Title: Quantum Field Theory II - Lecture 221202

Speakers:

Collection: Quantum Field Theory II (2022/2023)

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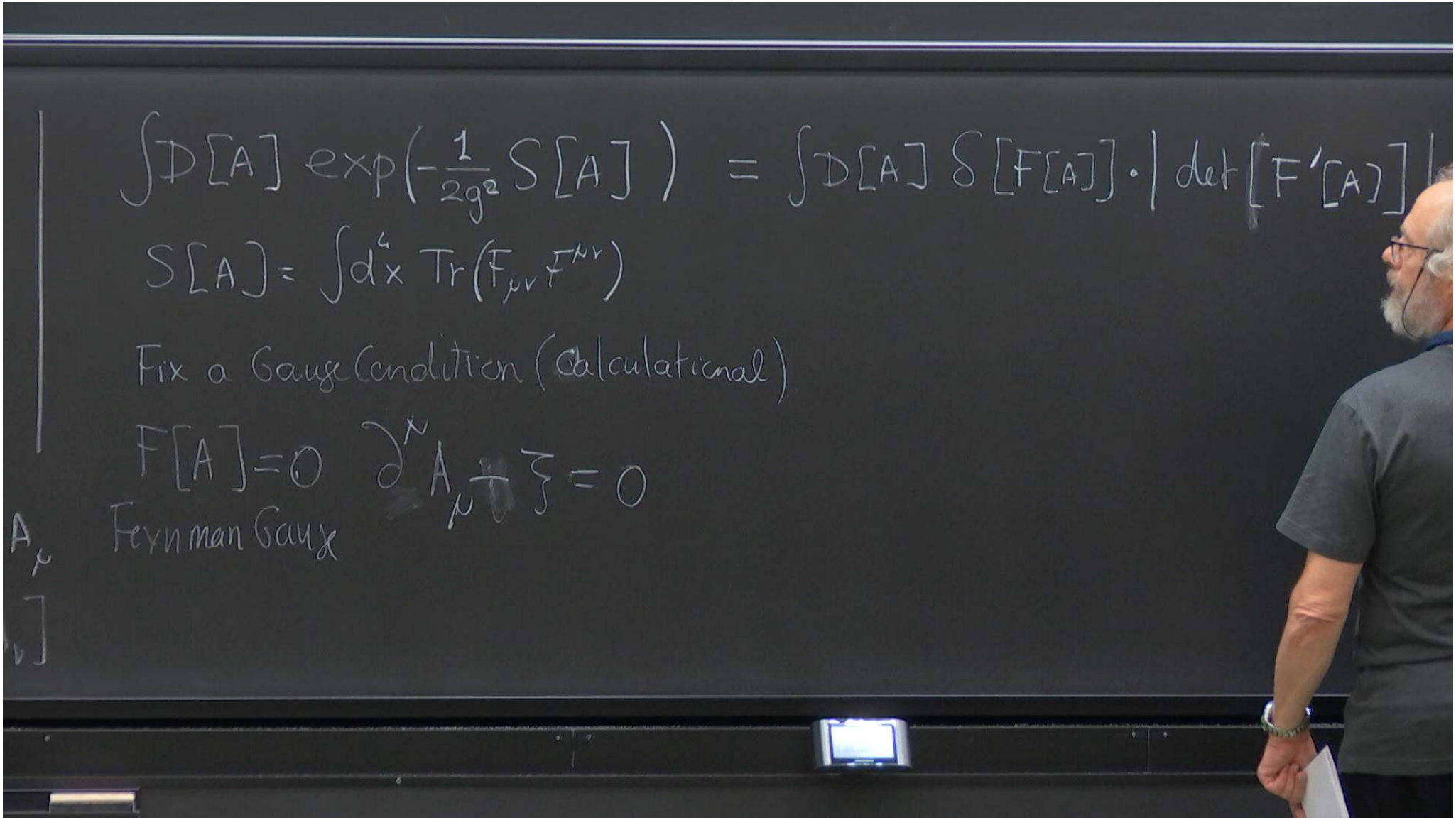

$\int \mathcal{D}[A] \exp\left(-\frac{1}{2g^2} S[A]\right) = \int \mathcal{D}[A]\, \delta[F[A]] \cdot \left|\det\left[F'[A]\right]\right|$
$S[A] = \int d^4x \, \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$
Fix a Gauge Condition (Calculational)
$F[A] = 0 \quad \partial^\mu A_\mu + \xi = 0$
Feynman Gauge

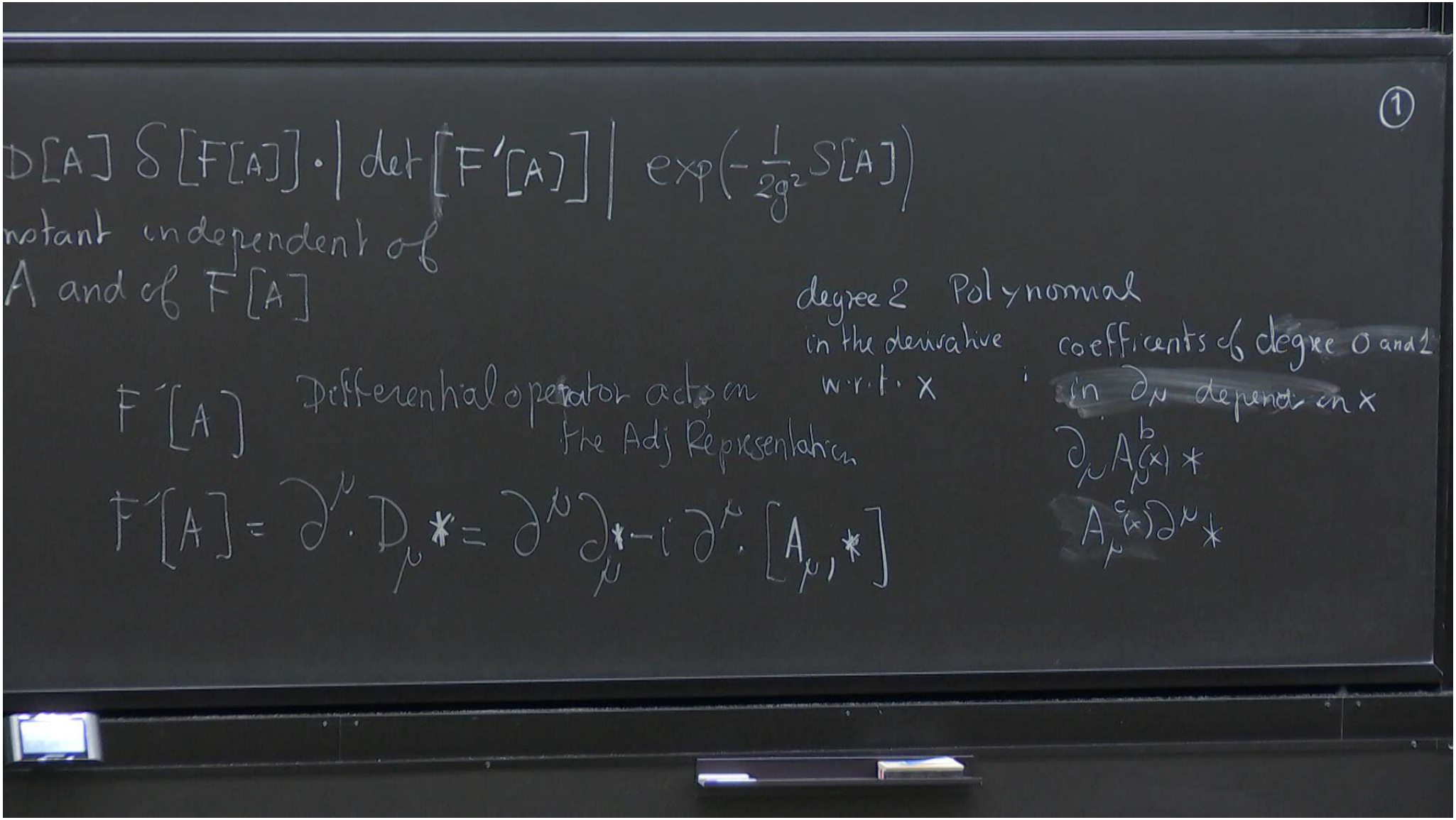
$\frac{1}{2g^2} S[A]\Big) \propto \int \mathcal{D}[A]\, \delta[F[A]] \cdot \left|\det[F'[A]]\right| \exp\left(-\frac{1}{2g^2} S[A]\right)$

$(F_{\mu\nu} F^{\mu\nu})$

constant independent of A and of $F[A]$

tion (calculational)

$\partial^\mu \xi = 0$


①
$D[A]\,\delta[F[A]]\cdot\left|\det[F'[A]]\right|\exp\left(-\frac{1}{2g^2}S[A]\right)$
notant independent of
A and of F[A]
degree 2 Polynomial
in the derivative
w.r.t. x
coefficients of degree 0 and 1
in ∂_ν depend on x
$F'[A]$ Differential operator acts on
the Adj Representation
$\partial_\mu A^b_\mu(x)\,*$
$A^c_\mu(x)\partial_\mu *$
$F'[A] = \partial^\mu \cdot D_\mu * = \partial^\mu \partial_\mu * - i\,\partial^\mu \cdot [A_\mu, *]$

Gauge Theories:

$SU(2)$ $\quad A_\mu = A_\mu^a t_a$

$a = 1, 2, 3$

$$A_\mu \longrightarrow A_\mu + D_\mu \alpha$$

$$F_{\mu\nu} = i\left[D_\mu, D_\nu\right] = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right]$$

$$\int \mathcal{D}[A] \exp\left(-\frac{1}{2g^2} S[A]\right) \alpha \int$$

$$S[A] = \int d^4x \, \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$$

Fix a Gauge Condition (Calculational)

$F[A] = 0 \quad \partial^\mu A_\mu + \xi = 0$

Feynman Gauge

$$\det\left[-\partial^\mu D_\mu\right] = \int \mathcal{D}[\bar{\mathbb{C}}, \mathbb{C}]\ \exp\left(-\bar{\mathbb{C}}\cdot\left(-\partial^\mu D_\mu\right)\cdot\mathbb{C}\right)$$

$$\bar{\mathbb{C}}\cdot\left(-\partial^\mu D_\mu\right)\cdot\mathbb{C} = \int d^4x\ \mathrm{Tr}\ \bar{\mathbb{C}}(x)\cdot\left(-\partial^\mu D_\mu\right)\mathbb{C}(x)$$

acts $\longrightarrow$

Gauge Theories :

$SU(2)$ $\quad A_\mu = A_\mu^a t_a$

$a = 1, 2, 3$

$$A_\mu \longrightarrow A_\mu + D_\mu q$$

$$F_{\mu\nu} = i\left[D_\mu, D_\nu\right] = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right]$$

$$\int \mathcal{D}[A] \exp\left(-\frac{1}{2g^2} S[A]\right) \alpha \int_{\text{con}}$$

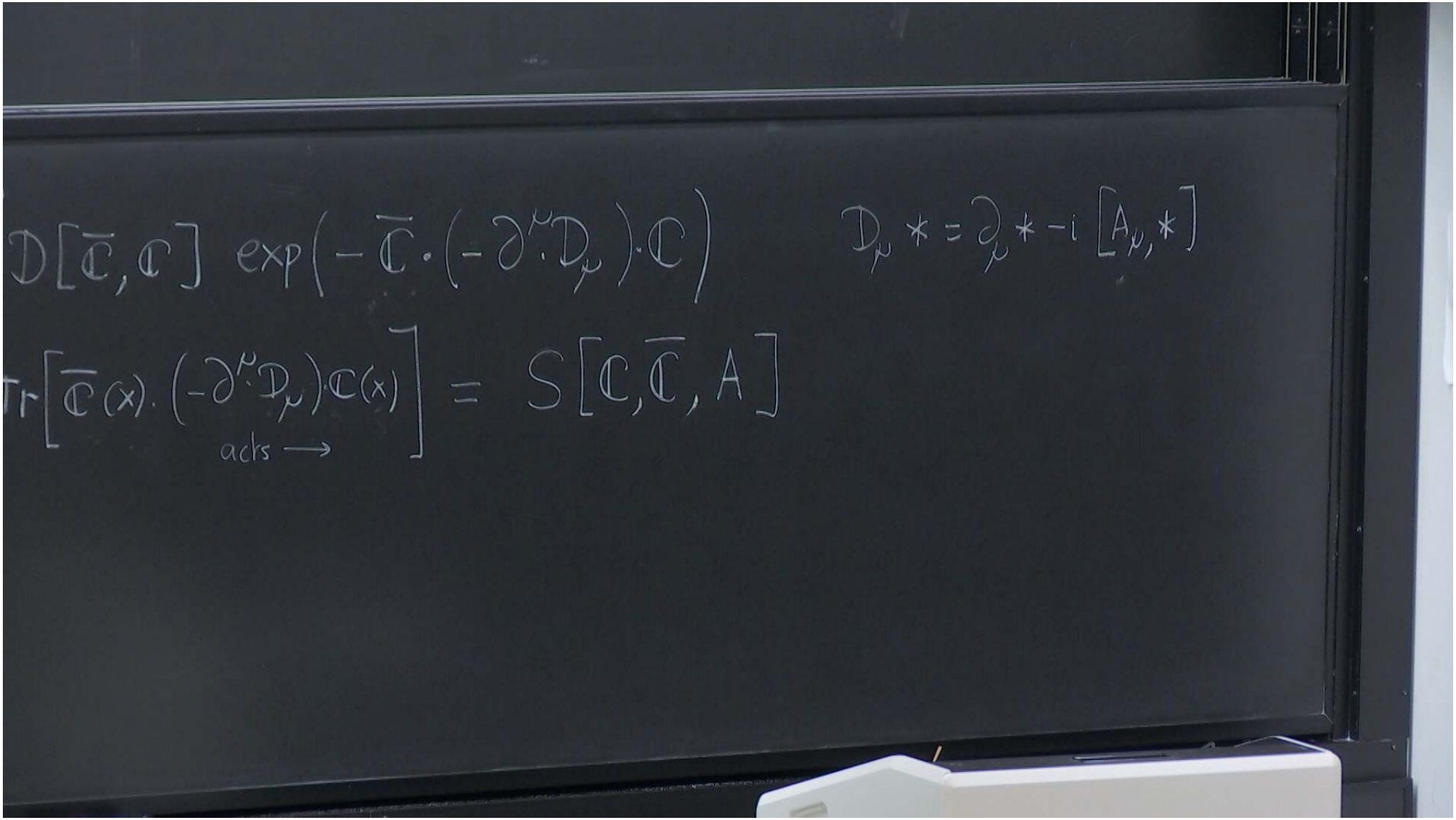
$$S[A] = \int d^4x \, \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$$

Fix a Gauge Condition (Calculational)

$F[A] = 0 \quad \partial^\mu A_\mu + \xi = 0$

Feynman Gauge

$\mathrm{Tr}(t_a t_b) = \frac{1}{2}\delta_{ab}$


$\mathcal{D}[\bar{\mathbb{C}},\mathbb{C}]\ \exp\left(-\bar{\mathbb{C}}\cdot(-\partial^\mu\cdot D_\mu)\cdot\mathbb{C}\right)$
$D_\mu * = \partial_\mu * - i\,[A_\mu, *]$
$\mathrm{Tr}\left[\bar{\mathbb{C}}(x)\cdot(-\partial^\mu\cdot D_\mu)\cdot\mathbb{C}(x)\right] = S[\mathbb{C},\bar{\mathbb{C}},A]$
acts

② Fadeev-Popov Ghosts

1.- $|\det(F'[A])| = \det(F'[A])$

2.- 1 solution to $F[A]=0$

$$\det\left[-\partial^\nu D_\nu\right] = \int \mathcal{D}$$

$$\overline{\mathbb{C}}\cdot(-\partial^\nu D_\nu)\cdot\mathbb{C} = \int d^4x \, \mathrm{Tr}$$

A pair of Grassmann Conjugate Fields

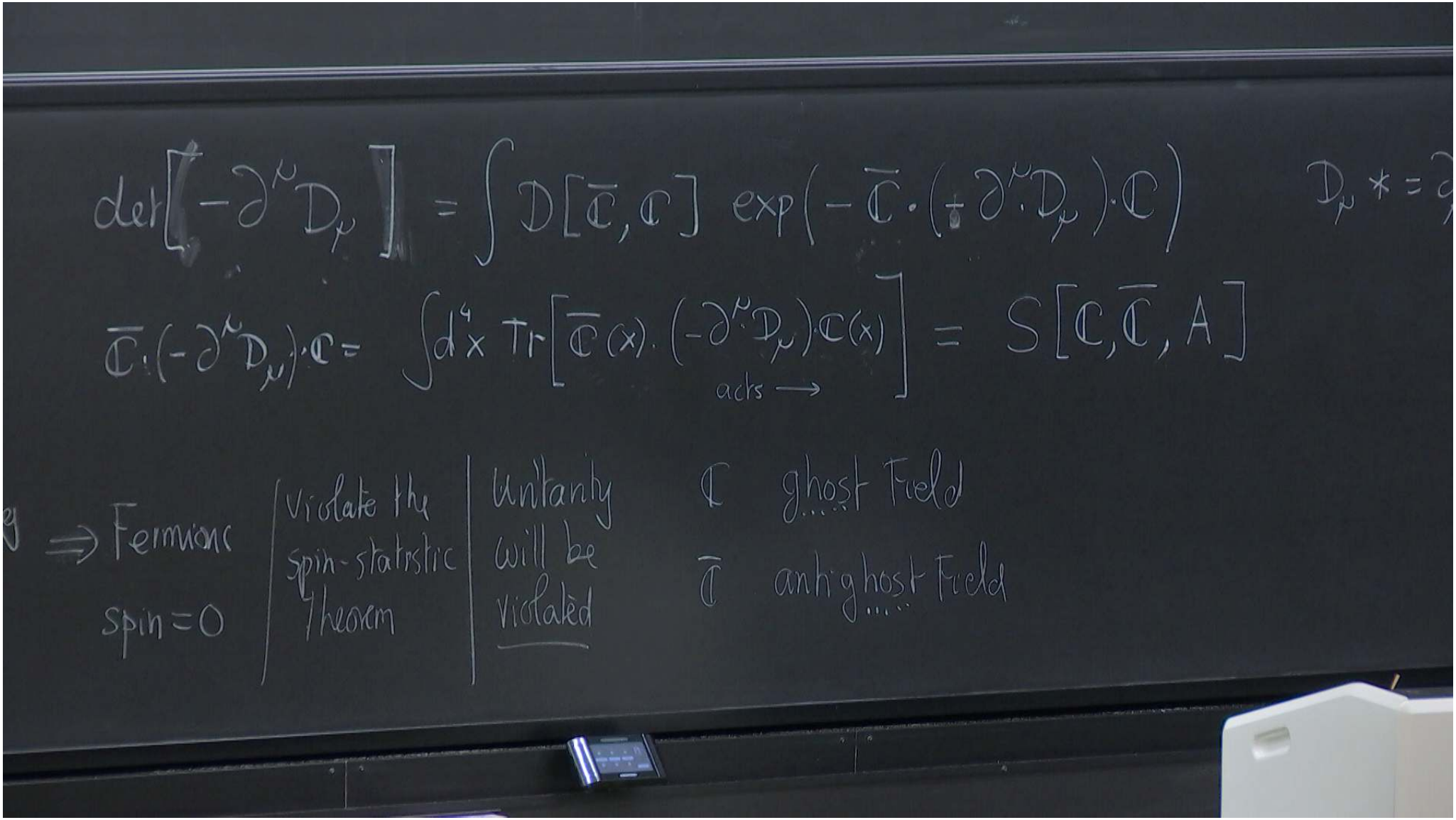
$\mathbb{C}(x)$, $\overline{\mathbb{C}}(x)$ Adj Repr. of $SU(2)$ Anticommuting Fields

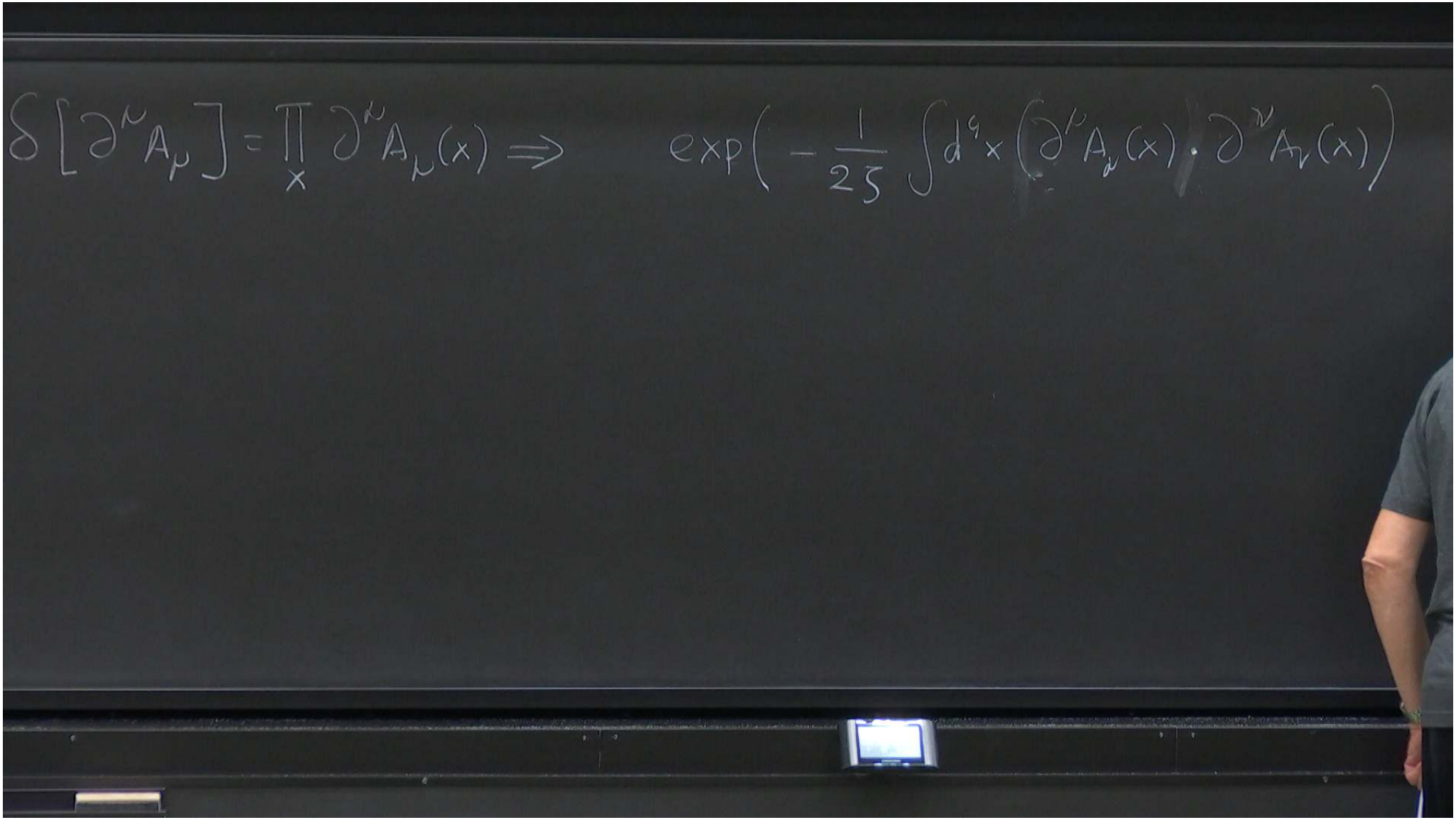
$\mathbb{C}(x) = \mathbb{C}^a(x) t_a$

$\overline{\mathbb{C}}(x) = \overline{\mathbb{C}}^a(x) t_b$

For Lorentz Scalors

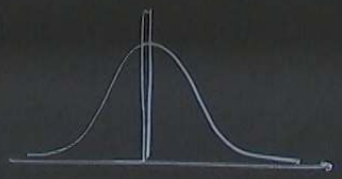
Spin 0

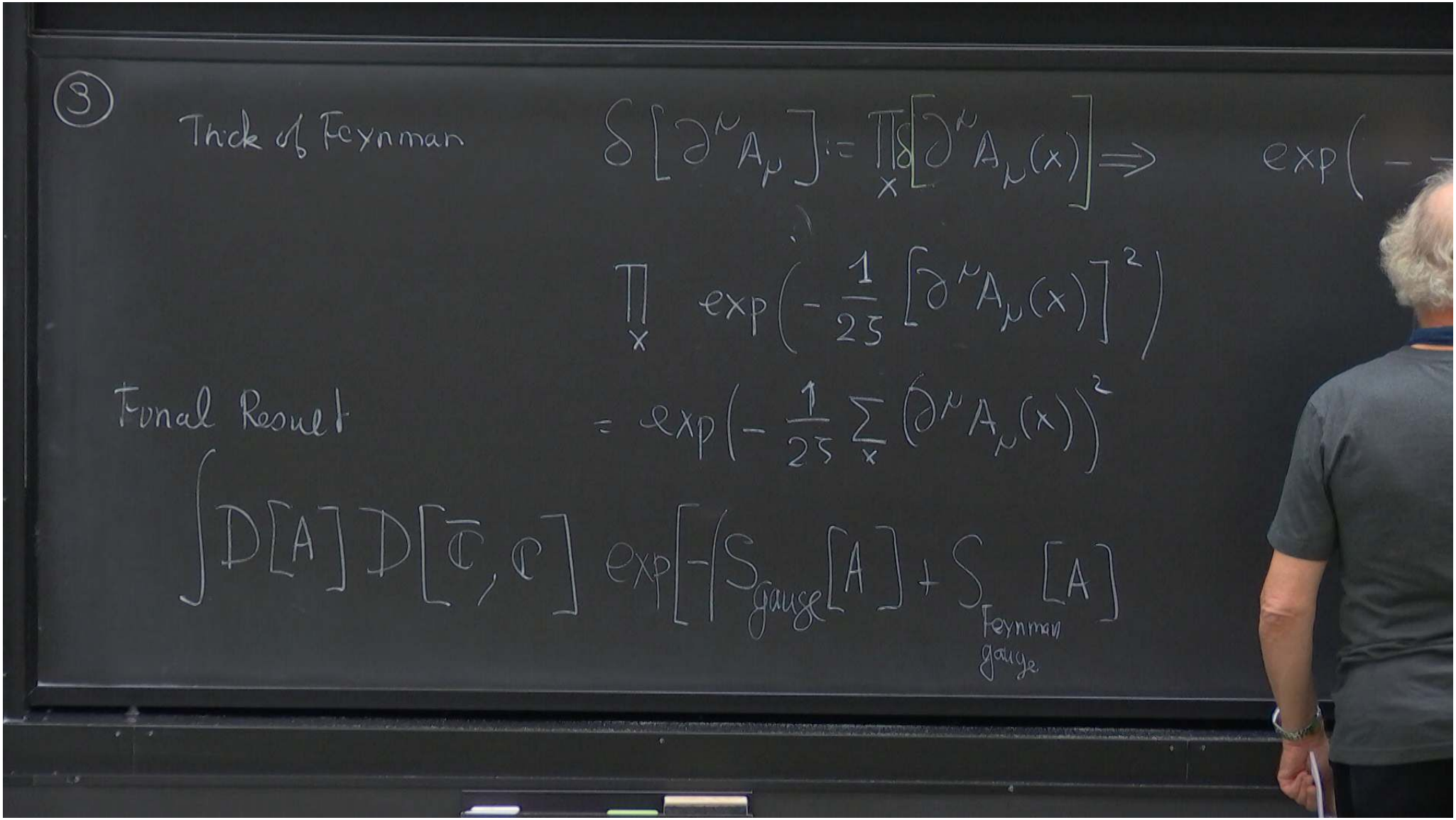

$\det\left[-\partial^\nu D_\nu\right] = \int \mathcal{D}[\bar{\mathbb{C}},\mathbb{C}]\ \exp\left(-\bar{\mathbb{C}}\cdot(-\partial^\nu D_\nu)\cdot\mathbb{C}\right)$
$D_\nu * = \partial$
$\bar{\mathbb{C}}\cdot(-\partial^\nu D_\nu)\cdot\mathbb{C} = \int d^4x\ \mathrm{Tr}\left[\bar{\mathbb{C}}(x)\cdot(-\partial^\nu D_\nu)\,\mathbb{C}(x)\right] = S[\mathbb{C},\bar{\mathbb{C}},A]$
acts
$\Rightarrow$ Fermionic spin = 0
violate the spin-statistic Theorem
Unitarity will be violated
$\mathbb{C}$ ghost Field
$\bar{\mathbb{C}}$ antighost Field


$\delta[\partial^\nu A_\mu] = \prod_x \partial^\nu A_\mu(x) \Rightarrow \quad \exp\left(-\frac{1}{2\xi}\int d^4x \left(\partial^\mu A_\mu(x)\right)\cdot\partial^\nu A_\nu(x)\right)$

Δ Laplace-Beltrami op.

$\frac{1}{25} \int d^4x \left(\partial^\mu A_\mu(x) \cdot \partial^\nu A_\nu(x) \right)$




③
Trick of Feynman
$\delta[\partial^\mu A_\mu] = \prod_x \delta[\partial^\mu A_\mu(x)] \Rightarrow$
$\exp($
$\prod_x \exp\left(-\frac{1}{2\xi}[\partial^\mu A_\mu(x)]^2\right)$
Final Result
$= \exp\left(-\frac{1}{2\xi}\sum_x (\partial^\mu A_\mu(x))^2\right.$
$\int \mathcal{D}[A]\,\mathcal{D}[\bar{c}, c]\,\exp[-(S_{gauge}[A] + S_{Feynman\ gauge}[A]$

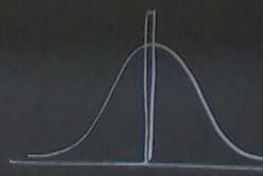
$$\prod_x \delta\left[\partial^\nu A_\nu(x)\right] \Rightarrow \exp\left(-\frac{1}{25}\int d^4x \left(\partial^\nu A_\nu(x)\right)\cdot\partial^\nu A_\nu(x)\right)$$

$$S_{\text{gauge Fixing}}[A]$$

$$\frac{1}{25}\left[\partial^\nu A_\nu(x)\right]^2\Big)$$

$$\sum_x \left(\partial^\nu A_\nu(x)\right)^2$$

$$A] + S_{\substack{\text{Feynman}\\ \text{gauge}}}[A] + S_{\text{ghost}}[\mathbb{C},\bar{\mathbb{C}},A]\Big)$$



$\times \left(\partial^\mu A_\mu(x) \cdot \partial^\nu A_\nu(x)\right)$

fixing $[A]$

$$Z_\xi = \int D[A]\ \delta[\partial^\mu A_\mu - \xi]$$

The final result do not depend on ξ

$$Z_\xi = Z \quad \text{for all } \xi$$

$$Z = \int d\xi\, Z_\xi \exp\left(-\frac{\xi^2}{2\zeta}\right)$$

$$\int d\xi\, \delta(\partial^\mu A_\mu - \xi) \exp\left(-\frac{\xi^2}{2\zeta}\right) = \exp\left(-\frac{(\partial^\mu A_\mu)^2}{\zeta}\right)$$

Start From Action: $S_{gauge} + S_{gaugeFixing} + S_{ghosts}$

Feynman diagramatics

$$\frac{1}{g^2}\left[\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 + \right.$$

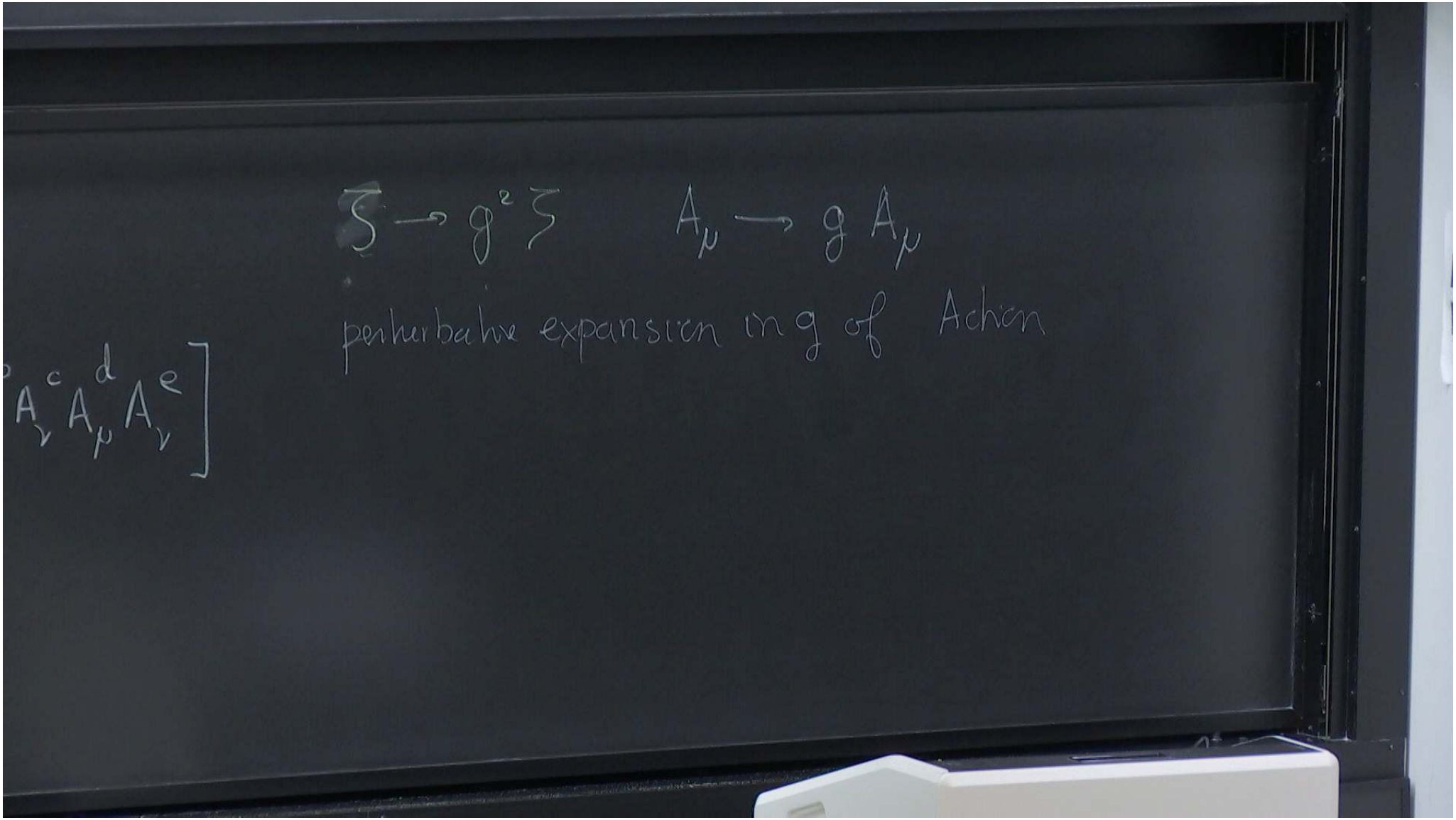
Start From Action: $S_{gauge} + S_{gaugeFixing} + S_{ghosts}$

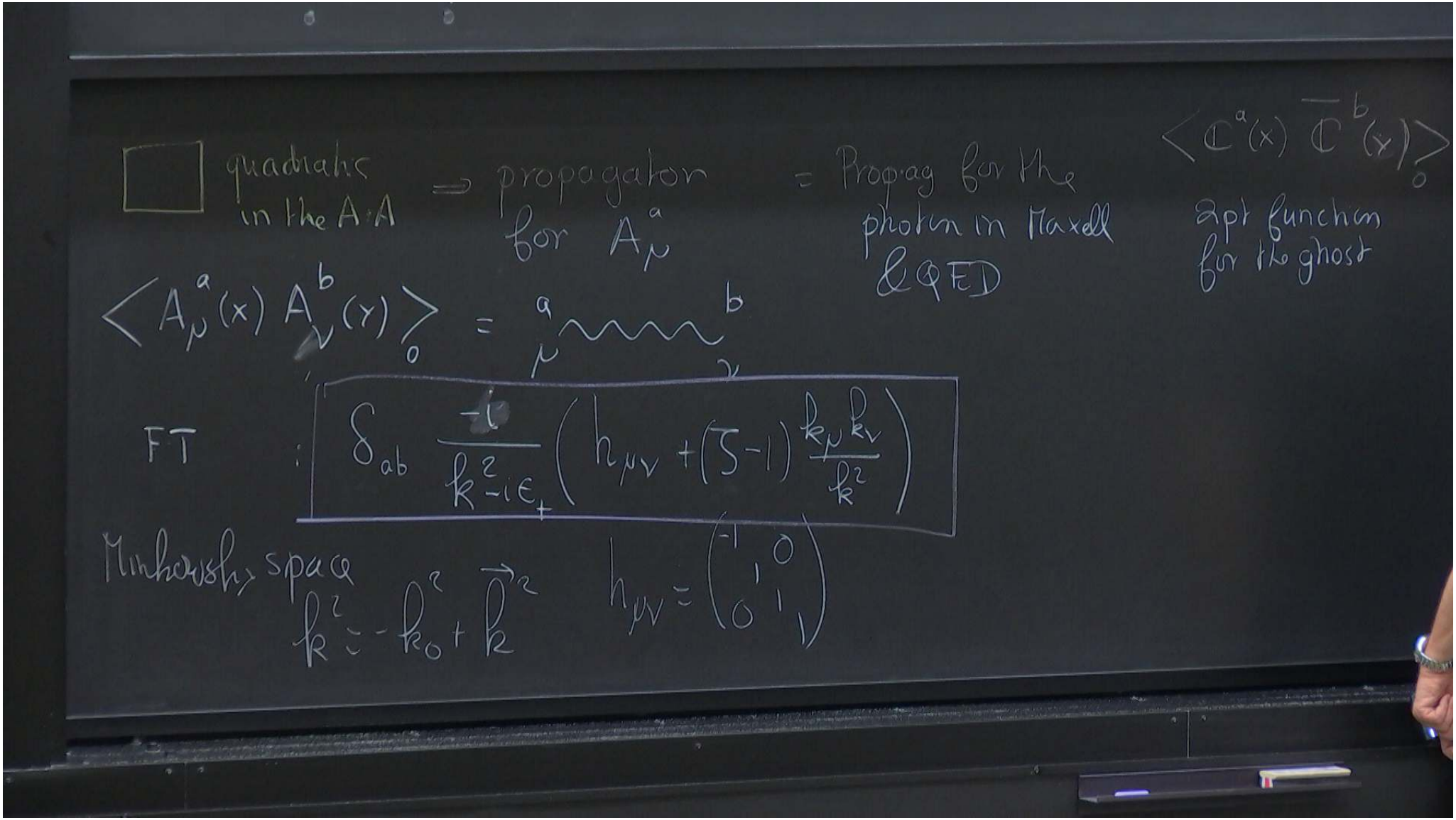
Feynman diagramatics

Gauge $\frac{1}{g^2}\left[\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 + 2\,\partial_\mu A^a_\nu\, f_{abc} A^b_\mu A^c_\nu + \frac{1}{2} f_{abc} f_{ade} A^b_\mu A^c_\nu A^{\;}\right.$

GF $+ \frac{1}{2\xi}\,\partial_\mu A^a_\mu\;\partial_\nu A^a_\nu$

Ghosts $+\;\partial_\mu \bar{C}^a \cdot \partial_\mu C^a + f_{abc}\cdot \partial_\mu \bar{C}^a \cdot A^b_\mu \cdot C^c$


$\xi \to g^2 \xi \qquad A_\mu \to g A_\mu$
perturbative expansion in g of Action
$A_\nu^c A_\mu^d A_\nu^e]$


quadratic in the A·A
⇒ propagator for A^a_μ
= Propag for the photon in Maxell & QED
$\langle \mathbb{C}^a(x)\, \bar{\mathbb{C}}^b(y) \rangle_0$?
2pt function for the ghost
$\langle A^a_\mu(x)\, A^b_\nu(y) \rangle_0 = \mu\, a \sim\sim b\, \nu$
FT : $\delta_{ab} \frac{-i}{k^2 - i\epsilon} \left(h_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2} \right)$
Minkowski space
$k^2 = -k_0^2 + \vec{k}^2$
$h_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

$f_{abc} \cdot \partial_\mu \bar{C}^a \cdot A_\mu^b \cdot C^c$

= Propag for the photon in Maxell & QED

b

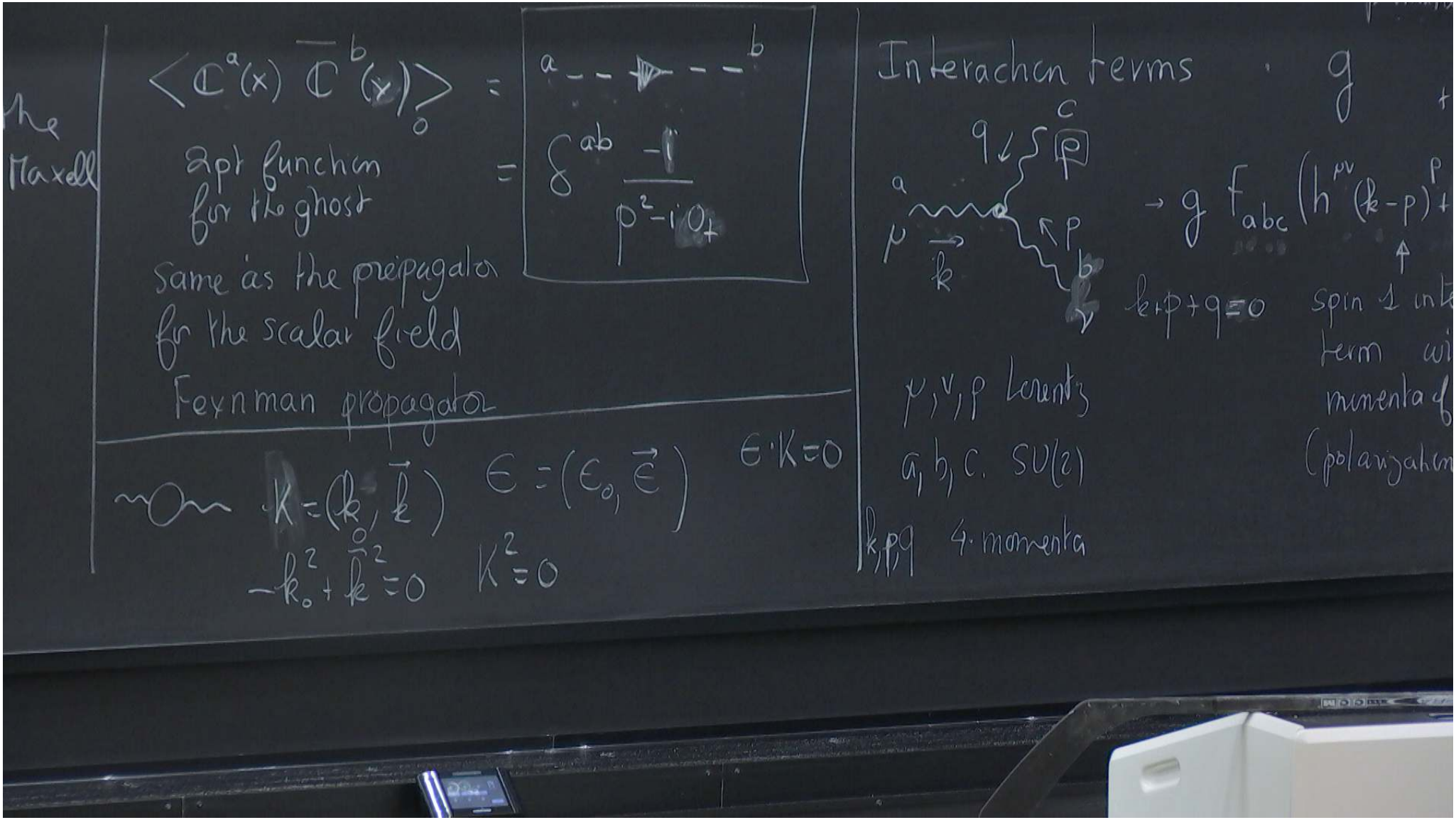
$+\left(5-1\right)\frac{k_\mu k_\nu}{k^2}\Big)$

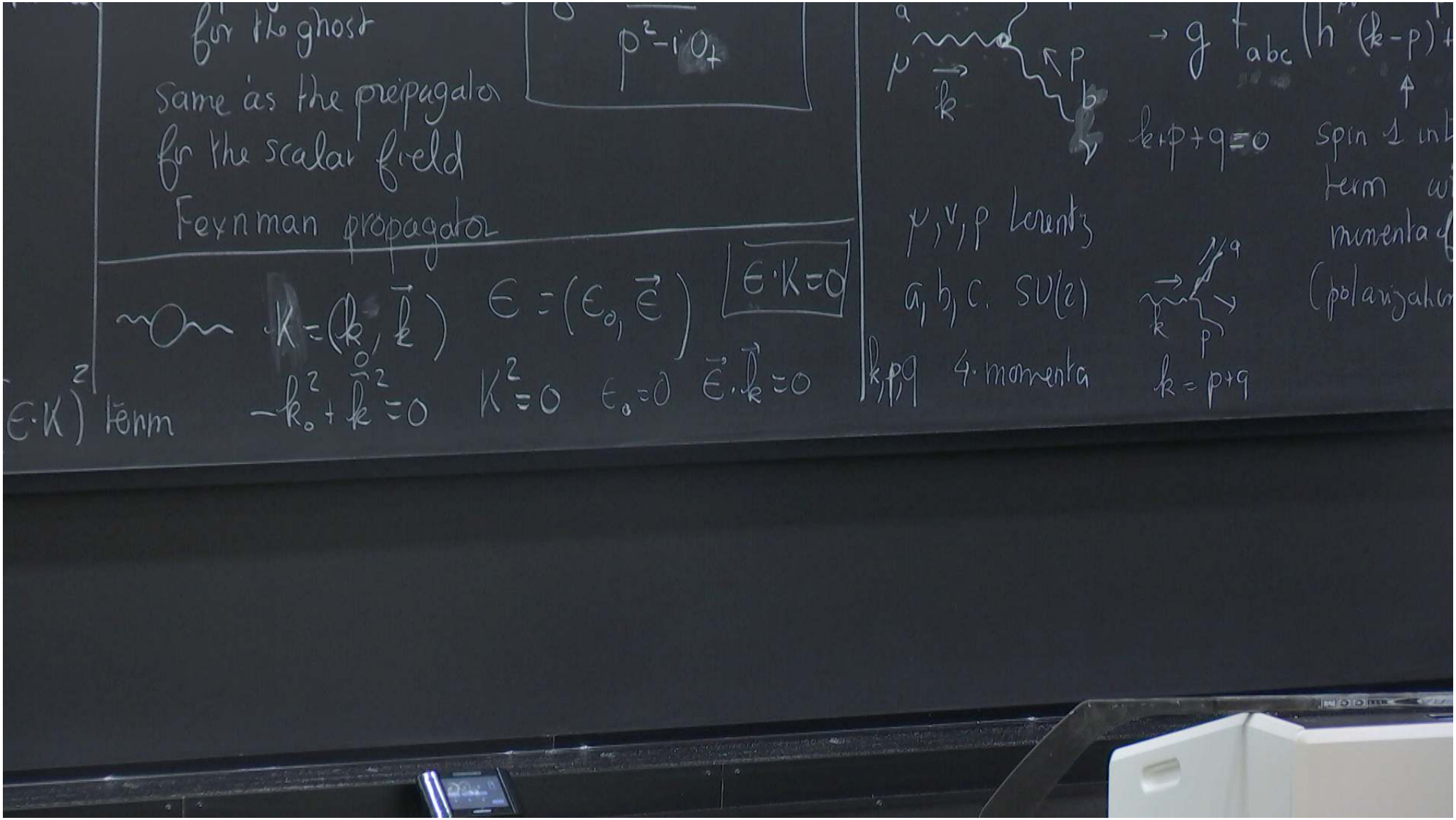
$\begin{pmatrix} -1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}$

$\langle C^a(x)\, \bar{C}^b(x) \rangle_0$ = 2pt function for the ghost

Same as the propagator for the scalar field

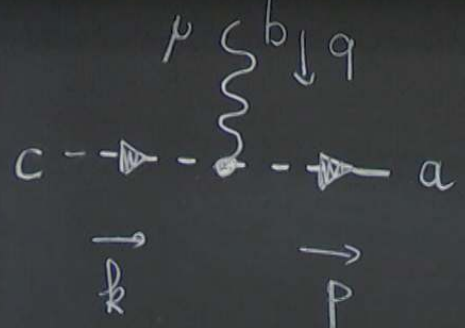
$= \delta^{ab} \frac{-i}{p^2 - i\epsilon_+}$


the Maxell
$\langle \mathbb{C}^a(x)\, \bar{\mathbb{C}}^b(y) \rangle_0 =$
a --- b
$\delta^{ab} \frac{-i}{p^2 - i0_+}$
2pt function for the ghost
Same as the propagator for the scalar field
Feynman propagator
$K = (k_0, \vec{k})$ $\epsilon = (\epsilon_0, \vec{\epsilon})$ $\epsilon \cdot K = 0$
$-k_0^2 + \vec{k}^2 = 0$ $K^2 = 0$
Interaction terms
g
c
q
p
a
μ
k
b
$\to g f_{abc} (h^{\mu\nu}(k-p)^{\rho}$
$k+p+q=0$
Spin 1 in
term wi
momenta
(polariza
μ, ν, ρ Lorentz
a, b, c. SU(2)
k, p, q 4-momenta


for the ghost
Same as the propagator
for the scalar field
Feynman propagator
$\frac{i}{p^2 - i0_+}$
$K = (k_0, \vec{k})$
$\epsilon = (\epsilon_0, \vec{\epsilon})$
$\epsilon \cdot K = 0$
$-k_0^2 + \vec{k}^2 = 0$
$K^2 = 0$
$\epsilon_0 = 0$
$\vec{\epsilon} \cdot \vec{k} = 0$
$(\epsilon \cdot K)^2$ term
μ
k
p
q
$\rightarrow g f_{abc}$
$k + p + q = 0$
μ, ν, ρ Lorentz
a, b, c. $SU(2)$
k, p, q 4-momenta
$k = p + q$
spin 1
term
momenta
(polarizat

Feynman gauge

ghost



SU(2) group

$-g f_{abc} p_\mu$

Similar Fermion and Photon in QED

For Scalar electrodynamics

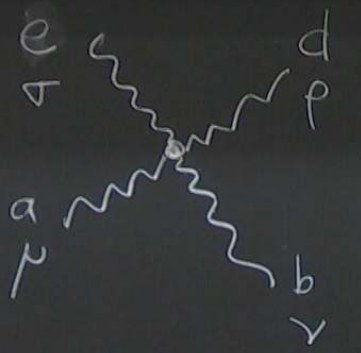
$\vec{k} + \vec{q} = \vec{p}$

4 momenta

Feynman gauge

ghost

$\int \mathcal{D}[A] \int \mathcal{D}[\xi] = \int \mathcal{D}[$



$g^2 \; h_{\mu\rho} \; h_{\nu\sigma} \; f_{abc} \; f_{dec}$

$\delta_{ad}\delta_{bc} - \delta_{ae}\delta_{bd}$

exchange of q. numbers

μ, ν, ρ, σ Lorentz

a, b, c, d SU(2)

$\partial^\nu D_\nu q) = 0$

$$\int d\xi\, \delta(\partial^\nu A_\nu - \xi) \exp\left(-\frac{\xi^2}{\zeta}\right) = \exp\left(-\frac{(\partial^\nu A_\nu)^2}{\zeta}\right)$$

$$\int D[A] \int D[\xi] = \int D[\xi] \int D[A]$$

$h_{\nu\sigma}$ f_{abc} f_{dec}

$\delta_{ad}\delta_{bc} - \delta_{ae}\delta_{bd}$

exchange of q. numbers

$g \approx$ unit of charge $\leftrightarrow$ e

same for gauge field, and the ghosts

↳ SU(2) symm.

$\frac{g}{4!}\phi^4 \rightarrow \times g$

U(1)
no need for ghosts

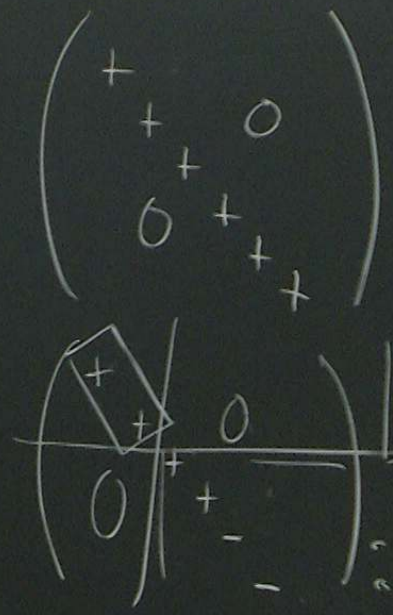
creation annihil

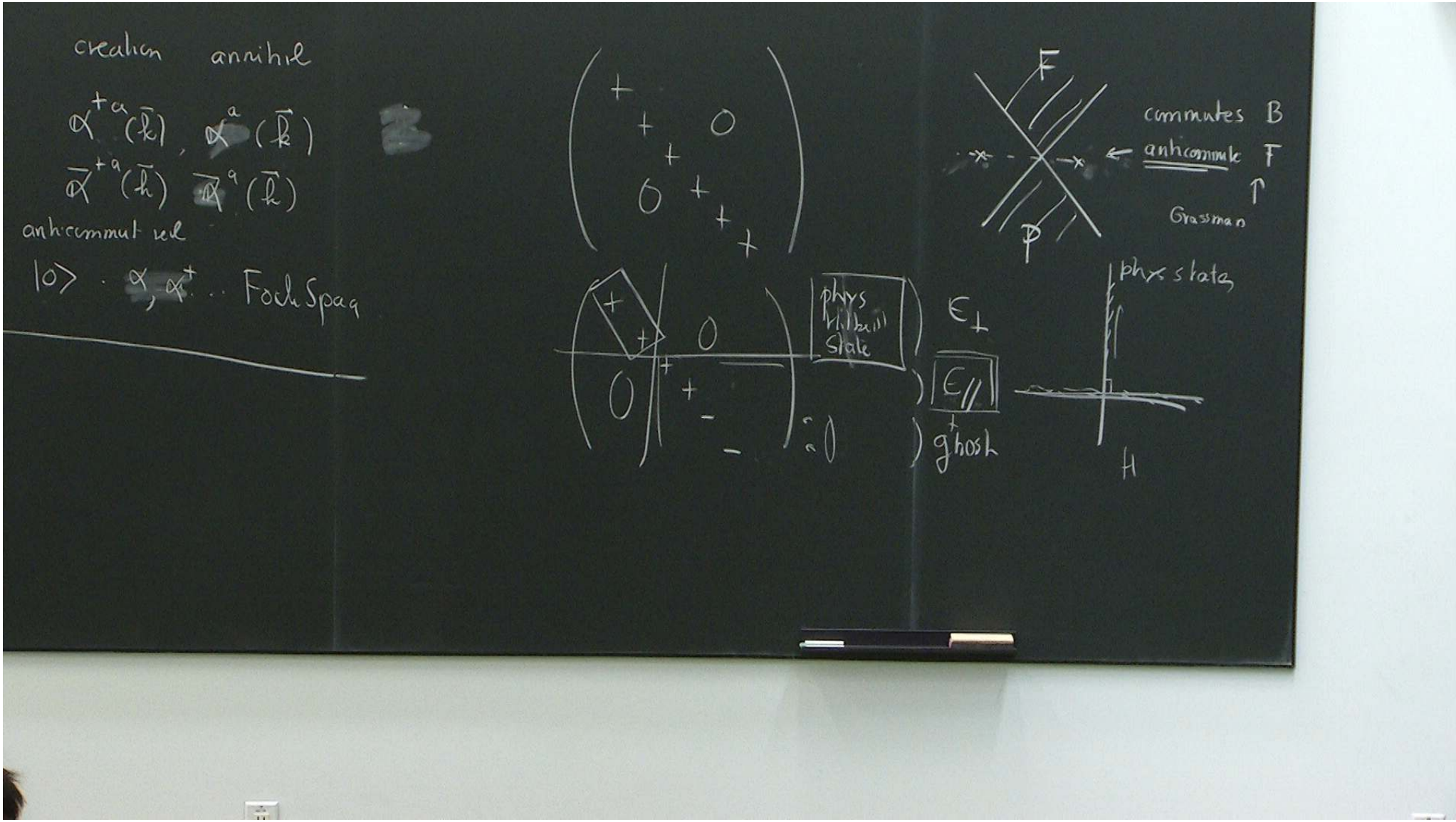
c $\alpha^{+a}(\vec{k})$, $\alpha^{a}(\vec{k})$

$\bar{c}$ $\bar{\alpha}^{+a}(\vec{k})$ $\bar{\alpha}^{a}(\vec{k})$

anticommutation

$|0\rangle$. α, α^{+} .. Fock Space




creation annihil
$\alpha^{+a}(\vec{k})$, $\alpha^{a}(\vec{k})$
$\bar{\alpha}^{+a}(\vec{k})$ $\bar{\alpha}^{a}(\vec{k})$
anticommut rel
$|0\rangle$. α, α^{+} .. Fock Space
F
commutes B
anticommute F
Grassman
P
phys
phys state
$\epsilon_\perp$
$\epsilon_{//}$
ghost
H