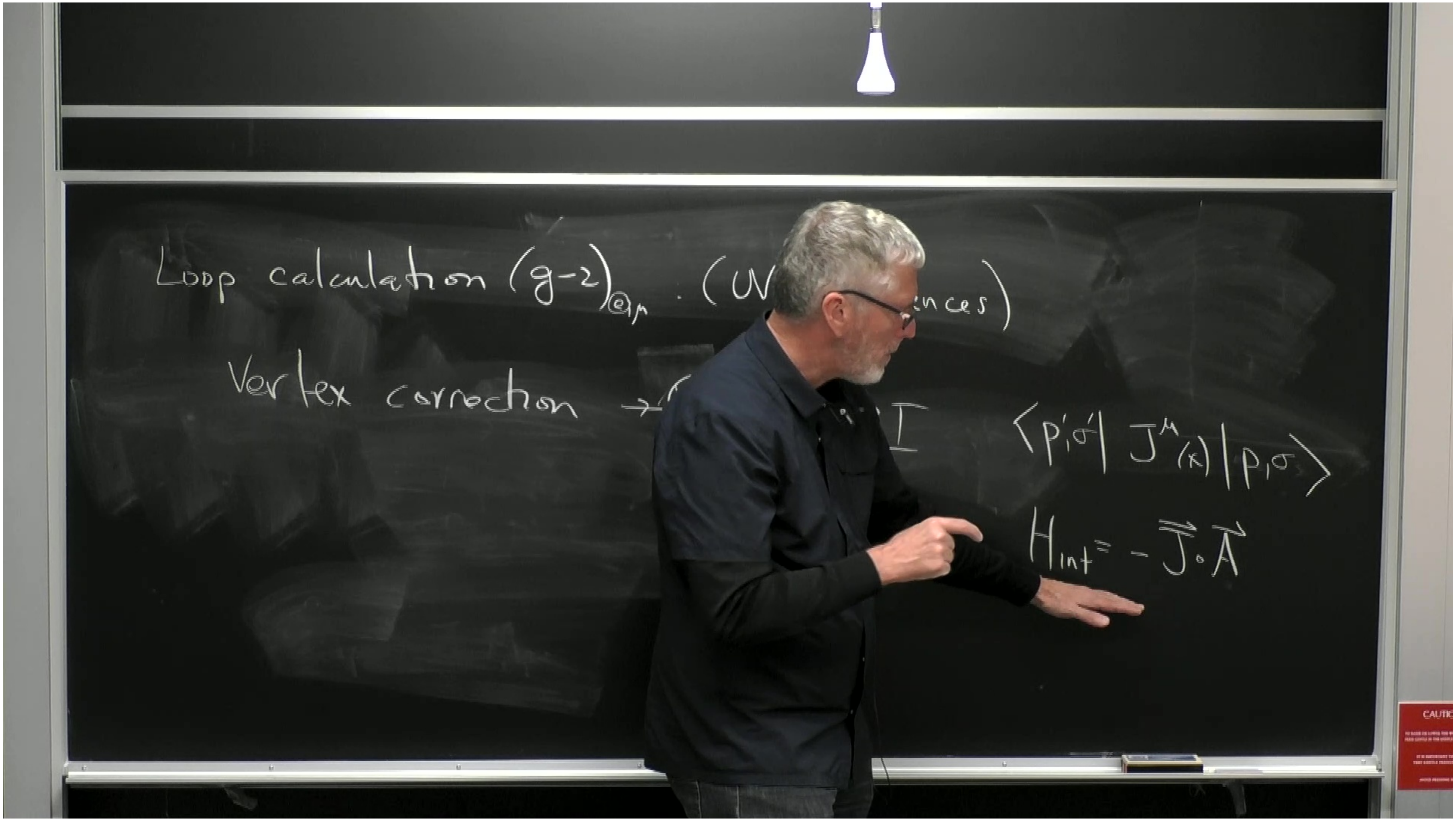
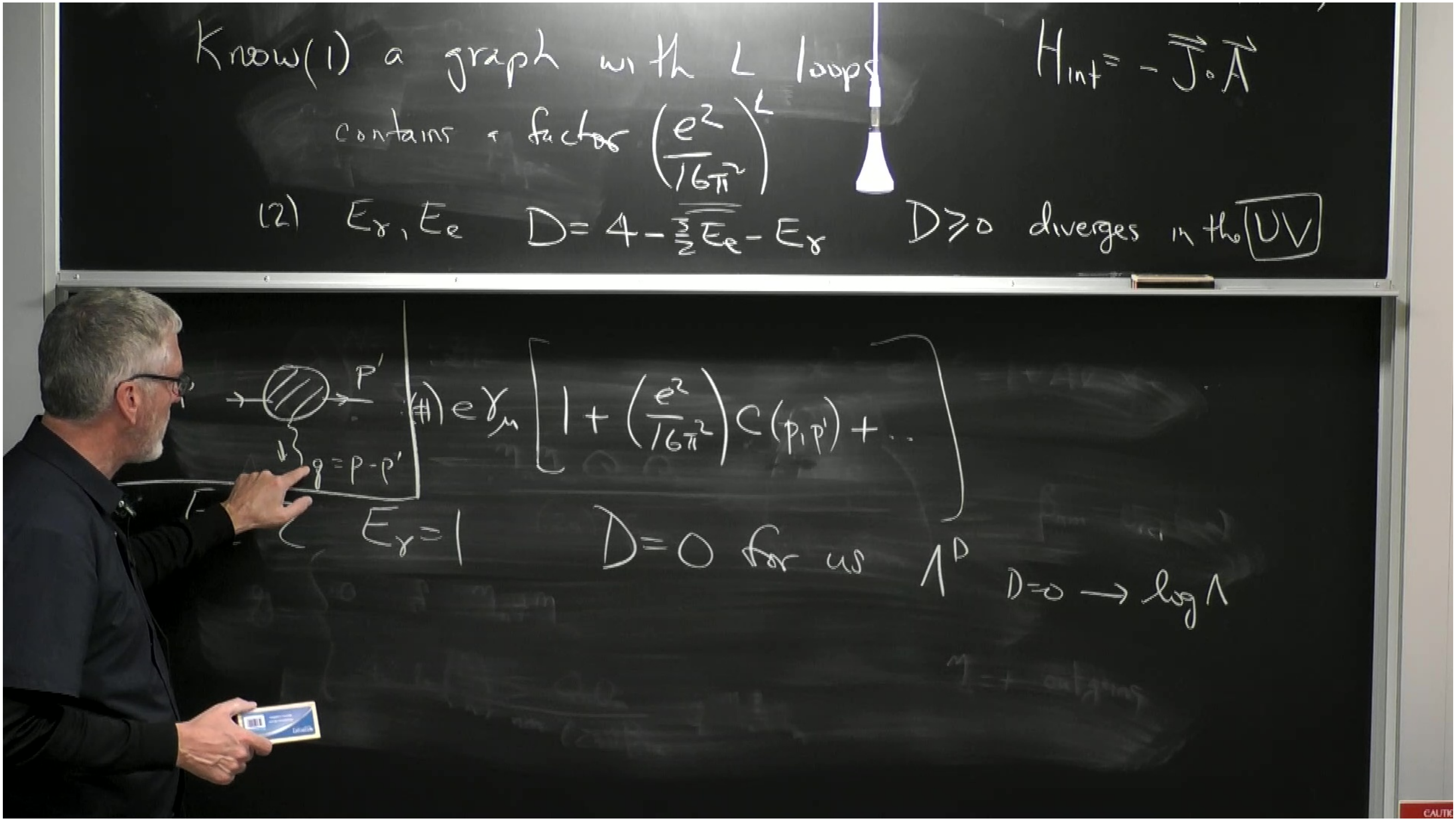
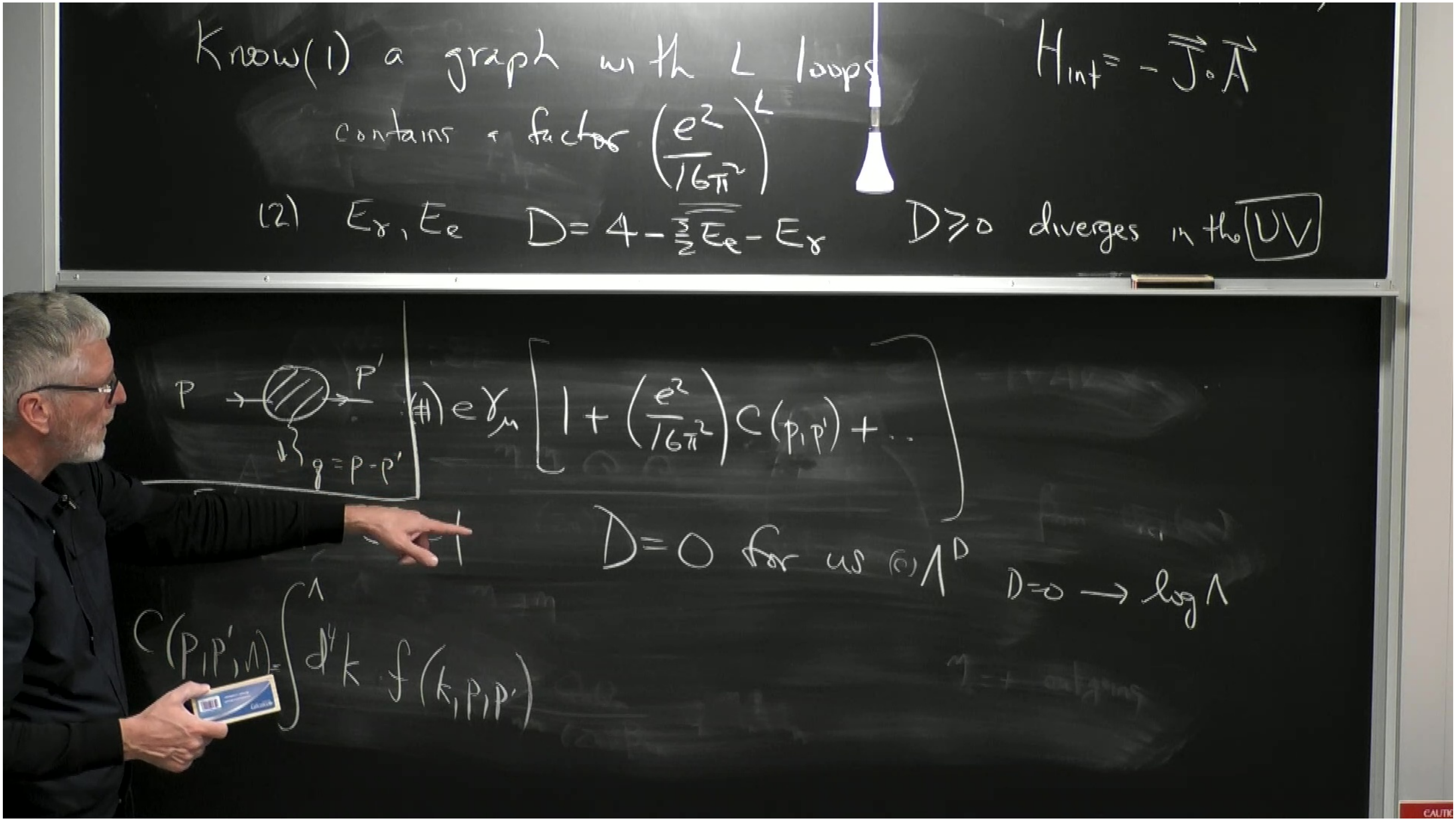
$\Gamma^{SE}_{\beta\alpha} = \Gamma_{\beta\alpha} \left(\frac{E}{\Lambda}\right)^A b(A, E/\lambda)$ = finite as $\lambda \to 0$
$\frac{\alpha}{4\pi} \ln\left(\frac{E_1}{E_2}\right)$
$\frac{\alpha}{4\pi} \approx 10^{-3}$
90GeV


Loop calculation $(g-2)_{e,\mu}$. (UV
Vertex correction →
$\langle p',\sigma'|\, J^{\mu}(x)\, |p,\sigma\rangle$
$H_{int} = -\vec{J}\cdot\vec{A}$



Know (1) a graph with L loops
contains a factor $\left(\frac{e^2}{16\pi^2}\right)^L$
(2) E_γ, E_e $D = 4 - \frac{3}{2}E_e - E_\gamma$
$D \geq 0$ diverges in the $\boxed{UV}$
$H_{int} = -\vec{J}\cdot\vec{A}$
p'
$q = p - p'$
$(\pm)\, e\gamma_\mu \left[1 + \left(\frac{e^2}{16\pi^2}\right) C(p,p') + \dots\right]$
$E_\gamma = 1$
$D = 0$ for us
Λ^D
$D = 0 \to \log\Lambda$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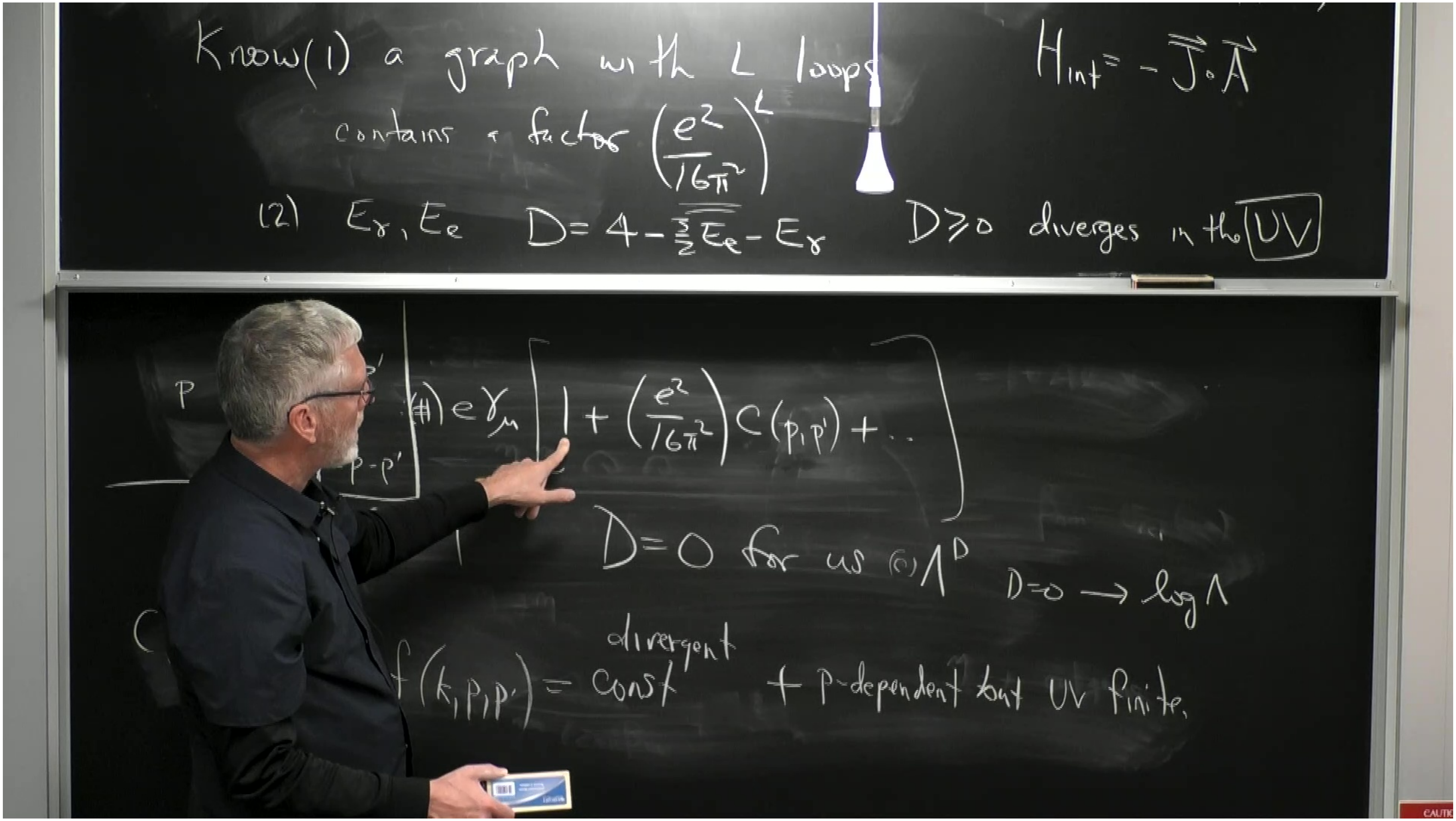
p p'
$q = p - p'$
$(-i)e\gamma_\mu\left[1 + \left(\frac{e^2}{16\pi^2}\right)C(p,p') + \ldots\right]$
$D = 0$ for us Λ^D
$D=0 \longrightarrow \log\Lambda$
$C(p,p',\Lambda) = \int^\Lambda d^4k\, f(k,p,p')$



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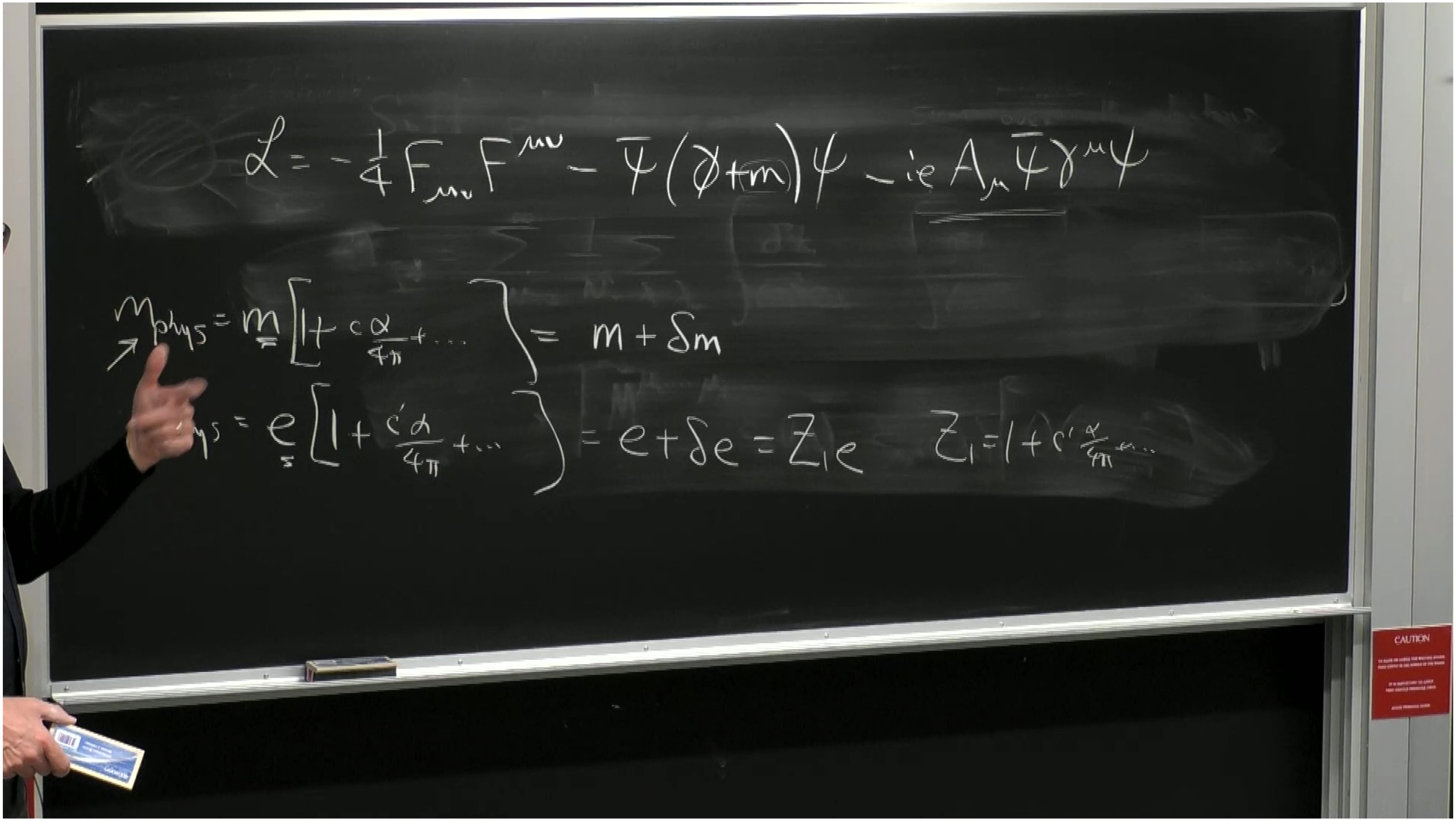
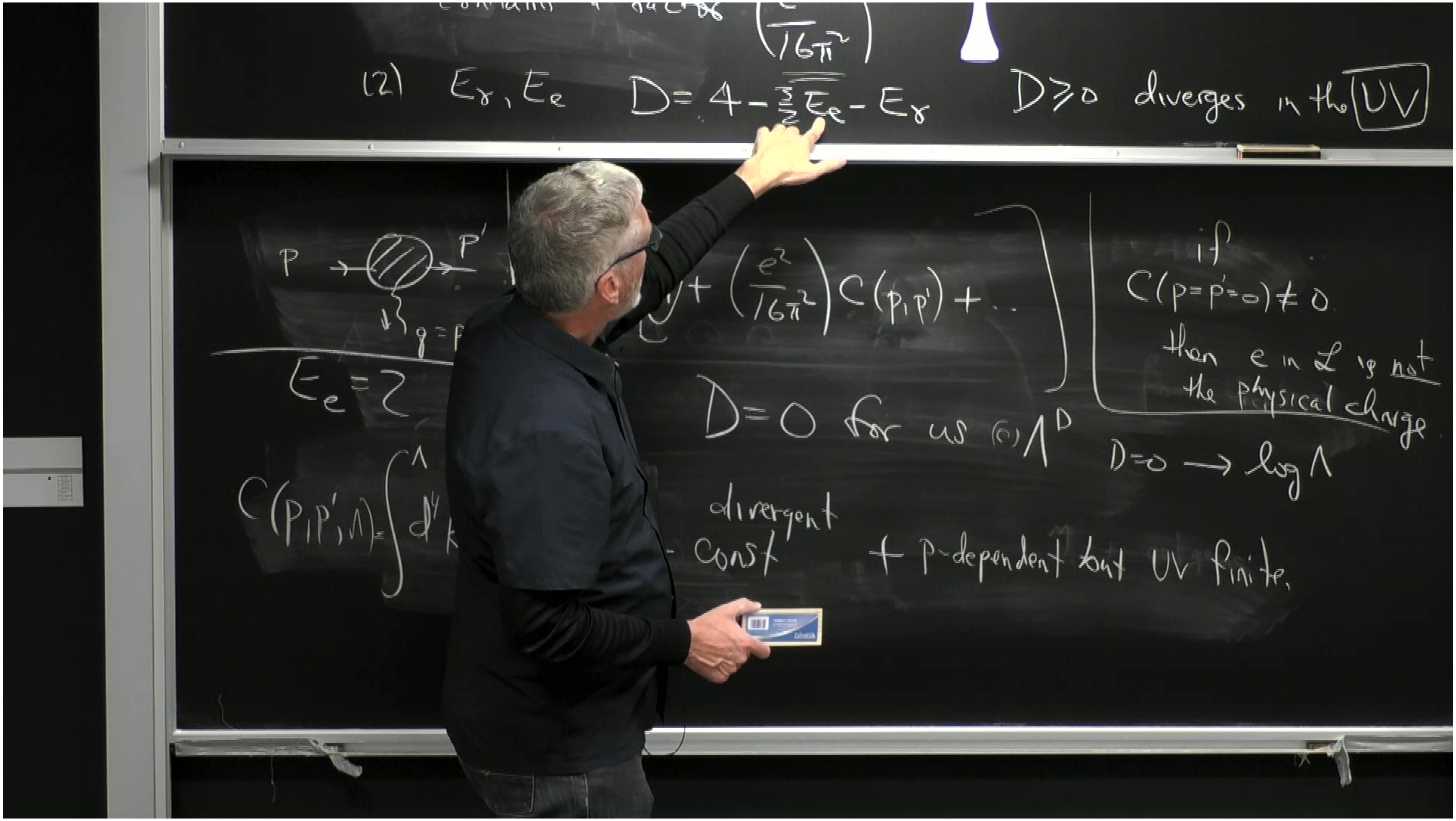
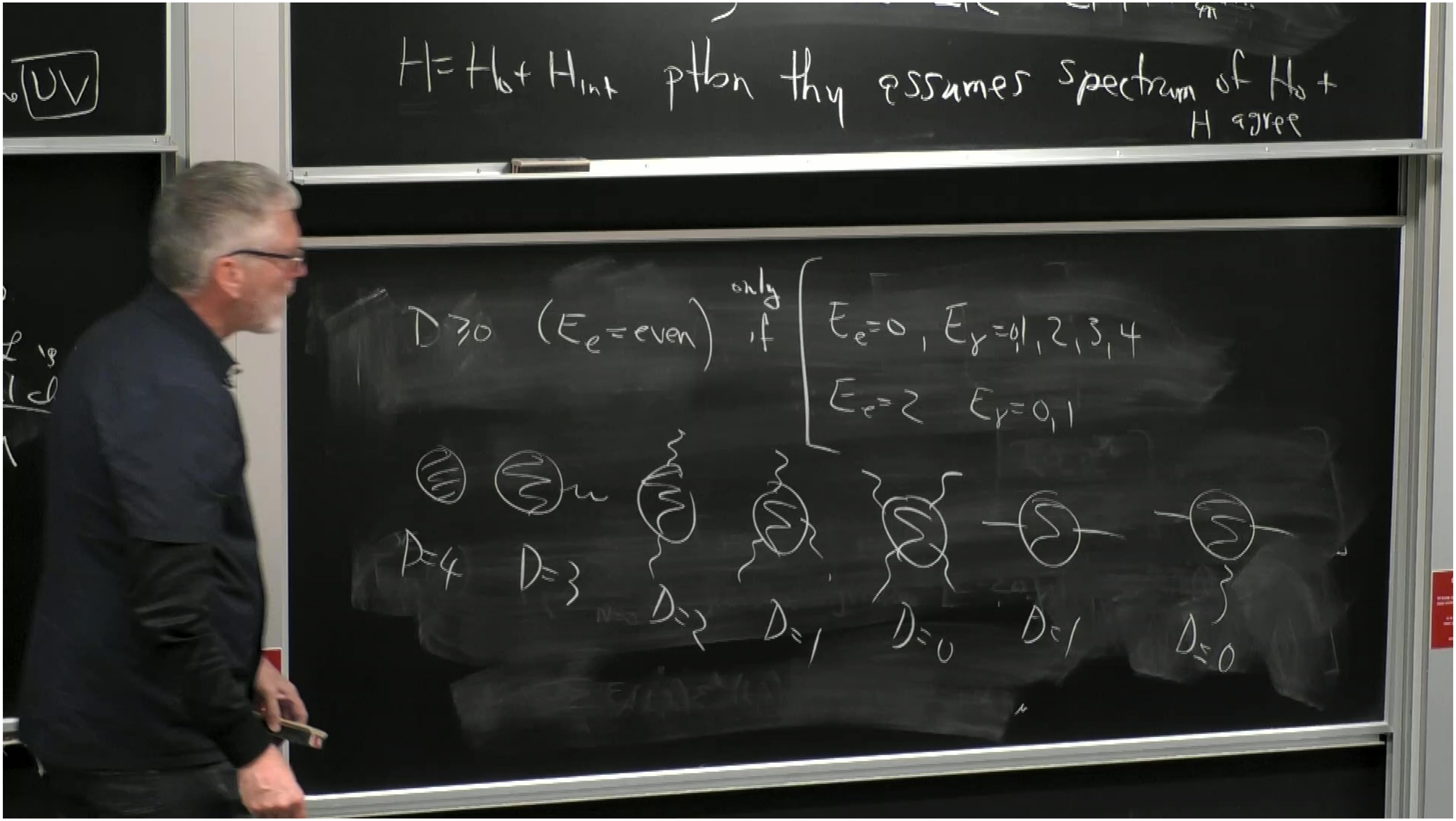
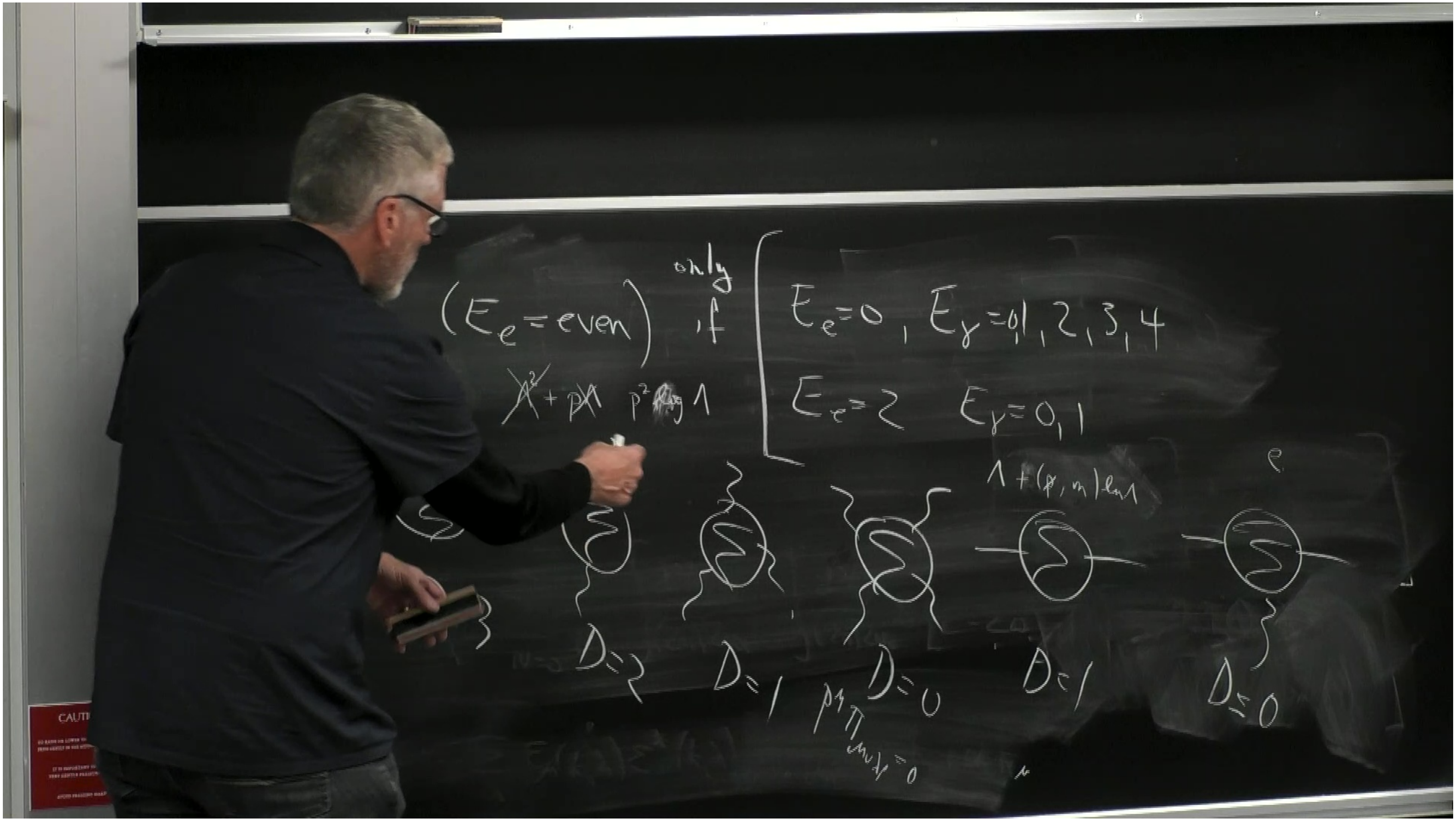
$(-i) e \gamma_\mu \left[1 + \left(\frac{e^2}{16\pi^2}\right) C(p, p') + \dots \right]$
$D = 0$ for us C Λ^D
$D = 0 \longrightarrow \log \Lambda$
$f(k, p, p') = $ Const divergent + p-dependent but UV finite.

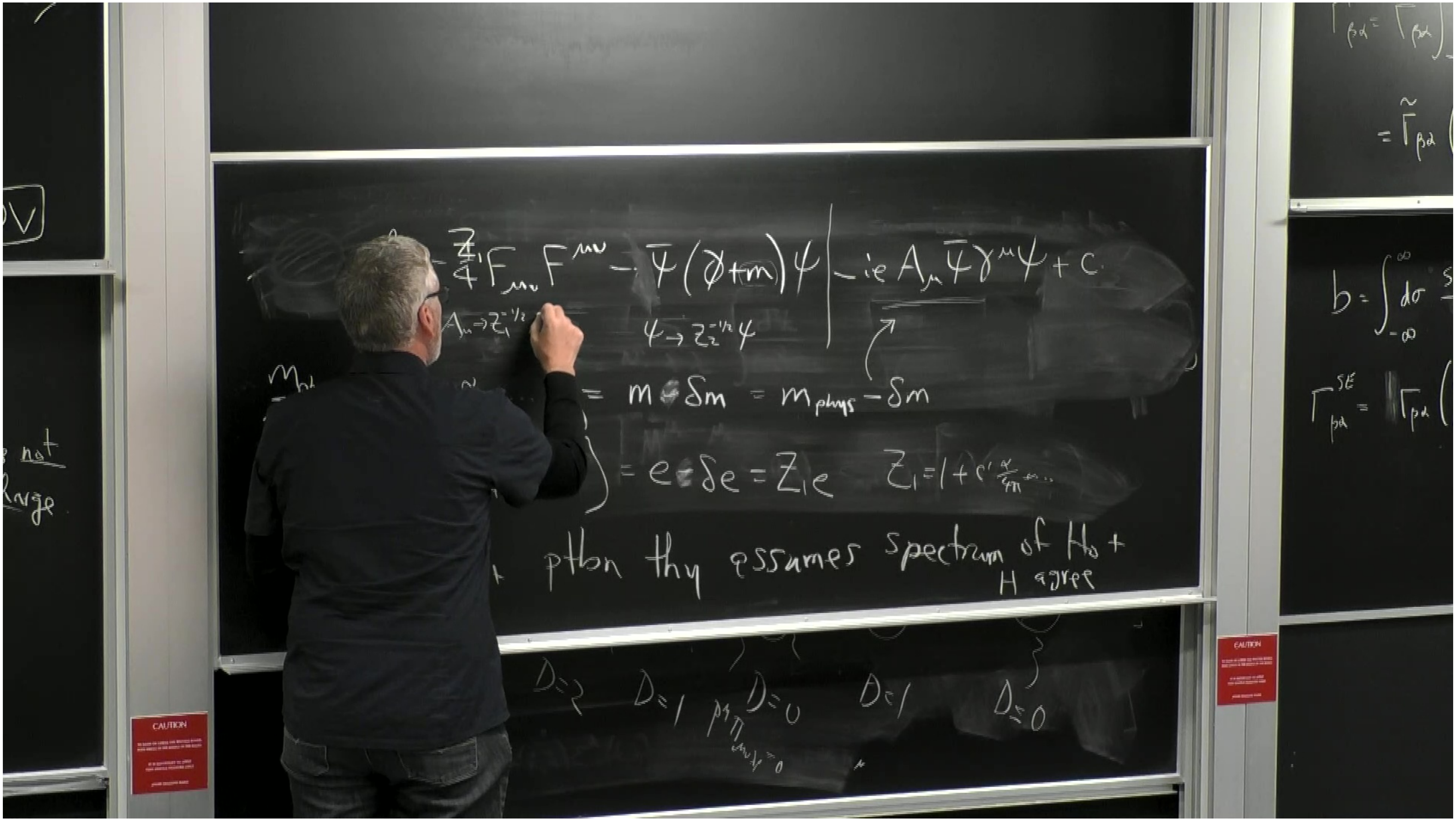

$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{\partial} + m)\psi - ie A_\mu \bar{\psi}\gamma^\mu\psi$
$m_{phys} = m\left[1 + c\frac{\alpha}{4\pi} + \dots\right] = m + \delta m$
$= e\left[1 + c'\frac{\alpha}{4\pi} + \dots\right] = e + \delta e = Z_1 e$
$Z_1 = 1 + c'\frac{\alpha}{4\pi} + \dots$

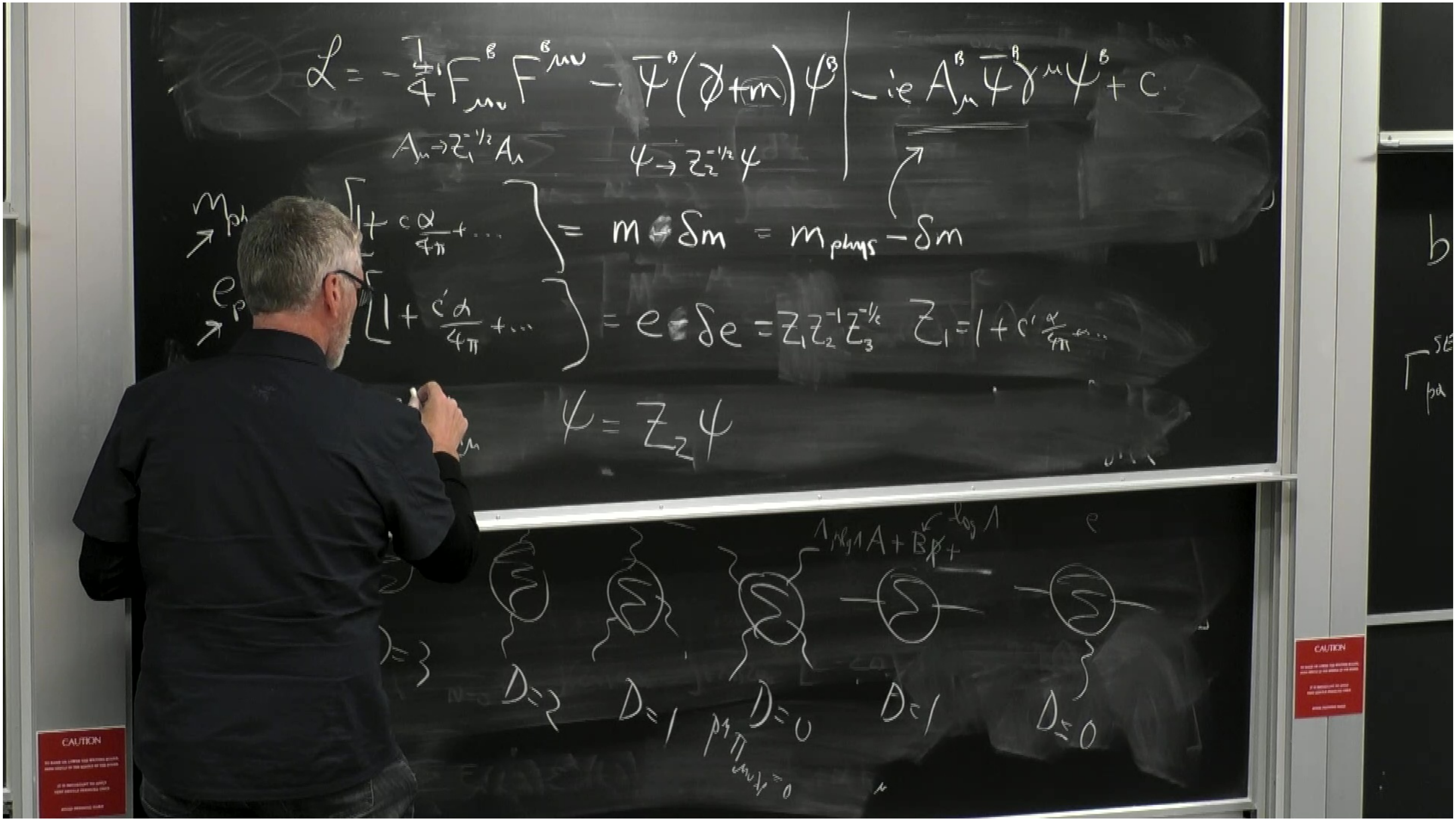

(2) Eγ, Ee D= 4 - 3/2 Ee - Eγ D≥0 diverges in the UV
p p'
+ (e²/16π²) C(p,p') + ...
if
C(p=p'=0) ≠ 0
then e in ℒ is not
the physical charge
Ee=2
D=0 for us Λ^D
D=0 → log Λ
C(p,p',Λ)= ∫^Λ d⁴k
divergent
Const
+ p-dependent but UV finite.

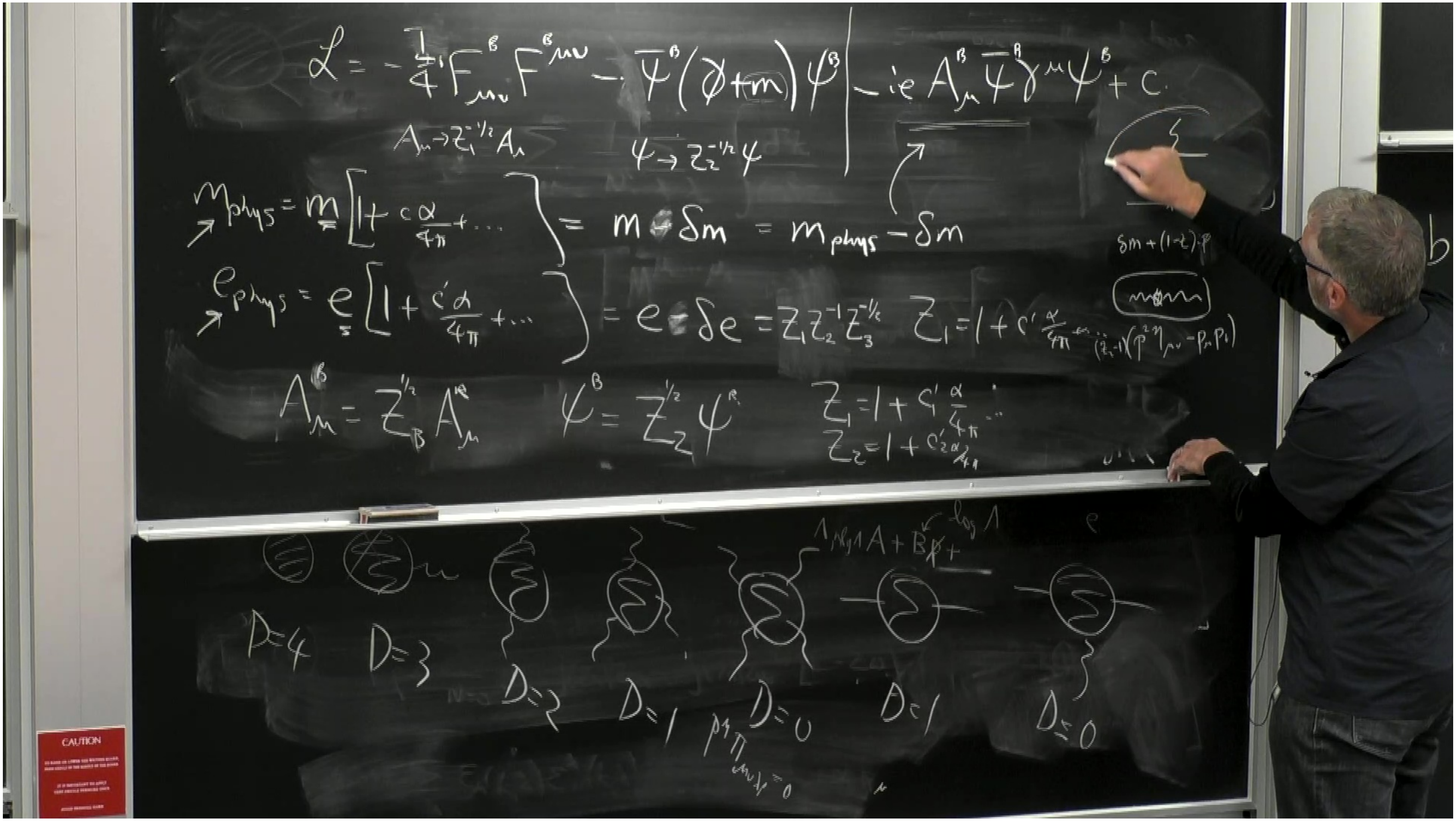

UV
H = H0 + Hint ptbn thy assumes spectrum of H0 + H agree
D ≥ 0 (Ee = even) only if
Ee = 0 , Eγ = 0, 1, 2, 3, 4
Ee = 2 Eγ = 0, 1
D=4
D=3
D=2
D=1
D=0
D=1
D=0



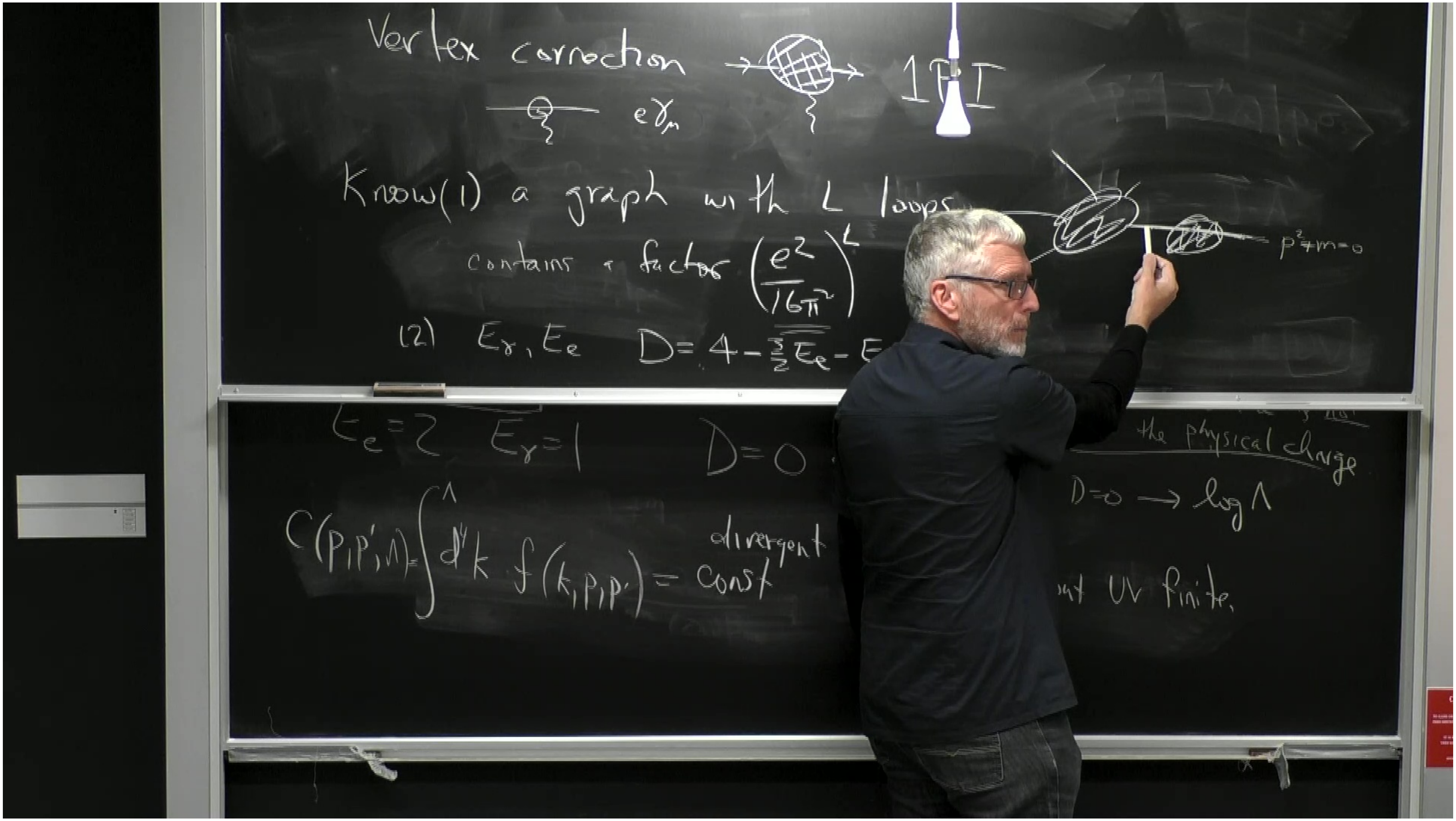
only
(Ee = even) if
Ee = 0 , Eγ = 0, 1, 2, 3, 4
Ee = 2 Eγ = 0, 1
D = 2
D = 1
D = 0
D = 1
D = 0



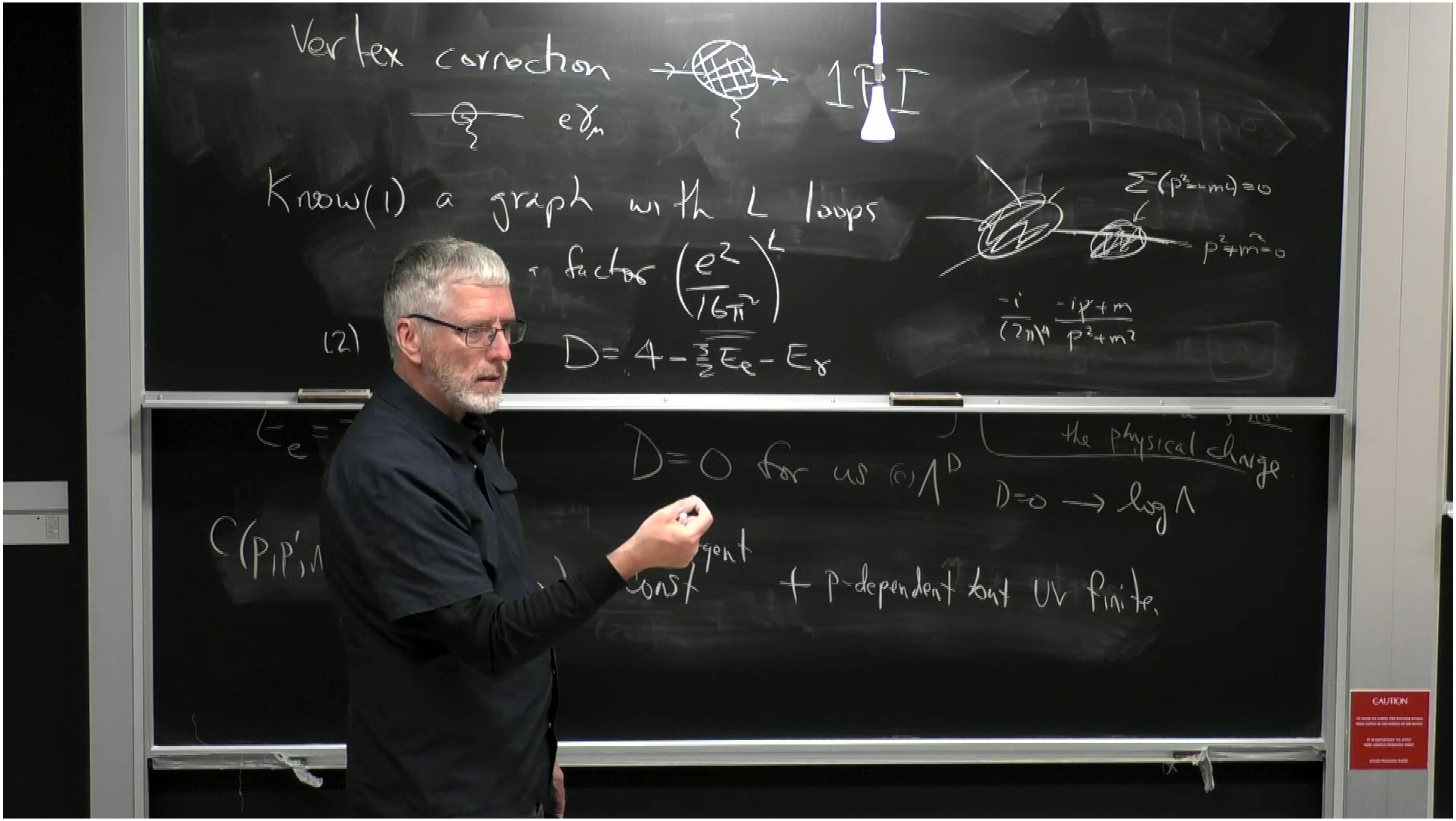

$\mathcal{L} = -\frac{1}{4}F^B_{\mu\nu}F^{B\mu\nu} - \bar\psi^B(\partial\!\!\!/ + m)\psi^B - ieA^B_\mu\bar\psi^B\gamma^\mu\psi^B + C$
$A_\mu \to Z_1^{-1/2} A_\mu$
$\psi \to Z_2^{-1/2}\psi$
$= m + \delta m = m_{phys} - \delta m$
$= e + \delta e = Z_1 Z_2^{-1} Z_3^{-1/2}$
$Z_1 = 1 + c'\frac{\alpha}{4\pi} + \ldots$
$\psi = Z_2 \psi$
$D=2$
$D=1$
$D=0$
$D=1$
$D=0$



ℒ = −¼ F^B_μν F^Bμν − Ψ̄^B(∂̸ + m)ψ^B − ie A^B_μ Ψ̄^B γ^μ ψ^B + C
A_μ → Z_1^{-1/2} A_μ
ψ → Z_2^{-1/2} ψ
m_phys = m[1 + c α/4π + …] = m + δm = m_phys − δm
e_phys = e[1 + c′ α/4π + …] = e + δe = Z_1 Z_2^{-1} Z_3^{-1/2}
Z_1 = 1 + c′ α/4π
A^B_μ = Z_B^{1/2} A^R_μ
ψ^B = Z_2^{1/2} ψ^R
Z_1 = 1 + c_1′ α/4π
Z_2 = 1 + c_2′ α/4π
D=4
D=3
D=2
D=1
D=0
D=1
D=0



Vertex correction
1PI
eγ_μ
Know (1) a graph with L loops
contains a factor (e²/16π²)^L
(2) E_γ, E_e D = 4 − 3/2 E_e − E
p² + m = 0
E_e = 2 E_γ = 1 D = 0
the physical charge
D = 0 → log Λ
C(p, p', Λ) = ∫^Λ d⁴k f(k, p, p') = const
divergent
UV finite.


Vertex correction
1PI
eγμ
Know (1) a graph with L loops
factor (e²/16π²)^L
(2)
D = 4 − 3/2 Ee − Eγ
Σ(p²=−m²)=0
p²+m²=0
the physical charge
D=0 for us
D=0 → log Λ
const
+ p-dependent but UV finite.