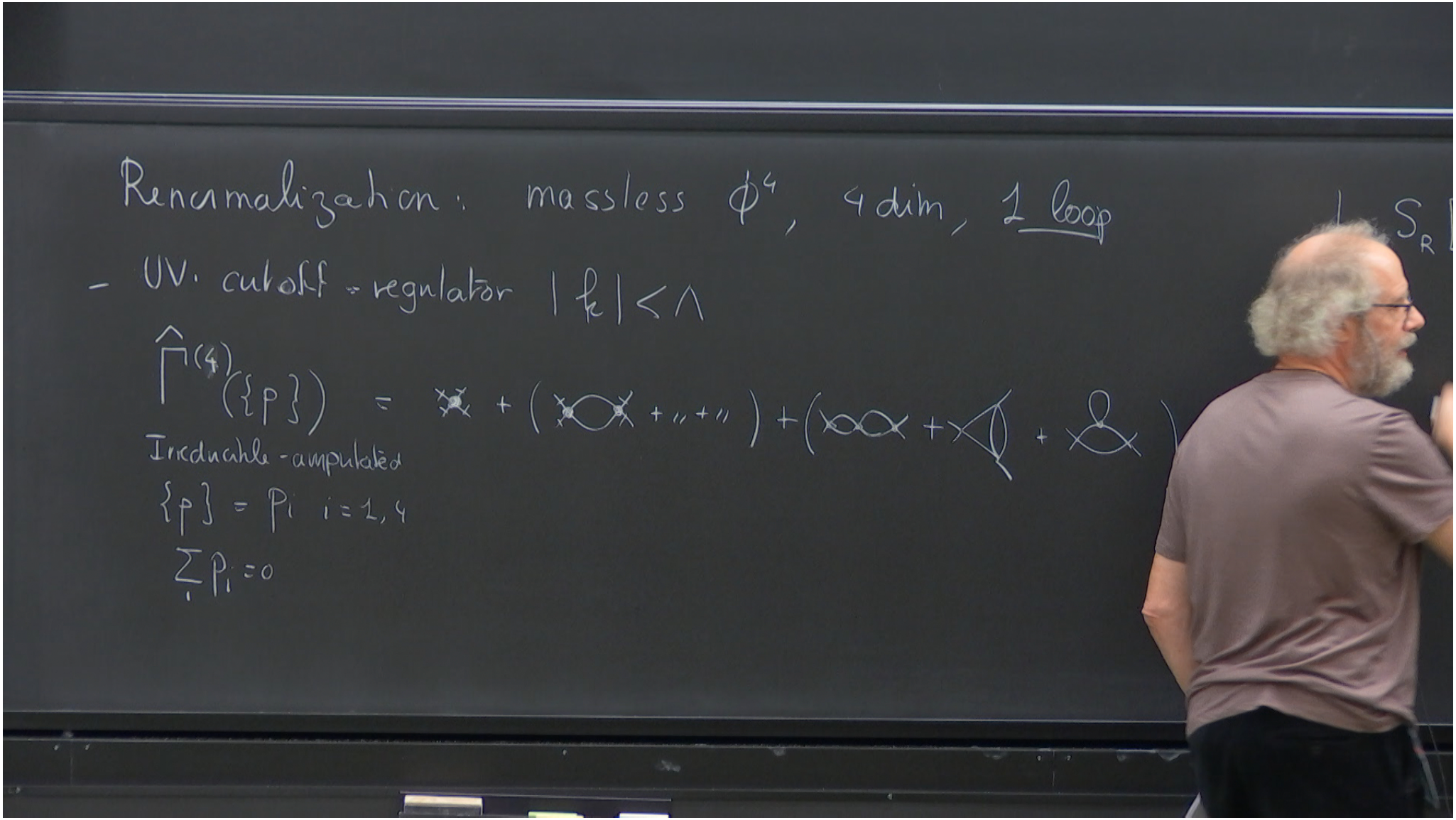
Title: Quantum Field Theory II - Lecture 221207

Speakers:

Collection: Quantum Field Theory II (2022/2023)

Date: December 07, 2022 - 10:45 AM

URL: https://pirsa.org/22120003


Renormalization: massless ϕ^4, 4 dim, 1 loop
S_R
- UV cutoff - regulator $|k| < \Lambda$
$\hat{\Gamma}^{(4)}(\{p\})$ =
+ (+ ... + ...) + (+ +)
Irreducible - amputated
$\{p\} = p_i \quad i = 1, 4$
$\sum_i p_i = 0$

$$S_R[\phi] = \int \left(\frac{A}{2} (\partial_\mu \phi)^2 + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4 \right)$$

$A = 1$, B

$$S_R[\phi] = \int \left(\frac{A}{2} (\partial_\mu \phi)^2 + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4 \right)$$

$A = 1$, $B = \Lambda^2$, $C = ?$

* $S_R[\phi] = \int \left(\frac{A}{2} (\partial_\mu \phi)^2 + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4 \right)$

$A = 1$, $B = \cdots \Lambda^2$, $C = ?$

* (re)normalization scale μ

* $S_R[\phi] = \int \left(\frac{A}{2} (\partial_\mu \phi)^2 + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4 \right)$

$A = 1$, $B = \cdot \Lambda^2$, $C = ?$

* (re)normalization scale μ (momentum)
renormalized coupling constant g_R w.r.t. μ

$g_R := \Gamma^{(4)}(\{p_i^{\text{conf}}\})$ $\quad p_i \simeq \mu$

My choice $|p_1 + p_2|^2 = |p_1 + p_3|^2 = |p_1 + p_4|^2 = \mu^2$

$$* \; S_R[\phi] = \int \left(\frac{A}{2} (\partial_\mu \phi)^2 + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4 \right)$$

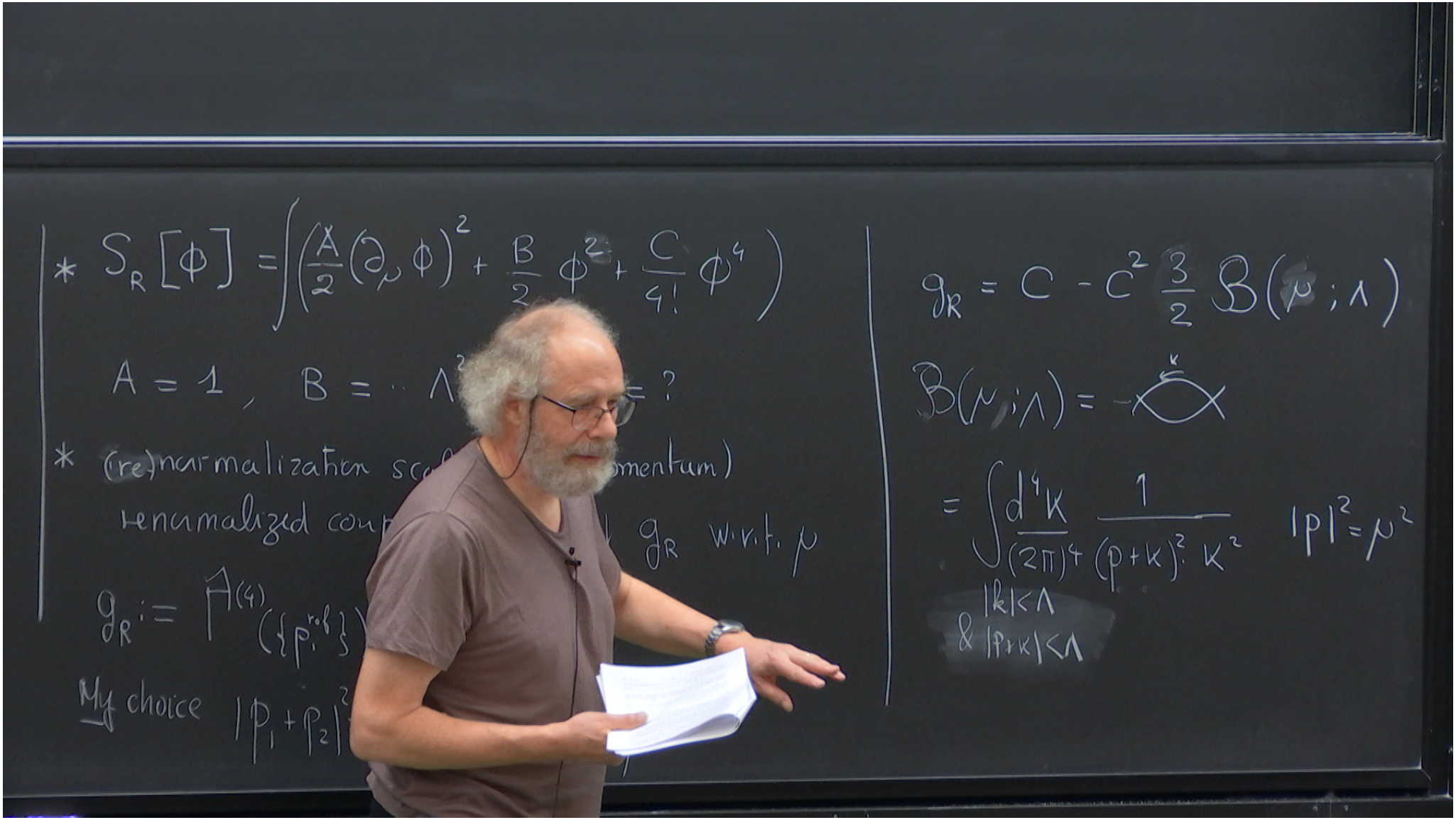
$A = 1$, $B = \cdots \Lambda^2$, $C = ?$

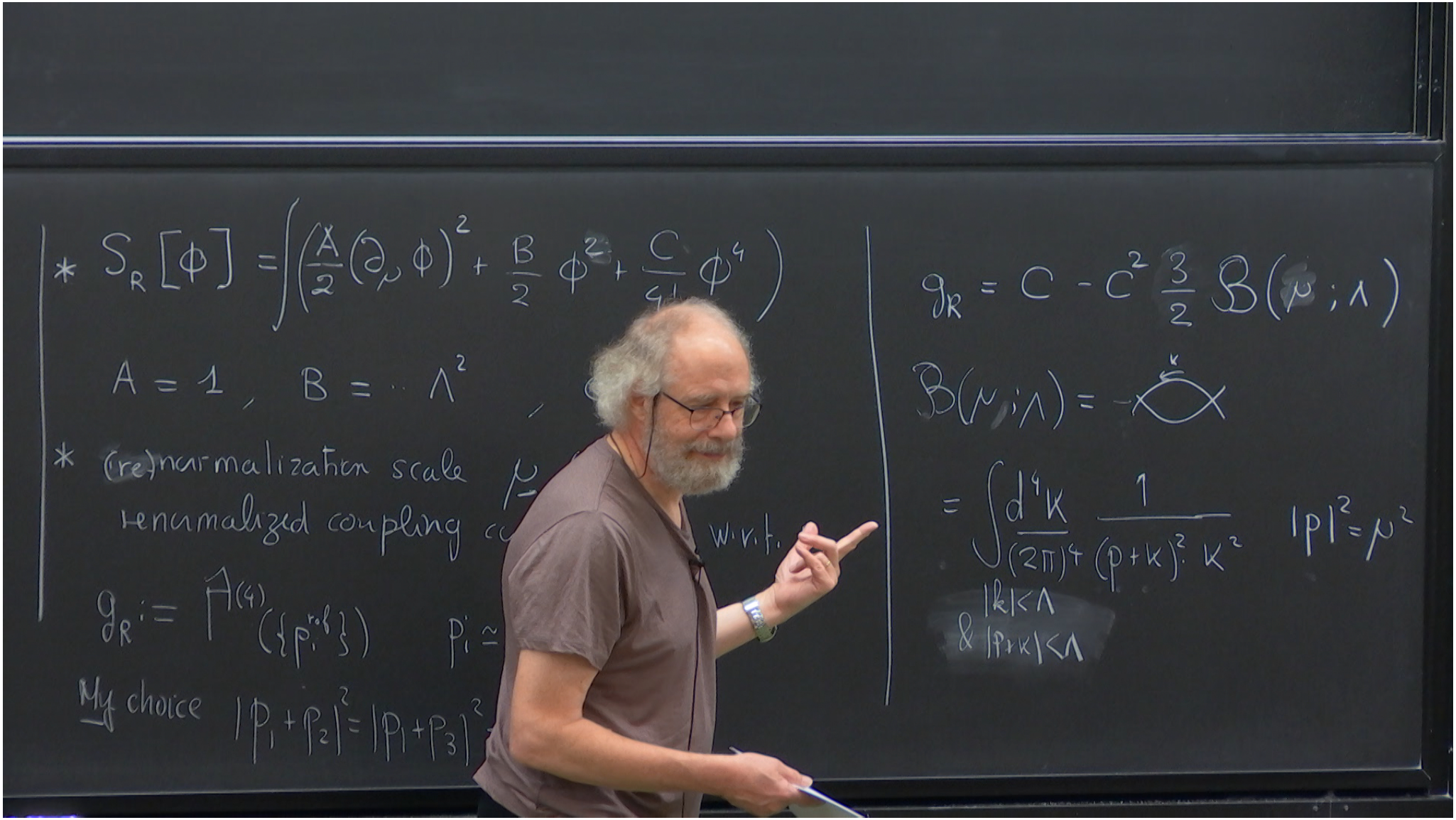
$g_R = C$

* (re)normalization scale μ (momentum)
renormalized coupling constant g_R w.r.t. μ

$$g_R := \Gamma^{(4)}(\{p_i^{ref}\}) \qquad p_i \simeq \mu$$

My choice $|p_1 + p_2|^2 = |p_1 + p_3|^2 = |p_1 + p_4|^2 = \mu^2$


$* S_R[\phi] = \int \left(\frac{A}{2} (\partial_\mu \phi)^2 + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4 \right)$
$A = 1$
renormalized
g_R w.r.t. μ
$g_R := \Gamma^{(4)}$
My choice
$g_R = C - C^2 \frac{3}{2} B(\mu; \Lambda)$
$B(\mu; \Lambda) =$
$= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p+k)^2 k^2}$
$|p|^2 = \mu^2$
$|k| < \Lambda$
$|p+k| < \Lambda$


$* S_R[\phi] = \int \left(\frac{A}{2} (\partial_\mu \phi)^2 + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4 \right)$
$A = 1$, $B = \cdot \Lambda^2$
* (re)normalization scale μ
renormalized coupling
w.r.t.
$g_R := \Gamma^{(4)}(\{p_i^{ref}\})$
$p_i \simeq$
My choice $|p_1 + p_2|^2 = |p_1 + p_3|^2$
$g_R = C - C^2 \frac{3}{2} B(\mu; \Lambda)$
$B(\mu; \Lambda) =$
$= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p+k)^2 k^2}$ $|p|^2 = \mu^2$
$|k| < \Lambda$
& $|p+k| < \Lambda$

[2] which divergence for $\mathcal{B}$

$|p| = \mu$ fixed, $\Lambda \to \infty$

$$\mathcal{B} \simeq \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}$$

[2] which divergence for $\mathcal{B}$

$|p| = \mu$ fixed, $\Lambda \to \infty$

unit sphere in $\mathbb{R}^4$: S_3

$$\mathcal{B} \simeq \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{(K^2)^2} = \int_0^\Lambda dk \, \frac{1}{(2\pi)^4} \frac{k^3}{k^4} \underbrace{\int_{S_3} d\Omega}_{\text{Vol}(S_3) = 2\pi^2}$$

2 which divergence for $\mathcal{B}$

$|p| = \mu$ fixed, $\Lambda \to \infty$

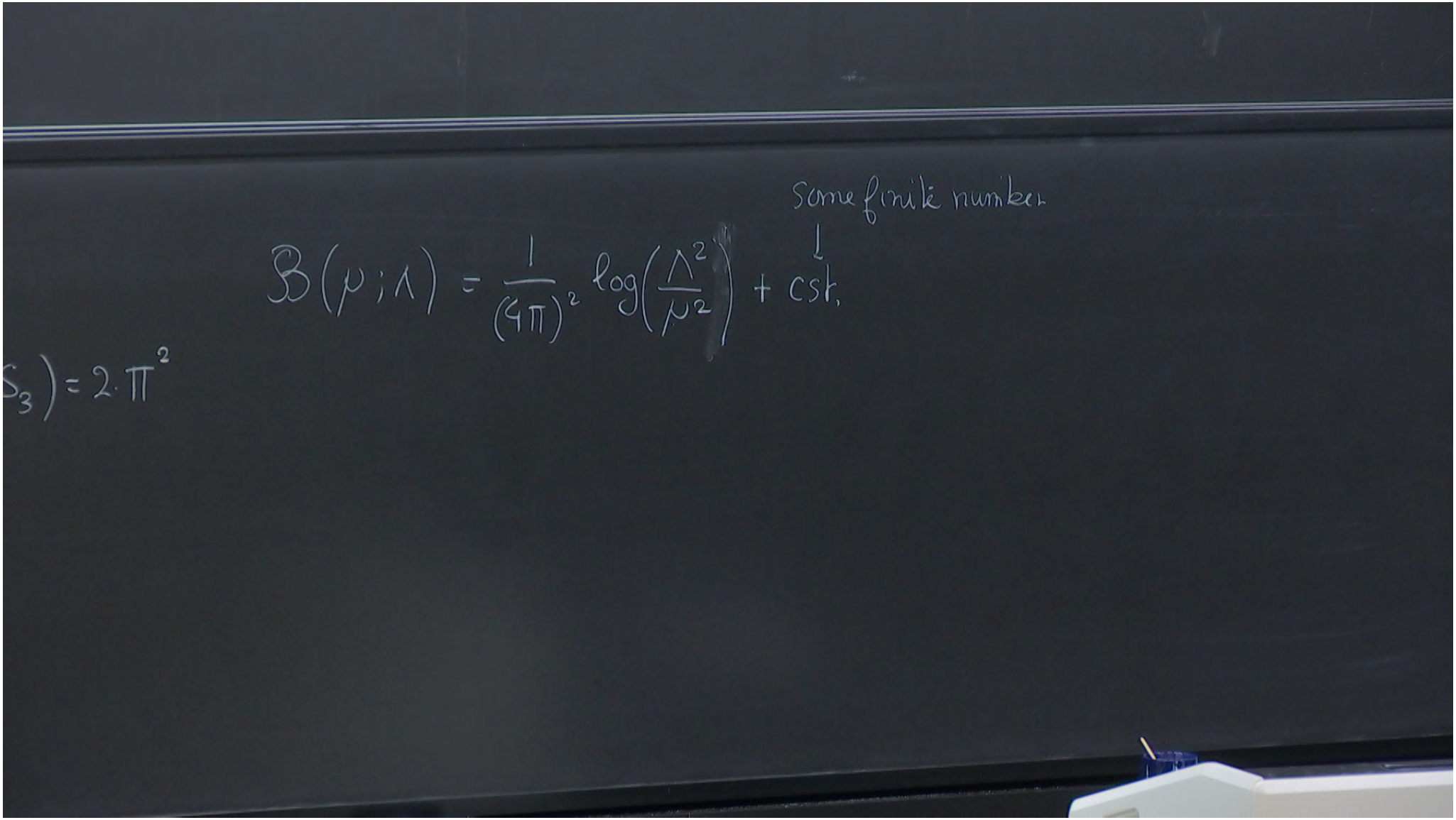
$$\mathcal{B} \simeq \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2)^2} = \int_0^\Lambda dk \, \frac{1}{(2\pi)^4} \frac{k^3}{k^4} \int_{S_3} d\Omega$$

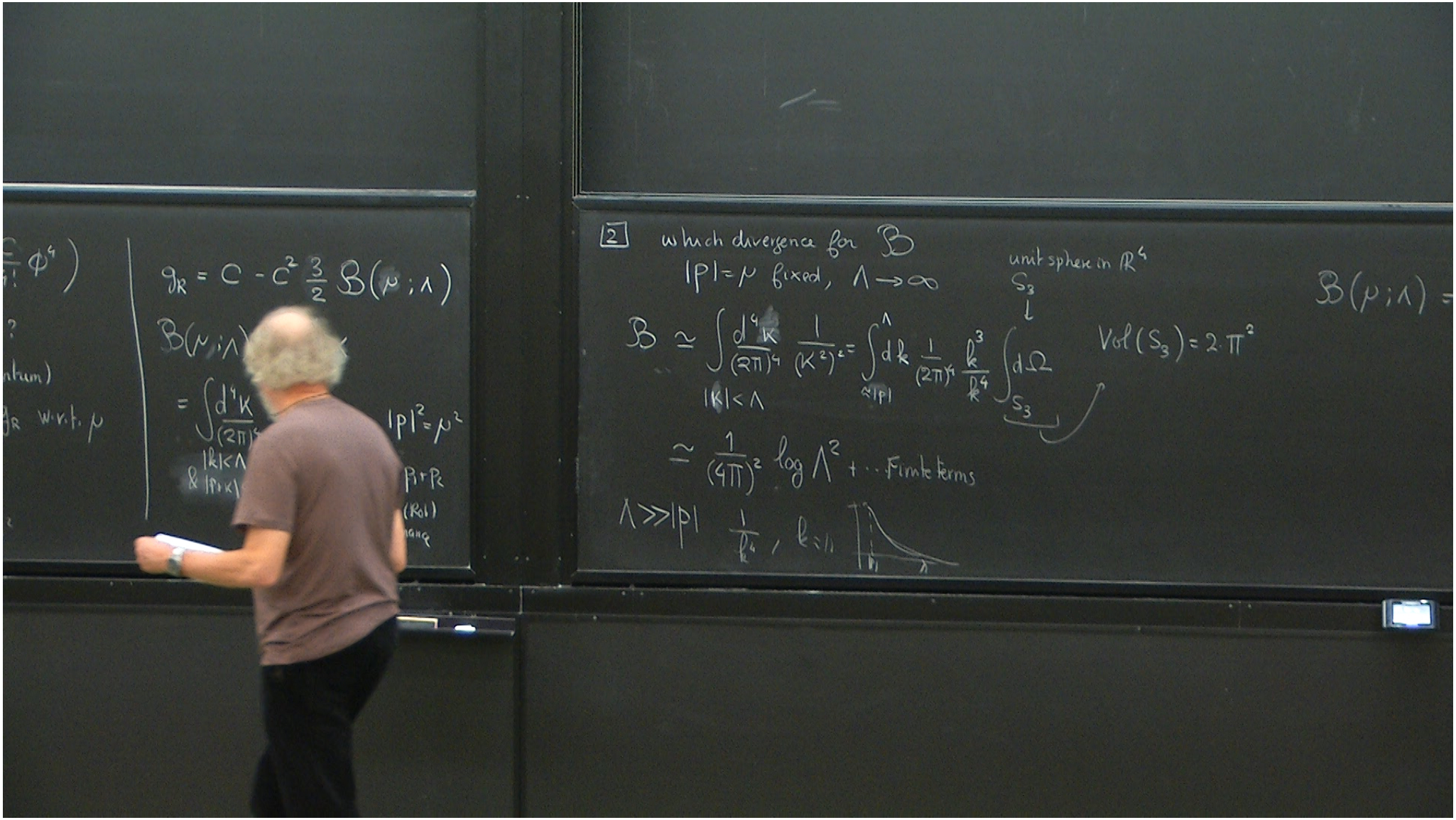
$|k| < \Lambda$

S_3 unit sphere in $\mathbb{R}^4$

$$\mathrm{Vol}(S_3) = 2\cdot\pi^2$$

$$\simeq \frac{1}{(4\pi)^2} \log \Lambda^2 + \ldots \text{Finite terms}$$


Some finite number
$\mathcal{B}(\mu;\Lambda) = \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{cst.}$
$S_3) = 2 \cdot \pi^2$


$g_R = C - C^2 \frac{3}{2} \mathcal{B}(\mu;\Lambda)$
$\mathcal{B}(\mu;\Lambda)$
$= \int \frac{d^4K}{(2\pi)^4}$
$|k|<\Lambda$
& $|p+k|$
$|p|^2=\mu^2$
$\beta_1 r \beta_2$
(RGI)
2 which divergence for $\mathcal{B}$
$|p|=\mu$ fixed, $\Lambda \to \infty$
unit sphere in $\mathbb{R}^4$
S_3
$\mathcal{B} \simeq \int_{|K|<\Lambda} \frac{d^4K}{(2\pi)^4} \frac{1}{(K^2)^2} = \int_{\simeq|p|}^{\Lambda} dk \frac{1}{(2\pi)^4} \frac{k^3}{k^4} \int_{S_3} d\Omega$
$Vol(S_3) = 2\cdot\pi^2$
$\simeq \frac{1}{(4\pi)^2} \log \Lambda^2 + \cdots$ Finite terms
$\Lambda \gg |p|$ $\frac{1}{k^4}$, $k=|$
$\mathcal{B}(\mu;\Lambda) =$

$$* \; S_R[\phi] = \int \left(\frac{A}{2} (\partial_\mu \phi)^2 + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4 \right)$$

$A = 1$, $B = \cdots \Lambda^2$, $C = ?$

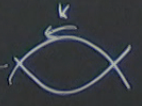
* (re)normalization scale μ (momentum) $\mu \neq 0$

renormalized coupling constant g_R w.r.t. μ

$$g_R := \Gamma^{(4)}(\{p_i^{=\mu}\}) \qquad p_i \simeq \mu$$

My choice $|p_1 + p_2|^2 = |p_1 + p_3|^2 = |p_1 + p_4|^2 = \mu^2$

$$g_R = C - C^2 \frac{3}{2} B(\mu; \Lambda)$$

$B(\mu; \Lambda) =$ 

$$= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p+k)^2 \, k^2} \qquad |p|^2 = \mu^2$$

$|k| < \Lambda$ & $|p+k| < \Lambda$

$p = p_1 + p_2$

Lorentz (Rot.) invariance

no div $k \to 0$ $p \neq 0$

$$B(\mu;\Lambda) = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right) + \underbrace{\text{cst.}}_{\text{Some finite number}}$$

$(S_3) = 2\cdot\pi^2$

Back to g_R

$$g_R = C - C^2\frac{3}{2}\frac{1}{(4\pi)^2}\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{cst.}\right] + O(C^3)$$

$$C = g_R + g_R^2\frac{3}{4}\frac{1}{(4\pi)^2}\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{cst.}\right] + O(g_R^3)$$

Finite renormalized theory

$\Lambda \to \infty$, g_R fixed

$$B(\mu;\Lambda) = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right) + \underbrace{\text{csk.}}_{\text{some finite number}}$$

$(S_3) = 2\cdot\pi^2$

Back to g_R

$$g_R = C - C^2\frac{3}{2}\frac{1}{(4\pi)^2}\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{csk.}\right] + O(C^3)$$

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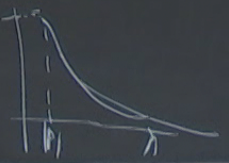
Finite renamalized theory

$\Lambda \to \infty$, g_R fixed

then $C \to \infty$ at that order

$\simeq \frac{1}{(4\pi)^2} \log \Lambda^2 + \cdots$ Finite terms

$\Lambda \gg |p|$ $\frac{1}{k^4}$, $k \simeq \mu$



$C = g_R + g_R^2 \frac{3}{4} \frac{1}{(4\pi)^2} \left[\log \left(\frac{\Lambda^2}{\mu^2} \right.$

$C = g_R + C^2 \frac{3}{2}(\cdots) + O(C^3)$

$= g_R + \left(g_R + C^2 \cdots\right)^2 (\cdots) + O(C^3)$

$g_R^2 + O(C^3)$

in $\mathbb{R}^4$

$$\mathcal{B}(\mu;\Lambda) = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right) + \underbrace{1}_{\text{Some finite number}} + \text{cst.}$$

$\text{Vol}(S_3) = 2\cdot\pi^2$

Back to g_R

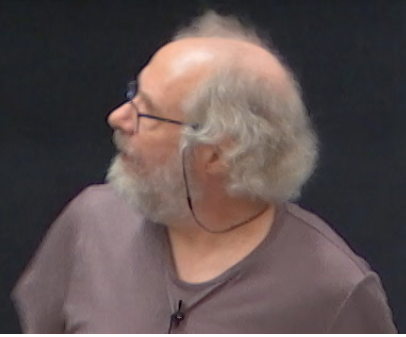
$$g_R = C - C^2\frac{3}{2}\frac{1}{(4\pi)^2}\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{cst.}\right] + O(C^3)$$

$$C = g_R + g_R^2\frac{3}{2}\frac{1}{(4\pi)^2}\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{cst.}\right] + O(g_R^3)$$

Finite renormalized theory

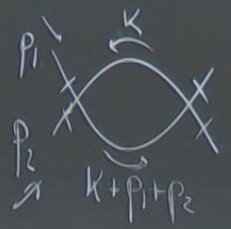
$\Lambda \to \infty$, g_R fixed

then $C \to \infty$ at that order



$\{p_i\} = p_1 \quad i = 1, 4$

$\sum_i p_i = 0$



$g_R := \Gamma^{(4)}(\{p_i^{sym}\})$ $\quad p_i \simeq \mu$

My choice $|p_1 + p_2|^2 = |p_1 + p_3|^2 = |p$

general irreducible 4 point function

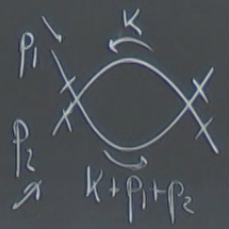
$$\Gamma^{(4)}(p_1 \cdots p_4) = C - C^2 \frac{1}{2}\left[\mathcal{B}(p_1+p_2;\Lambda) + \mathcal{B}(p_1+p_3;\Lambda) + \mathcal{B}(p_1+p_4;\Lambda)\right] + O(C^3)$$

$$= g_k - g_R^2 \frac{1}{2}\left[\left[\mathcal{B}(|p_1+p_2|;\Lambda) - \mathcal{B}(\mu;\Lambda)\right] + \text{//} + \text{//}\right] + O(g_R^3)$$

$\Lambda \to \infty$ this is finite

$\{p_i\} = p_1 \; i = 1,4$

$\sum_i p_i = 0$



$g_R := \Gamma^{(4)}(\{p_i^{=\mu}\})$ $\quad p_i \simeq \mu$

My choice $|p_1+p_2|^2 = |p_1+p_3|^2 = |p$

general irreducible 4 point function

$$\Gamma^{(4)}(p_1 \cdots p_4) = C - C^2 \frac{1}{2}\left[B(p_1+p_2;\Lambda) + B(p_1+p_3;\Lambda) + B(p_1+p_4,\Lambda)\right] + O(C^3)$$

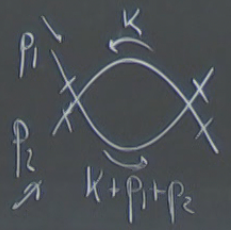
$$= g_k - g_R^2 \frac{1}{2}\left[\left[B(|p_1+p_2|,\Lambda) - B(\mu;\Lambda)\right] + \text{//} + \text{//}\right] + O(g_R^3)$$

$\Lambda \to \infty$ this is finite

$$= \boxed{g_R - g_R^2 \frac{1}{2}\frac{1}{(4\pi)^2}\left[\log\frac{\mu^2}{(p_1+p_2)^2} + \log\frac{\mu^2}{(p_1+p_3)^2} + \log\frac{\mu^2}{(p_1+p_4)^2}\right] + O(g_R^3)}$$

$\{p_i\} = p_1 \; i = 1,4$

$\sum_i p_i = 0$



$g_R := \Gamma^{(4)}(\{p_i^{=\mu}\})$ $\quad p_i \simeq \mu$

My choice $|p_1+p_2|^2 = |p_1+p_3|^2 = |p$

general irreducible 4 point function

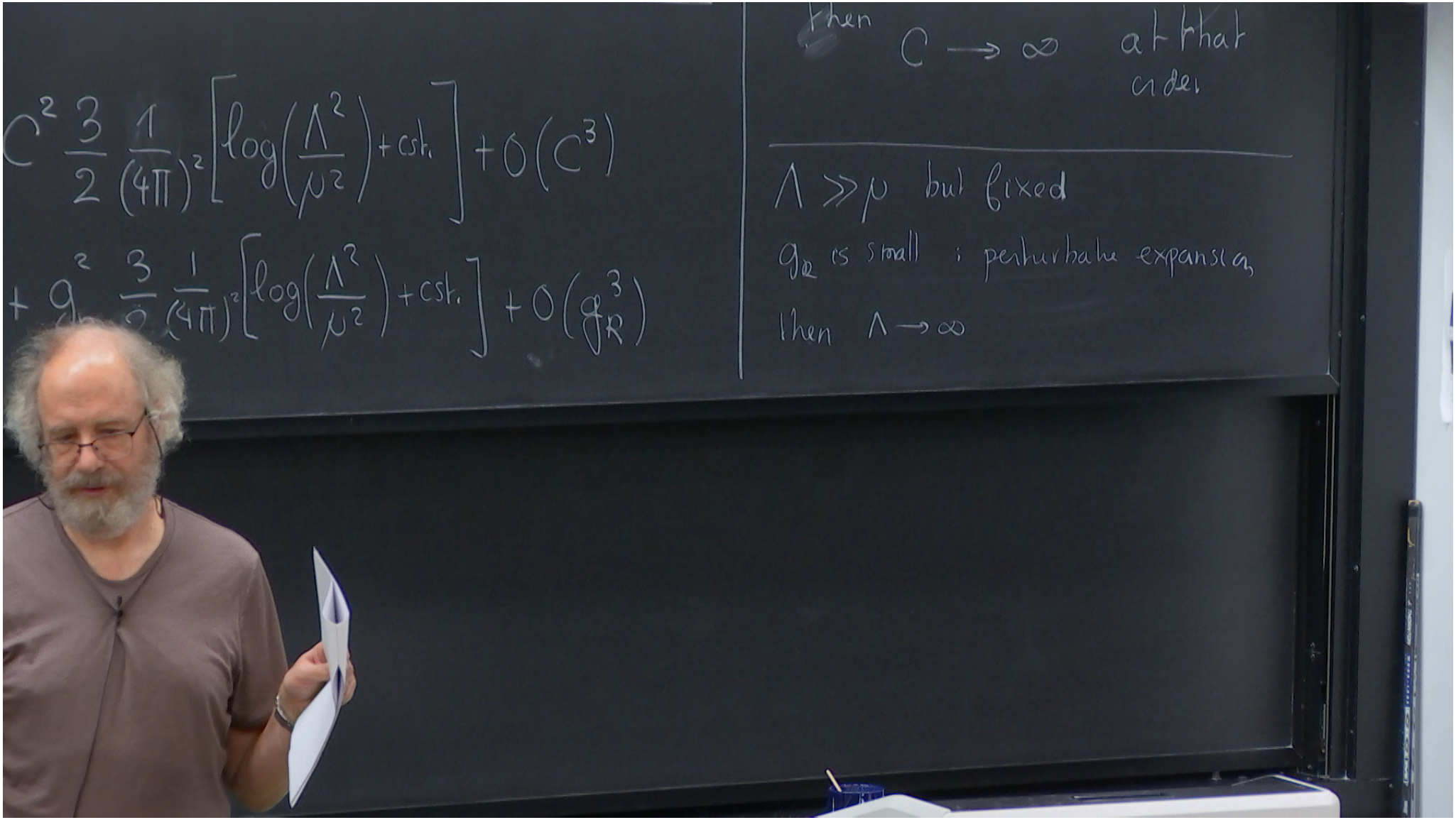
$$\Gamma^{(4)}(p_1 \cdots p_4) = C - C^2 \frac{1}{2}\left[\mathcal{B}(p_1+p_2;\Lambda) + \mathcal{B}(p_1+p_3;\Lambda) + \mathcal{B}(p_1+p_4,\Lambda)\right] + O(C^3)$$

renormalized 4 pt function

$$= g_R - g_R^2 \frac{1}{2}\left[\left[\mathcal{B}(|p_1+p_2|;\Lambda) - \mathcal{B}(\mu;\Lambda)\right] + \text{//} + \text{//}\right] + O(g_R^3)$$

$\Lambda \to \infty$ this is finite

$$\Gamma_R^{(4)}(\{p_i\}; g_R, \mu) = \left[g_R - g_R^2 \frac{1}{2}\frac{1}{(4\pi)^2}\left[\log\frac{\mu^2}{|p_1+p_2|^2} + \log\frac{\mu^2}{(p_1+p_3)^2} + \log\frac{\mu^2}{(p_1+p_4)^2}\right] + O(g_R^3)\right]$$


$c^2 \frac{3}{2} \frac{1}{(4\pi)^2} \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{cst.} \right] + O(c^3)$
$+ g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2} \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{cst.} \right] + O(g_R^3)$
then $c \to \infty$ at that order
$\Lambda \gg \mu$ but fixed
g_R is small : perturbative expansion
then $\Lambda \to \infty$

choice $|p_1+p_2|^2=|p_1+p_3|^2=|p_1+p_4|^2=\mu^2$

no div $k\to 0$ $p\neq 0$ invariance

$+P_4,\Lambda)\Big]+O(C^3)$

$\Big]+O(g_R^3)$

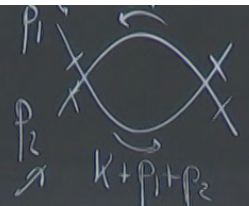
$)$

Λ finite $\gg p$ $\Rightarrow$ express everything in term of g_R

series-expansion in g_R instead of C

check: each term g_R^k is finite when $\Lambda\to\infty$

$\sum_i p_i = 0$



$g_R := \Gamma^{(4)}(\{p_i^{sym}\})$

My choice $|p_1+p_2|^2 = |p_1+$

general irreducible 4 point function

$$\Gamma^{(4)}(p_1 \cdots p_4) = C - C^2 \frac{1}{2}\left[B(p_1+p_2;\Lambda) + B(p_1+p_3;\Lambda) + B(p_1+p_4,\Lambda)\right] + O(C^3)$$

$$= g_R - g_R^2 \frac{1}{2}\left[\left[B(|p_1+p_2|,\Lambda) - B(\mu;\Lambda)\right] + \text{//} + \text{//}\right] + O(g_R^3)$$

renormalized 4pt function

$\Lambda \to \infty$ this is finite

$$\Gamma_R^{(4)}(\{p_i\}; g_R, \mu) = g_R - g_R^2 \frac{1}{2}\frac{1}{(4\pi)^2}\left[\log\frac{\mu^2}{|p_1+p_2|^2} + \log\frac{\mu^2}{|p_1+p_3|^2} + \log\frac{\mu^2}{|p_1+p_4|^2}\right] + O(g_R^3)$$

any $\{p_i\}$

$g_R = \Gamma^{(4)}(\{p_i^{=\mu}\})$ $\quad p_i = \mu$

My choice $|p_1+p_2|^2 = |p_1+p_3|^2 = |p_1+p_4|^2 = \mu^2$

$|k|<\Lambda$ & $|p+k|<\Lambda$

no div $k\to 0$ $p \neq 0$

$p = p_1+p_2$

Lorentz (Rot) invariance

$+ B(p_i+p_4,\Lambda)\Big] + O(c^3)$

$+ \text{//} \Big] + O(g_R^3)$

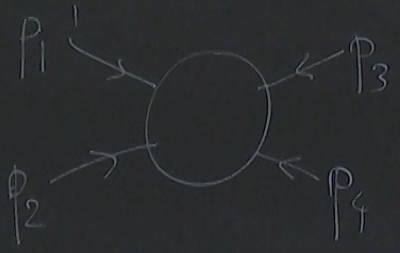
any $\{p_i\}$

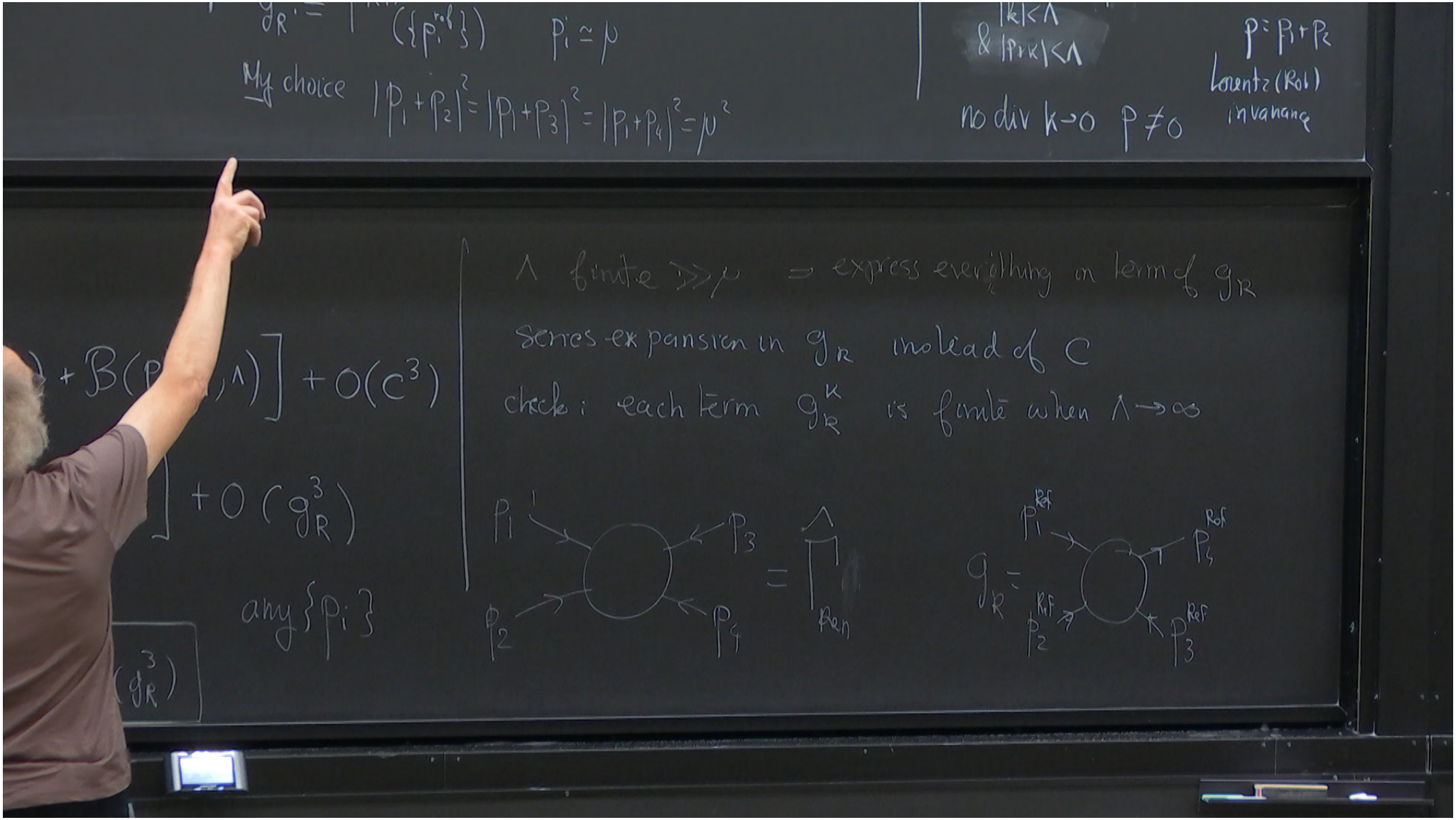
(g_R^3)

Λ finite $\gg \mu$ $\Rightarrow$ express everything in term of g_R

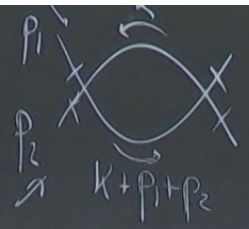
series expansion in g_R instead of c

check: each term g_R^k is finite when $\Lambda \to \infty$




$g_R = \Gamma^{(4)}(\{p_i^{ref}\})$ $p_i \simeq \mu$
My choice $|p_1+p_2|^2 = |p_1+p_3|^2 = |p_1+p_4|^2 = \mu^2$
$|k|<\Lambda$ & $|p+k|<\Lambda$
$p = p_1+p_2$
Lorentz (Rot) invariance
no div $k\to 0$ $p \neq 0$
$+ B(p, \Lambda)] + O(c^3)$
$+ O(g_R^3)$
any $\{p_i\}$
Λ finite $\gg \mu$ $\Rightarrow$ express everything in term of g_R
series expansion in g_R instead of c
check: each term g_R^k is finite when $\Lambda \to \infty$
p_1 p_2 p_3 p_4 $= \Gamma_{Ren}$
$g_R =$ p_1^{Ref} p_2^{Ref} p_3^{Ref} p_4^{Ref}

$\sum_i p_i = 0$



$g_R := \Gamma^{(4)}(\{p_i^{\nu_i}\})$

My choice $|p_1+p_2|^2 = |p_1$

general irreducible 4 point function $\Lambda \gg \mu$ and all momenta

$$\Gamma^{(4)}(p_1 \cdots p_4) = C - C^2 \frac{1}{2}\left[B(p_1+p_2;\Lambda) + B(p_1+p_3;\Lambda) + B(p_1+p_4;\Lambda)\right] + O(C^3)$$

$$= g_R - g_R^2 \frac{1}{2}\left[\left[B(|p_1+p_2|;\Lambda) - B(\mu;\Lambda)\right] + \text{//} + \text{//}\right] + O(g_R^3)$$

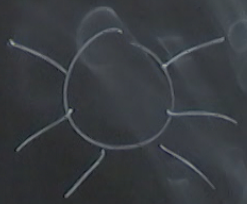
renormalized 4 pt function

$\Lambda \to \infty$ this is finite

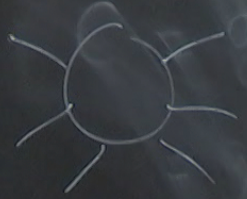
$$\Gamma_R^{(4)}(\{p_i\}; g_R, \mu) = g_R - g_R^2 \frac{1}{2}\frac{1}{(4\pi)^2}\left[\log\frac{\mu^2}{(p_1+p_2)^2} + \log\frac{\mu^2}{(p_1+p_3)^2} + \log\frac{\mu^2}{(p_1+p_4)^2}\right] + O(g_R^3)$$

any $\{p_i\}$

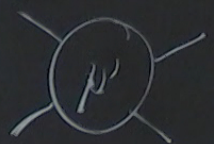
1 loop : all correlation functions are UV. finite



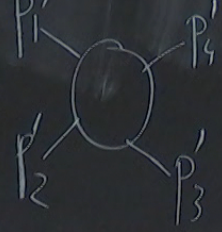
1 loop : all correlation functions are UV. finite



What if we choose $\mu' \neq \mu$?

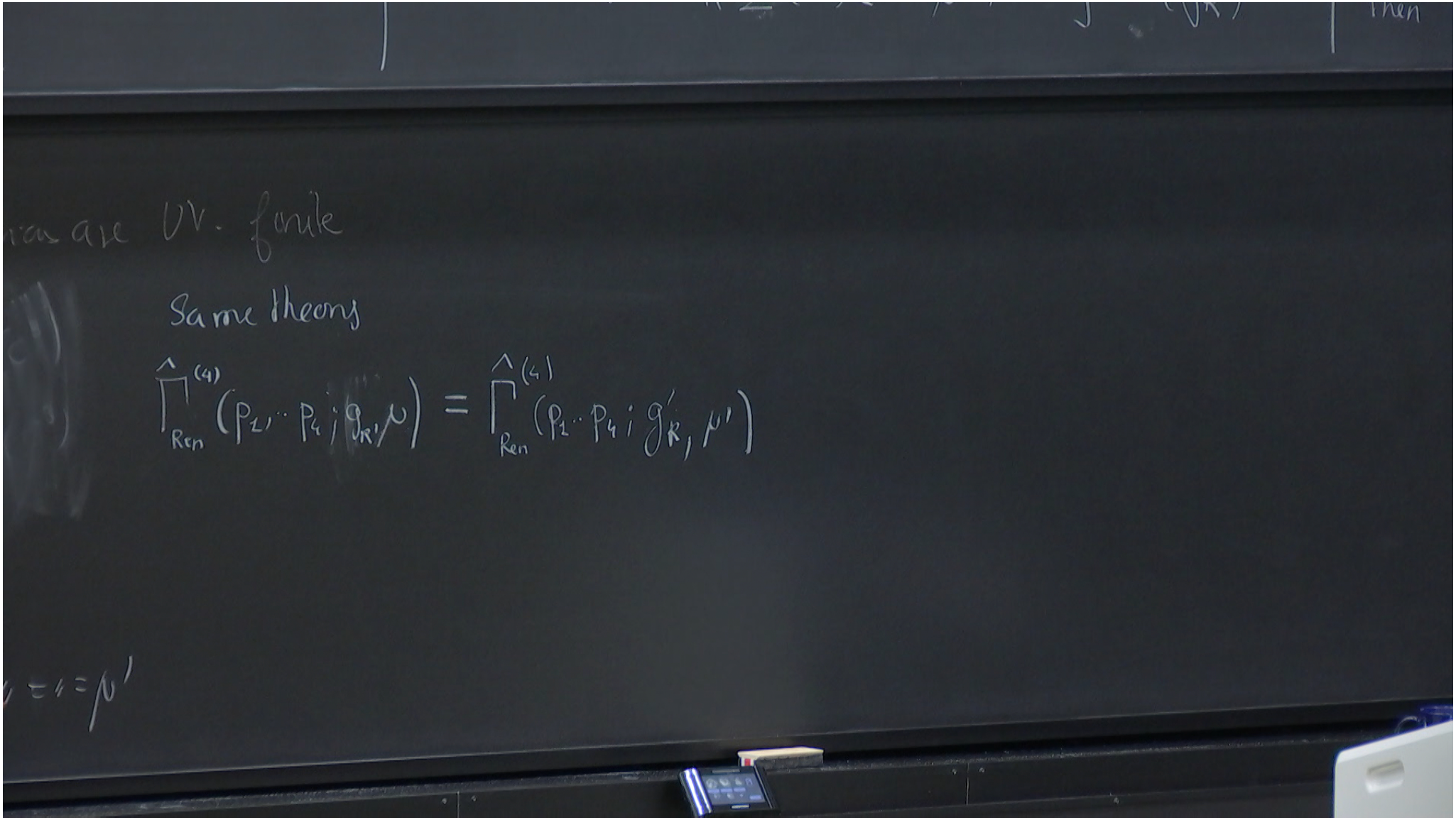


$= g_R' =$



such that

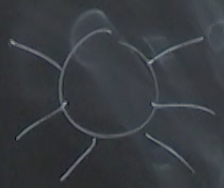
$|p_1' + p_2'| = \ldots = \mu'$


are UV. finite
Same theory
$\hat{\Gamma}^{(4)}_{Ren}(p_1 \dots p_4 ; g_R, \mu) = \hat{\Gamma}^{(4)}_{Ren}(p_1 \dots p_4 ; g'_R, \mu')$

$\Lambda \gg |p|$ $\frac{1}{k^4}$, $k \sim \Lambda$

$C = g_R + g_R \frac{3}{2} \frac{1}{(4\pi)^2} [\log($

1 loop : all correlation functions are UV. finite

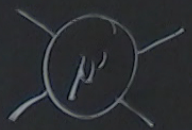


Same theory

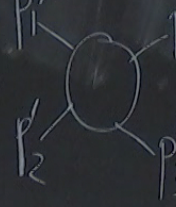
$\hat{\Gamma}^{(4)}_{Ren}(p_1 \dots p_4 ; g_R, \mu) = \hat{\Gamma}^{(4)}_{Ren}(p_1 \dots p_4 ; g'_R, \mu')$

for all $\{p_1 \dots p_4\}$

What if we choose $\mu' \neq \mu$?



$= g'_R =$

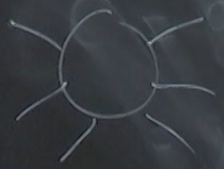


such that $|p'_1 + p'_2| = \mu = \mu = \mu'$

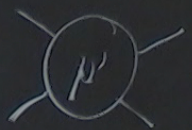
$\Lambda \gg |p|$ $\frac{1}{k^4}$, $k \simeq \Lambda$

$C = g_R + g_R \frac{3}{2} \frac{1}{(4\pi)^2} \left[\log \right.$

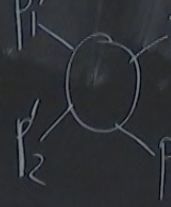
1 loop : all correlation functions are UV. finite



What if we choose $\mu' \neq \mu$?



$= g'_R =$



such that $|p_1' + p_2'| = \Lambda = \Lambda = \mu'$

Same theory

$$\hat{\Gamma}^{(4)}_{Ren}(p_1 \dots p_4; g_R, \mu) = \hat{\Gamma}^{(4)}_{Ren}(p_1 \dots p_4; g'_R, \mu')$$

for all $\{p_1 \dots p_4\}$

Solution:

$$g'_R - g'^2_R \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\mu'}{\Lambda}\right)^2 + O(g'^3_R) = g_R$$

$$C = g_R + g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2} \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{cst.} \right] + O(g_R^3)$$

$\Lambda \gg \mu$ but fixed

g_R is small ; perturbative exp

then $\Lambda \to \infty$

UV. finite

if $g'_R = \text{Function of} \left[g_R \text{ and } \left(\frac{\mu'}{\mu}\right) \right]$

Same theory

$$\hat{\Gamma}^{(4)}_{\text{Ren}}(p_1 \cdots p_4 ; g_R, \mu) = \hat{\Gamma}^{(4)}_{\text{Ren}}(p_1 \cdots p_4 ; g'_R, \mu')$$

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Solution:

$$g'_R - g'^2_R \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\mu'}{\mu}\right)^2 + O(g'^3_R) = g_R$$

if $g_R' = \text{Function of } \left[g_R \text{ and } \left(\frac{\mu'}{\mu}\right) \right]$

OR: start from (μ_0, g_R^0)

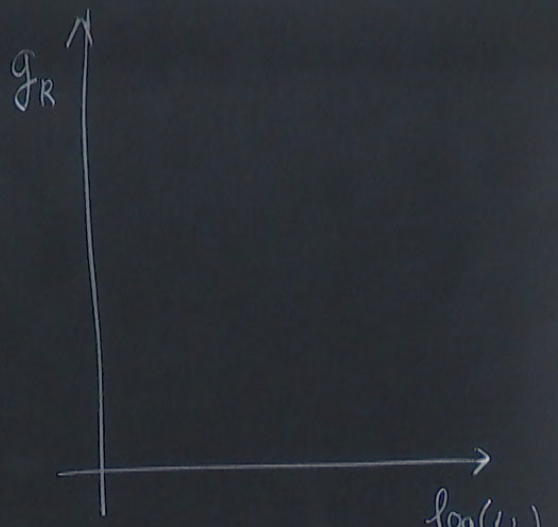
$\left(g_R(\mu), \mu\right)$

$g_R(\mu) - g_R^2(\mu) \frac{3}{2}\frac{1}{(4\pi)^2} \log\left(\frac{\Lambda}{\mu_0}\right)^2 = g_R^0$

$\hat{\Gamma}^{(4)}_{\text{Ren}}(p_1 \cdots p_4 ; g_R', \mu')$

$\frac{1}{(4\pi)^2} \log\left(\frac{\mu'}{\mu}\right)^2 + O(g_R'^3) = g_R$

$g_R(\mu) = g_R^0 + g_R^{0\,2} \frac{3}{2}\frac{1}{(4\pi)^2} \log\left(\frac{\Lambda}{\mu_0}\right)^2$



if $g_R' = \text{Function of } \left[g_R \text{ and } \left(\frac{\mu'}{\mu}\right) \right]$

$$\hat{\Gamma}^{(4)}_{\text{Ren}}(p_1 \cdots p_4 ; g_R', \mu')$$

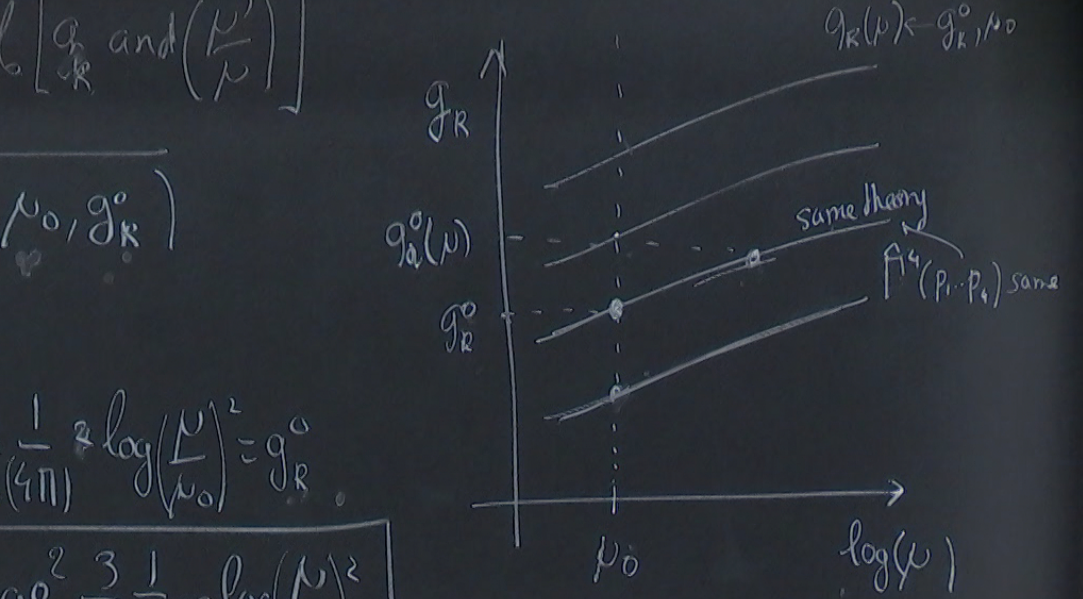
OR: start from (μ_0, g_R^0)

$$(g_R(\mu), \mu)$$

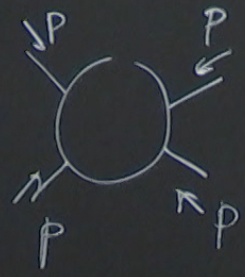
$$g_R(\mu) - g_R^2(\mu)\,\frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\mu}{\mu_0}\right)^2 = g_R^0$$

$$\left[g_R(\mu) = g_R^0 + g_R^{0\,2}\,\frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\mu}{\mu_0}\right)^2 \right]$$

$$\cdots \log\left(\frac{\mu'}{\mu}\right)^2 + O(g_R'^3) = g_R$$

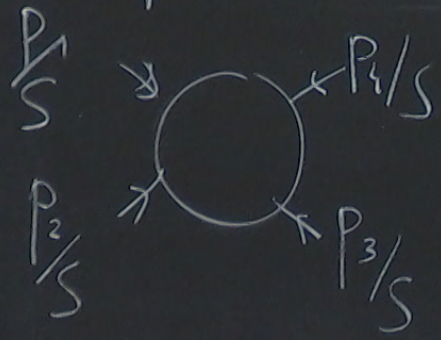


"Effective cc" or "Running cc"



distance → S·distance

momenta → $\frac{1}{S}$ momenta

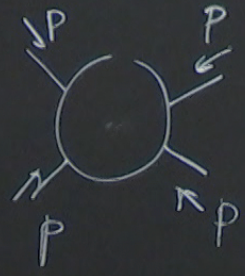


S > 1 small energy

S < 1 larger energy

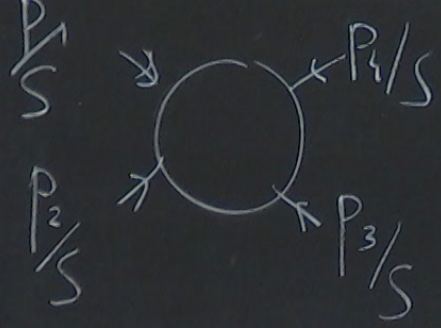
"Effective c.c." or "Running c.c."

"effective ccc" depends on the energy



distance → S·distance

momenta → $\frac{1}{S}$ momenta



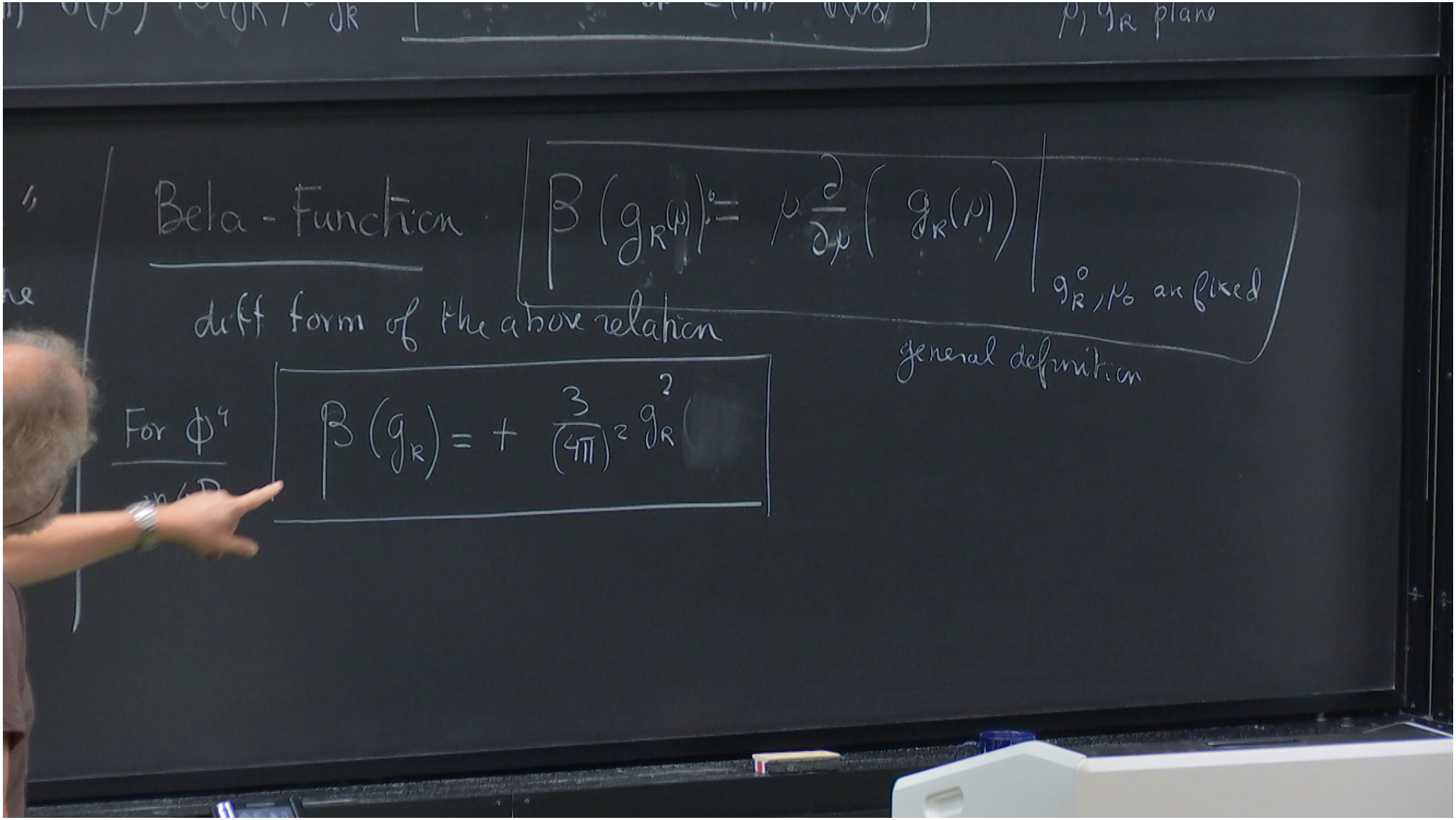
$S > 1$ small energy

$S < 1$ larger energy

g_R

↓

$g_R(s)$


Beta-Function
$\beta(g_R(\mu)) \equiv \mu \frac{\partial}{\partial \mu} g_R(\mu)$
g_R^0, μ_0 are fixed
general definition
diff form of the above relation
For ϕ^4
$\beta(g_R) = + \frac{3}{(4\pi)^2} g_R^2$

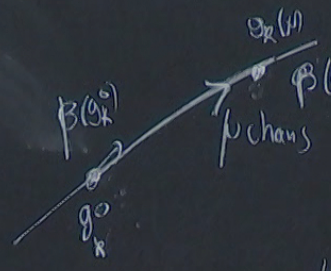
Beta-Function $\beta(g_R(\mu)) := \mu \frac{\partial}{\partial \mu} g_R(\mu)$ g_R^0, μ_0 are fixed

diff form of the above relation

For ϕ^4 in 4D $\beta(g_R) = + \frac{3}{(4\pi)^2} g_R^2$ (

Flow equation for $g_R(\mu)$

general definition



Higher loops Ren Program Wks . . -

Eff. coc $g_R(\mu)$

still Flow Equ. $\mu \frac{\partial}{\partial \mu} g_R(\mu) = \beta(g_R(\mu))$

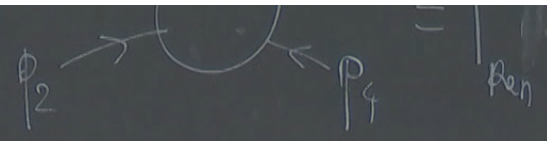
$\beta(g_R) = * g_R^2 + * g_R^3 + * g_R^4 + \ldots$

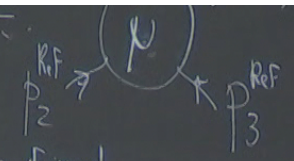
1 loop diagrams (1 loop +2 loop) 1/2/3 loop

finite

$\left[\log \frac{\mu^2}{(p_1+p_2)^2} \right] + O(g_R^3)$

any $\{p_i\}$





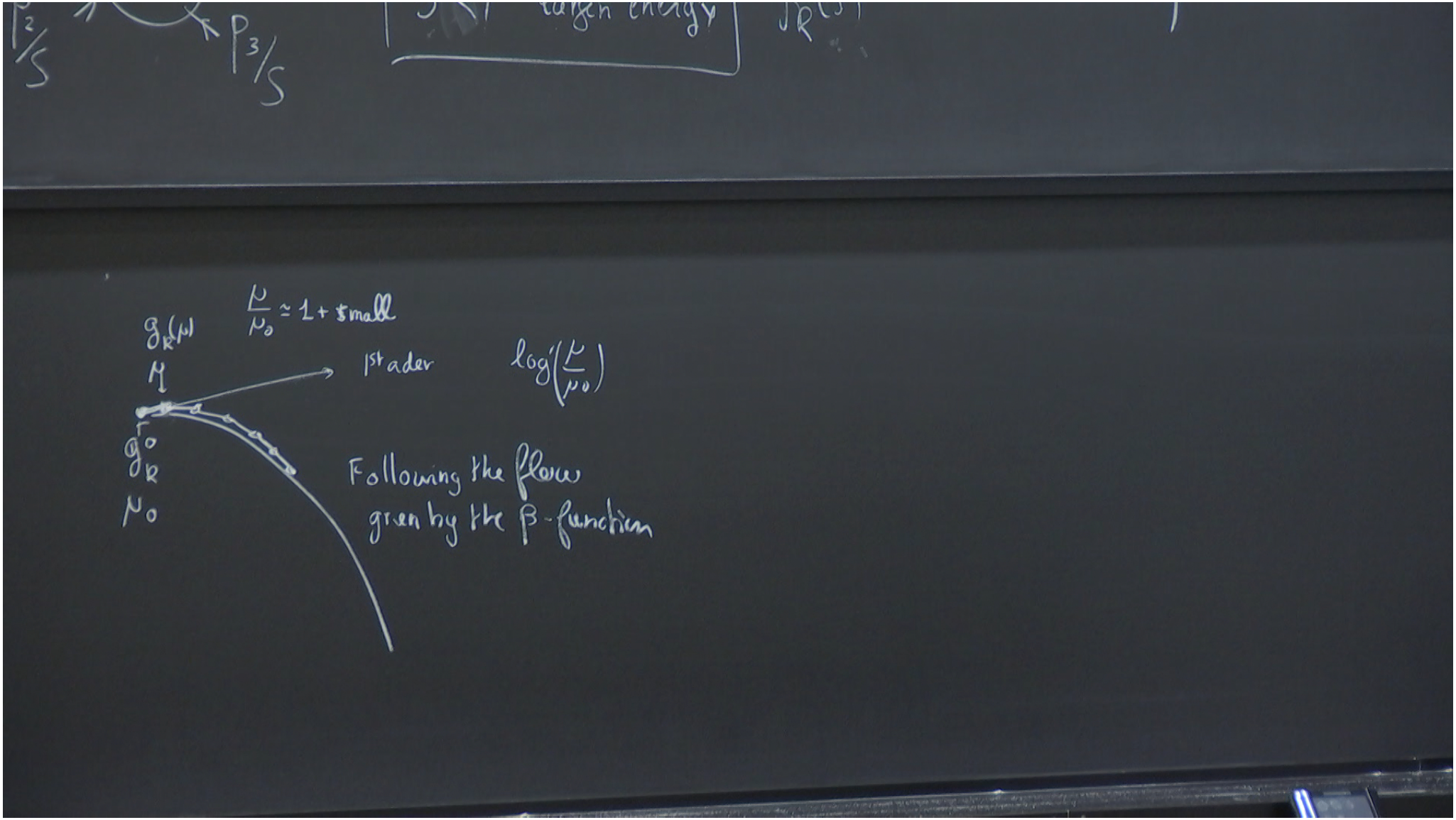
μ fixed

$$\mu \frac{\partial}{\partial \mu} g_R(\mu) = b_2 \, g_R(\mu)^2 \qquad b_2 = + \frac{3}{(4\pi)^2}$$

1 loop

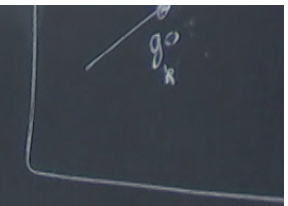
$$\frac{d\, g_R(\mu)}{g_R(\mu)^2} = b_2 \frac{d\mu}{\mu} \Rightarrow \frac{1}{g_R^0} - \frac{1}{g_R(\mu)} = b_2 \log\left(\frac{\mu}{\mu_0}\right) \Rightarrow \boxed{g_R(\mu) = \frac{g_R^0}{1 - g_R^0 \log\left(\frac{\mu}{\mu_0}\right)}}$$

$g_R(\mu)$ ↗ with μ high energy ⇒ stronger the effective coupling


$\frac{\mu}{\mu_0} \simeq 1 + \text{small}$
$g_R(\mu)$
μ
1st order
$\log\left(\frac{\mu}{\mu_0}\right)$
g_0
g_R
μ_0
Following the flows
given by the β-function

$g_R(s)$

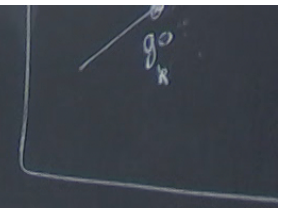
Flow equation for $g_R(\mu)$



If $\log\left(\frac{\mu}{\mu_0}\right) = \frac{1}{b_2\, g_R^0} \Rightarrow g_R(\mu) = \infty$

$g_R(s)$

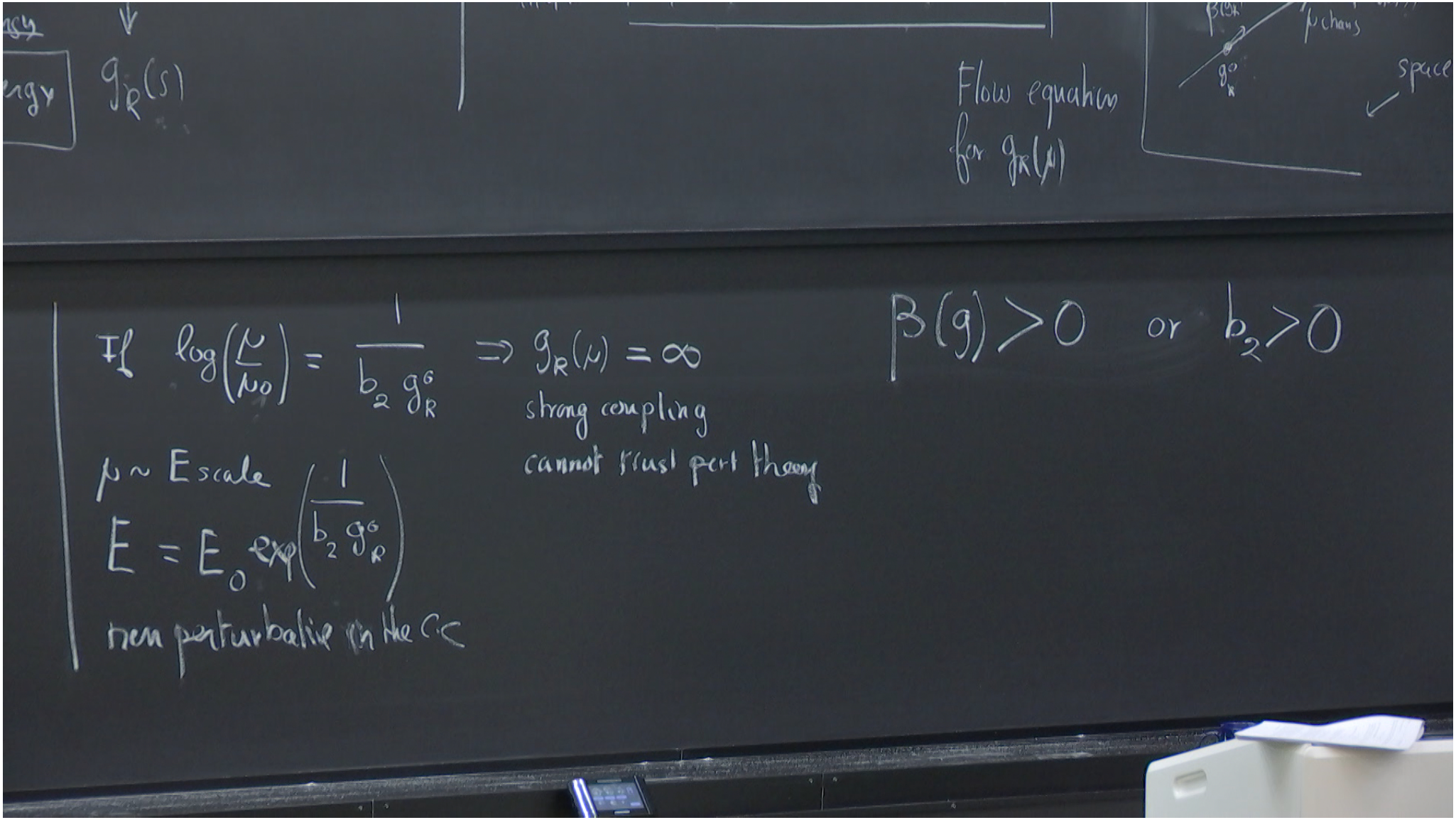
Flow equation for $g_R(\mu)$



If $\log\left(\frac{\mu}{\mu_0}\right) = \frac{1}{b_2\, g_R^0} \Rightarrow g_R(\mu) = \infty$

strong coupling

cannot trust pert theory


$g_R(s)$
Flow equation for $g_R(\mu)$
μ chaos
space
If $\log\left(\frac{\mu}{\mu_0}\right) = \frac{1}{b_2\, g_R^0} \Rightarrow g_R(\mu) = \infty$
strong coupling
cannot trust pert theory
$\beta(g) > 0$ or $b_2 > 0$
$\mu \sim$ E scale
$E = E_0 \exp\left(\frac{1}{b_2\, g_R^0}\right)$
non perturbative in the c.c