Title: Quantum Field Theory II - Lecture 221205

Speakers:

Collection: Quantum Field Theory II (2022/2023)

Date: December 05, 2022 - 10:45 AM

URL: https://pirsa.org/22120001

Y.M. theories: (discussion)

A^a_μ gauge fields, $\bar{c}^a$, c^a "ghost fields"

a SU(2) group index

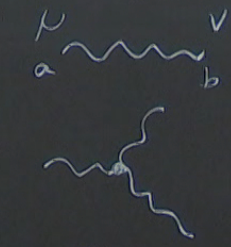
μ Lorentz indices

+ Gauge fixing condition $\partial^\mu A^a_\mu = 0$

$$\Rightarrow \mathcal{L}_{\text{gauge fixed}} = \underbrace{\frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu}}_{=} + \frac{1}{2\alpha}(\partial^\mu A_\mu)^2 + (\partial^\mu \bar{c}^a)(D_\mu c_a)$$

contains A

gauge field



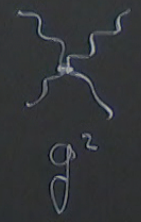
g

ghost



as in QED

δ_{ab} $G_{\mu\nu}$



g^2

Y.M. theories: (discussion)

A^a_μ gauge fields / $\bar{c}^a$, c^a "ghost fields"

a SU(2) group index

μ Lorentz indices

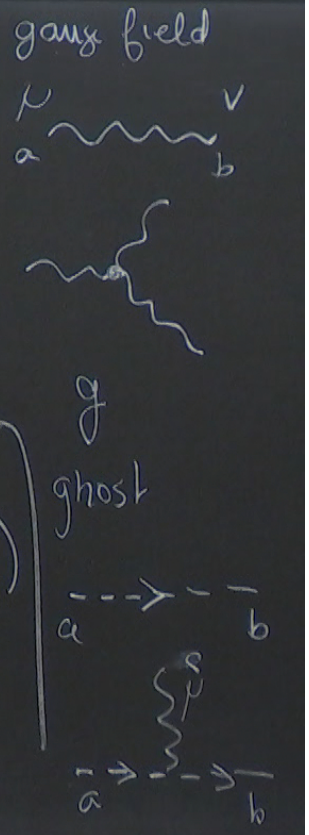
+ Gauge fixing condition

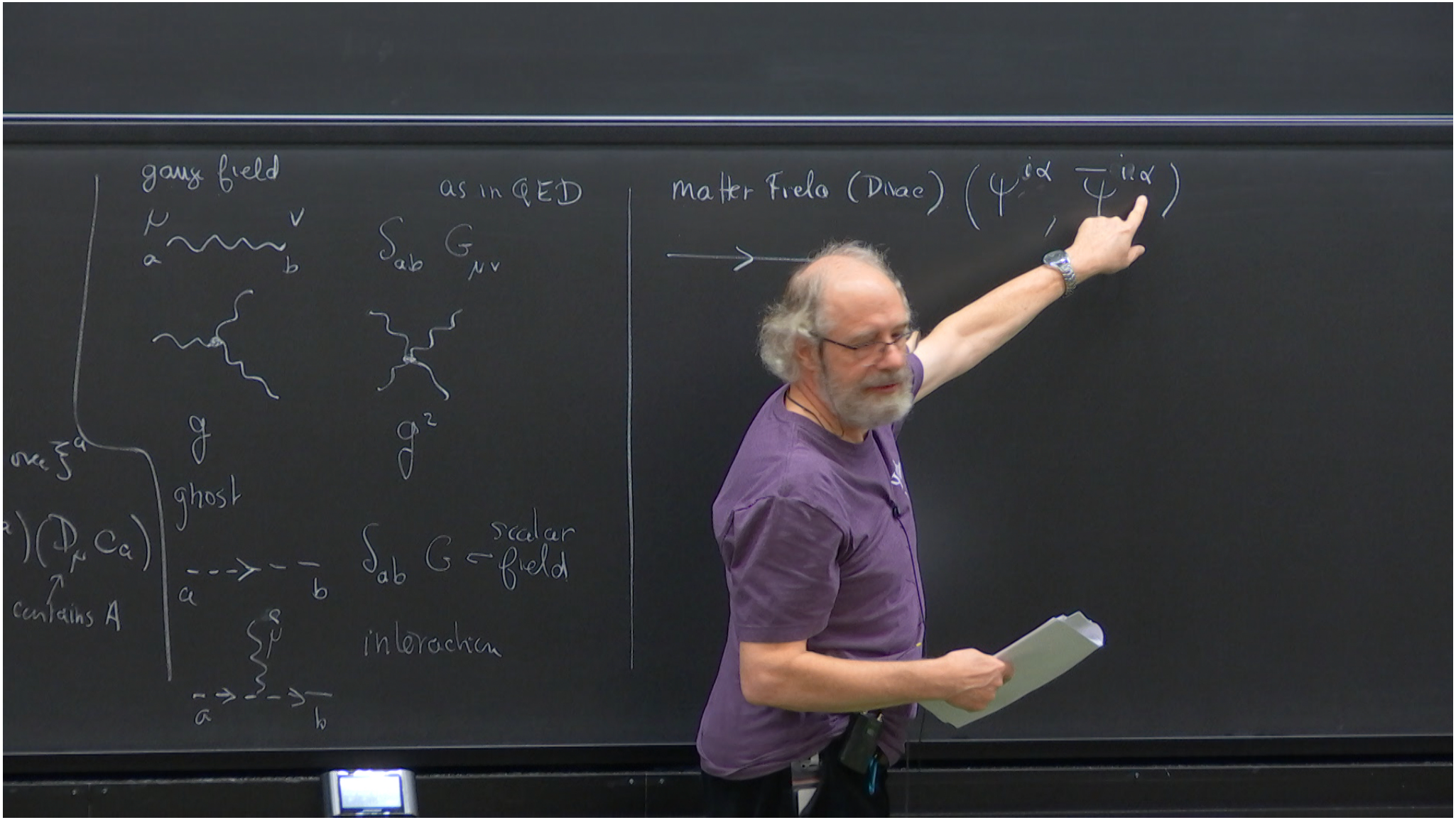
$\partial^\mu A^a_\mu = \xi^a$; integration over ξ^a

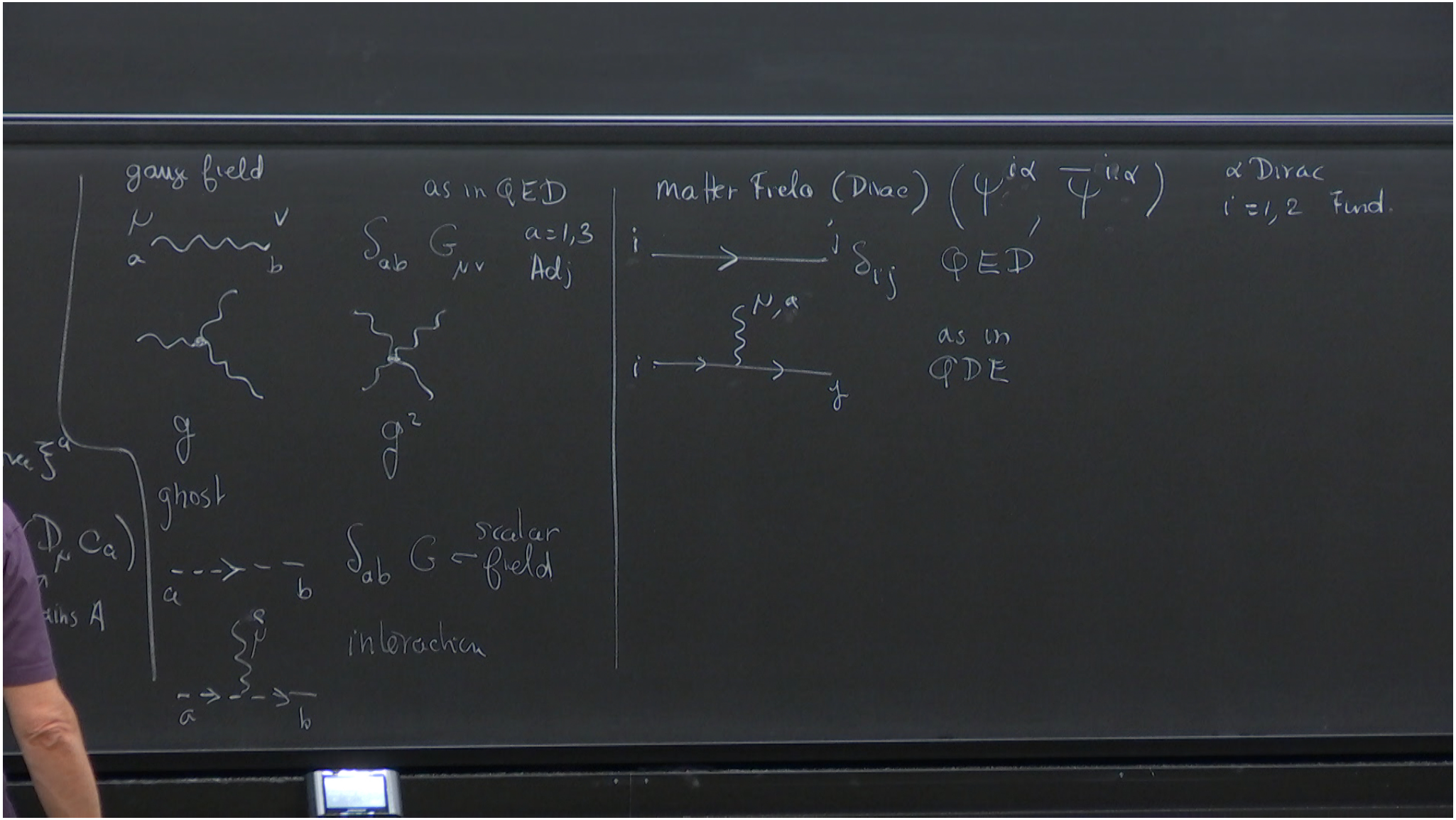
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Feynman Gauge "Sym. Braking" term

contains A



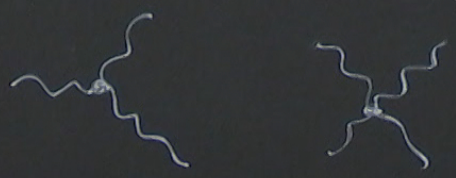

gauge field
as in QED
matter Field (Dirac)
$\delta_{ab}\, G_{\mu\nu}$
g
g^2
ghost
$\delta_{ab}\, G$ ← scalar field
interaction
contains A


gauge field
as in QED
δ_{ab} $G_{\mu\nu}$ a=1,3 Adj
g
g^2
ghost
δ_{ab} G ← scalar field
interaction
matter Fields (Dirac) $(\psi^{i\alpha}, \bar{\psi}^{i,\alpha})$
α Dirac
i=1,2 Fund.
δ_{ij} QED
as in QDE

gauge field

as in QED

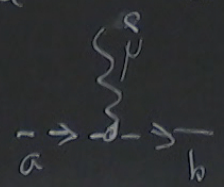
$\delta_{ab}\ G_{\mu\nu}$ $a=1,3$ Adj



g $\quad g^2$

ghost

$\delta_{ab}\ G \leftarrow$ scalar field

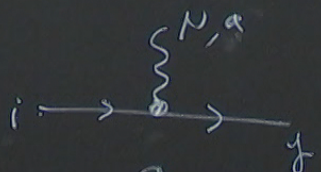


interaction $\quad g$

matter Fields (Dirac) $\left(\psi^{i\alpha},\ \bar{\psi}^{i\alpha}\right)$ α Dirac $i=1,2$ Fund.



δ_{ij} QED



g $\quad$ as in QDE

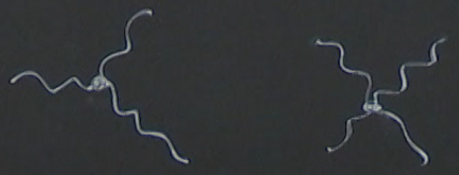
one coupling constant g = charge

gauge field



as in QED

δ_{ab} $G_{\mu\nu}$ $a=1,3$ Adj

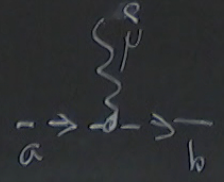


g g^2

ghost



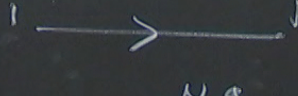
δ_{ab} G ← scalar field



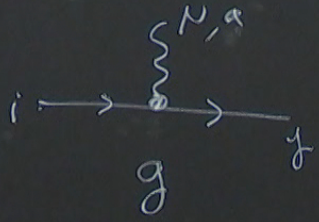
interaction

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matter Fields (Dirac) $(\psi^{i\alpha}, \bar{\psi}^{i,\alpha})$ α Dirac $i=1,2$ Fund.



δ_{ij} QED



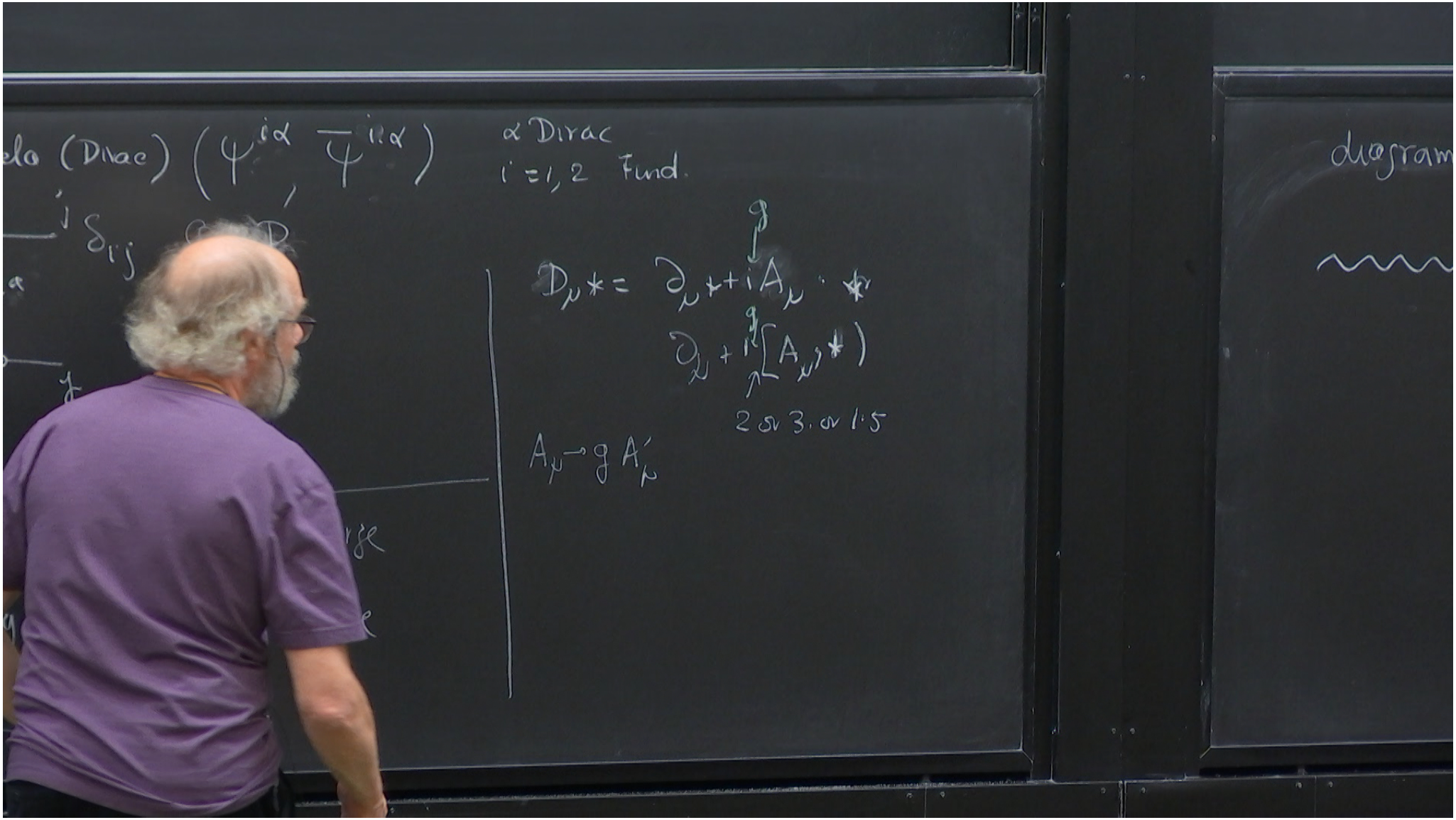
g

as in QDE

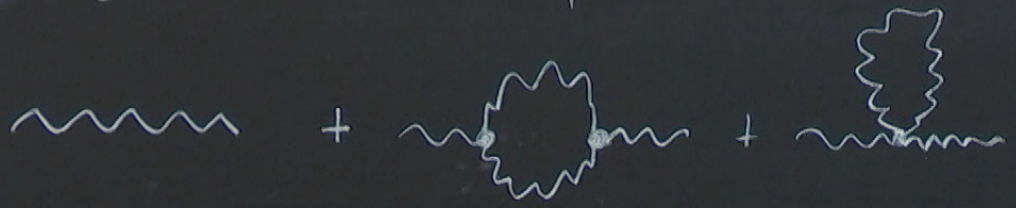
one coupling constant g = charge

everything else ← group structure

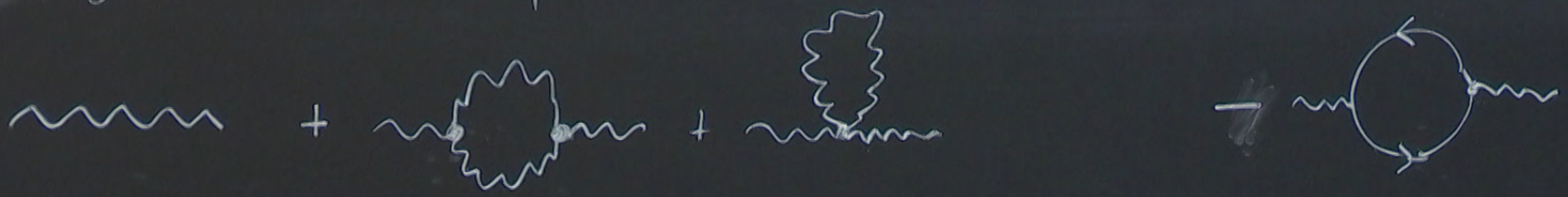
"charge is unique"

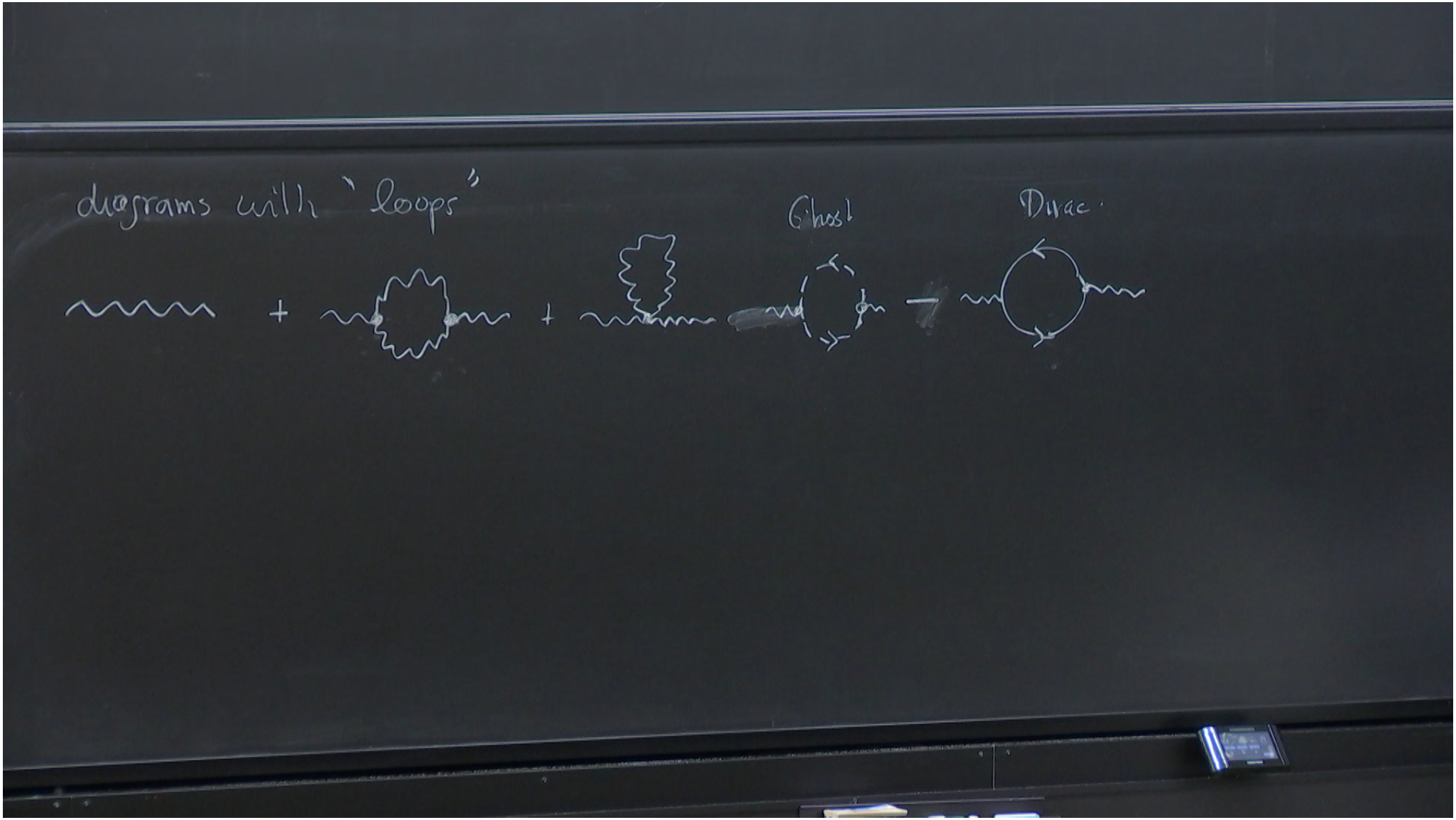

(Dirac)
α Dirac
i = 1, 2 Fund.
$D_\mu * = \partial_\mu * + i A_\mu \cdot *$
$\partial_\mu + i[A_\mu, *)$
2 or 3. or 1.5
$A_\mu \to g A'_\mu$
diagram

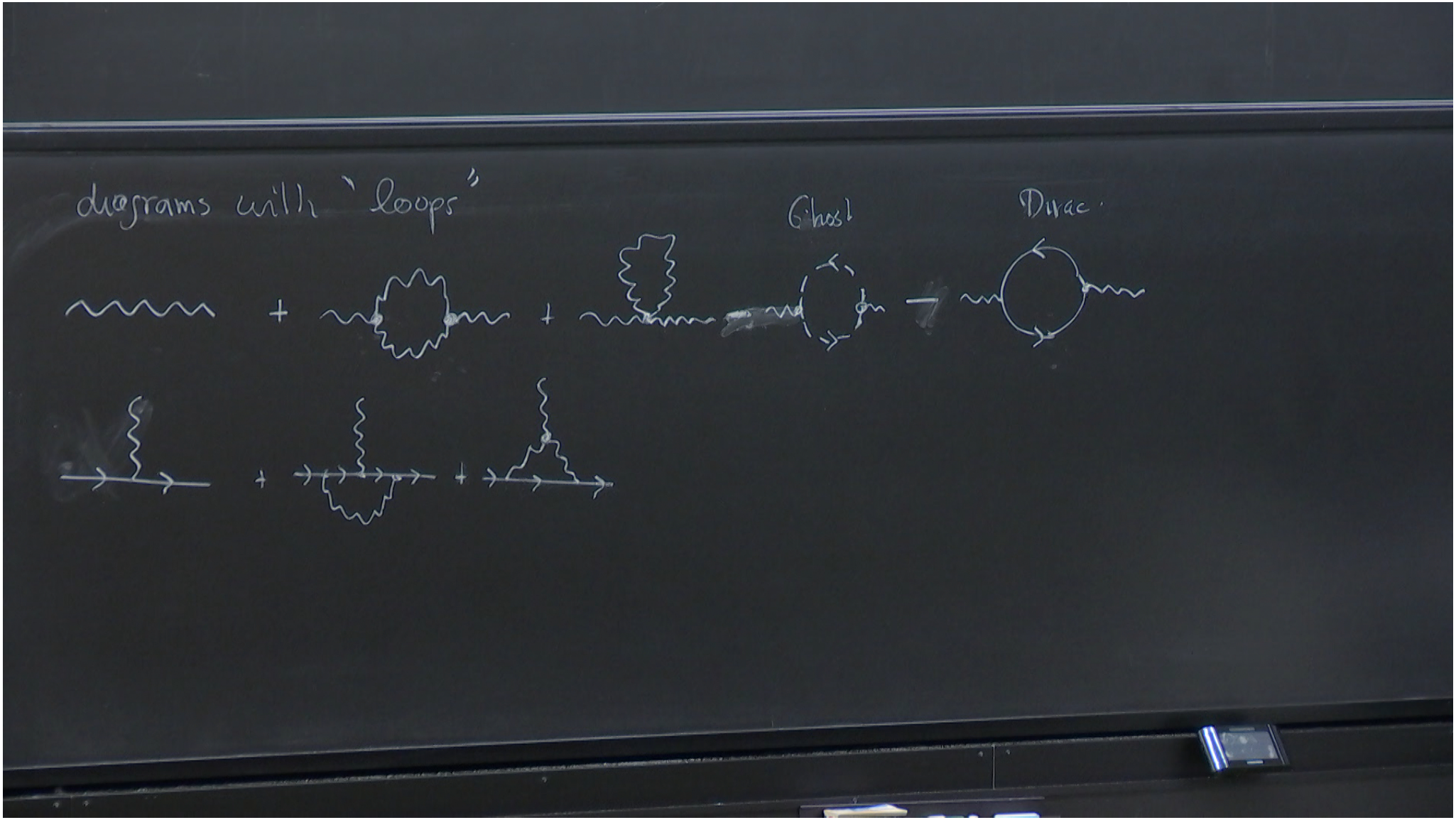
diagrams with "loops"

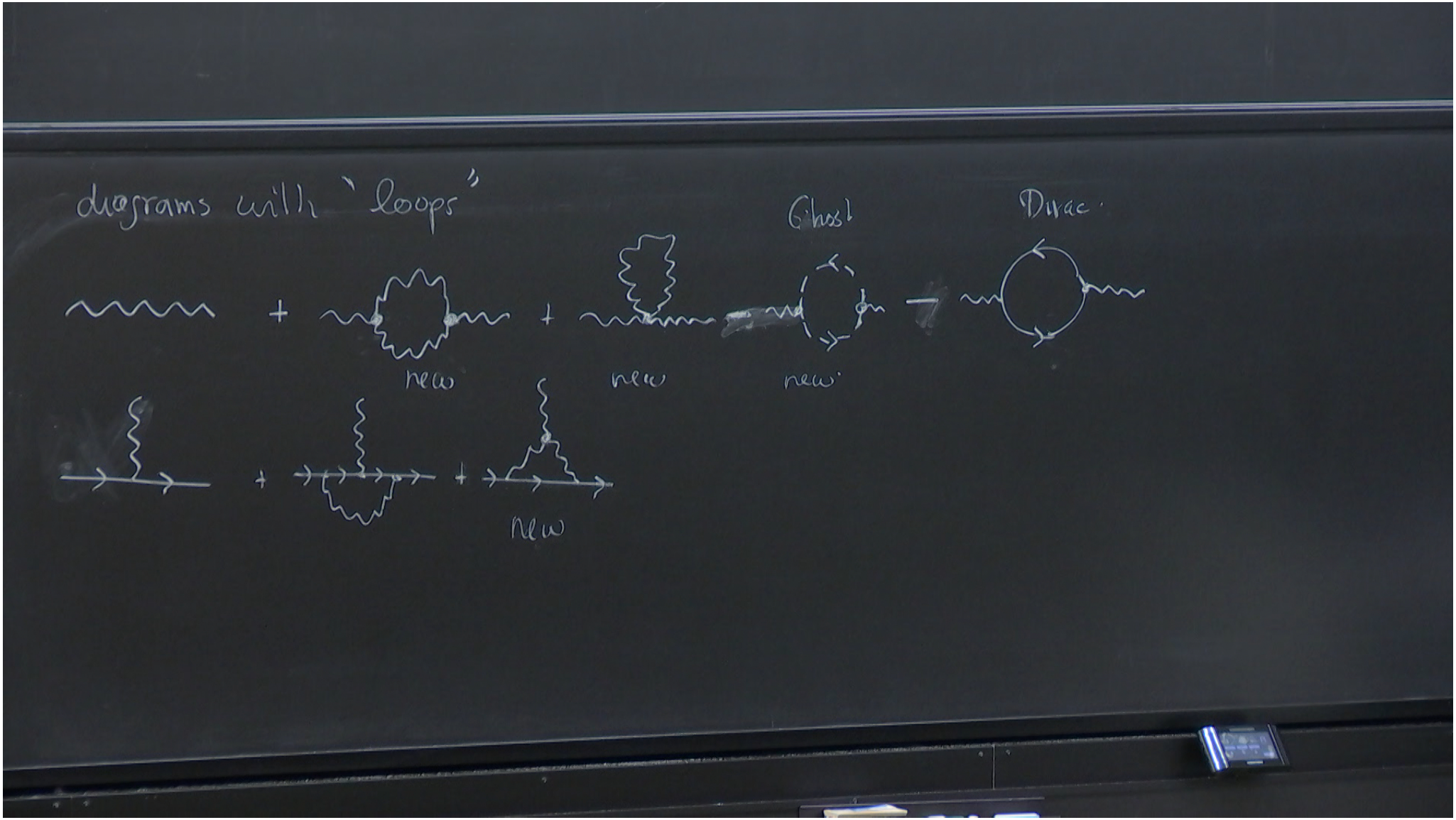


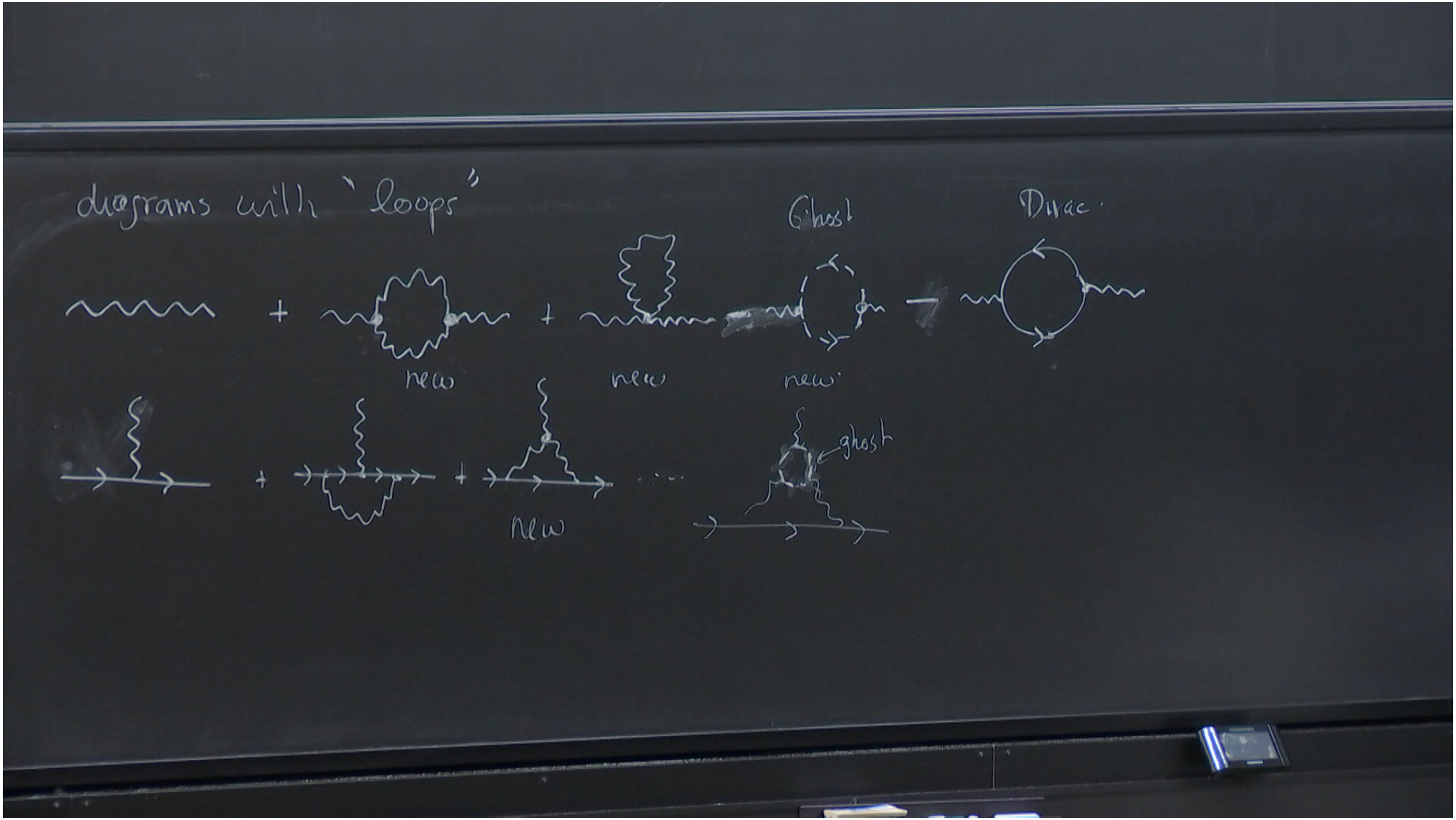
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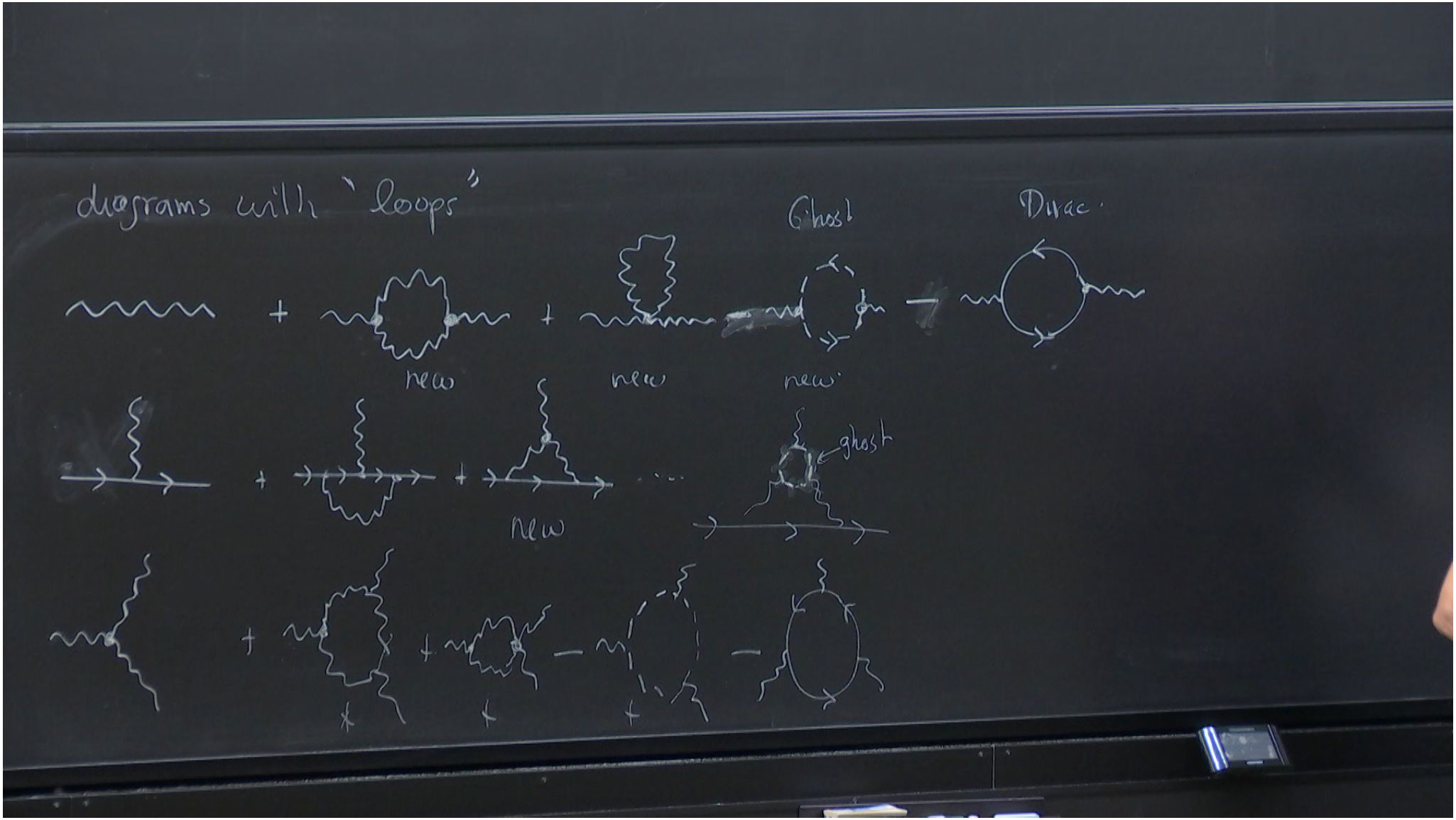


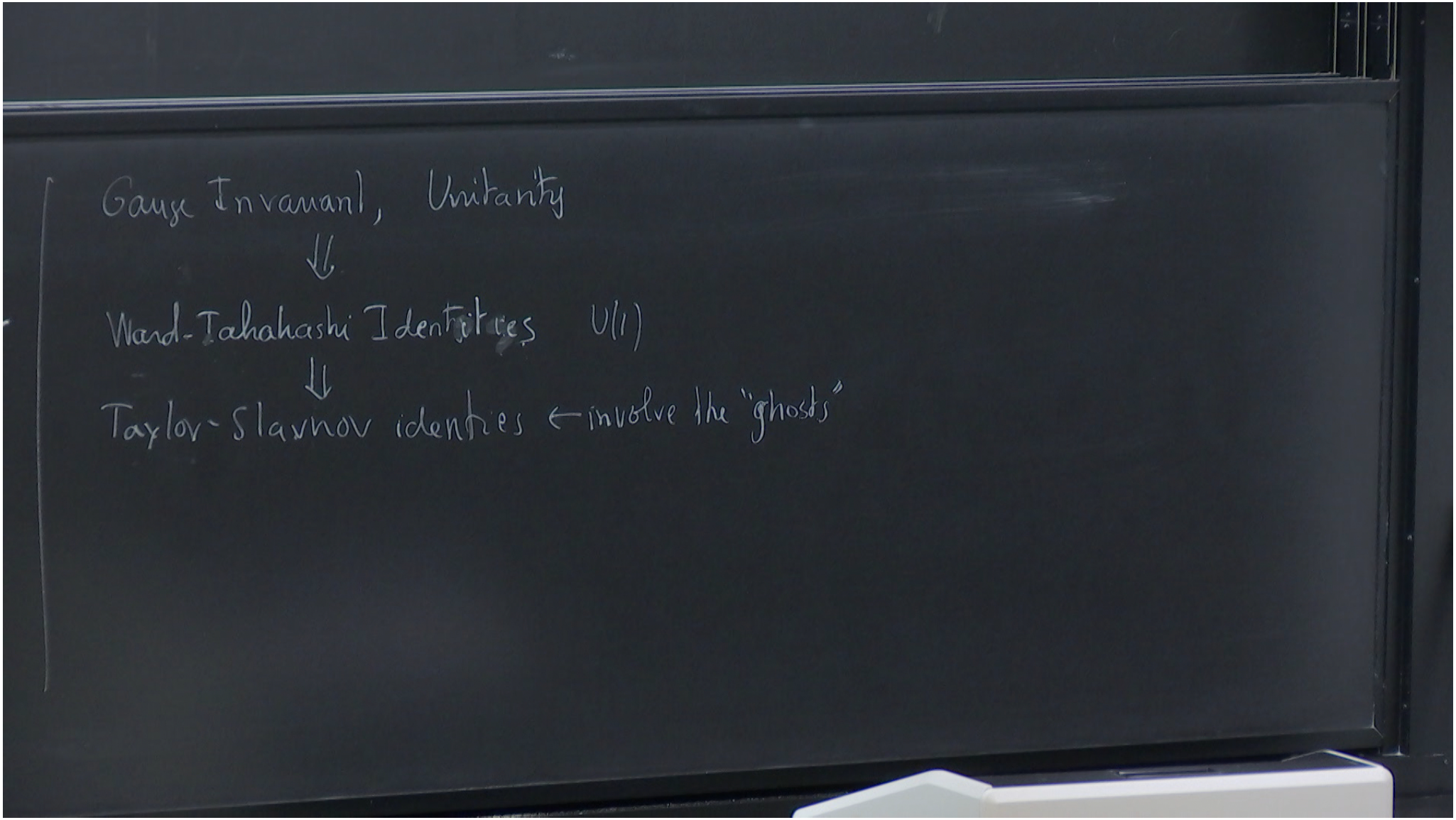

diagrams with "loops"
Ghost
Dirac

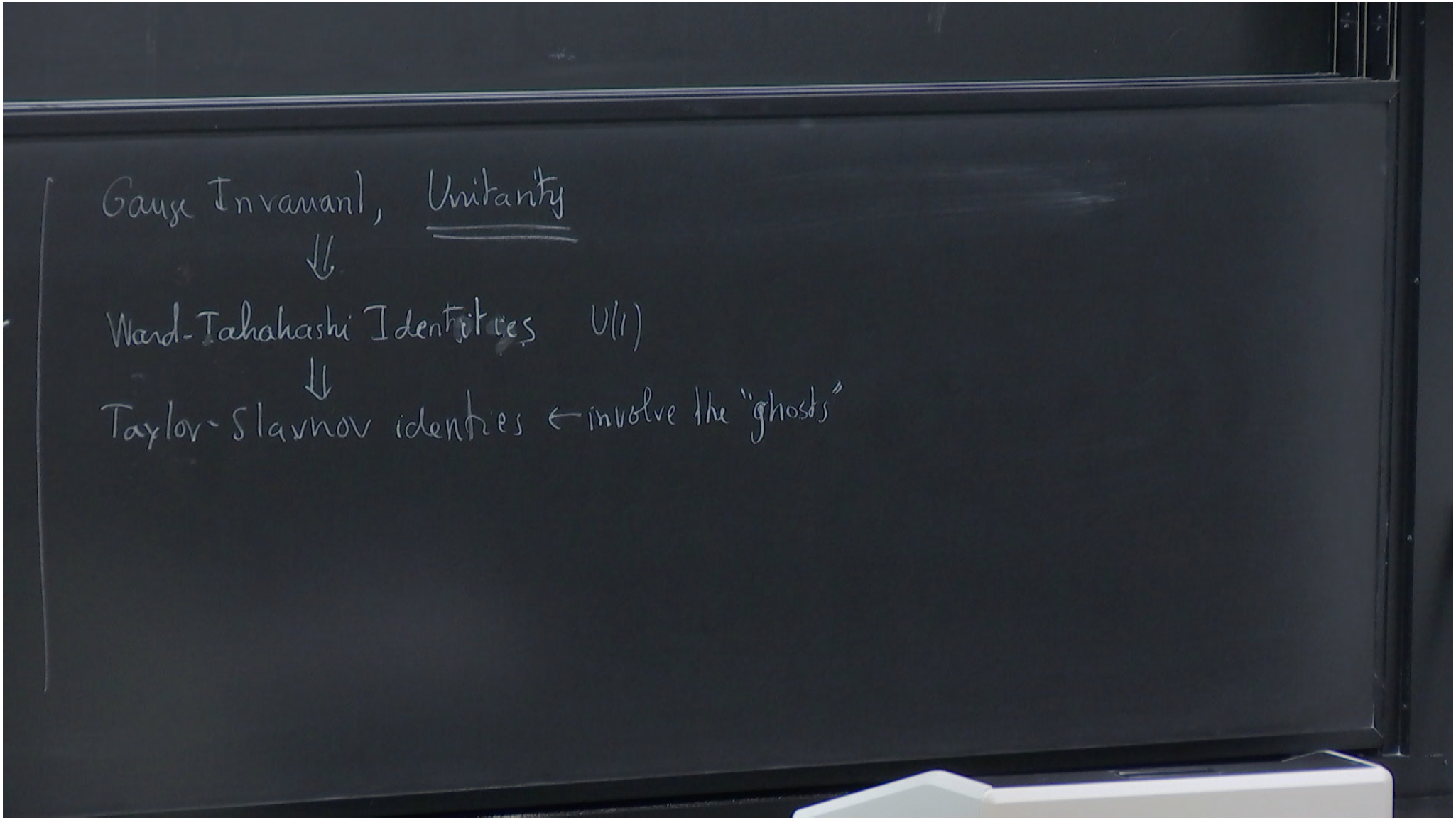

diagrams with "loops"
Ghost
Dirac


diagrams with "loops"
Ghost
Dirac
new
new
new
new


diagrams with "loops"
Ghost
Dirac
new
new
new
ghost
new


diagrams with "loops"
Ghost
Dirac
new
new
new
ghost
new


Gauge Invariant, Unitarity
⇓
Ward-Takahashi Identities U(1)
⇓
Taylor-Slavnov identies ← involve the "ghosts"


Gauge Invariant, Unitarity
Ward-Takahashi Identities U(1)
Taylor-Slavnov identies ← involve the "ghosts"

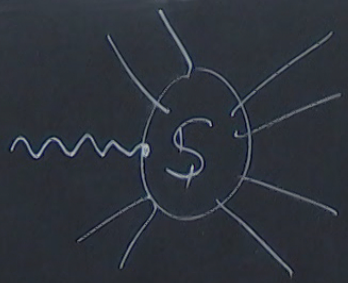
Gauge Invariant, Unitarity

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Ward-Takahashi Identities U(1)

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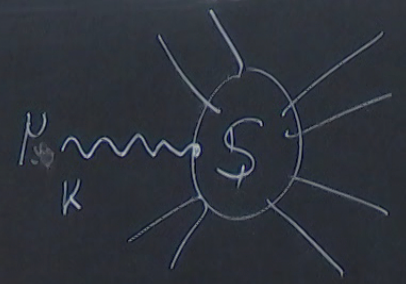
Gauge Invariant, Unitarity

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Ward-Takahashi Identities U(1)

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plane wave

$$A_\mu(x) = \epsilon_\mu e^{ik\cdot x}$$

polarization vector

$$k^2 = 0 \qquad \epsilon \cdot k = 0$$

Gauge Invariant, Unitarity

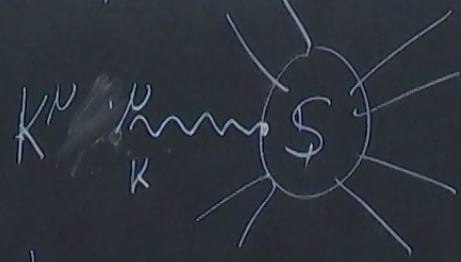
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$\epsilon_\mu \propto k_\mu$



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Gauge Invariant, Unitarity

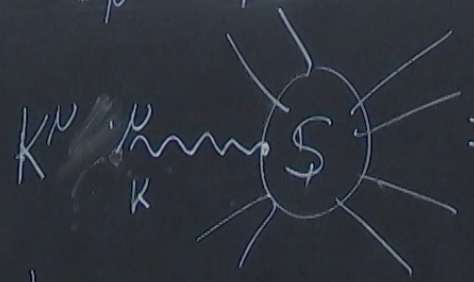
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polarzation

$k^2 = 0$

Dimension of c.c g^2

$e^{iS[A]}$

$S[A] = \frac{1}{2g^2}\int d^D x\, F_{\mu\nu} F^{\mu\nu}$

D: dim. of spacetime

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$

$[A_\mu] = [\text{length}]$

Gauge Invariant, Unitarity

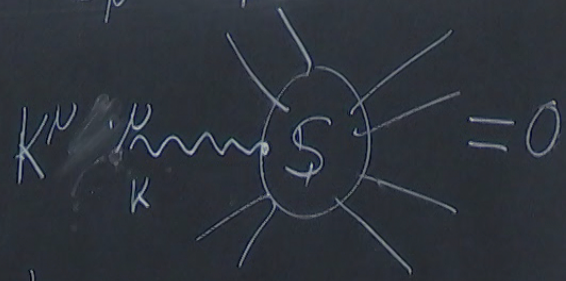
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$$A_\mu(x) = \epsilon_\mu e^{ik\cdot x}$$

polarization vector

$k^2 = 0 \qquad \epsilon \cdot k = 0$

Dimension of c.c g^2

dim. of spacetime

$$S[A] = \frac{1}{2g^2}\int d^D x\, F_{\mu\nu}F^{\mu\nu} \qquad \frac{i S[A]}{\hbar}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$$

$[A_\mu] = [\text{length}]^{-1}$

$[F_{\mu\nu}] = [\text{length}]^{-2}$

$[\text{length}]^{D-4}$

$$[g^2] = [\text{length}]^{4-D}$$

Gauge Invariant, Unitarity

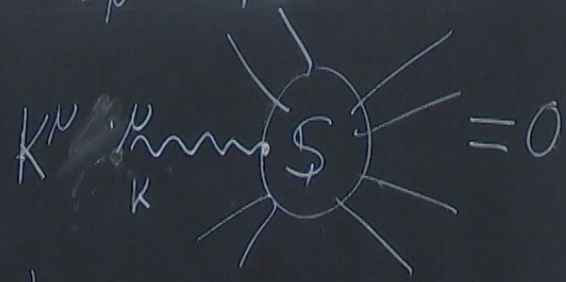
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Ward-Takahashi Identities U(1)

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$K^2 = 0 \quad \epsilon \cdot K = 0$

Dimension of c.c g^2

$e^{iS[A]}$

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D: dim. of spacetime

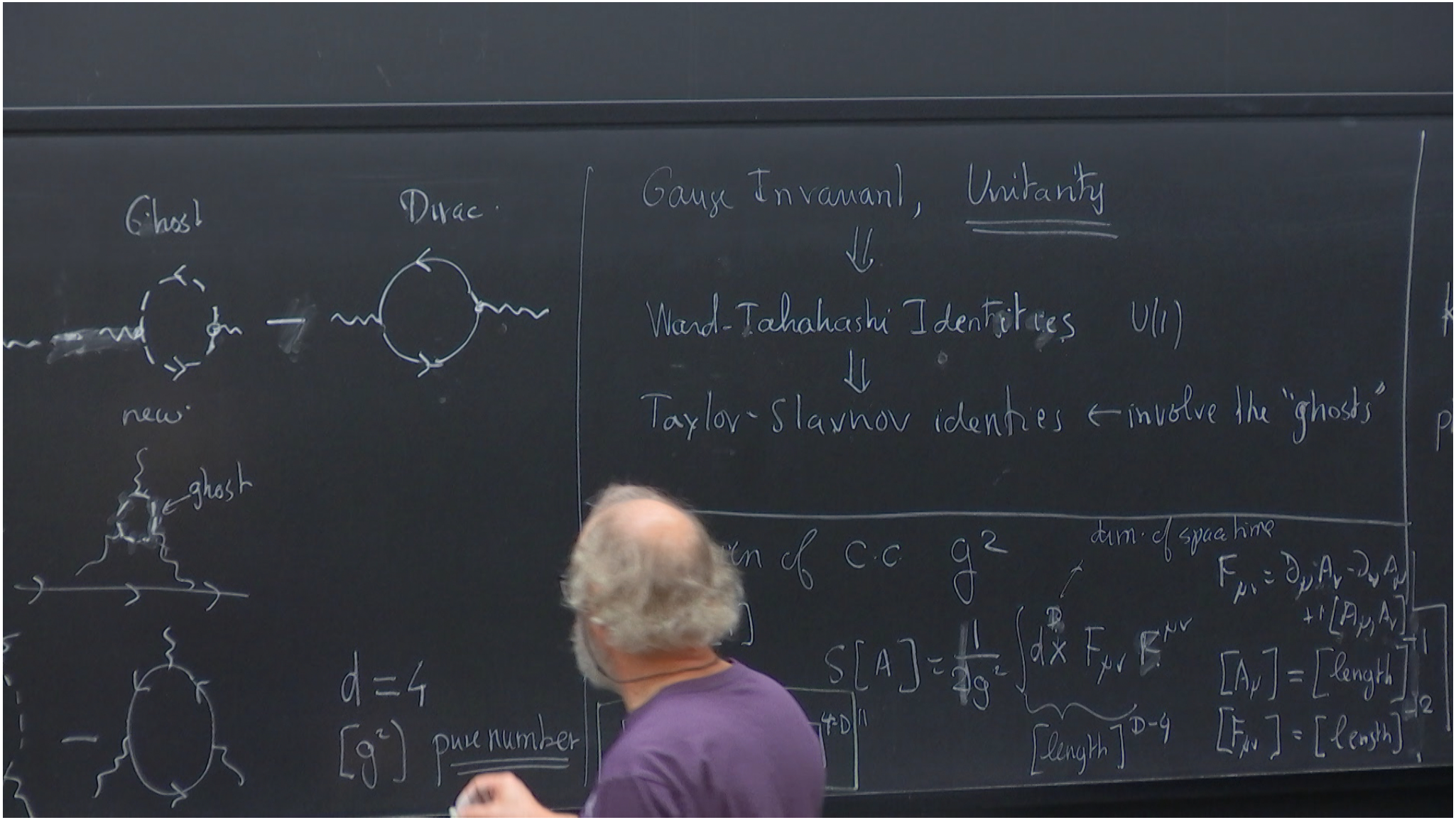
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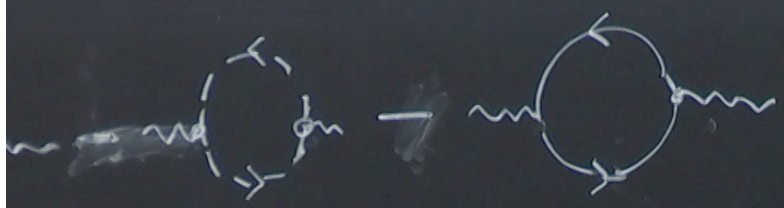
$[\text{length}]^{D-4}$

$[g^2] = [\text{length}]^{4-D}$

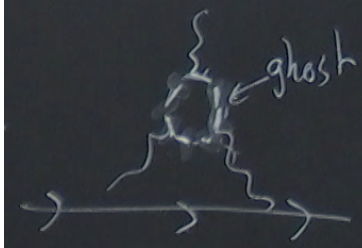

Ghost
Dirac
new
ghost
d = 4
[g²] pure number
Gauge Invariant, Unitarity
Ward-Takahashi Identities U(1)
Taylor-Slavnov identies ← involve the "ghosts"
dim. of spacetime
S[A] = 1/2g² ∫ d^D x F_μν F^μν
[length]^D-4
F_μν = ∂_μ A_ν − ∂_ν A_μ + i[A_μ, A_ν]
[A_μ] = [length]^-1
[F_μν] = [length]^-2

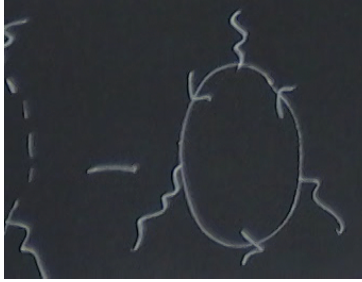
Ghost

Dirac



new:





Gauge Invariant, Unitarity

⇓

Ward-Takahashi Identities U(1)

⇓

Taylor-Slavnov identies ← involve the "ghosts"

Renormalizability

⇑

$d = 4$

$[g^2]$ pure number

Dimension of c.c g^2

$iS[A]$

$S[A] = \frac{1}{2g^2}\int d^D x\, F_{\mu\nu} F^{\mu\nu}$

dim. of space time

$[g^2] = [\text{length}]^{4-D}$

$[\text{length}]^{D-4}$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$

$[A_\mu] = [\text{length}]^{-1}$

$[F_{\mu\nu}] = [\text{length}]^{-2}$

$\Rightarrow \mathcal{L}_{\text{gauge fixed}} = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\alpha}(\partial^\mu A^a_\mu)^2 + (\partial^\mu \bar{C}^a)(D_\mu C_a)$

$\partial^\mu A^a_\mu = \xi^a$ integration over ξ^a

Fermion Gauge Sym. Breaking term

contains A

ghost

δ_{ab} G ← scalar field

interaction g

one coupling constant g =

everything else ← group str

"charge is unique"

Renormalization theory in QFT

Scalar Field spin 0 $\phi(x)$ space time dim D

Free Field (Gaussian)

$$S_0[\phi] = \int d^D x \left(\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{m^2}{2}\phi^2\right)$$

$$\langle \phi(x)\phi(y)\rangle_0 = G_F(x-y) = \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2+m^2} \qquad \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2+m^2-i0_+}$$

Feynman

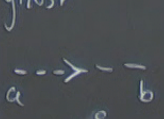
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$\partial^\mu A^a_\mu = \xi^a$; integration over ξ^a

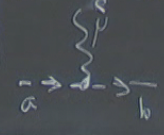
Fermion Gauge Sym. Breaking term

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δ_{ab} $G \leftarrow$ scalar field



interaction

g

one coupling constant $g =$

everything else $\Leftarrow$ group str

"charge is unique"

Renormalization theory in QFT

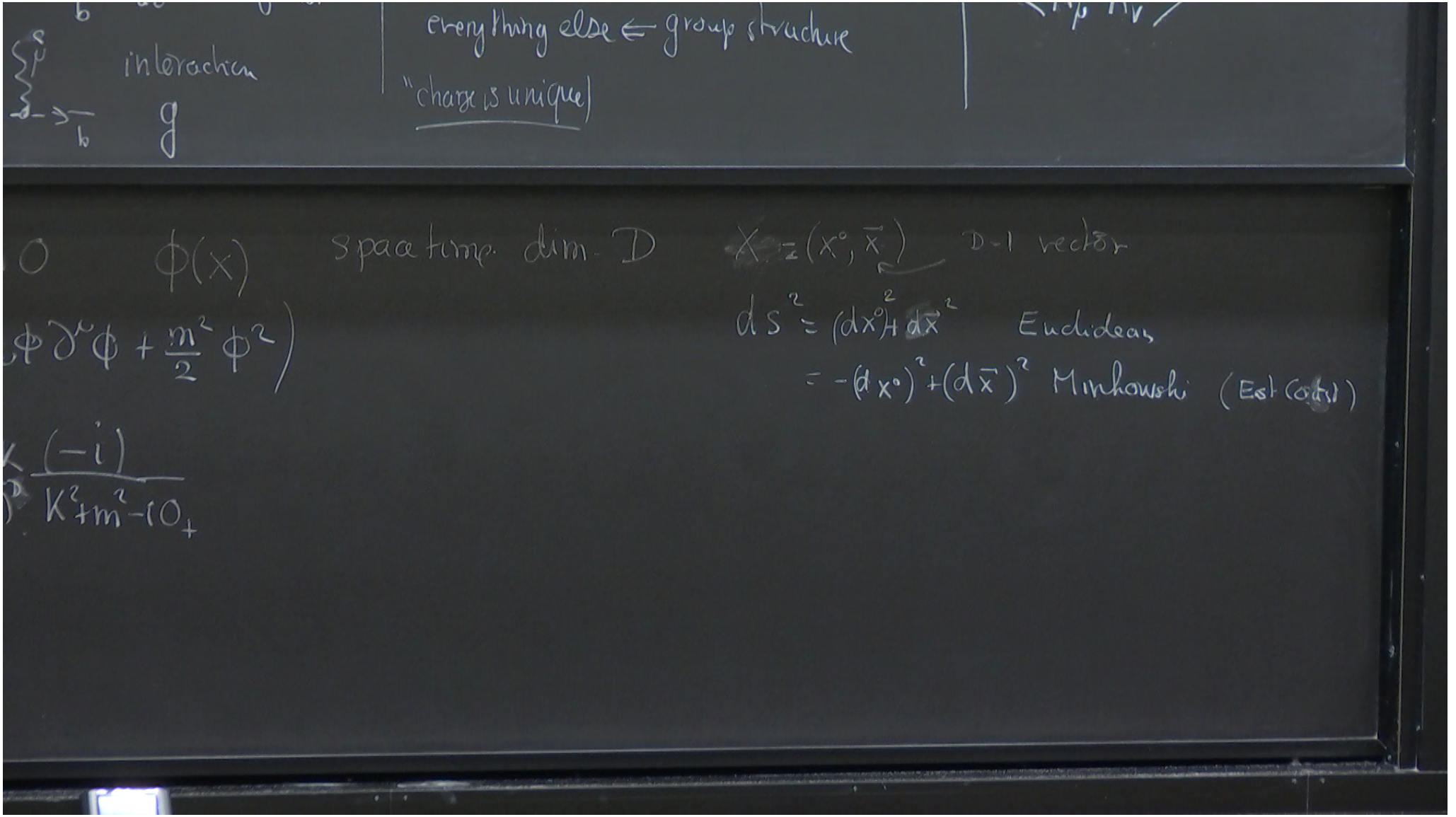
Scalar Field spin 0 $\phi(x)$ space time dim D $x =$

Free Field (Gaussian)

$$S_0[\phi] = \int d^D x \left(\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{m^2}{2}\phi^2\right)$$

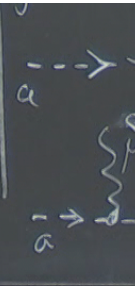
$$\langle \phi(x)\phi(y)\rangle_0 = G_F(x-y) = \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2+m^2} \qquad \int \frac{d^D k}{(2\pi)^D} \frac{-i}{k^2+m^2-i0_+}$$

G_F: Feynman


interaction
g
everything else ⇐ group structure
"charge is unique"
$\phi(x)$
spacetime dim D
$X = (x^0, \vec{x})$
D-1 vector
$ds^2 = (dx^0)^2 + d\vec{x}^2$ Euclidean
$= -(dx^0)^2 + (d\vec{x})^2$ Minkowski
$\phi \partial^\nu \phi + \frac{m^2}{2} \phi^2$
$\frac{(-i)}{K^2 + m^2 - i0_+}$

$\frac{1}{2\alpha}(\partial^\mu A_\mu)^2 + (\partial^\mu \bar{C}^a)(D_\mu C_a)$

Feynman Gauge "Sym. Braking" term — contains A



δ_{ab} G ← scalar field

interaction

g

one

er

"cha

Renormalization theory in QFT

Scalar Field spin 0 $\phi(x)$ space-t

Free Field (Gaussian)

$$S_0[\phi] = \int d^4x \left(\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{m^2}{2}\phi^2\right)$$

$$\langle \phi(x)\phi(y)\rangle_0 = G_F(x-y) = \int \frac{d^Dk}{(2\pi)^D} \frac{e^{ik\cdot(x-y)}}{k^2+m^2}$$

G_F: Feynman

well defined $x \neq y$

Euclidean

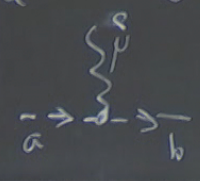
$$\int \frac{d^Dk}{(2\pi)^D} \frac{(-i)e^{ik\cdot(x-y)}}{k^2+m^2-i0_+}$$

Minkowski

$x \to y$ diverges like $|x-y|^{2-D}$ $D>2$ / $\frac{1}{2\pi}\log\left(\frac{1}{|x-y|}\right)$ $D=2$ naive $\phi(x)^2$ ill defined

ghost

$\delta_{ab}\, G$ ← scalar field



interaction g

one coupling constant g = charge

everything else ⇐ group structure

"charge is unique"

$\langle A^a_\mu A^b_\nu \rangle$

spin 0 $\phi(x)$ spacetime dim D $X = (x^0, \vec{x})$ D-1 vector

$\left(\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{m^2}{2}\phi^2\right)$

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$\int \frac{d^D k}{(2\pi)^D} \frac{(-i)e^{i\,k\cdot(x-y)}}{k^2+m^2-i0_+}$

Minkowski

In Fourier Momentum space

$\hat{G}(k) \simeq \frac{1}{k^2}$ when $k \nearrow \infty$

$D=2$ naive $\phi(x)^2$ ill defined

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Minkowski

Fourier Momentum space

$G(k) \simeq \frac{1}{k^2}$ when $k \nearrow \infty$

ient time space

$G_F(x,0)$

T

X

$D=2$ naive $\phi(x)^2$ i

ghost

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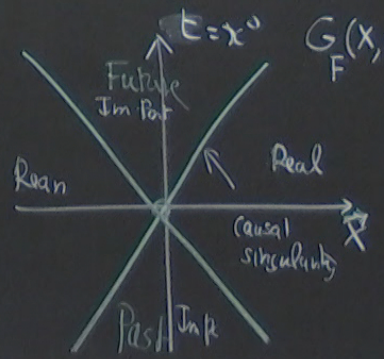
Minkowski

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Positivent time space

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$G_F(x,0)$

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Minkowski

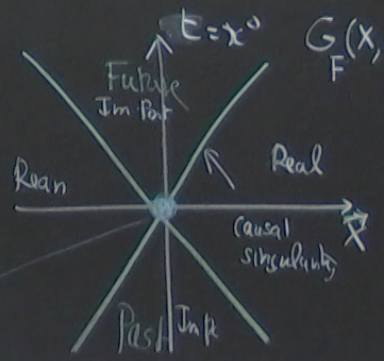
In Fourier Momentum space

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Positivent time space

UV singularity

$D=2$ naive $\phi(x)^2$ ill defined



$G_F(x,0)$

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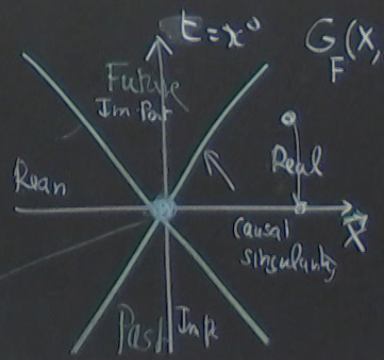
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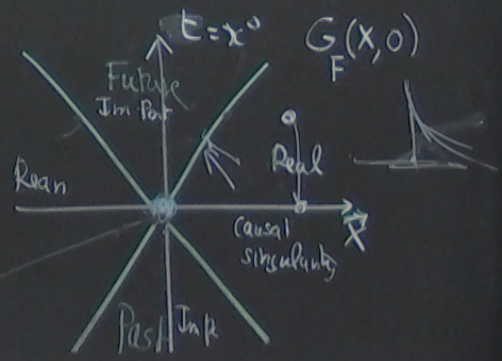
Minkowski

In Fourier Momentum space

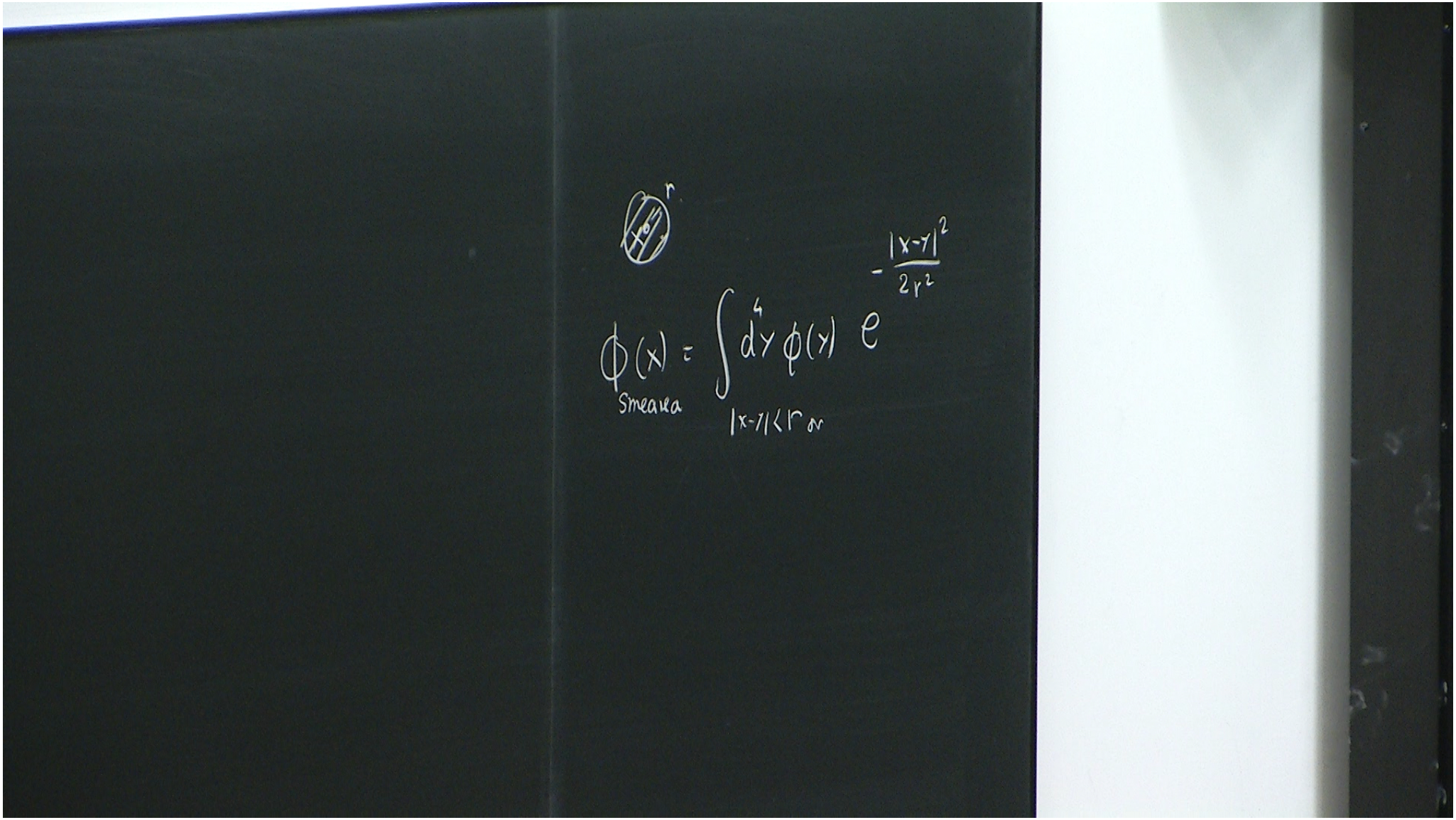
$\hat{G}(k) \simeq \frac{1}{k^2}$ when $k \nearrow \infty$

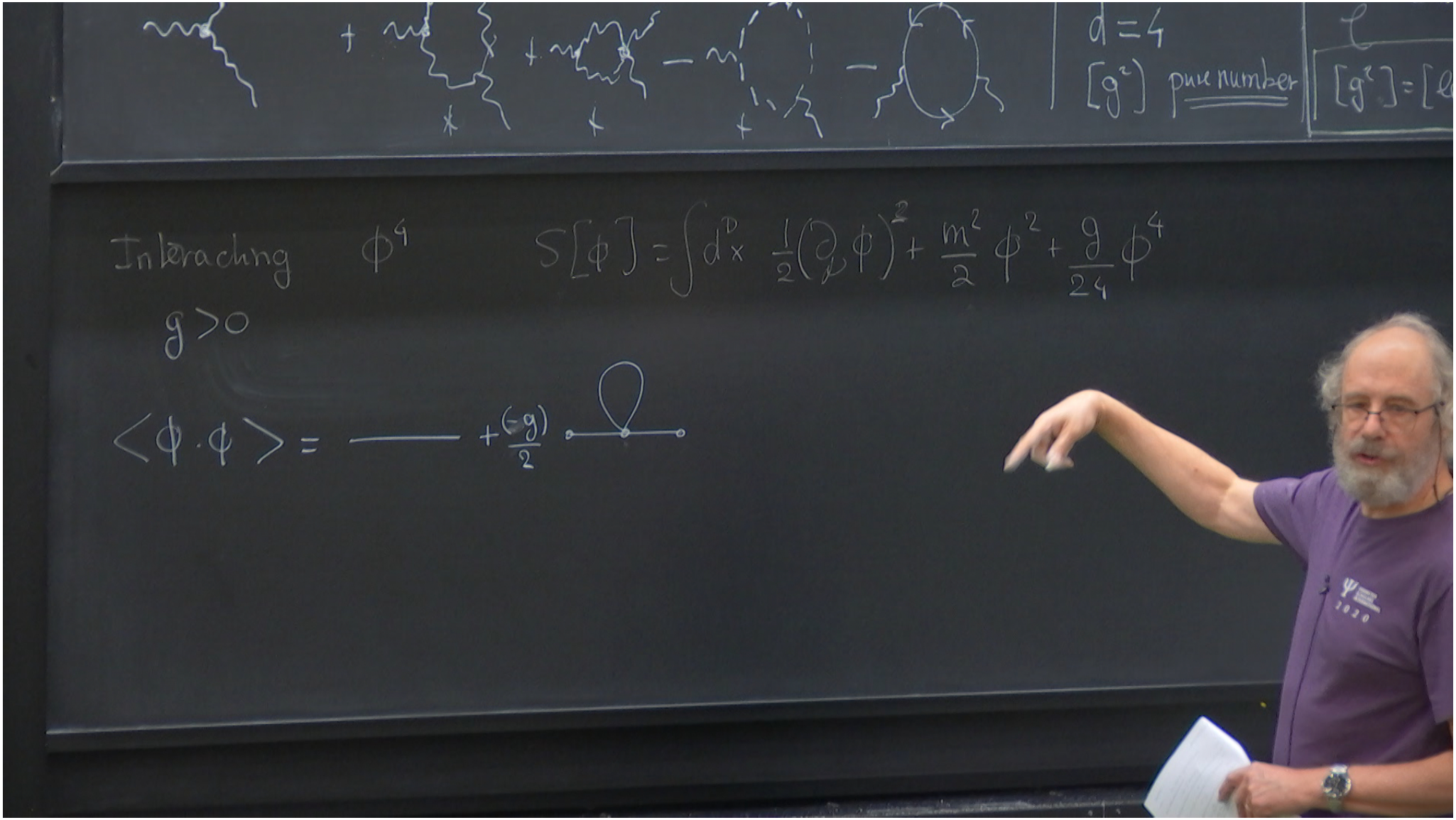
Positivent time space

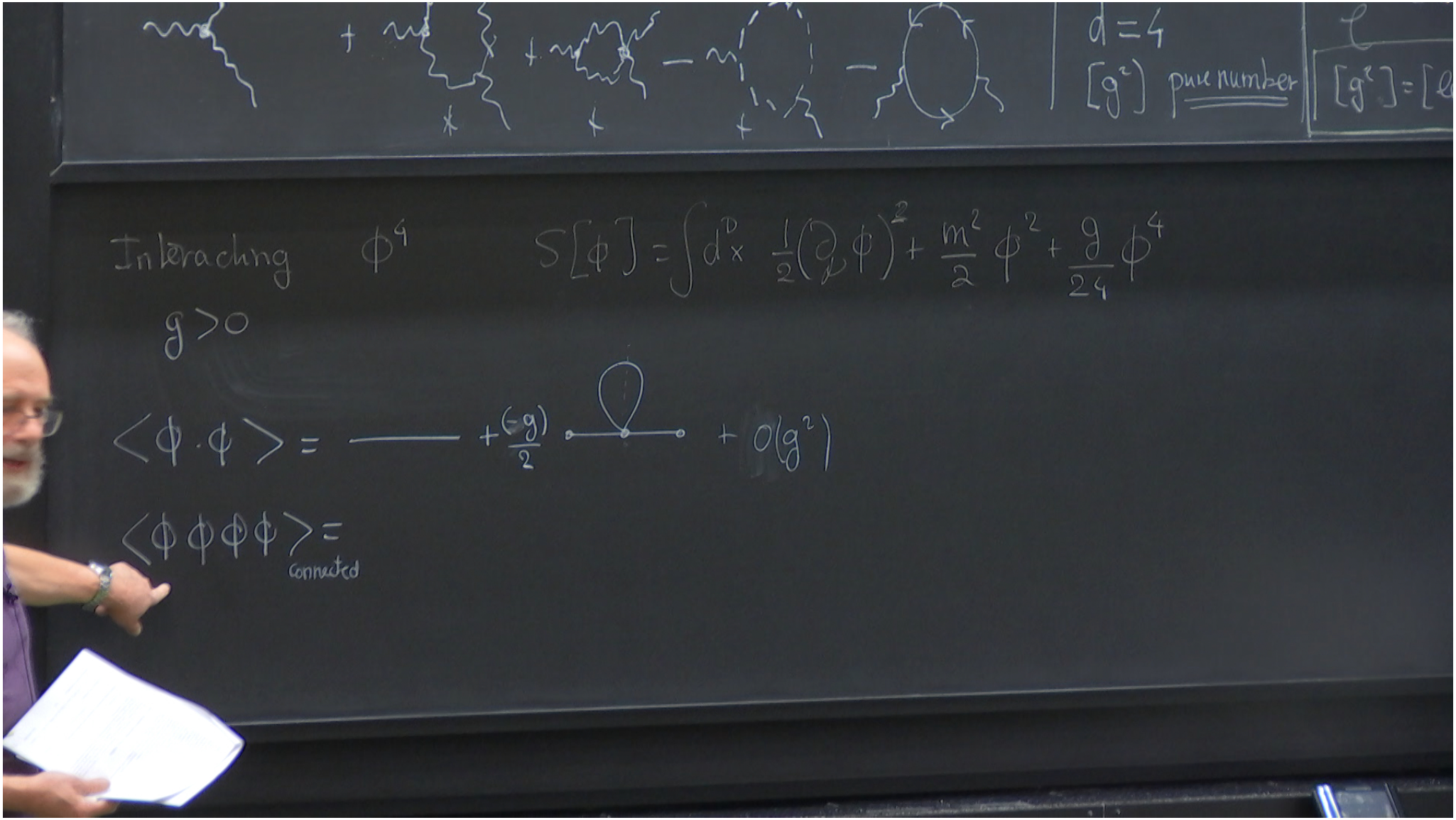
UV singularity

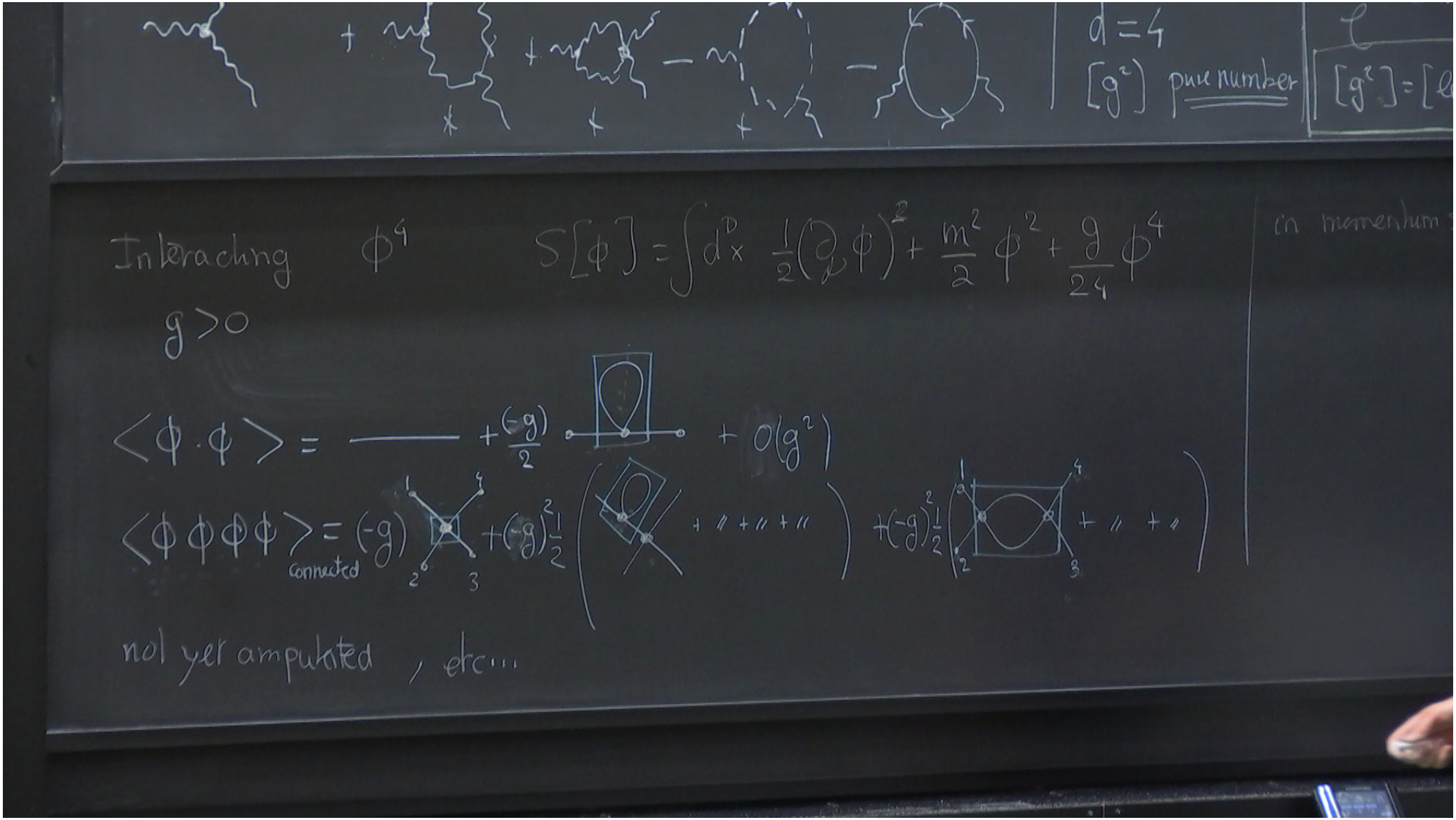


$D=2$ naive $\phi(x)^2$ ill defined


r
$\phi(x) = \int_{|x-y|<r} d^4y\, \phi(y)\, e^{-\frac{|x-y|^2}{2r^2}}$
Smeared


d=4
[g²] pure number
Interacting ϕ^4
$S[\phi] = \int d^D x \, \frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{24}\phi^4$
$g > 0$
$\langle \phi \cdot \phi \rangle =$
$+ \frac{(-g)}{2}$


d=4
$[g^2]$ pure number
$[g^2]=[$
Interacting ϕ^4
$S[\phi]=\int d^Dx\ \frac{1}{2}(\partial_\mu\phi)^2+\frac{m^2}{2}\phi^2+\frac{g}{24}\phi^4$
$g>0$
$\langle\phi\cdot\phi\rangle=$
$+\frac{(-g)}{2}$
$+O(g^2)$
$\langle\phi\phi\phi\phi\rangle=$
connected


d=4
$[g^2]$ pure number
$[g^2]=[$
Interacting ϕ^4
$S[\phi]=\int d^Dx\ \frac{1}{2}(\partial\phi)^2+\frac{m^2}{2}\phi^2+\frac{g}{24}\phi^4$
in momentum
$g>0$
$\langle\phi\cdot\phi\rangle =$
$+\frac{(-g)}{2}$
$+O(g^2)$
$\langle\phi\phi\phi\phi\rangle_{\text{connected}} = (-g)$
$+(-g)^2\frac{1}{2}$
$+(-g)^2\frac{1}{2}$
not yet amputated, etc...

$d=4$

$[g^2]$ pure number

$iS[A]$

$S[A] = \frac{1}{2g^2}\int d^D x\, F_{\mu\nu}F^{\mu\nu}$

$[g^2] = [\text{length}]^{4-D}$

$[\text{length}]^{D-4}$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$

$[A_\mu] = [\text{length}]^{-1}$

$[F_{\mu\nu}] = [\text{length}]^{-2}$

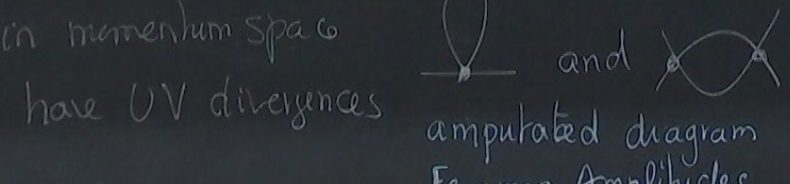
$k^2 = 0$

polarization vector

$\epsilon \cdot k = 0$

$+\frac{g}{24}\phi^4$

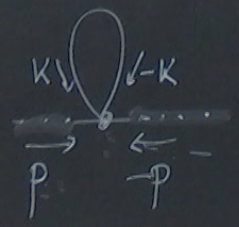
in momentum space have UV divergences



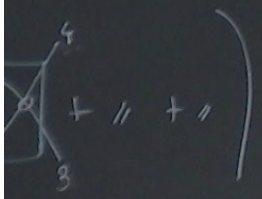
and

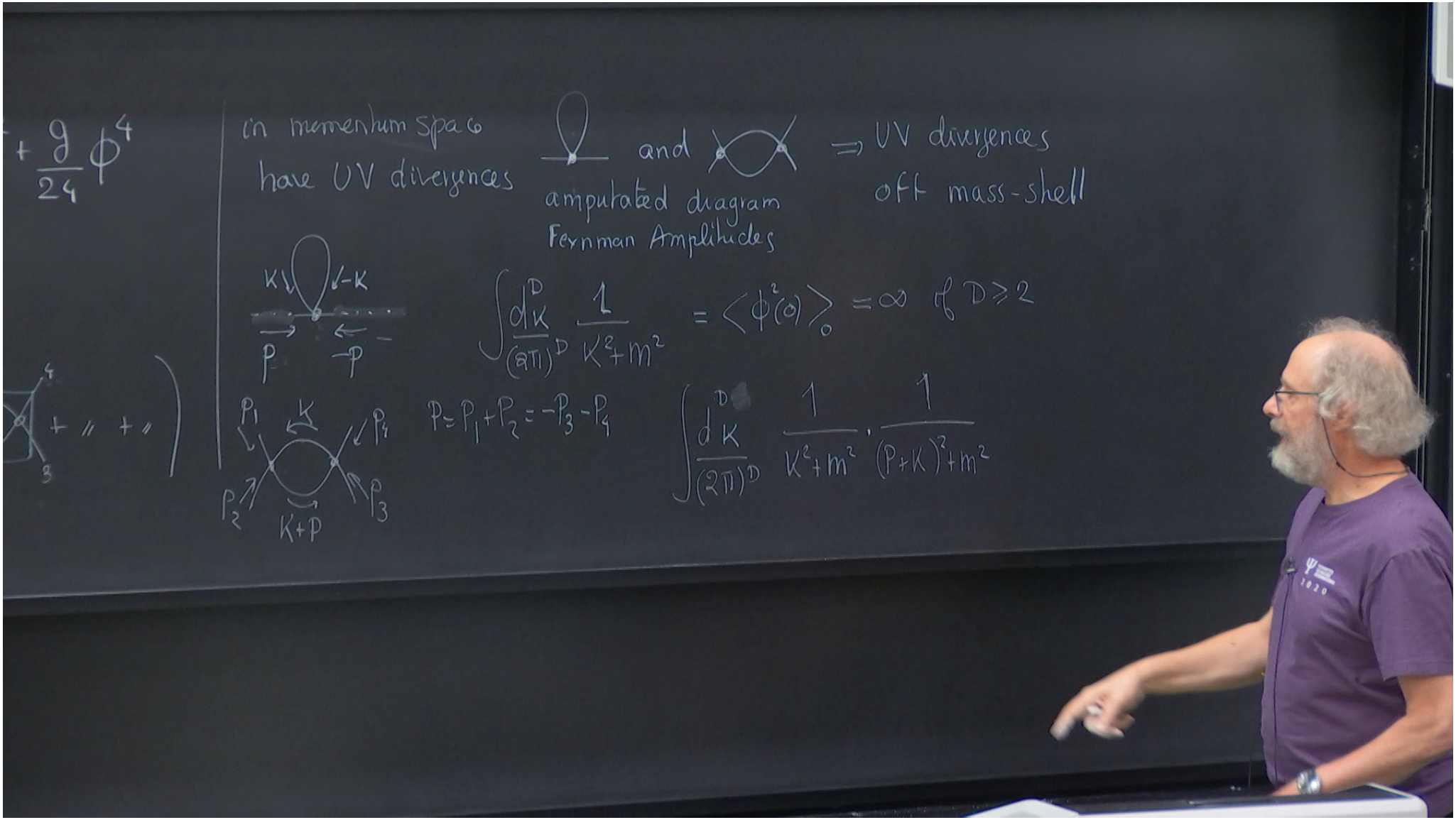
amputated diagram

Feynman Amplitudes



$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2+m^2} = \langle \phi^2(0) \rangle_0$$




+ g/24 φ^4
in momentum space
have UV divergences
and
amputated diagram
Feynman Amplitudes
⇒ UV divergences
off mass-shell
K
-K
P
-P
∫ d^D K / (2π)^D · 1/(K^2+m^2) = ⟨φ^2(0)⟩_0 = ∞ if D ≥ 2
P1
K
P4
P2
P3
K+P
P = P1+P2 = -P3-P4
∫ d^D K / (2π)^D · 1/(K^2+m^2) · 1/((P+K)^2+m^2)

$d = 4$

$[g^2]$ pure number

$iS[A]$

$S[A] = \frac{1}{2g^2}\int d^D x \, F_{\mu\nu} F^{\mu\nu}$

$[g^2] = [\text{length}]^{4-D}$

$[\text{length}]^{D-4}$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$

$[A_\mu] = [\text{length}]^{-1}$

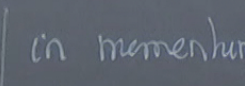
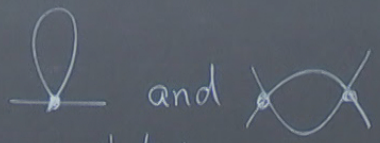
$[F_{\mu\nu}] = [\text{length}]^{-2}$

$k^2 = 0$

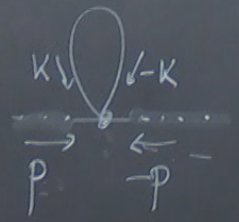
polarizacion vector

$\epsilon \cdot k = 0$

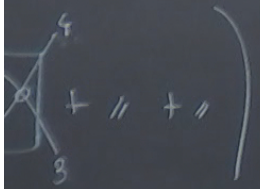
$+ \frac{g}{24}\phi^4$

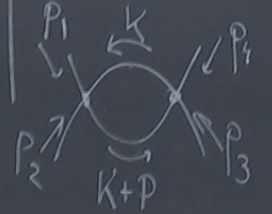
in momentum space have UV divergences  and  $\Rightarrow$ UV divergences off mass-shell

amputated diagram Feynman Amplitudes



$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + m^2} = \langle \phi^2(0) \rangle_0 = \infty \text{ if } D \geq 2$$





$P = P_1 + P_2 = -P_3 - P_4$

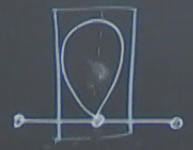
$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + m^2} \cdot \frac{1}{(P+k)^2 + m^2} = \infty \text{ if } D \geq 4$$

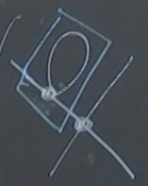
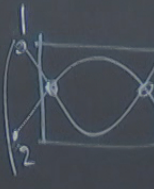
$k \to \infty$ ultravioleta

$[g]$ pure number $\|$ $[g^2]$

Interacting ϕ^4 $\quad S[\phi] = \int d^D x \; \frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{24}\phi^4$

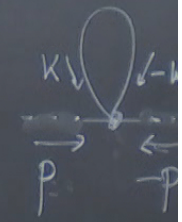
$g > 0$

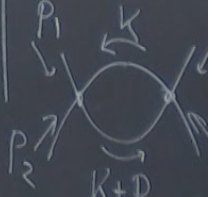
$\langle \phi \cdot \phi \rangle =$ ——— $+ \frac{(-g)}{2}$  $+ O(g^2)$

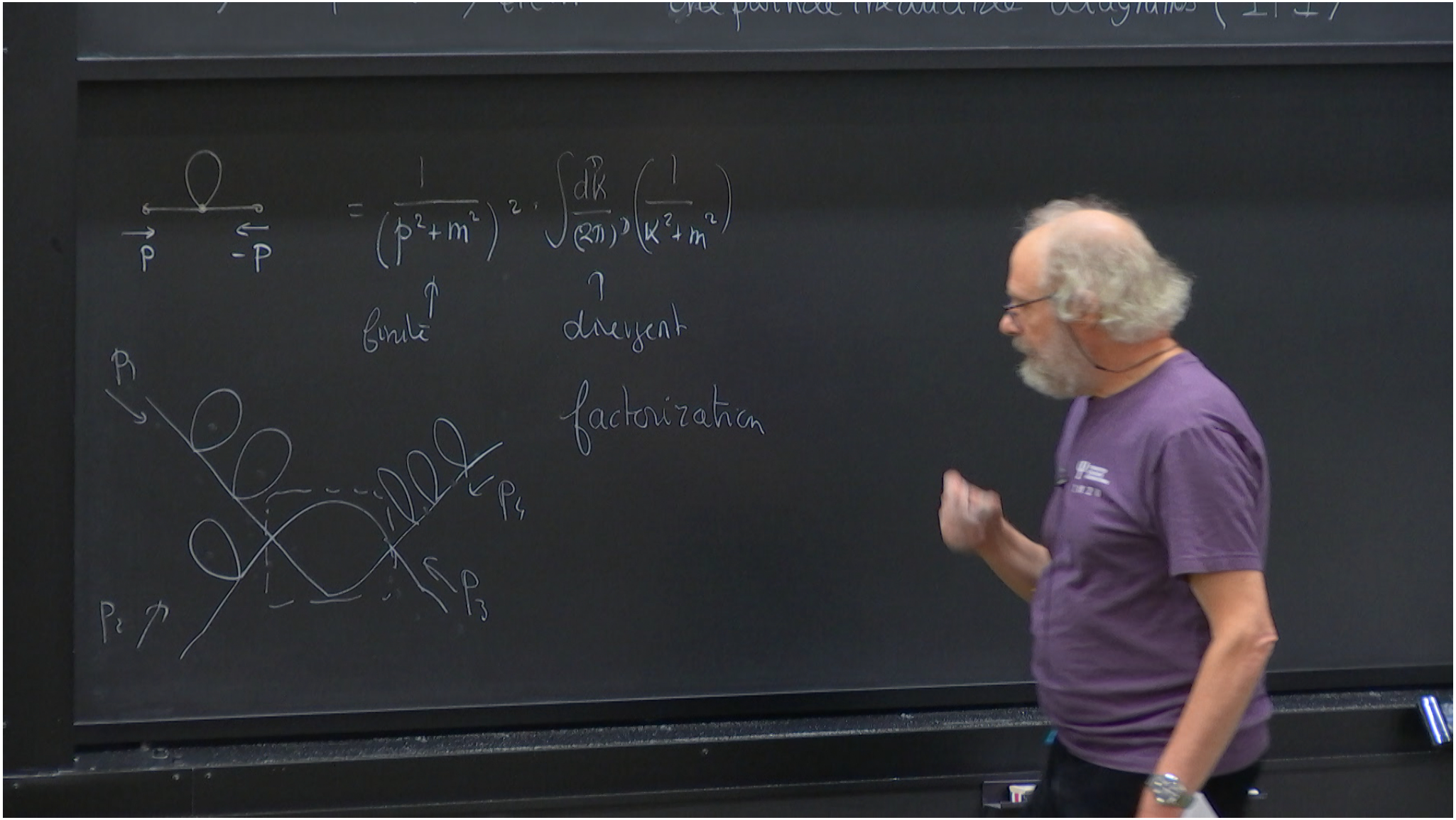
$\langle \phi\phi\phi\phi \rangle_{\text{connected}} = (-g)$  $+ (-g)^2 \frac{1}{2}\Big($  $+ \text{//} + \text{///} + \text{///}\Big) + (-g)^2 \frac{1}{2}\Big($  $+ \text{//} + \text{//}\Big)$

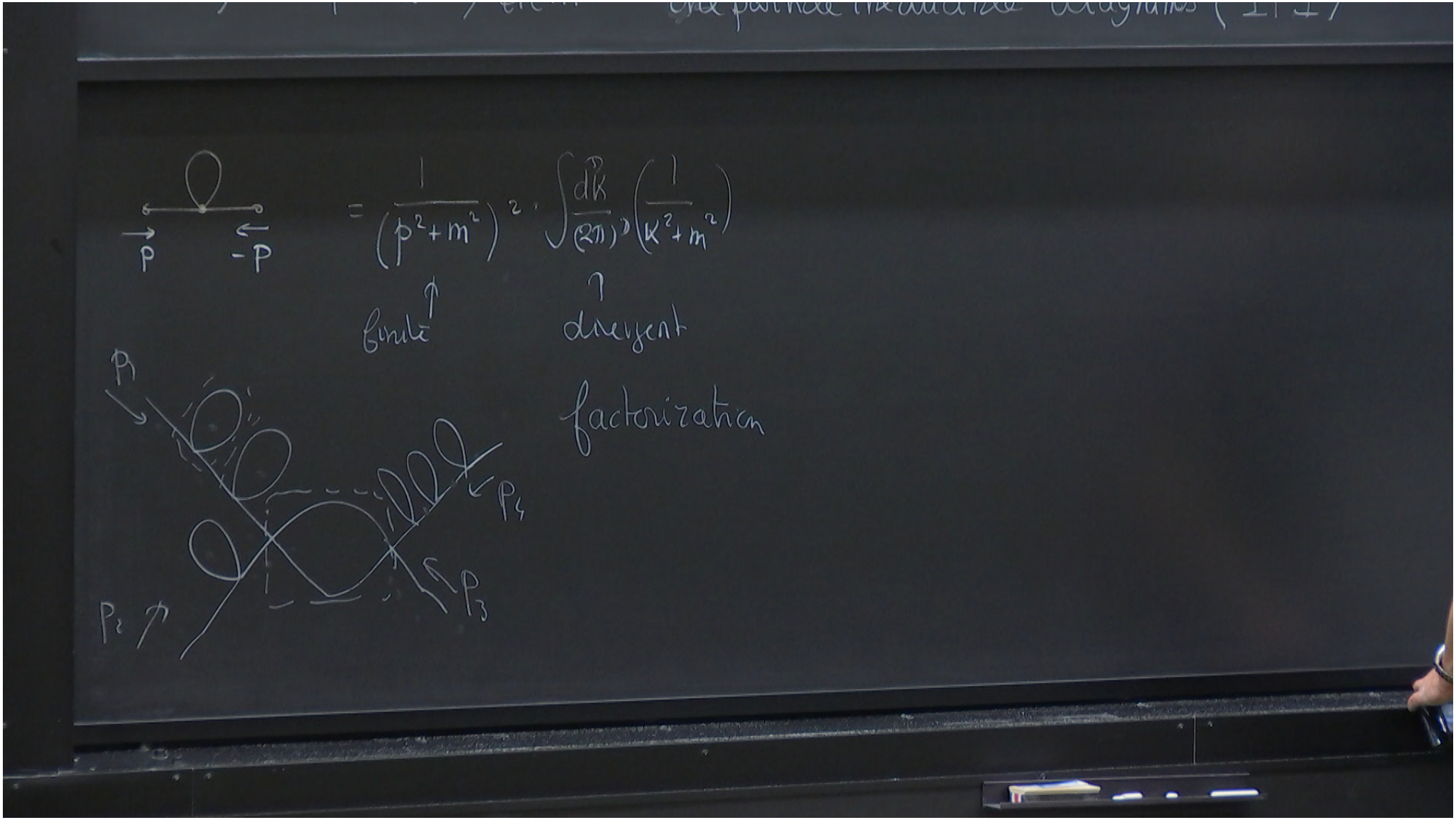
not yet amputated, etc... one particle irreducible diagrams (1PI)

in momen
have U






p
-p
$= \frac{1}{(p^2+m^2)^2} \cdot \int \frac{d^D k}{(2\pi)^D} \left(\frac{1}{k^2+m^2}\right)$
finite
divergent
factorization
p_1
p_2
p_3
p_4


p
-p
$= \frac{1}{(p^2+m^2)^2} \cdot \int \frac{d^D k}{(2\pi)^D} \left(\frac{1}{k^2+m^2}\right)$
finite
divergent
factorization
p_1
p_2
p_3
p_4

in momentum space have UV divergences



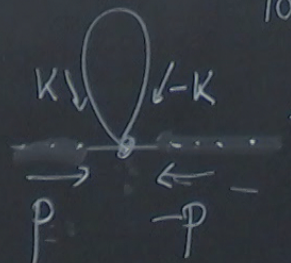
and



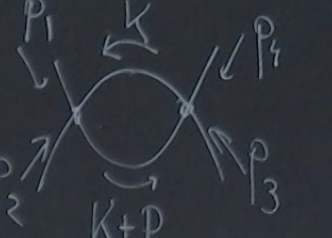
$\Rightarrow$ UV divergences off mass-shell

amputated diagram

"Tadpole" diagram Feynman Amplitudes



$$T = \int \frac{d^D K}{(2\pi)^D} \frac{1}{K^2 + m^2} = \langle \phi^2(0) \rangle_0 = \infty \quad \text{if } D \geq 2$$



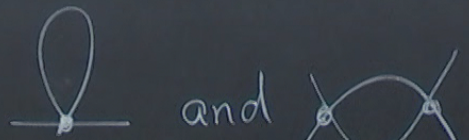
$P = P_1 + P_2 = -P_3 - P_4$

$$\int \frac{d^D K}{(2\pi)^D} \frac{1}{K^2 + m^2} \cdot \frac{1}{(P+K)^2 + m^2} = \infty \quad \text{if } D \geq 4$$

$K \to \infty$ UV divergence

PI)

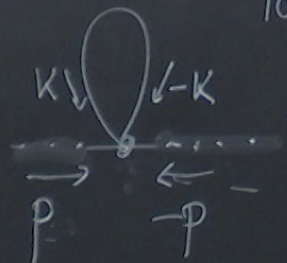
in momentum space have UV divergences



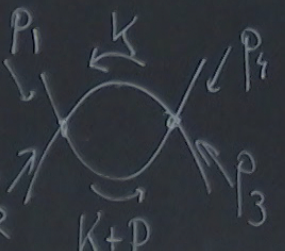
amputated diagram

$\Rightarrow$ UV divergences off mass-shell

"Tadpole" diagram Feynman Amplitudes



$$T = \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + m^2} = \langle \phi^2(0) \rangle_0 = \infty \text{ if } D \geq 2$$

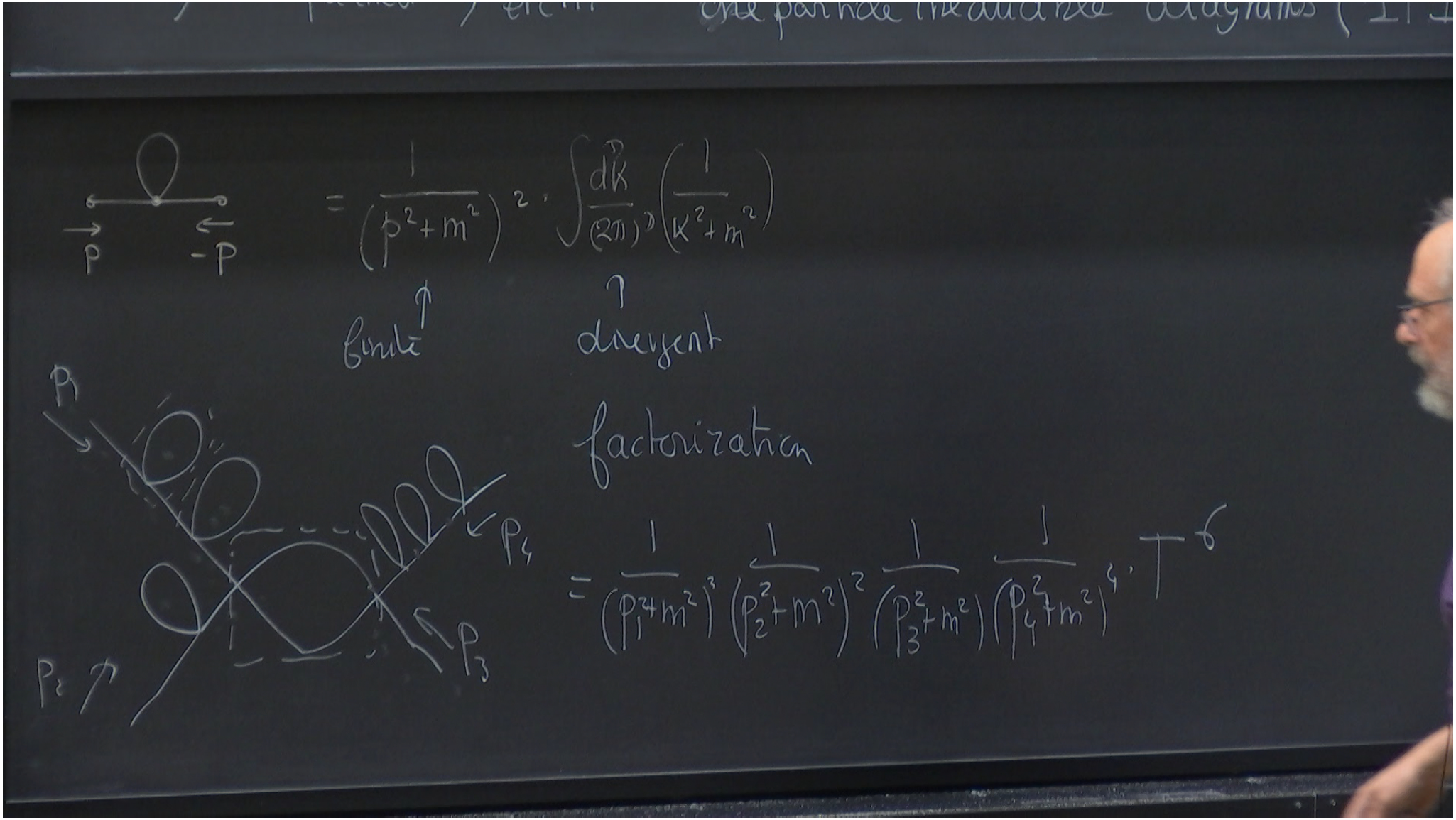


$$P = p_1 + p_2 = -p_3 - p_4$$

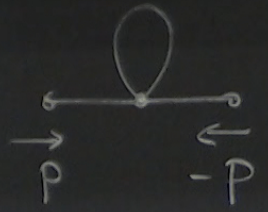
$$B(p_1+p_2) = \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 + m^2} \cdot \frac{1}{(P+k)^2 + m^2} = \infty \text{ if } D \geq 4$$

$k \to \infty$ ultraviolet

Loop = Boucle in French

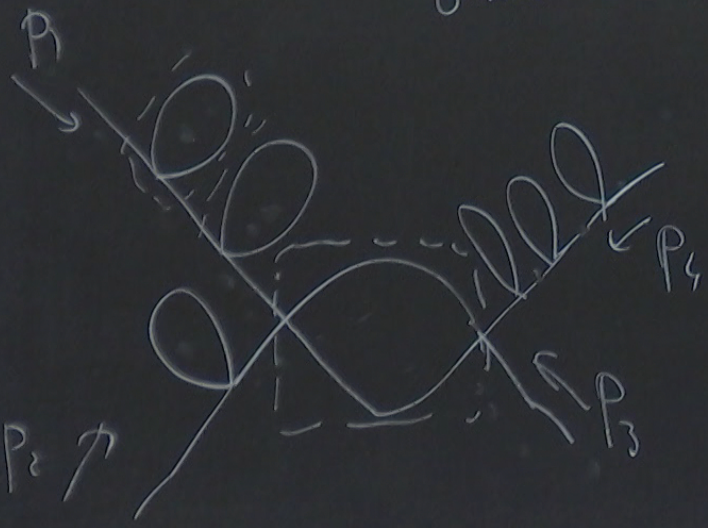

p
-p
finite
divergent
factorization
p_1
p_2
p_3
p_4

one particle irreducible diagrams (1 l



$$= \frac{1}{(p^2+m^2)^2} \cdot \int \frac{d^D k}{(2\pi)^D} \left(\frac{1}{k^2+m^2} \right)$$

finite ↑ ? divergent



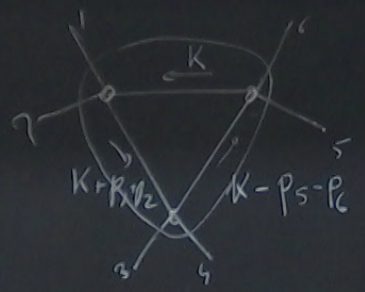
factorization

$$= \frac{1}{(p_1^2+m^2)^3} \frac{1}{(p_2^2+m^2)^2} \frac{1}{(p_3^2+m^2)} \frac{1}{(p_4^2+m^2)^4} \cdot T^{\sigma} \; B(p_1+p_2)$$

PI) p_2 K+P p_3

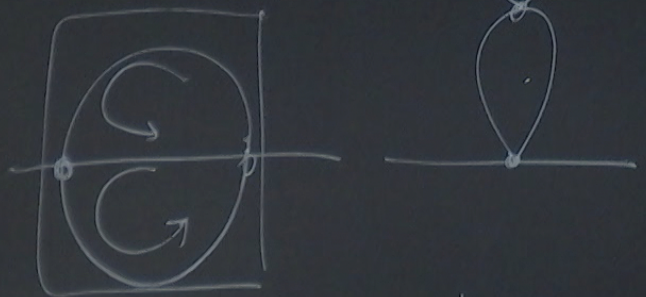
Loop = Boucle in French

Integration



triangle diagram

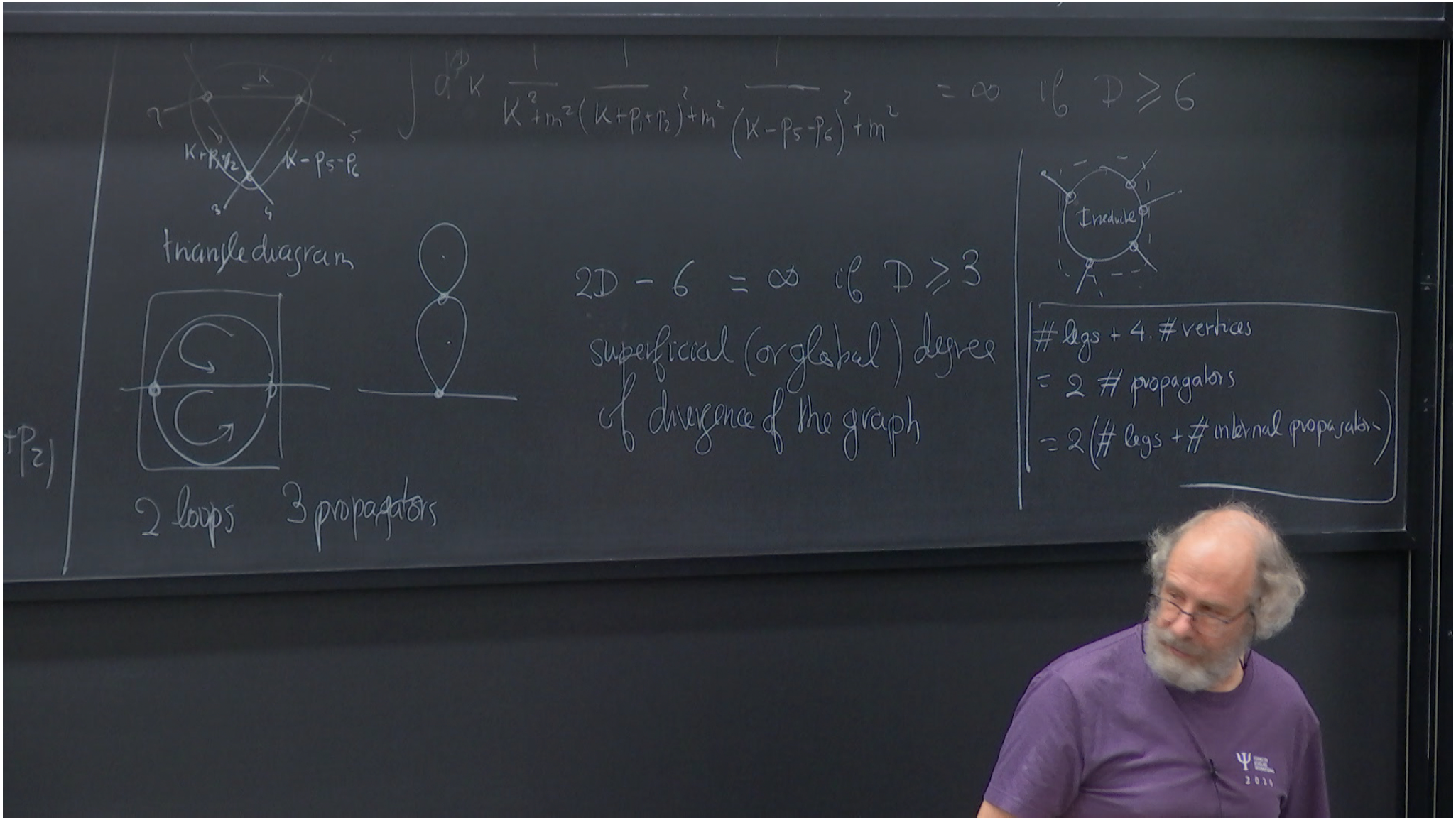
$$\int d^D K \; \frac{1}{K^2+m^2} \; \frac{1}{(K+P_1+P_2)^2+m^2} \; \frac{1}{(K-P_5-P_6)^2+m^2} = \infty \text{ if } D \geq 6$$



2 loops 3 propagators

$2D - 6 = \infty$ if $D \geq 3$

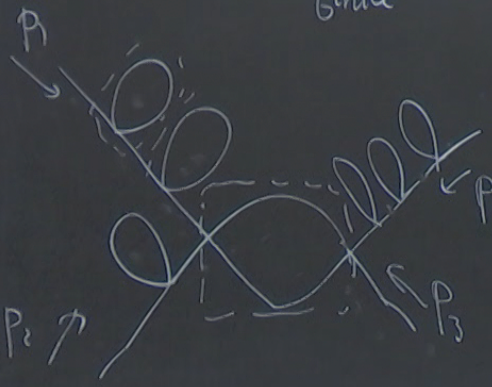
superficial (or global) degree of divergence of the graph


$\int d^D k \frac{1}{k^2+m^2} \frac{1}{(k+p_1+p_2)^2+m^2} \frac{1}{(k-p_5-p_6)^2+m^2} = \infty$ if $D \geq 6$
triangle diagram
$2D - 6 = \infty$ if $D \geq 3$
superficial (or global) degree of divergence of the graph
Irreducible
legs + 4. # vertices
= 2 # propagators
= 2(# legs + # internal propagators)
2 loops 3 propagators

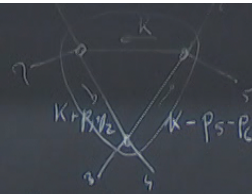
finite

divergent

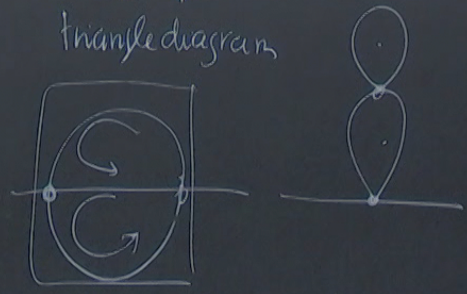
factorization



$$= \frac{1}{(P_1^2+m^2)^3}\left(\frac{1}{P_2^2+m^2}\right)^2 \frac{1}{(P_3^2+m^2)}\frac{1}{(P_4^2+m^2)^4}\cdot T^6\, B(P_1+P_2)$$



triangle diagram



2 loops 3 propagators

2D

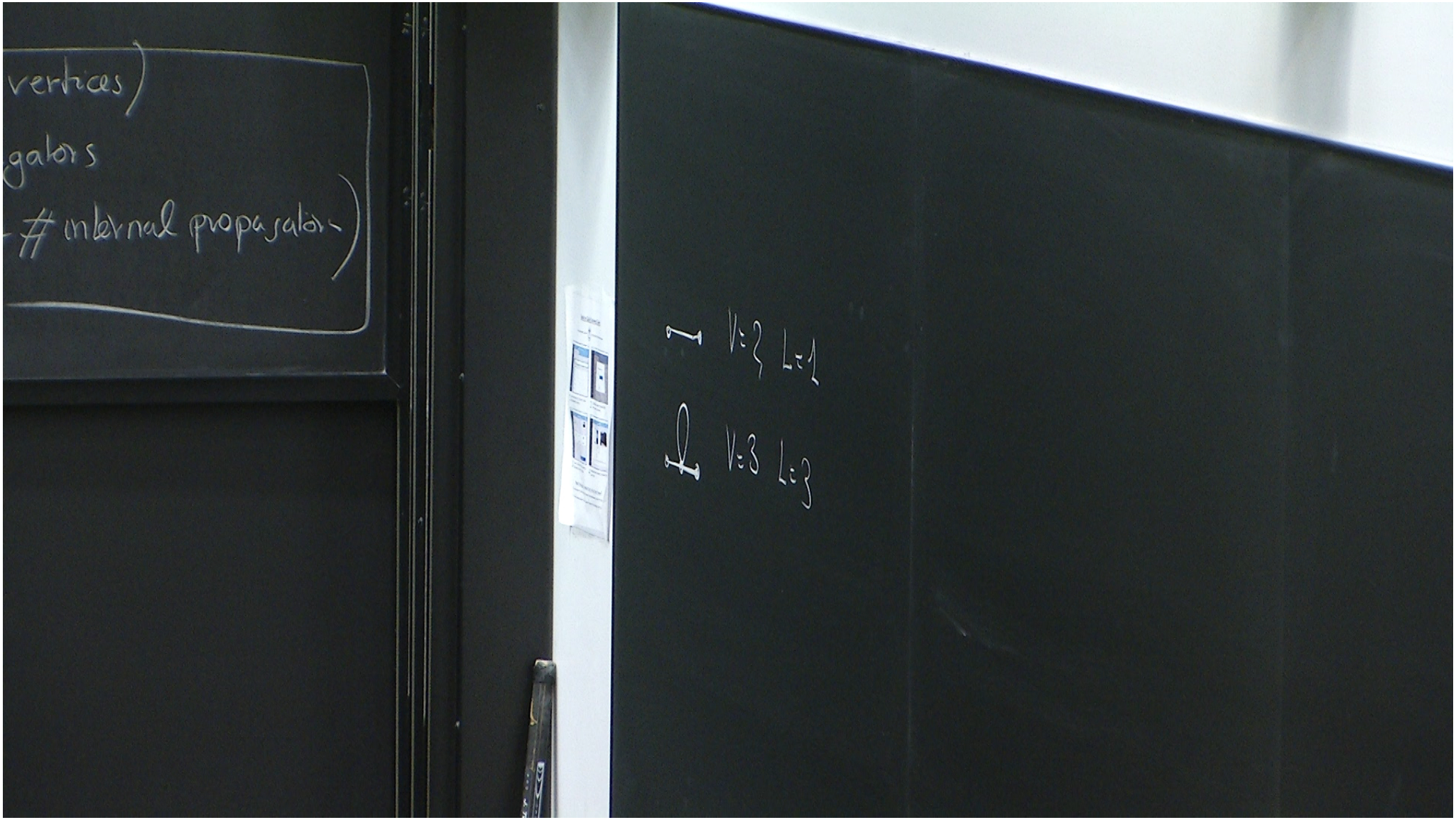
Su

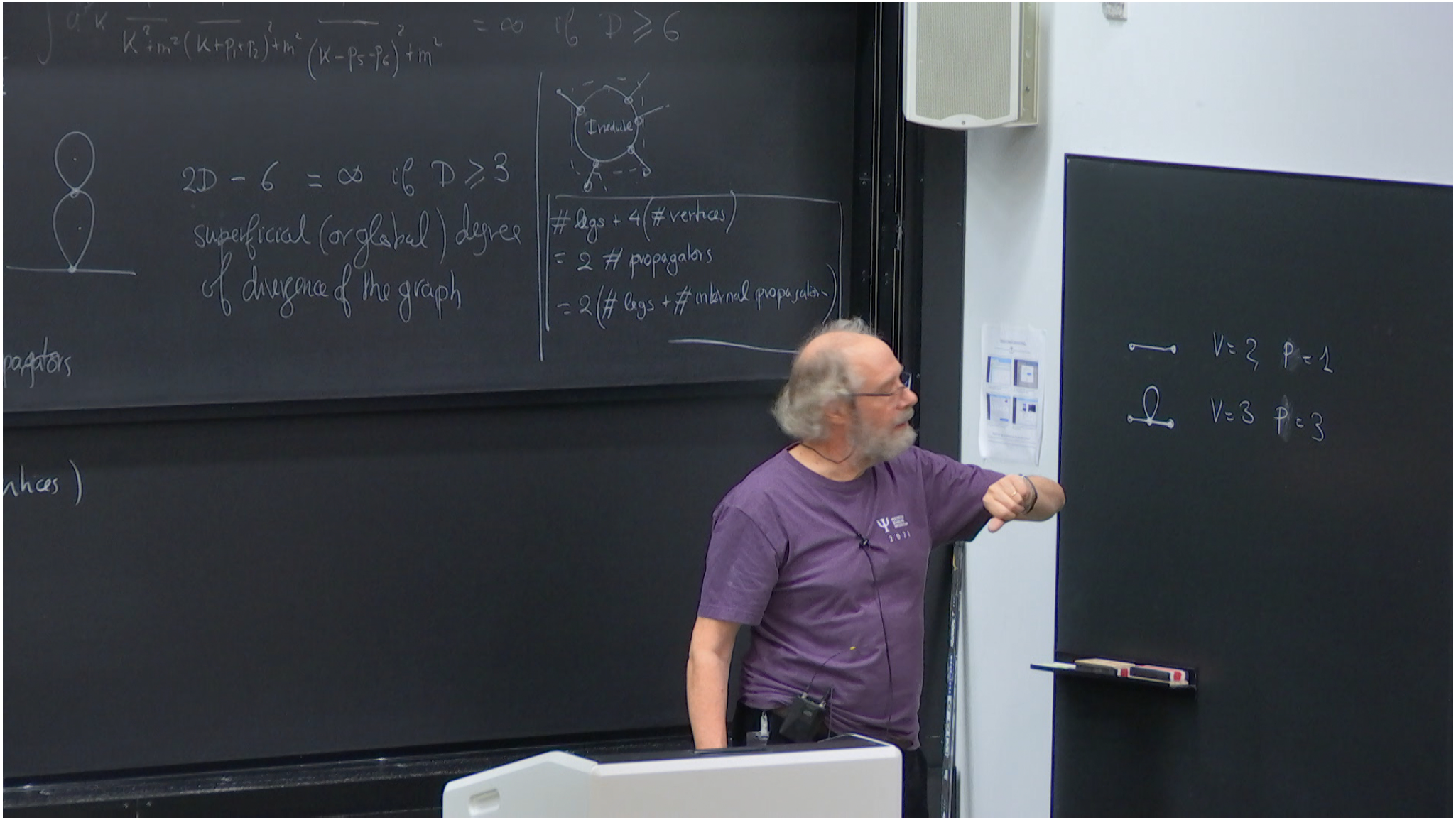
G

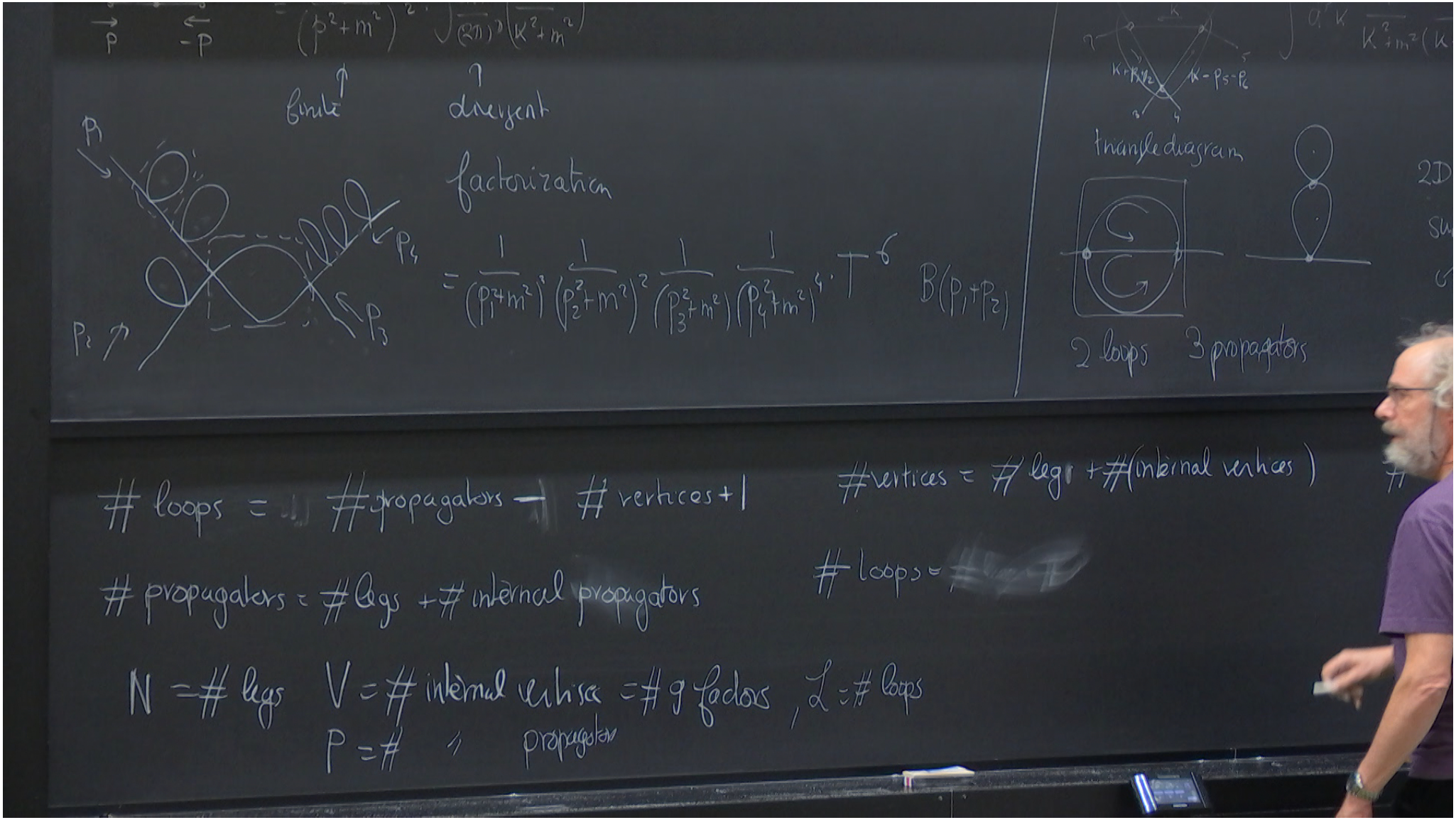
loops = # propagators − # vertices + 1

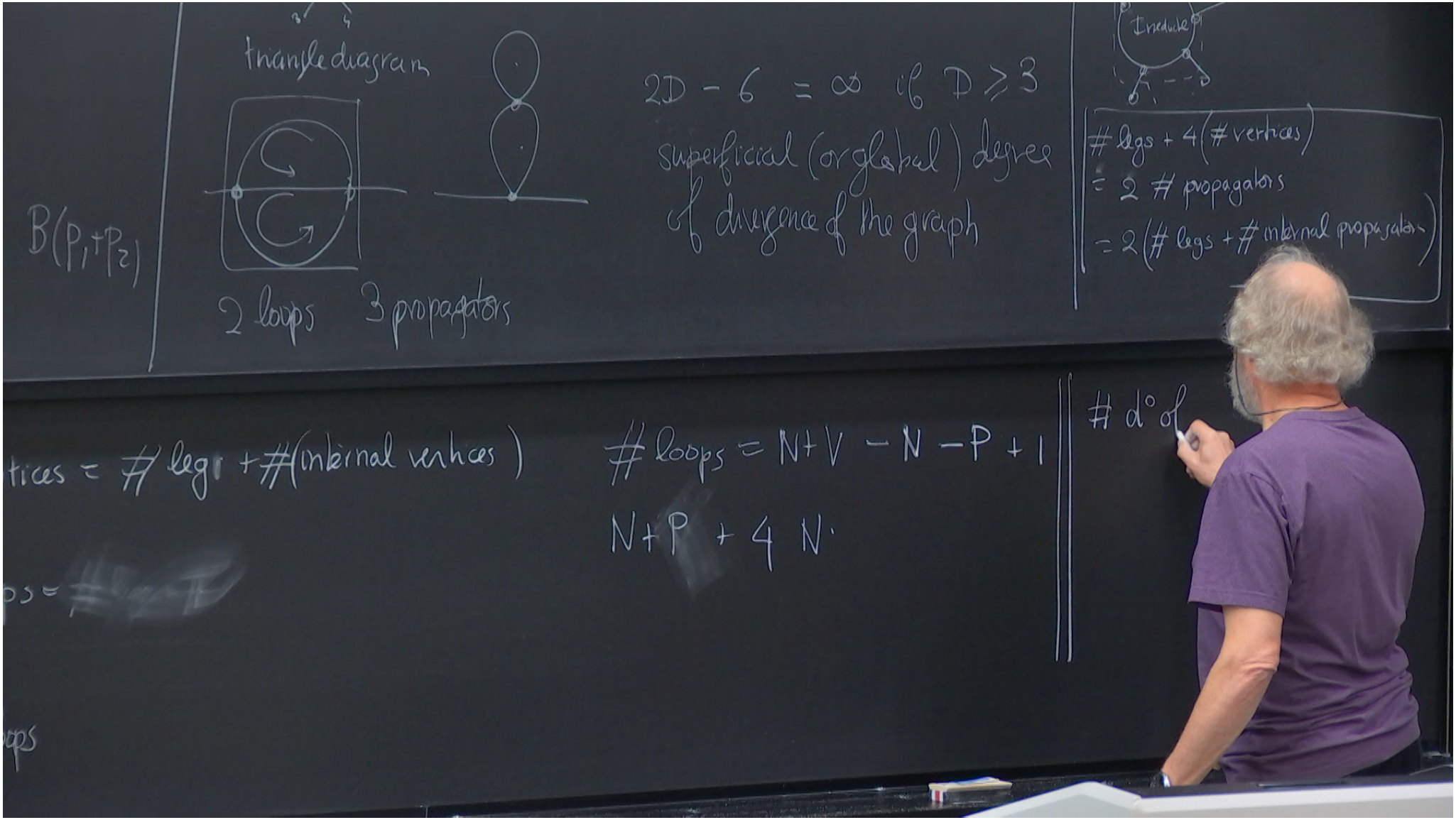
vertices = # legs + # (internal vertices)

propagators = # legs + # internal propagators


vertices)
galors
# internal propagators)
V=2 L=1
V=3 L=3


$\int d^4k \; \frac{1}{k^2+m^2}\frac{1}{(k+p_1+p_2)^2+m^2}\frac{1}{(k-p_5-p_6)^2+m^2} = \infty$ if $D \geq 6$
$2D - 6 = \infty$ if $D \geq 3$
superficial (or global) degree of divergence of the graph
Irreducible
legs + 4(# vertices)
= 2 # propagators
= 2(# legs + # internal propagators)
V=2 P=1
V=3 P=3


finite
divergent
factorization
triangle diagram
2 loops
3 propagators
# loops = # propagators − # vertices + 1
# vertices = # legs + # (internal vertices)
# propagators = # legs + # internal propagators
# loops =
N = # legs
V = # internal vertices = # g factors
L = # loops
P = # propagators


triangle diagram
2 loops 3 propagators
$2D - 6 = \infty$ if $D \geq 3$
superficial (or global) degree of divergence of the graph
Irreducible
legs + 4(# vertices)
= 2 # propagators
= 2(# legs + # internal propagators)
$B(p_1+p_2)$
tices = # legs + #(internal vertices)
# loops = N+V − N − P + 1
N+P + 4 N·
d° of

of divergence of the graph

$= 2(\#\text{ legs} + \#\text{ internal propagators})$

3 propagators

nal vertices)

$\#\text{loops} = N + V - N - P + 1$

$N + P + 4N$

# of Feynman graphs depends on the dimension

$x \; y = \langle \phi(x)\phi(y) \rangle_0 = G_F(x-y) = \int \frac{d^D k}{(2\pi)^D} \frac{e^{ik\cdot(x-y)}}{k^2+m^2}$ Euclidean $\int \frac{d^D k}{(2\pi)^D} \frac{(-i)e^{ik\cdot(x-y)}}{k^2+m^2-i0_+}$ Minkowski

well defined $x \neq y$

Feynman

In Fourier Momentum space

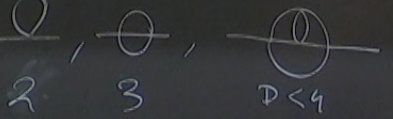
$\hat{G}(k) \simeq \frac{1}{k^2}$ when $k \nearrow \infty$

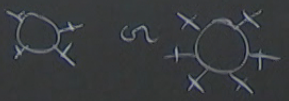
Positivity time space

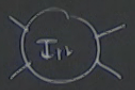
$x \to y$ diverges like $|x-y|^{2-D}$ $D>2$ $\frac{1}{2\pi}\log\left(\frac{1}{|x-y|}\right)$ $D=2$ naive $\phi(x)^2$ ill defined

UV singular

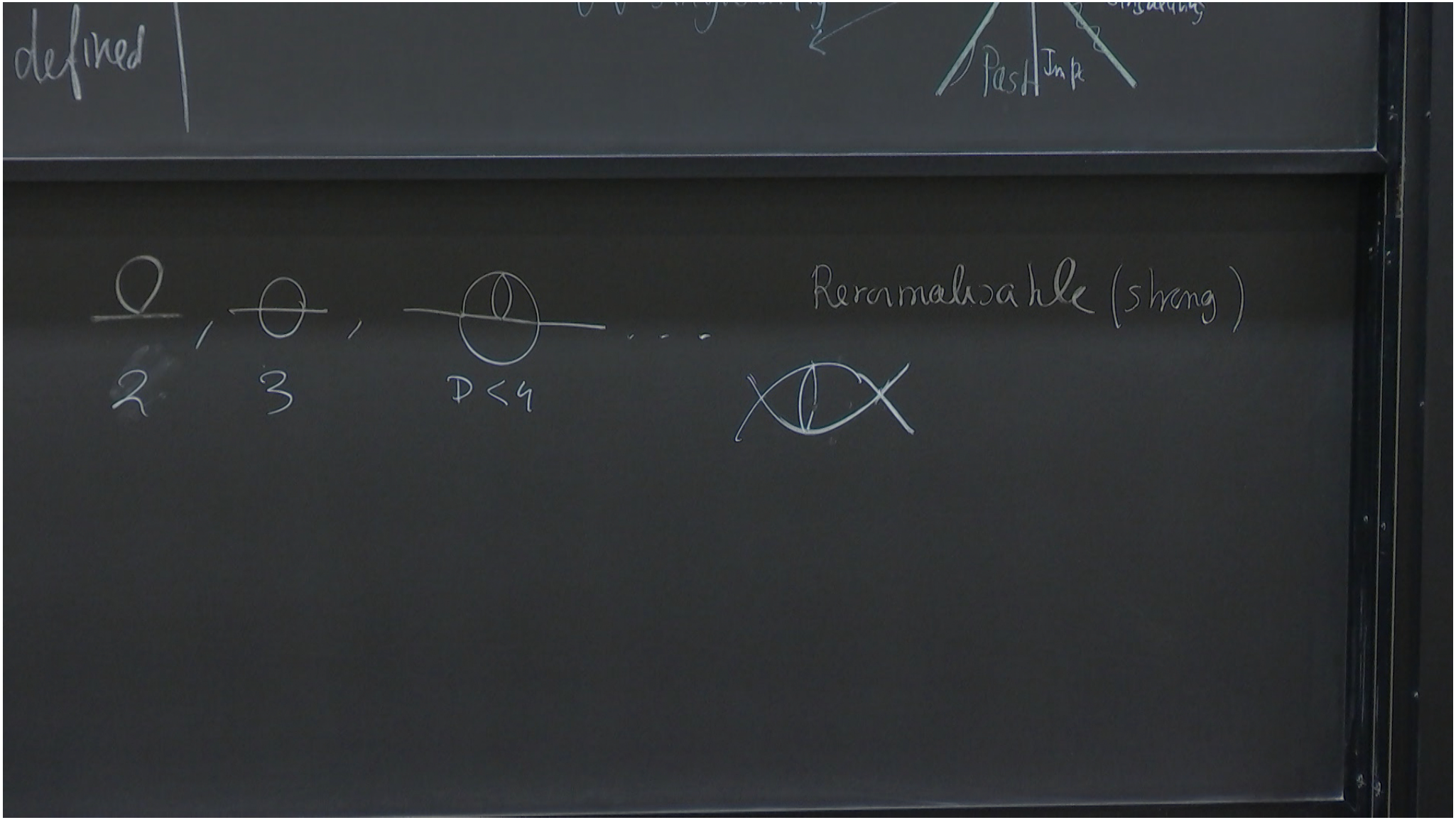
$2 \leq D < 4$ only have a finite # of 2 pants irreducible diagrams divergent

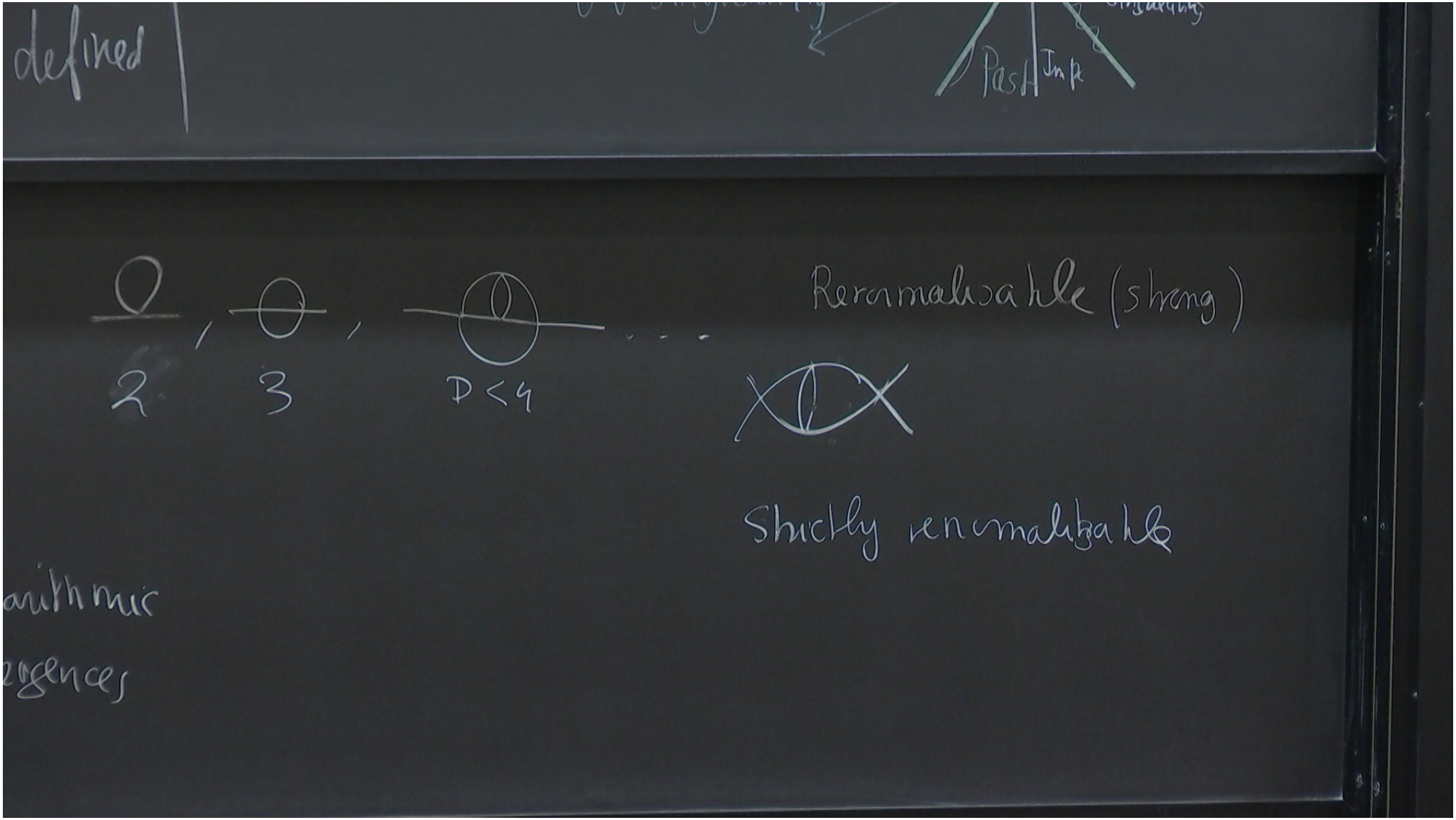


no  or divergents 

$D=4$ 

are divergents like $\mathcal{L}\cdot 4 - 2\mathcal{P}$


defined
Past
Imp
Renormalizable (strong)
2
3
D<4


defined
Past
Im f
2
3
D<4
Renormalizable (strong)
Strictly renormalizable
arithmic
ergences

$x \;\; y = \langle \phi(x)\phi(y) \rangle_0 = G_F(x-y) = \int \frac{d^D k}{(2\pi)^D} \frac{e^{ik\cdot(x-y)}}{k^2+m^2}$

well defined $x \neq y$

Feynman

Euclidean

$\int \frac{d^D k}{(2\pi)^D} \frac{(-i)e^{ik\cdot(x-y)}}{k^2+m^2-i0_+}$

Minkowski

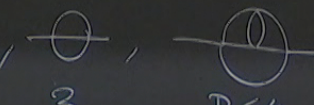
In Fourier Momentum

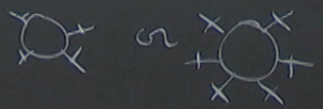
$\hat{G}(k) \simeq \frac{1}{k^2}$

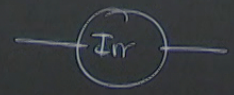
Position time space

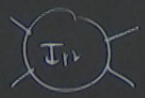
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$2 \leq D < 4$ only have a finite # of 2 pants irreducible diagrams divergent



no  or divergents



$D=4$ 

are divergents like $\mathcal{L} \cdot 4 - 2\mathcal{P} = 0$ $\frac{d^{4\alpha}k}{k^{4\alpha}} \Rightarrow$ logarithmic divergences

$\mathcal{N}$ external legs $\mathcal{N}=4$

$\alpha = 4$

$\mathcal{L}, \mathcal{N} \Rightarrow \mathcal{V}, \mathcal{P} \Longrightarrow$ degree of divergence

$+m-i0_+$

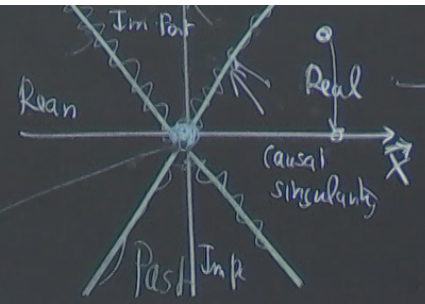
inkowski

$G(k) \simeq \frac{1}{k^2}$ when $k \to \infty$

Positivent time space

UV singularity

$d=2$ naive $\phi(x)^2$ ill defined



diagrams divergent $\underset{2}{\bigcirc}$, $\underset{3}{\bigcirc}$, $\underset{D<4}{\bigcirc}$ ... Renormalizable (strong)

Err

Strictly renormalizable

$\frac{d^{4L}k}{k^{4\alpha}} \Rightarrow$ logarithmic divergences

V = # internal vertices

L = # loops

I = # internal propagators

$k + m - i0_+$

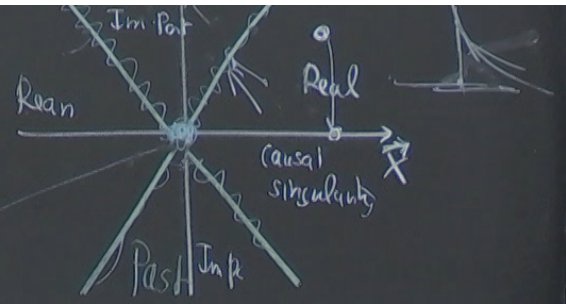
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Positivent time space

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diagrams divergent $\bigcirc_2$, $\bigcirc_3$, $\bigcirc_{D<4}$ ... Renormalizable (strong)

Err

$\bigcirc$ $\frac{d^{4\mathcal{L}}k}{k^{4\alpha}} \Rightarrow$ logarithmic divergences

Strictly renormalizable

$\mathcal{V}$ = # internal vertices

$\mathcal{L}$ = # loops

$\mathcal{P}$ = # internal propagators

$\int d^D k \times \cdots \int d^D k \; \frac{1}{\frac{1}{k^2} \cdots \frac{1}{k^2}}$

$\mathcal{L}$ $\mathcal{P}$