Title: From KMOC to WQFT in Yang-Mills and gravity

Speakers: Leonardo de la Cruz

Series: Particle Physics

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Abstract: Recently, powerful quantum field theory techniques, originally developed to calculate observables in colliders, have been applied to describe classical observables relevant to gravitational wave physics. This has motivated a proliferation of approaches to extract classical information from quantum scattering amplitudes. Since the double copy suggests that the basis of the dynamics of general relativity is Yang-Mills theory, in this talk I will first discuss scattering in Yang-Mills theory as a toy model to study the connection between the framework by Kosower-Maybee-O'Connell (KMOC), the language of effective field theory (EFT) and the eikonal phase. After a brief review of the KMOC formalism to compute classical observables from scattering amplitudes, I will consider the dynamics of colour-charged particle scattering and explain how to compute the change of colour, and the radiation of colour, during a classical collision. Finally, moving on to gravity, I will discuss the deflection of light by a massive spinless/spinning object using the novel worldline quantum field theory (WQFT) formalism for classical scattering.

Zoom link: https://pitp.zoom.us/j/98649931693?pwd=Z2s1MlZvSmFVNEFqdjk2dlZNRm9PQT09

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From KMOC to WQFT in Yang-Mills and gravity

[arxiv:2009.03842 with Maybee-O'Connell-Ross] [arXiv:2108.02178 with Luna-Scheopner] [arXiv:2112.05013 with Bastianelli-Comberiati]

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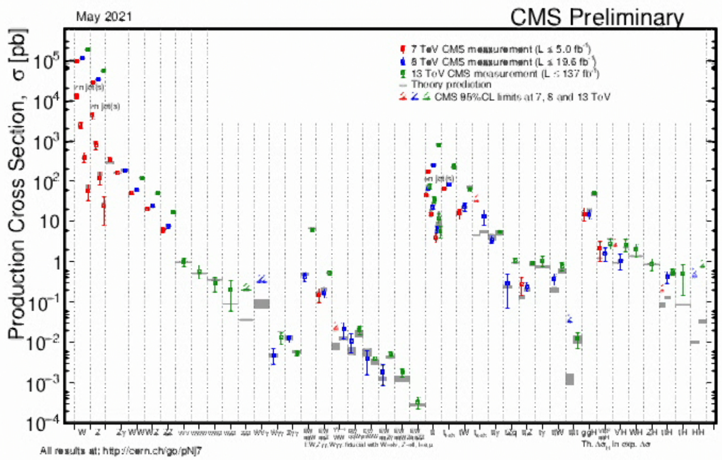
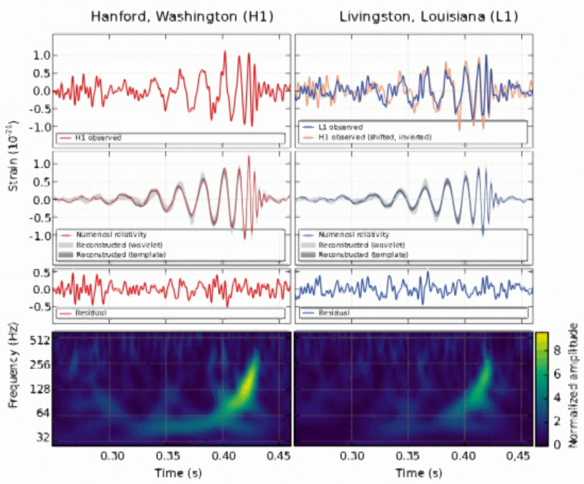
Seminar at Perimeter Institute

Scattering amplitudes and observables





Cross sections

- Modern approach: Unitarity, spinor-helicity, color-kinematics duality, double copy etc.
- Feynman integrals through differential equations

Waveforms

- Two-body classical bounded problem
- Solutions of Einstein equations
- Numerical GR, EFT approach, amplitudes, etc

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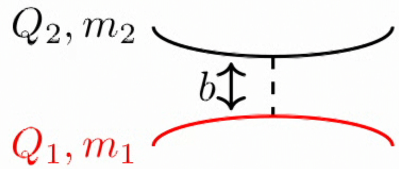
Invitation: Scattering in electrodynamics



- Assuming power series expansion for trajectories ($\alpha = 1, 2$)

$$x^\mu_\alpha(\tau_\alpha) = b^\mu_\alpha + u_\alpha \tau_\alpha + \Delta^{(1)} x^\mu_\alpha(\tau_\alpha) + \Delta^{(2)} x^\mu_\alpha(\tau_\alpha) + \cdots$$

$$v^\mu_\alpha(\tau_\alpha) = u_\alpha + \Delta^{(1)} v^\mu_\alpha(\tau_\alpha) + \Delta^{(2)} v^\mu_\alpha(\tau_\alpha) + \cdots$$

$\Delta^{(i)} x^\mu_\alpha$ is $\mathcal{O}(g^{2i})$

- We are interested in solving Maxwell equations and Lorentz force

$$\partial_\mu F^{\mu\nu} = J^\nu(x) = e \sum_i Q_\alpha \int \mathrm{d}\tau_\alpha \; \delta^4(x - x(\tau_\alpha)) v^\nu_\alpha(\tau_\alpha)$$

$$\frac{\mathrm{d}p^\mu_\alpha}{\mathrm{d}\tau_\alpha} = e Q_\alpha F^{\mu\nu}(x_\alpha(\tau_\alpha)) v_{\alpha\nu}(\tau_\alpha)$$

- The total change in momentum of the particle during the collision

$$\Delta p^\mu_1 \equiv m_1 \int_{-\infty}^{\infty} \mathrm{d}\tau_1 \frac{\mathrm{d}v^\mu_1}{\mathrm{d}\tau_1}$$

Classical Impulse

Invitation: Classical scattering in QED

- The amplitudes approach Kosower-Maybee-O'Connell (KMOC), 2019

$$\langle \Delta p_1^\mu \rangle = \langle \psi | S^\dagger \mathbb{P}_1^\mu S | \psi \rangle - \langle \psi | \mathbb{P}_1^\mu | \psi \rangle$$

- Restore $\hbar$: Units with $\hbar \neq 1$ but $c = 1$
- Dimensionless couplings

$$\frac{e}{\sqrt{\hbar}}, \qquad \frac{\kappa}{\sqrt{\hbar}}, \qquad g\sqrt{\hbar}$$

Colour

- Distinction between momentum and wavenumber

$$k \equiv \hbar \bar{k}$$

E.g., Massless particles

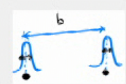
- Quantum states $|\psi\rangle$ with sharply peaked quantum wavepackets: sharply-defined position and momenta

Point particle

$$\phi(p) \sim m^{-1} \exp[-p \cdot u/(\hbar \ell_{\text{Compton}} \ell_{\text{spread}}^2)] \to \langle p^\mu \rangle = m u^\mu, \langle p^2 \rangle = m^2$$

- Scales of the problem satisfy the inequalities

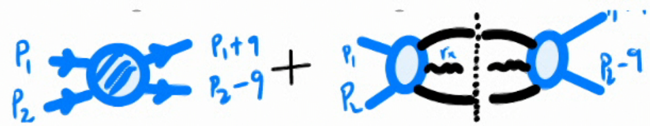
$$\ell_{\text{Compton}} \ll \ell_{\text{spread}} \ll \sqrt{-b^2}$$



Invitation: Classical scattering from QED

- $S = 1 + \mathrm{i}T$ and unitarity

$$\langle \Delta p_1^\mu \rangle = \langle \psi | \mathrm{i}[\mathbb{P}_1^\mu, T] | \psi \rangle + \langle \psi | T^\dagger [\mathbb{P}_1^\mu, T] | \psi \rangle \sim$$

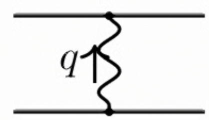


- At leading order ($\hbar$ in coupling already taken into account)

$$\langle \Delta p_1^\mu \rangle = \mathrm{i} \left\langle\!\!\left\langle \hbar^2 \int \hat{\mathrm{d}}^4 \bar{q} \hat{\delta}(2\bar{q} \cdot p_1) \hat{\delta}(2\bar{q} \cdot p_2) e^{-\mathrm{i}b\cdot\bar{q}} \bar{q}^\mu \bar{\mathcal{A}}^{(0)}(p_1, p_2 \to p_1 + \hbar\bar{q}, p_2 - \hbar\bar{q}) \right\rangle\!\!\right\rangle$$

- Restore $\hbar$ in amplitude and Laurent expansion

$$\mathrm{i}\bar{\mathcal{A}}^{(0)}(p_1, p_2 \to p_1 + \hbar\bar{q}, p_2 - \hbar\bar{q}) \sim$$



$$\sim \mathrm{i}e^2 Q_1 Q_2 \frac{4p_1 \cdot p_2 + \hbar^2 \bar{q}^2}{\hbar^2 \bar{q}^2}$$

- Quantity inside double braket must be independent of $\hbar$
- In the classical limit then $p_i = m_i u_i$ ($\alpha = e^2/4\pi, \sigma = u_1 \cdot u_2$)

$$\Delta p_1^\mu = \mathrm{i}e^2 Q_1 Q_2 \int \hat{\mathrm{d}}^4 \bar{q}\ \hat{\delta}(\bar{q} \cdot u_1) \hat{\delta}(\bar{q} \cdot u_2) e^{-\mathrm{i}b\cdot\bar{q}} \bar{q}^\mu \frac{u_1 \cdot u_2}{\bar{q}^2} \to -2\alpha\, Q_1 Q_2 \frac{\sigma}{\sqrt{\sigma^2 - 1}} \frac{b^\mu}{b^2}$$

KMOC formalism

- Classical Yang-Mills observables from amplitudes

$$\langle \Delta O \rangle = \langle S^\dagger \mathbb{O} S \rangle - \langle \mathbb{O} \rangle$$

- This talk: $\mathbb{C}_1^a$, $\mathbb{P}_1^\mu$, $\mathbb{F}^a$
- Conventions

$$S = \int \mathrm{d}^4 x \left(\sum_{\alpha=1}^{2} \left[(D_\mu \varphi_\alpha^\dagger) D^\mu \varphi_\alpha - \frac{m_\alpha^2}{\hbar^2} \varphi_\alpha^\dagger \varphi_\alpha \right] - \frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu} \right),$$

$$D_\mu = \partial_\mu + igA_\mu^a T^a, \qquad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

Summary KMOC algorithm

- Momentum transfer q scales as wavenumber
- Restore $\hbar$'s in couplings g
- Massless loop momenta scales as wavenumber
- Perform a Laurent expansion in powers of $\hbar$

Problem: Classical limit of colour?

Classical Yang-Mills theory

- Wong[1970] equations — Classical

$$D^\mu F^a_{\mu\nu} = J^a_\nu(x) = g\sum_\alpha \int \mathrm{d}\tau_\alpha\; c^a_\alpha(\tau_\alpha)\delta^4(x - x_\alpha(\tau_\alpha))v_{\alpha\nu}(\tau_\alpha)$$

$$\frac{\mathrm{d}p^\mu_\alpha}{\mathrm{d}\tau_\alpha} = g c^a_\alpha(\tau_\alpha)F^{a\mu\nu}(x_\alpha(\tau_\alpha))v_{\alpha\nu}(\tau_\alpha)$$

$$\frac{\mathrm{d}c^a_\alpha}{\mathrm{d}\tau_\alpha} = g f^{abc} v^\mu_\alpha A^b_\mu(x_\alpha(\tau_\alpha))c^c_\alpha(\tau_\alpha)$$

- Yang-Mills field coupled to point-like particles carrying colour charges c^a
- Noether charge operator (dimensions of angular momentum)

$$\mathbb{C}^a = \hbar\int \mathrm{d}\Phi(p)\left(a^\dagger(p)T^a_R a(p) + b^\dagger(p)T^a_{\bar{R}} b(p)\right), \quad \mathrm{d}\Phi(p) = \frac{\mathrm{d}^4 p}{(2\pi)^3}\Theta(p^0)\delta(k^2 - m^2)$$

- Satisfies the Lie algebra of the generators of the representation R

$$[\mathbb{C}^a, \mathbb{C}^a] = \mathrm{i}\hbar f^{abc}\mathbb{C}^c$$

- Classical limit of the colour charge — colour wavefunctions $|\psi\rangle$

$$c^a \equiv \langle\psi|\mathbb{C}^a|\psi\rangle$$

KMOC formalism

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- Conventions

$$S = \int \mathrm{d}^4 x \left(\sum_{\alpha=1}^{2} \left[(D_\mu \varphi_\alpha^\dagger) D^\mu \varphi_\alpha - \frac{m_\alpha^2}{\hbar^2} \varphi_\alpha^\dagger \varphi_\alpha \right] - \frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu} \right),$$

$$D_\mu = \partial_\mu + igA_\mu^a T^a, \qquad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

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Problem: Classical limit of colour?

Classical Yang-Mills theory

- Single particle states are tensor products of colour and kinematics

$$|\psi\rangle = |\psi_{\text{colour}}\rangle \otimes |\psi_{\text{kin}}\rangle$$

- Colour states are coherent states (made up of Schwinger bosons$^{SU(2)\text{, Chapter 3, Sakurai QM}}$)

$$|\psi_{\text{colour}}\rangle \to |\xi\,\zeta\rangle_{[n_1,n_2]} \equiv \frac{1}{\sqrt{(n_1!n_2!)}}\left(\zeta\cdot b^\dagger\right)^{n_2}\left(\xi\cdot a^\dagger\right)^{n_1}|0\rangle$$

- Size of representation labelled by n_1, n_2 must be large in the classical limit
- In the classical limit colour operators are finite and factorise

$$\langle\psi|\mathbb{C}^a|\psi\rangle = \text{finite}\,,$$
$$\langle\psi|\mathbb{C}^a\mathbb{C}^b|\psi\rangle = \langle\psi|\mathbb{C}^a|\psi\rangle\langle\psi|\mathbb{C}^b|\psi\rangle + \text{negligible}$$

- Multi-particle states (χ_i labels parameters ξ, ζ)

$$|\Psi\rangle = \int \mathrm{d}\Phi(p_1)\mathrm{d}\Phi(p_2)\,\phi_1(p_1)\phi_2(p_2)\,e^{ib\cdot p_1/\hbar}\,|p_1\,\chi_1;\,p_2\,\chi_2\rangle$$
$$= \int \mathrm{d}\Phi(p_1)\mathrm{d}\Phi(p_2)\,\phi_1(p_1)\phi_2(p_2)\,e^{ib\cdot p_1/\hbar}\,\chi_{1i}\,\chi_{2j}|{p_1}^i;\,{p_2}^j\rangle$$

Colourful scattering observables

- Colour impulse

$$\langle \Delta c_1^a \rangle = \langle \Psi | S^\dagger \mathbb{C}_1^a S | \Psi \rangle - \langle \Psi | \mathbb{C}_1^a | \Psi \rangle$$
$$= \mathrm{i} \langle \Psi | [\mathbb{C}_1^a, T] | \Psi \rangle + \langle \Psi | T^\dagger [\mathbb{C}_1^a, T] | \Psi \rangle \,,$$

- Momentum operator $\mathbb{P}_1^\mu$ is a colour singlet so momentum impulse differs from QED by colour expectation values
- Both impulse observables are well defined in the quantum and classical regime
- Total colour charge radiated from a scattering event

$$\langle R^a_{\text{col}} \rangle = \langle \Psi | T^\dagger \mathbb{F}^a T | \Psi \rangle \,, \quad \mathbb{F}^a = \mathrm{i}\hbar f^{bac} \sum_{\sigma=\pm} \int \mathrm{d}\Phi(k)\, a_\sigma^{b\dagger}(k)\, a_\sigma^c(k)$$

- Colour conservation

$$[\mathbb{C}_1^a + \mathbb{C}_2^a + \mathbb{F}^a, T] = 0 \Rightarrow \langle \Delta c_1^a \rangle + \langle \Delta c_2^a \rangle = -\langle R^a_{\text{col}} \rangle$$

- Useful notation $(C_i^a \to c_i^a,\, p_i \to m_i u_i)$

$$\left\langle\!\!\left\langle f(p_1, p_2, \cdots) \right\rangle\!\!\right\rangle = \int \mathrm{d}\Phi(p_1)\mathrm{d}\Phi(p_2) |\phi(p_1)|^2 |\phi(p_2)|^2 \langle \chi_1\, \chi_2 | f(p_1, p_2, \cdots) | \chi_1\, \chi_2 \rangle$$

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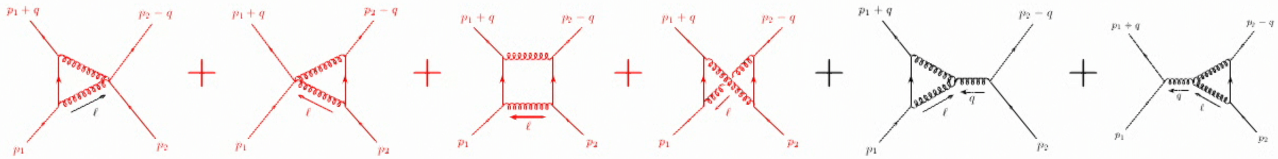
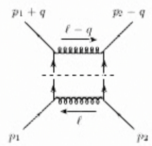
Colour impulses at NLO

- Kinematics

$\mathcal{A}^{(1)}_{\text{QCD}} \sim$



$i\not{\mathcal{B}} \sim$



- At NLO colour and kinematics combine in a highly non-trivial way because

$$\mathcal{A}^{(1)}_{\text{QCD}} = \mathcal{C}\left(\square\right) \mathcal{A}^{(1,\text{QED})} + \mathcal{O}(\hbar)$$

3-gluon vertex

- "Singular" terms $\mathcal{O}(\hbar^{-2})$ in $\mathcal{A}^{(1,\text{QED})}$ mix with $\mathcal{O}(\hbar^2)$ in $\mathcal{C}\left(\square\right)$
- Colour

$$[C_1^a, \mathcal{C}\left(\square\right)] = [C_1^a, C_1^b C_1^c] C_2^b C_2^c$$
$$= i\hbar f^{acd}\left(2C_1^d C_2^c (C_1 \cdot C_2) - i\hbar f^{dbe}\left(C_1^e C_2^b C_2^c - C_2^e C_1^b C_1^c\right) + \mathcal{O}(\hbar^2)\right)$$

- Final result matches iterative solutions of Wong equations $(\alpha = g^2/(4\pi))$ DLC-Luna-Scheopner,'21

$$\Delta c_1^{a,(1)} = \alpha^2 \Big\{ \pi \frac{f^{acd} c_1^c c_2^d (c_1 \cdot c_2)}{\sqrt{\sigma^2 - 1}|b|} \Big(\frac{1}{m_1} + \frac{1}{m_2} \Big) + \frac{1}{2} \frac{\sigma^2}{(\sigma^2 - 1)} \log^2 \Big(\frac{|b|^2}{L^2} \Big) \Big[f^{acd} f^{dbe} c_1^e c_2^b c_2^c - f^{acd} f^{dbe} c_1^b c_1^c c_2^e \Big] \Big\}$$

- Structure constants appear due to colour not to kinematics!
- Indeed for QCD, Δp_1^μ is identical to QED case with colour replacement $Q_1 Q_2 \to c_1 \cdot c_2$

$$\Delta p_1^{\mu,(1)} = (c_1 \cdot c_2)^2 \frac{2\pi\alpha^2}{m_1 m_2} \Big\{ - \frac{1}{4\sqrt{\sigma^2 - 1}} (m_1 + m_2) \frac{b^\mu}{|b|^3} - \frac{1}{\pi} \frac{1}{b^2} \frac{\sigma^2}{(\sigma^2 - 1)^2} \left[(m_2 + \sigma m_1) u_1^\mu - (m_1 + \sigma m_2) u_2^\mu \right] \Big\}$$

- QCD is similar to QED in the classical limit
- ... if radiation is not included

as in thermal QCD... another talk

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Radiation

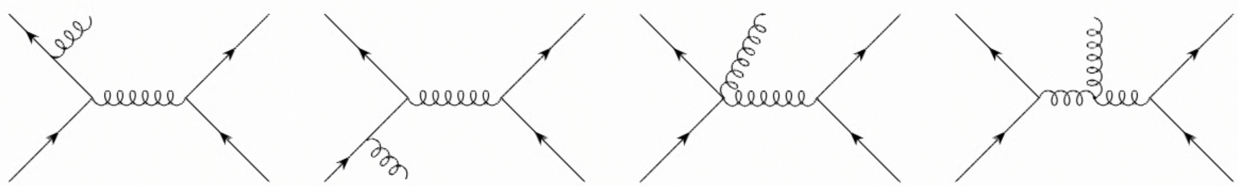
- Radiation simpler as it depends on amplitude squared

$$\mathcal{R}^a_{\text{col}} = -\mathrm{i} f^{abc} \sum_\sigma \left\langle\!\!\left\langle \hbar^{-2} \mathrm{d}\Phi(k) \mathcal{R}^{*b}(k,\sigma) \mathcal{R}^c(k,\sigma) \right\rangle\!\!\right\rangle$$

- Radiation kernel

$$\mathcal{R}^a(k,\sigma) = \hbar^{\frac{3}{2}} \int \mathrm{d}^4 q_1 \mathrm{d}^4 q_2 \, \hat{\delta}(2p_1 \cdot q_1 + q_1^2) \hat{\delta}(2p_2 \cdot q_2 + q_2^2) \, \hat{\delta}^{(4)}(k - q_1 - q_2) \, e^{ib \cdot q_1/\hbar} \times \sum_D \mathcal{C}^a(D) \, A_D(p_1 + q_1, p_2 + q_2 \to p_1, p_2; k, \sigma)$$

- Leading order $\bar{A}^{(0)}_D(p_1 + q_1, p_2 + q_2 \to p_1, p_2; k, \sigma)$



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Effective field theory approach

- EFT approach by Cheung-Rothstein-Solon, '18: $\hbar/\hbar$ stuff not present
- Suppose I am given a perturbative Hamiltonian

$$H \equiv H(\boldsymbol{r}, \boldsymbol{p}, \mathcal{C}_i) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{r}^2, \boldsymbol{p}^2, \mathcal{C}_i) + \dots ,$$

- Then we can solve the problem in perturbation theory

$$\dot{\boldsymbol{r}} = \frac{\partial H}{\partial \boldsymbol{p}} , \qquad \dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{r}} , \qquad \dot{c}_i^a = f^{abc} c_i^b \frac{\partial H}{\partial c_i^c} , \quad i = 1, 2 .$$

Assuming power expansion in $\alpha = g^2/4\pi$

$$\boldsymbol{r}(t) = \boldsymbol{r}_0(t) + \alpha \boldsymbol{r}_1(t) + \alpha^2 \boldsymbol{r}_2(t) + \dots ,$$
$$\boldsymbol{p}(t) = \boldsymbol{p}_0(t) + \alpha \boldsymbol{p}_1(t) + \alpha^2 \boldsymbol{p}_2(t) + \dots ,$$
$$c_i^a(t) = c_{i,0}^a(t) + \alpha c_{i,1}^a(t) + \alpha^2 c_{i,2}^a(t) + \dots .$$

- Observables

$$\Delta O^{(n)} = \int_{-T}^{T} \mathrm{d}t \, \frac{\mathrm{d}O^{(n)}}{\mathrm{d}t} = O^{(n)}(t = T) - O^{(n)}(t = -T)$$

- Center of momentum frame

$$p_1 = -(E_1, \boldsymbol{p}) , \qquad p_2 = -(E_2, -\boldsymbol{p}) , \qquad q = (0, \boldsymbol{q}) , \qquad \boldsymbol{p} \cdot \boldsymbol{q} = \boldsymbol{q}^2/2$$
$$\sigma = \frac{p_1 \cdot p_2}{m_1 m_2} \qquad \nu = \frac{m_1 m_2}{m^2} \qquad \gamma = \frac{E}{m} , \qquad E = E_1 + E_2 , \qquad \xi = \frac{E_1 E_2}{E^2}$$

Computation of Hamiltonian I

Full theory

Classical bits

$\lambda_1 = -4m_1 m_2 \sigma$

- LO

$$\mathcal{A}^{\text{tree}} = -\frac{4\pi\alpha}{q^2}\lambda_1 \mathcal{C}\left(\rangle\!-\!\langle\right) + \ldots$$

- NLO (unitarity)

$$\mathcal{A}^{\text{1-loop}} = \mathcal{C}\left(\boxtimes\right)\mathcal{A}^{\text{1-loop, QED}} + \ldots$$

Amplitude

$$i\mathcal{A}^{\text{1-loop, QED}} = d_{\square} I_{\square} + d_{\bowtie} I_{\bowtie} + c_{\triangle} I_{\triangle} + c_{\bigtriangledown} I_{\bigtriangledown}$$

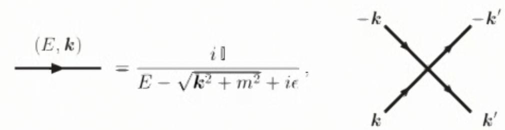
Effective theory

- Effective action

$$S = \int \hat{\mathrm{d}}^{D-1}\boldsymbol{k} \sum_{a=1,2} \xi_a^\dagger(-\boldsymbol{k})\left(i\partial_t - \sqrt{\boldsymbol{k}^2 + m_a^2}\right)\xi_a(\boldsymbol{k})$$

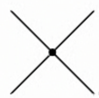
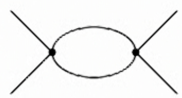
$$-\int \hat{\mathrm{d}}^{D-1}\boldsymbol{k}\int \hat{\mathrm{d}}^{D-1}\boldsymbol{k}'\,\xi_1^\dagger(\boldsymbol{k}')\xi_2^\dagger(-\boldsymbol{k}')\,V(\boldsymbol{k}', \boldsymbol{k}, \hat{\mathcal{C}}_i)\,\xi_1$$

- Feynman rules



$$= \frac{i\mathbb{1}}{E - \sqrt{\boldsymbol{k}^2 + m^2} + i\epsilon}, \qquad = -iV(\boldsymbol{k}', \boldsymbol{k}, \hat{\mathcal{C}}_i).$$

Amplitude in EFT

- Relevant diagrams: LO , NLO: 
- Ansatz:

$$\hat{V}(\boldsymbol{k}', \boldsymbol{k}, \hat{\mathcal{C}}_i) = \frac{4\pi\alpha}{\hat{\boldsymbol{q}}^2} d_1\left(\hat{\boldsymbol{p}}^2\right)\hat{\mathcal{C}}\left(\rangle\!-\!\langle\right) + \frac{2\pi^2\alpha^2}{|\hat{\boldsymbol{q}}|} d_2\left(\hat{\boldsymbol{p}}^2\right)\hat{\mathcal{C}}\left(\boxtimes\right) + \mathcal{O}(\alpha^3)$$

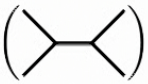
Computation of Hamiltonian II

Full theory

Classical bits

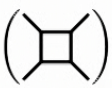
- LO

$$\frac{\mathcal{A}^{\text{tree}}}{4E_1E_2} = \frac{4\pi\alpha}{\boldsymbol{q}^2}\Lambda_1\mathcal{C}$$



- NLO

$$\frac{\mathcal{A}^{\triangle+\triangledown}}{4E_1E_2} = \frac{2\pi^2\alpha^2}{|\boldsymbol{q}|}\Lambda_2\mathcal{C}$$



$$\Lambda_1 = -\frac{\nu\sigma}{\gamma^2\xi}, \qquad \Lambda_2 = \frac{1}{2m\gamma^2\xi}$$

Result of matching

$$d_1 = -\frac{\nu\sigma}{\gamma^2\xi},$$

$$d_2 = \frac{1}{m\xi}\left(\frac{1}{2\gamma^2} - \frac{\nu\sigma}{\xi\gamma^3} + \frac{(1-\xi)\nu^2\sigma^2}{2\xi^2\gamma^5}\right)$$

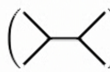
Effective theory

- Amplitude

$$\hat{\mathcal{A}}^{\text{EFT}} = -\hat{V}(\boldsymbol{p}', \boldsymbol{p}, \hat{\mathcal{C}}_i) - \int \hat{d}^{D-1}\boldsymbol{k} \frac{\hat{V}(\boldsymbol{p}', \boldsymbol{k}, \hat{\mathcal{C}}_i)\,\hat{V}(\boldsymbol{k}, \boldsymbol{p}, \hat{\mathcal{C}}_i)}{E_1 + E_2 - \sqrt{\boldsymbol{k}^2 + m_1^2} - \sqrt{\boldsymbol{k}^2 + m_2^2}}$$

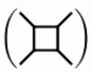
- $\mathcal{O}(\alpha)$

$$\mathcal{A}^{\text{EFT}}_{\mathcal{O}(\alpha)} = -\frac{4\pi\alpha}{\boldsymbol{q}^2}d_1\mathcal{C}$$

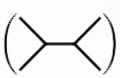


- $\mathcal{O}(\alpha^2)$

$$\mathcal{A}^{\text{EFT}}_{\mathcal{O}(\alpha^2)} = \frac{2\pi^2\alpha^2}{|\boldsymbol{q}|}\Lambda_2\mathcal{C}$$



$$+(4\pi\alpha)^2\,\Lambda_{\text{iter}}\,\mathcal{C}(\ldots)^2 \int \hat{d}^{D-1}\boldsymbol{\ell}\frac{2\xi E}{\boldsymbol{\ell}^2(\boldsymbol{\ell}+\boldsymbol{q})^2(\boldsymbol{\ell}^2 + 2\boldsymbol{p}\cdot\boldsymbol{\ell})}$$



with

$$\Lambda_2 = -d_2 + \frac{1-3\xi}{2\xi E}d_1^2 + \xi E\partial_{\boldsymbol{p}^2}d_1^2$$

Eikonal phase

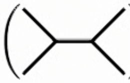
- Following conjectures by Bern-Luna-Roiban-Shen-Zeng, 20', we define $(\chi = \chi_1 + \chi_2 + \dots)$

$$\chi_1 = \frac{1}{4m_1 m_2\sqrt{\sigma^2-1}} \int \hat{\mathrm{d}}^2\boldsymbol{q}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{A}^{\text{tree}}(\boldsymbol{q})\,,$$

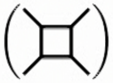
$$\chi_2 = \frac{1}{4m_1 m_2\sqrt{\sigma^2-1}} \int \hat{\mathrm{d}}^2\boldsymbol{q}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \mathcal{A}^{\triangle+\triangledown}(\boldsymbol{q})$$

- Integration leads to DLC-Luna-Scheopner, 2021

$$\chi_1 = -\frac{\xi E\alpha}{|\boldsymbol{p}|}\Lambda_1\left(\ln\frac{\boldsymbol{b}^2}{L^2}\right)\mathcal{C}\left(\right)$$



$$\chi_2 = \frac{\pi\xi E\alpha^2}{|\boldsymbol{p}|}\frac{\Lambda_2}{|\boldsymbol{b}|}\mathcal{C}\left(\right)\,,$$



$$\Delta\boldsymbol{p} = \Delta p_{\parallel}\frac{\boldsymbol{p}}{|\boldsymbol{p}|} + \Delta\boldsymbol{p}_{\perp}$$

$$\{\boldsymbol{p}_\perp, g\} \equiv -\frac{\partial g}{\partial\boldsymbol{b}}\,, \qquad \{c_1^a, g\} \equiv f^{abc}\frac{\partial g}{\partial c_1^b}c_1^c$$

Results

$$\Delta\boldsymbol{p}_\perp = -\{\boldsymbol{p}_\perp, \chi\} - \frac{1}{2}\{\chi, \{\boldsymbol{p}_\perp, \chi\}\}$$

$$\Delta c_1^a = -\{c_1^a, \chi\} - \frac{1}{2}\{\chi, \{c_1^a, \chi\}\}$$

Maxwell-Einstein Theory

- Einstein Hilbert action (de Donder gauge)

$$S_{\mathsf{EH}} = -\frac{2}{\kappa^2}\int \mathrm{d}^4x\sqrt{-g}R, \quad S_{\mathsf{gf}} = \int \mathrm{d}^4x\,(\partial^\nu h_{\mu\nu} - 1/2\partial_\mu h^\nu{}_\nu)^2$$

- and Maxwell theory minimally coupled to gravity

$$S_\gamma = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}\,g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

- Feynman gauge

$$S_{\gamma,gf} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\nabla^\mu A_\mu)^2\right] = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}A^\mu \hat{H}_\mu{}^\nu A_\nu\right]$$

- Second order differential operator

$$\hat{H}_\mu{}^\nu = \delta_\mu{}^\nu\nabla^2 - R_\mu{}^\nu$$

- We interpret as a first-quantized Hamiltonian ($g_{\mu\nu}(x) = \eta_{ab} e^a_\mu(x) e^b_\nu(x)$)

$$\hat{H}_\mu^{\ \nu} \to \hat{H}_a^{\ b} = \delta_a^{\ b} \nabla^2 - R_a^{\ b}, \qquad \nabla_\mu = \partial_\mu - \frac{\mathrm{i}}{2} \omega_\mu^{\ ab} S_{ab}$$

- Matrix valued action ($\partial_\mu \to -\mathrm{i} p_\mu$)

$$(S_{\mathsf{p}}[x;g])_a^{\ b} = \int_0^1 \mathrm{d}\tau \left(-\frac{1}{4T} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \delta_a^{\ b} - \frac{1}{2} \dot{x}^\mu \omega_\mu^{cd} (S_{cd})_a^{\ b} + T R_a^{\ b} - \frac{1}{4} T R \delta_a^{\ b} \right)$$

- Dressed propagator(Schwinger proper time T) Worldline formalism, review Schubert' 01

$$D_a^{\ b}(x_0, y_0; g) = \int_0^\infty \mathrm{d}T \, \langle y_0, a | e^{-\mathrm{i} T \hat{H}} | x_0, b \rangle = \int_0^\infty \mathrm{d}T \int_{x(0)=x_0}^{x(1)=y_0} \mathcal{D}x \, \mathbf{T} \left(e^{\mathrm{i} S_{\mathsf{p}}[x;g]} \right)_a^{\ b}$$

- Introducing auxiliary variables $\bar{Q}^a(\tau) = z \bar{u}^a + \bar{\lambda}^a(\tau), \quad Q_a(\tau) = u_a + \lambda_a(\tau)$

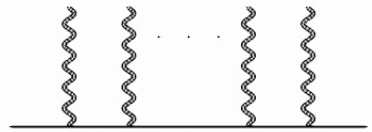
$$S = \int_0^1 \mathrm{d}\tau \Big(-\frac{1}{4T} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + \mathrm{i} \bar{\lambda}^a \dot{\lambda}_a - \frac{1}{2} \dot{x}^\mu \omega_\mu^{cd} (S_{cd})_a^{\ b} \bar{Q}^a Q_b + T R_a^{\ b} \bar{Q}^a Q_b - \frac{3}{4} T R \Big)$$

Dressed propagator

- Time ordering exchanged for modular integration over z

$$D(x_0, y_0, u, \bar{u}; g) = \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} \frac{e^{z\bar{u}\cdot u}}{z^2} \int_0^\infty \mathrm{d}T \int_{x(0)=x_0}^{x(1)=y_0} \mathcal{D}x \int_{\lambda(0)=0}^{\bar{\lambda}(1)=0} D\lambda D\bar{\lambda}\; e^{\mathrm{i}S}$$

- $\bar{u}$, u are placeholders for polarisation vectors
- Dressed propagator with external photons (solid line) off-shell



- Prescription for on-shell amputated dressed propagator

$$D^c(p, p', u, \bar{u}; g) := \lim_{p^2, p'^2 \to 0} (\mathrm{i}p^2\ \mathrm{i}p'^2) \int \mathrm{d}^4x_0 \mathrm{d}^4y_0\, e^{\mathrm{i}p\cdot x_0 - ip'\cdot y_0} D(x_0, y_0, u, \bar{u}; g)$$

- Overall effect: eliminate the proper time integral and set $\tau \in (-\infty, +\infty)$in the action
Fradkin, 60's, see also Fabbrichesi-Pettorino-Veneziano-Vilkovisky,'93Mogull-Plefka-Steinhoff,

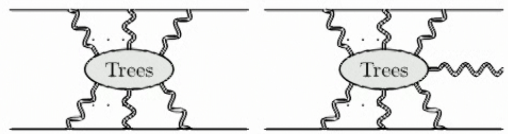
Mirrored Displays

Relation to amplitudes

- Following Fabbrichesi-Pettorino-Veneziano-Vilkovisky, '93 consider the correlator

$$\langle \mathrm{T} A_\mu(x_1) A_\nu(x_2) \varphi(x_1') \varphi^\dagger(x_2') \rangle \sim \int \mathcal{D}g \mathcal{D}A \mathcal{D}\varphi \mathcal{D}\varphi^* \, e^{\mathrm{i}S} A_\mu(x_1) A_\nu(x_2) \varphi(x_1') \varphi^*(x_2')$$

- Disregarding diagrams with scalar and graviton loops (classical limit)



$$\langle \Omega | \mathrm{T} A_\mu(x_1) A_\nu(x_2) \varphi(x_1') \varphi^\dagger(x_2') | \Omega \rangle \sim \int \mathrm{d}g \, e^{\mathrm{i}S_g} D_{\mu\nu}[x_1, x_2; g] G[x_1', x_2'; g]$$

- $G[x_1', x_2'; g]$ is graviton dressed scalar propagator
- Amplitude

$$\mathcal{A}(\gamma\varphi \to \gamma\varphi) \sim \int \Big(\prod_{i=1}^{2} \mathrm{d}x_i \mathrm{d}x'_i \Big) e^{-\mathrm{i}p_1 x_1 + \mathrm{i}p_2 x_2} e^{-\mathrm{i}p_1' x_1' + \mathrm{i}p_2' x_2'} \times \int \mathcal{D}g \, e^{\mathrm{i}S_{\mathsf{EH}}} \varepsilon^{*\mu}(p_1) \varepsilon_\nu(p_2) D^c{}_\mu{}^\nu [x_1, x_2; g] G^c[x_1', x_2'; g]$$

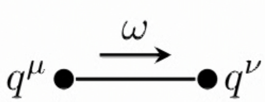
Classical limit

- Shortcut is WQFT for classical observables Berlin collaboration: Jakobsen-Mogull-Shi-Plefka-Steinhoff,'20-22 (massive scalar and spinning)
- Using our photon propagator WQFT is[Bastianelli-Comberiati-DLCD'21] (massless photon)

$$\mathcal{Z}[u,\bar{u}] = \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} \frac{e^{z\bar{u}\cdot u}}{z^2} \int \mathcal{D}g_{\mu\nu} \int \mathcal{D}x_1 \int \mathcal{D}x_2 \int D\lambda \int D\bar{\lambda}\, e^{\mathrm{i}S_{\text{full}}}$$

$$S_{\text{full}} = S_{\text{Einstein-Hilbert}}[g] + S_{\text{Einstein-Maxwell}}[x_1, \lambda, \bar{\lambda}, z; g] + S_{\text{Massive-scalar}}[x_2; g]$$

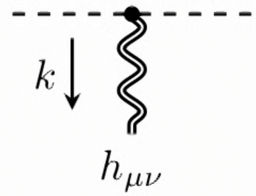
- Background expansion of: e^a_μ, $\omega_\mu{}^{ab}$, x^μ, and Fourier expansion of graviton field $h_{\mu\nu}(x)$. $x^\mu = b^\mu + p^\mu \tau + q^\mu$
- Worldline Feynman rules: Propagators



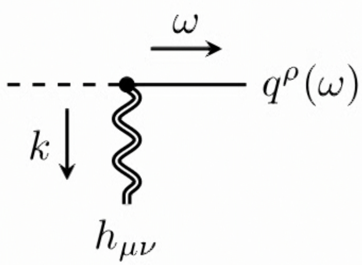
$$= -\mathrm{i}\frac{\eta^{\mu\nu}}{2}\left(\frac{1}{(\omega+\mathrm{i}\epsilon)^2}\right),$$

$$= \mathrm{i}\frac{P_{\mu\nu\rho\sigma}}{k^2+\mathrm{i}\epsilon}$$

- Scalar interactions

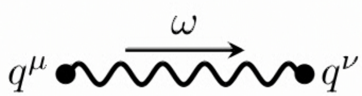


$$= -\mathrm{i}\frac{\kappa}{2} e^{\mathrm{i}k\cdot b}\hat{\delta}(k\cdot p) p^\mu p^\nu ,$$

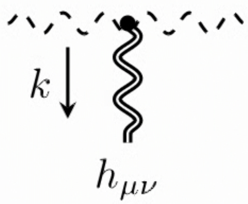


$$= \frac{\kappa}{2} e^{\mathrm{i}k\cdot b}\hat{\delta}(k\cdot p+\omega)\left(2\omega p^{(\mu}\delta^{\nu)}_{\rho} + p^\mu p^\nu k_\rho\right).$$

- Photon-graviton interactions

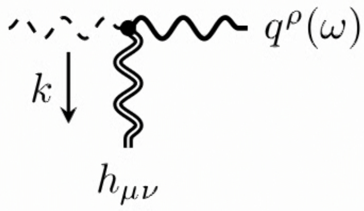


$$= -\mathrm{i}\frac{\eta^{\mu\nu}}{2}\left(\frac{1}{(\omega+\mathrm{i}\epsilon)^2} + \frac{1}{(\omega-\mathrm{i}\epsilon)^2}\right)$$



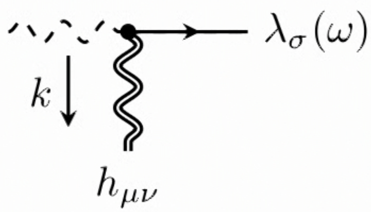
$$= \frac{\mathrm{i}\kappa}{2} e^{\mathrm{i}k\cdot b}\hat{\delta}(k\cdot p)\left(-p^\mu p^\nu + \mathrm{i}zk^\alpha p^{(\nu}\eta^{\mu)\beta}(S_{\alpha\beta})_\rho{}^\sigma \bar{u}^\rho u_\sigma\right),$$

- Worldline Feynman rules (fluctuations)

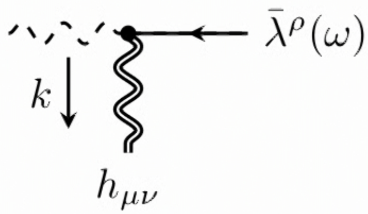


$$= \frac{\kappa}{2} e^{\mathrm{i}k\cdot b} \hat{\delta}\left(k\cdot p + \omega\right) \left[\left(p^{\mu} p^{\nu} k_{\rho} + 2\omega p^{(\mu} \delta^{\nu)}_{\rho} \right) - \mathrm{i} z k^{\alpha} \left(\eta^{\beta(\mu} (p^{\nu)} k_{\rho} + \omega \delta^{\nu)}_{\rho}) \right) (S_{\alpha\beta})_{\lambda}{}^{\delta} \bar{u}^{\lambda} u_{\delta} \right],$$

- Auxiliary variables produce additional rules analogous to spin



$$= -\frac{\kappa}{2} e^{\mathrm{i}k\cdot b} \hat{\delta}(\omega + k\cdot p) z\, k^{\alpha} p^{(\nu} \eta^{\mu)\beta} (S_{\alpha\beta})_{\rho}{}^{\sigma} \bar{u}^{\rho}$$



$$= -\frac{\kappa}{2} e^{\mathrm{i}k\cdot b} \hat{\delta}(\omega - k\cdot p) k^{\alpha} p^{(\nu} \eta^{\mu)\beta} (S_{\alpha\beta})_{\rho}{}^{\sigma} u_{\sigma}$$

Eikonal phase

- Classical limit of the amplitude taken care by $\mathcal{Z}[u, \bar{u}]$
- Geometric optics limit $(\varepsilon^{(\lambda)}(k_{\text{out}}) = \varepsilon^{(\lambda)}(k_{\text{in}}) + \hbar\Delta\varepsilon^{(\lambda)}(k_{\text{out}})$
  Waveforms from amplitudes: Cristofoli-Gonzo-Kosower-O'Connell, '21

$$\varepsilon^{(\lambda)}(k_{\text{out}}) \to \varepsilon^{(\lambda)}(k_{\text{in}})$$

- Shortcut (minus sign is due $u \cdot \bar{u} = -1$,)

$$\mathcal{Z}_{\text{geom-opt}} := -\mathcal{Z}(u, \bar{u})\Big|_{\bar{u} \to u}$$

- Matching with the amplitude as usual

$$\begin{aligned}
\mathcal{Z}_{\text{geom-opt}} &= e^{\mathrm{i}\chi} \\
e^{\mathrm{i}\chi} &:= \frac{1}{4mE}\int \hat{\mathrm{d}}^d q\, \hat{\delta}(q \cdot v_1)\hat{\delta}(q \cdot v_2) e^{\mathrm{i}q \cdot b}\mathcal{A}(\phi\gamma \to \phi\gamma) \\
\mathrm{i}\chi &= \mathrm{i}(\chi_1 + \chi_2 + \dots)
\end{aligned}$$

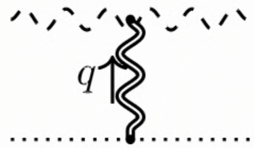
- Deflection angle

$$\theta_i = -\frac{1}{E}\frac{\partial \chi_i}{\partial |b|}$$

Example: LO

- Spinless

$i\chi_1 =$

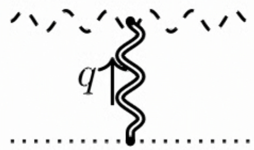


$$= -i\kappa^2 \frac{(p_1 \cdot p_2)^2}{4} \int \hat{d}^4 q\, \hat{\delta}(q \cdot p_1) \hat{\delta}(q \cdot p_2) \frac{e^{iq \cdot b}}{q^2}$$

$$= -2iG_N (p_1 \cdot p_2) \log \left(\frac{|b|^2}{L^2} \right) \to \theta_1 = -\frac{1}{E} \frac{\partial \chi_1}{\partial |b|} = \frac{4G_N m}{|b|}$$

- Spinning

$i\chi_1 =$

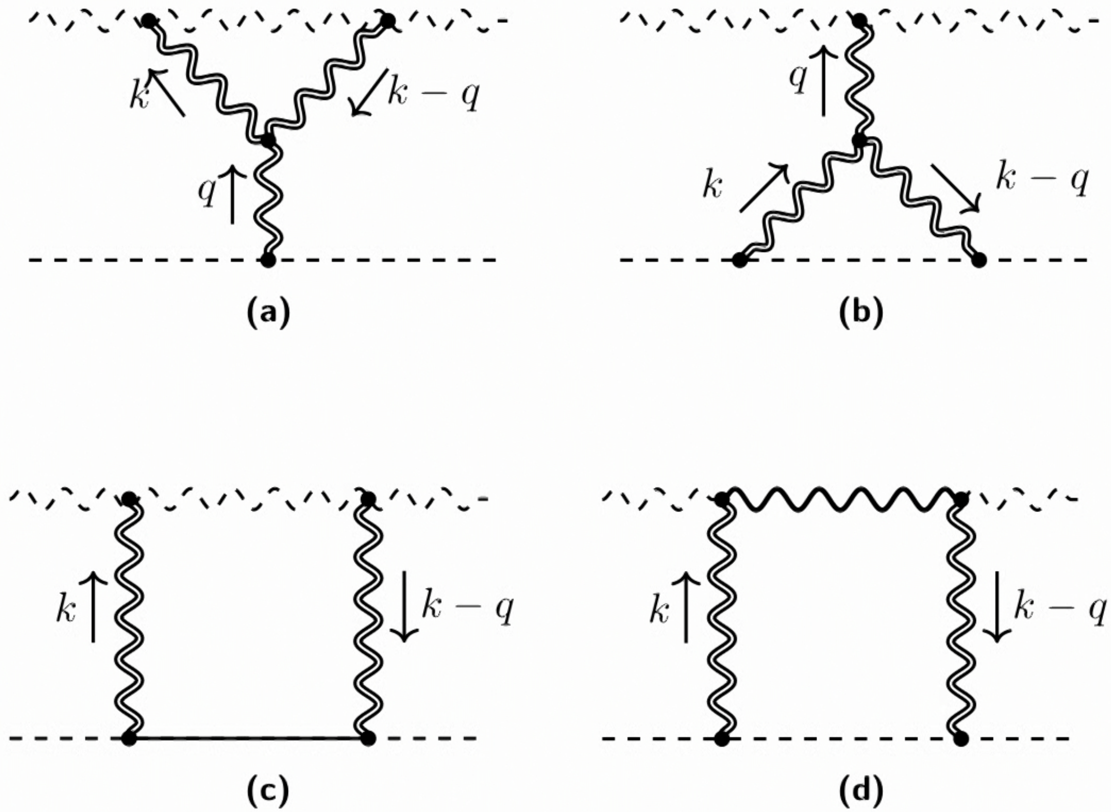


$$= -i\kappa^2 \frac{(p_1 \cdot p_2)^2}{4} \int \hat{d}^4 q\, \hat{\delta}(q \cdot p_1) \hat{\delta}(q \cdot p_2) \frac{e^{iq \cdot b}(1 + N_S)}{q^2}$$

$$N_S = -\frac{im}{p_1 \cdot p_2} (p_1 \cdot \mathcal{S} \cdot q) - \frac{m^2}{2(p_1 \cdot p_2)^2} (p_1 \cdot \mathcal{S} \cdot q)^2$$

$$\chi_1 = \kappa^2 \frac{p_1 \cdot p_2}{8\pi} \left(-\frac{1}{2} \log \left(\frac{|b^2|}{L^2} \right) - \frac{s}{|b|} + \frac{s^2}{2|b|^2} \right) \to \theta_1 = 4 \left(\frac{1}{|b|} - \frac{s}{|b|^2} + \frac{s^2}{|b|^3} \right) G_N m$$

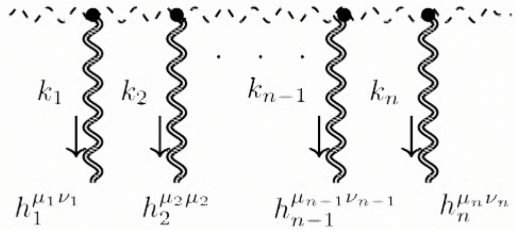
Example: NLO



NLO diagrams

Results

- Diagram (a) vanish identically in geometric optics limit
- Special case of



$$\sim (-1)^n p^{\mu_1}p^{\nu_1}\cdots p^{\mu_n}p^{\nu_n} - z\sum_{i=1}^{n} p^{\mu_1}p^{\nu_1}\cdots\widehat{p^{\mu_i}p^{\nu_i}}\cdots p^{\mu_1}p^{\nu_1}B_i^{\mu_i\nu_i}$$

- Masless limit equivalent to spin-tensor independence $\sim$ thanks to moduli integral over z
- Diagram (c) vanish due to the presence of double poles
- Integral reduction very simple: Passarino-Veltman
- Result only depends only on

$$I_{\triangleright} = \frac{1}{16\pi|b|}$$

- After adding up diagrams (b) and (d)

$$\chi_2 = \kappa^4\frac{15}{256}m(p_1\cdot p_2)\frac{1}{16\pi|b|} \to \theta_2 = -\frac{1}{E}\frac{\partial\chi_2}{\partial|b|} = \frac{15\pi}{4}\frac{G_N^2m^2}{|b|^2}$$

Mirrored Displays

Conclusions

KMOC and EFT

- KMOC extended to Yang-Mills (already existent for QED and gravity)
- Impulses are QED-like (3-gluon vertex subleading) but not colour radiation
- Construction of EFT hamiltonian including colour

On WQFT...

- Gravitationally dressed photon propagator
- WQFT for photons coupled to gravitons

Outlook

- Gravity and double copy: Double copy better understood in QFT: our amplitudes approach makes easier to implement double copy. For WQFT see[Shi-Plefka'21]
- In progress: off-shell story and beyond geometric optics

Thanks!