Title: w(1+infinity) in 4D Gravity

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Abstract: In this talk, I will show that the action of soft graviton operators generates a w(1+infinity) symmetry in gravitational theories minimally coupled to massless and massive scalar matter in 4D asymptotically flat spacetimes. I will discuss how the symmetry action follows from an infinite tower of soft graviton theorems in momentum space. By generalizing the previous analyses of w(1+infinity) symmetry in massless amplitudes, these results clarify that the symmetry emerges in 4D asymptotically flat gravitational theories without any additional technical ingredients. This talk is based on a forthcoming paper with Monica Pate.

Zoom link: https://pitp.zoom.us/j/91448920819?pwd=cFY0L1JPVFF2bDUwc3JiVlRlbE5ldz09

$W_{1+\infty}$ IN 4D GRAVITY

OUTLINE

1. REVIEW
2. INFINITE SOFT THEOREMS
3. MASSLESS $W_{1+\infty}$
4. MASSIVE $W_{1+\infty}$

MASSLESS PARTICLES.

$$|p_k^\mu\rangle \rightarrow |\omega_k, s_k, z_k, \bar{z}_k\rangle$$

$$p_k^\mu = \omega_k \hat{q}^\mu(z_k, \bar{z}_k)$$

$$= \omega_k (1 + z_k \bar{z}_k, \bar{z}$$

$_k, z_k, \bar{z}_k \rangle$

$$p^\mu_k = \omega_k \hat{q}^\mu(z_k, \bar{z}_k)$$

$$= \omega_k \left(1 + z_k \bar{z}_k, \ \bar{z}_k + \bar{z}_k, -i(z_k - \bar{z}_k), 1 - z_k \bar{z}_k\right)$$

z —

$\ldots_k, \bar{z}_k)$

$\ldots_k \bar{z}_k,\ \bar{z}_k + \bar{z}_k,\ -i(z_k - \bar{z}_k),\ 1 - z_k \bar{z}_k)$

$$z \to z' = \frac{az+b}{cz+d} \qquad ad - bc = 1$$

$$|p_k^\mu\rangle \rightarrow |\omega_k, s_k, z_k, \bar{z}_k\rangle$$

$$p_k^\mu = \omega_k \hat{q}^\mu(z_k, \bar{z}_k)$$

$$= \omega_k(1 + z_k\bar{z}_k,$$

$$p^\mu(\omega_k, z_k \bar{z}_k) \rightarrow p^\mu(\omega_k', z_k', \bar{z}_k') = \Lambda^\mu{}_\nu\, p_k^\nu(\omega_k, z_k, \bar{z}_k)$$

$$\hat{q}^\mu(z, \bar{z}) \longrightarrow \frac{1}{(cz+d)(\bar{c}\bar{z}+\bar{d})} \Lambda^\mu{}_\nu\, \hat{q}^\nu(z, \bar{z})$$

$$\omega_k \longrightarrow (cz_k + d)(\bar{c}\bar{z}_k + \bar{d})\,\omega_k$$

$(z_k, \bar{z}_k)$

$$z \longrightarrow z' = \frac{az+b}{cz+d} \qquad ad-bc=1$$

$$+ z_k\bar{z}_k,\ \bar{z}_k + \bar{z}_k,\ -i(z_k - \bar{z}_k),\ 1 - z_k\bar{z}_k)$$

BOOST EIGENSTATE MELLIN TRANSFORM

$$\int_0^\infty \frac{d\omega_k}{\omega_k}\, \omega_k^{\Delta_k} \,|\omega_k, s_k, z_k, \bar{z}_k\rangle \equiv |\Delta_k, s_k, z_k, \bar{z}_k\rangle$$

POLARIZATION

$$\varepsilon^{\mu}_{k,+} = \frac{1}{\sqrt{2}} \frac{1}{(z_0 - z_k)} \hat{q}^{\mu}(z_0, \bar{z}_k)$$

$$\varepsilon^{\mu}_{k,-} = \frac{1}{\sqrt{2}} \frac{1}{(\bar{z}_0 - \bar{z}_k)} \hat{q}^{\mu}(z_k, \bar{z}_0)$$

$$\mathcal{O}_{\Delta_k, s_k}(z_k, \bar{z}_k) \to (c z_k + d)^{\Delta_k + s_k} (\bar{c} \bar{z}_k + \bar{d})^{\Delta_k - s_k} \, \mathcal{O}_{\Delta_k, s_k}(z_k, \bar{z}_k)$$

$$(h_k, \bar{h}_k) = \left(\frac{\Delta_k + s_k}{2}, \frac{\Delta_k - s_k}{2} \right)$$

$$\mathcal{A} \xleftrightarrow{\text{MELLIN}} \langle \mathcal{O} \ldots \rangle$$

$$\varepsilon^{\mu}_{k,+} = \frac{1}{\sqrt{2}} \frac{1}{(z_0 - z_k)} \hat{q}^{\mu}(z_0, \bar{z}_k)$$

$$\varepsilon^{\mu}_{k,-} = \frac{1}{\sqrt{2}} \frac{1}{(\bar{z}_0 - \bar{z}_k)} \hat{q}^{\mu}(z_k, \bar{z}_0)$$

$$\varepsilon^{\mu}_{+}(z, \bar{z}, z_0) \longrightarrow \varepsilon'^{\mu}_{+}(z', \bar{z}'; z'_0) = \frac{(cz+d)}{(\bar{c}\bar{z}+\bar{d})} \Lambda^{\mu}{}_{\nu}\, \varepsilon^{\nu}_{+}(z, \bar{z}; z_0)$$

MASSIVE PARTICLES

$|p_k^\mu\rangle \qquad |m_k, \underbrace{y_k, w_k, \bar{w}_k}_{AdS_3}\rangle \qquad p_k^\mu = m_k \hat{p}_k^\mu \qquad \hat{p}_k^2 = -1$

$$\hat{p}_k^\mu = \frac{1}{2y_k}\big(y_k^2 \underset{\substack{\downarrow \\ n^\mu}}{n^\mu}$$

$$|m_k, \underbrace{y_k, w_k, \bar{w}_k}_{AdS_3}\rangle \qquad p_k^\mu = m_k \hat{p}_k^\mu \qquad \hat{p}_k^2 = -1$$

$$\hat{p}_k^\mu = \frac{1}{2y_k}\left(y_k^2 n^\mu + \hat{q}^\mu(w_k, \bar{w}_k)\right)$$

$$\downarrow$$

$$n^\mu = \partial_z \partial_{\bar{z}} \hat{q}(z, \bar{z})$$

$$N \quad A(\omega\hat{q}\,; p_1 \cdots ; p_n) = \frac{\kappa}{2} \sum_{\ell=-1}^{\infty} \omega^{\ell} A^{(\ell)}(\hat{q}\,; p_1, \cdots, p_n)$$

$$\cdots) = \lim_{\omega\to 0} \frac{\partial_\omega^{\ell+1}\left(\omega A(\omega\hat{q})\right)}{(\ell+1)!} = \underline{\lim_{\varepsilon\to 0} \varepsilon \int_0^\infty \frac{d\omega}{\omega}\, \omega^{-\ell+\varepsilon} A(\omega\hat{q}, \cdots)}$$

$$(h,\bar{h}) = \left(-\frac{\ell+2}{2}, -\frac{\ell-2}{2}\right) \quad H_{-\ell}(z,\bar{z}) \equiv \lim_{\varepsilon\to 0} \varepsilon\, G^{+}_{1-\ell+\varepsilon}(z,\bar{z})$$

INFINITE TOWER OF SOFT THMS

LEADING $A^{(-1)} = \sum_{k=1}^{n} S_k^{''(-1)} A(p_1, \ldots, p_n)$ $\qquad S_k^{''(-1)} =$

SUB LEADING $A^{(0)} = \sum_{k=1}^{n} S_k^{''(0)} A(\ldots)$ $\qquad S_k^{''(0)} =$

$$S_k^{\parallel(-1)} = \frac{\varepsilon^+_{\mu\nu}\, p^\mu p^\nu}{\hat{q}\cdot p_k}$$

$$S_k^{\parallel(0)} = \frac{\varepsilon^+_{\mu\nu}\, p_k \left(q_\sigma L_k^{\nu\sigma} \right)}{\hat{q}\cdot p_k}$$

$$L_{k\mu\nu} \propto p_{k\mu}\frac{\partial}{\partial p_k^\nu} - p_{k\nu}\frac{\partial}{\partial p_k^\mu}$$

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$$A^{(l>0)} = \sum_{k=1}^{n} S_k^{\prime\prime(l)} A(p_1, \ldots, p_n) + \overbrace{\varepsilon_{\mu\nu} N^{\mu\nu}(\hat{q}; p_1, \ldots, p_n)}^{\text{NON-UNIVERSAL}}$$

$$\downarrow$$

$$S_k^{\prime\prime(l)} = \frac{1}{(l+1)!} \frac{\varepsilon^+_{\mu\nu} (\hat{q}_\rho \mathcal{J}_k^{\mu\rho})(\hat{q}_\sigma \mathcal{J}_k^{\nu\sigma})}{\hat{q} \cdot p_k} \left(\hat{q} \cdot \frac{\partial}{\partial p_k}\right)^{l-1}$$

$$\hat q\cdot\frac{\partial}{\partial p_k}\to\frac{1}{(\varepsilon\cdot p_k)}\overbrace{\left(\varepsilon_+\cdot p_k\,\hat q\cdot\frac{\partial}{\partial p_k}-\hat q\cdot p\,\varepsilon_+\cdot\frac{\partial}{\partial p_k}\right)}^{=\varepsilon^\mu q^\nu \mathcal{L}_{\mu\nu}}$$

$$F_\pm^{\mu\nu}=\varepsilon_\pm^{(\mu}\hat q^{\nu)}$$

$$S_k^{\prime(\ell)}(z,\bar z)=\frac{(\varepsilon_+\cdot p)^2}{\hat q\cdot p}\frac{1}{(\ell+1)!}\left(\frac{F_+\cdot\mathcal{L}_k}{\varepsilon_+\cdot p_k}\right)^{\ell+1}$$

$$S_k^{\prime\ell}(z',\bar z')=(cz+d)^{-\ell+2}(\bar c\bar z+\bar d)^{-\ell-2}S^{\prime\ell}(z,\bar z)$$

MASSLESS WIth ∞

PRIMARY DESCENDANT $\underbrace{\partial_{\bar z}^{\ell+3} S_k^{1(\ell)}(z,\bar z)}_{\left(\frac{-\ell+2}{2},\frac{\ell+4}{2}\right)} \longrightarrow (cz+d)$

$$\underbrace{\partial_{\bar z}^{\,\ell+3} S_k^{1(\ell)}(z,\bar z)}_{\left(\frac{-\ell+2}{2},\,\frac{\ell+4}{2}\right)} \longrightarrow (cz+d)^{-\ell+2}(\bar c\bar z+\bar d)^{\ell+4}\,\partial_{\bar z}^{\,\ell+3} S_k^{1(\ell)}(z,\bar z)$$

PRIMARY DESCENDANT $\underline{\partial_{\bar{z}}^{\ell+3} \, S_k^{1(\ell)}(z,\bar{z})}$

$$p = \bar{h} = \frac{\ell+4}{2}$$

$$\left(\frac{-\ell+2}{2}, \frac{\ell+4}{2} \right)$$

$$\delta_m^p \, |p_k\rangle \;\propto\; \int d^2 z \; \bar{z}^{\,p+m-1} \, \partial_{\bar{z}}^{2p-1} \, S_k^1($$

$\frac{\ell+4}{2})$

$\partial_{\bar{z}}^{2p-1} \; S_k^{1(2p-4)}(z,\bar{z})\,|p_k\rangle$

$p = 1, \frac{3}{2}, 2, \frac{5}{2}$

$1-p \leq m \leq p-1$

$(\ell+1)!$

$\int_0$

$p = \bar{h} = \frac{l+4}{2}$

$\left(\frac{-l+2}{2}, \frac{l+4}{2}\right)$

$\delta^p_m |p_k\rangle \propto \int d^2z \; \bar{z}^{p+m-1} \partial_{\bar{z}}^{2p-1} S_k^{1(2p-4)}(z,\bar{z}) |p_k\rangle$

$p = 1, \frac{3}{2}, 2, \frac{5}{2}$

$$\boxed{\left[\delta^p_m, \delta^q_n\right] = \left[m(q-1) - n(p-1)\right] \delta^{p+q-2}_{m+n}}$$

$1 - p \le m \le p-1$

$(l+1)$

J_0

$(h,\bar{h}) = \left(-\frac{l+2}{2}, -\frac{l-2}{2}\right) \; H_{-l}(z,\bar{z}) \equiv \lim_{\epsilon \to 0} \epsilon \, G^+_{p-l+\epsilon}(z,\bar{z})$