Title: Quantum Field Theory II - Lecture 221202

Speakers:

Collection: Quantum Field Theory II (2022/2023)

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URL: https://pirsa.org/22120000

$$\int \mathcal{D}[A] \exp\left(-\frac{1}{2g^2} S[A]\right) = \int \mathcal{D}[A]\, \delta[F[A]] \cdot \left|\det\left[F'[A]\right]\right|$$

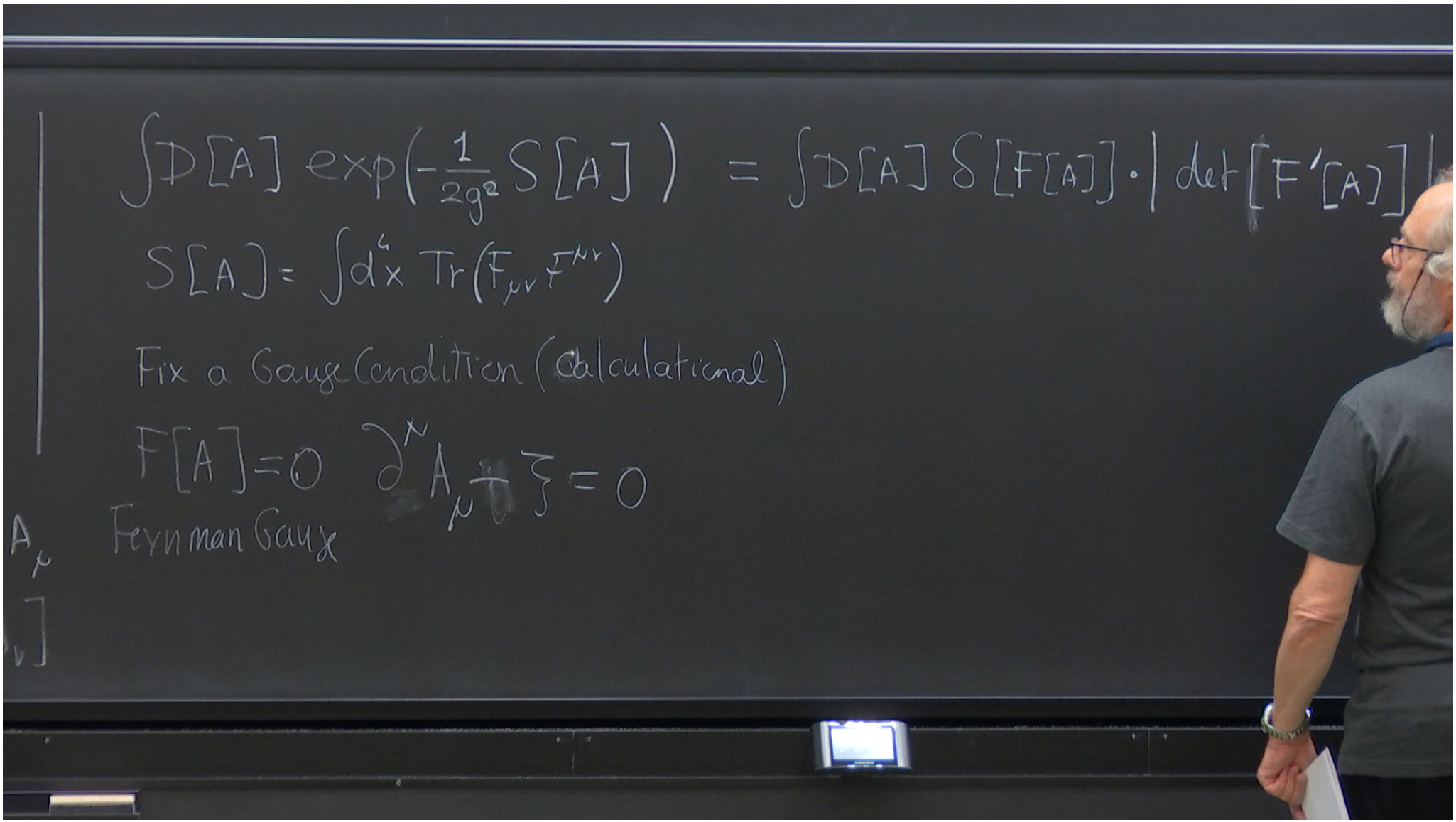
$$S[A] = \int d^4x \, \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$$

Fix a Gauge Condition (Calculational)

$F[A] = 0 \quad \partial^\mu A_\mu + \xi = 0$

A_μ Feynman Gauge

$\nu]$



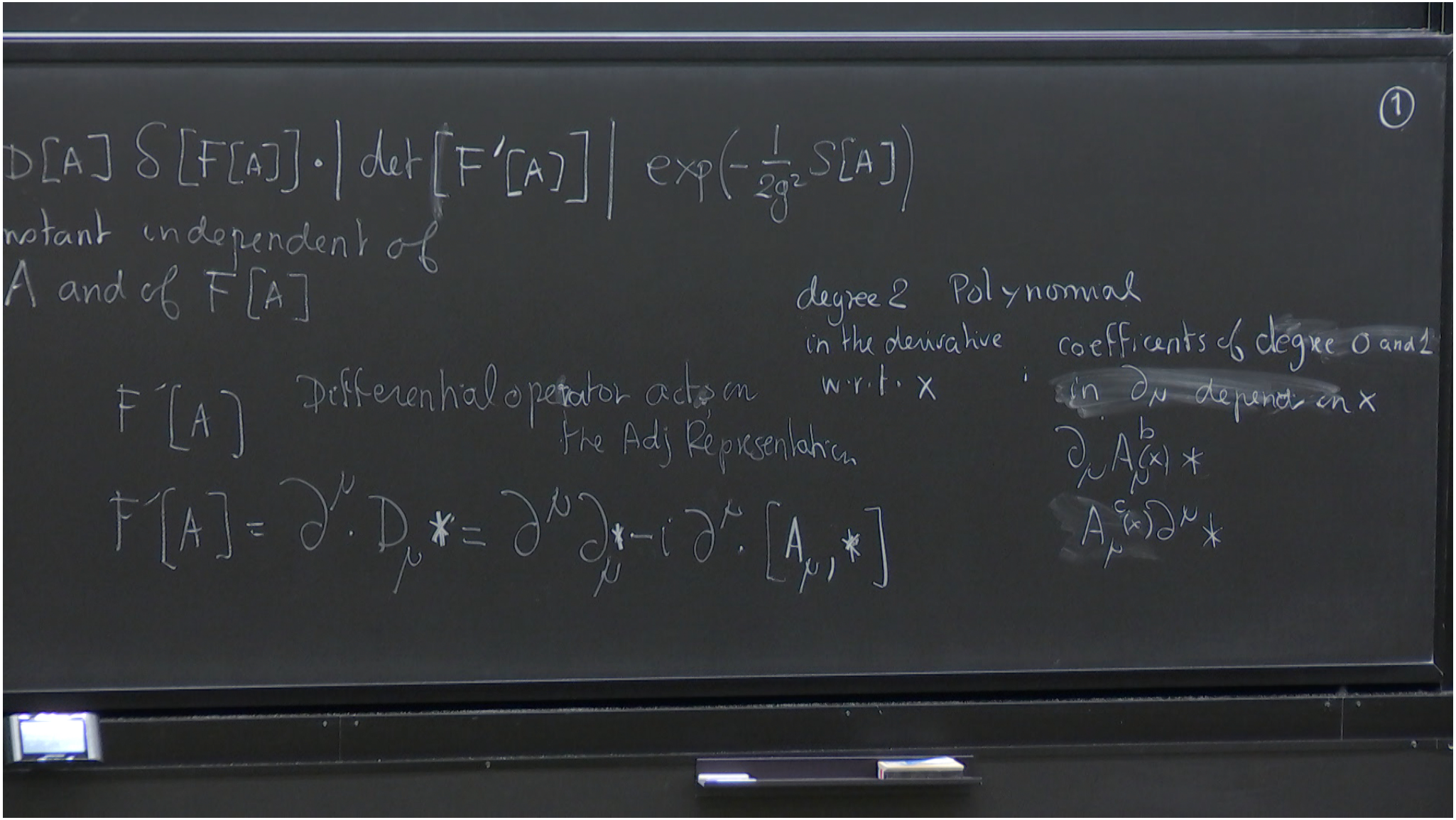
$\frac{1}{2g^2} S[A]\Big) \propto \int \mathcal{D}[A]\, \delta[F[A]] \cdot \left|\det\left[F'[A]\right]\right| \exp\left(-\frac{1}{2g^2} S[A]\right)$

$(F_{\mu\nu} F^{\mu\nu})$

constant independent of A and of $F[A]$

tion (calculational)

$\xi = 0$


①
$D[A]\,\delta[F[A]]\cdot\left|\det\left[F'[A]\right]\right|\exp\left(-\frac{1}{2g^2}S[A]\right)$
notant independent of
A and of F[A]
degree 2 Polynomial
in the derivative
w.r.t. x
coefficients of degree 0 and 1
in ∂_μ depend on x
$F'[A]$ Differential operator acts on
the Adj Representation
$\partial_\mu A^b_\mu(x)\,*$
$A^c_\mu(x)\,\partial_\mu *$
$F'[A] = \partial^\mu \cdot D_\mu * = \partial^\mu \partial_\mu * - i\,\partial^\mu \cdot [A_\mu, *]$

Gauge Theories:

$SU(2)$ $\quad A_\mu = A_\mu^a t_a$

$a = 1, 2, 3$

$$A_\mu \longrightarrow A_\mu + D_\mu \alpha$$

$$F_{\mu\nu} = i\left[D_\mu, D_\nu\right] = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right]$$

$$\int \mathcal{D}[A] \exp\left(-\frac{1}{2g^2} S[A]\right) \alpha \int_{\text{con}}$$

$$S[A] = \int d^4x \, \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$$

Fix a Gauge Condition (Calculational)

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Feynman Gauge

$$\det\left[-\partial^\mu D_\mu\right] = \int \mathcal{D}[\bar{c}, c]\, \exp\left(-\bar{c}\cdot\left(-\partial^\mu D_\mu\right)\cdot c\right)$$

$$\bar{c}\cdot\left(-\partial^\mu D_\mu\right)\cdot c = \int d^4x\, \mathrm{Tr}\, \bar{c}(x)\cdot\left(-\partial^\mu D_\mu\right) c(x)$$

acts $\longrightarrow$

Gauge Theories :

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$$\int \mathcal{D}[A] \exp\left(-\frac{1}{2g^2} S[A]\right) \alpha \int_{\text{con}}$$

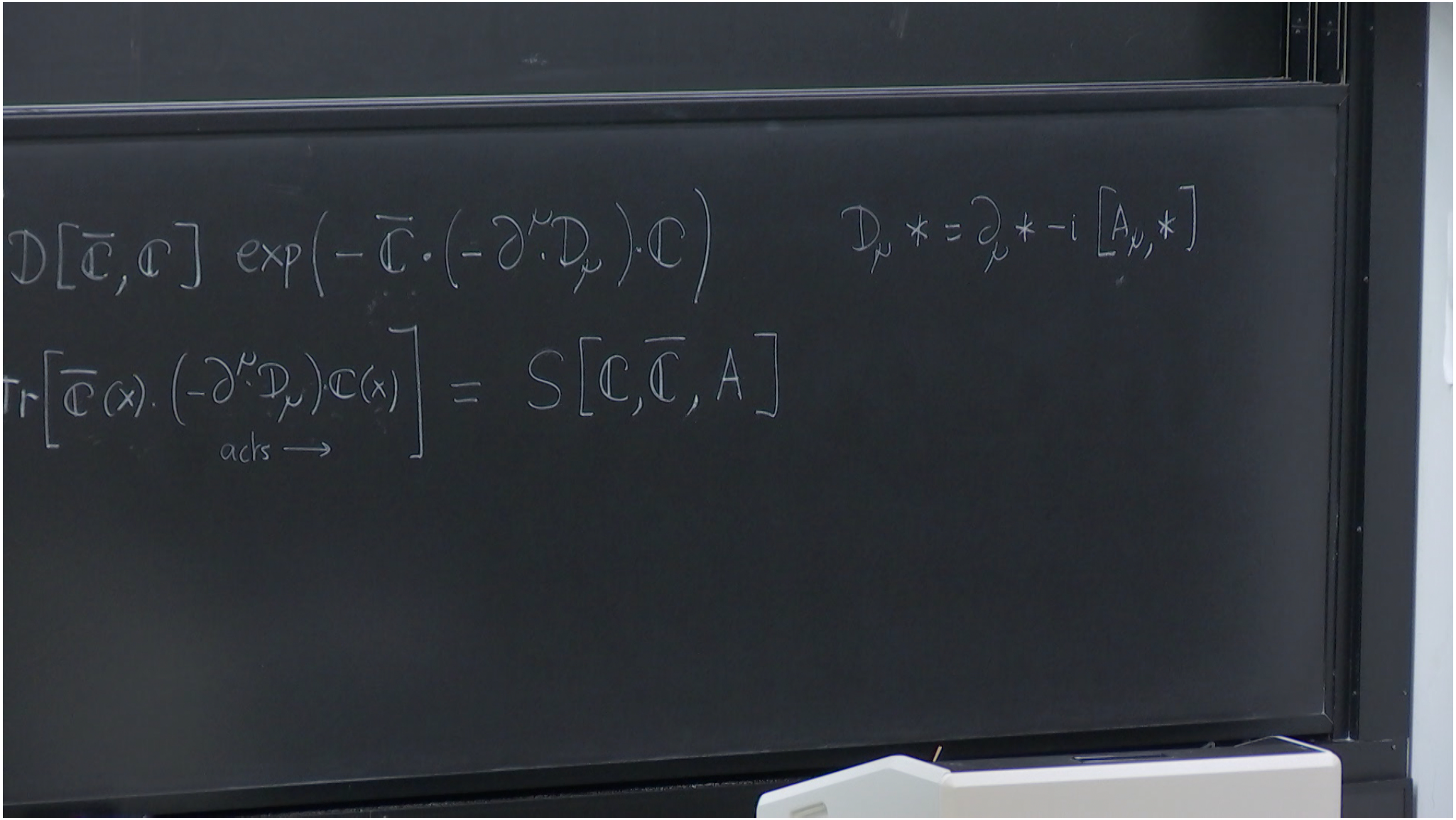
$$S[A] = \int d^4x \, \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$$

Fix a Gauge Condition (Calculational)

$F[A] = 0 \qquad \partial^\mu A_\mu + \xi = 0$

Feynman Gauge

$\mathrm{Tr}(t_a t_b) = \frac{1}{2} \delta_{ab}$


$\mathcal{D}[\bar{\mathbb{C}},\mathbb{C}]\ \exp\left(-\bar{\mathbb{C}}\cdot(-\partial^\mu\cdot D_\mu)\cdot\mathbb{C}\right)$
$D_\mu * = \partial_\mu * - i\,[A_\mu, *]$
$\mathrm{Tr}\left[\bar{\mathbb{C}}(x)\cdot(-\partial^\mu\cdot D_\mu)\cdot\mathbb{C}(x)\right] = S[\mathbb{C},\bar{\mathbb{C}},A]$
acts

② Fadeev-Popov Ghosts

1.- $|\det(F'[A])| = \det(F'[A])$

2.- 1 solution to $F[A]=0$

$$\det\left[-\partial^\nu D_\nu\right] = \int \mathcal{D}$$

$$\overline{\mathbb{C}}\cdot(-\partial^\nu D_\nu)\cdot\mathbb{C} = \int d^4x\ \mathrm{Tr}$$

A pair of Grassmann Conjugate Fields

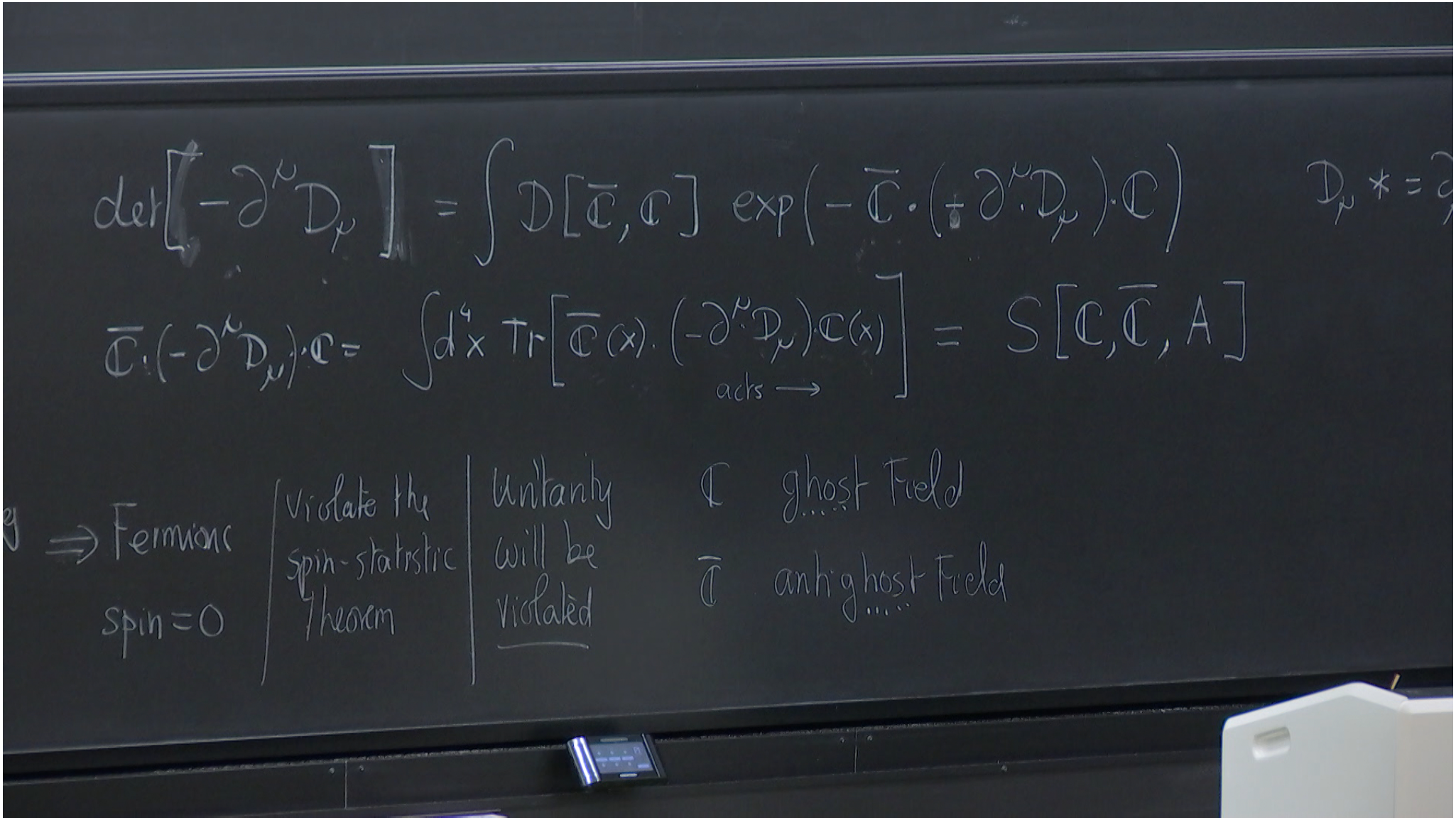
$\mathbb{C}(x)$, $\overline{\mathbb{C}}(x)$ Adj Repr. of $SU(2)$ Anticommuting Fields

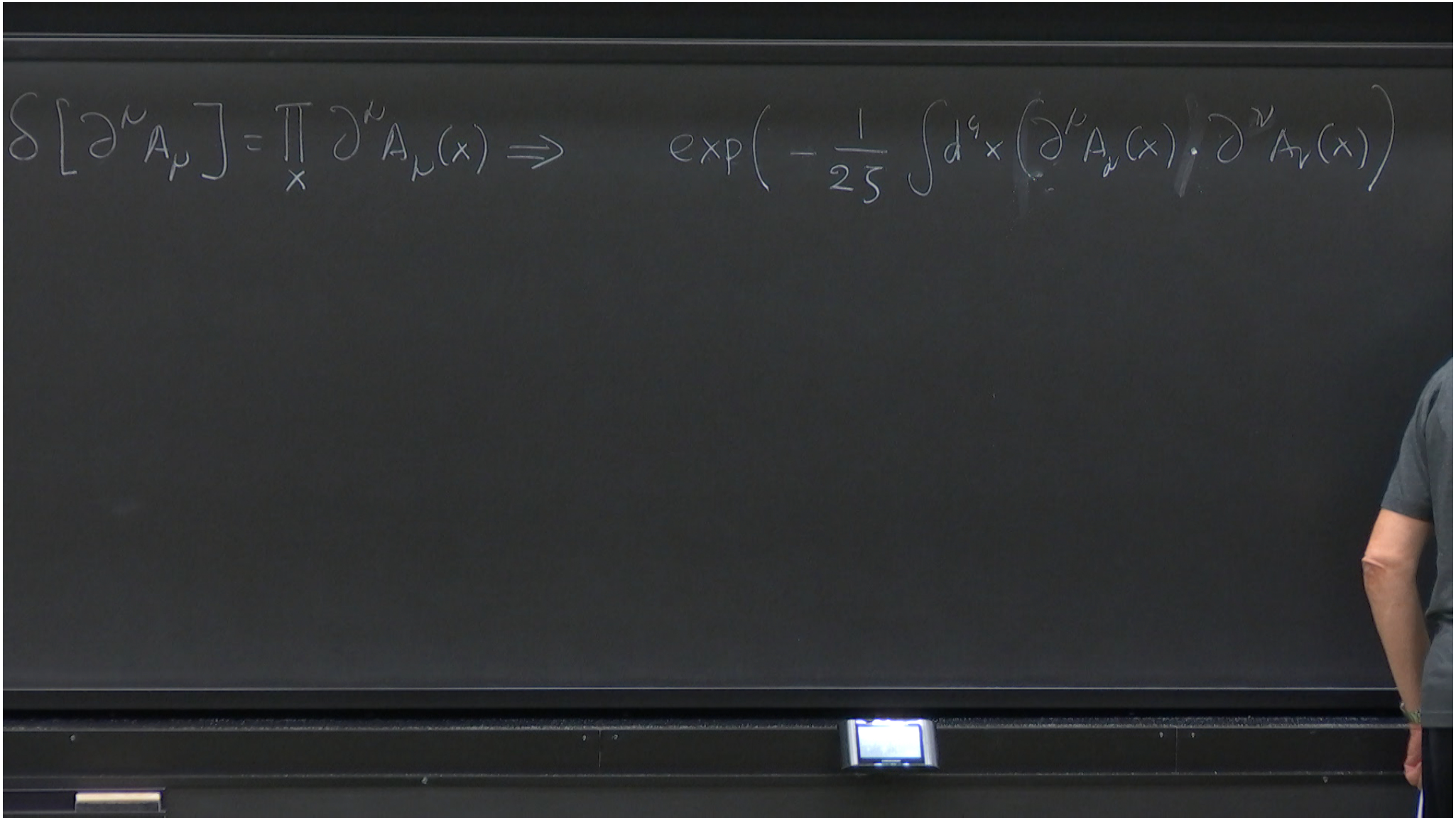
$\mathbb{C}(x) = \mathbb{C}^a(x)\, t_a$

$\overline{\mathbb{C}}(x) = \overline{\mathbb{C}}^a(x)\, t_b$

For Lorentz Scalors

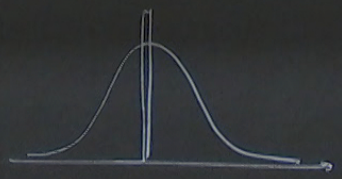
Spin 0

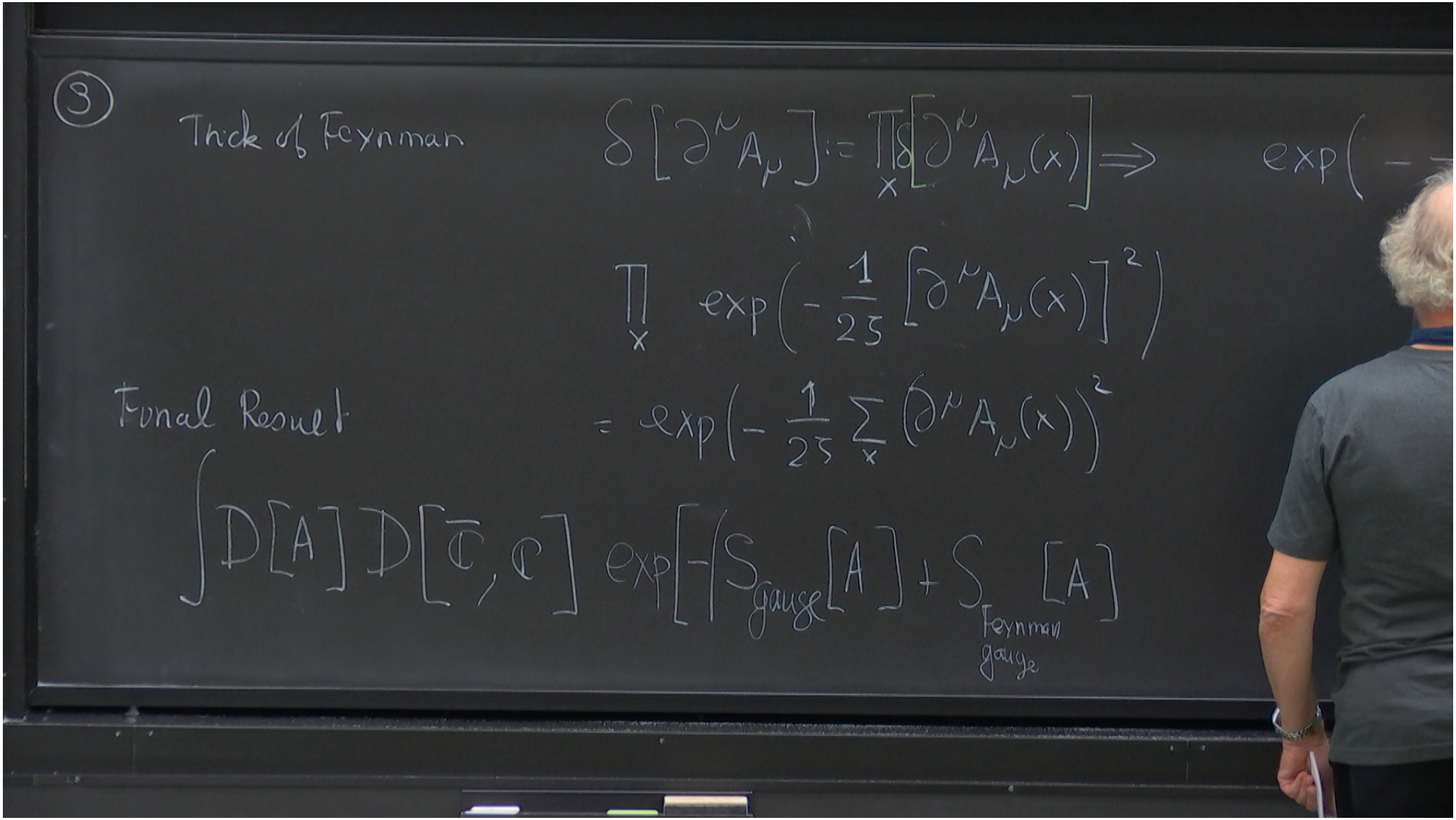

$\det\left[-\partial^\nu D_\nu\right] = \int \mathcal{D}[\bar{\mathbb{C}},\mathbb{C}]\, \exp\left(-\bar{\mathbb{C}}\cdot(-\partial^\nu D_\nu)\cdot\mathbb{C}\right)$
$D_\nu * = \partial$
$\bar{\mathbb{C}}\cdot(-\partial^\nu D_\nu)\cdot\mathbb{C} = \int d^4x\, \mathrm{Tr}\left[\bar{\mathbb{C}}(x)\cdot(-\partial^\nu D_\nu)\mathbb{C}(x)\right] = S[\mathbb{C},\bar{\mathbb{C}},A]$
acts
$\Rightarrow$ Fermions
spin = 0
violate the spin-statistic Theorem
Unitarity will be violated
$\mathbb{C}$ ghost Field
$\bar{\mathbb{C}}$ antighost Field


$\delta[\partial^\nu A_\mu] = \prod_x \partial^\nu A_\mu(x) \Rightarrow$
$\exp\left(-\frac{1}{2\xi}\int d^4x \left(\partial^\mu A_\mu(x)\right)\cdot\partial^\nu A_\nu(x)\right)$

Δ Laplace-Beltrami op.:

$$\frac{1}{25}\int d^4x \left(\partial^\mu A_\mu(x) \cdot \partial^\nu A_\nu(x)\right)$$




③
Trick of Feynman
$\delta[\partial^\mu A_\mu] := \prod_x \delta[\partial^\mu A_\mu(x)] \Rightarrow$
$\exp($
$\prod_x \exp\left(-\frac{1}{2\xi}[\partial^\mu A_\mu(x)]^2\right)$
Final Result
$= \exp\left(-\frac{1}{2\xi}\sum_x (\partial^\mu A_\mu(x))^2\right.$
$\int \mathcal{D}[A]\,\mathcal{D}[\bar{c}, c]\,\exp[-(S_{gauge}[A] + S_{Feynman\ gauge}[A]$

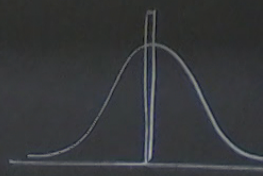
$$\prod_x \delta\left[\partial^\nu A_\nu(x)\right] \Rightarrow \exp\left(-\frac{1}{25}\int d^4x \left(\partial^\nu A_\nu(x)\right)\cdot\partial^\nu A_\nu(x)\right)$$

$$S_{\text{gauge Fixing}}[A]$$

$$\frac{1}{25}\left[\partial^\nu A_\nu(x)\right]^2\Big)$$

$$\sum_x \left(\partial^\nu A_\nu(x)\right)^2$$

$$A] + S_{\substack{\text{Feynman}\\ \text{gauge}}}[A] + S_{\text{ghost}}[\mathbb{C},\bar{\mathbb{C}},A]\Big)$$



$\times \left(\partial^\mu A_\mu(x) \cdot \partial^\nu A_\nu(x)\right)$

fixing $[A]$

$$Z_\xi = \int D[A]\ \delta[\partial^\nu A_\nu - \xi]$$

The final result do not depend on ξ

$$Z_\xi = Z \quad \text{for all } \xi$$

$$Z = \int d\xi\ Z_\xi \exp\left(-\frac{\xi^2}{2\zeta}\right)$$

$$\int d\xi\ \delta(\partial^\nu A_\nu - \xi) \exp\left(-\frac{\xi^2}{2\zeta}\right) = \exp\left(-\frac{(\partial^\nu A_\nu)^2}{\zeta}\right)$$

Start From Action: $S_{gauge} + S_{gaugeFixing} + S_{ghosts}$

Feynman diagramatics

$$\frac{1}{g^2}\left[\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 + \right.$$

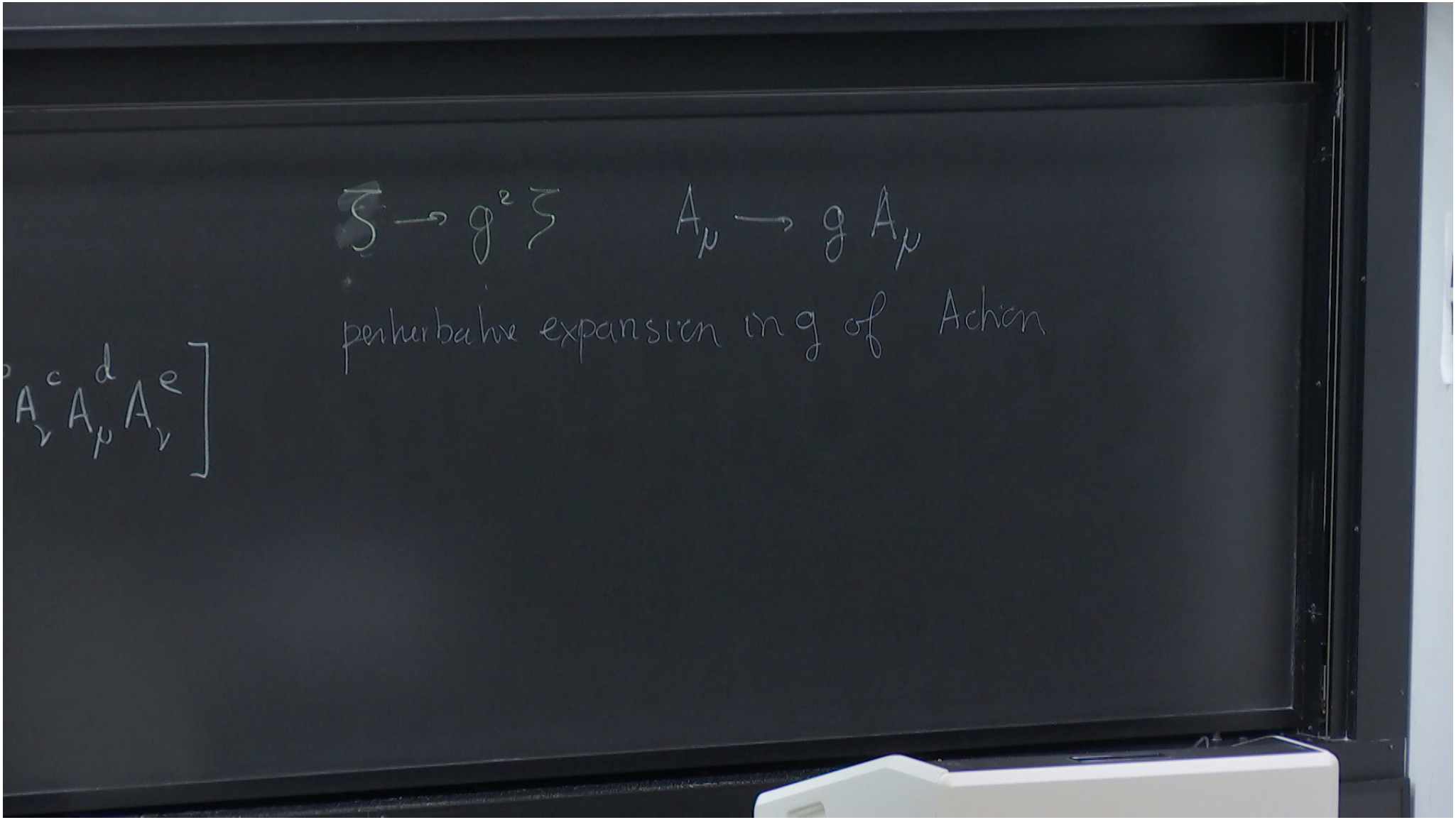
Start From Action: $S_{gauge} + S_{gaugeFixing} + S_{ghosts}$

Feynman diagramatics

Gauge $\frac{1}{g^2}\left[\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 + 2\,\partial_\mu A^a_\nu\, f_{abc} A^b_\mu A^c_\nu + \frac{1}{2} f_{abc} f_{ade} A^b_\mu A^c_\nu A^{\;}\right.$

GF $+ \frac{1}{2\xi}\partial_\mu A^a_\mu\, \partial_\nu A^a_\nu$

Ghosts $+ \partial_\mu \bar{C}^a \cdot \partial_\mu C^a + f_{abc} \cdot \partial_\mu \bar{C}^a \cdot A^b_\mu \cdot C^c$


$\xi \to g^2 \xi$
$A_\mu \to g A_\mu$
perturbative expansion in g of Action
$A^c_\nu A^d_\mu A^e_\nu]$

□ quadratic in the $A \cdot A$ $\Rightarrow$ propagator for A^a_μ = Propag for the photon in Maxell & QED

$\langle \mathbb{C}^a(x)\, \bar{\mathbb{C}}^b(y) \rangle_0$? 2pt function for the ghost

$$\langle A^a_\mu(x)\, A^b_\nu(y) \rangle_0 = \mu^a \sim\sim \nu^b$$

FT : $$\boxed{\delta_{ab} \frac{-i}{k^2 - i\epsilon} \left(h_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2} \right)}$$

Minkowski space $k^2 = -k_0^2 + \vec{k}^2$ $\quad h_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

$f_{abc} \cdot \partial_\mu \bar{C}^a \cdot A_\mu^b \cdot C^c$

= Propag for the photon in Maxell & QED

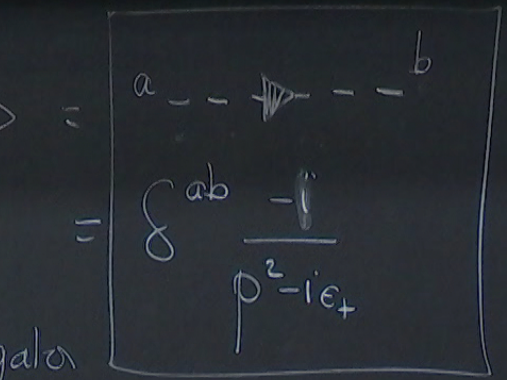


$+\left(5-1\right)\frac{k_\mu k_\nu}{k^2}\Big)$

$\begin{pmatrix} -1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}$

$\langle C^a(x)\, \bar{C}^b(x) \rangle_0$ = 2pt function for the ghost

= $\delta^{ab} \frac{-i}{p^2 - i\epsilon_+}$



same as the propagator for the scalar field

the Maxell

$\langle \mathbb{C}^a(x)\, \bar{\mathbb{C}}^b(x) \rangle_0 =$ a ---▶--- b $= \delta^{ab} \frac{-i}{p^2 - i0_+}$

2pt function for the ghost

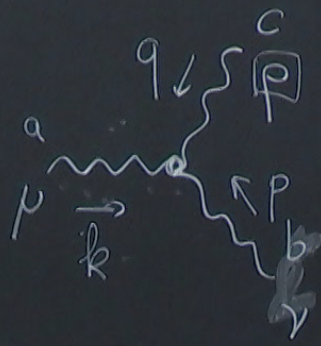
Same as the propagator for the scalar field

Feynman propagator

$K = (k_0, \vec{k})$ $\epsilon = (\epsilon_0, \vec{\epsilon})$ $\epsilon \cdot K = 0$

$-k_0^2 + \vec{k}^2 = 0$ $K^2 = 0$

Interaction terms · g



$\to g f_{abc} \left(h^{\mu\nu}(k-p)^{\rho} + \right.$

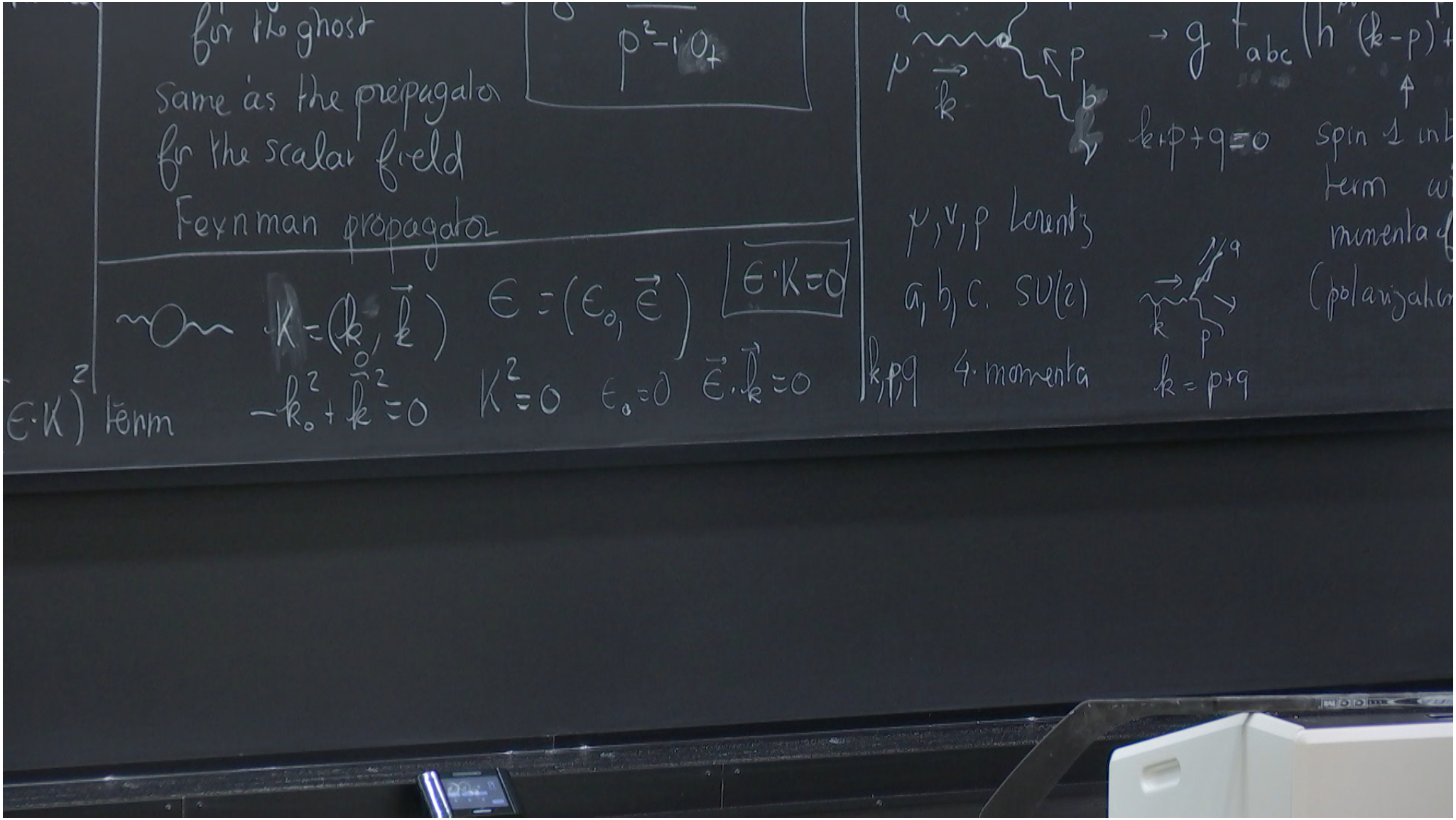
$k+p+q=0$

spin 1 in
term wi
momenta
(polarizatio

μ, ν, ρ Lorentz

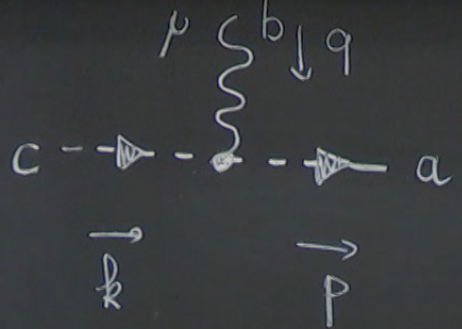
a, b, c SU(2)

k, p, q 4-momenta


for the ghost
Same as the propagator
for the scalar field
Feynman propagator
$\frac{i}{p^2 - i0_+}$
$K = (k_0, \vec{k})$
$\epsilon = (\epsilon_0, \vec{\epsilon})$
$\epsilon \cdot K = 0$
$-k_0^2 + \vec{k}^2 = 0$
$K^2 = 0$
$\epsilon_0 = 0$
$\vec{\epsilon} \cdot \vec{k} = 0$
$(\epsilon \cdot K)^2$ term
μ
k
p
q
a
b
c
$\rightarrow g f_{abc} (k-p)$
$k+p+q=0$
μ, ν, ρ Lorentz
a, b, c SU(2)
k, p, q 4-momenta
$k = p+q$
spin 1 in
term
momenta
(polarizat

Feynman gauge

ghost



SU(2) group

$-g f_{abc} p_\mu$

Similar Fermion and Photon in QED

For Scalar electrodynamics

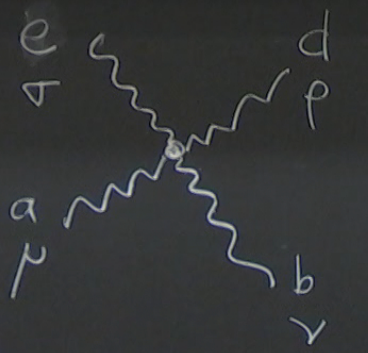
$k + q = p$

4 momenta

Feynman gauge

ghost

$\int \mathcal{D}[A] \int \mathcal{D}[\xi] = \int \mathcal{D}[$



$g^2 \, h_{\mu\rho} \, h_{\nu\sigma} \, f_{abc} \, f_{dec}$

$\delta_{ad}\delta_{bc} - \delta_{ae}\delta_{bd}$

exchange of q. numbers

μ, ν, ρ, σ Lorentz

a, b, c, d SU(2)

$\partial^\nu D_\nu q) - 0$

$$\int dA\, \delta(\partial^\nu A_\nu - \xi) \exp\left(-\frac{\xi^2}{\zeta}\right) = \exp\left(-\frac{(\partial^\nu A_\nu)^2}{\zeta}\right)$$

$$\int D[A] \int D[\xi] = \int D[\xi] \int D[A]$$

$h_{\nu\sigma}\ f_{abc}\ f_{dec}$

$\delta_{ad}\delta_{bc} - \delta_{ae}\delta_{bd}$

exchange of q. numbers

$g \approx$ unit of charge $\longleftrightarrow$ e

same for gauge field, and the ghosts

↳ SU(2) symm.

$\frac{g}{4!}\phi^4 \longrightarrow \times g$

U(1)
no need for ghosts

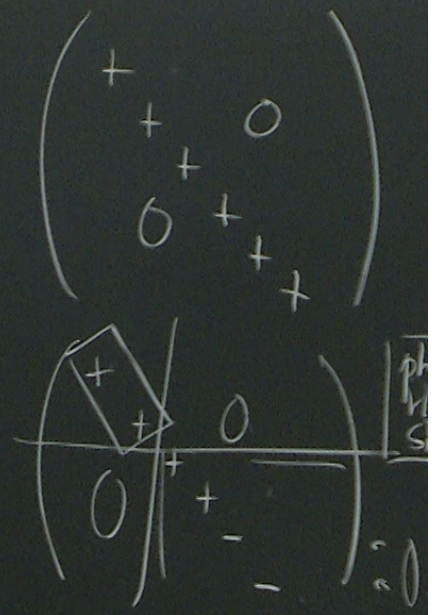
creation annihil

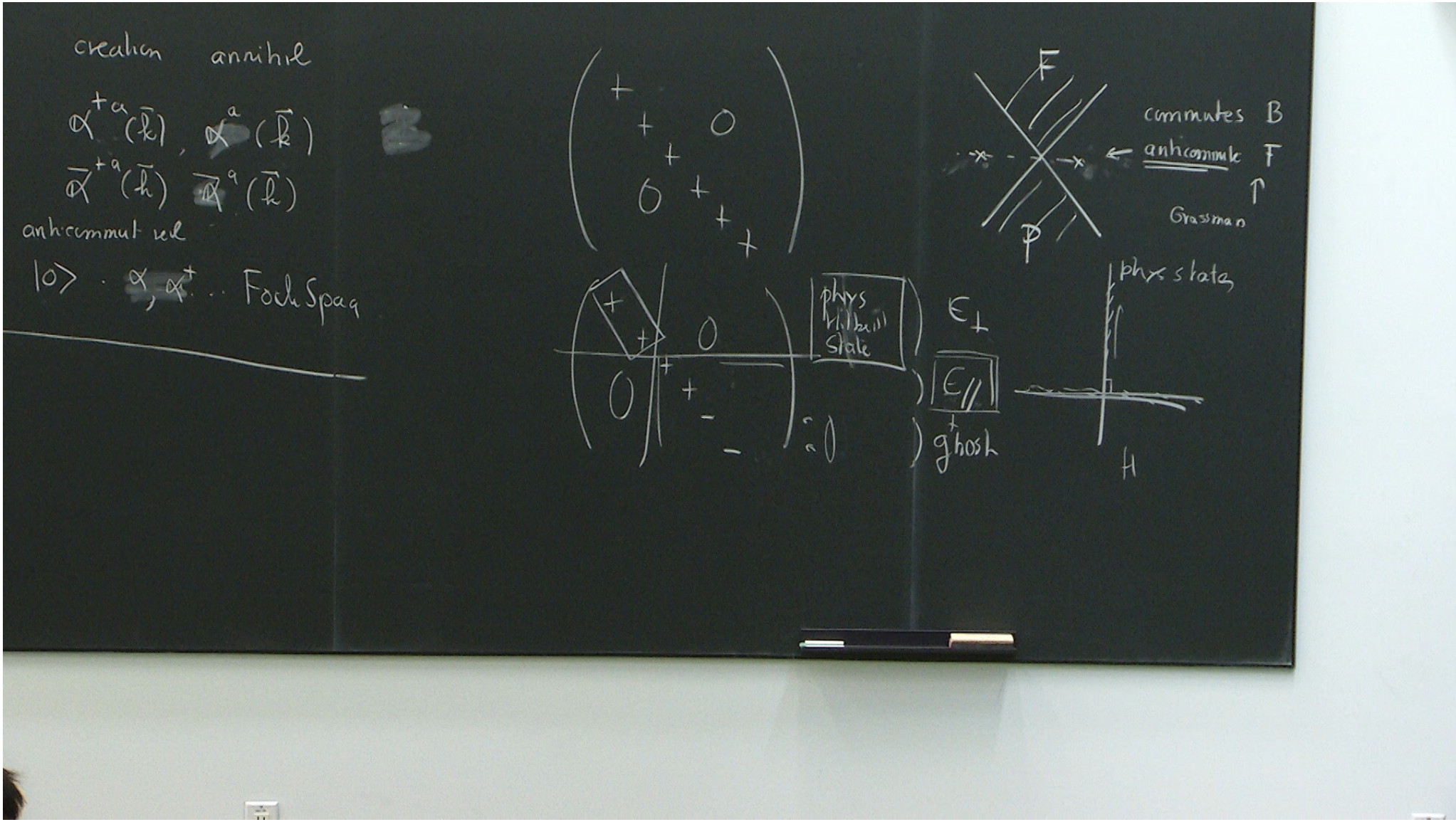
C $\alpha^{+a}(\vec{k})$, $\alpha^{a}(\vec{k})$

$\bar{C}$ $\bar{\alpha}^{+a}(\vec{k})$ $\bar{\alpha}^{a}(\vec{k})$

anticommut rel

$|0\rangle$. α, α^{+} .. Fock Space




creation annihil
$\alpha^{+a}(\vec{k})$, $\alpha^{a}(\vec{k})$
$\bar{\alpha}^{+a}(\vec{k})$ $\bar{\alpha}^{a}(\vec{k})$
anticommut rel
$|0\rangle$. α, α^{+} .. Fock Space
F
commutes B
anticommute F
Grassman
P
phys
phys state
$\epsilon_\perp$
$\epsilon_{//}$
ghost
H