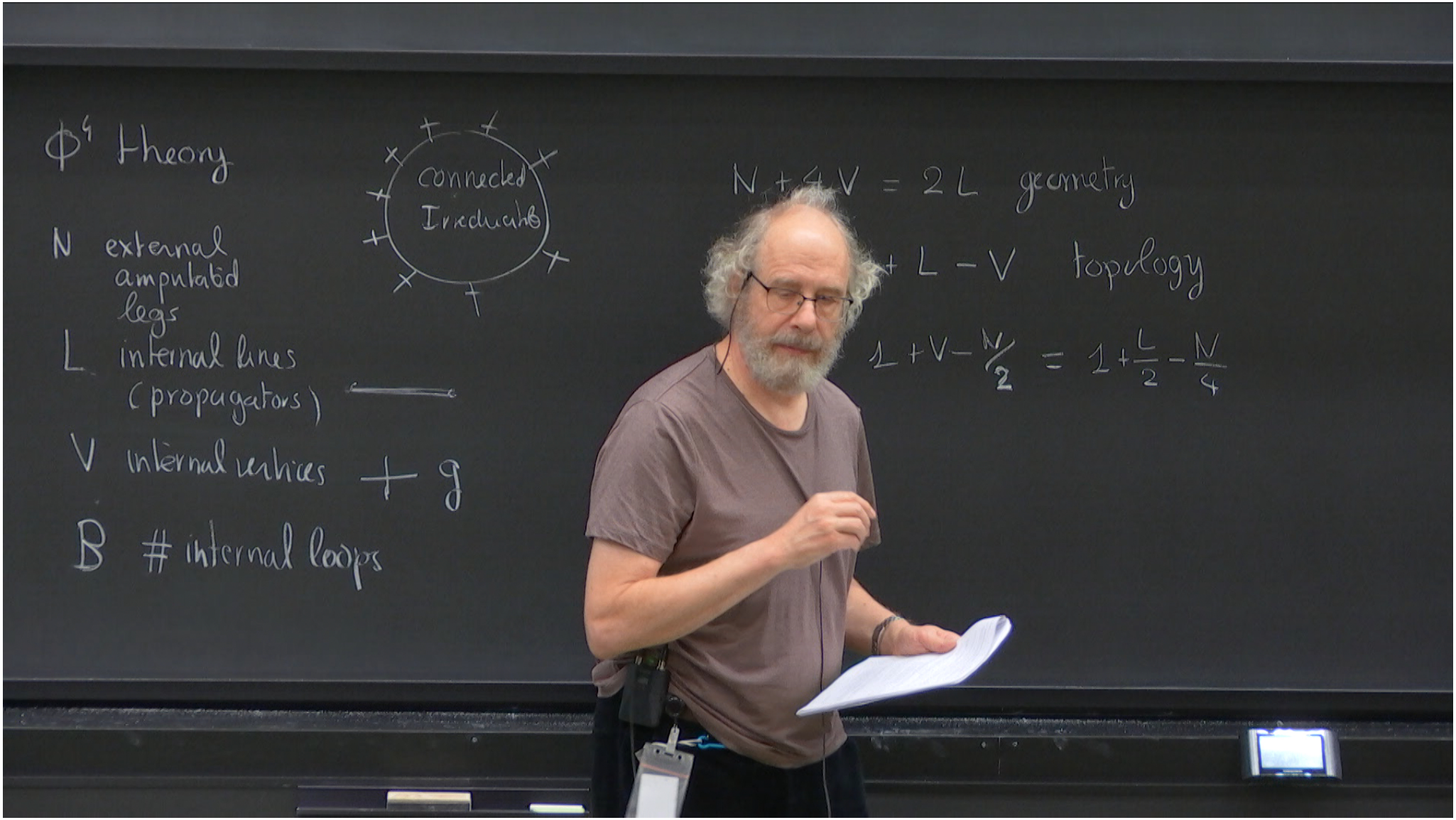
Title: Quantum Field Theory II - Lecture 221206

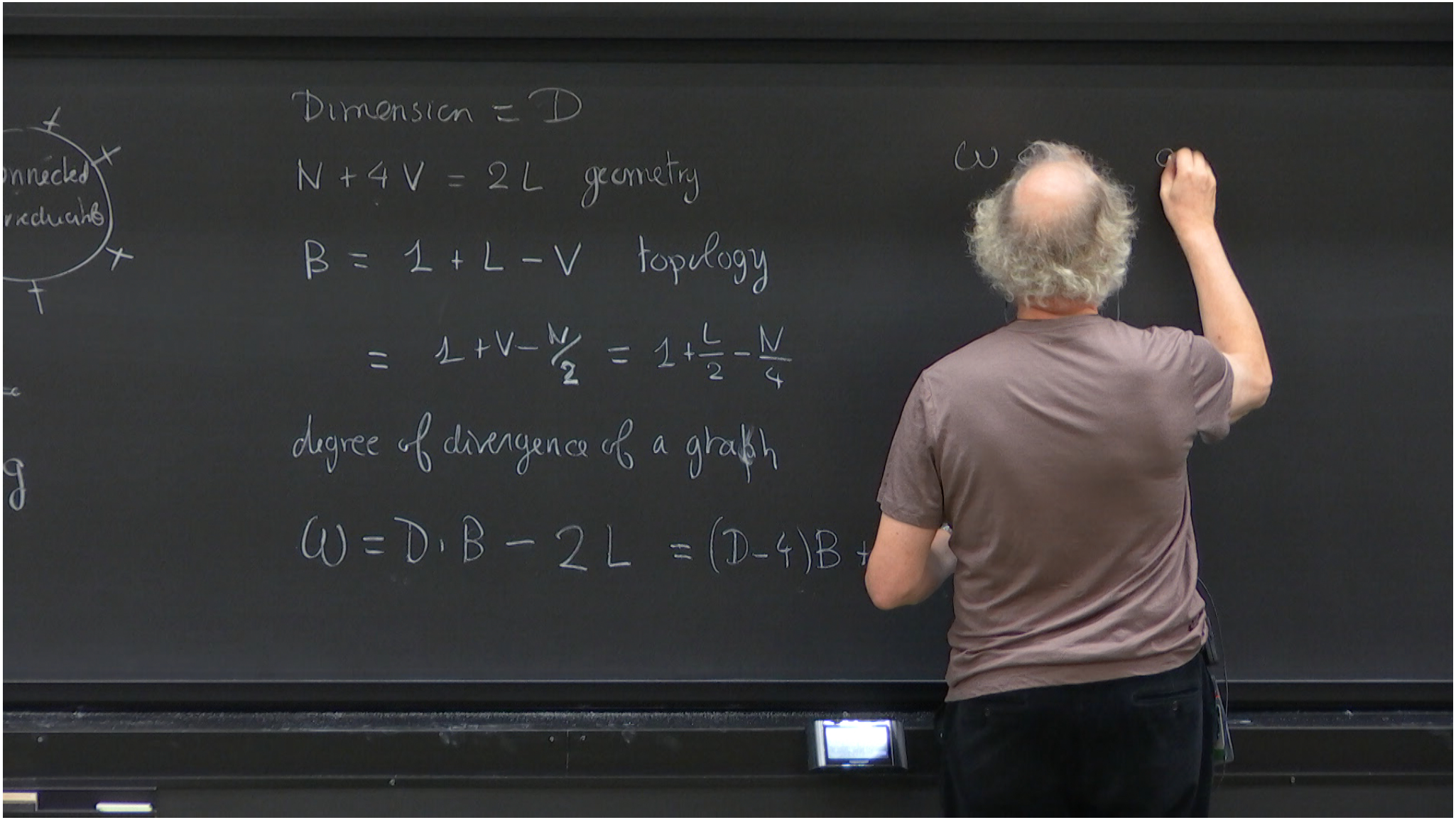
Speakers:

Collection: Quantum Field Theory II (2022/2023)

Date: December 06, 2022 - 10:45 AM

URL: https://pirsa.org/22120002


ϕ^4 theory
connected
Irreducible
N external amputated legs
L internal lines (propagators)
V internal vertices
B # internal loops
$N + 4V = 2L$ geometry
$+ L - V$ topology
$1 + V - \frac{N}{2} = 1 + \frac{L}{2} - \frac{N}{4}$

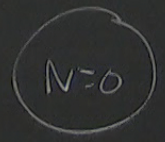

Dimension = D
N + 4V = 2L geometry
B = 1 + L − V topology
= 1 + V − N/2 = 1 + L/2 − N/4
degree of divergence of a graph
ω = D · B − 2L = (D − 4)B +

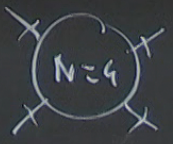
$\omega < 0$ graph convergent

$\omega = 0$ marginally divergent

$\omega > 0$ divergent



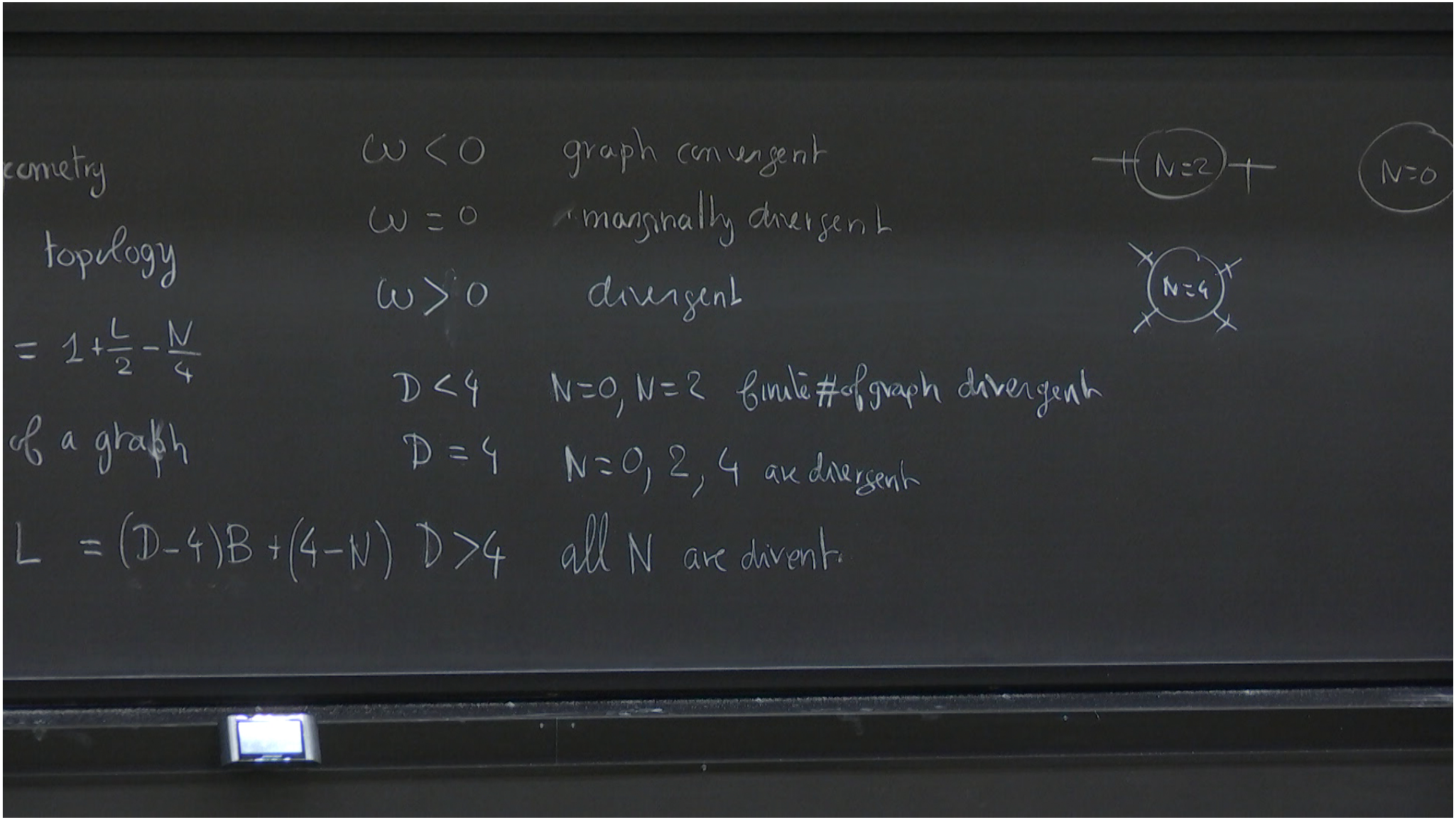




$D < 4$ $N = 0, N = 2$ finite # of graph divergent

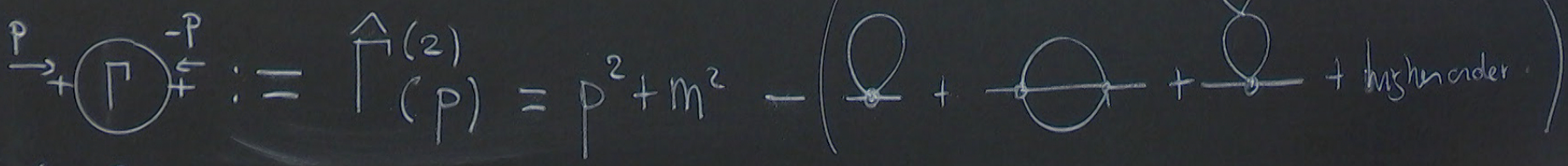
$D = 4$ $N = 0, 2, 4$ are divergent

$)B + (4 - N)\ D > 4$

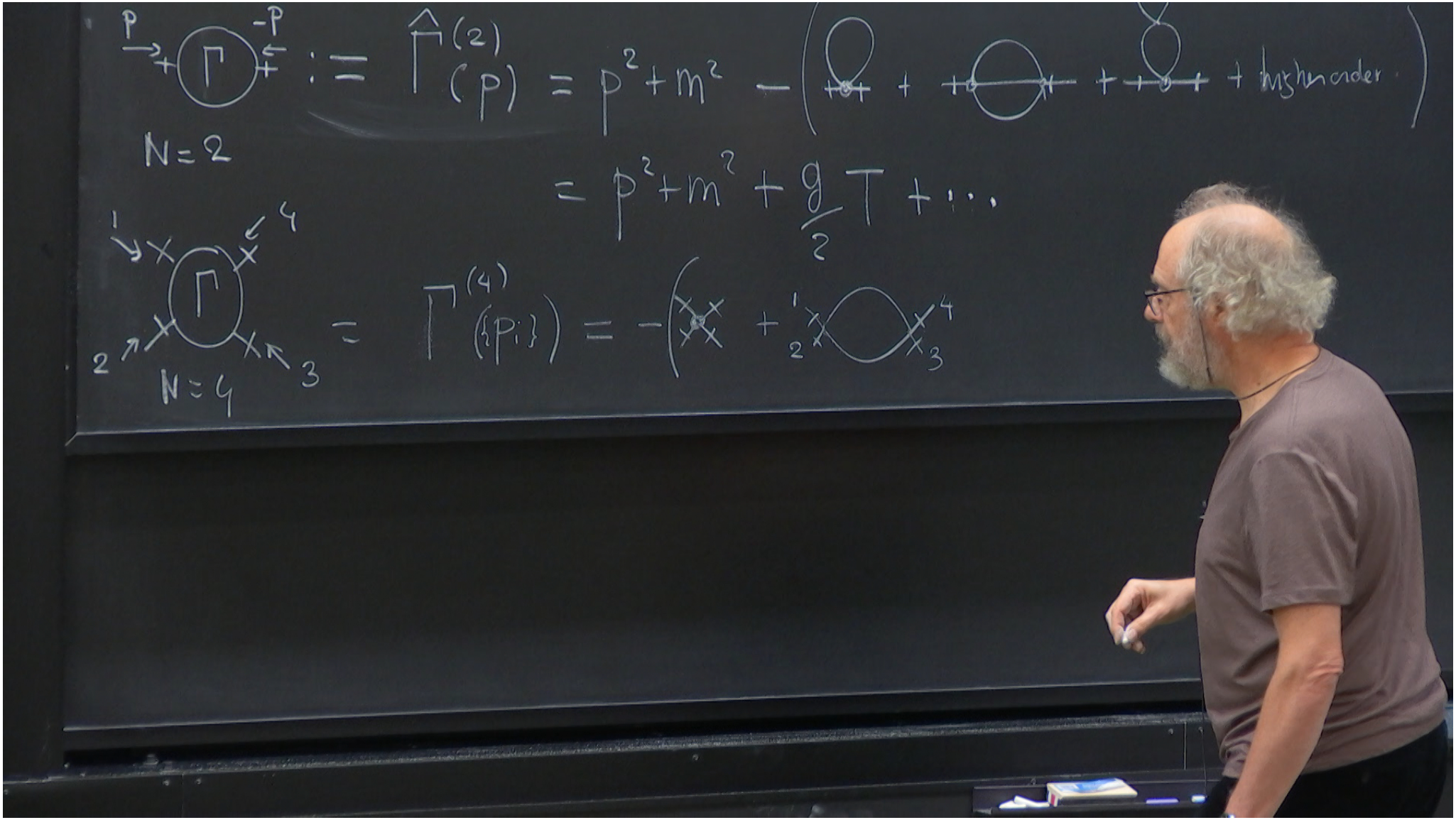

ometry
topology
= 1 + L/2 - N/4
of a graph
L = (D-4)B + (4-N)
ω < 0 graph convergent
ω = 0 marginally divergent
ω > 0 divergent
D < 4 N=0, N=2 finite # of graph divergent
D = 4 N=0, 2, 4 are divergent
D > 4 all N are divert.
N=2
N=0
N=4

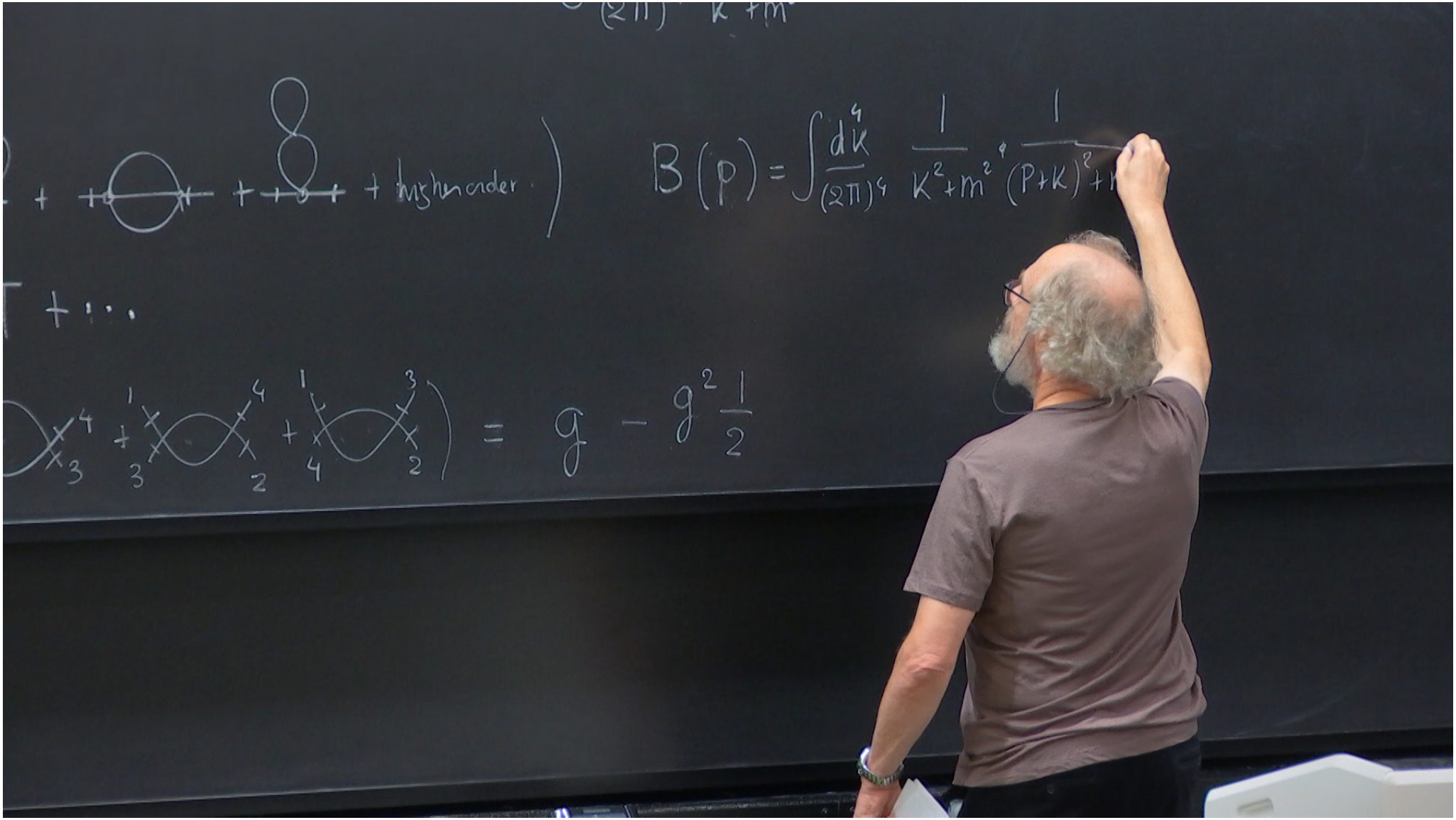
ϕ^4 in $D=4$, massless Euclidean

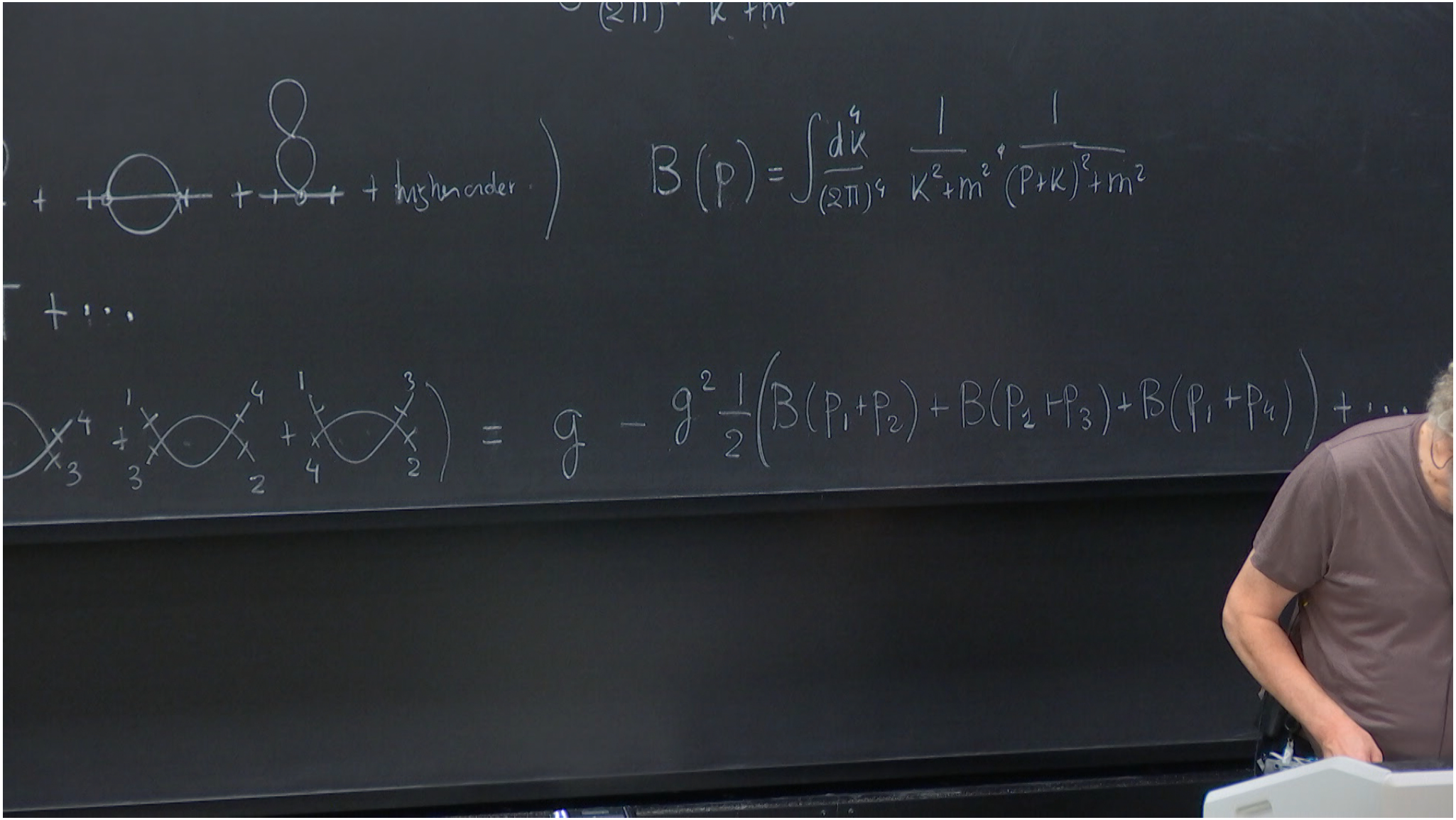
Irreducible Functions $\Gamma^{(N)}$

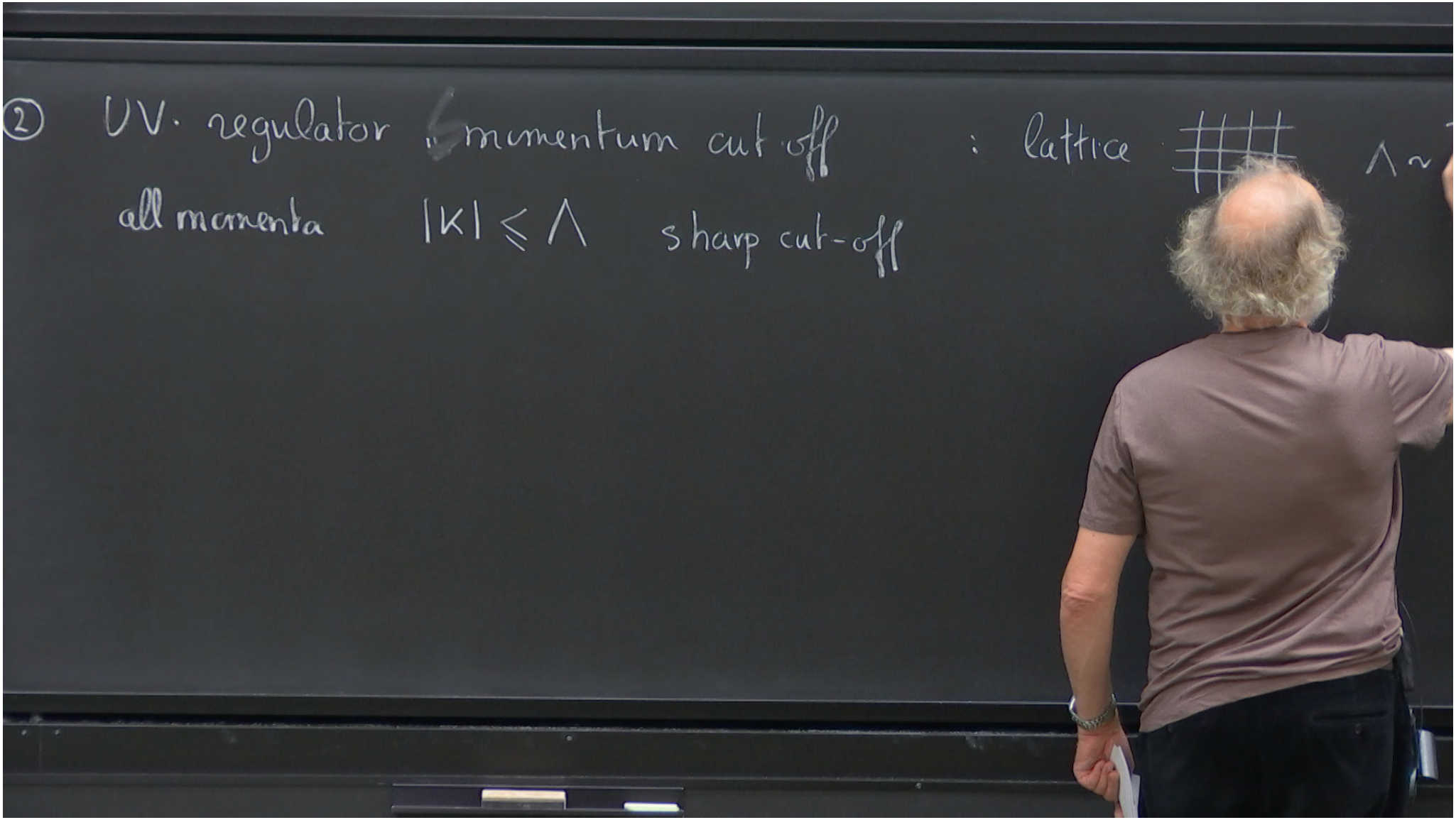


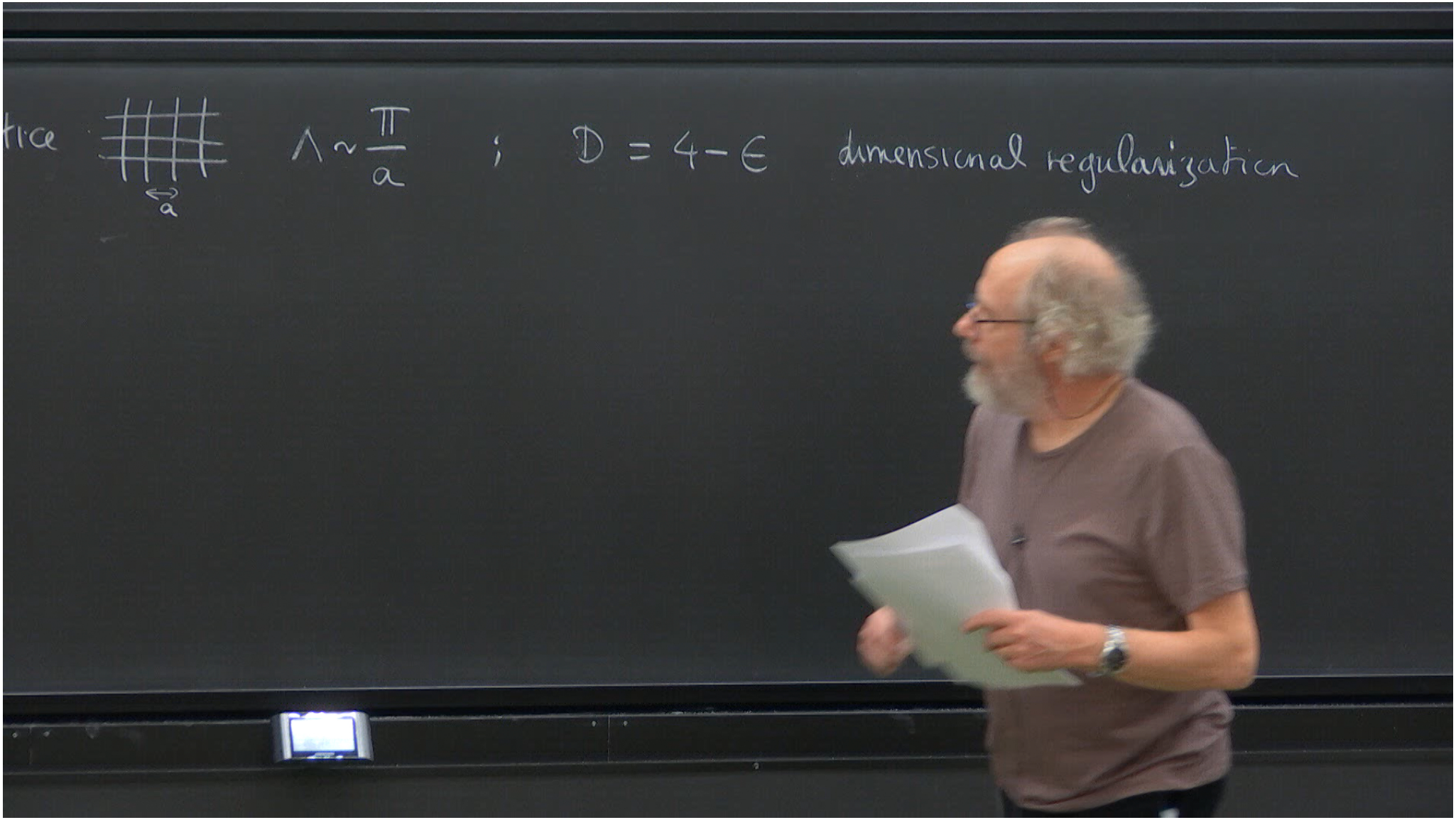
$N=2$

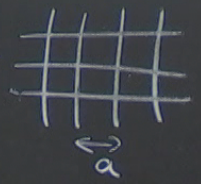

p
-p
Γ
:= Γ̂(2)(p) = p² + m² − (+ + + higher order)
N = 2
= p² + m² + g/2 T + ...
1
2
3
4
Γ
= Γ(4)({pi}) = −(+ 1 2 3 4)
N = 4


+ + + + higher order)
B(p) = ∫ d⁴k/(2π)⁴ 1/(k²+m²) · 1/((p+k)²+
+ ...
= g − g² 1/2

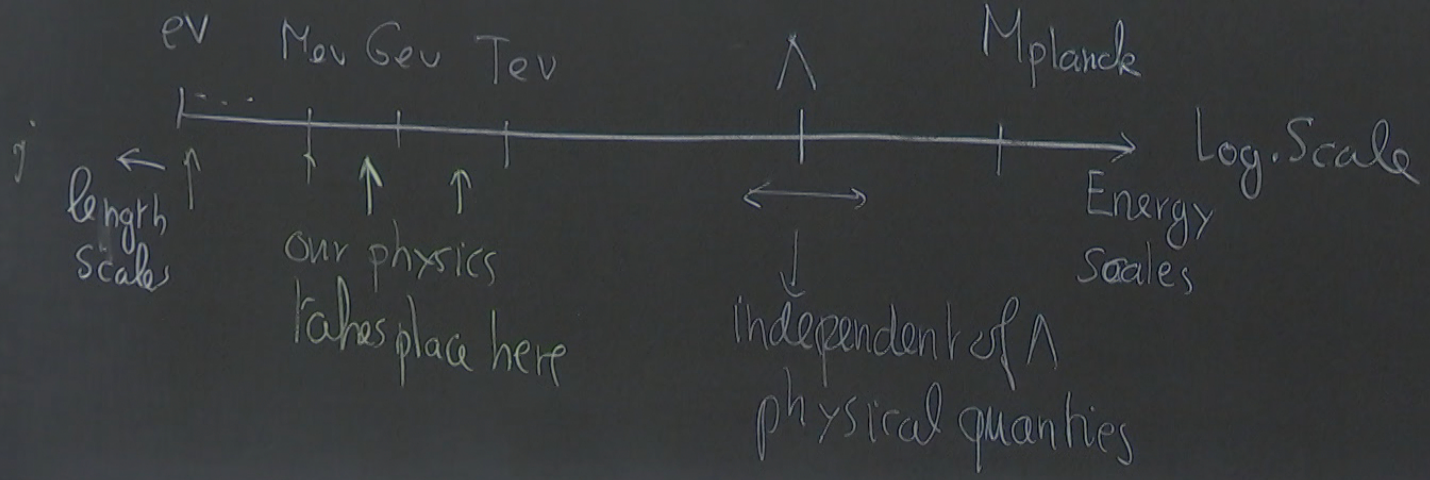

+ + + + + higher order)
B(p) = ∫ d^4k/(2π)^4 1/(k^2+m^2) · 1/((p+k)^2+m^2)
+ ...
= g − g^2 1/2 (B(p_1+p_2) + B(p_2+p_3) + B(p_1+p_4)) + ...


② UV. regulator : momentum cut-off : lattice ### $\Lambda \sim$
all momenta $|k| \leqslant \Lambda$ sharp cut-off


tice
a
$\Lambda \sim \frac{\pi}{a}$; $D = 4 - \epsilon$ dimensional regularization

② UV. regulator : momentum cut-off : lattice 

all momenta $|k| \leq \Lambda$ sharp cut-off



$\Lambda \sim \frac{\pi}{a}$; $D = 4 - \epsilon$ dimensional regularization

n calculation

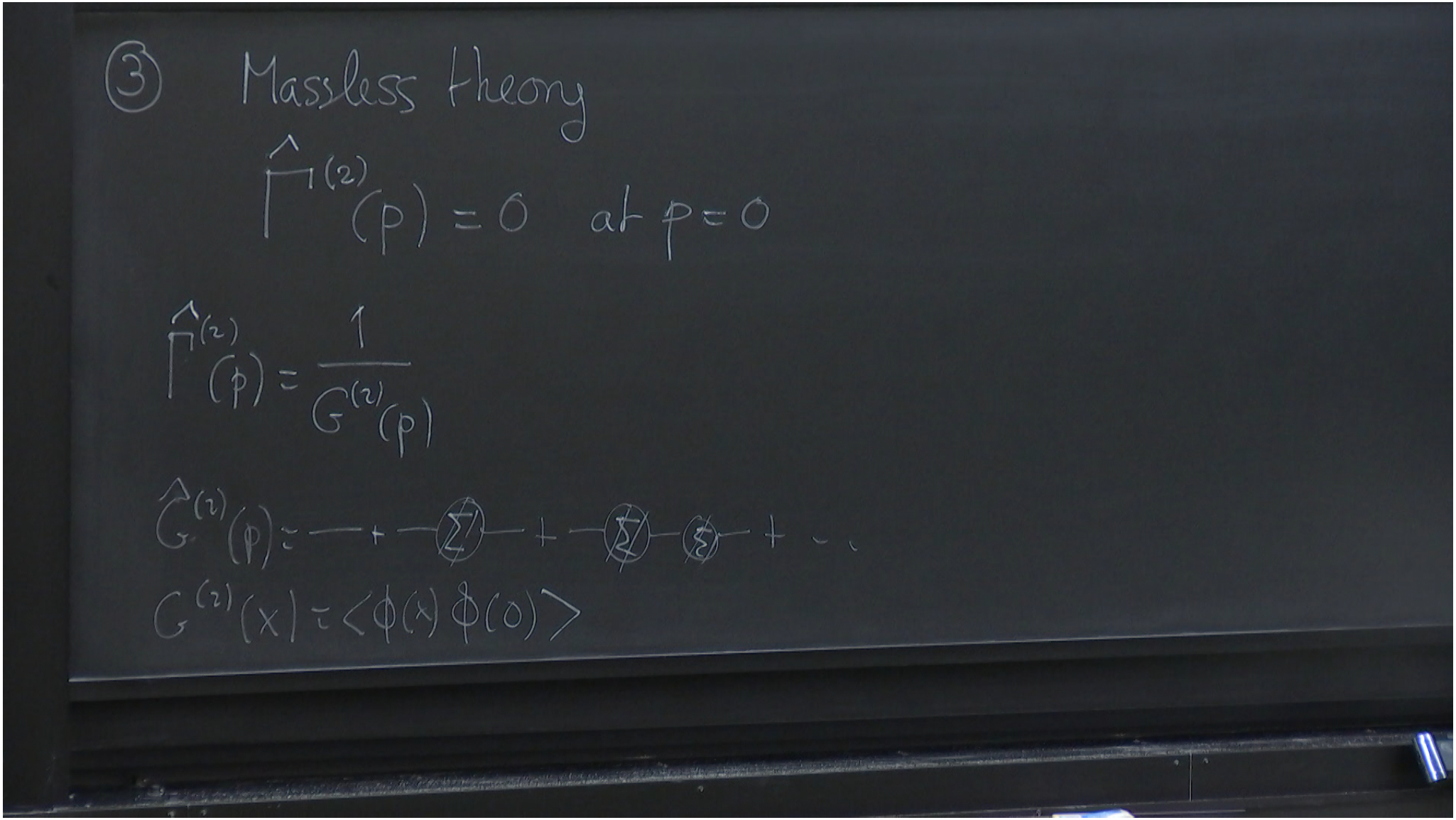
energy/mass scale

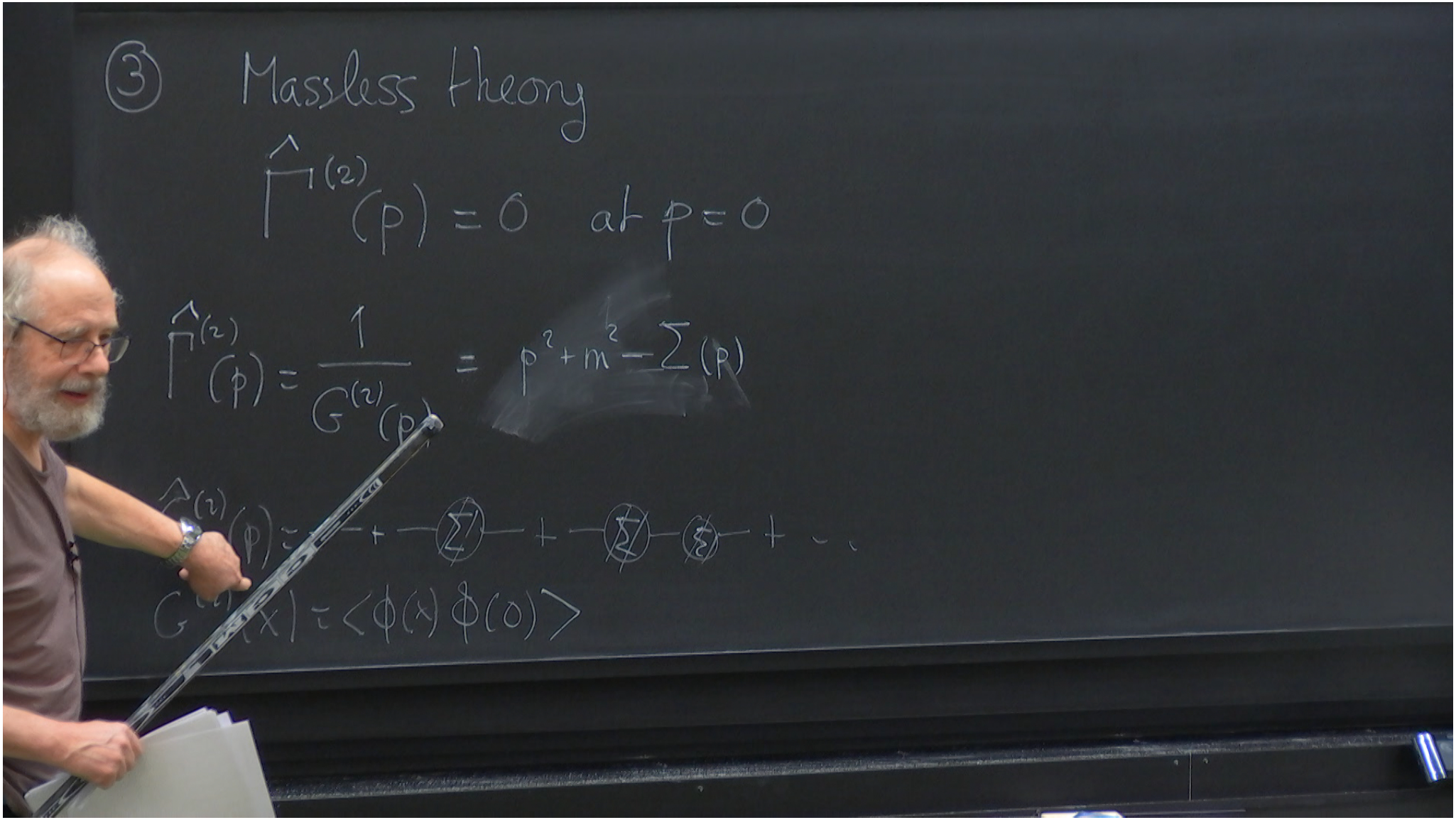
chian scale $\ll \Lambda$

in QED $m_e \neq 0$

in YM, in massless ϕ^4

no "natural" choice of Λ

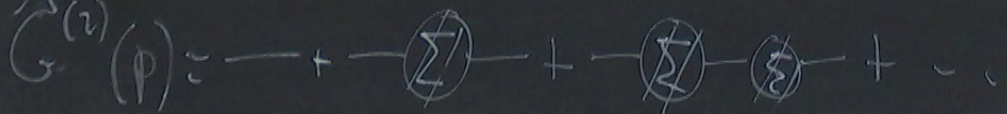

③ Massless theory
$\hat{\Gamma}^{(2)}(p) = 0$ at $p = 0$
$\hat{\Gamma}^{(2)}(p) = \frac{1}{G^{(2)}(p)}$
$\hat{G}^{(2)}(p) =$ + + + + ...
$G^{(2)}(x) = \langle \phi(x)\, \phi(0) \rangle$


③ Massless theory
$\hat{\Gamma}^{(2)}(p) = 0$ at $p = 0$
$\hat{\Gamma}^{(2)}(p) = \frac{1}{G^{(2)}(p)} = p^2 + m^2 - \Sigma(p)$
$G^{(2)}(x) = \langle \phi(x)\phi(0) \rangle$

③ Massless theory ; physical mass = 0

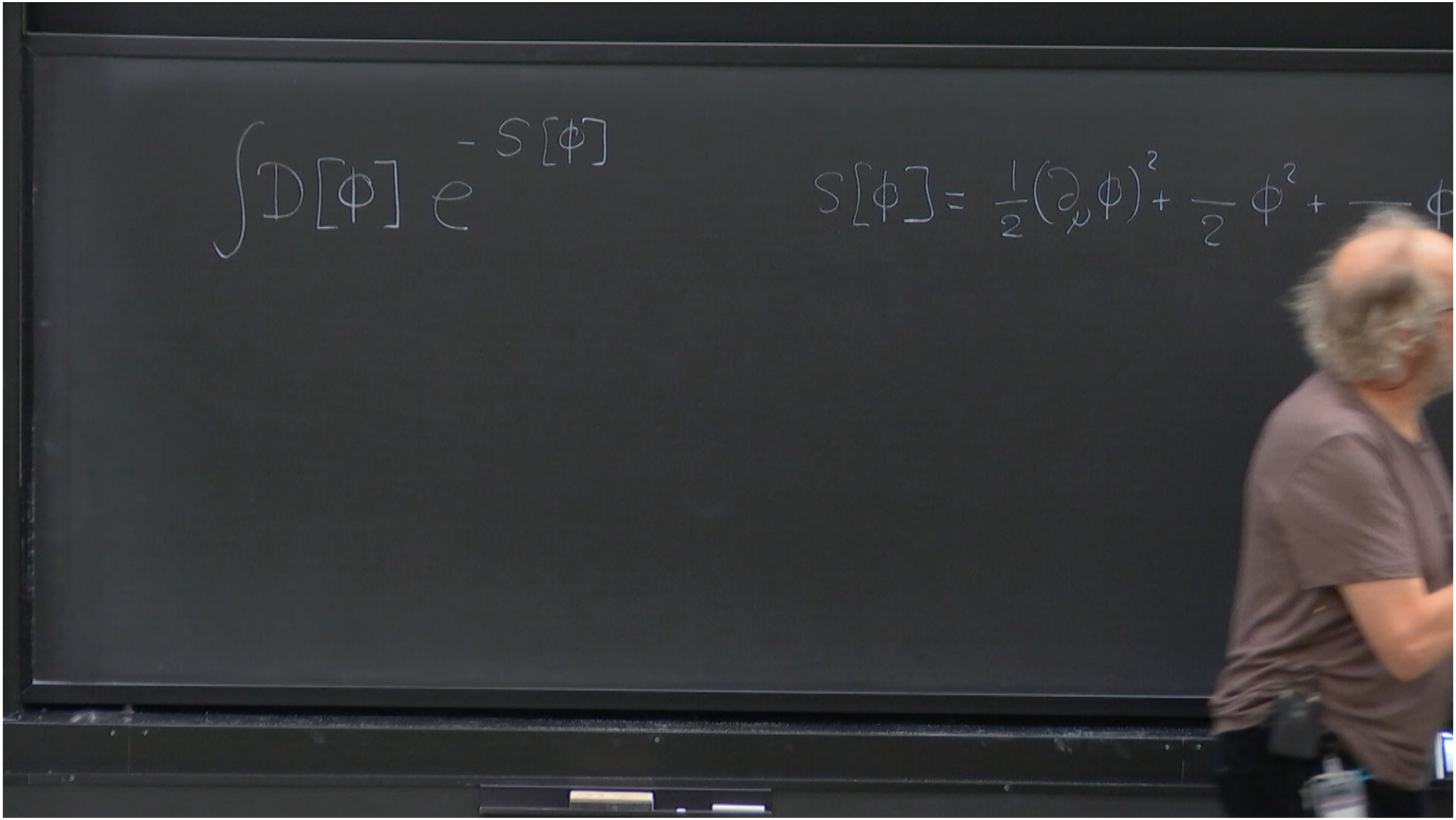
$\hat{\Gamma}^{(2)}(p) = 0$ at $p = 0$

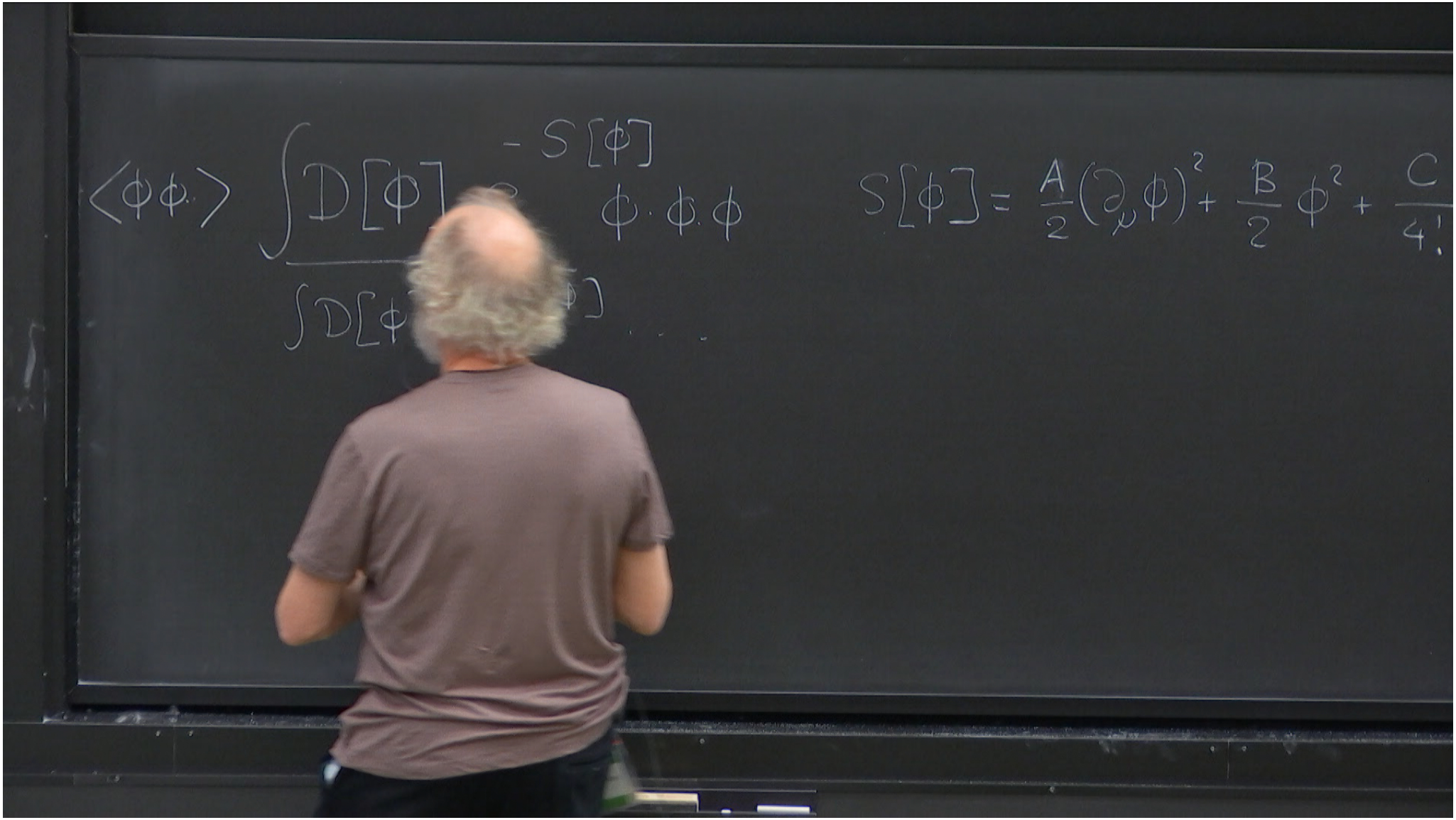
$\hat{\Gamma}^{(2)}(p) = \frac{1}{G^{(2)}(p)} = p^2 + m^2 - \Sigma(p)$

$\hat{G}^{(2)}(p) =$  $+ \ldots$

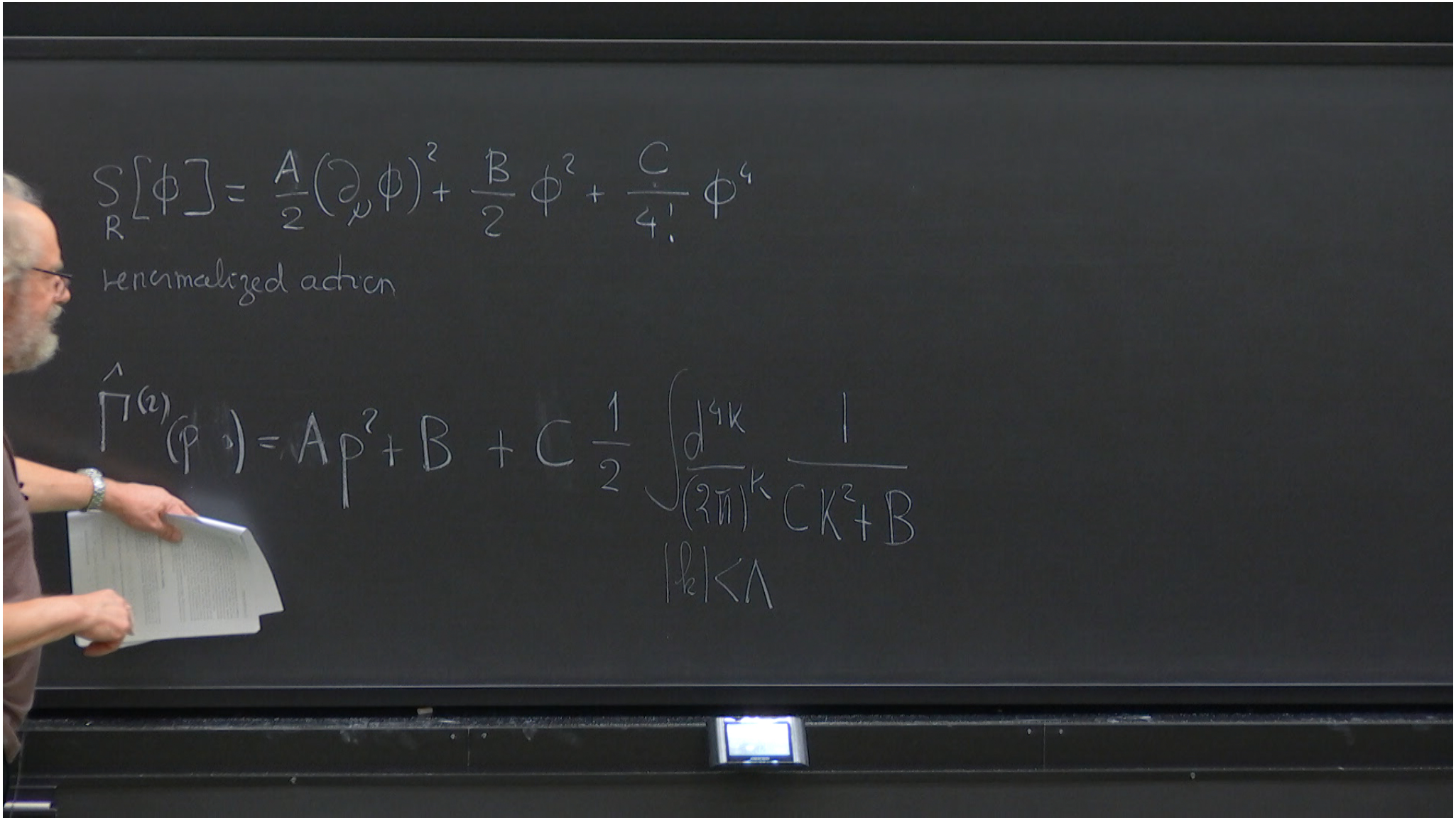
$G^{(2)}(x) = \langle \Omega | \phi(x)\, \phi(0) | \Omega \rangle$

Renormalization of mass


$\int \mathcal{D}[\phi]\, e^{-S[\phi]}$
$S[\phi] = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{}{2}\phi^2 +$


$\langle \phi\phi\cdot \rangle \int \mathcal{D}[\phi]$
$- S[\phi]$
$\phi \cdot \phi \cdot \phi$
$S[\phi] = \frac{A}{2}(\partial_\mu \phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}$

$$\langle \phi\phi\ldots \rangle = \frac{\int \mathcal{D}[\phi]\, e^{-S[\phi]}\, \phi\cdot\phi\ldots}{\int \mathcal{D}[\phi]\, e^{-S[\phi]}} \qquad S[\phi] = \frac{A}{2}(\partial_\mu\phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}$$


$S_R[\phi] = \frac{A}{2}(\partial_\mu \phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4$
renormalized action
$\hat{\Gamma}^{(2)}(p) = Ap^2 + B + C\frac{1}{2}\int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{Ck^2 + B}$

$$\langle \phi\phi \rangle \quad \frac{\int \mathcal{D}[\phi]\, e^{-S_R[\phi]}\, \phi\cdot\phi}{\int \mathcal{D}[\phi]\, e^{-S[\phi]}}$$

$$S_R[\phi] = \frac{A}{2}(\partial_\mu \phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi$$

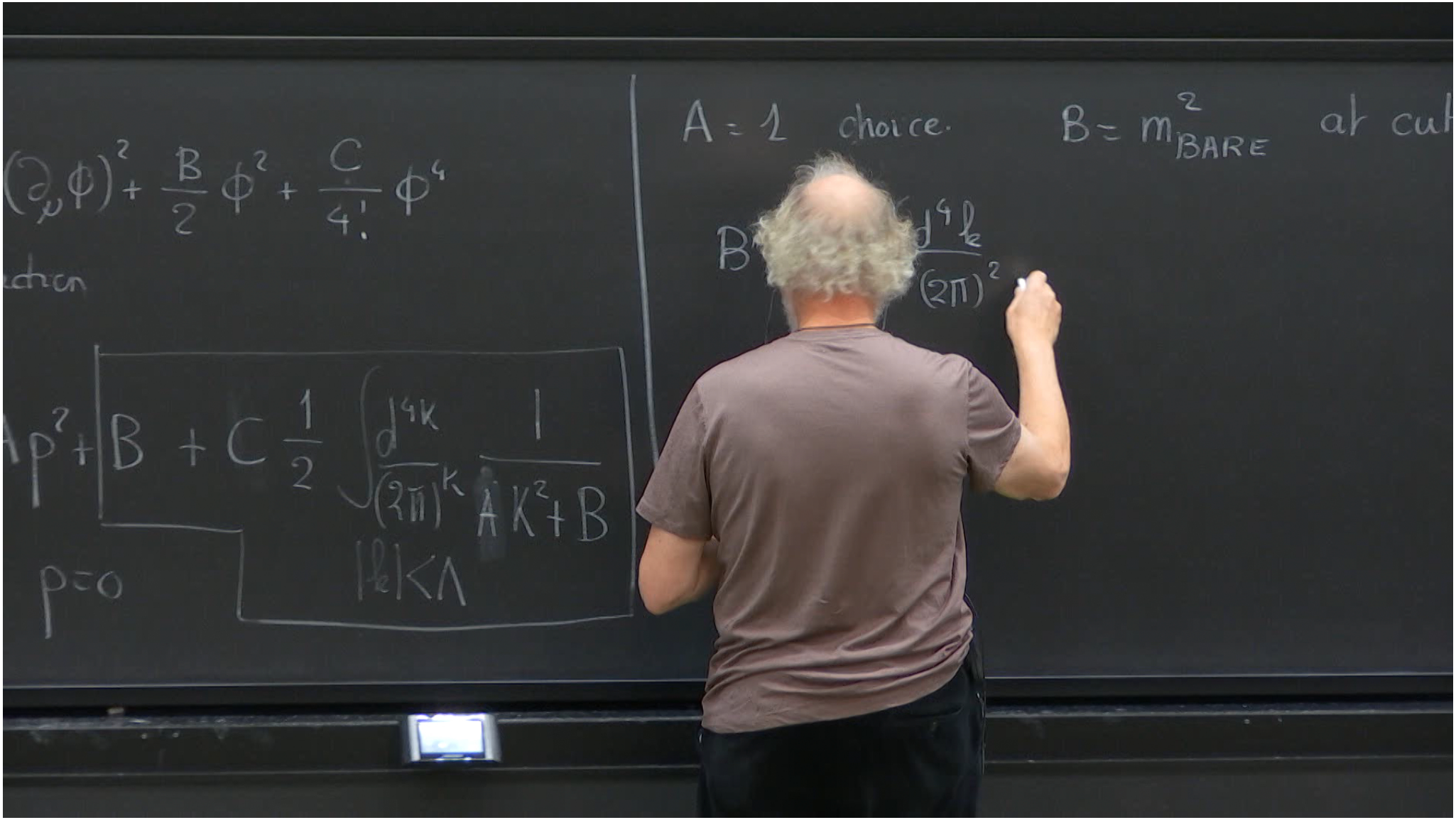
renormalized action

2 renormalization conditions

$$-A\cdot\Delta_x + B \qquad Ap^2 + B$$

$$\hat{\Gamma}^{(2)}(p) = Ap^2 + B + C\frac{1}{2}\int \frac{d^4}{(2\pi}$$

$$\hat{\Gamma}^{(2)}(p) = 0 \quad p = 0 \qquad |k| <$$


$(\partial_\mu\phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4$
action
$A = 1$ choice.
$B = m^2_{BARE}$ at cut
$B + C\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{Ak^2+B}$
$|k_E| < \Lambda$
$p = 0$

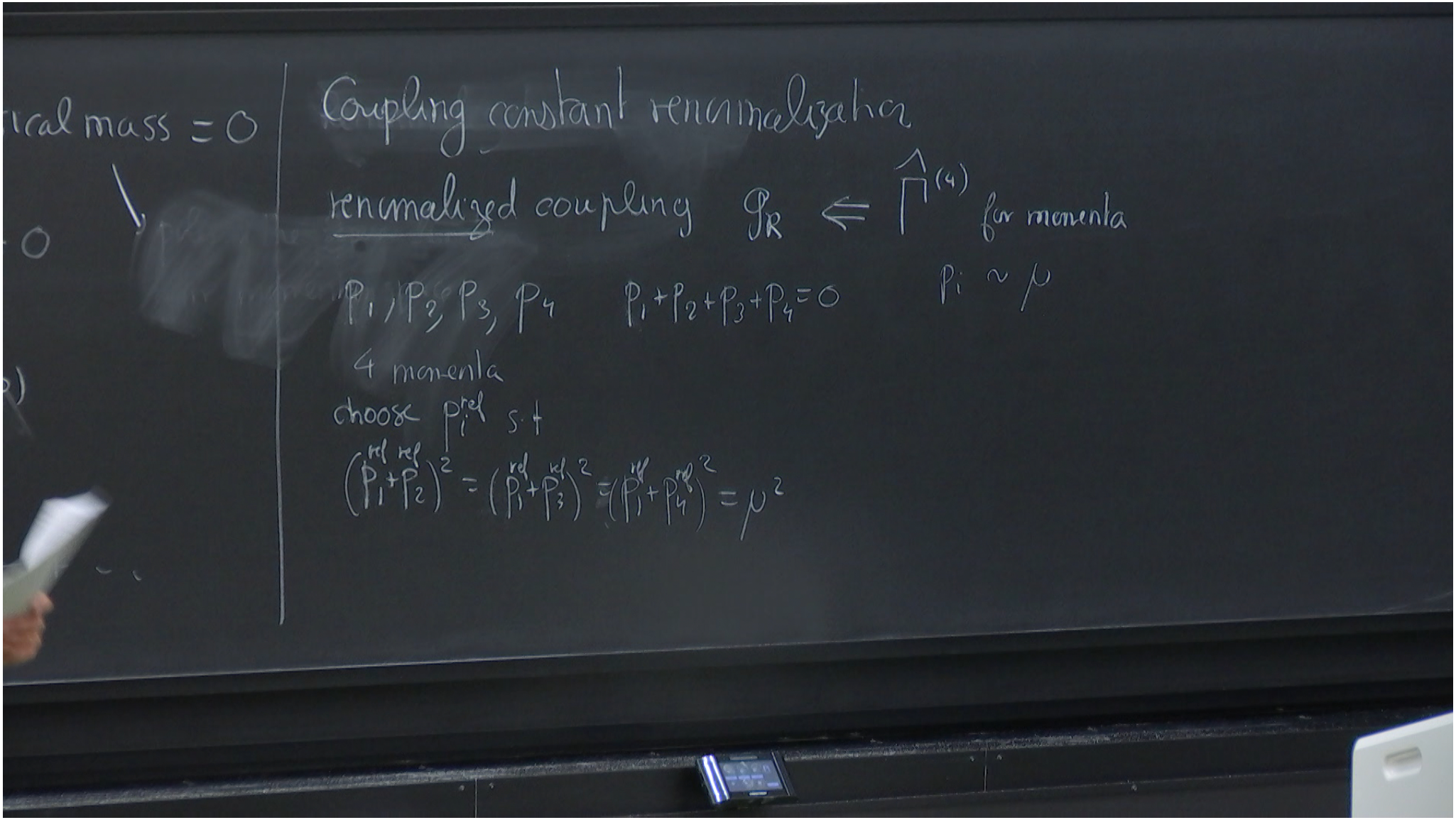
$A = 1$ choice. $B = m^2_{BARE}$ at cut-off scale Λ

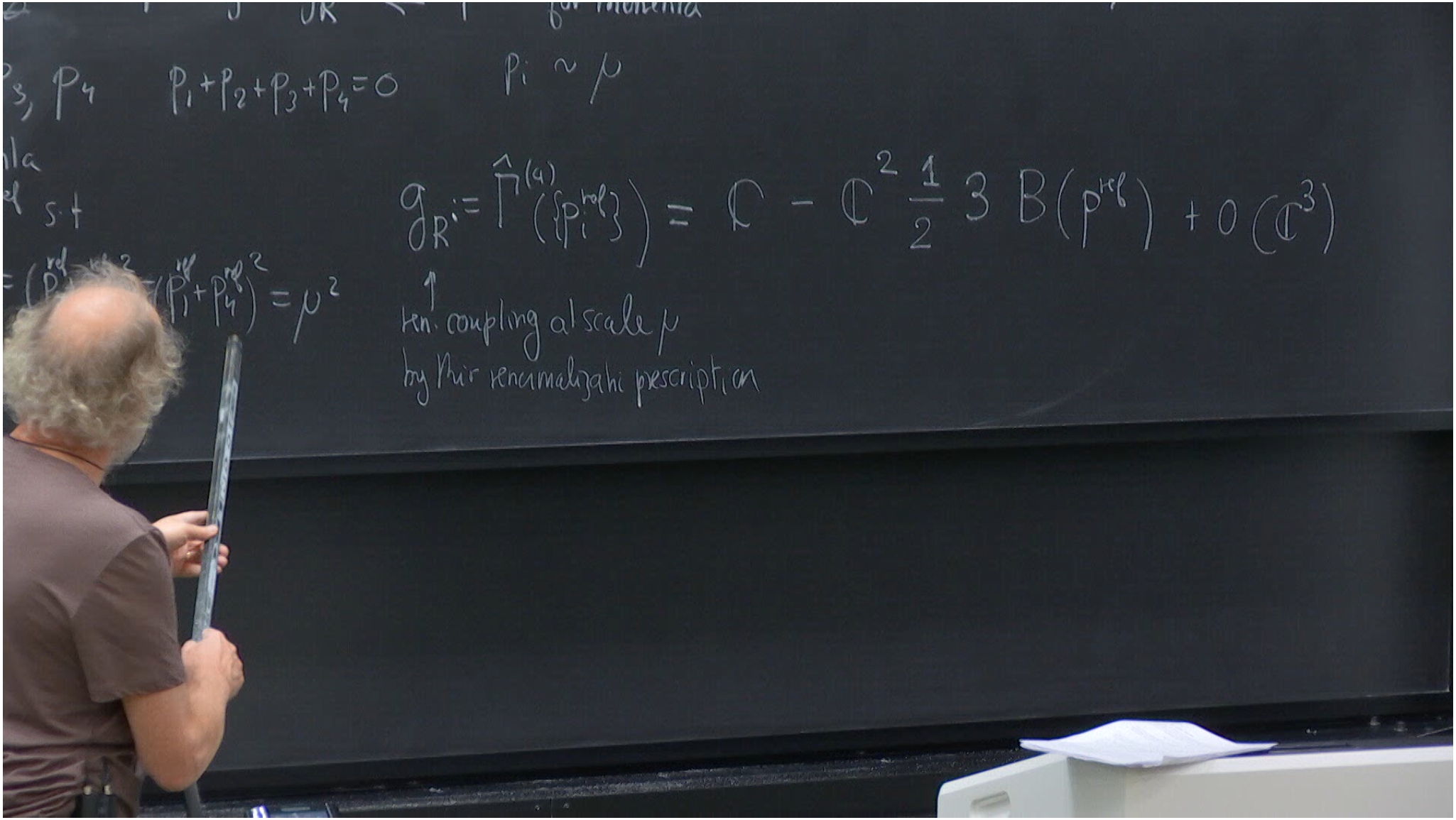
$$B + C\frac{1}{2}\int_{|k|<\Lambda}\frac{d^4K}{(2\pi)^4}\frac{1}{K^2} \quad \text{forget} \quad = B + C\frac{1}{2}\frac{1}{(4\pi)^2}\Lambda^2 = 0$$

$$\Rightarrow B = -C\,\frac{1}{2}\frac{1}{(4\pi)^2}\Lambda^2$$

$$B = m^2_{BARE} \neq 0$$

$$\frac{1}{K^2 + B} = 0$$


ical mass = 0
Coupling constant renormalization
renormalized coupling $g_R \Leftarrow \hat{\Gamma}^{(4)}$ for momenta
p_1, p_2, p_3, p_4 $p_1+p_2+p_3+p_4=0$ $p_i \sim \mu$
4 momenta
choose p_i^{ref} s.t
$(p_1^{ref}+p_2^{ref})^2 = (p_1^{ref}+p_3^{ref})^2 = (p_1^{ref}+p_4^{ref})^2 = \mu^2$

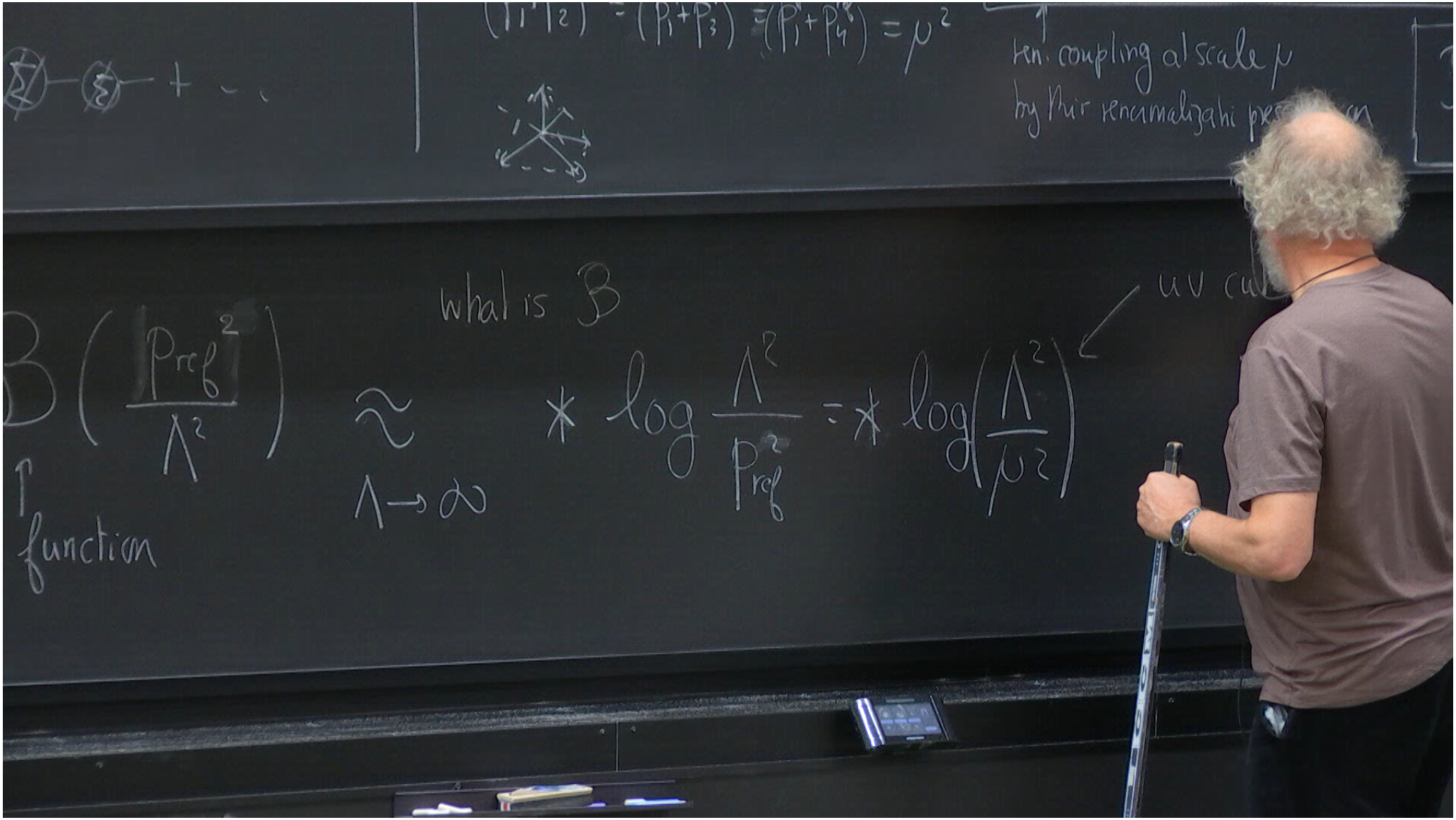

$p_1+p_2+p_3+p_4=0$
$p_i \sim \mu$
$g_{R^i} = \hat{\Gamma}^{(4)}(\{p_i^{ref}\}) = C - C^2 \frac{1}{2} 3 B(p^{ref}) + O(C^3)$
$(p_1+p_4)^2 = \mu^2$
ren. coupling at scale μ
by their renormalization prescription

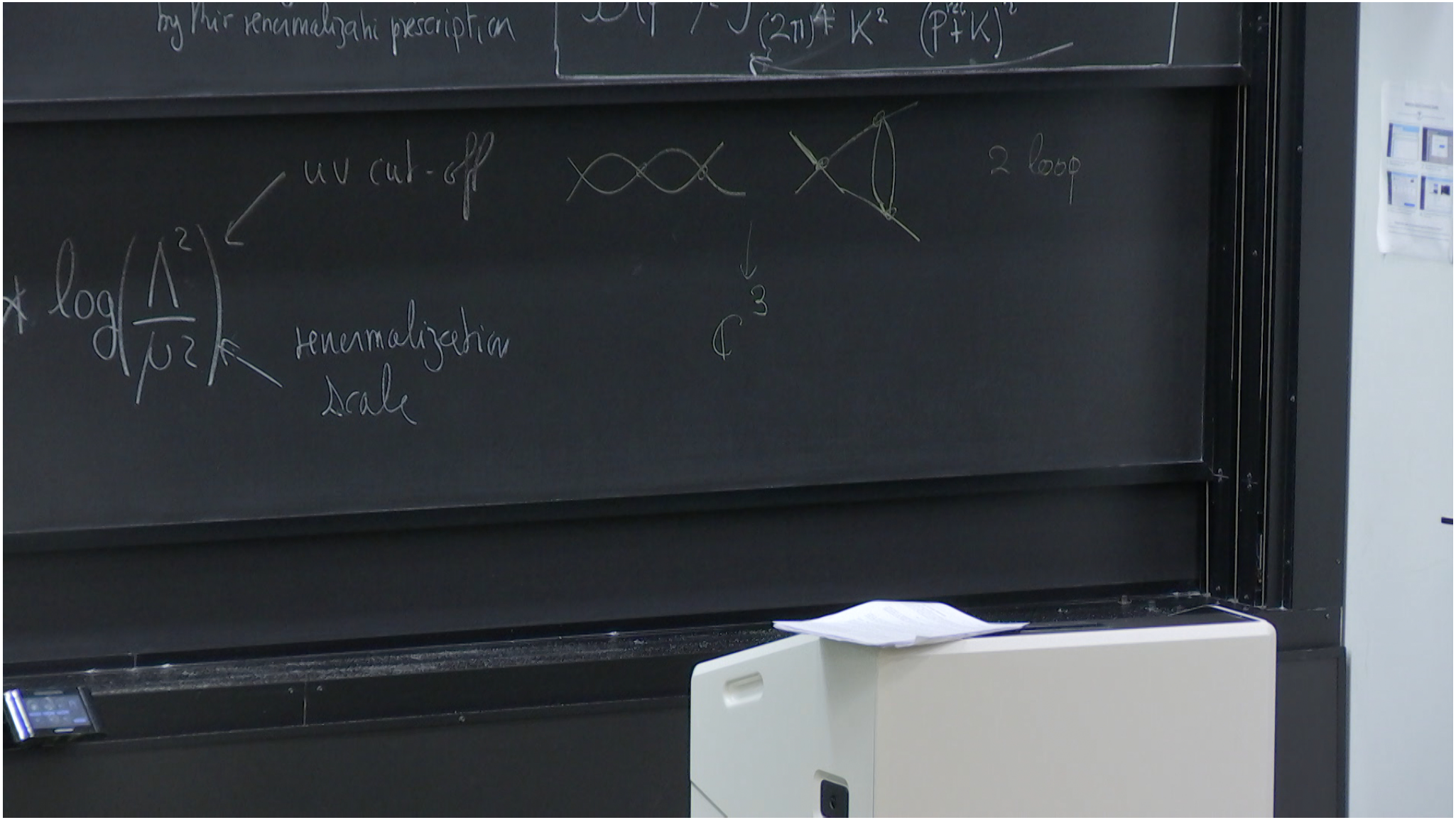
p_4 $p_1+p_2+p_3+p_4=0$ $p_i \sim \mu$

$$g_R := \hat{\Gamma}^{(4)}(\{p_i^{rel}\}) = \mathbb{C} - \mathbb{C}^2 \frac{1}{2} 3\, \mathcal{B}(p^{rel}) + O(\mathbb{C}^3)$$

ren. coupling at scale μ by their renormalization prescription

$$\mathcal{B}(p^{rel}) = \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^2} \frac{1}{(p^{rel}+K)^2} \qquad |k| < \Lambda$$


ren. coupling at scale μ
by Wilsonian renormalization
what is B
B(Proj² / Λ²)
≈
Λ→∞
* log Λ²/Proj² = * log(Λ²/μ²)
β function
UV cut


by their renormalizabi prescription
uv cut-off
2 loop
renormalization scale