Title: Holographic cameras: an eye for the bulk

Speakers: Simon Caron-Huot

Series: Quantum Fields and Strings

Date: December 13, 2022 - 2:00 PM

URL: https://pirsa.org/22120013

Abstract: We consider four-point correlators in an arbitrary excited state of a quantum field theory. We show that when the theory and state are holographic, such correlators can produce high-quality movies of point-like bulk particles, revealing the geometry in which they move. In some situations, Einstein's equations amount to a local differential equation on the correlator data. In theories or states that are not holographic, images are too blurry to extract a bulk geometry. Calculations are performed by adapting formulas from conformal Regge theory, to excited states and out-of-time-order correlators.

Zoom link: https://pitp.zoom.us/j/94153545930?pwd=YUFsUW44S1Z5Ri83a2xQdEN0Vk9XZz09

Holographic cameras: an eye for the bulk

Simon Caron-Huot
McGill University

Seminar at Perimeter Institute, December 13, 2022 strongly interacting QFT ⟷ gravity

finite temperature ⟷ black hole

? ⟵ geometry

strongly interacting QFT ⟷ gravity

finite temperature ⟷ black hole

? ⟵ geometry

A: $\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle_\Psi$

+ Simply related to $g_{ab}(r, x)$, but *always defined*
+ All orders in $\langle T^{\mu\nu} \rangle$
+ Easily detects small-scale features (*sub-AdS)*
+ Constrained non-perturbatively

Outline

1. Holographic cameras: Fun with four points
   - bulk reconstruction

2. Pictures & movies in geometrical optics approximation

3. Corrections: why *large-N large-gap* is crucial
   - conformal Regge theory

1. cannon

(familiar)

Start with $|\Psi\rangle$ = arbitrary state of boundary QFT

Excite with high-frequency operator $\mathcal{O}$:

$$|\Psi\rangle \mapsto \int d^d x \, \psi_{p,L}(x) \; \mathcal{O}(x) |\Psi\rangle$$

$$\psi_{p,L}(x) \sim e^{i\omega t - i\vec{p}\cdot\vec{x} - \frac{|x|^2}{2L^2}}$$

plane wave with
Gaussian envelope

Today: $\mathcal{O}$ = simple $(T^{\mu\nu}, J^{\mu}, \text{light scalar}\ldots)$

PI
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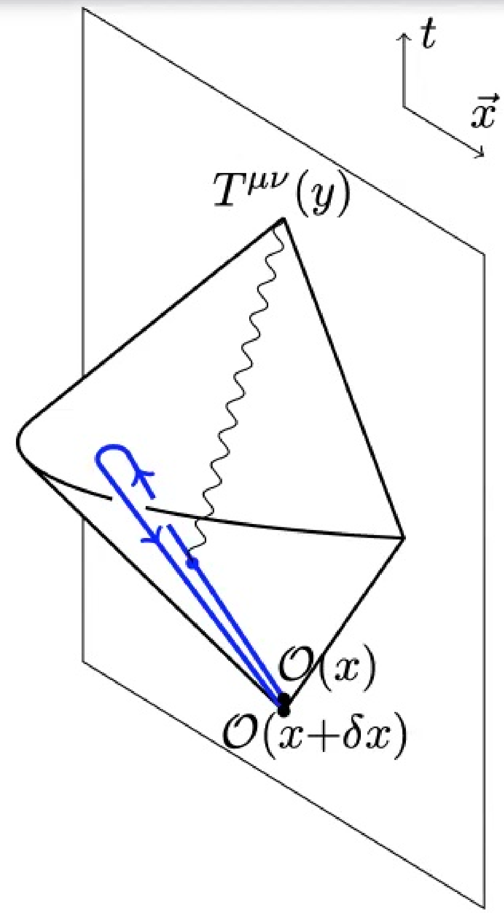
Rechercher

How much energy is in $\mathcal{O}\,|\,\Psi\rangle$?

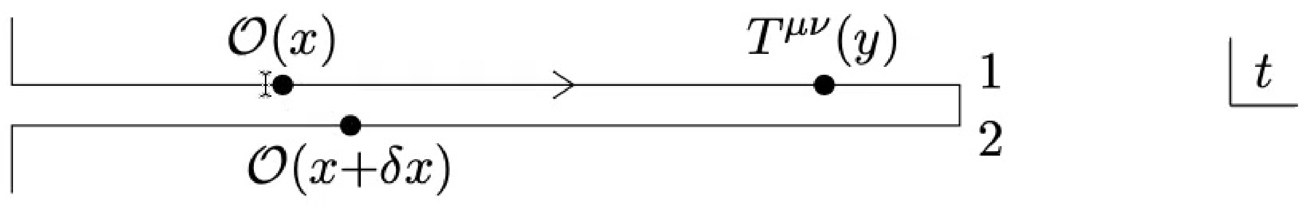
$$\int_{\delta x} \psi_{p,L}(\delta x)\ \langle\Psi|\ \mathcal{O}(x+\delta x)\ T^{\mu\nu}(y)\ \mathcal{O}(x)\ |\Psi\rangle$$

- In practice, single δx integral suffices
- Not T-ordered!



cf: [Hoffman& Maldacena '08]
[Arnold, Vaman '11]

(OTO can be computed on standard Schwinger-Keldysh timefold:



$\delta x \to 0$ singularity is caused by energetic excitations going around)

How much energy is in $\mathcal{O}\,|\Psi\rangle$?

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- In practice, single δx integral suffices
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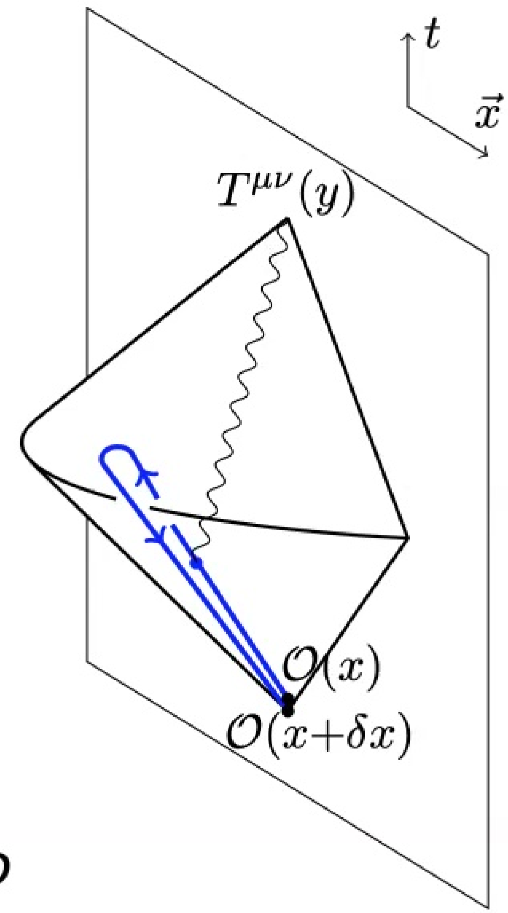
- If theory holographic & p is timelike: $\mathcal{O}$ follows bulk null geodesic
- Initial conditions accurately controlled by p
- $\langle T^{\mu\nu}(y)\rangle$ ~ expanding shell of energy



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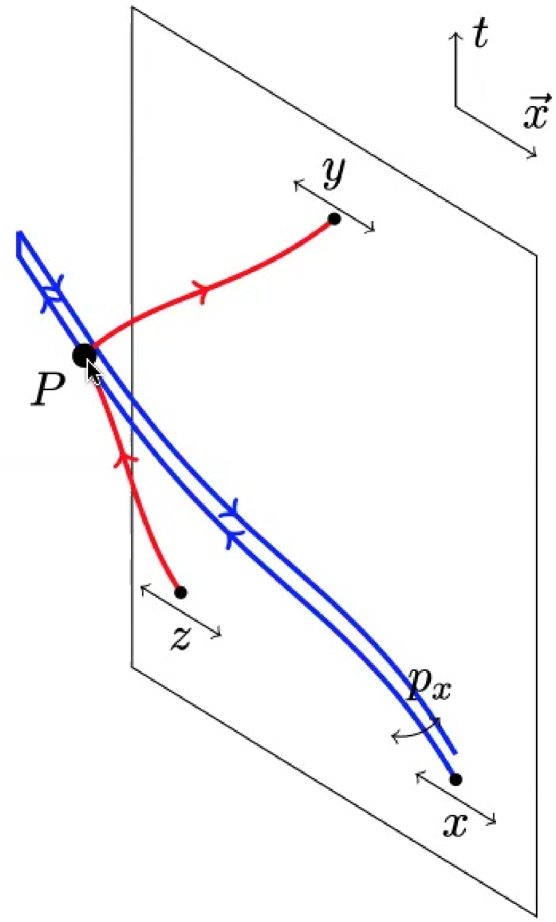
The cannon creates a point particle in the bulk,
but $\langle T^{\mu\nu}(y) \rangle$ is too crappy a camera to reveal it.

I will now present two variants of *holographic cameras:*
radar and active,
which aim to observe bulk physics more finely.

a. Radar camera:

$$\int_{\delta x} \psi_{p,L}(\delta x)\, \langle \Psi | \mathcal{O}(x+\delta x)\ \mathcal{O}'(y)\mathcal{O}'(z)\ \mathcal{O}(x) | \Psi \rangle$$

- Send pulse from z, record reflection off P at y



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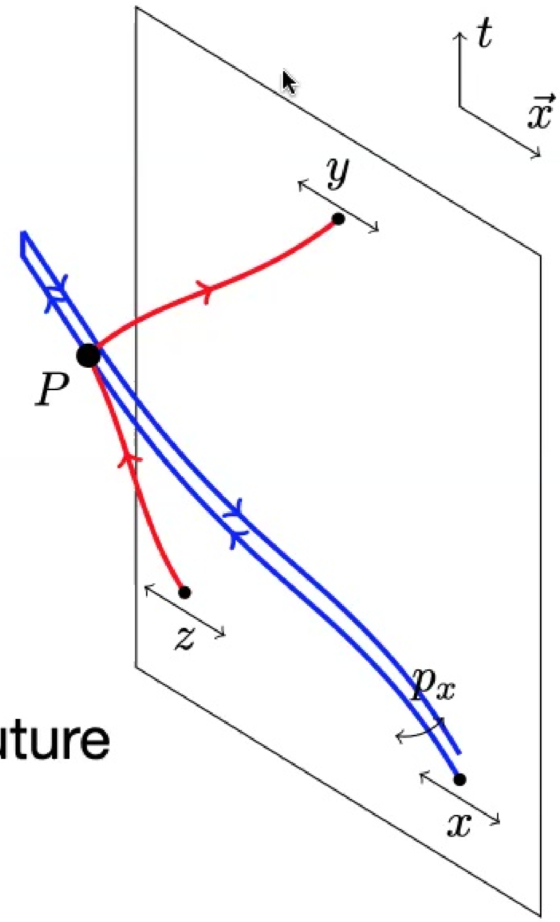


- Send pulse from z, record reflection off P at y

- Ideal regime: three null geodesics
- Signal = singularity as y → lightcone of P
- Similar to 'bulk point' *except* don't track $\mathcal{O}$'s future

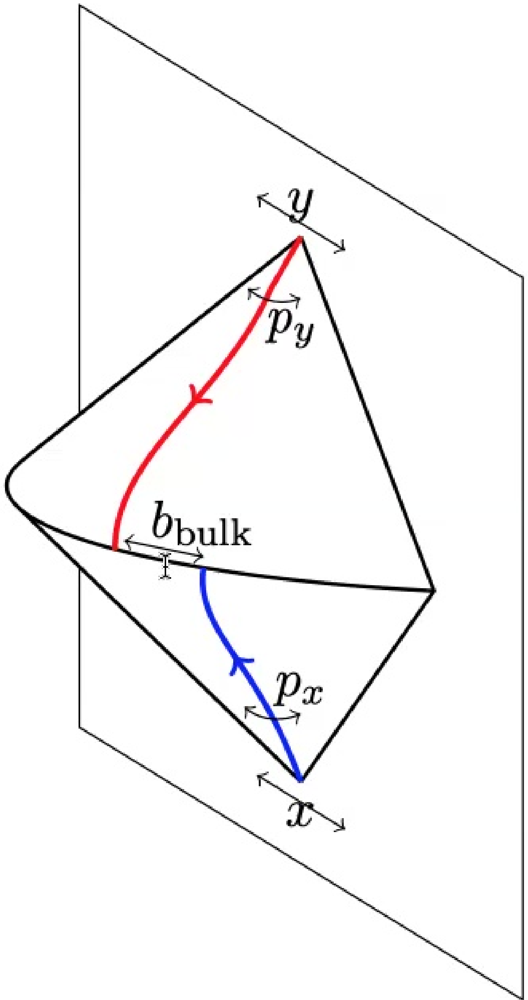
[Maldacena,Simmons-Duffin&Zhiboedov'15]

b. Active camera:

$$\int_{\delta x} \psi_{p_x, L_x}(\delta x) \int_{\delta y} \psi_{p_y, L_y}(\delta y)$$

$$\langle \Psi | \mathcal{O}'(y + \delta y)\ \mathcal{O}(x + \delta x)\ \mathcal{O}'(y)\ \mathcal{O}(x) | \Psi \rangle$$

- OTOC with high energy, early times
- Ideal regime: two null geodesics



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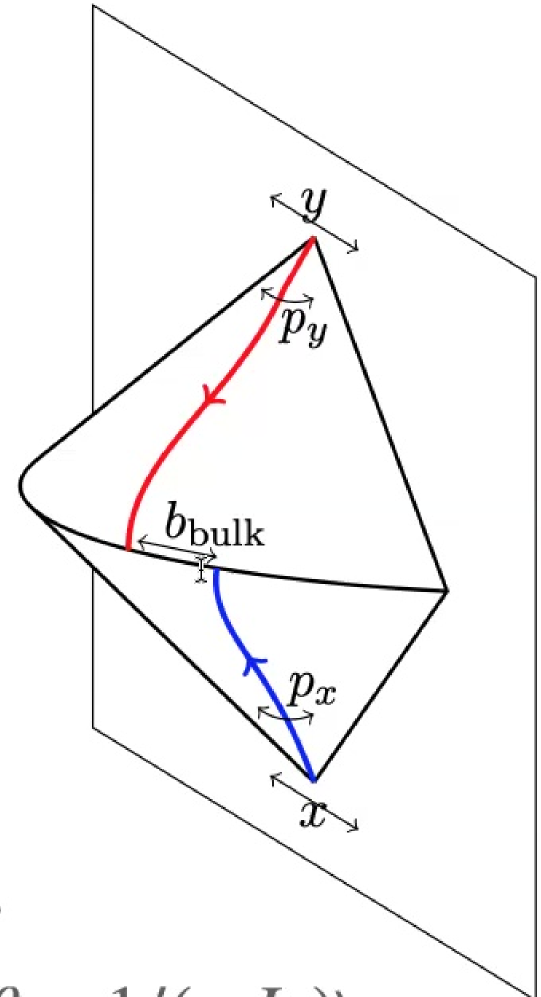


- OTOC with high energy, early times
- Ideal regime: two null geodesics

- Signal = singularity as $b_{\text{bulk}} \to 0$
- Knobs: x^μ, y^μ = spacetime shooting points

  p_x^μ, p_y^μ = shooting directions+energies

  L_x, L_y = Gaussian widths (optics: $\delta\theta \sim 1/(p_x L_x)$)

Don't we already know how to reconstruct bulk?

Fefferman-Graham: $g_{ab}(r,x) \xleftarrow{\text{Einstein}} \langle T^{\mu\nu}(y)\rangle_\Psi$

HKLL: $\phi(r,x) = \int_\Diamond d^d y\; K_\Psi(x,r\,|\,y)\;\langle \mathcal{O}(y)\rangle_\Psi$

[Hamilton,Kabat,Lifschytz& Lowe '06]

…

heavy two-point functions → geodesic lengths] bulk geometry
entanglement entropy → areas of surfaces]

…

1. ‘Linear inverses' need exponential accuracy to detect local features:

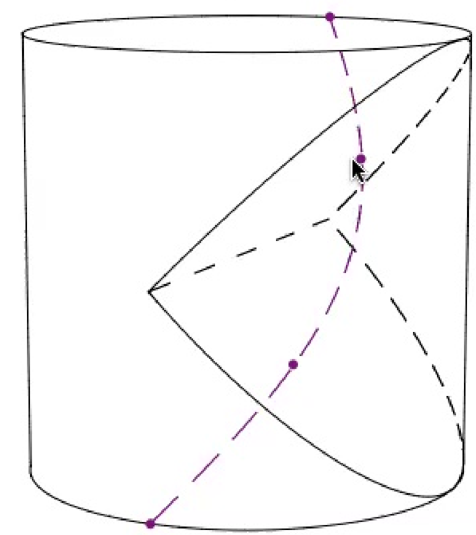
$$K_{\mathrm{HKLL}}(x, r \,|\, y) \propto \int d\nu \; K_\nu(x, r \,|\, y)$$

AdS momentum $\sim e^{+\pi\nu}$

[HKLL ’06: AdS-Rindler kernel]

[Bousso, Freivogel, Leichenauer, Rosenhaus, Zukowski ’12]

obvious why: one-particle states can have exponential small wavefunction near boundary.



2. Reconstructions can ‘work’ even when nonsensical

(ex: bulk metric from 3D Ising $\langle T^{\mu\nu} \rangle$??)

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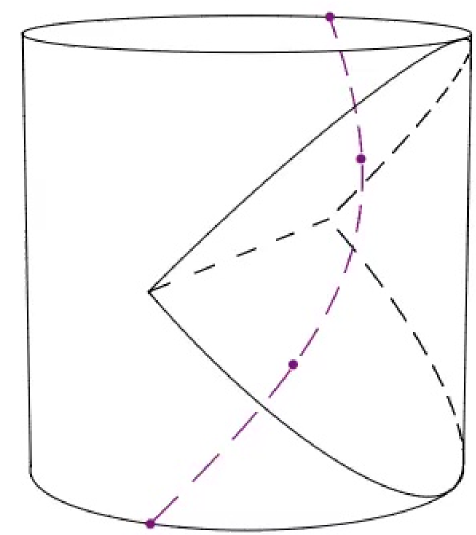

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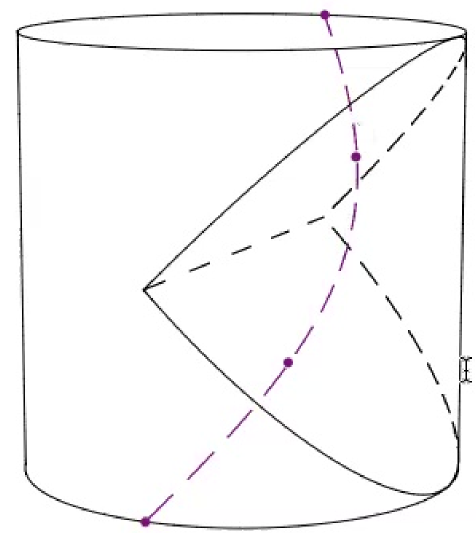

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(ex: bulk metric from 3D Ising $\langle T^{\mu\nu} \rangle$??)

interactions:
+ signal is never exponentially small
+ closer to how we actually observe particles
+ obvious from data if metric ill-defined (blurry images)

1. 'Linear inverses' need exponential accuracy to detect local features:

$$K_{\mathrm{HKLL}}(x, r\,|\,y) \propto \int d\nu \; K_\nu(x, r\,|\,y)$$

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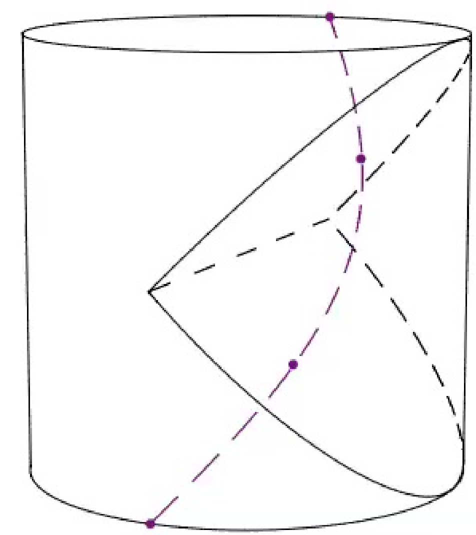

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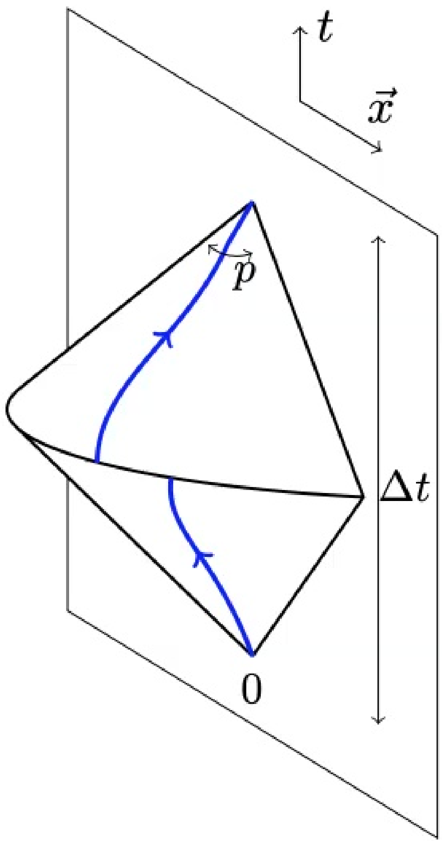
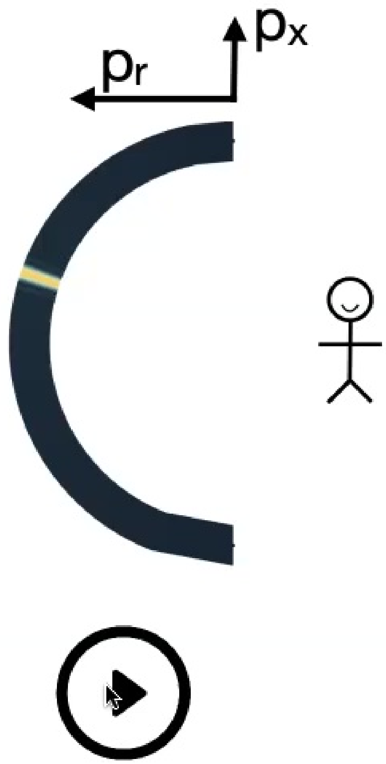
Vacuum AdS_3/CFT_2

$$p_r = \sqrt{p_0^2 - p_x^2}$$

($p_r L \sim 50$)





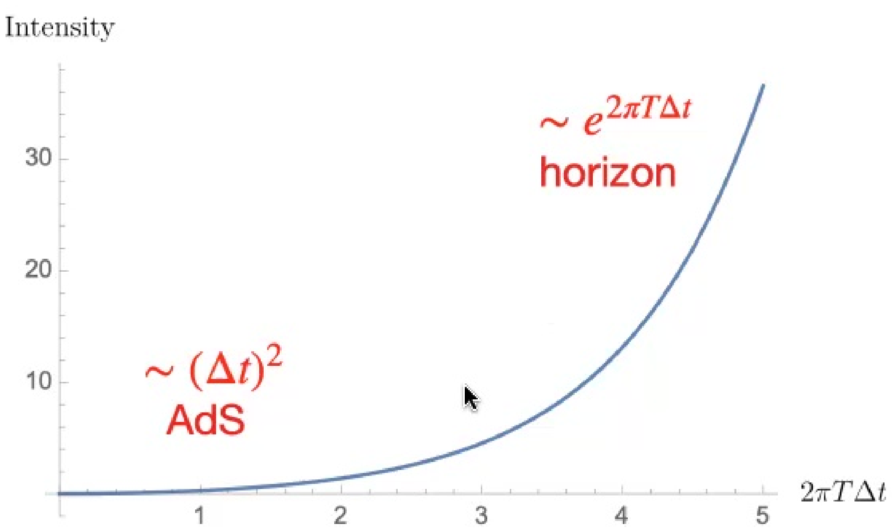
Thermal state in CFT$_2$

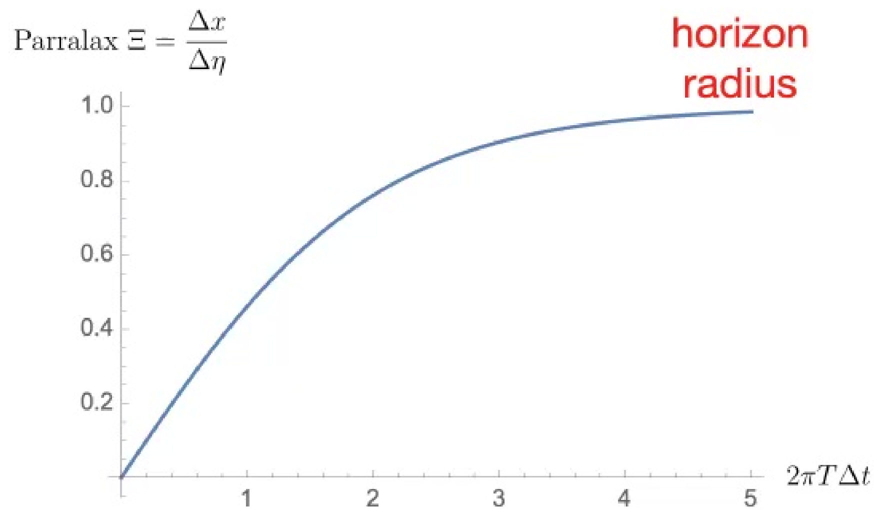
$$\sqrt{z\bar{z}} \to \frac{\pi^2 T_R T_L \;\delta x \delta y}{\sinh(\pi T_R(\Delta t - y)\sinh(\pi T_L(\Delta t + y))}, \quad \sqrt{\frac{z}{\bar{z}}} \to e^{-\varphi_x - \varphi_y} \frac{T_R \sinh(\pi T_L(\Delta t + y))}{T_L \sinh(\pi T_R(\Delta t - y))}.$$

Thermal state in CFT$_2$

$$\sqrt{z\bar{z}} \to \frac{\pi^2 T_R T_L \ \delta x \delta y}{\sinh(\pi T_R(\Delta t - y) \sinh(\pi T_L(\Delta t + y))}, \quad \sqrt{\frac{z}{\bar{z}}} \to e^{-\varphi_x - \varphi_y} \frac{T_R \sinh(\pi T_L(\Delta t + y))}{T_L \sinh(\pi T_R(\Delta t - y))}.$$

BTZ black hole (AdS_3/CFT_2)





Bulk metric from 4pt function

Metric ansatz: $$ds^2 = \frac{R_{\text{AdS}}^2}{r^2}\left(-A(r)^2 dt^2 + \frac{dr^2}{B(r)} + dx^2\right)$$

OTOC data $\Rightarrow$ geometrical optics

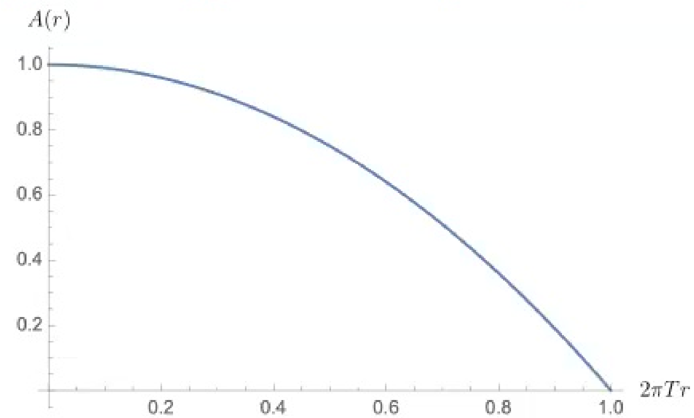
$$r(\Delta t) \propto \frac{I(\Delta t)}{\Xi(\Delta t)^{3-d}} \frac{d\Xi(\Delta t)}{d\Delta t}$$

$$A(\Delta t)/r(\Delta t) = \Xi(\Delta t)/I(\Delta t)$$

$$\sqrt{A(r)B(r)} = 2\frac{dr}{d\Delta t}$$

$\Rightarrow$ BTZ black hole metric!

$$A(r) = B(r) = 1 - (2\pi T r)^2$$



$$ds^2 = \frac{R_{\mathrm{AdS}}^2}{r^2}\left(-A(r)^2 dt^2 + \frac{dr^2}{B(r)} + dx^2\right)$$

For thermal states in CFT$_d$, the OTOC data $I(\Delta t), \Xi(\Delta t)$ is quite literally the bulk metric.

$$r(\Delta t) \propto \frac{I(\Delta t)}{\Xi(\Delta t)^{3-d}} \frac{d\Xi(\Delta t)}{d\Delta t}$$

$$A(\Delta t)/r(\Delta t) = \Xi(\Delta t)/I(\Delta t)$$

$$\sqrt{A(r)B(r)} = 2\frac{dr}{d\Delta t}$$

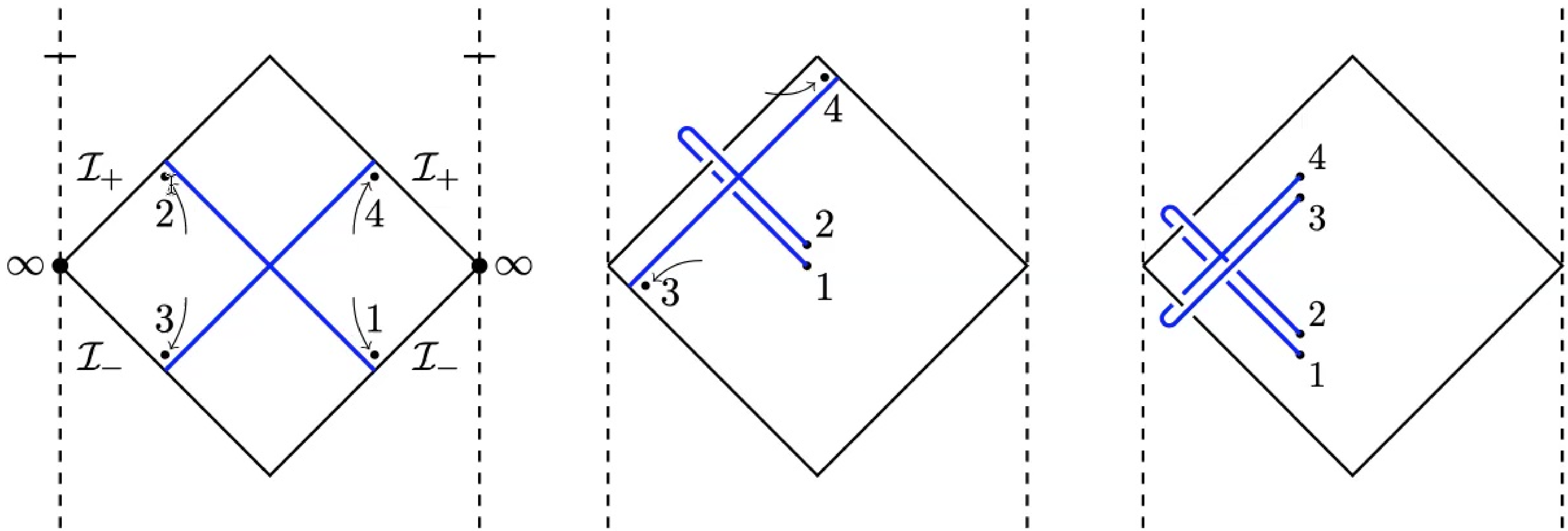
Einstein's equations can be formulated directly on OTOC:

$$\Xi(\Delta t)^{-d}\left(2\frac{d\Xi(\Delta t)}{d\Delta t} - 1\right) = \text{Constant}$$

$$I(\Delta t)\Xi(\Delta t)^{d-4}\,\frac{d\Xi(\Delta t)}{dt} = \text{Constant}$$

Away from pure GR, bootstrapping the OTOC may be more fruitful than 'constructing metric' ?

In vacuum, high-energy OTOC = Regge correlator = detector



(a) Regge correlator (b) Detectors (c) OTOC

[going past Poincaré patches: Kravchuk& Simmons-Duffin '18]

Conformal Regge theory

$$\mathcal{G}(z,\bar{z})^{\circlearrowleft} - 1 \approx 2\pi i \int_0^\infty \frac{d\nu}{2\pi} \rho(\nu)\alpha(\nu)\ (z\bar{z})^{\frac{1-J(\nu)}{2}} \mathcal{P}_{\frac{2-d}{2}+i\nu}\left(\frac{z+\bar{z}}{2\sqrt{z\bar{z}}}\right)$$

J=2 for graviton

$\frac{\delta x \cdot I \cdot \delta y}{|\delta x||\delta y|} \sim b_{bulk}$

$\alpha \sim \frac{G_N^{(d+1)}}{\nu^2 + \frac{d^2}{4}} \Gamma(\text{double-traces})$

Crucial fact: $\alpha \lesssim e^{-\pi|\nu|}$. (Known in examples.)
Integral commutes with Fourier transform.

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J=2 for graviton

$\frac{\delta x \cdot I \cdot \delta y}{|\delta x||\delta y|}$ ~ b_{bulk}

$\alpha \sim \frac{G_N^{(d+1)}}{\nu^2+\frac{d^2}{4}}\Gamma$(double-traces)

Crucial fact: $\alpha \lesssim e^{-\pi|\nu|}$. (Known in examples.)
Integral commutes with Fourier transform.

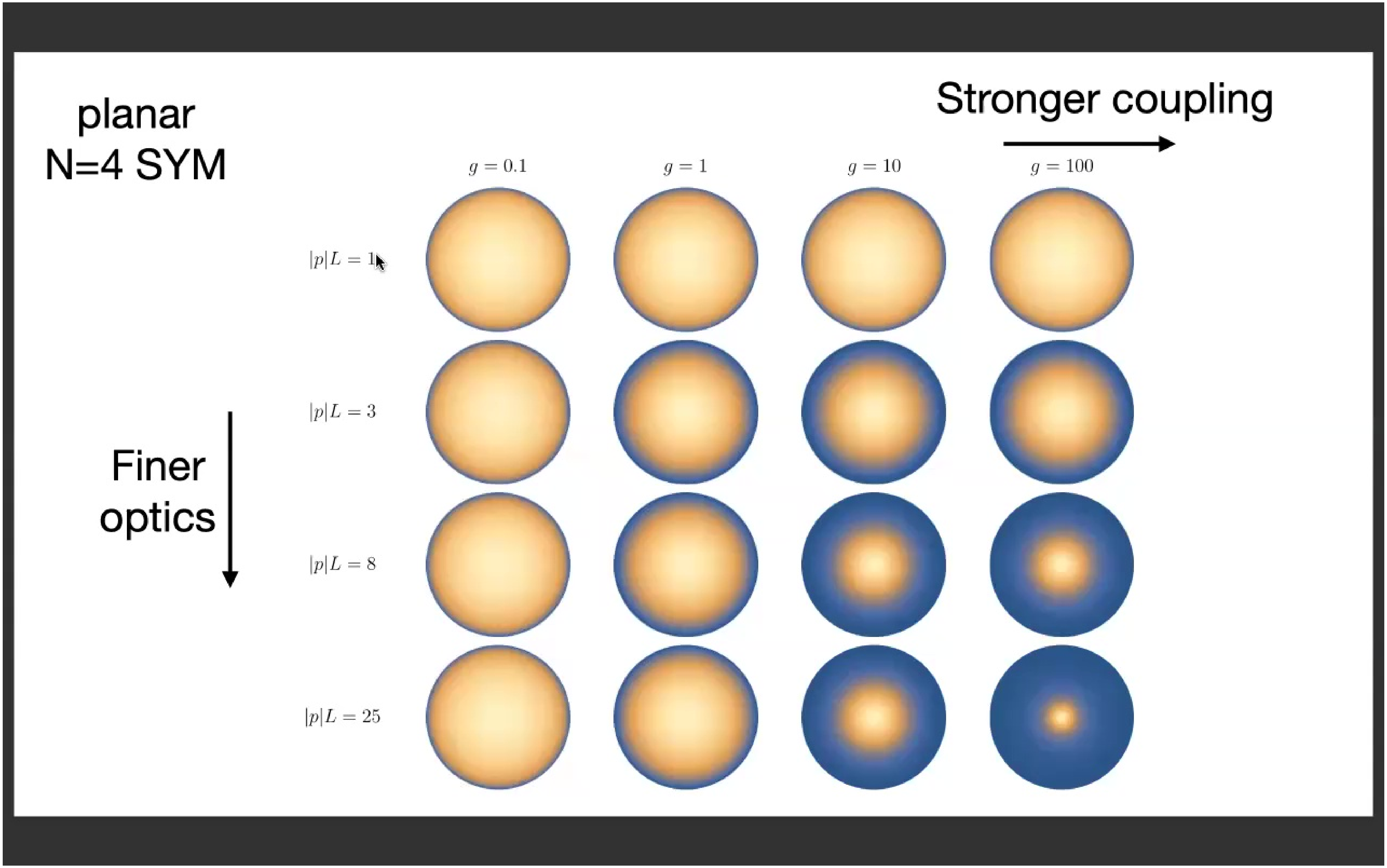
$$G(0,p,L;e,p',L')\big|_{\text{CRT}} \approx 1 - i\int_0^\infty \frac{d\nu}{2\pi}\rho(\nu)\tilde{\alpha}(\nu)\ \big(|p||p'|\big)^{J(\nu)-1}\mathcal{P}_{\frac{d-2}{2}+i\nu}(\hat{p}\cdot\mathcal{I}_e\cdot\hat{p}')e^{-\frac{\nu^2}{2}\sigma_{\text{eff}}^2}$$

$\tilde{\alpha} = \frac{4\pi G_N^{(d+1)}}{\nu^2+\frac{d^2}{4}}$

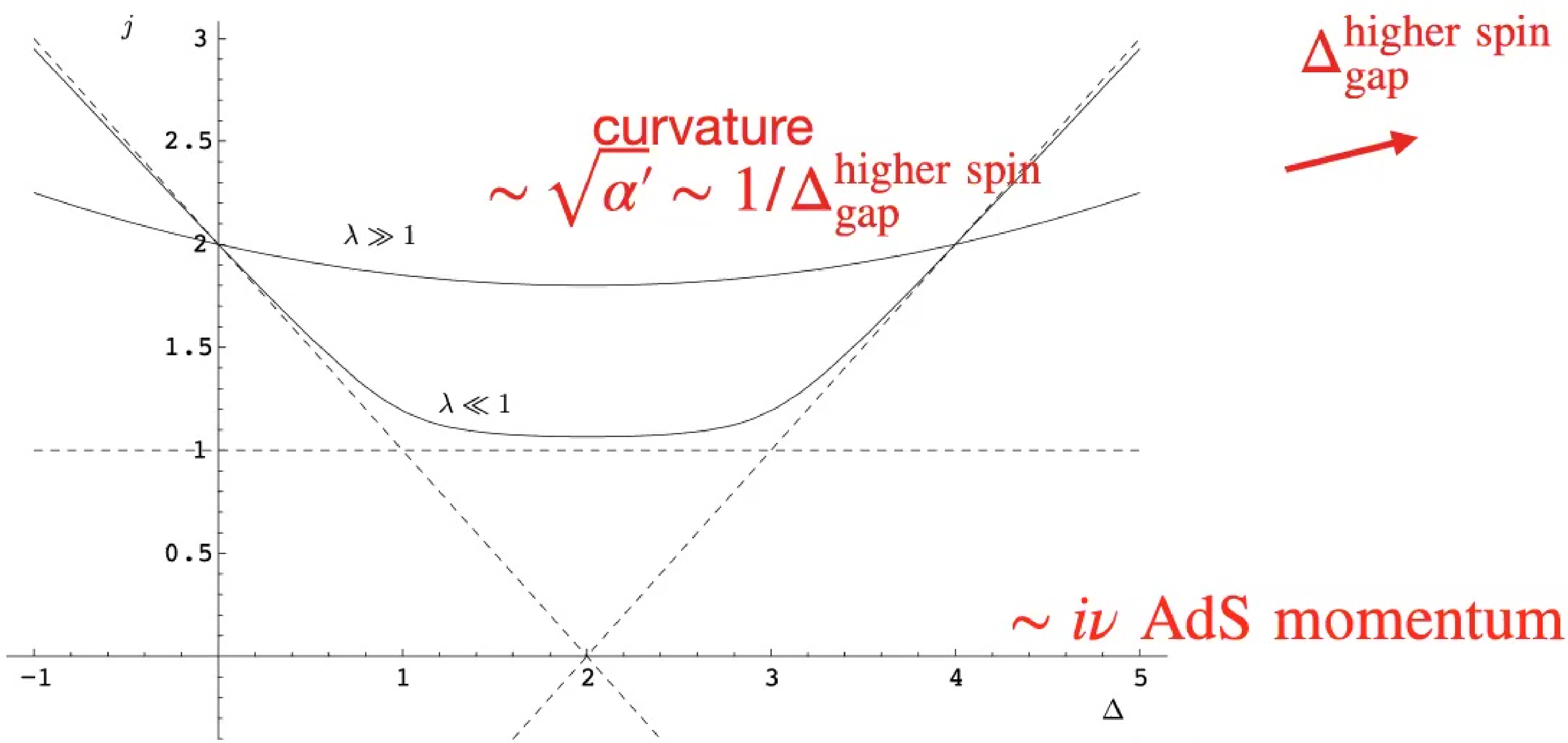
~flat space amplitude.
easy to add stringy effects.

$\sigma_{\text{eff}} \sim \frac{1}{|p|L}$

optical limit on angular resolution

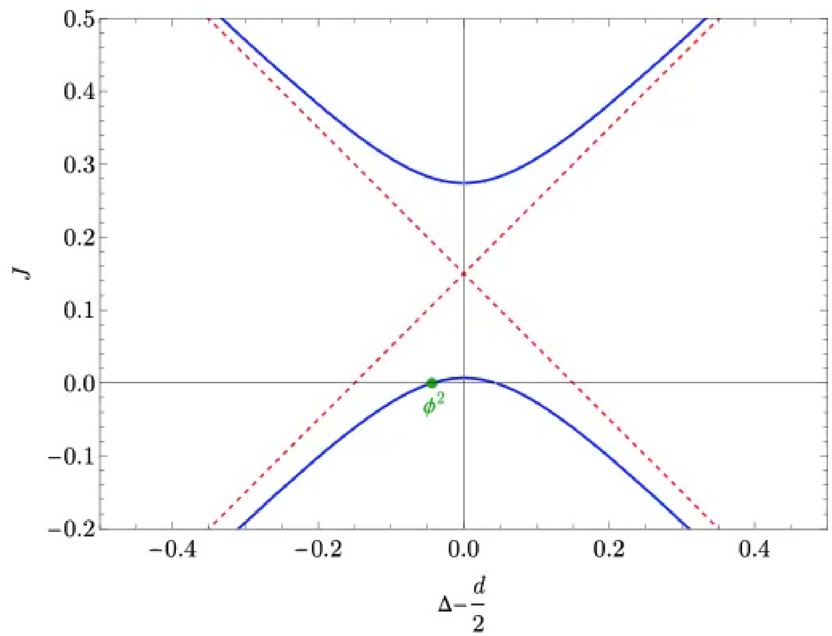

planar
N=4 SYM
Stronger coupling
$g = 0.1$
$g = 1$
$g = 10$
$g = 100$
$|p|L = 1$
$|p|L = 3$
$|p|L = 8$
$|p|L = 25$
Finer
optics

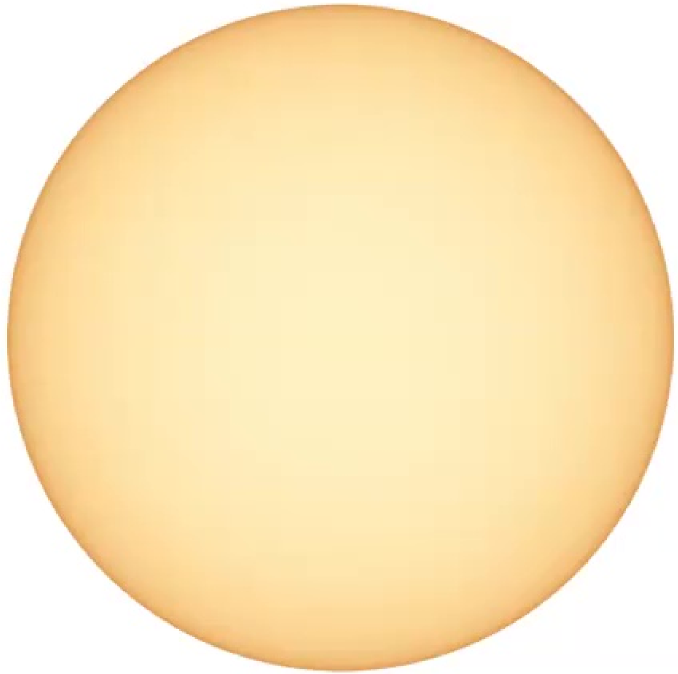
Large AdS momenta require flat single-trace Regge trajectory = large higher-spin gap. Can't probe scales shorter than $\sqrt{\alpha'}$.



[Brower,Polchinski,Strasser& Tan '06]

critical Wilson-Fisher theory / 3D Ising





Steep Regge trajectories prevent access to high momenta.
No hint of local bulk physics.

Summary

- Four-point correlators $\leftrightarrow$ *local* bulk metric.
- Works in arbitrary states, but images only sharp in holographic states & theories.

Many open questions…

- Other bulk fields?
- Trying it in more interesting geometries?
- Bootstrap:
  - why bulk EOMs locally the same everywhere? (equivalence principle)?
  - what to do at large*ish* N, λ ?
- Flat space version: image classical geometries?

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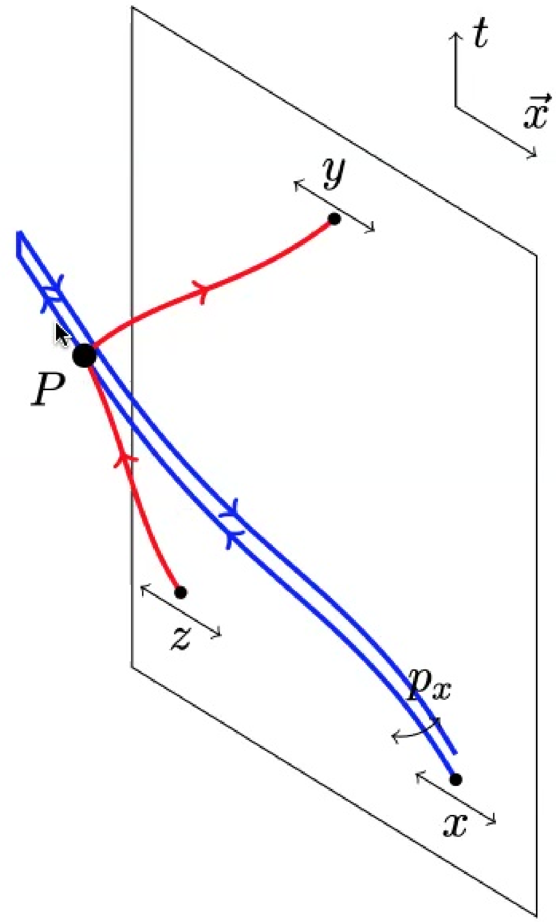

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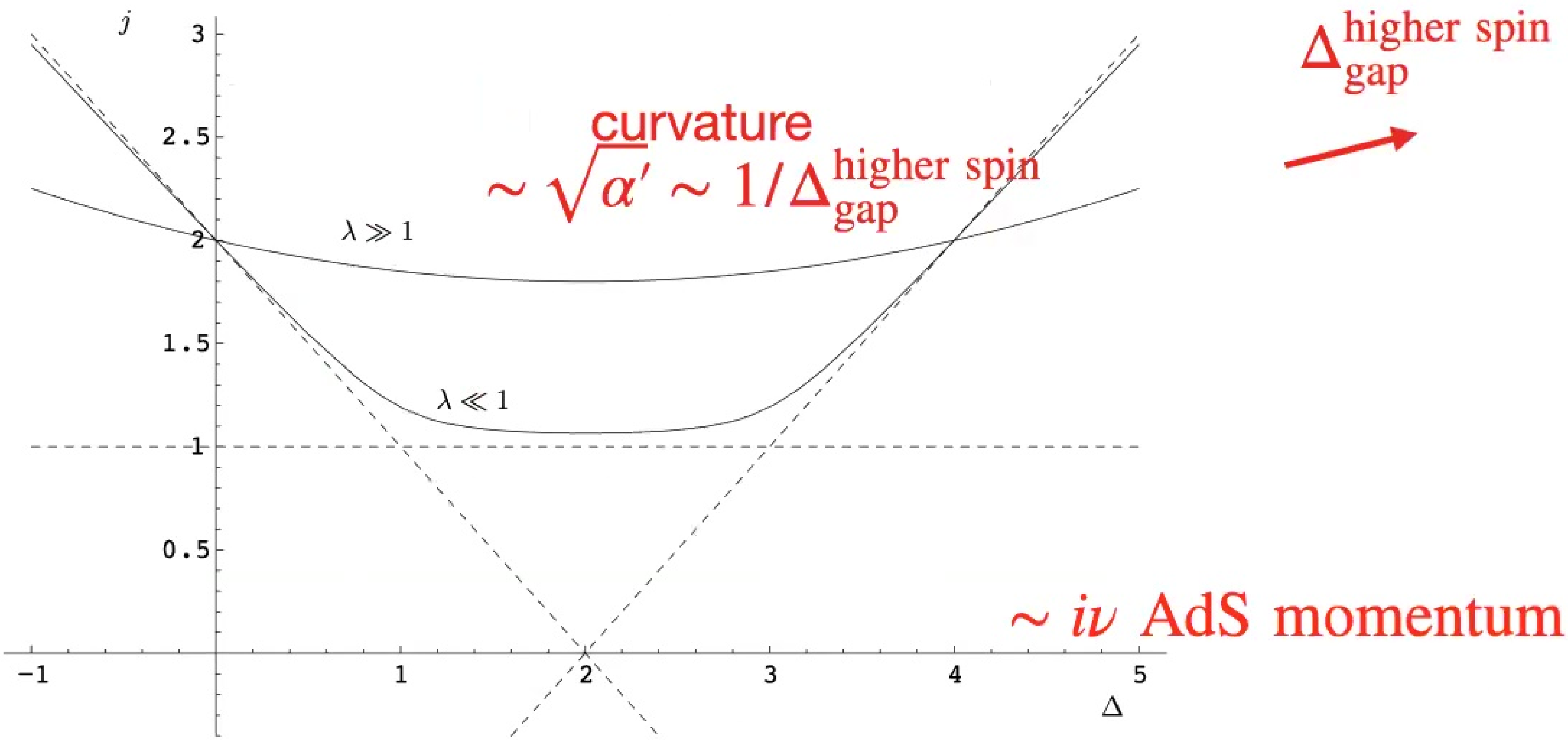

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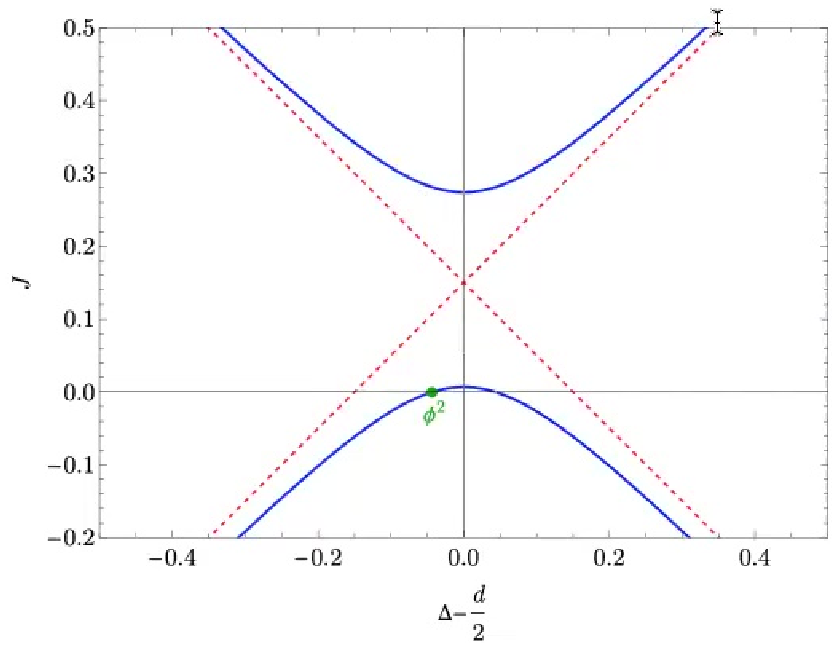
critical Wilson-Fisher theory / 3D Ising



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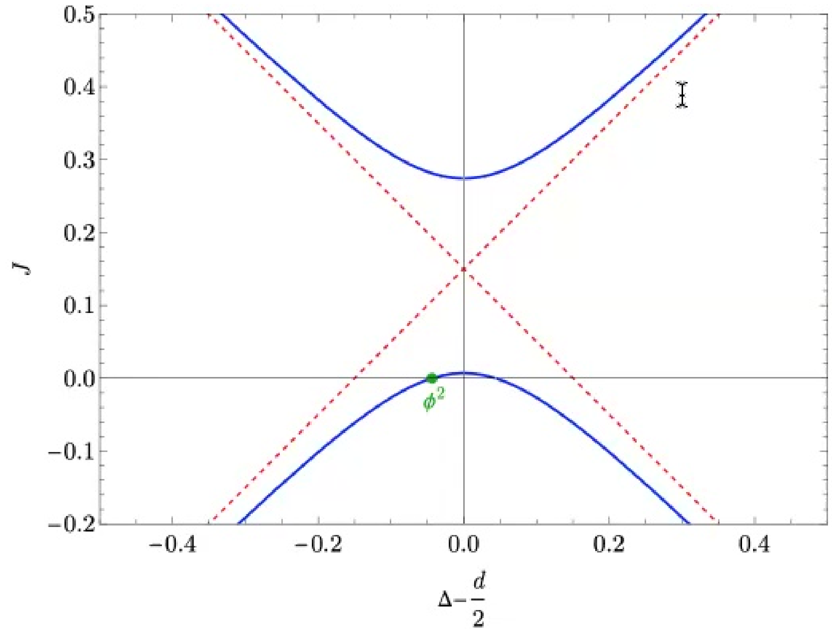

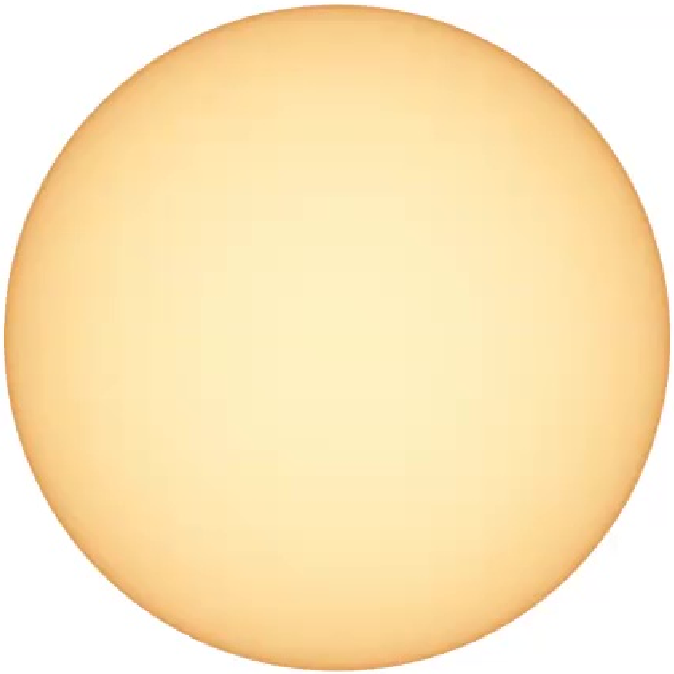

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