Title: Staying Ahead of the Curve(ature) in Topological Phases

Speakers: Julian May-Mann

Series: Quantum Matter

Date: November 17, 2022 - 2:00 PM

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Abstract: Many topological phases of lattice systems display quantized responses to lattice defects. Notably, 2D insulators with C_n lattice rotation symmetry hosts a response where disclination defects bind fractional charge. In this talk, I will show that the underlying physics of the disclination-charge response can be understood via a theory of continuum fermions with an enlarged SO(2) rotation symmetry. This interpretation maps the response of lattice fermions to disclinations onto the response of continuum fermions to spatial curvature. Additionally, in 3D, the response of continuum fermions to spatial curvature predicts a new type of lattice response where disclination lines host a quantized polarization. This disclination-polarization response defines a new class of topological crystalline insulator that can be realized in lattice models. In total, these results show that continuum theories with spatial curvature provide novel insights into the universal features of topological lattice systems. In total, these results show that theories with spatial curvature provide novel insights into the universal features of topological lattice systems.

Zoom link: https://pitp.zoom.us/j/97325013281?pwd=MU5tdFYzTFljMGdaelZtNjJqbmRPZz09

STAYING AHEAD OF THE CURVE(ATURE) IN TOPOLOGICAL PHASES

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NOVEMBER 17, 2022

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

NSF

FOCUS OF THIS TALK

The responses of topological phases to disclination defects

Equivalently, responses to fluxes of C_n rotation symmetry

- The responses of lattice fermions to disclinations can be understood using continuum theories of fermions in curved space
  - Correctly reproduces the disclination responses of two-dimensional higher-order topological insulators
  - Predicts a new type of disclination responses in 3D, which we verify using lattice models

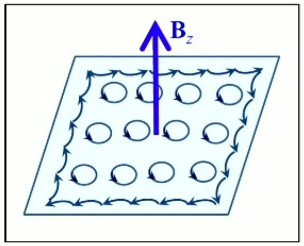
THE BIG QUESTION

How do we understand topological phases of matter?

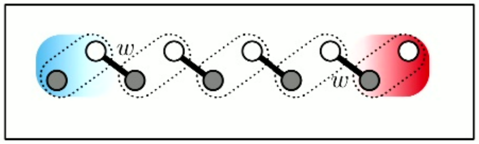
Focus on fermionic symmetry protected topological phases (**SPTs**) without interactions

SPTs: gapped systems that cannot be deformed into a trivial system without breaking symmetry or closing the band gap

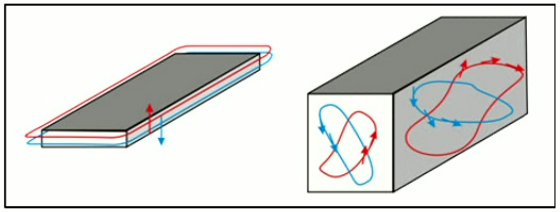
Integer quantum Hall Insulator



Su-Schrieffer–Heeger chain



Topological Insulators (TIs) in 2D and 3D



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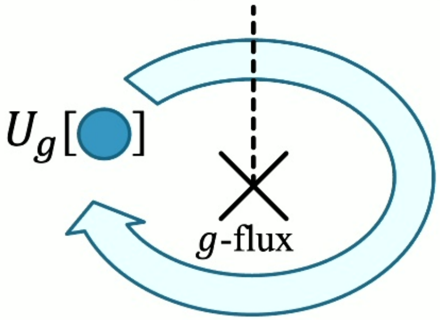
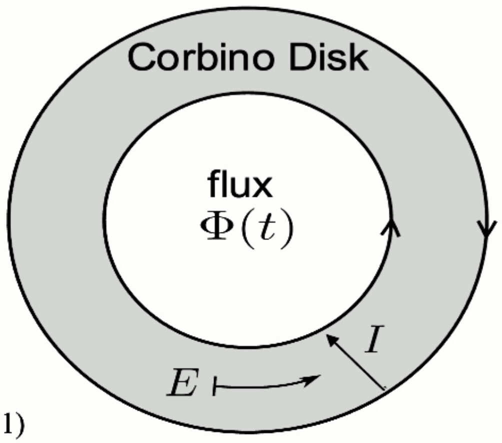
Many SPTs can be understood through their responses to symmetry fluxes

Symmetry G with element g
U_g : representation of g



Example: response of quantum Hall insulators to fluxes of U(1) charge symmetry

Laughlin pumping argument: Hall conductance = charge is bound to magnetic fluxes



As the flux increases, current flows to the flux
$\Rightarrow$ charge accumulates at the flux

A 2π U(1) flux will bind charge $Q_{2\pi-\text{flux}} \in \mathbb{Z}$

Laughlin PRB (1981)

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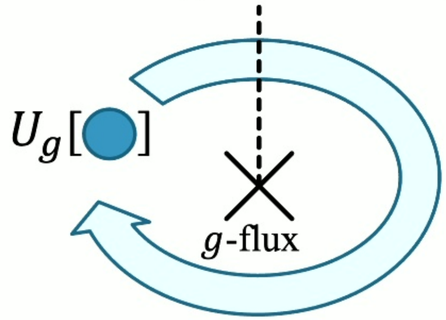

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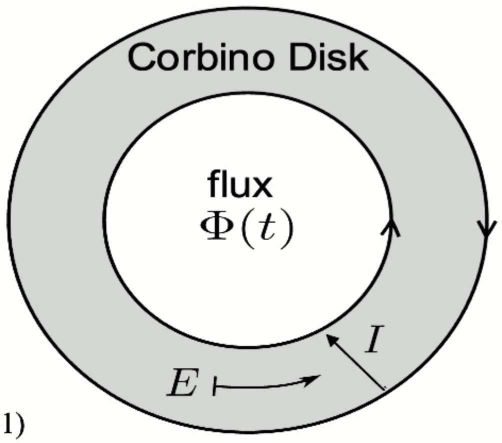

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$\Longrightarrow$ charge accumulates at the flux

A 2π U(1) flux will bind charge $Q_{2\pi-\text{flux}} \in \mathbb{Z}$

$Q_{2\pi-\text{flux}}$ is the Chern number

Laughlin PRB (1981)

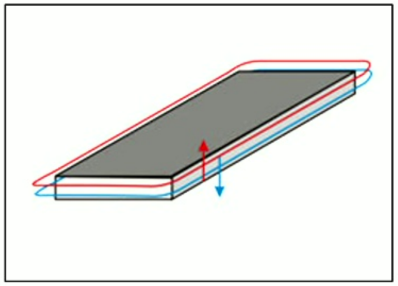
SYMMETRY FLUXES IN OTHER TOPOLOGICAL INSULATORS

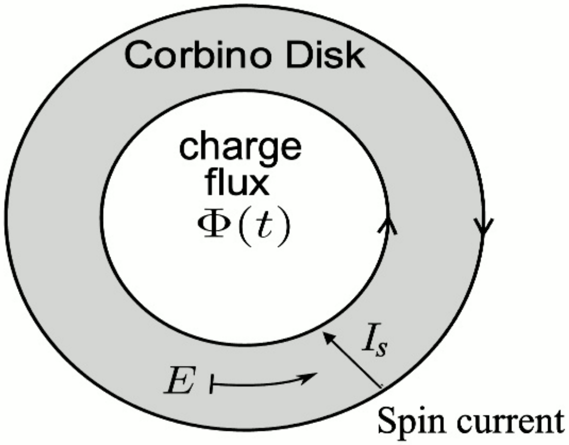
2D TI (AKA Quantum spin Hall insulators)

U(1) charge and U(1) S_z spin

Mixed responses:

- charge flux binds spin
- spin flux binds charge

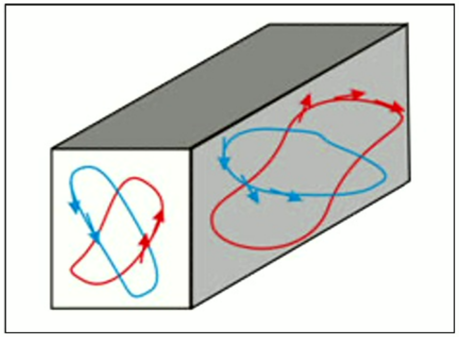


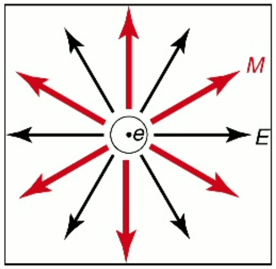


3D TIs/Axion insulators

Quantized magnetoelectric effect

- Witten effect: charged magnetic monopoles





Qi and Zhang, PRL 2008

Qi, Hughes, and Zhang, PRB 2008

Topological Field Theories

2D Quantized Hall Conductance

$$\mathcal{L} = \frac{C}{4\pi}\, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda,$$

$$j_x = \frac{C}{2\pi} E_y \qquad \rho = \frac{C}{2\pi} B_z$$

3D Magnetoelectric Effect

$$\mathcal{L} = \frac{\Theta}{32\pi^2} \epsilon^{\mu\nu\lambda\delta}\, \partial_\mu A_\nu \partial_\lambda A_\delta,$$

$$\rho = -\frac{\Theta}{4\pi^2}\, \boldsymbol{\nabla} \cdot \boldsymbol{B}$$

A_μ: U(1) charge gauge field

Gauge fields in this talk are <u>non-dynamic</u> background probes

Topological terms appear in effective response theory after integrating out the microscopic fermions

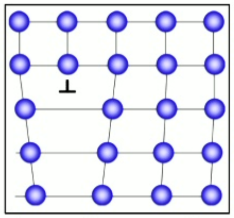
$$Z[A_\mu, \dots] = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{i\int dtd\boldsymbol{x}\, \mathcal{L}[\bar{\Psi},\Psi,A_\mu,\dots]} \qquad \int dtd\boldsymbol{x}\, \mathcal{L}_{eff}[A_\mu, \dots] = \log Z[A_\mu, \dots]$$

Zee, Springer 1995 (review) Qi, et al., PRB 2008

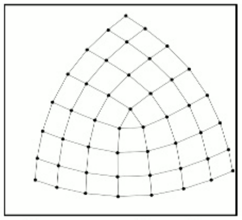
Topological phases of lattice system can also host quantized responses to fluxes of lattice symmetries

Lattice symmetries: $\boldsymbol{C_n}$**-rotation**, discrete translation, mirror

Fluxes of a lattice symmetry = lattice defects



Translation flux = dislocation

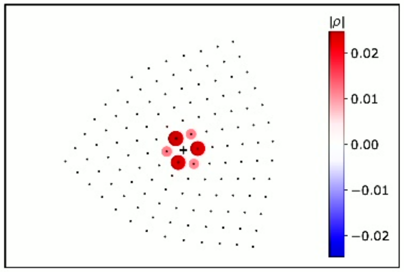


C_n flux = disclination

Example of a geometric response:

C_n symmetry fluxes bind charge

Equiv. disclination defects bind charge

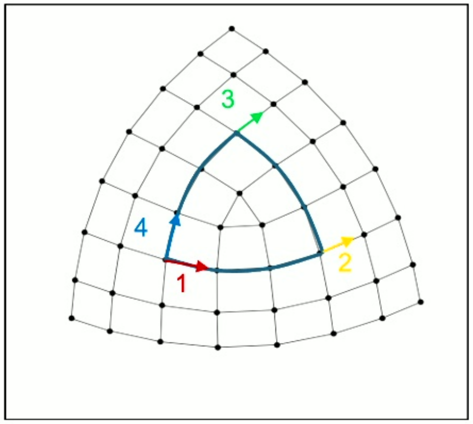


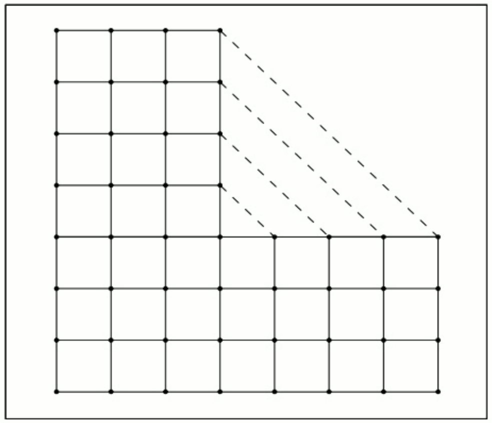
charge bound to disclination

Disclination defects

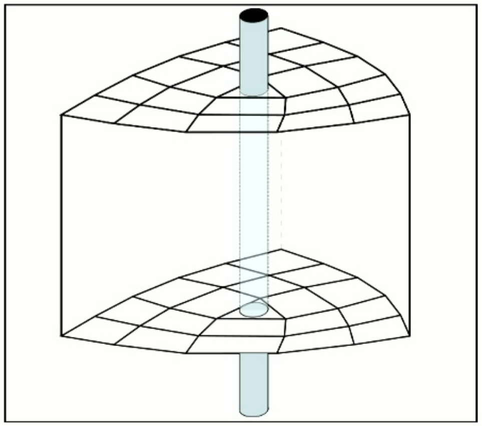
Take $\vec{v}$ that transforms under a representation of C_n
Parallel transport of $\vec{v}$ around a disclination rotates $\vec{v}$

2D disclination





3D Disclination-line



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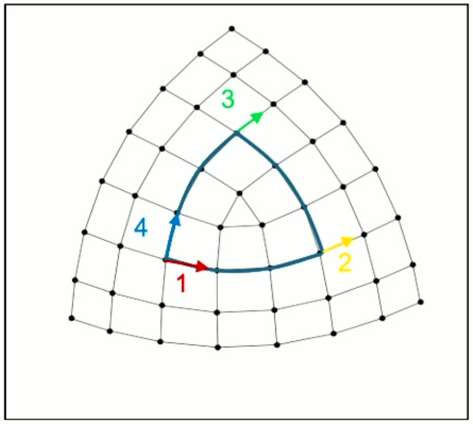

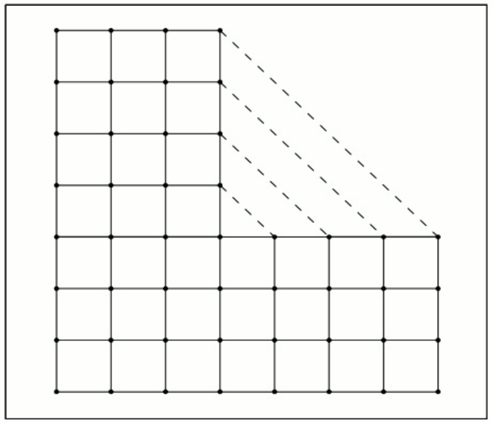

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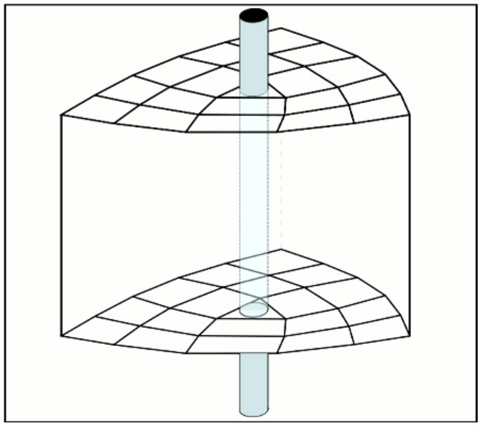


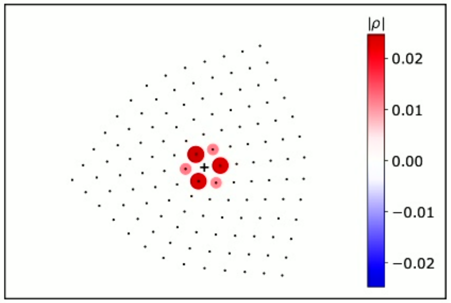
Disclinations are characterized by a **Frank angle:** $\boldsymbol{\theta_F}$

Equal to angle the lattice vectors rotate around the disclination

Disclination-Charge Response

C_n rotation symmetry fluxes are encoded in fluxes of a discrete gauge field: C

Disclination-Charge field theory term: $\frac{\mathfrak{s}}{2\pi} A \wedge dC$



- $\mathfrak{s} \in \frac{1}{2}\mathbb{Z}$ for 2D spinless fermions without TRS
- $\mathfrak{s} \in \mathbb{Z}$ for 2D spin-1/2 fermions without TRS
- $\mathfrak{s} \in \mathbb{Z}$ for 2D spinless fermions with TRS
- $\mathfrak{s} \in 2\mathbb{Z}$ for 2D spin-1/2 fermions with TRS

Discrete nature of C makes analytic analysis difficult

Can't use perturbative approaches

See Manjunath and Barkeshli, PRR 2021

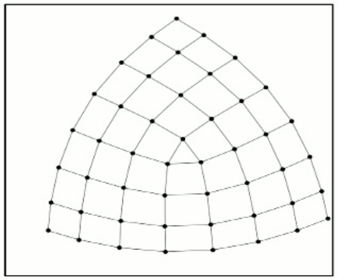
Responses to fluxes $SO(2)$ continuous rotation symmetry

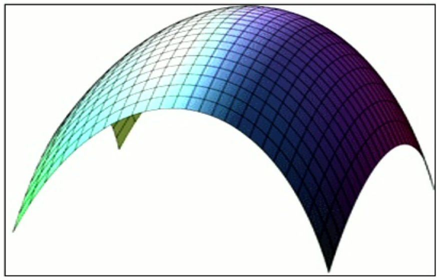
Fluxes of $SO(2) = U(1)$ are continuous and can be added perturbatively

Allows for linear response theory, adiabatic approximations, etc.

Fluxes of $SO(2)$ are sources of spatial curvature

Responses of lattice fermions to disclination defects ? ⟷ Responses to continuum fermions to spatial curvature

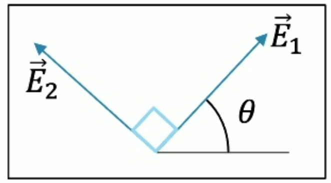


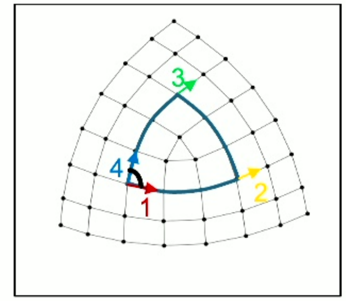


Assume the spatial manifold is locally flat, $g_{ij} = \delta_{ij}$

$$\vec{E}_1 = (\cos(\theta), \sin(\theta)),$$

$$\vec{E}_2 = (-\sin(\theta), \cos(\theta))$$





$SO(2)$ Gauge redundancy of $\vec{E}_A$: $\theta \to \theta + \varphi(x),\ \varphi(x) \in [0, 2\pi)$

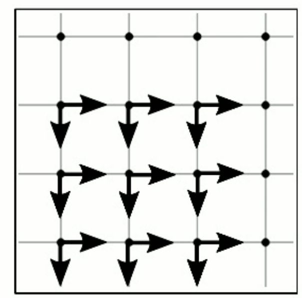
Related to ambiguity in defining primitive vectors

θ winds by θ_F around a disclination

Spin connection definition: $\omega_\mu \equiv E_2^i \partial_\mu E_i^1 = \partial_\mu \theta$

$SO(2)$ gauge transformation: $\omega_\mu \to \omega_\mu + \partial_\mu \varphi$

Curvature tensor $R_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$



$$\oint_{disc} \omega_i dl_i = \oint_{disc} \partial_i \theta \, dl_i = \theta_F$$

Lattice with Disclinations → Manifold with singular points of curvature

CURVATURE-CHARGE FIELD THEORY

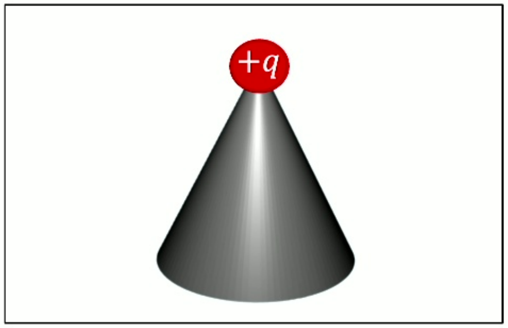
Curvature response: charge bound to curvature

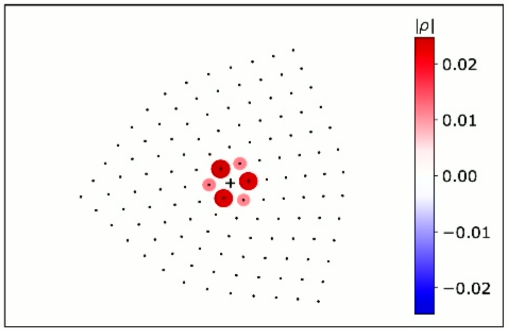
Effective field theory

Wen-Zee term: $\mathcal{L}_{WZ} = \frac{\mathcal{S}}{2\pi}\,\epsilon^{\mu\nu\lambda}\omega_\mu\partial_\nu A_\lambda$

Wen and Zee, PRL 1991

$$\rho = \frac{\mathcal{S}}{2\pi}\left[\,\partial_x\omega_y - \partial_y\omega_x\right] = \frac{\mathcal{S}}{2\pi}\,R_{xy}$$





$\mathcal{S}$ (shift) is quantized, $\mathcal{S} \in \frac{1}{2}\mathbb{Z}$

- Gauss-Bonnet Theorem: total curvature of a closed surface is a multiple of 4π ($\frac{1}{4\pi}\int d\boldsymbol{x}\,R_{xy} = \frac{1}{2}\chi \in \mathbb{Z}$)
- Total charge on any closed surface must be an integer ($\int d\boldsymbol{x}\,\rho \in \mathbb{Z}$)

THE QUADRUPOLE INSULATOR HOTI

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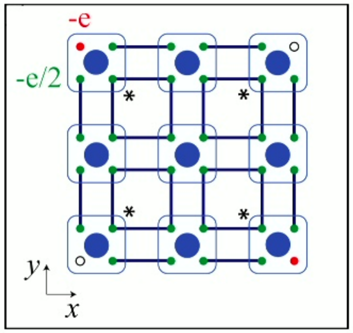
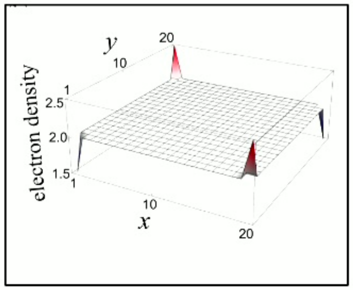
THE QUADRUPOLE INSULATOR HOTI

The quadrupole insulator (**QI**) AKA Benalcazar, Bernevig, Hughes (BBH) model

- First example of a 2D higher-order topological insulator (HOTI)
- Model of spinless fermions with time-reversal symmetry (TRS)
- C_4 symmetric topological phase
  - Half-integer charges at corner
  - Half-integer charges at $\pi/2$ disclinations



Is there a theory of fermions in curved space that capture the disclination response of the QI?

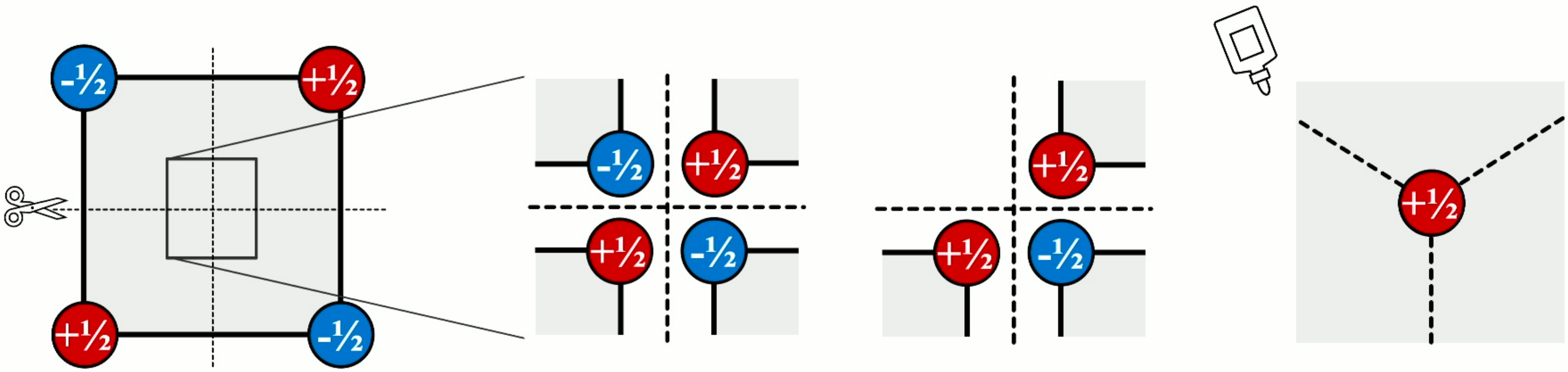


Expected Continuum Response:

$$\mathcal{L}_{QI-WZ} = \frac{1}{\pi}\,\epsilon^{\mu\nu\lambda}\omega_\mu\partial_\nu A_\lambda, \qquad \mathcal{S} = 2$$

Benalcazar et al., Science 2017

How to teach your child about disclination responses in HOTIs



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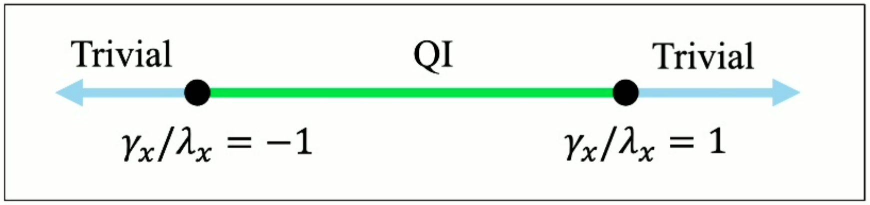
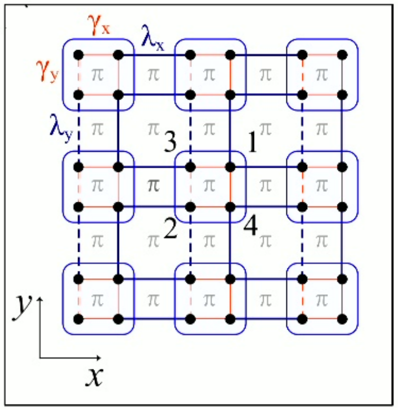
QUADRUPOLE INSULATOR

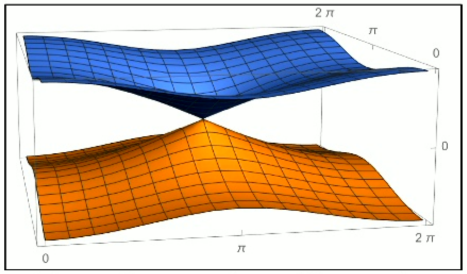
$$h^q(\boldsymbol{k}) = \lambda_y \sin(k_y)\Gamma^1 + [\gamma_y + \lambda_y \cos(k_y)]\Gamma^2 + \lambda_x \sin(k_x)\Gamma^3 + [\gamma_x + \lambda_x \cos(k_x)]\Gamma^4$$

C_4 rotation symmetry when $\lambda_x = \lambda_y$ and $\gamma_x = \gamma_y$

QI phase occurs for $\left|\frac{\gamma_x}{\lambda_x}\right| = \left|\frac{\gamma_y}{\lambda_y}\right| < 1$







Spectrum at $\gamma_x/\lambda_x = 1$

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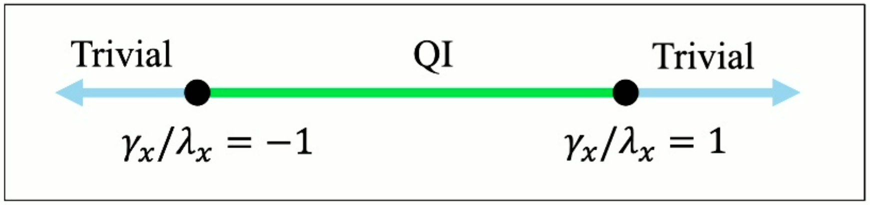
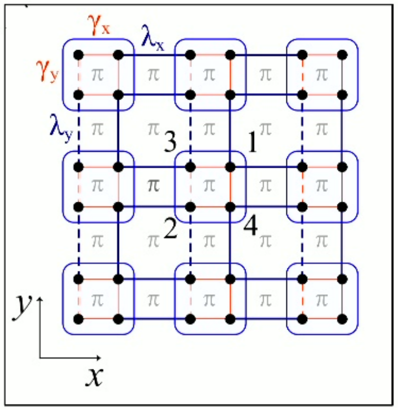
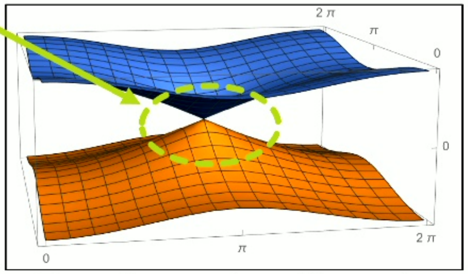




Dirac-like 4-band crossing



Spectrum at $\gamma_x/\lambda_x = 1$

Proposed continuum theory

$$\mathcal{L}^q = \bar{\psi}\left[i\,\sigma^z\tau^0 D_t + i\,E_A^x\sigma^A\tau^0 D_x + i\,E_A^y\sigma^A\tau^0 D_y + m\sigma^0\tau^z\right]\psi$$

$$D_\mu = \partial_\mu - i\,A_\mu - i\,\omega_\mu\left(\frac{1}{2}\sigma^z\tau^0 + \sigma^0\tau^z\right)$$

Angular momentum of the continuum model
$\equiv C_4$ angular momentum of lattice model
mod(4)

$SO(2)$ gauge transformation

$\vec{E}_1 = (\cos(\theta), \sin(\theta))$
$\vec{E}_2 = (-\sin(\theta), \cos(\theta))$
$\omega_\mu = \partial_\mu\theta$

$\theta \to \theta + \varphi(x_\mu),\ \varphi(x_\mu) \in [0, 2\pi)$

$\mathbf{\Psi} \to e^{i\varphi(x_\mu)\times\left(\frac{i}{4}[\gamma^1,\gamma^2]+\gamma^3\right)}\mathbf{\Psi}$

JMM and Hughes, Arxiv 2021

BOUNDARY ANOMALY AND CORNER CHARGES

For a system with boundary, the Wen-Zee term is not invariant under U(1) gauge transformations

$$\mathcal{L}_{QI-WZ} = \frac{1}{\pi}\, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu \omega_\lambda$$

$$A_\mu \to A_\mu + \partial_\mu \theta$$

$$\mathcal{L}_{QI-WZ} \to \mathcal{L}_{QI-WZ} + \frac{1}{\pi}\, \epsilon^{\mu\nu\lambda} \partial_\mu (\theta \partial_\nu \omega_\lambda)$$

Charge is not conserved at the boundary

$$\partial_\mu j^\mu|_{QI-bdry} = \frac{1}{\pi} \epsilon^{\mu\nu} \partial_\mu \omega_\nu|_{QI-bdry}$$

$$\Delta Q|_{QI-bdry} = \Delta \frac{1}{2} \int dx \frac{2}{\pi}\, R_{xy}|_{QI-bulk}$$

JMM and Hughes, Arxiv 2021

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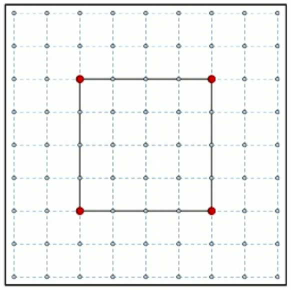
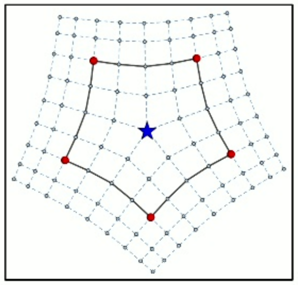


$N_{\pi/2-disc} = 0$

4 corners

$Q|_{QI-bdry} = 0\ mod\ (1)$



$N_{\pi/2-disc} = 1$

5 corners

$Q|_{QI-bdry} = 1/2\ mod\ (1)$

JMM and Hughes, Arxiv 2021

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CURVATURE RESPONSES IN 3D

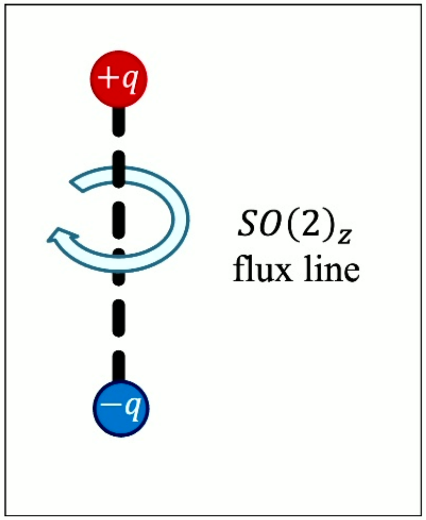
In 2D curvature ($SO(2)$ fluxes) can bind charge

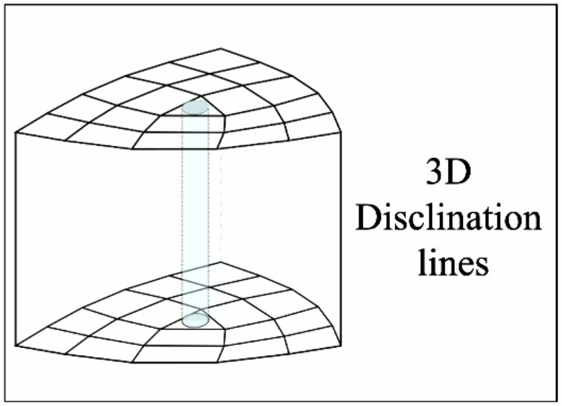
What can occur in 3D?

For simplicity, we will only consider flux of $SO(2)_z$ symmetry, that rotates the xy-planes

Equiv. only consider curvature of xy-panes

Possible response: $SO(2)_z$ flux-lines have an electric polarization





JMM et al., Arxiv 2022

Field Theory Term

$SO(2)_z$ gauge field: ω_μ,

xy-plane curvature tensor $R_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$

The $R \wedge F$-term: $\mathcal{L}_{RF} = \frac{\Phi}{16\pi^2} \epsilon^{\mu\nu\lambda\rho} R_{\mu\nu} F_{\lambda\rho} = \frac{\Phi}{4\pi^2} \epsilon^{\mu\nu\lambda\rho} \partial_\mu \omega_\nu \partial_\lambda A_\rho$

$SO(2)_z$ flux line: $R_{xy} = 2\pi\delta(x)\delta(y)$

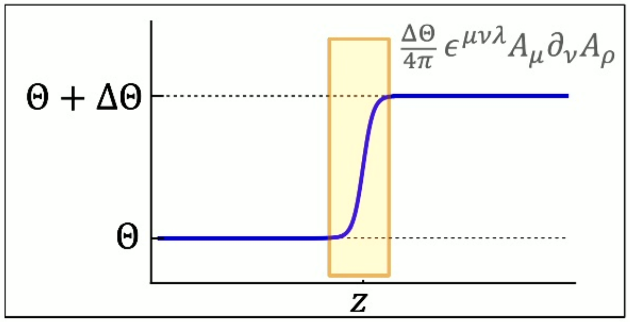
$\Rightarrow \int dt\, d\boldsymbol{x}\, \mathcal{L}_{RF} = \int dt\, dz\, \frac{\Phi}{2\pi} [\partial_z A_t - \partial_t A_z]$ ← Action for a 1D object with polarization $P_z = \frac{\Phi}{2\pi}$

JMM et al., Arxiv 2022

The $R\wedge F$ term is similar to the 3D Θ-term that describes 3D TIs/Axion insulators

3D Θ-term: $\frac{\Theta}{32\pi^2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$

- Total derivative
- Θ is periodic
- Domain walls of Θ bind unquantized Chern-Simons terms: $\frac{\Delta\Theta}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\rho$



Qi, et. al, PRB 2008

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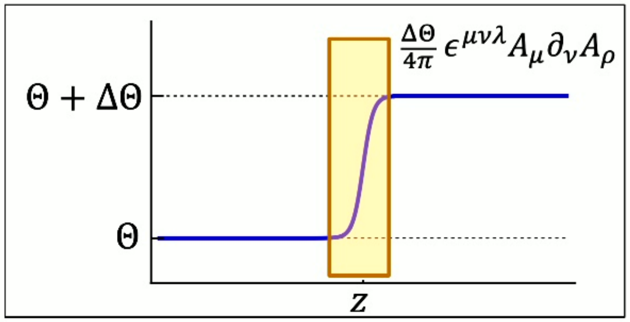
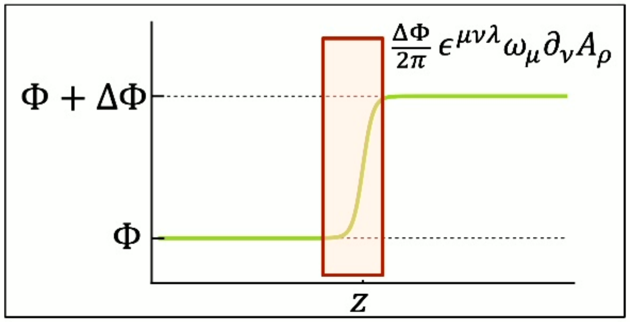
- Witten Effect: monopoles of A_μ carry charge



Qi, et. al, PRB 2008

$R \wedge F$ term: $\frac{\Phi}{16\pi^2}\epsilon^{\mu\nu\lambda\rho}R_{\mu\nu}F_{\lambda\rho}$

- Total derivative
- Φ is periodic
- Domain walls of Φ bind unquantized Wen-Zee terms: $\frac{\Delta\Phi}{2\pi}\epsilon^{\mu\nu\lambda}\omega_\mu\partial_\nu A_\rho$



JMM et al., Arxiv 2022

$R \wedge F$ term predicts a new type of topological crystalline insulator

- For spinless fermions with TRS, Φ is 2π periodic*
- The $R \wedge F$ term is odd under particle-hole symmetry (PHS) and z-mirror symmetry (M_z) $\Rightarrow$ **$\boldsymbol{\Phi = 0, \pi}$ for symmetric insulators**

JMM et al., Arxiv 2022

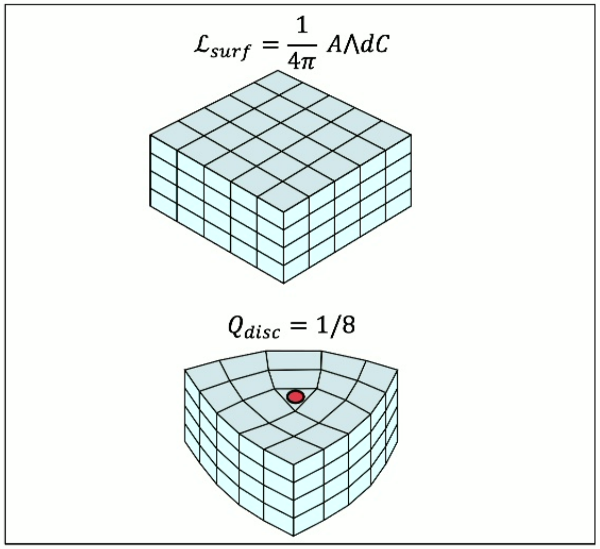
*In general, the periodicity of Φ depends on the fermion spin, and if TRS is present

Physical properties of the rTCI

Domain walls between $\Phi = 0$ and $\Phi = \pi$ bind a Wen-Zee term $\frac{\mathcal{S}}{4\pi}\,\epsilon^{\mu\nu\lambda}\omega_\mu\partial_\nu A_\rho$, $\mathcal{S} = 1/2 \bmod(1)$

Gapped surfaces of the lattice rTCI should have disclination response $\frac{\mathfrak{s}}{4\pi}\,A\wedge dC$, $\mathfrak{s} = 1/2 \bmod(1)$

- mod(1) comes from 2D surface effects
- Half the $\mathfrak{s}$ that can occur in 2D
- Gapped surfaces break PHS but can preserve mirror symmetry



$\mathfrak{s} \in \frac{1}{2}\mathbb{Z}$ for 2D spinless fermions without TRS

$\mathfrak{s} \in \mathbb{Z}$ for 2D spin-1/2 fermions without TRS

$\mathfrak{s} \in \mathbb{Z}$ for 2D spinless fermions with TRS

$\mathfrak{s} \in 2\mathbb{Z}$ for 2D spin-1/2 fermions with TRS

JMM et al., Arxiv 2022

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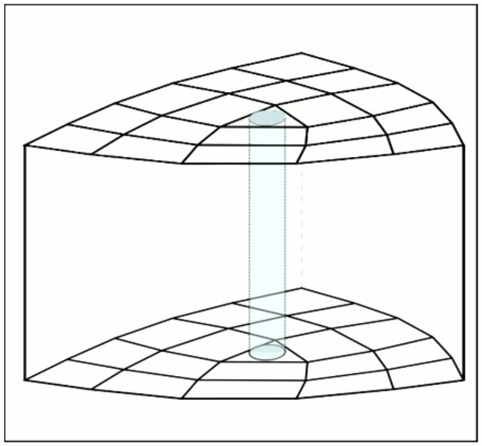
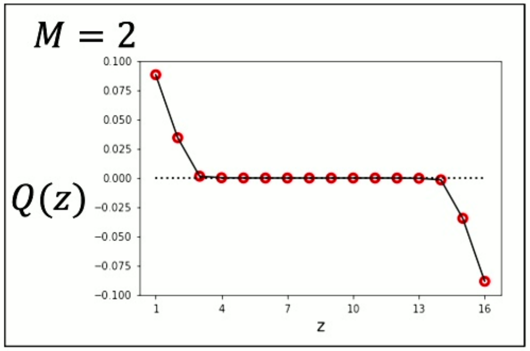
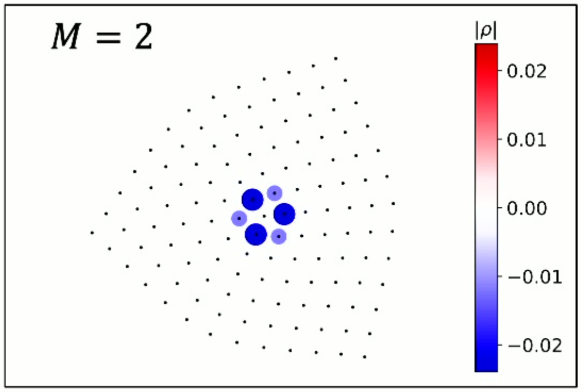
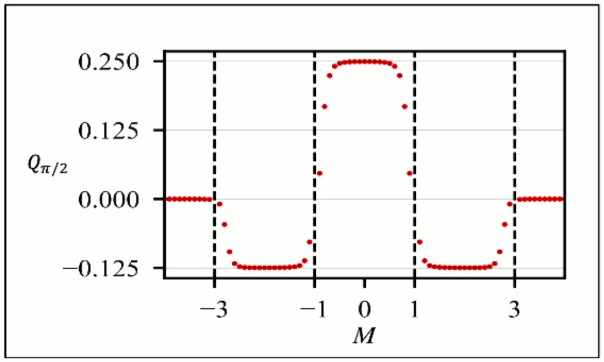
Numeric analysis of the disclination response of h^{rTCI}









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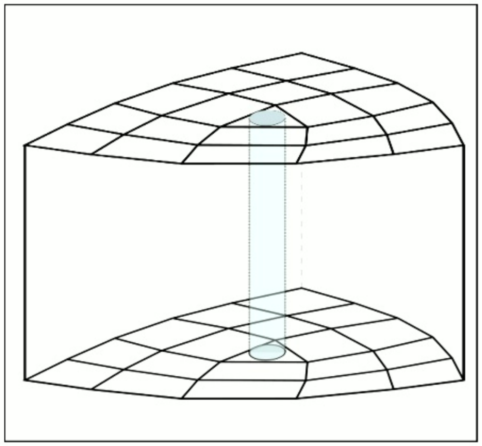
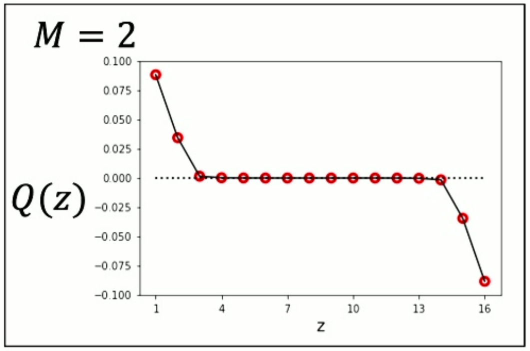
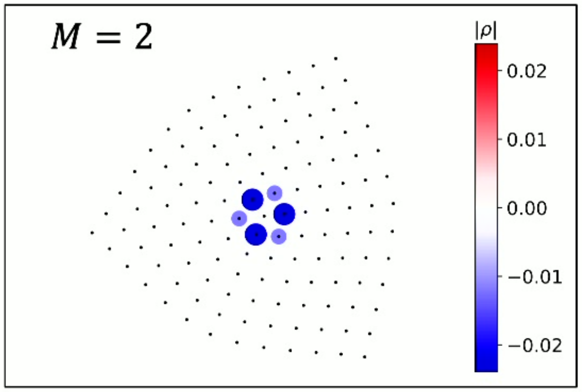
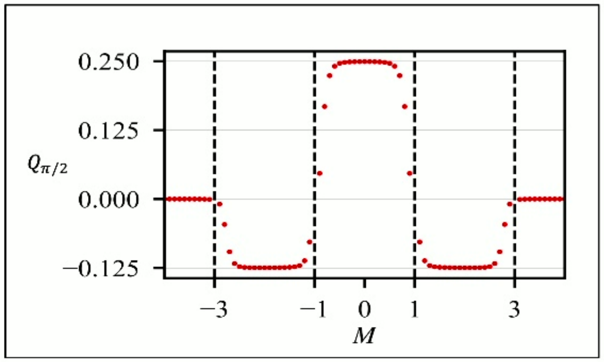
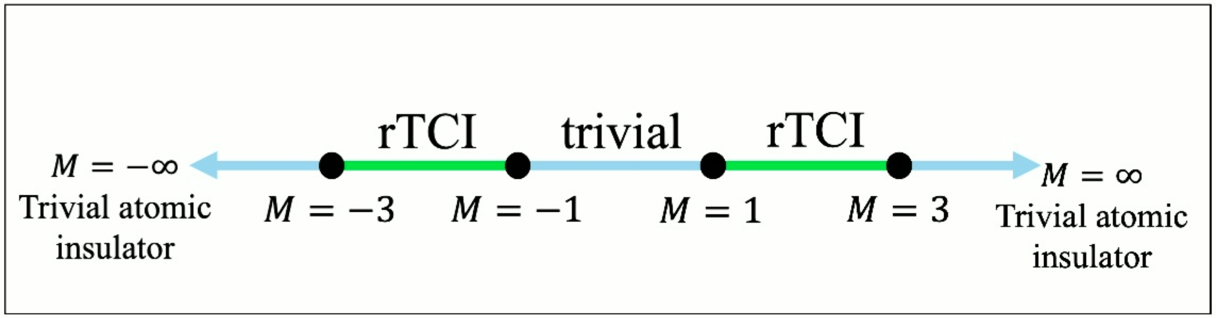




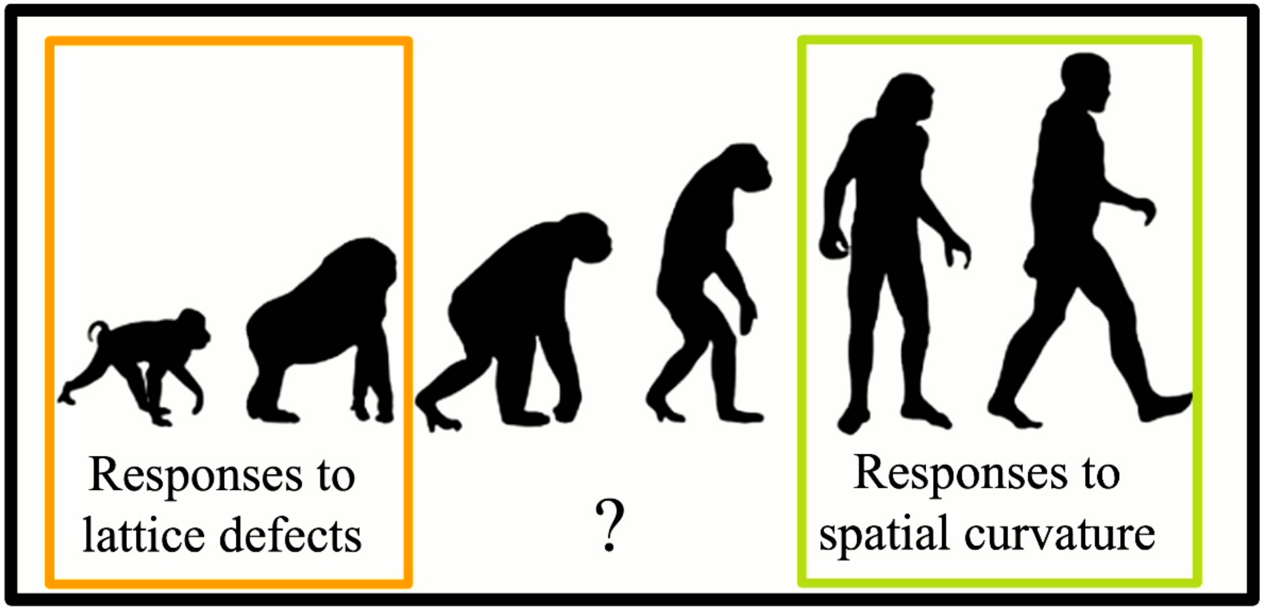






JMM et al., Arxiv 2022

THE MISSING LINK



$SO(2) \rightarrow C_n$

Spontaneous symmetry breaking in topological fluid

$C_n \rightarrow SO(2)$

Melting a topological crystal

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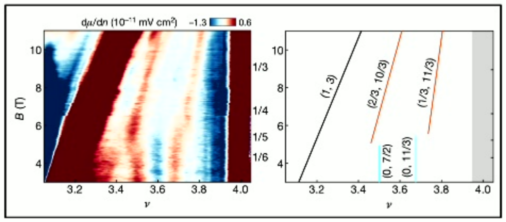
CONCLUSION AND FUTURE DIRECTIONS

Geometric responses (responses to crystal symmetry fluxes) can be understood int terms of theories of fermions in curves space

Useful for understanding known geometric responses, and predicting new geometric responses



Xie, et al. Nature 2021

Other Future Directions

- Geometric responses in strongly correlated systems
  - Non-Abelian lattice defects in 2D systems with topological order
  - Applications to recently discovered fractional Chern insulator states in MATBG
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- Incorporating translation symmetry fluxes
  - Adding dislocations in the lattice theory
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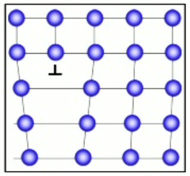

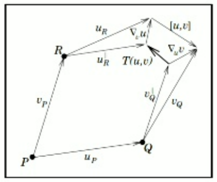

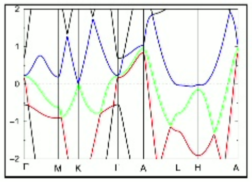

Xu, et al. Nature 2020

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JMM et al., Arxiv 2022

The $R \wedge F$-term in a fermionic theory

The $R\wedge F$-term in a fermionic theory

Two non-relativistic 3D Dirac fermions in curved space

Coupled to a non-dynamic scalar boson Φ

$$\mathcal{L} = \bar{\Psi}\left[i\,\Gamma^0 \otimes \sigma^0\, D_t + i\,\Gamma^i \otimes \sigma^0\, D_i + m(\cos(\Phi)\mathrm{I} \otimes \sigma^z + \sin(\Phi)\Gamma^4 \otimes \sigma^0)\right]\Psi$$

$$D_\mu = \partial_\mu - i\,A_\mu - i\,\omega_\mu\left(\tfrac{i}{4}[\Gamma^1, \Gamma^2] \otimes \sigma^0 + \tfrac{1}{2}\mathrm{I} \otimes \sigma^z\right)$$

$$i\int dtd\boldsymbol{x}\,\mathcal{L}_{eff}\left[A_\mu, \omega_\mu, \Phi\right] = \log\int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{i\int dtd\boldsymbol{x}\mathcal{L}\,[\bar{\Psi},\Psi,A_\mu,\omega_\mu,\Phi\,]}$$

$$\mathcal{L}_{eff}\left[A_\mu, \omega_\mu, \Phi\right] = \frac{\Phi}{16\pi^2}\epsilon^{\mu\nu\lambda\rho}R_{\mu\nu}F_{\lambda\rho} + \ \ldots$$

JMM et al., Arxiv 2022

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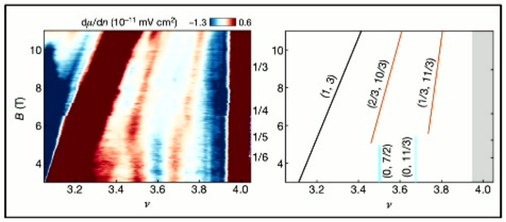

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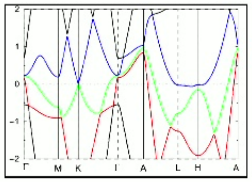
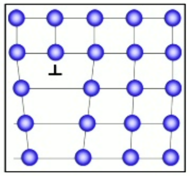
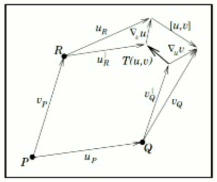


Xu, et al. Nature 2020





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