Title: Quantum-enhanced telescopy

Speakers: Yunkai Wang

Series: Perimeter Institute Quantum Discussions

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Abstract: Optical astronomical imaging looks for better imaging quality in extreme cases of weak and subdiffraction limits. I focus on the quantum enhancement of astronomical interferometric imaging, including its fundamental limit and practical issues. For the fundamental aspects, I ignore any resource limit and noise and consider the ideal imaging problems. I show that the resolution limit can be enhanced with more carefully chosen measurement strategies and the general imaging quality can be enhanced by postprocessing the stellar photons with a quantum computer. For the practical aspects, I try to overcome the transmission loss suffered by interferometric imaging using quantum network, consider the possibility to implement a local scheme with better performance, and discuss the feasibility of decomposing thermal states into temporally localized pulses.

Outline

Quantum Algorithm

Advisor:
Shengjun Wu

- Quantum search on hierarchical database [PRR (2019), PRA (2020)]
- Dimension reduction of quantum walk [SPIN (2021)]

Nonlinear Optics

Advisor:
Kejie Fang

- Few-photon transportation via a multimode nonlinear cavity [PRA (2022)]
- Continuous-variable graph states for quantum metrology [PRA (2020)]

Quantum Imaging

Advisor:
Virginia Lorenz
Eric Chitambar

Fundamental limit

- Resolution limit of interferometric imaging [PRA (2021)]
- Improve bandwidth extrapolation method
- Avoid requirement of prior information for superresolution
- Enhance imaging by entangled measurements

Practical issues

- A generalized intensity interferometer
- A quantum-network-based interferometry
- Decomposition of thermal states into localized pulses

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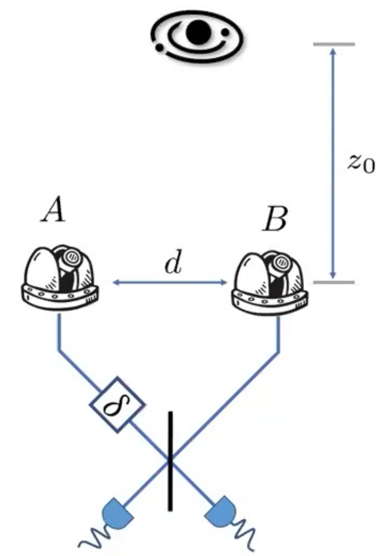


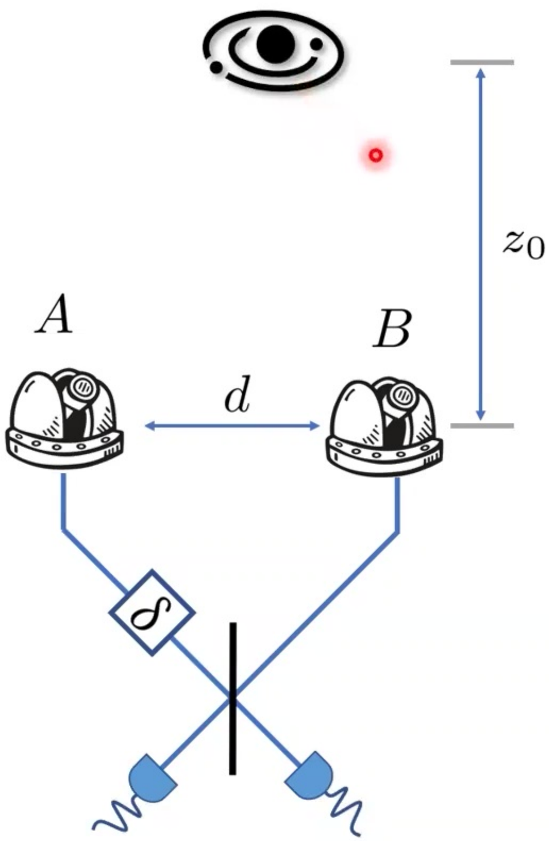
Quantum-enhanced telescopy

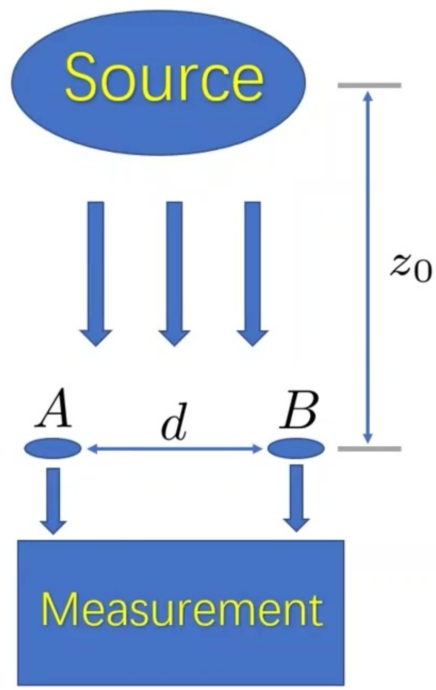
Yunkai Wang

Department of Physics, UIUC

November 30, 2022

General set up



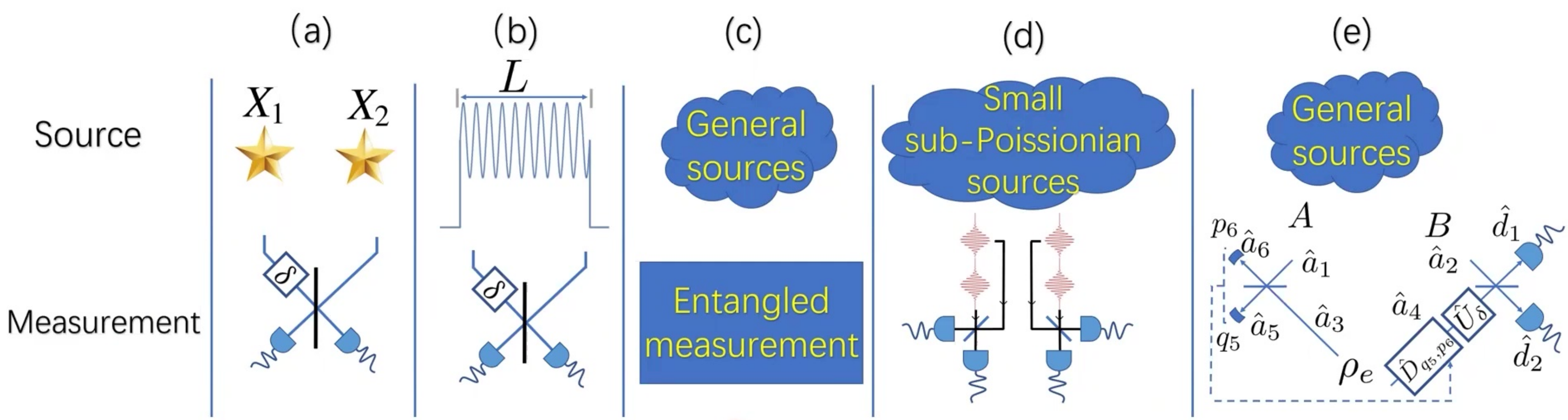


Van Cittert-Zernike theorem: Coherence function g is a Fourier component of the intensity distribution of the sources.

Received state for an astronomical interferometer

$$\rho = \frac{1}{2}\begin{bmatrix} 1 & g \\ g^* & 1 \end{bmatrix}$$

Different scenarios considered in our work



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Quantum Estimation Theory

Lower bounds of estimating unknown parameters for given probes and encoding process can be calculated.

Classical Cramer-Rao bound — Quantum Cramer-Rao bound

$$\delta\phi \geq 1/\sqrt{I(\phi|\mathcal{P},\hat{\rho})} \geq 1/\sqrt{F(\phi|\hat{\rho})}$$

Fisher information (FI) — Quantum Fisher information (QFI)

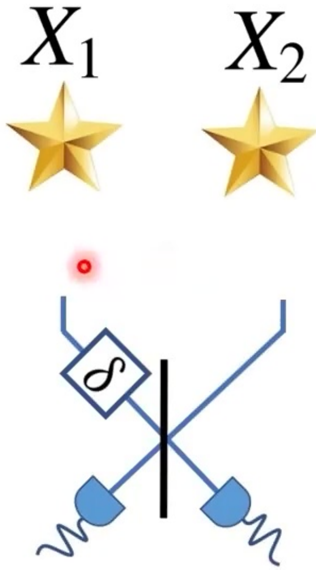
ϕ Unknown parameter

$\delta\phi$ Variance of the estimation

$\mathcal{P}$ POVM

$\hat{\rho}$ Probe state

Quantum Imaging



- Fundamental aspects:
  - Resolution limit of interferometric imaging
  - Improve bandwidth extrapolation method
  - Avoid requirement of prior information for superresolution
  - Enhance imaging by entangled measurements

- Imaging under practical constraints and assumptions
  - A generalized intensity interferometer
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Resolution limit of interferometric imaging

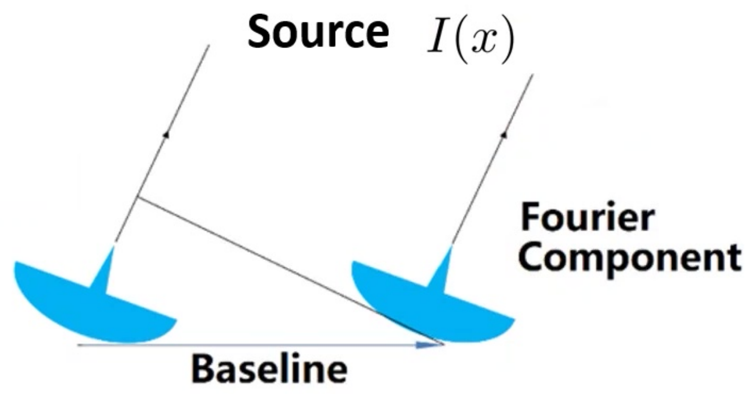
Reconstructed image is the convolution between $I(x)$ and an effective PSF

$$I'(x) = (I * PSF_{\text{eff}})(x)$$

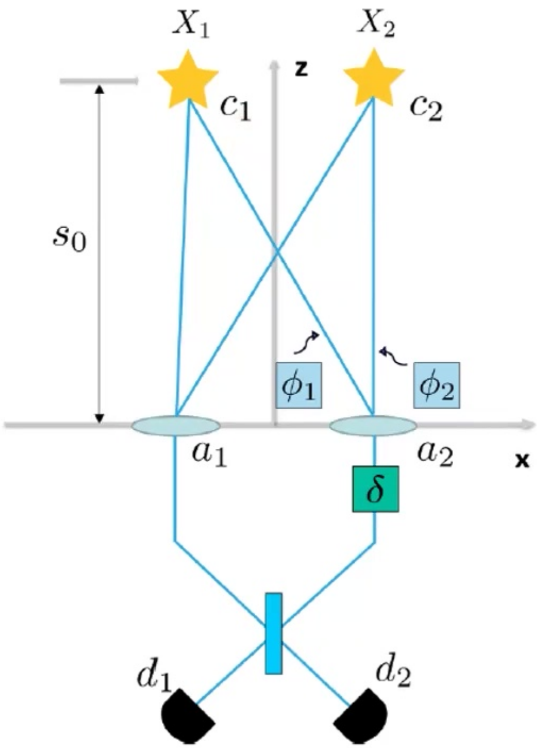
Resolution is roughly determined by the longest baseline.

Can we avoid the resolution limit?

Yes! We just need to carefully design the measurement.



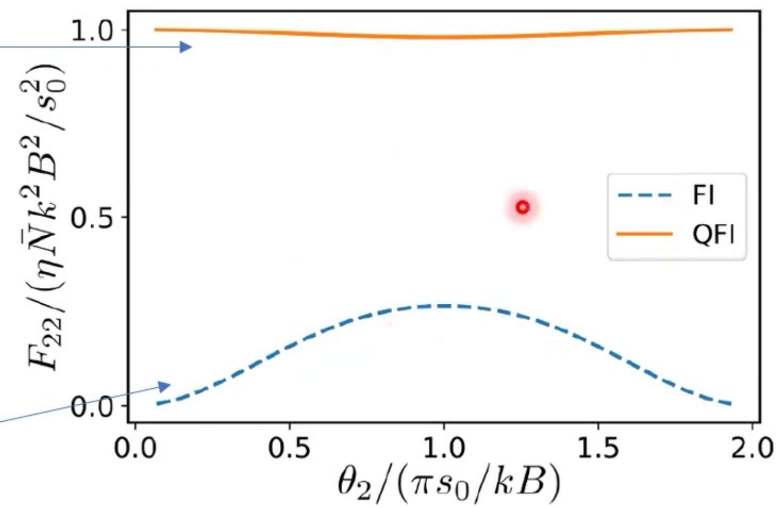
Superresolution can be achieved in interferometric imaging



Consider the resolution limit of imaging two strong thermal sources of equal strength with an interferometer.

QFI for estimating the separation remains a constant.

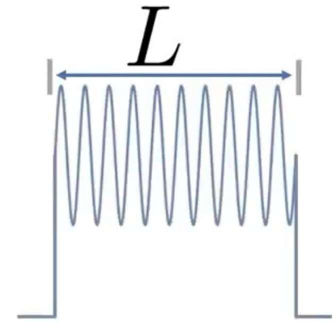
Conventionally, the resolution limit is related to baseline between two spatial modes.



Separation between two point sources

[1] Wang, Y., Zhang, Y., & Lorenz, V. O. Physical Review A, 104(2), 022613. (2021).

Quantum Imaging



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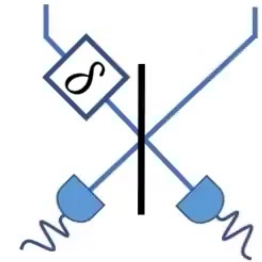

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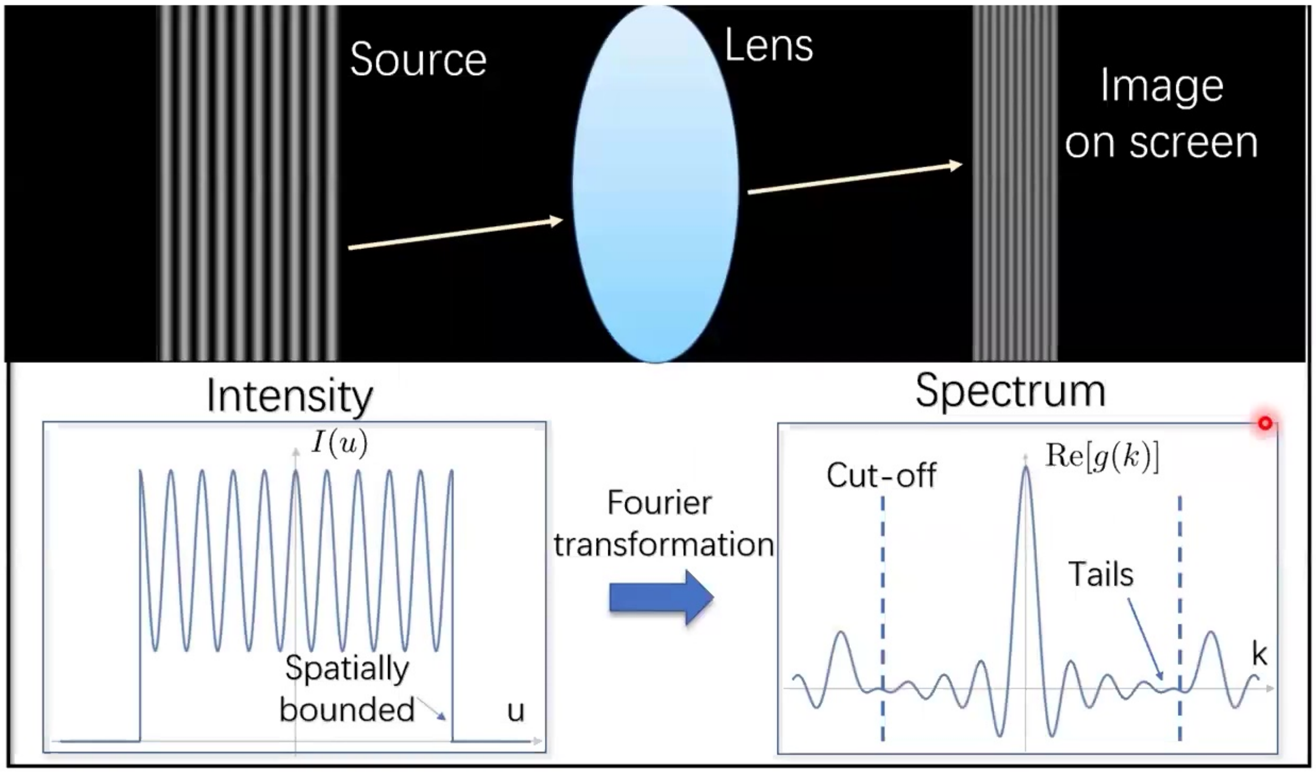
Basic ideas of bandwidth-extrapolation-based method



The tails on the low-frequency side are related to high-frequency side.

Problem:
The extrapolation is vulnerable to the errors of estimating lower frequency information.

Set up for the simplest case of bandwidth-extrapolation-based superresolution

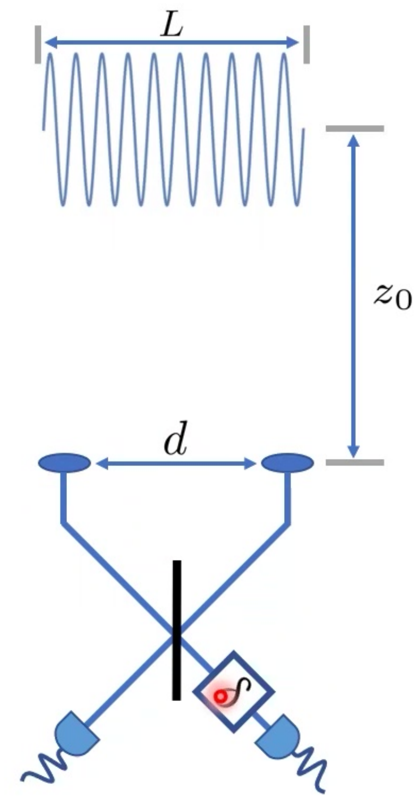
Consider the simplest case:

An interferometer array with two detector and a source whose intensity distribution

$$I(u) = \frac{1}{L} + \frac{a}{L}\cos k_1(u-u_0) + \frac{b}{L}\sin k_2(u-u_0)$$

only for $u_0 - \frac{L}{2} \leq u \leq u_0 + \frac{L}{2}$

The unknown parameters we want to measure are a, b



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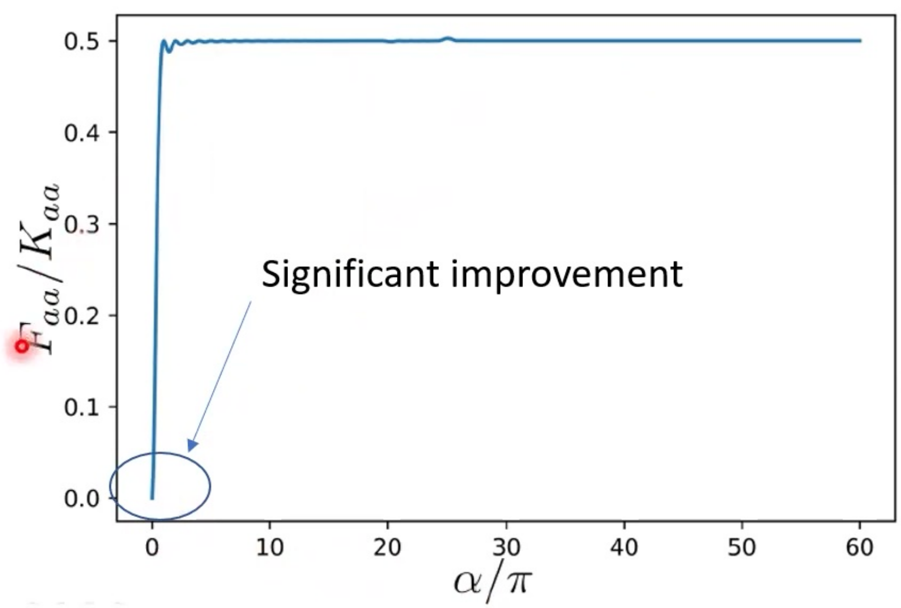
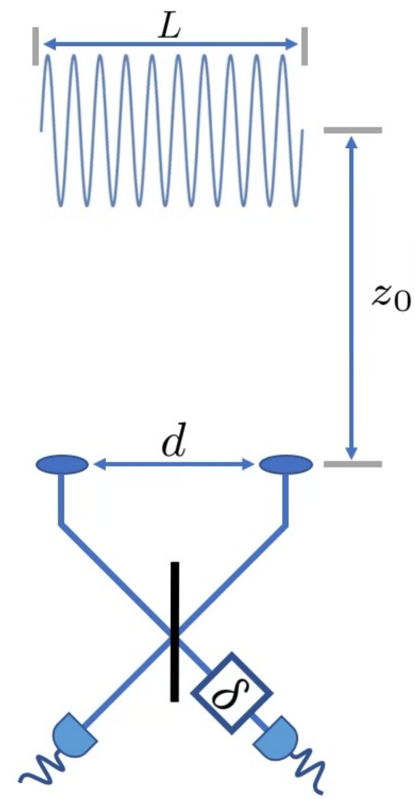
Improvement compared to a fixed phase measurement

Carefully designed measurement can have significant improvement

Ratio between QFI and FI





Quantum Imaging

General sources

- Fundamental aspects:
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Entangled measurement

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Superresolution of imaging extended sources in the small source limit

We model the source in terms of its moments, the received state is

$$\rho = \rho^{(0)} + \sum_{n=1}^{\infty} \frac{i^n}{n!} x_n \rho^{(n)} \qquad \rho^{(n)} \sim O(\alpha^n)$$

α = (size of source) / (resolution limit)

Moment $$x_\mu = \left(\frac{2}{L}\right)^\mu \int du I(u)(u-u_0)^\mu$$

If we choose the measurement naively, probability of getting the outcome is

$$P \sim \underline{k_0 + k_1 x_1 \alpha} + k_2 x_2 \alpha^2 + \cdots$$

lower order term introduces strong noise

We can construct the measurement such that $k_{n\leq m} = 0$ [2]

e.g. $P \sim \cancel{k_0 + k_1 x_1 \alpha} + k_2 x_2 \alpha^2 + \cdots$ Estimation of x_2 is improved.

[2] Zhou, S., & Jiang, L. Physical Review A, 99(1), 013808. (2019).

Avoid requirement of prior information in a simple case

Inspired by Ref. [3], we can avoid this requirement by entangled measurement.

When we have two detectors, two photons $\rho^{\otimes 2}$ and want to measure x_1

$$|b_1\rangle = (|0_{1A}1_{1B}\rangle\,|1_{2A}0_{2B}\rangle - |1_{1A}0_{1B}\rangle\,|0_{2A}1_{2B}\rangle)/\sqrt{2}$$ Do not need prior information.

$$\langle b_1|\,\rho^{\otimes 2}\,|b_1\rangle \propto x_1\alpha + O(\alpha^2)$$ No zeroth order term $O(1)$

This construction must have two photon go to different telescope and hence waste half of the photons.

Waste lots of photons when we have more telescopes.

[3] Parniak, M., Borówka, S., Boroszko, K., Wasilewski, W., Banaszek, K., & Demkowicz-Dobrzański, R. Physical review letters, 121(25), 250503. (2018).

Construction of measurement without prior information for the general cases

Solution is to use more photons and measure them together.

Intuition for typical set:
If we flip a coin for 10000 times, we will very likely get similar number of heads and tails.

For the case when we have $N \to \infty$ photons, we will very likely get equal number of photons at each telescope.

Part 2: Quantum Imaging

General sources

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Entangled measurement

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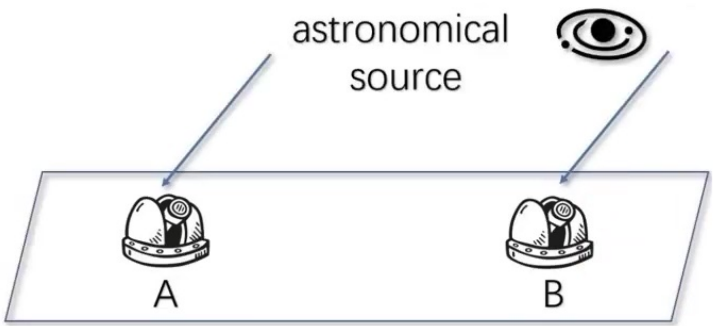
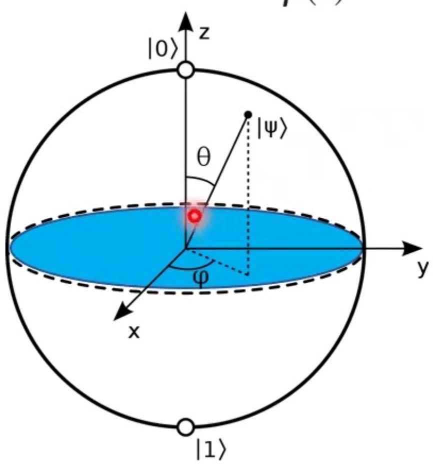
Reduce interferometric imaging to qubit state estimation



Received state for an astronomical interferometer

$$\rho = \frac{1}{2}\begin{bmatrix} 1 & g \\ g^* & 1 \end{bmatrix}$$

$$\rho(\vec{r}) = \frac{\mathbb{1} + \vec{r}\cdot\vec{\sigma}}{2}$$



If we are given (1) N copies of the identically prepared qubit states
(2) Some prior knowledge about the distribution of $\vec{r}$.

This is a qubit state estimation problem aiming at maximizing the average fidelity

Entangled measurement can outperform separable measurement

For any prior, optimal **separable adaptive** measurement can have fidelity [4]

$$F = 1 - \frac{1}{N} + o(1/N)$$

For any prior, optimal **collective** measurement can have fidelity [5]

$$F = 1 - \frac{1}{2N} + o(1/N)$$

These results immediately imply entangled measurement can outperform any adaptive separable measurement for imaging.

[4] Bagan, E., Ballester, M. A., Gill, R. D., Muñoz-Tapia, R., & Romero-Isart, O. Physical review letters, 97(13), 130501. (2006).
[5] Bagan, E., Ballester, M. A., Gill, R. D., Monras, A., & Munoz-Tapia, R. Physical Review A, 73(3), 032301. (2006).

Significant improvement in the subdiffraction limit

Consider the imaging in the subdiffraction limit, $|g| \to 1$

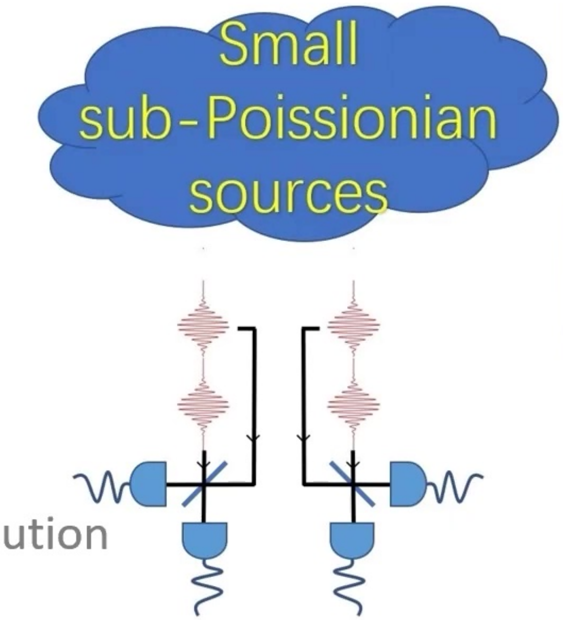
We can assume the prior distribution $d\rho = A(1-r)^{\alpha} d\theta dr \qquad \alpha < 0$

For a fixed separable measurement

$$F = 1 - \frac{C}{N^{1+\alpha/2}} + o(1/N)$$

This shows in the subdiffraction imaging, adaptive measurement and entangled measurement can significantly improve the imaging quality.

Quantum Imaging



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Local schemes performs worse than nonlocal schemes for imaging weak thermal sources

Fisher information for estimating the coherence function g [6]

Nonlocal measurement $F \sim O(\epsilon)$

Local measurement on a single temporal mode $F \sim O(\epsilon^2)$

Mean photon number $\epsilon \ll 1$

Is there any way to get around this result and have a local scheme with good performance?
If we change the statistic of the source, we can have a good local scheme.

[6] Tsang, M. Physical review letters, 107(27), 270402. (2011)

Superresolution may help local scheme in the subdiffraction limit

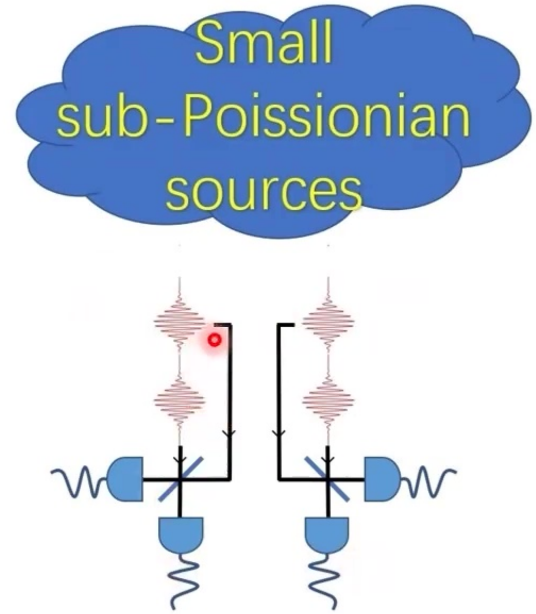
For a weak thermal source

$$F_{|g|} \sim O(\epsilon^2)$$

For a sub-Poissonian source where there is no two photon terms

$$F_{|g|} \sim O(\epsilon^2/(1-|g|))$$

Significant improvement for local scheme in the subdiffraction limit



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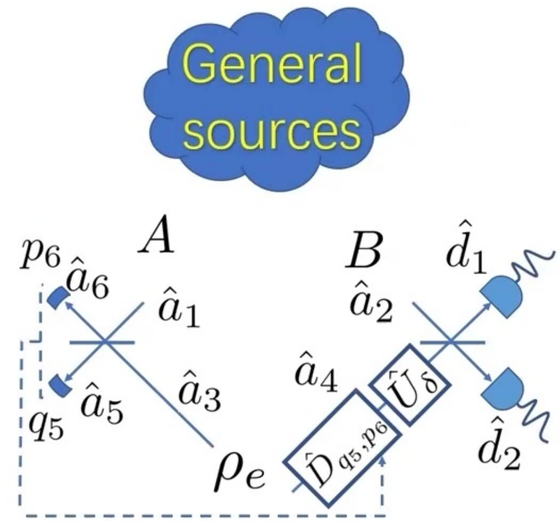

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Interferometric imaging based on continuous-variable quantum network

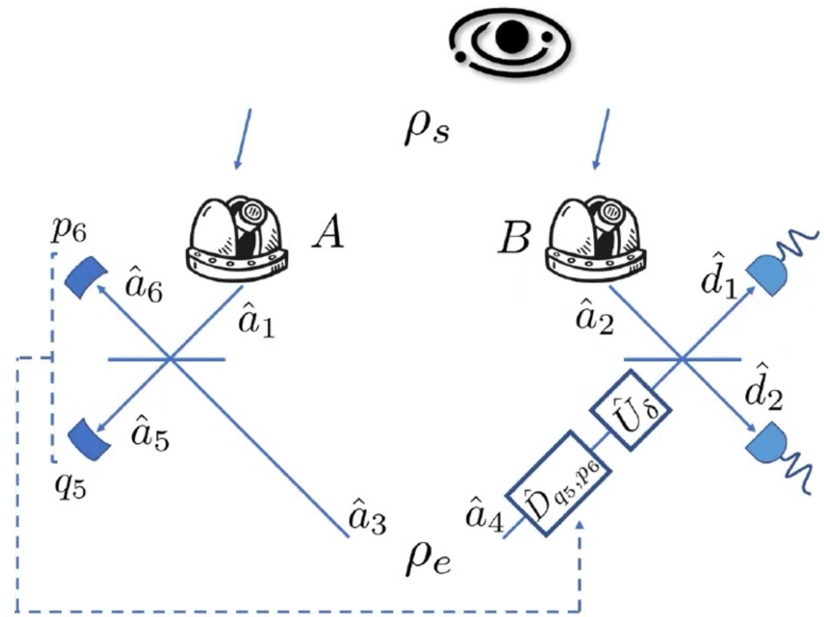
An alternative scheme to Ref. [7]

We distribute two mode squeezed state as the entanglement resources.

FI of estimating the coherence function

$$F_{\theta\theta} = \frac{2\epsilon^2|g|^2}{2y + \epsilon(2 + \epsilon - \epsilon|g|^2 + 2y)}$$

$$y = 2e^{-2r'}$$



Squeezing level should be high enough such that $y \sim O(\epsilon)$

[7] Gottesman, D., Jennewein, T., & Croke, S. Physical review letters, 109(7), 070503. (2012)

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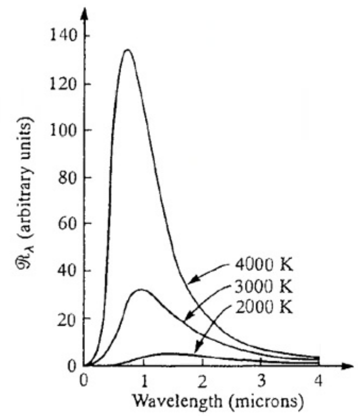
Are thermal states a set of independent localized pulses?

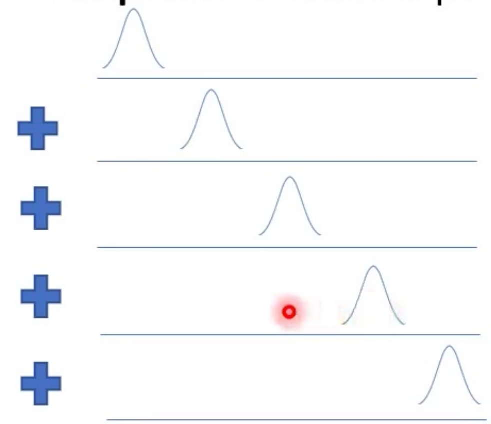
The decomposition of thermal light into a mixture of independent localized pulses are not in general allowed [8].

We here find for thermal state with flat spectrum, this decomposition is allowed.

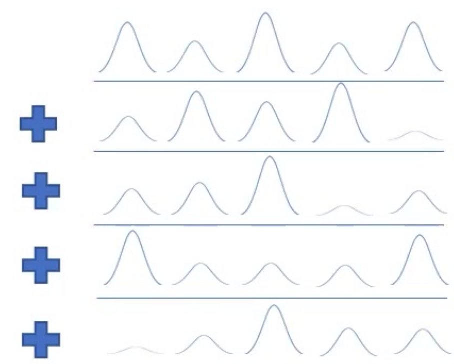


[8] Chenu, A., Brańczyk, A. M., Scholes, G. D., & Sipe, J. E. Physical review letters, 114(21), 213601. (2015)

Decomposition as a set of **independent** localized pulses



Decomposition as a set of **correlated** localized pulses

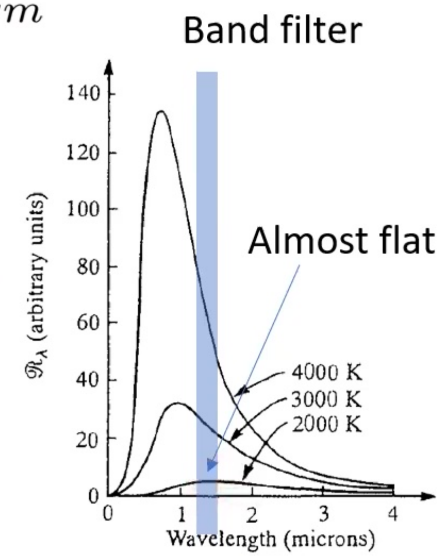


Rewrite thermal states in a set of spatially localized modes

Consider direct product of thermal states in a set of frequency modes a_m

$$\rho = \bigotimes_m \rho_m$$

ρ_m is a thermal state in mth mode with mean photon number n_m



Wavefunction of frequency modes a_m

$$\phi_m(z) = e^{i\tilde{k}z}\chi_m(z), \quad \chi_m(z) = e^{i\kappa_m z}/\sqrt{L}$$

Spatial modes c_s is the Fourier transformation of frequency modes a_m

$$c_s = \sum_{m=-n}^{n} C_{sm} a_m \ , \quad C_{sm} = \exp(2\pi i s m/N)/\sqrt{N}$$

Wave function $\omega_s(z) = \omega(z - sl) \qquad \omega(z) \sim \mathrm{sinc}\,\frac{\pi z}{l}$ ← c_s are localized spatial modes

Thermal states with flat spectrum can be decomposed as localized pulses

If we consider the **flat spectrum** $n_m = n$, all the localized modes c_s are independent.

$$\rho = \otimes_s \rho_s \qquad \rho_s = \int \frac{d^2\gamma_s}{\pi n} \exp\left(-|\gamma_s|^2/n\right) |\gamma_s\rangle \langle\gamma_s|$$

Each mode c_s is in a thermal state with mean photon number n

In the weak limit, we expand the above decomposition for a flat spectrum

$$\rho = \left(\frac{1}{1+n}\right)^N \left[\otimes_s \rho_s^{(0)} + \epsilon \boxed{\sum_s \rho_1^{(0)} \otimes \rho_2^{(0)} \otimes \cdots \otimes \rho_s^{(1)} \otimes \cdots \otimes \rho_N^{(0)}} + o(\epsilon^2) \right]$$

$$\epsilon = n/(1+n) \to 0 \qquad \rho_s^{(0)} = |0\rangle\langle 0| \qquad \rho_s^{(1)} = c_s^\dagger |0\rangle\langle 0| c_s$$

We can use the picture that stellar light comes as localized temporal pulses, which may contain vacuum or single photon states.

Research Interests

Quantum Computing

Quantum algorithms: phase estimation, Schur sampling, quantum walk
Near-term quantum computing
Quantum error correction

Quantum Sensing

Imaging improvement based on quantum state estimation
Quantum synthetic aperture LIDAR
Revisit Fourier optics

Summary

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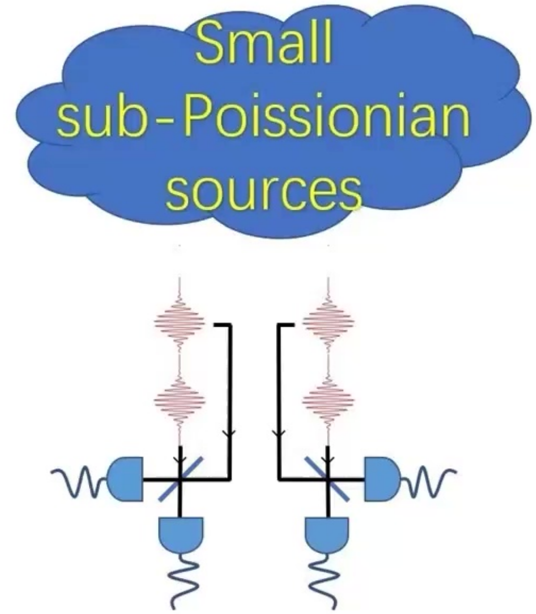
Superresolution may help local scheme in the subdiffraction limit

For a weak thermal source

$$F_{|g|} \sim O(\epsilon^2)$$

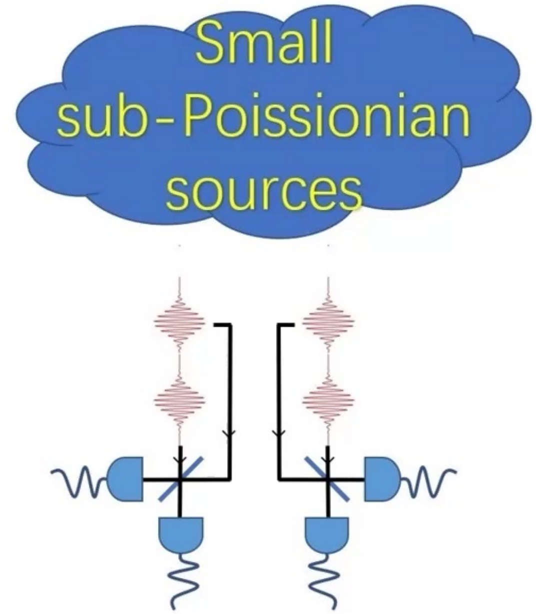
For a sub-Poissonian source where there is no two photon terms

$$F_{|g|} \sim O(\epsilon^2/(1-|g|))$$



Significant improvement for local scheme in the subdiffraction limit

Superresolution may help local scheme in the subdiffraction limit



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$P = 1 - |g|$

$$F_{|g|} \sim O(\epsilon^2/(1-|g|))$$

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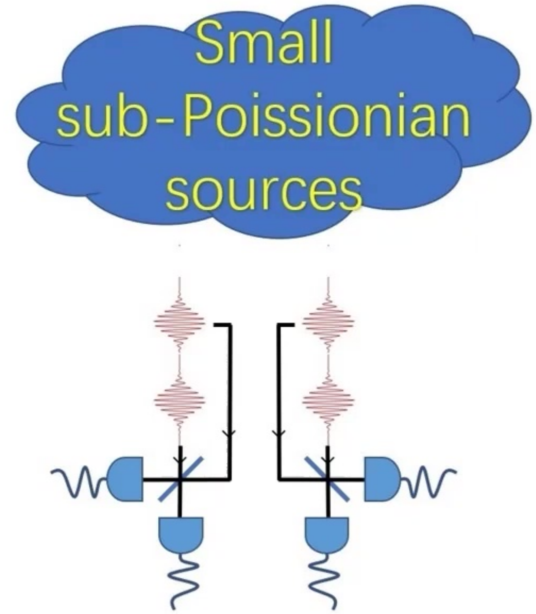
$P = 1-|g|\cos\theta$

$g \rightarrow 0$

$g \rightarrow 1$



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