Title: Statistical Physics - Lecture 221201

Speakers:

Collection: Statistical Physics (2022/2023)

Date: December 01, 2022 - 9:00 AM

URL: https://pirsa.org/22120006

Population Dynamics

- logistic growth
- 2-species competition
- " " symbiosis
- prey predor models

$\boxed{\text{perfect mixing assumption}}$

Lotka-Volterra Model

$$\dot{A} = A - AB$$

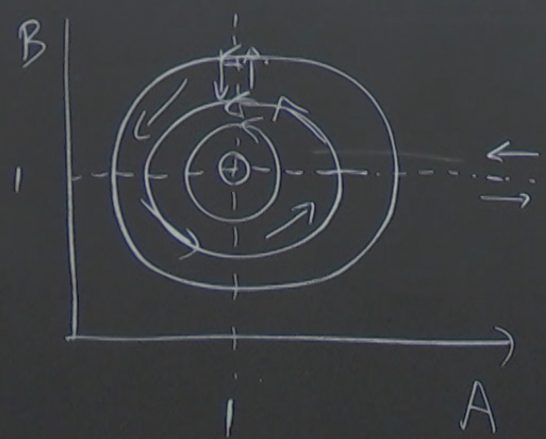
$$\dot{B} = -B + AB$$

nullclines: $\dot{A} = 0 \Rightarrow A - AB = 0$

$\rightarrow A = 0,\ B = 1$

$\dot{B} = 0 \Rightarrow -B + AB = 0$

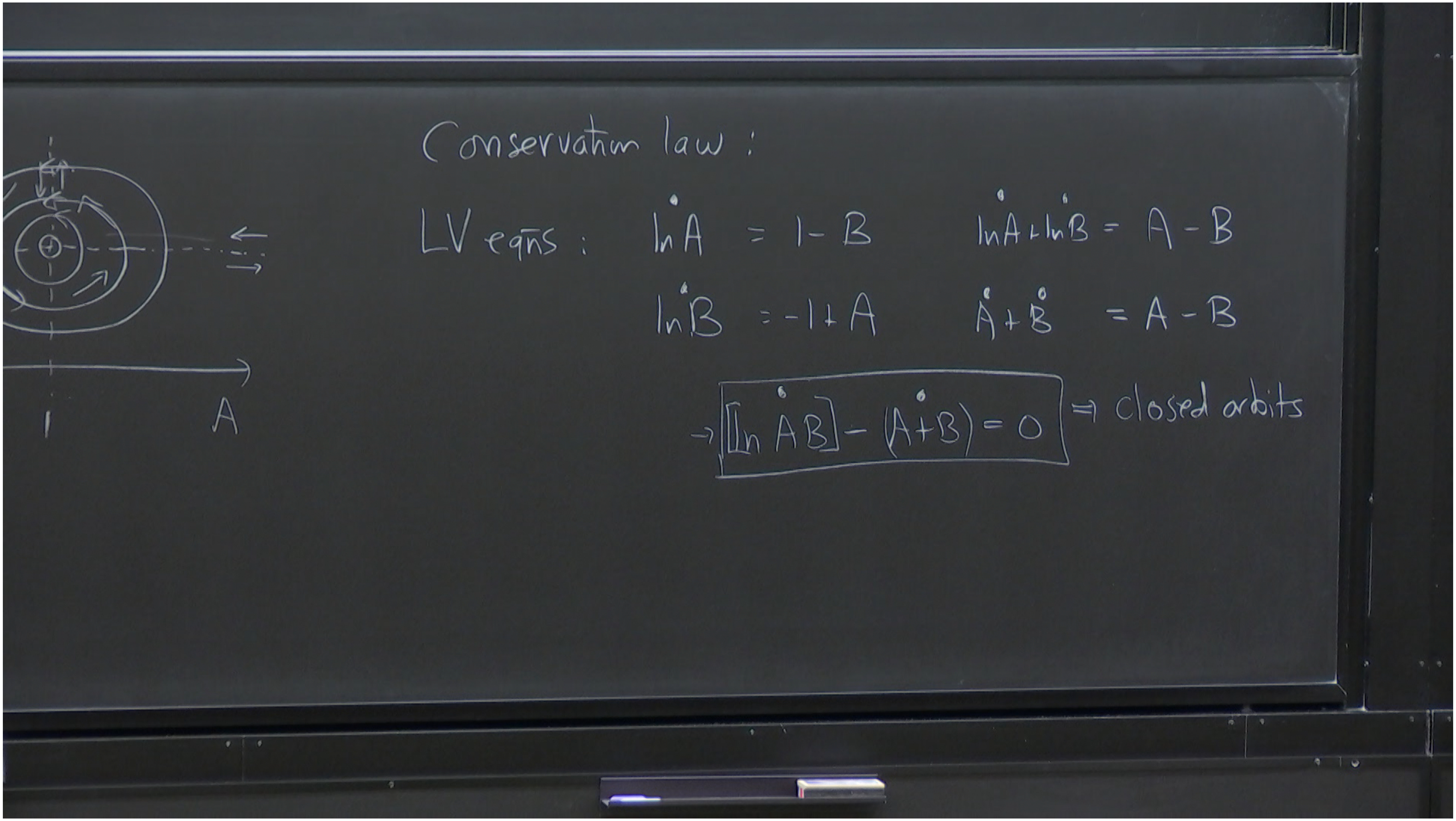
$\rightarrow B = 0\ \ A = 1$



Conservation law:

LV eqns: $\dot{\ln A} = 1 - B$ $\qquad \dot{\ln A} + \dot{\ln B} =$

$\dot{\ln B} = -1 + A$


Conservation law :
LV eqns : $\dot{\ln A} = 1 - B$
$\dot{\ln B} = -1 + A$
$\dot{\ln A} + \dot{\ln B} = A - B$
$\dot{A} + \dot{B} = A - B$
$\rightarrow \boxed{[\dot{\ln AB}] - (\dot{A+B}) = 0} \Rightarrow$ closed orbits
1
A

Lotka-Volterra Model

$$\dot{A} = A - AB - A^2$$

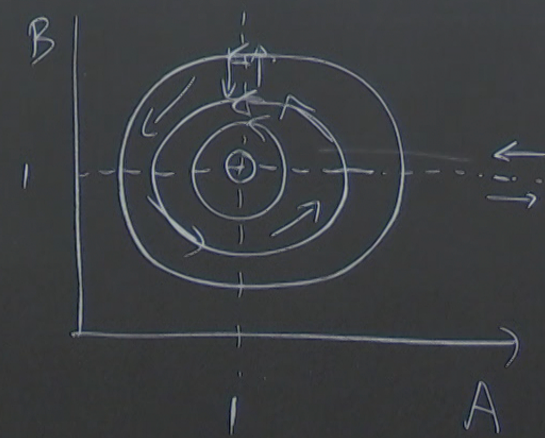
$$\dot{B} = -B + AB - B^2$$

nullclines: $\dot{A} = 0 \Rightarrow A - AB = 0$

$\rightarrow A = 0,\ B = 1$

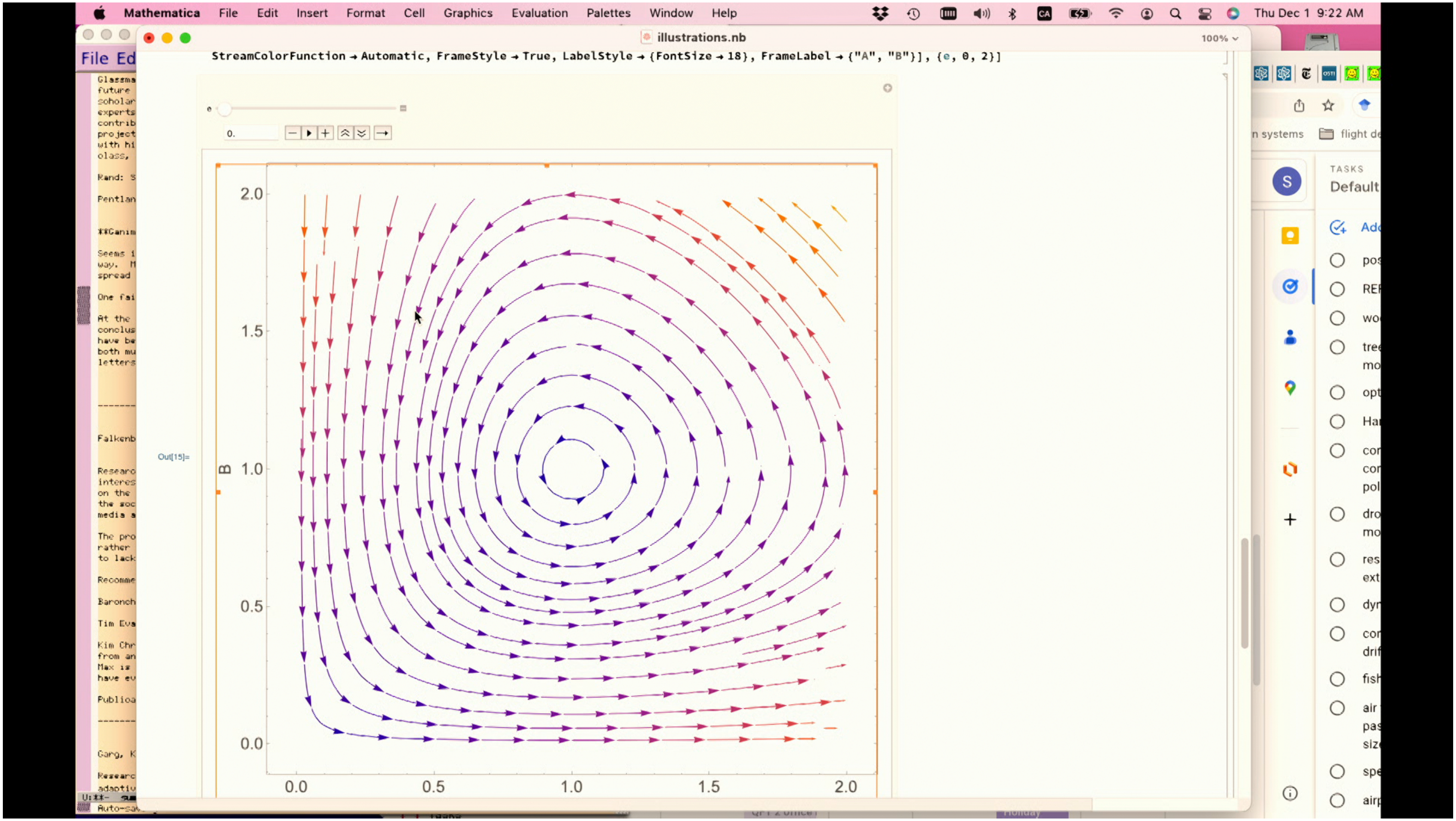
$\dot{B} = 0 \Rightarrow -B + AB = 0$

$\rightarrow B = 0\ \ A = 1$



Cons

LV eq


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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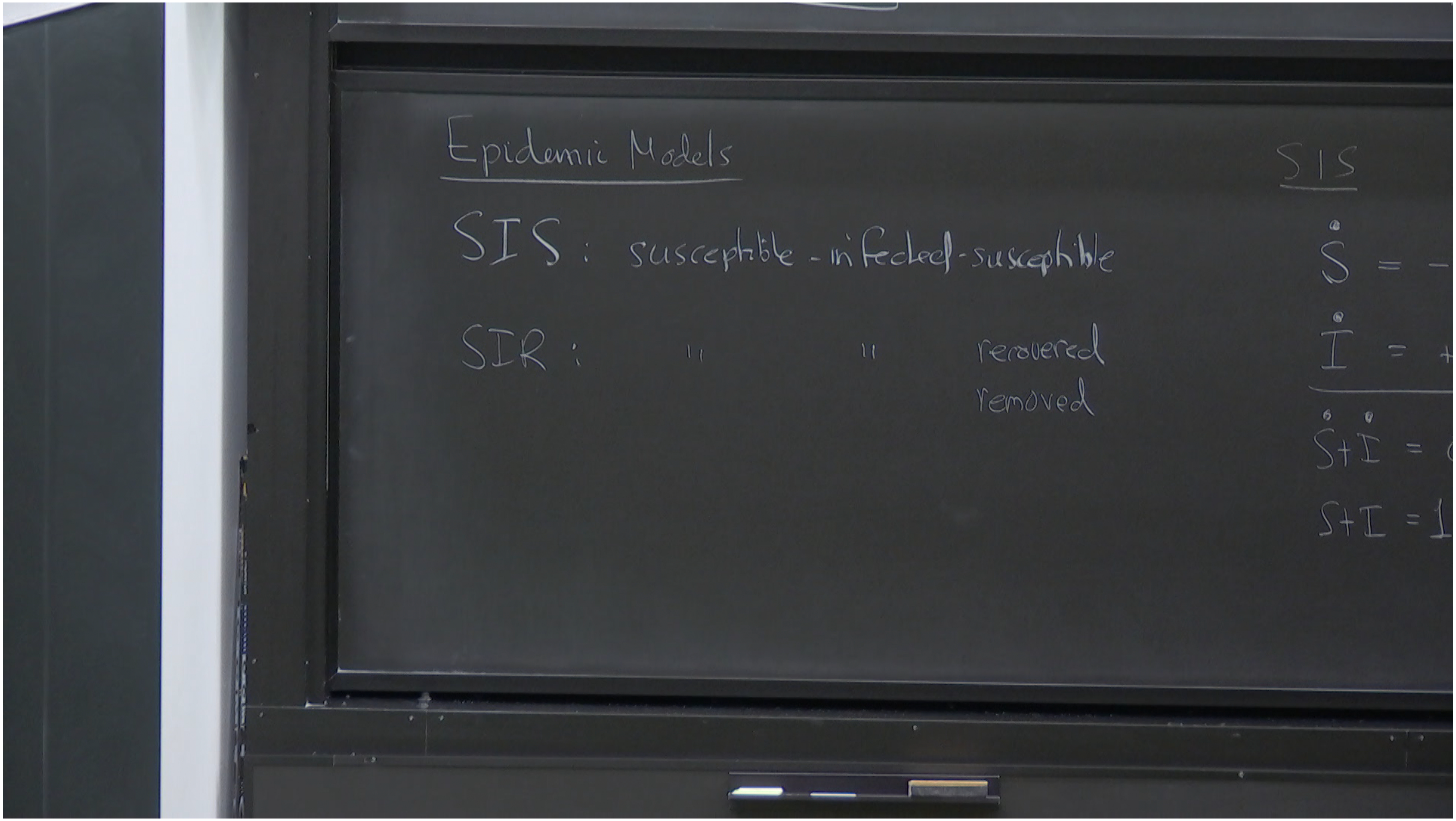
illustrations.nb
StreamColorFunction → Automatic, FrameStyle → True, LabelStyle → {FontSize → 18}, FrameLabel → {"A", "B"}], {e, 0, 2}]
Out[15]=
B
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
TASKS
Default


Epidemic Models
SIS : susceptible - infected - susceptible
SIR : " " recovered
removed
SIS
$\dot{S} = -$
$\dot{I} = +$
$\dot{S} + \dot{I} =$
$S + I = 1$

SIS

$$\dot{S} = -kIS + rI$$

$$\dot{I} = +kIS - rI$$

$$\dot{S} + \dot{I} = 0$$

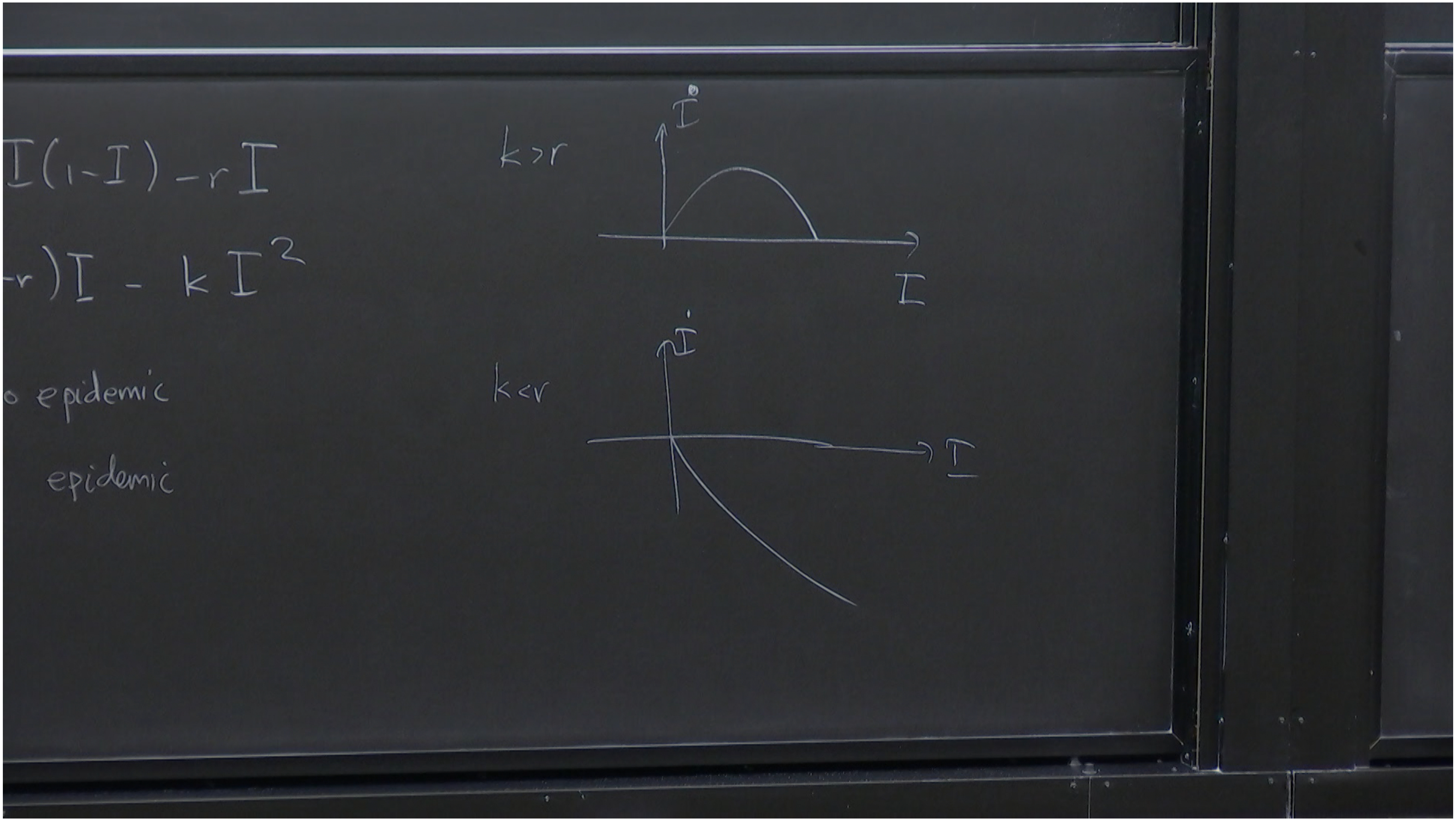
$$S + I = 1$$

$$\dot{I} = kI(1-I) - rI$$

$$= (k-r)I - kI^2$$

$k<r$ no epidemic

$k>r$ epidemic


I(1-I) - r I
-r) I - k I²
o epidemic
epidemic
k>r
İ
I
k<r
İ
I

SIR

$$\dot{S} = -kIS \longrightarrow \text{always decreasing}$$

$$\dot{I} = +kIS - rI \longrightarrow \begin{array}{l}\text{increasing} \Leftrightarrow \text{growing epidemic}\\ \text{decreasing} \Leftrightarrow \text{ebbing epidemic}\end{array}$$

$$\dot{R} = rI \longrightarrow \text{always increasing}$$

$$S + I + R = 1$$

idemic
pidemic

epidemic condition $\dot{I} > 0 \rightarrow \underbrace{kS}_{\text{input rate to } I} - \underbrace{r}_{\text{output rate to } I} > 0$

epidemic severity:

$$\frac{dI}{dS} = \frac{\dot{I}}{\dot{S}} = \frac{kIS - rI}{-kIS} = -1 + \frac{r}{k}\frac{1}{S}$$

$$\equiv -1 + \bar{r}/S$$

$$\underbrace{kS}_{\text{input rate to } I} - \underbrace{r}_{\text{output rate to } I} > 0$$

$$-1 + \frac{r}{k}\frac{1}{S}$$

$$-1 + \bar{r}/S$$

$$\frac{dI}{dS} = 0 \Rightarrow S = \bar{r}$$

epidemic maximum

$$I(t) - I(0) = -\left(S(t) - S(0)\right) + \bar{r}\ln S(t)/S(0)$$

$$S(t)+I(t) = \underbrace{S(0)+I(0)}_{1} + \bar{r}\ln\left[S(t)/S(0)\right]$$

$\underline{t\to\infty}$

$$\boxed{S(\infty) = 1 + \bar{r}\ln\left(S(\infty)/S(0)\right)}$$

$$\Rightarrow S(\infty) = 1 - R(\infty)$$

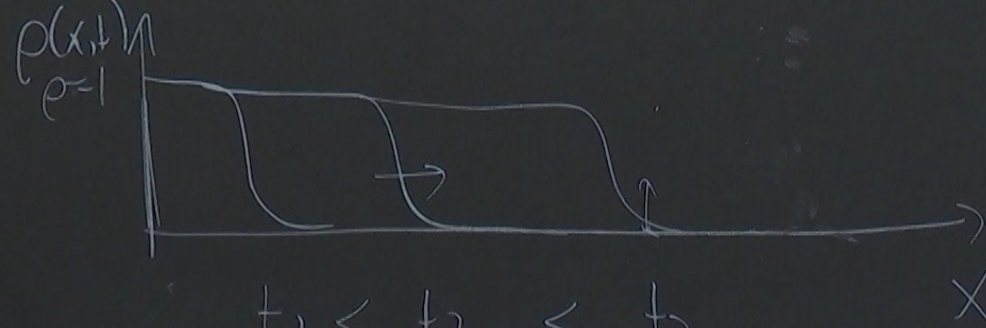
Invasion Dynamics

$$\frac{\partial \rho(x,t)}{\partial t} = k\rho(1-\rho) + D\frac{\partial^2 \rho(x,t)}{\partial x^2}$$

Invasion Dynamics

Assu

$$\frac{\partial \rho(x,t)}{\partial t} = k\rho(1-\rho) + D\frac{\partial^2 \rho(x,t)}{\partial x^2} \qquad \text{FKPP}$$



Assumption: $\rho(x,t) = e^{-\lambda(x-vt)}$

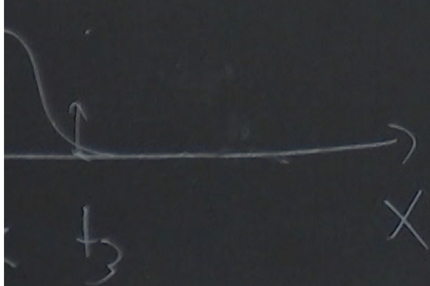
$$+ D \frac{\partial^2 \rho(x,t)}{\partial x^2} \qquad \text{FKPP}$$

Substitute in FKPP

$$\rightarrow \lambda v \cong D\lambda^2 + k$$

$$\boxed{v = D\lambda + \frac{k}{\lambda}}$$

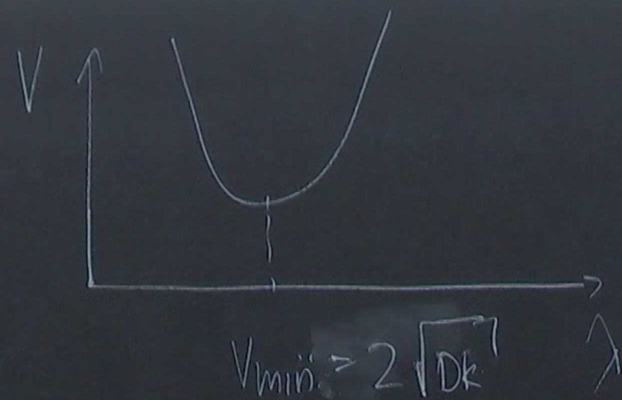
dispersion relation



$$\rho(x,t) = e^{-\lambda(x-vt)}$$

Substitute in FKPP

$$\rightarrow \lambda v \cong D\lambda^2 + k$$



$$\boxed{v = D\lambda + \frac{k}{\lambda}}$$

dispersion relation

$$\frac{dv}{d\lambda} = D - \frac{k}{\lambda^2} \qquad \lambda^* = \sqrt{\frac{k}{D}} \qquad v^* = 2\sqrt{Dk}$$

Aggregation

- gelation, gravitational accretion, rain

$$i, \; j \to K_{ij} \to i+j$$

→ epidemi

epidemi

$$\frac{dI}{dS} = \frac{\dot{I}}{\dot{S}} =$$

, rain

Basic observable

$C_k(t) \equiv$ density of clusters of mass k at time t

assumptions:

- perfect mixing
- bimolecular interactions (low density)
- no shape
- no discreteness