Title: QFT2 - Quantum Electrodynamics - Morning Lecture

Speakers:

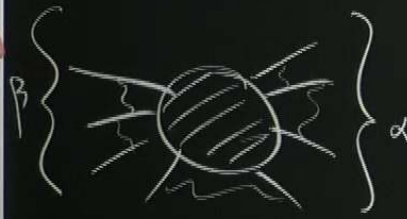
Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

Date: December 06, 2022 - 10:00 AM

URL: https://pirsa.org/22120014

Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.

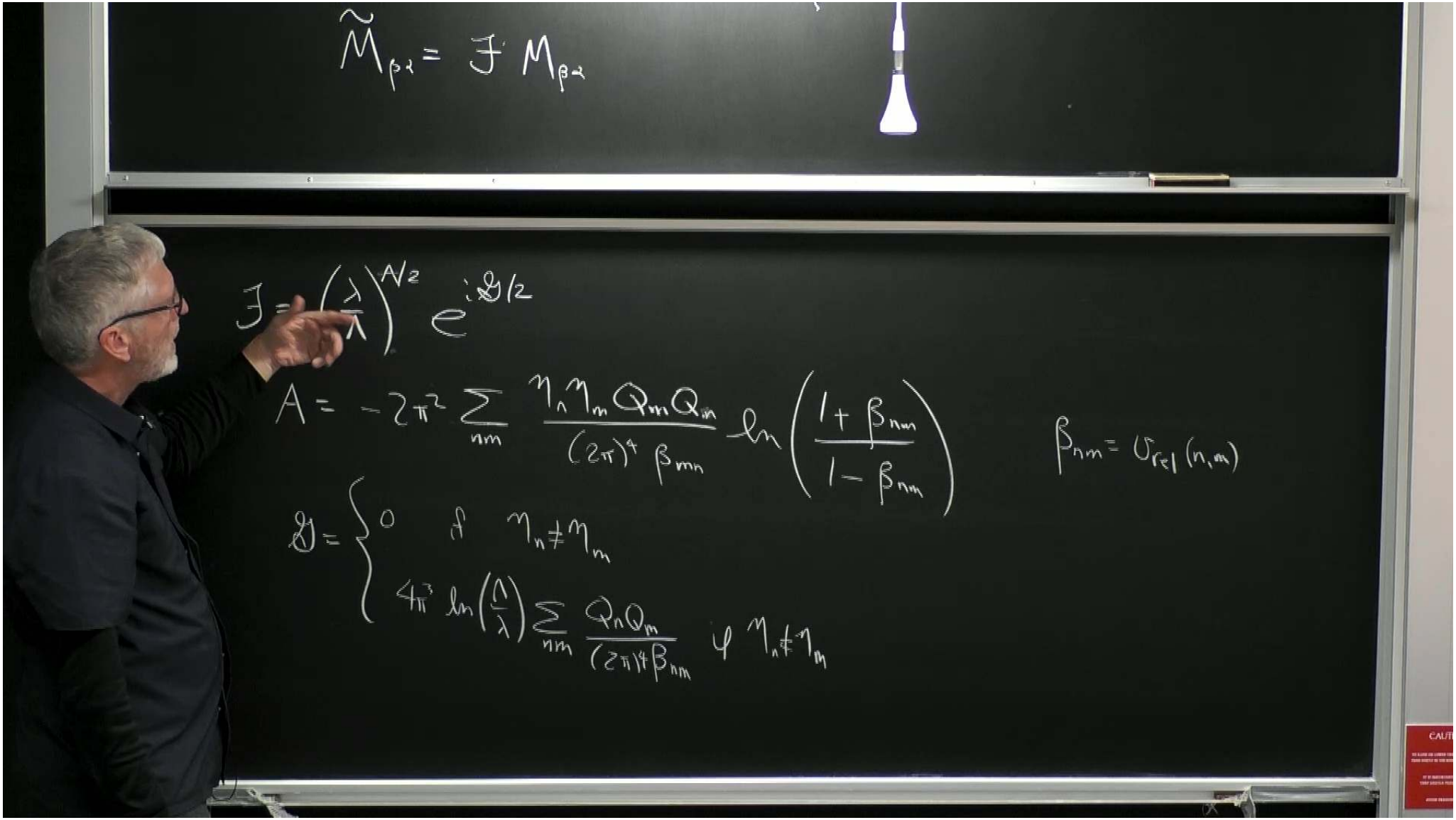
- IR divergences (Bloch-Nordsieck)

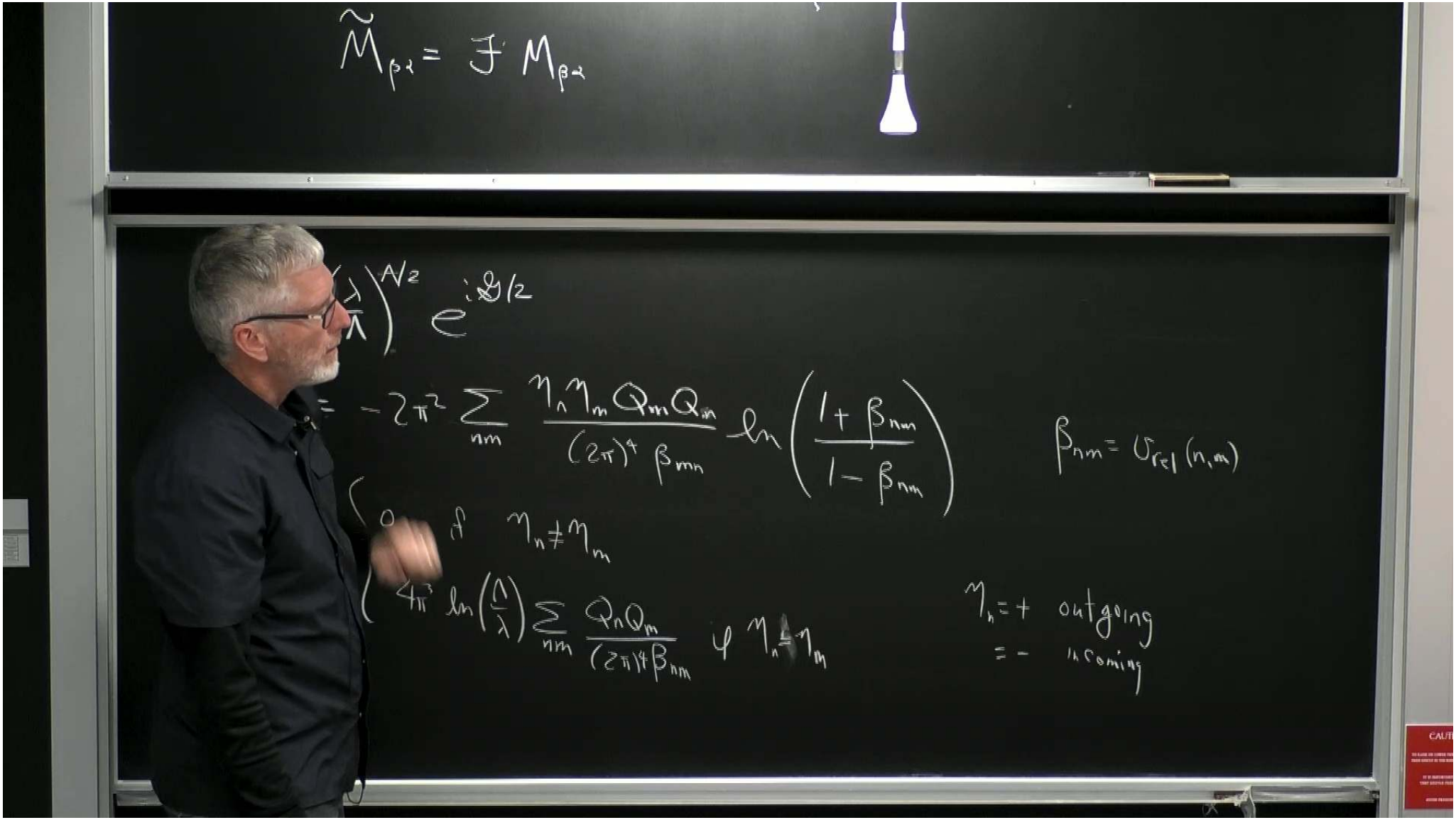


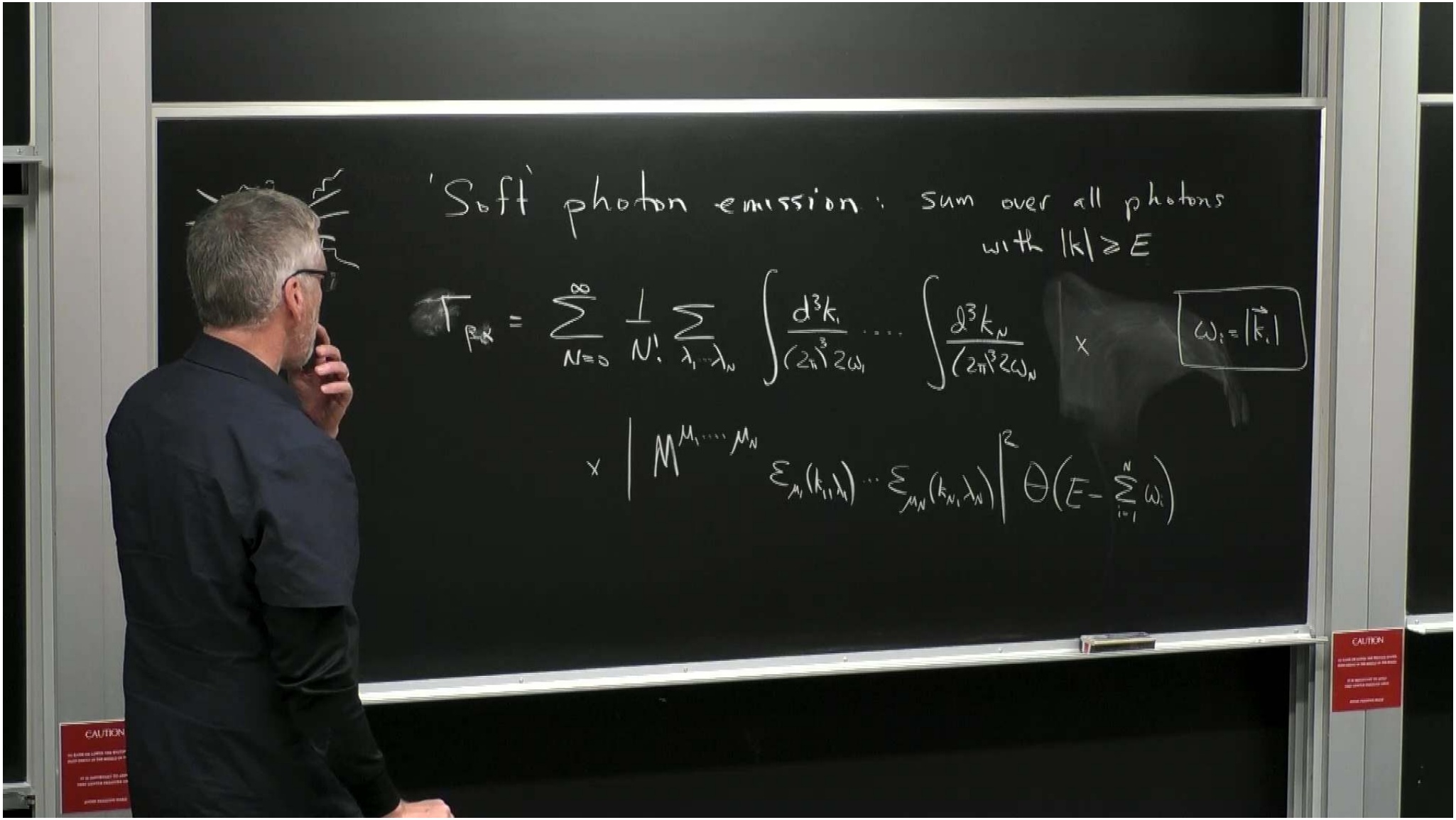
hard process $|k| \geq \Lambda$

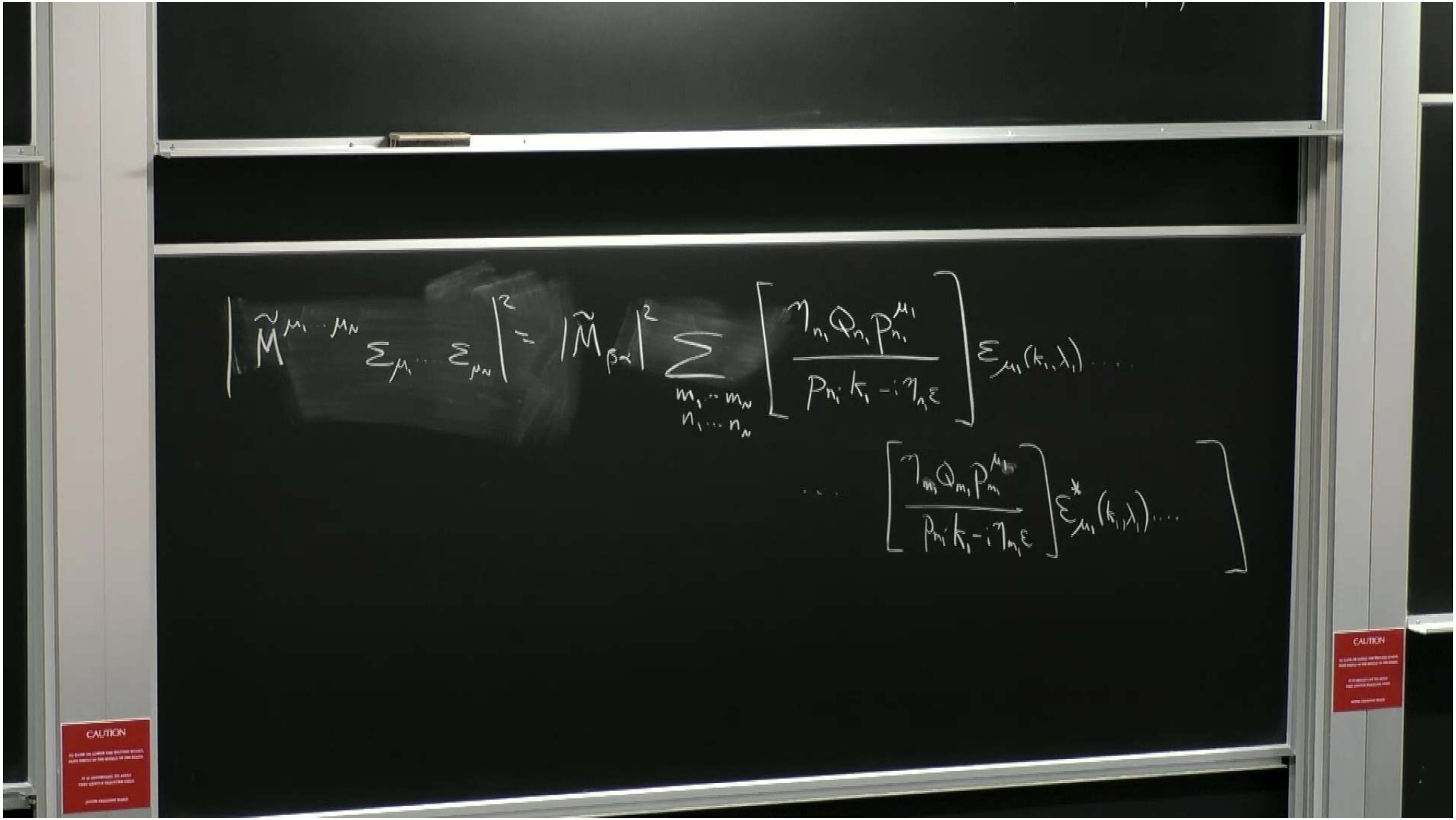
$M_{\beta\alpha}(\Lambda)$

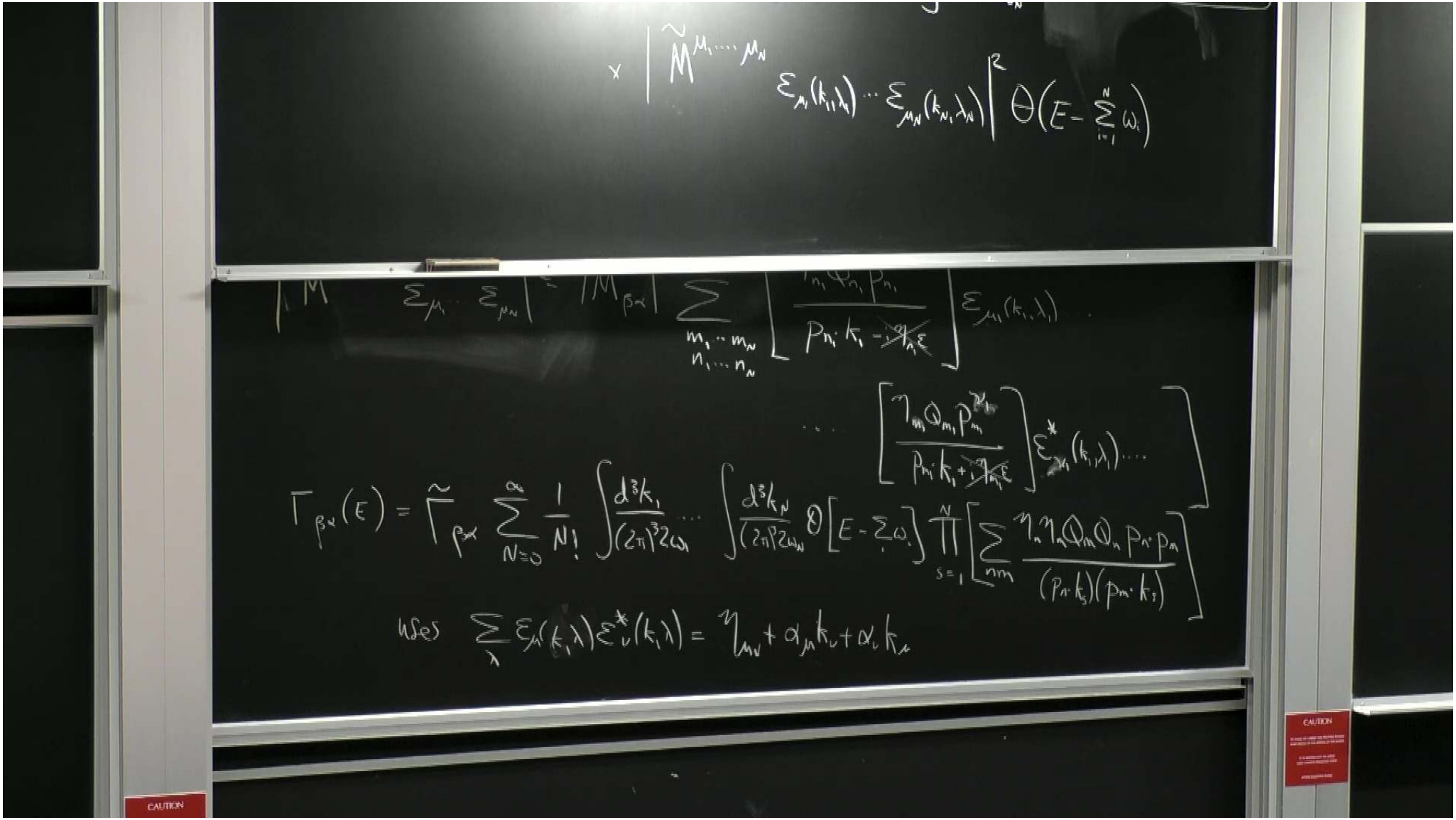
Soft photons with $\lambda < |k| < \Lambda$

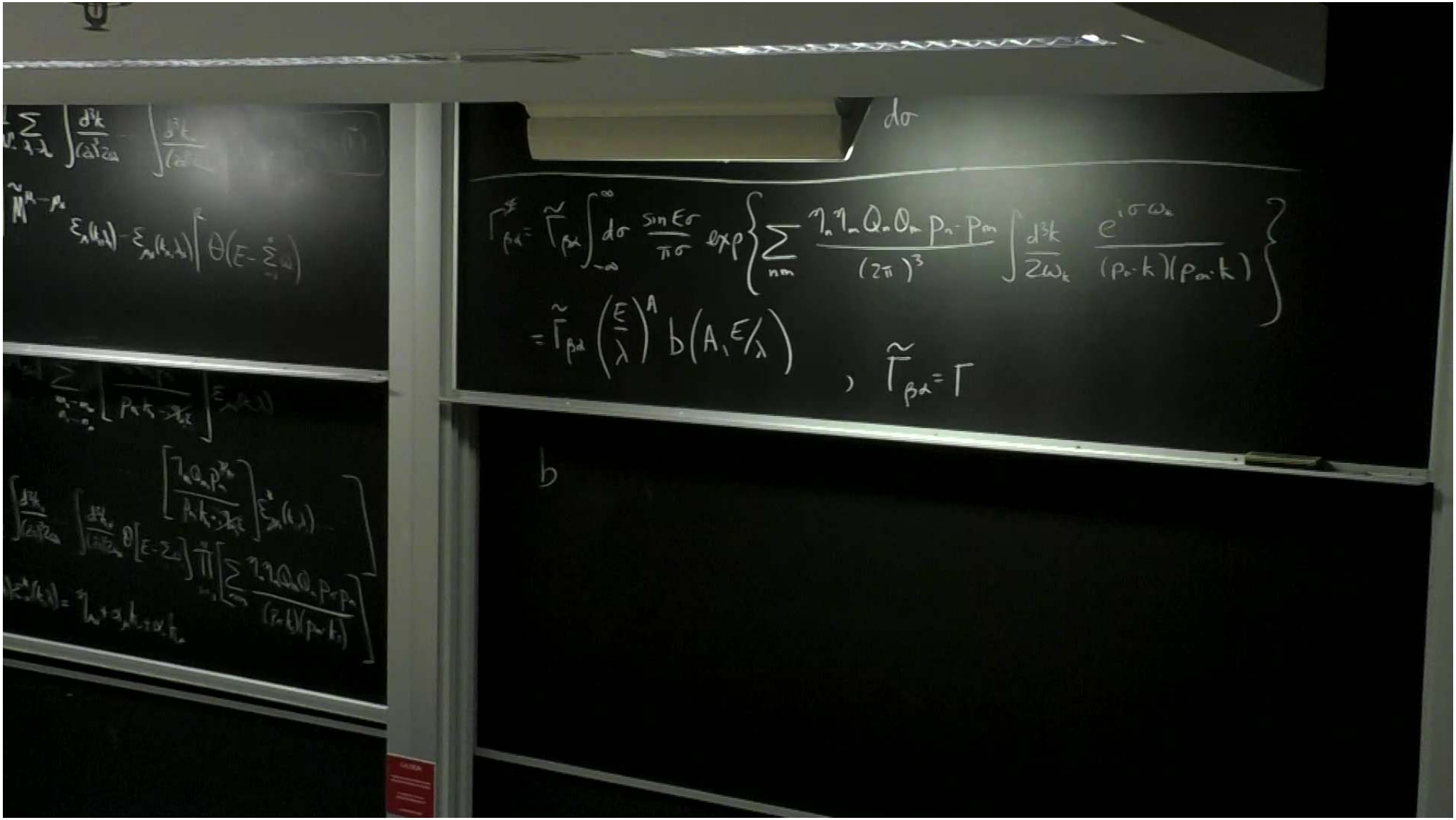

$\tilde{M}_{\beta_1} = \mathcal{J}\, M_{\beta_2}$
$\mathcal{J} = \left(\frac{\lambda}{\Lambda}\right)^{N_2} e^{i\vartheta/2}$
$A = -2\pi^2 \sum_{nm} \frac{\eta_n \eta_m Q_n Q_m}{(2\pi)^4 \beta_{nm}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right)$
$\beta_{nm} = v_{rel}(n,m)$
$\vartheta = \begin{cases} 0 & \text{if } \eta_n \neq \eta_m \\ 4\pi^3 \ln\left(\frac{\Lambda}{\lambda}\right) \sum_{nm} \frac{Q_n Q_m}{(2\pi)^4 \beta_{nm}} & \text{if } \eta_n \neq \eta_m \end{cases}$

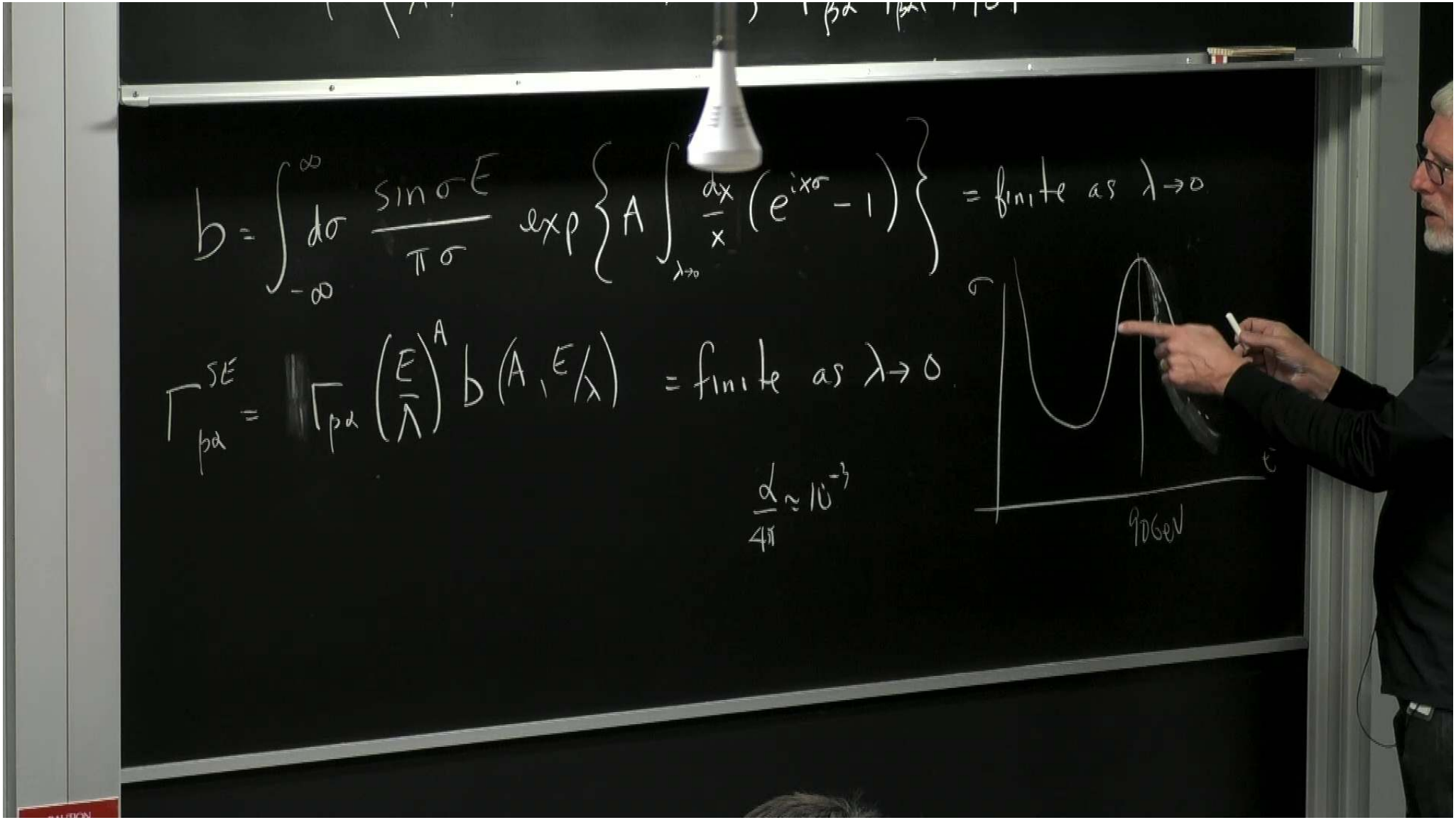

$\tilde{M}_{\beta_1} = \mathcal{F} M_{\beta_2}$
$\left(\frac{\lambda}{\Lambda}\right)^{N_2} e^{i\delta/2}$
$= -2\pi^2 \sum_{nm} \frac{\eta_n \eta_m Q_n Q_m}{(2\pi)^4 \beta_{nm}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right)$
$\beta_{nm} = v_{rel}(n,m)$
0 if $\eta_n \neq \eta_m$
$4\pi^3 \ln\left(\frac{\Lambda}{\lambda}\right) \sum_{nm} \frac{Q_n Q_m}{(2\pi)^4 \beta_{nm}}$ if $\eta_n = \eta_m$
$\eta_n = +$ outgoing
$= -$ incoming


'Soft' photon emission: sum over all photons with $|k| \leq E$
$\Gamma_{fi} = \sum_{N=0}^{\infty} \frac{1}{N!} \sum_{\lambda_1 \cdots \lambda_N} \int \frac{d^3k_1}{(2\pi)^3 2\omega_1} \cdots \int \frac{d^3k_N}{(2\pi)^3 2\omega_N} \times$
$\omega_i = |\vec{k}_i|$
$\times \left| M^{\mu_1 \cdots \mu_N} \varepsilon_{\mu_1}(k_1, \lambda_1) \cdots \varepsilon_{\mu_N}(k_N, \lambda_N) \right|^2 \theta\left(E - \sum_{i=1}^{N} \omega_i\right)$
CAUTION
CAUTION


$\left| \tilde{M}^{\mu_1 \cdots \mu_N} \varepsilon_{\mu_1} \cdots \varepsilon_{\mu_N} \right|^2 = \left| \tilde{M}_{\beta\alpha} \right|^2 \sum_{\substack{m_1 \cdots m_N \\ n_1 \cdots n_N}} \left[\frac{\eta_{n_1} Q_{n_1} p_{n_1}^{\mu_1}}{p_{n_1} k_1 - i\eta_1 \varepsilon} \right] \varepsilon_{\mu_1}(k_1, \lambda_1) \cdots$
$\cdots \left[\frac{\eta_{m_1} Q_{m_1} p_{m_1}^{\mu_1}}{p_{m_1} k_1 - i\eta_1 \varepsilon} \right] \varepsilon^*_{\mu_1}(k_1, \lambda_1) \cdots \Bigg]$
CAUTION
CAUTION


$\times \left| \tilde{M}^{\mu_1 \cdots \mu_N} \varepsilon_{\mu_1}(k_1,\lambda_1) \cdots \varepsilon_{\mu_N}(k_N,\lambda_N) \right|^2 \Theta\left(E - \sum_{i=1}^{N} \omega_i\right)$
$\left| M \quad \varepsilon_{\mu_1} \cdots \varepsilon_{\mu_N} \right|^2 \left| M_{\beta\alpha} \right| \sum_{\substack{m_1 \cdots m_N \\ n_1 \cdots n_N}} \left[\frac{\eta_{n_1} Q_{n_1} p_{n_1}}{p_{n_1} k_1 - q_1 \varepsilon} \right] \varepsilon_{\mu_1}(k_1,\lambda_1)$
$\cdots \left[\frac{\eta_{m_1} Q_{m_1} p^{\nu_1}_{m_1}}{p_{m_1} k_1 + q_1 \varepsilon} \right] \varepsilon^*_{\nu_1}(k_1,\lambda_1) \cdots \Big]$
$\Gamma_{\beta\alpha}(E) = \tilde{\Gamma}_{\beta\alpha} \sum_{N=0}^{\infty} \frac{1}{N!} \int \frac{d^3k_1}{(2\pi)^3 2\omega_1} \cdots \int \frac{d^3k_N}{(2\pi)^3 2\omega_N} \theta\left[E - \Sigma_i \omega_i\right] \prod_{s=1}^{N} \left[\sum_{nm} \frac{\eta_n \eta_m Q_m Q_n \, p_n \cdot p_m}{(p_n \cdot k_s)(p_m \cdot k_s)} \right]$
uses $\sum_{\lambda} \varepsilon_\mu(k,\lambda) \varepsilon^*_\nu(k,\lambda) = \eta_{\mu\nu} + \alpha_\mu k_\nu + \alpha_\nu k_\mu$

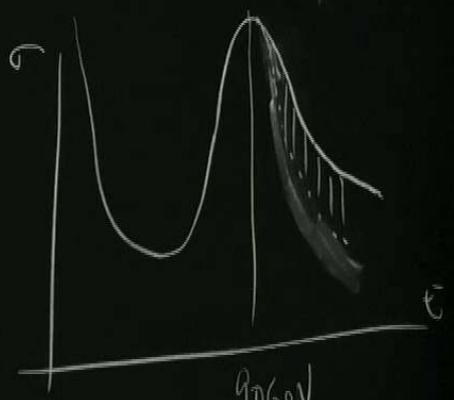

dσ
b


$b = \int_{-\infty}^{\infty} d\sigma \frac{\sin \sigma E}{\pi \sigma} \exp\left\{ A \int_{\lambda \to 0} \frac{dx}{x} \left(e^{ix\sigma} - 1 \right) \right\}$ = finite as $\lambda \to 0$
$\Gamma^{SE}_{\beta\alpha} = \Gamma_{\beta\alpha} \left(\frac{E}{\Lambda} \right)^A b(A, E/\lambda)$ = finite as $\lambda \to 0$
$\frac{\alpha}{4\pi} \simeq 10^{-3}$
σ
t
90GeV

$$b = \int_{-\infty}^{\infty} d\sigma \, \frac{\sin \sigma E}{\pi \sigma} \exp\left\{ A \int_{\lambda \to 0} \frac{dx}{x} \left(e^{ix\sigma} - 1 \right) \right\} = \text{finite as } \lambda \to 0$$

$$\Gamma_{\beta\alpha}^{SE} = \Gamma_{\beta\alpha} \left(\frac{E}{\Lambda} \right)^{A} b\left(A, E/\lambda \right) = \text{finite as } \lambda \to 0$$

$$\frac{\alpha}{4\pi} \ln\left(\frac{E_1}{E_2} \right) \qquad \frac{\alpha}{4\pi} \simeq 10^{-3}$$



$$= \tilde{\Gamma}_{\beta\alpha} \left(\frac{E}{\lambda}\right)^A b(A, E/\lambda) \quad , \quad \tilde{\Gamma}_{\beta\alpha} \Gamma_{\beta\alpha}(1) |\mathcal{F}|^2 \qquad d\Gamma_\alpha$$

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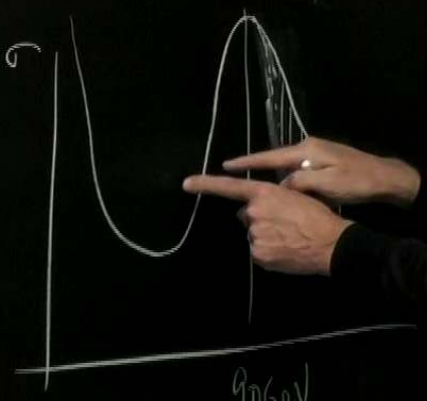

$$= \tilde{\Gamma}_{\beta\alpha} \left(\frac{E}{\lambda}\right)^A b(A, E/\lambda) \quad , \quad \tilde{\Gamma}_{\beta\alpha} \to \Gamma_{\beta\alpha}(1)\,|\mathcal{F}|^2 \qquad d\Gamma_\alpha$$

$$b = \int_{-\infty}^{\infty} d\sigma \, \frac{\sin\sigma E}{\pi\sigma} \exp\left\{ A \int_{\lambda\to 0} \frac{dx}{x} \left(e^{ix\sigma} - 1\right) \right\} = \text{finite as } \lambda \to 0$$

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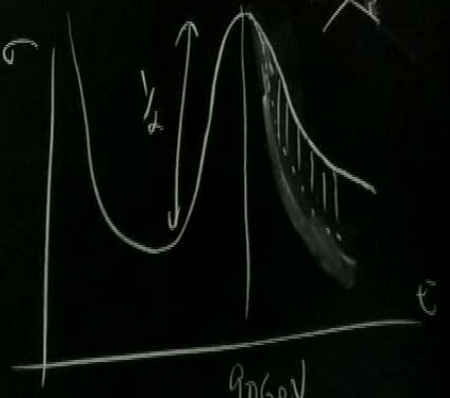
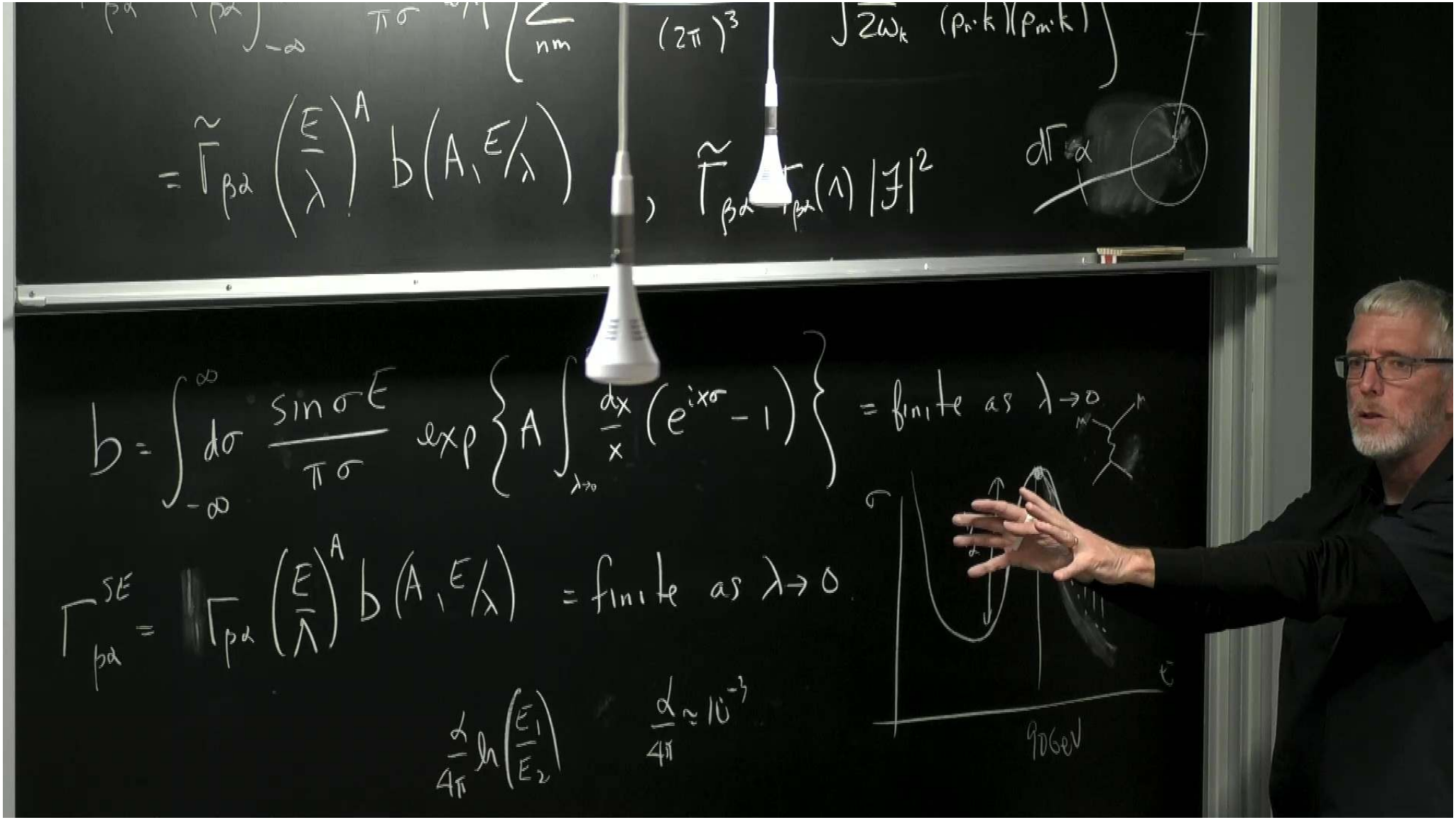
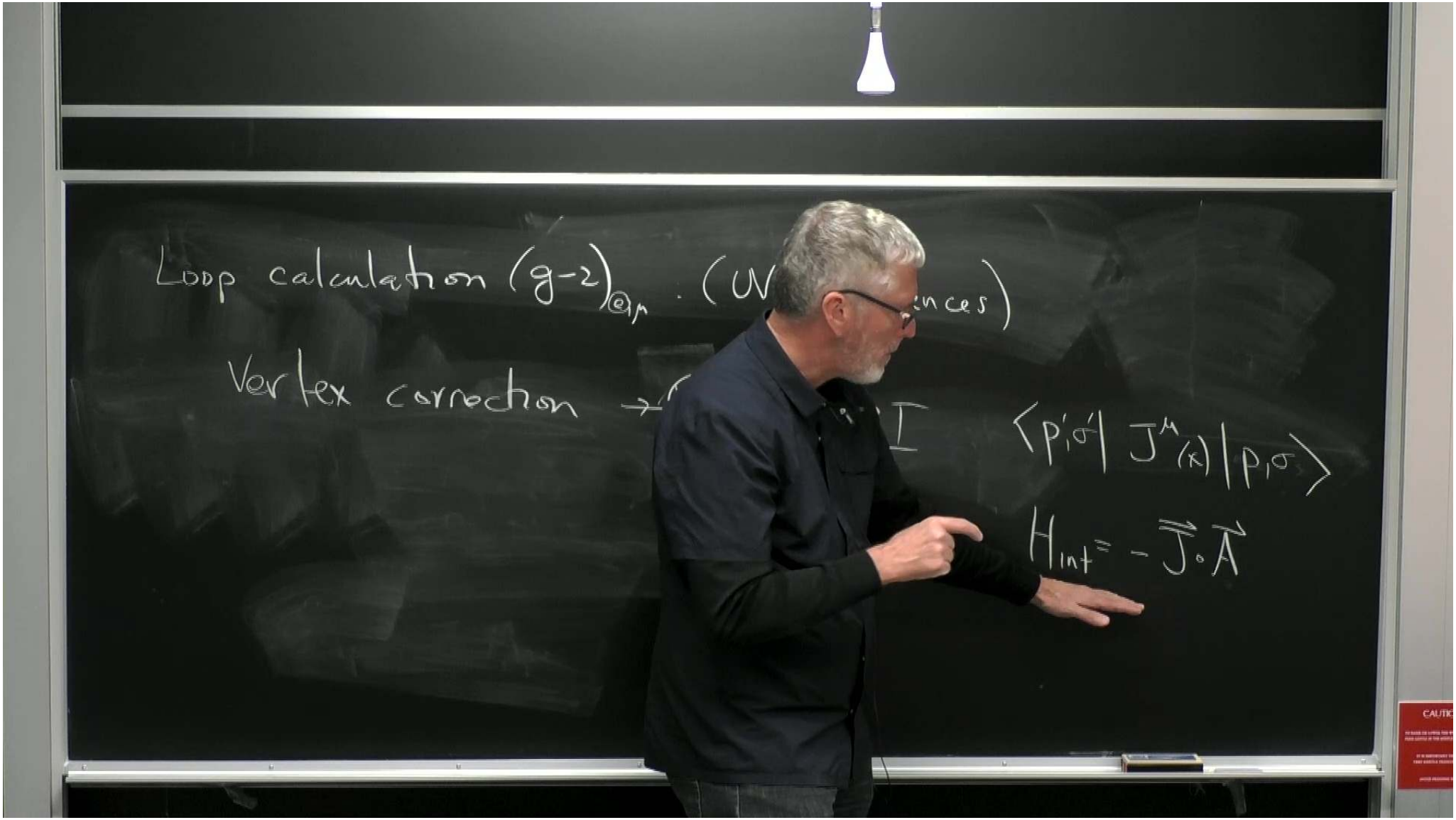
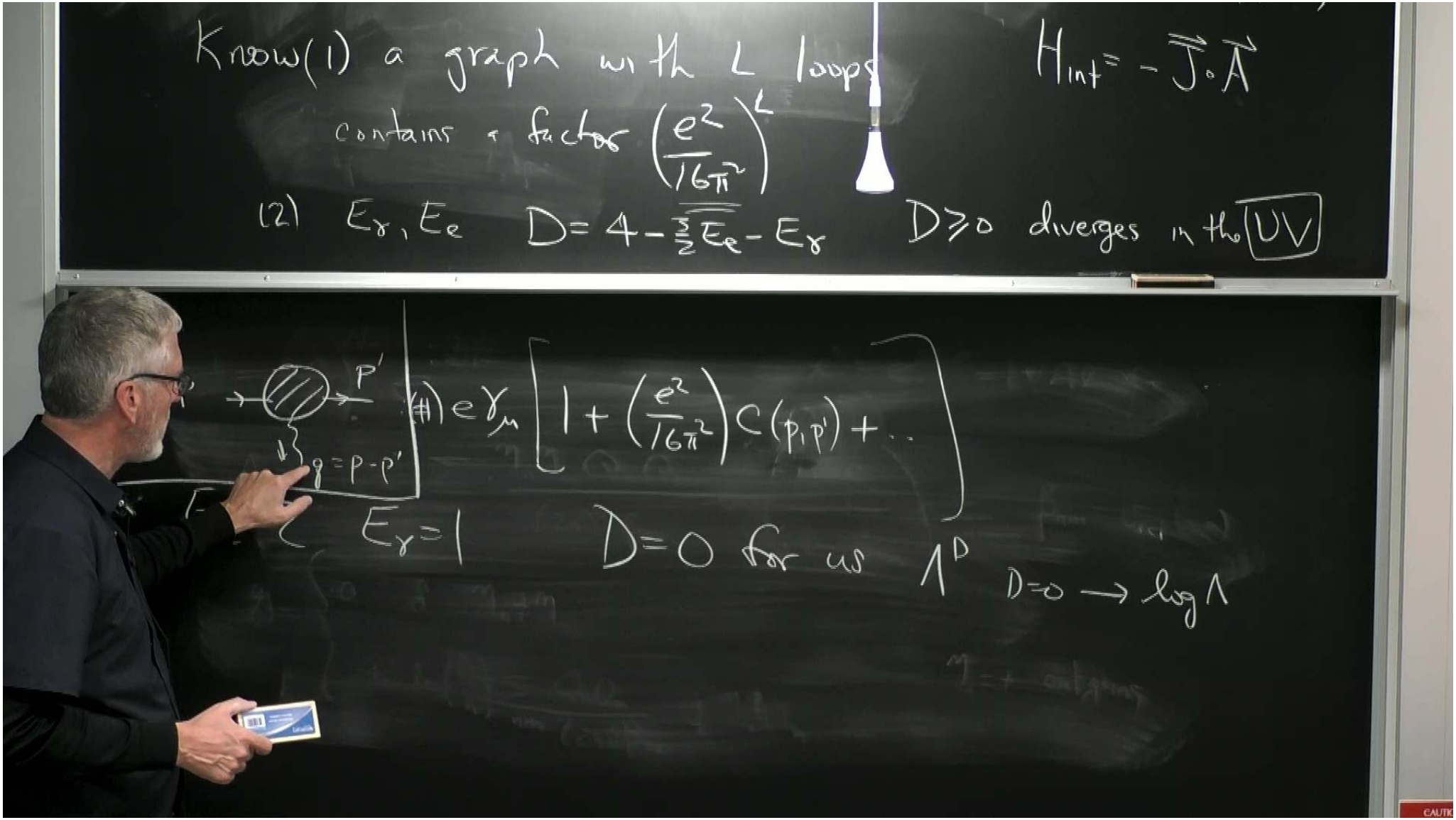



$\tilde{\Gamma}_{\beta\alpha}\left(\frac{E}{\lambda}\right)^A b(A, E/\lambda)$
$\tilde{\Gamma}_{\beta\alpha} = \Gamma_{\beta\alpha}(1)\,|\mathcal{F}|^2$
$d\Gamma_\alpha$
$b = \int_{-\infty}^{\infty} d\sigma \frac{\sin\sigma E}{\pi\sigma} \exp\left\{A \int_{\lambda\to 0} \frac{dx}{x}\left(e^{ix\sigma} - 1\right)\right\}$ = finite as $\lambda \to 0$
$\Gamma^{SE}_{\beta\alpha} = \Gamma_{\beta\alpha}\left(\frac{E}{\lambda}\right)^A b(A, E/\lambda)$ = finite as $\lambda \to 0$
$\frac{\alpha}{4\pi} \ln\left(\frac{E_1}{E_2}\right)$
$\frac{\alpha}{4\pi} \simeq 10^{-3}$
σ
E
90GeV

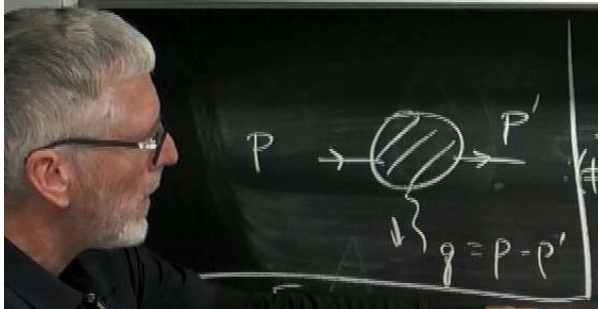

Loop calculation $(g-2)_{@1}$. (UV
Vertex correction
$\langle p',\sigma' | J^{\mu}(x) | p,\sigma \rangle$
$H_{int} = -\vec{J} \cdot \vec{A}$


Know(1) a graph with L loops
$H_{int} = -\vec{J}\cdot\vec{A}$
Contains a factor $\left(\frac{e^2}{16\pi^2}\right)^L$
(2) E_γ, E_e $D = 4 - \frac{3}{2}\overline{E_e} - E_\gamma$ $D \geq 0$ diverges in the UV
p'
$q = p - p'$
$(\#)\, e\gamma_\mu \left[1 + \left(\frac{e^2}{16\pi^2}\right) C(p,p') + \ldots\right]$
$E_\gamma = 1$ $D = 0$ for us 1^D $D=0 \rightarrow \log\Lambda$

Know(1) a graph with L loops $\quad H_{int} = -\vec{J}\cdot\vec{A}$

Contains a factor $\left(\frac{e^2}{16\pi^2}\right)^L$

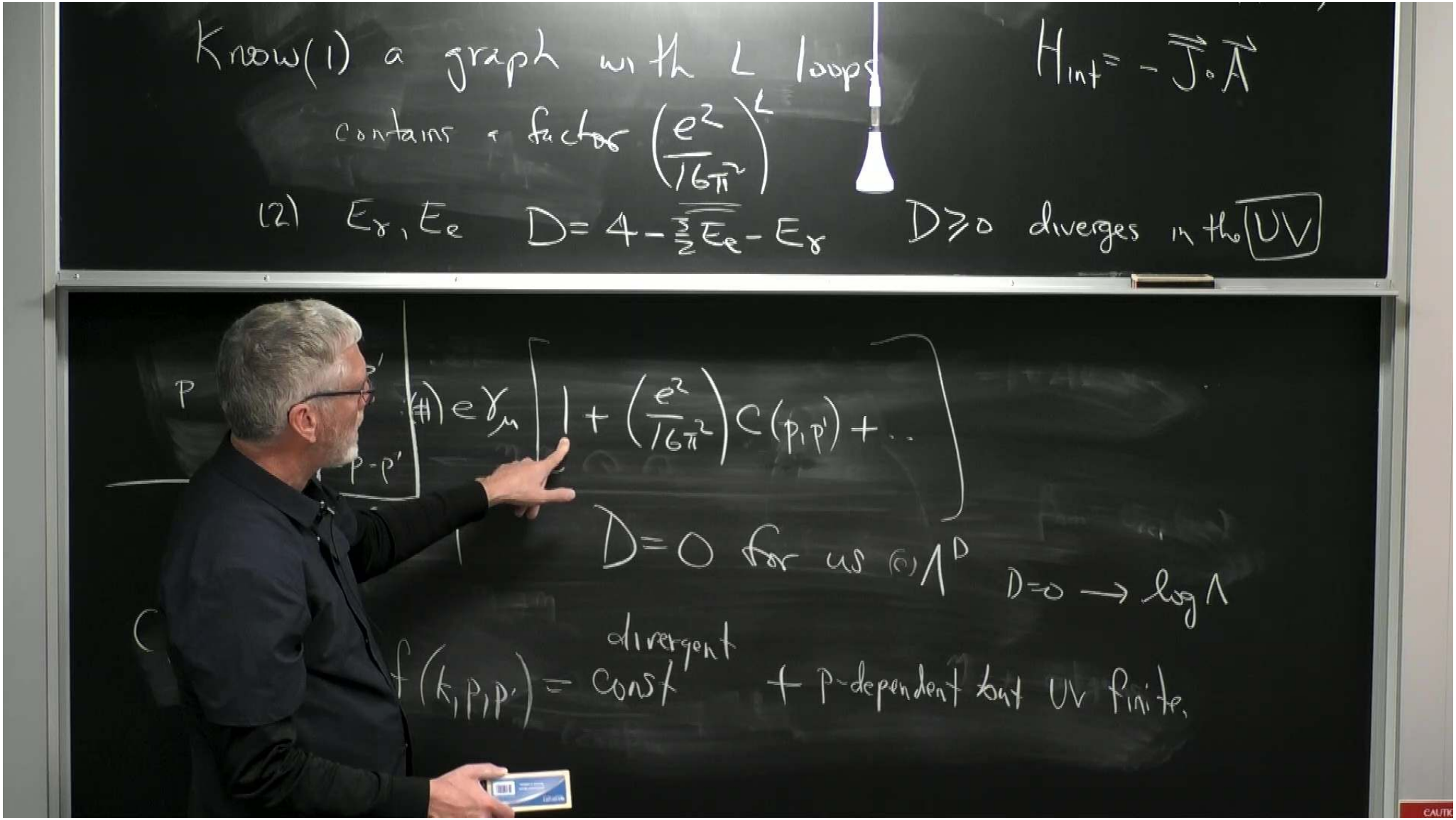
(2) $E_\gamma, E_e \quad D = 4 - \frac{3}{2}E_e - E_\gamma \quad D \geq 0$ diverges in the UV

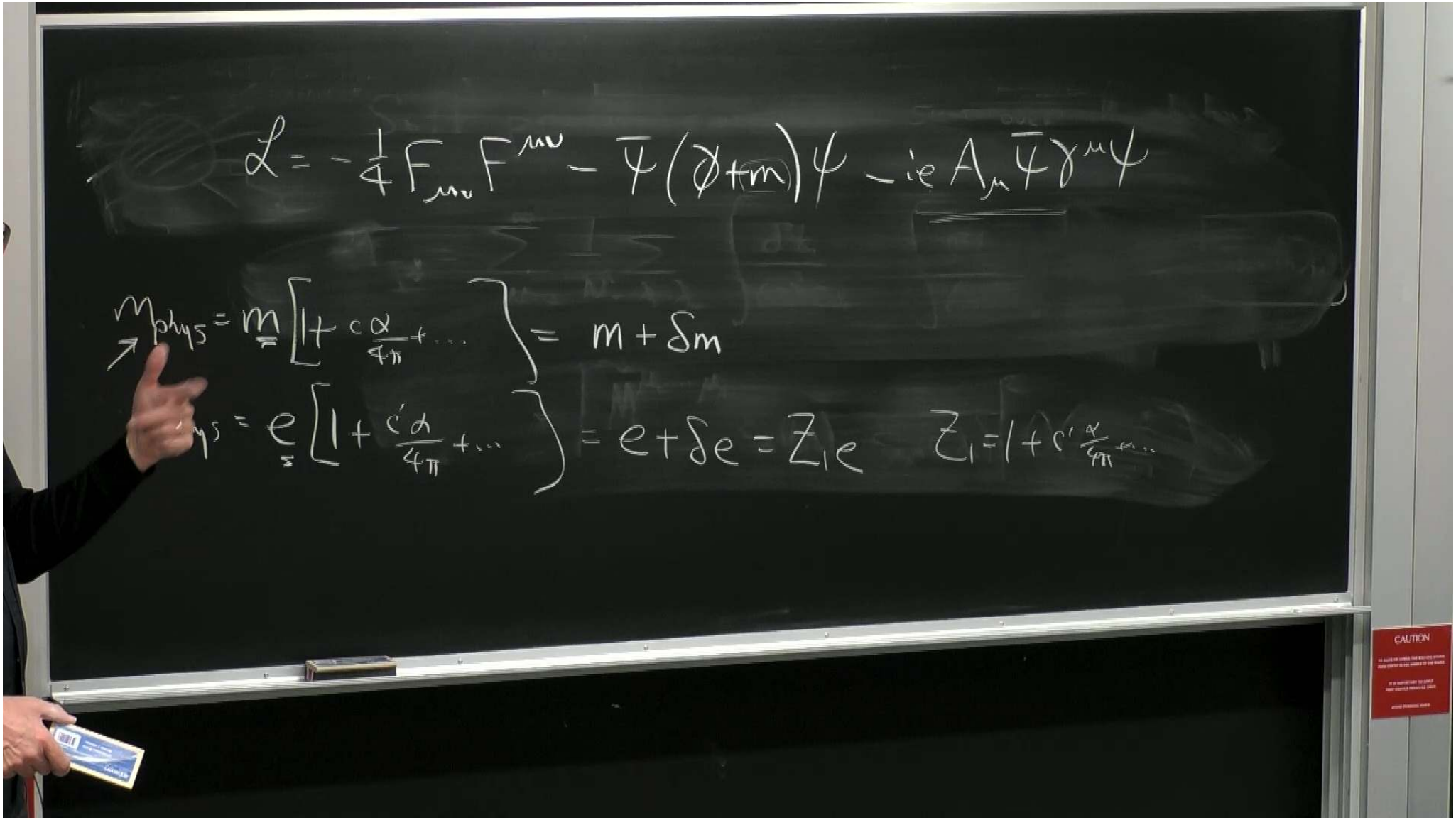


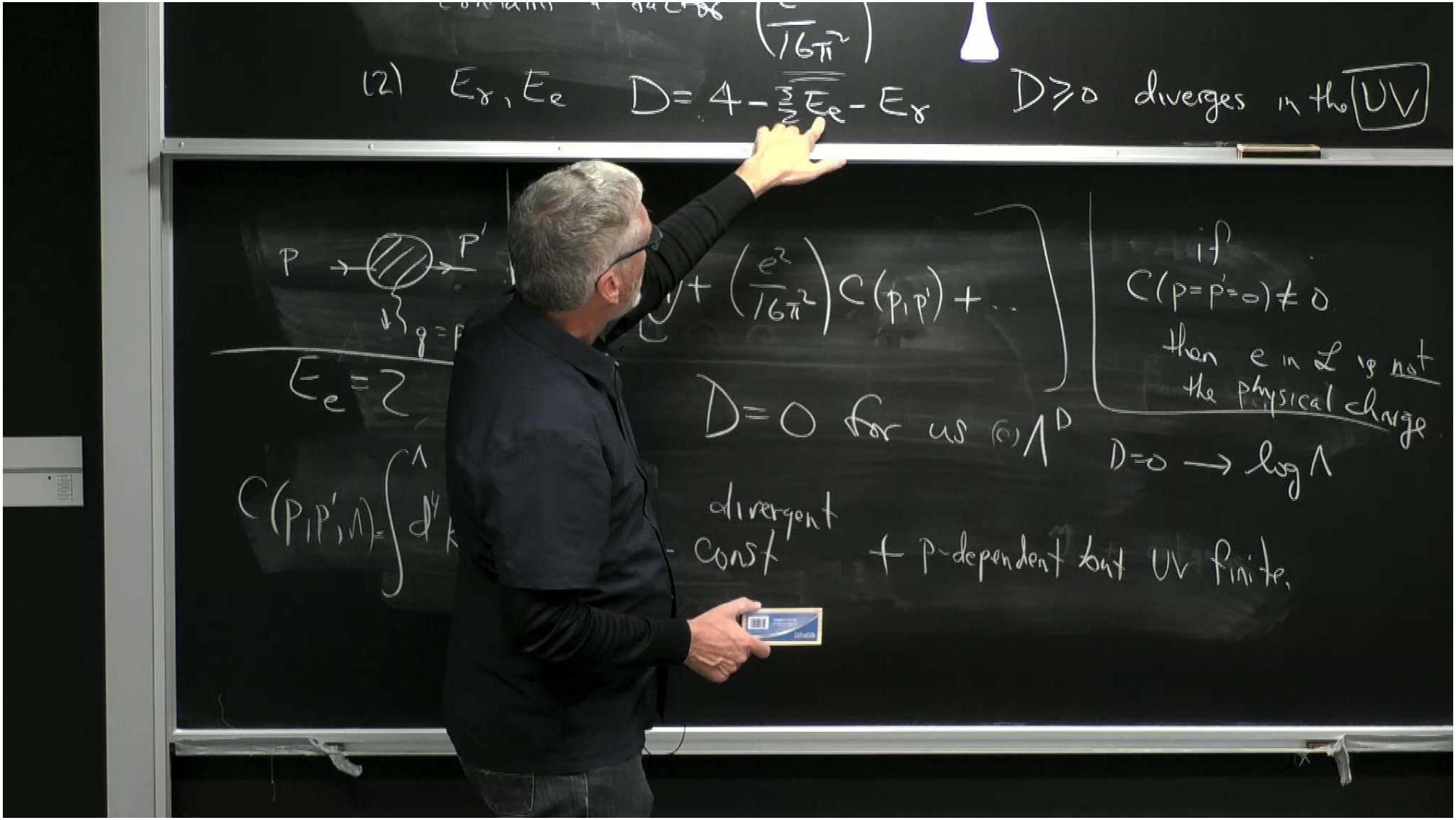
$(\#)\, e\gamma_\mu \left[1 + \left(\frac{e^2}{16\pi^2}\right) C(p,p') + \ldots\right]$

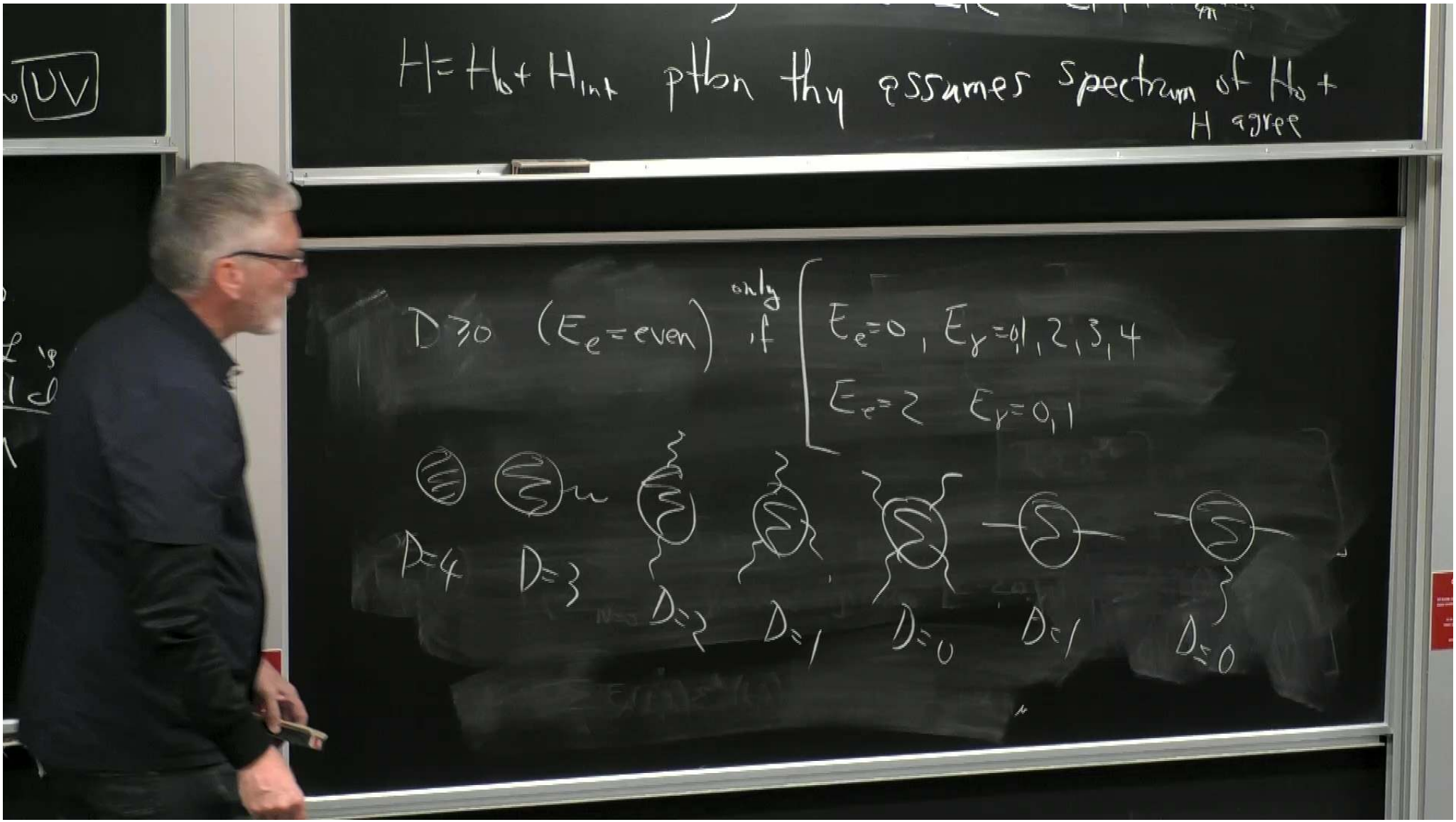
$D = 0$ for us $\quad \int^\Lambda 1^D \quad D=0 \rightarrow \log\Lambda$

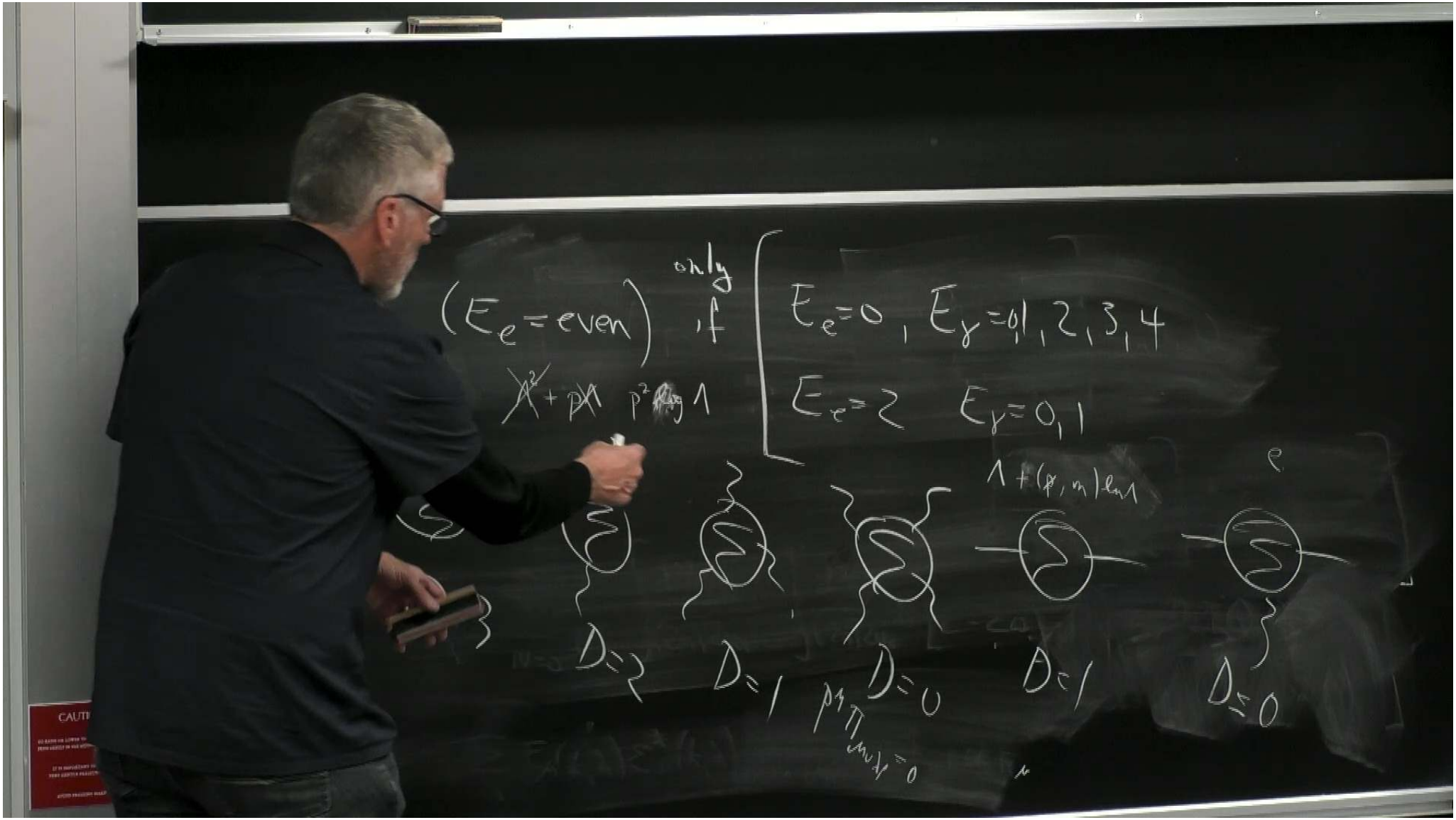
$C(p,p',\Lambda) = \int^\Lambda d^4k \; f(k,p,p')$

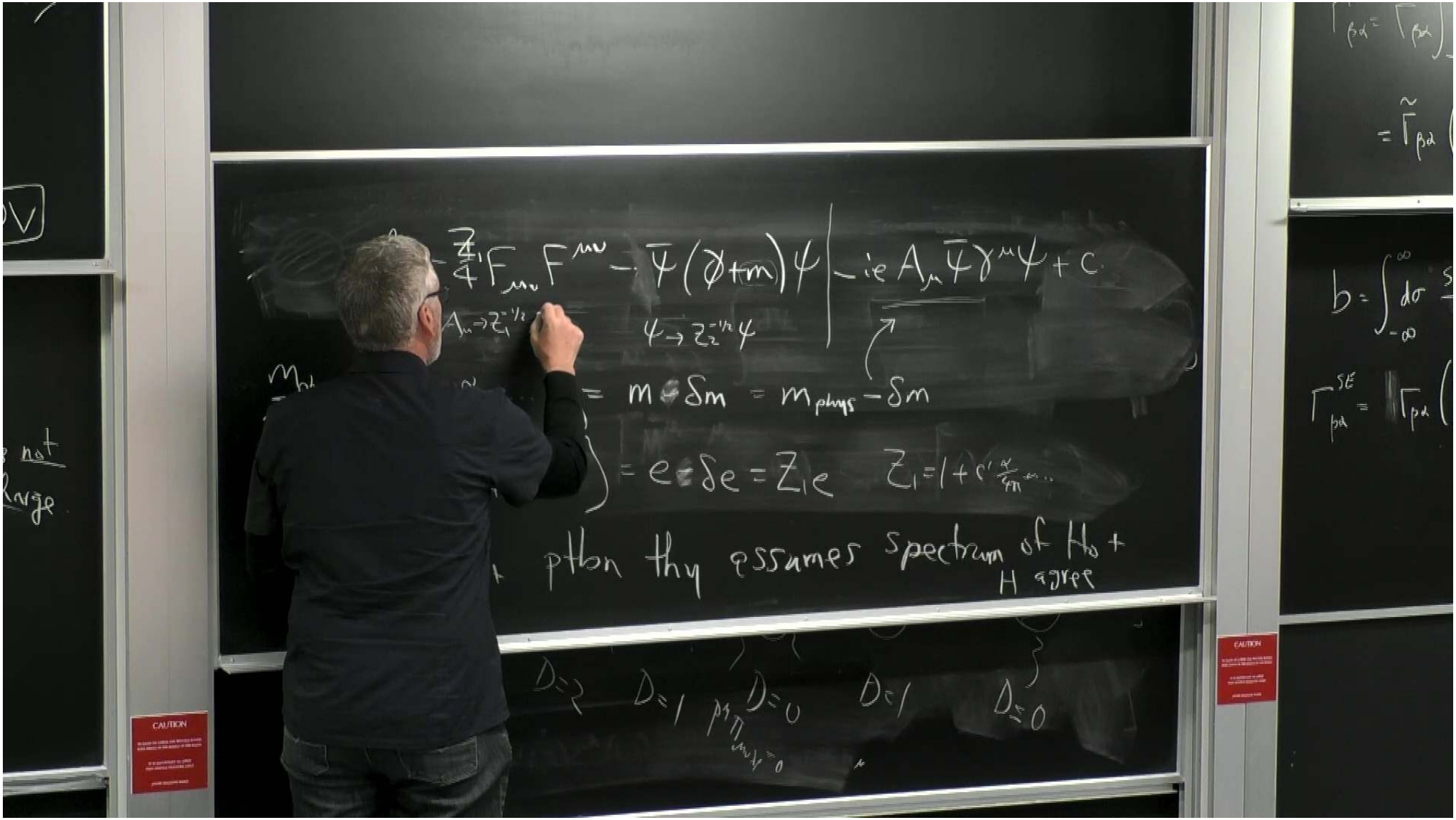

Know(1) a graph with L loops
$H_{int} = -\vec{J}\cdot\vec{A}$
contains a factor $\left(\frac{e^2}{16\pi^2}\right)^L$
(2) E_γ, E_e $D = 4 - \frac{3}{2}\overline{E_e} - E_\gamma$ $D \geq 0$ diverges in the UV
p
$p-p'$
$(\#)\, e\gamma_\mu \left[1 + \left(\frac{e^2}{16\pi^2}\right) C(p,p') + \ldots\right]$
$D=0$ for us Λ^D
$D=0 \rightarrow \log\Lambda$
$f(k,p,p') =$ divergent Const + p-dependent but UV finite.

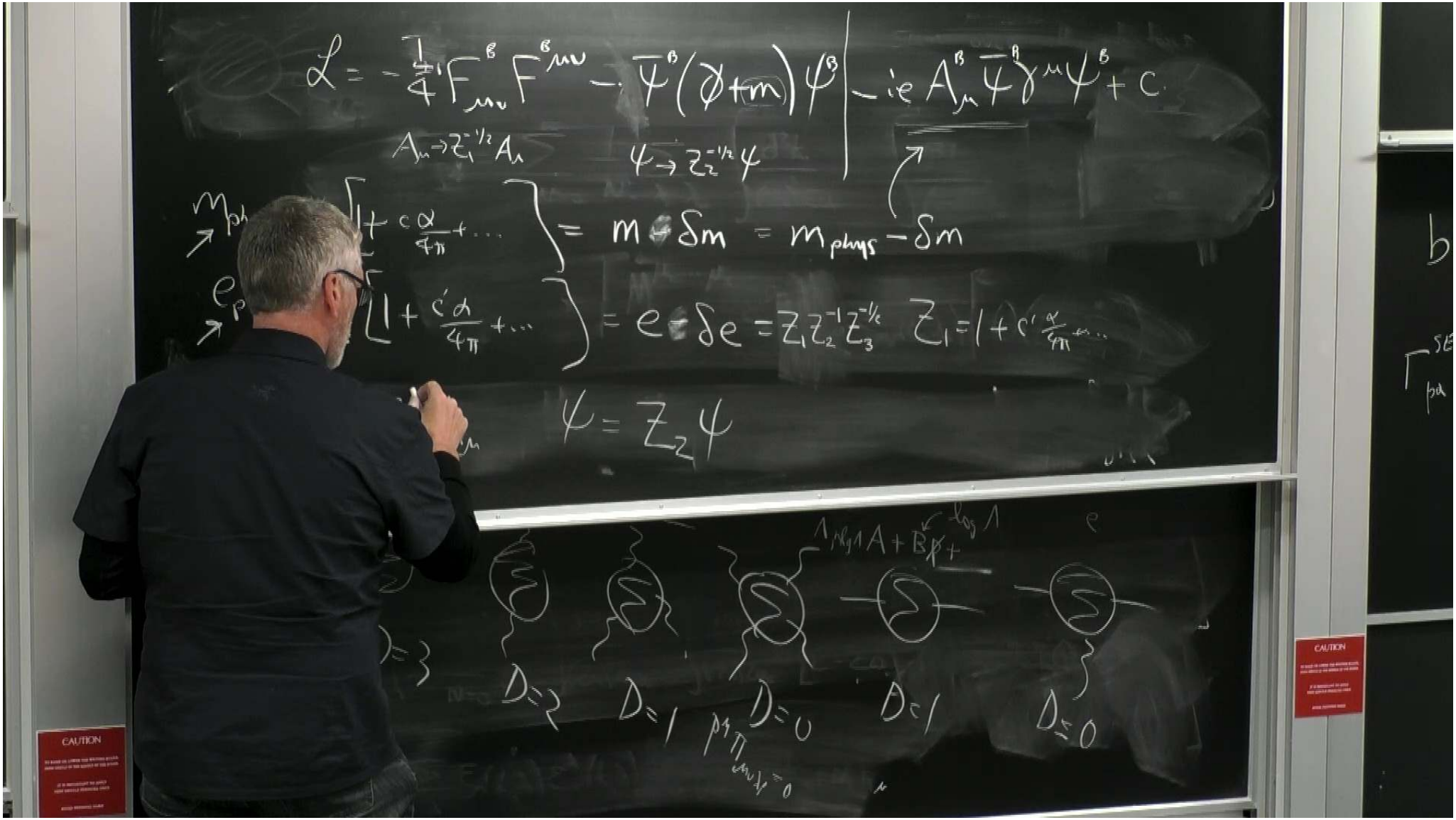

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{\partial} + m)\psi - ie A_\mu \bar{\psi} \gamma^\mu \psi$
$m_{phys} = m \left[1 + c \frac{\alpha}{4\pi} + \dots \right] = m + \delta m$
$e_{phys} = e \left[1 + c' \frac{\alpha}{4\pi} + \dots \right] = e + \delta e = Z_1 e \quad Z_1 = 1 + c' \frac{\alpha}{4\pi} + \dots$
CAUTION

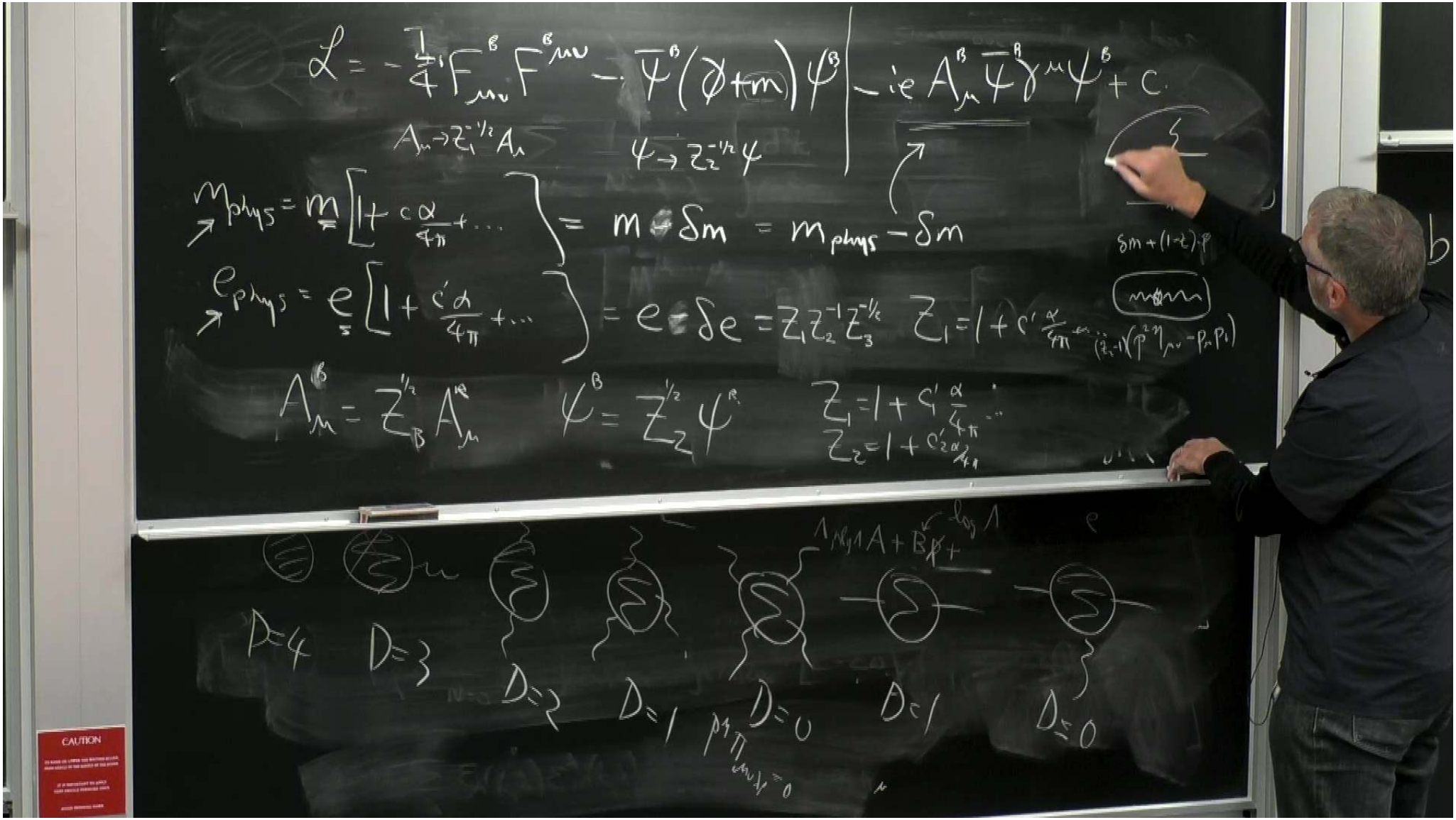

(2) E_γ, E_e $D = 4 - \frac{3}{2}E_e - E_\gamma$ $D \geq 0$ diverges in the UV
$\left(\frac{e^2}{16\pi^2}\right)C(p,p') + \ldots$
if $C(p=p'=0) \neq 0$ then e in $\mathcal{L}$ is not the physical charge
$D=0$ for us Λ^D
$D=0 \rightarrow \log\Lambda$
$C(p,p',\Lambda) = \int^\Lambda d^4k$
divergent
- Const + p-dependent but UV finite.
$E_e = 2$

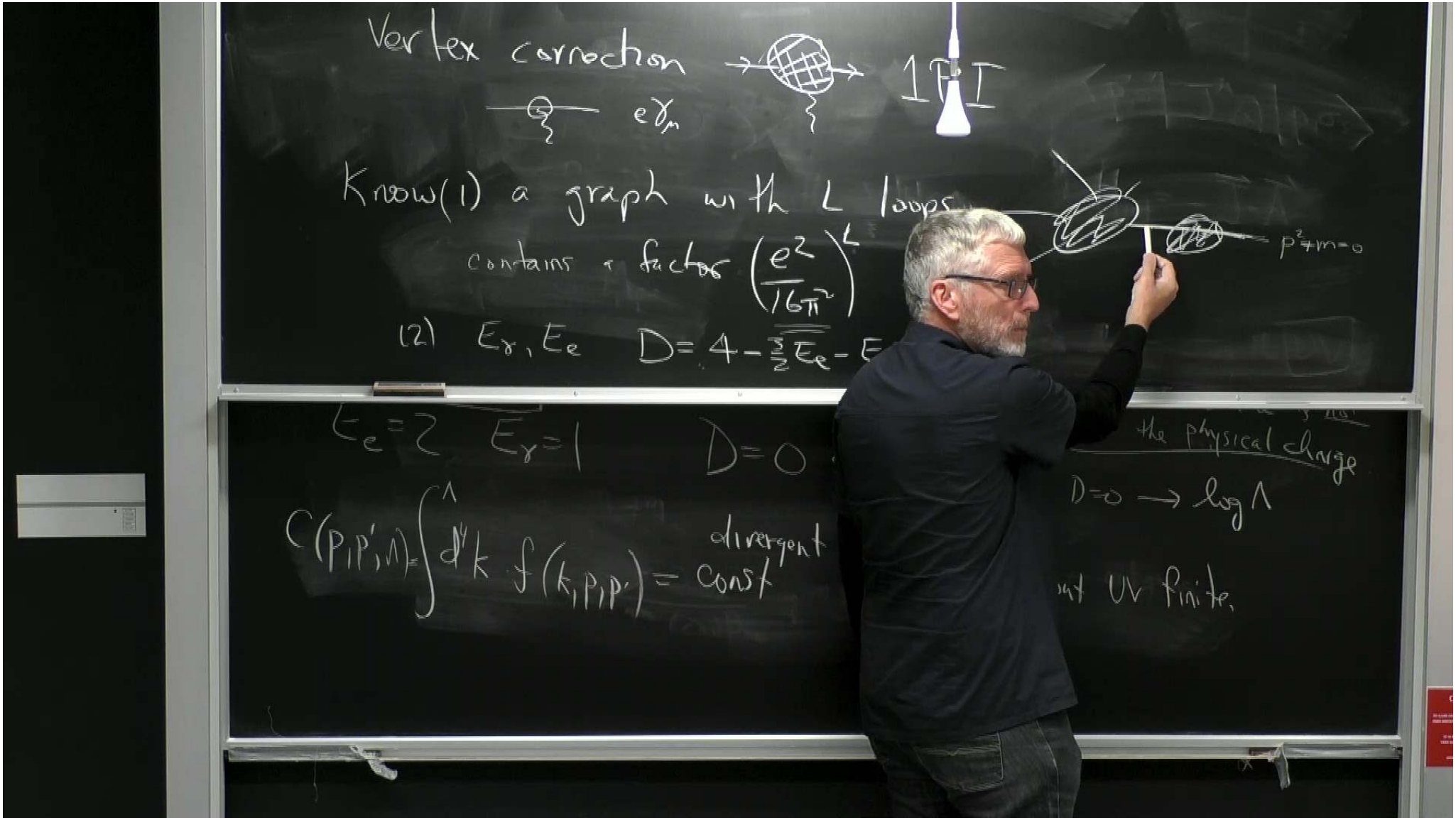

UV
H = H0 + Hint ptbn thy assumes spectrum of H0 + H agree
D ≥ 0 (Ee = even) if only
Ee = 0, Eγ = 0, 1, 2, 3, 4
Ee = 2 Eγ = 0, 1
D = 4
D = 3
D = 2
D = 1
D = 0
D = 1
D = 0

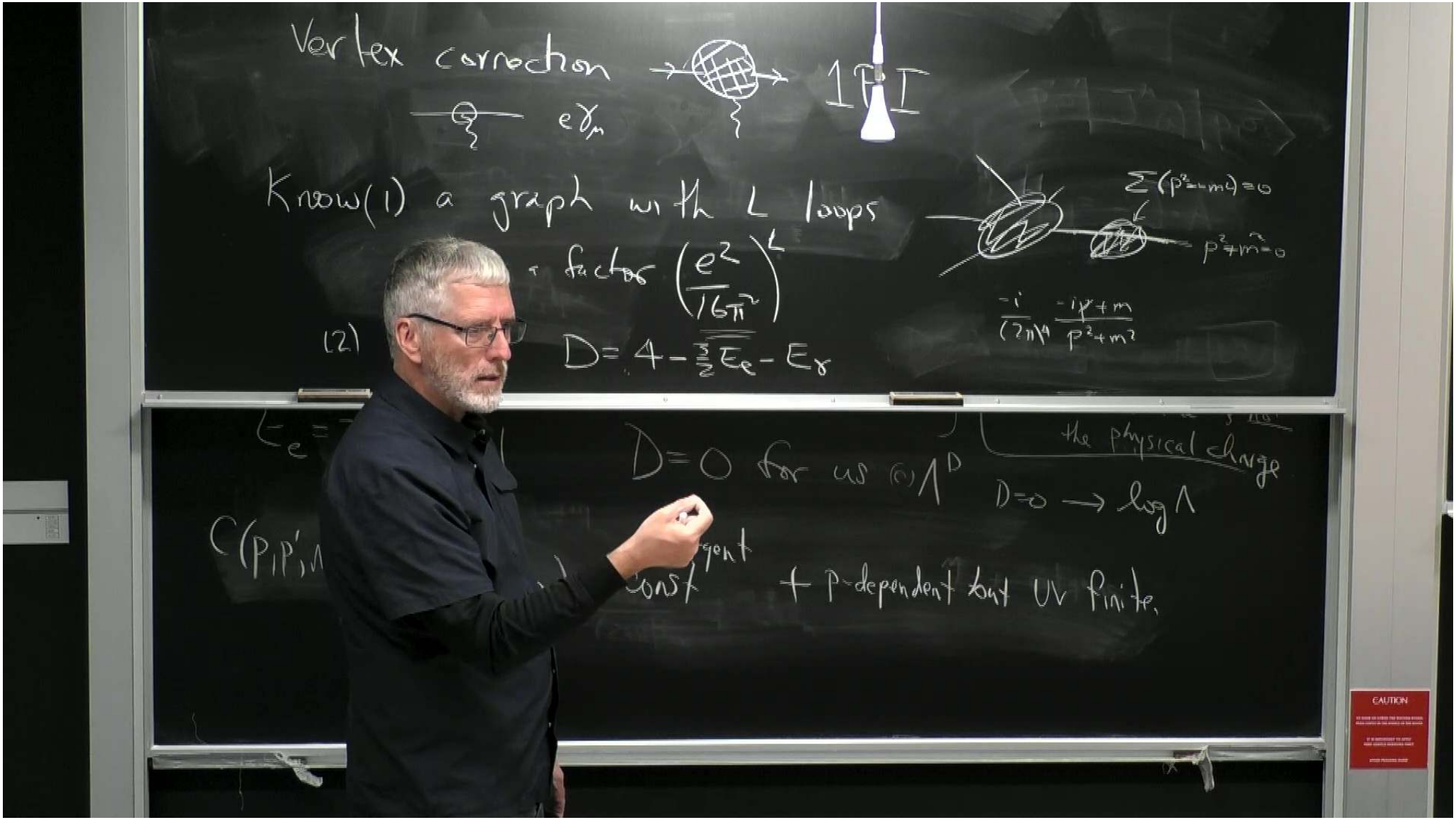

$(E_e = \text{even})$ only if
$E_e = 0, E_\gamma = 0,1,2,3,4$
$E_e = 2 \quad E_\gamma = 0,1$
$1 + (p,m)\ell_{m1}$
$D=2$
$D=1$
$D=0$
$D=1$
$D=0$


$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{\partial}+m)\psi - ie A_\mu \bar{\psi}\gamma^\mu\psi + c.$
$A_\mu \to Z_1^{-1/2} A_\mu$
$\psi \to Z_2^{-1/2}\psi$
$= m + \delta m = m_{phys} - \delta m$
$= e + \delta e = Z_1 e$
$Z_1 = 1 + c' \frac{\alpha}{4\pi} \ldots$
ption thy assumes spectrum of H_0 +
H agrees
$D=2$
$D=1$
$D=0$
$D=1$
$D=0$
CAUTION
CAUTION


$\mathcal{L} = -\frac{1}{4} F^B_{\mu\nu} F^{B\mu\nu} - \bar{\psi}^B(\not{\partial} + m)\psi^B - ie A^B_\mu \bar{\psi}^B \gamma^\mu \psi^B + c.$
$A_\mu \to Z_1^{-1/2} A_\mu$
$\psi \to Z_2^{-1/2} \psi$
m_{ph}
e_p
$= m + \delta m = m_{phys} - \delta m$
$= e + \delta e = Z_1 Z_2^{-1} Z_3^{-1/2}$
$Z_1 = 1 + c' \frac{\alpha}{4\pi} + \ldots$
$\psi = Z_2 \psi$
$D=3$
$D=2$
$D=1$
$D=0$
$D<1$
$D<0$
CAUTION
CAUTION


$\mathcal{L} = -\frac{1}{4} F^{B}_{\mu\nu} F^{B\mu\nu} - \bar{\psi}^{B}(\not{\partial} + m)\psi^{B} - ie A^{B}_{\mu} \bar{\psi}^{B} \gamma^{\mu} \psi^{B} + c.$
$A_\mu \to Z_1^{-1/2} A_\mu$
$\psi \to Z_2^{-1/2} \psi$
$m_{phys} = m\left[1 + c\frac{\alpha}{4\pi} + \dots\right] = m + \delta m = m_{phys} - \delta m$
$e_{phys} = e\left[1 + c'\frac{\alpha}{4\pi} + \dots\right] = e + \delta e = Z_1 Z_2^{-1} Z_3^{-1/2}$
$Z_1 = 1 + c'\frac{\alpha}{4\pi} \dots$
$A^{B}_{\mu} = Z_3^{1/2} A^{R}_{\mu}$
$\psi^{B} = Z_2^{1/2} \psi^{R}$
$Z_1 = 1 + c_1'\frac{\alpha}{4\pi} \dots$
$Z_2 = 1 + c_2'\frac{\alpha}{4\pi} \dots$
$\delta m + (1-Z_2)\not{p}$
$(Z_3 - 1)(p^2 \eta_{\mu\nu} - p_\mu p_\nu)$
$D=4$
$D=3$
$D=2$
$D=1$
$D=0$
$D=1$
$D=0$
CAUTION


Vertex correction
1PI
$e\gamma_m$
Know(1) a graph with L loops
contains a factor $\left(\frac{e^2}{16\pi^2}\right)^L$
(2) E_γ, E_e $D = 4 - \frac{3}{2}E_e - E$
$p^2+m=0$
$E_e=2$ $E_\gamma=1$ $D=0$
the physical charge
$D=0 \rightarrow \log\Lambda$
$C(p,p',1)=\int^\Lambda d^4k\, f(k,p,p') = \text{const}$
divergent
UV finite,


Vertex correction
1PI
eγm
Know(1) a graph with L loops
factor $\left(\frac{e^2}{16\pi^2}\right)^L$
(2) $D = 4 - \frac{3}{2}E_e - E_\gamma$
$\Sigma(p^2+m^2)=0$
$p^2+m^2=0$
$\frac{-i}{(2\pi)^4}\frac{-ip+m}{p^2+m^2}$
$D=0$ for us @ 1^D
$D=0 \rightarrow \log\Lambda$
the physical charge
+ p-dependent but UV finite.
CAUTION