Title: Towards a non-relativistic AdS/CFT duality

Speakers: Andrea Fontanella

Series: Quantum Fields and Strings

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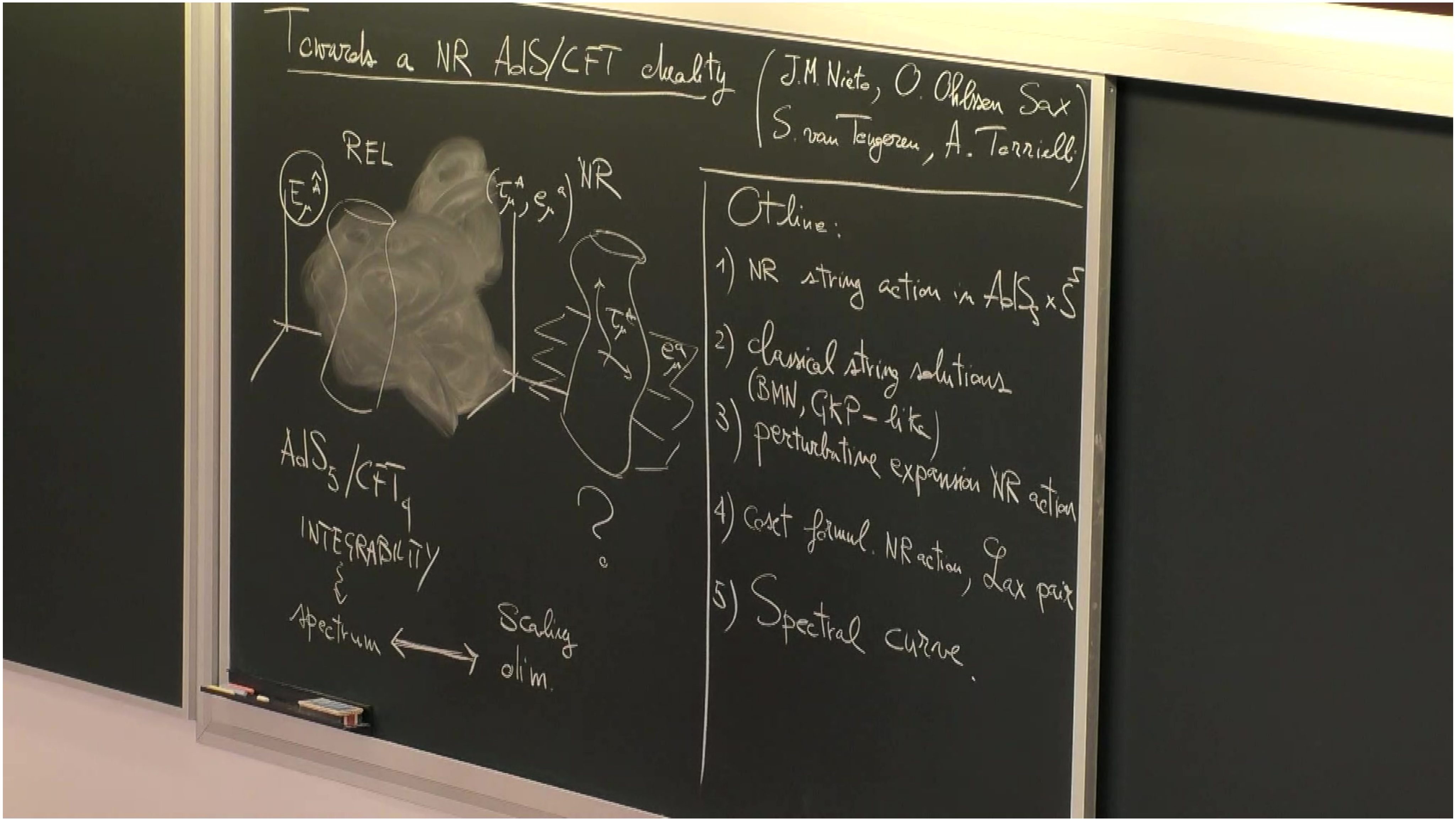
Abstract: The background geometry seen by a propagating string in the non-relativistic limit is non-Lorentzian. This motivates us to study the non-relativistic AdS/CFT correspondence as a new example of non-AdS holography. In this talk I will focus on the string side of the correspondence, keeping an integrability perspective, and illustrate remarkable differences from the relativistic theory. I will report on recent progress made in AdS5xS5 non-relativistic strings, in particular regarding their classical string solutions, semi-classical expansion of the action, coset space formulation of the action, Lax pair and some preliminary progress on the spectral curve. Based on work done in collaboration with J. M. Nieto, O. Ohlsson Sax, A. Torrielli, S. Van Tongeren.

Zoom link: https://pitp.zoom.us/j/99895811528?pwd=YWIreWtzdmRBanpIZXBLLzY3ZFNoZz09


Towards a NR AdS/CFT duality
(J.M. Nieto, O. Ohlsson Sax
S. van Tongeren, A. Torrielli)
REL
NR
AdS5/CFT4
INTEGRABILITY
Spectrum
Scaling
dim.
?
Outline:
1) NR string action in AdS5 x S5
2) classical string solutions
(BMN, GKP-like)
3) perturbative expansion NR action
4) Coset formul. NR action, Lax pair
5) Spectral curve.

1) Flat space

$$S = \int \partial X^\mu \partial X^\nu \eta_{\mu\nu}$$

[Gomis, Ooguri, 2000] : $X^0 \longrightarrow c X^0,\ X^1 \longrightarrow c X^1$

$c \longrightarrow \infty$

Curved spacetime $\quad S = \int \partial X^\mu \partial X^\nu g_{\mu\nu}$

rescaling ?

Homogeneous Space = G/H $\quad$ (e.g. $AdS_5 \times S^5$)

$$S = \int \langle A, PA \rangle$$

proj. into G/H

$A = g^{-1} dg$

$g \in G$

(J.M. Nieto, O. Ohlsson Sax, S. van Tongeren, A. Torrielli)

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time expansion NR action

NR action, Lax pair

curve.

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proj. into G/H

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$g \in G$

Choice of g = Choice of coords

(e.g. $g = e^{X \cdot T}$)

NR limit: $so(2,4) \oplus so(6) \longrightarrow$ st Newton Hooke$_5$ $\oplus$ Ecl$_5$

IW: transl. "2+8": $P_0 \to \frac{P_0}{c}$, $P_1 \to \frac{P_1}{c}$

resc. generators $\longleftrightarrow$ resc. coords.

$E_\mu{}^{\hat{A}}$

long $E_\mu{}^A = c\tau_\mu{}^A + \frac{1}{c} m_\mu{}^A$ $A = 0, 1$

transv. $E_\mu{}^a = e_\mu{}^a$

$a = 2, \dots, D-1$

(J.M Nieto, O. Ohlsson Sax, S. van Tongeren, A. Torrielli)

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trans $E_\mu^a = e_\mu^a$ $a = 2, \dots, D-1$

$g_{\mu\nu} = -c^2 \tau_\mu^A \tau_\nu^B \eta_{AB}$ + finite

$B_{\mu\nu} = c^2 \varepsilon_{AB} \tau_\mu^A \tau_\nu^B$

Divergent

$\Rightarrow$ st Newton Hooke$_5$ $\oplus$ Eucl$_5$

$\to \frac{P_0}{c}$, $P_1 \to \frac{P_1}{c}$

$A = 0, 1$

$a = 2, \dots, D-1$

finite

Divergent$(g_{\mu\nu} + B_{\mu\nu})$ = finite(λ_A) + subleading

$c \to \infty$ $\qquad$ λ_A: 2 w.s. scalars.

$$\boxed{S^{NR} = \int d^2\sigma\, \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu H_{\mu\nu} + \varepsilon^{\alpha\beta}\left(\lambda_+ e_\alpha^+ \tau_\mu^+ + \lambda_- e_\alpha^- \tau_\mu^-\right) \partial_\beta X^\mu}$$

$AdS_5 \times S^5 = (\underbrace{t, z}_{\text{long.}}, \underbrace{X^m) \oplus (y^i}_{\text{transv.}})$

$\tau_\mu{}^A$ spans AdS_2

$e_\mu{}^a$ " $f(z)\, \mathbb{R}^3 \times \mathbb{R}^5$

2) $\varepsilon^{\lambda_\pm} = 0$

$t = \kappa \tau \qquad z = 2 \tan\left(-\frac{\kappa}{2}\sigma\right)$

<u>winding</u>

leading
colors.

$\ldots^{+} + \lambda_- e_\alpha^- \tau_\mu^-)$
$\partial_\beta X^\mu$

BMN-like

$t = \kappa\tau$, $\boxed{z = 2\tan\left(-\frac{\kappa}{2}\sigma\right)}$

$\phi = \omega\tau$, $\lambda_\pm \sim \cos(\kappa\sigma)$

$E \sim J^2$

(GKP)

REL BMN

$t = \phi = \kappa\tau$

$E = J$

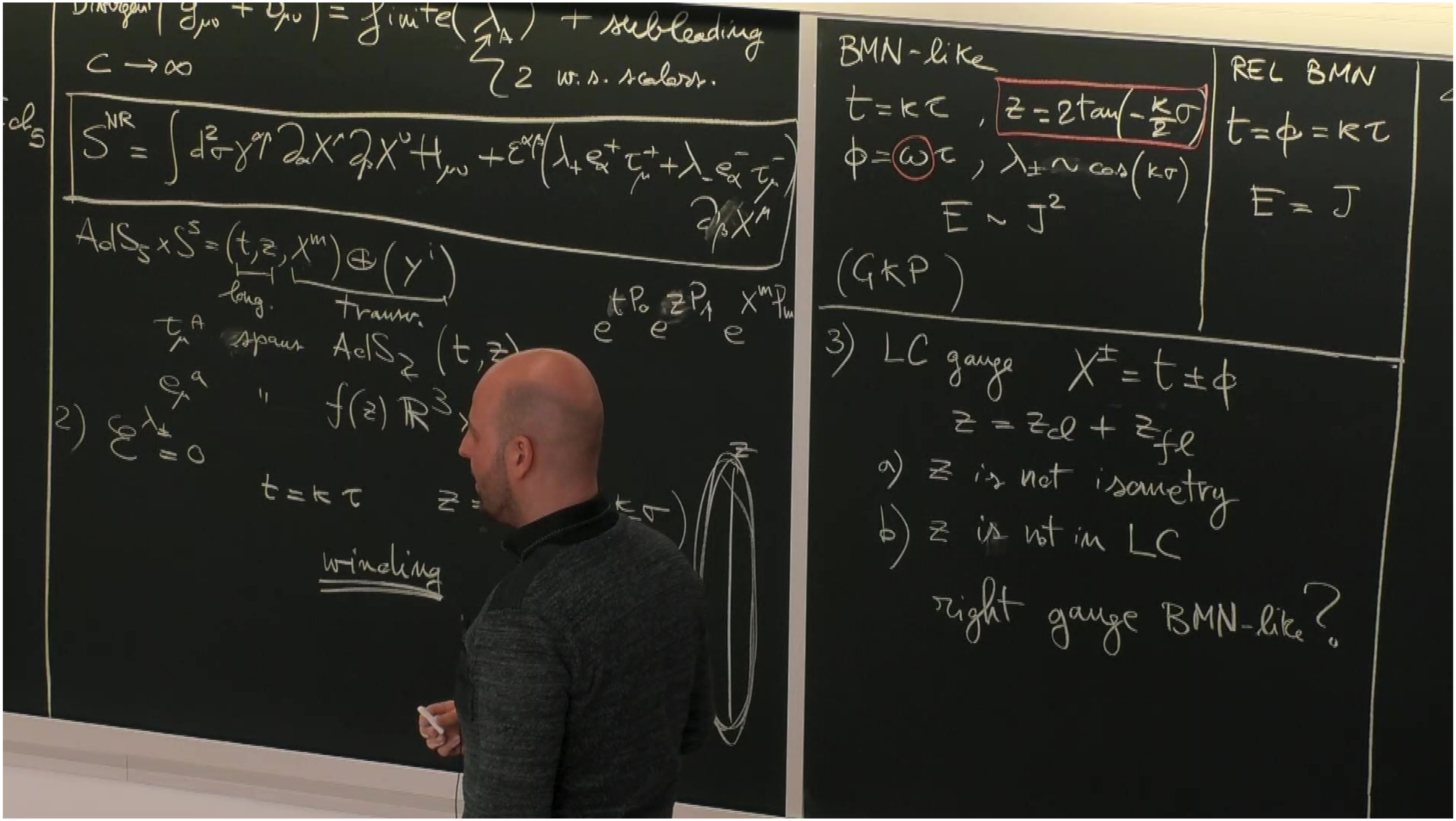
3) LC gauge $X^\pm = t \pm \phi$

$z = z_{cl} + z_{fl}$

a) z is not isometry

b) z is not in LC

right gauge BMN-like?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$AdS_5 \times S^5 = (t, z, X^m) \oplus (y^i)$
long.
transv.
τ_μ^A spans AdS_2 (t,z)
e_μ^a " $f(z)\,\mathbb{R}^3$
2) $\varepsilon^{\lambda_\pm} = 0$
$e^{tP_0} e^{zP_1} e^{X^m P_m}$
$t = k\tau$
winding
BMN-like
$t = k\tau$, $z = 2\tan(-\frac{k}{2}\sigma)$
$\phi = \omega\tau$, $\lambda_\pm \sim \cos(k\sigma)$
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4) NR $AdS_5 \times S^5 = \tilde{G}/\tilde{H}$

$$S = \int \langle J, PJ \rangle$$

Generalized MC $J = A + *_2 \Lambda$ $\lambda_{\pm}$

$$\mathcal{L}ax^{NR} = \mathcal{L}ax^{BRP}\left(A \to J \text{ in one place}\right)$$

$$\begin{bmatrix} \text{Flat } \mathcal{L}ax^{NR} \\ \mathcal{E}^{\lambda_{\pm}} = 0 \end{bmatrix} \longleftrightarrow \mathcal{E}_{.c.m.}$$

$$\left(\mathcal{E}^{\lambda_{\pm}} \sim A^{+\!\!+} = 0\right)$$

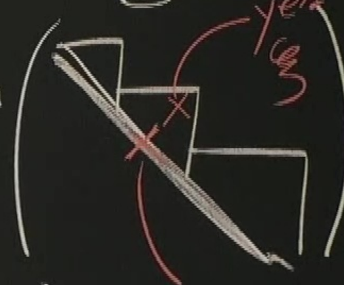
5) Spectral curve.

$$\mathcal{M} = P\exp\left(\int_0^{2\pi} d\sigma\, \mathcal{L}^{NR}(\xi)\right)$$

spectral par.

Th: On $\mathcal{E}^{\lambda_\pm}=0$, eigenvalues of $\mathcal{M}$ are ξ- indep.

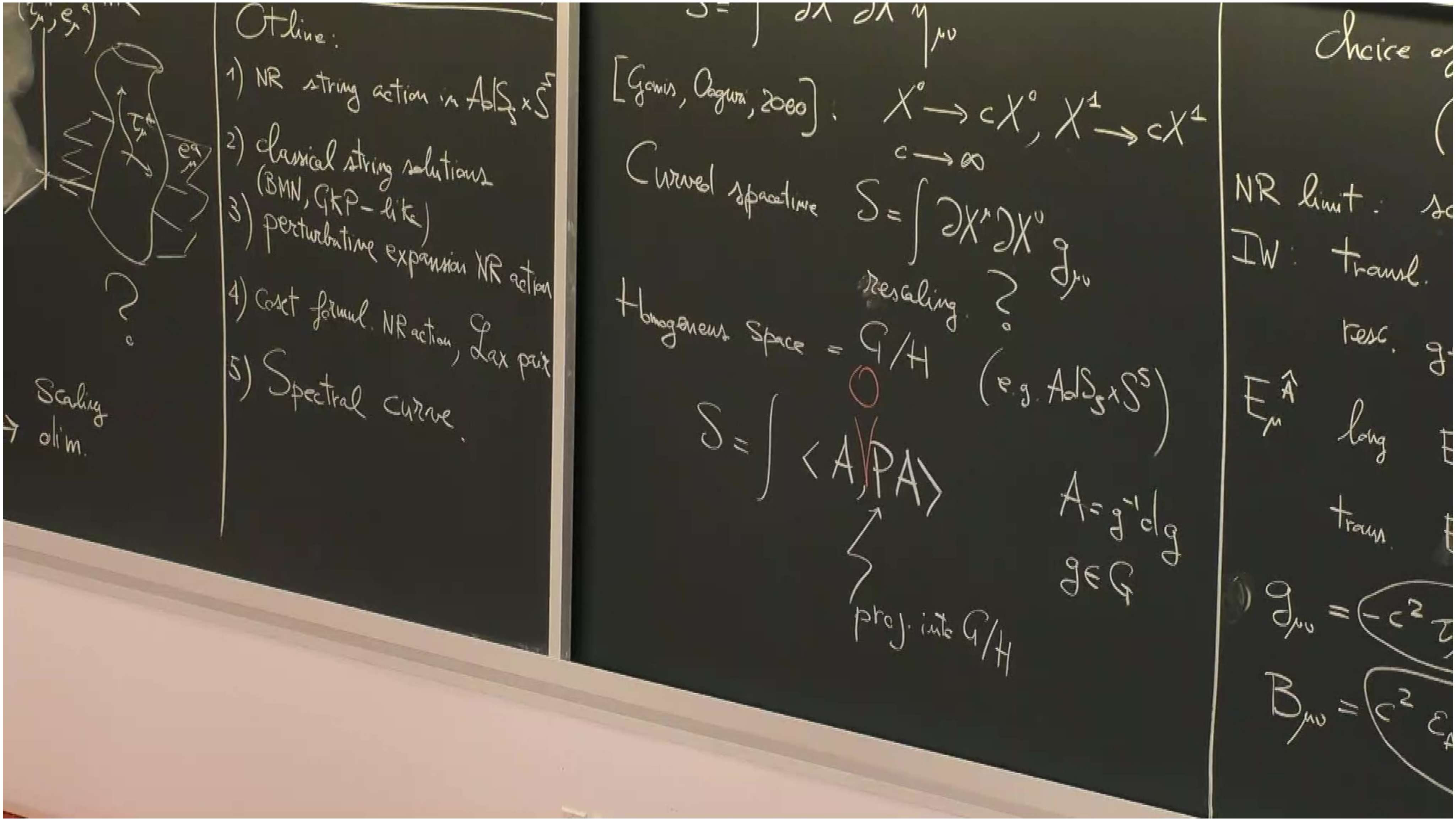
$$\mathcal{M}\Big|_{\text{BMN-like}} = \text{NON-DIAGON.} = U^{-1}$$



U

DIAG: $\mathcal{M} = U^{-1} e^{p_i C_i} U^{-1}$ $\{C_i\}$ Cartan no ξ

NON-DIAG: $\mathcal{M} = U^{-1} e^{q_i(\xi) W_i} U^{-1}$ $\{W_i\}$ MAS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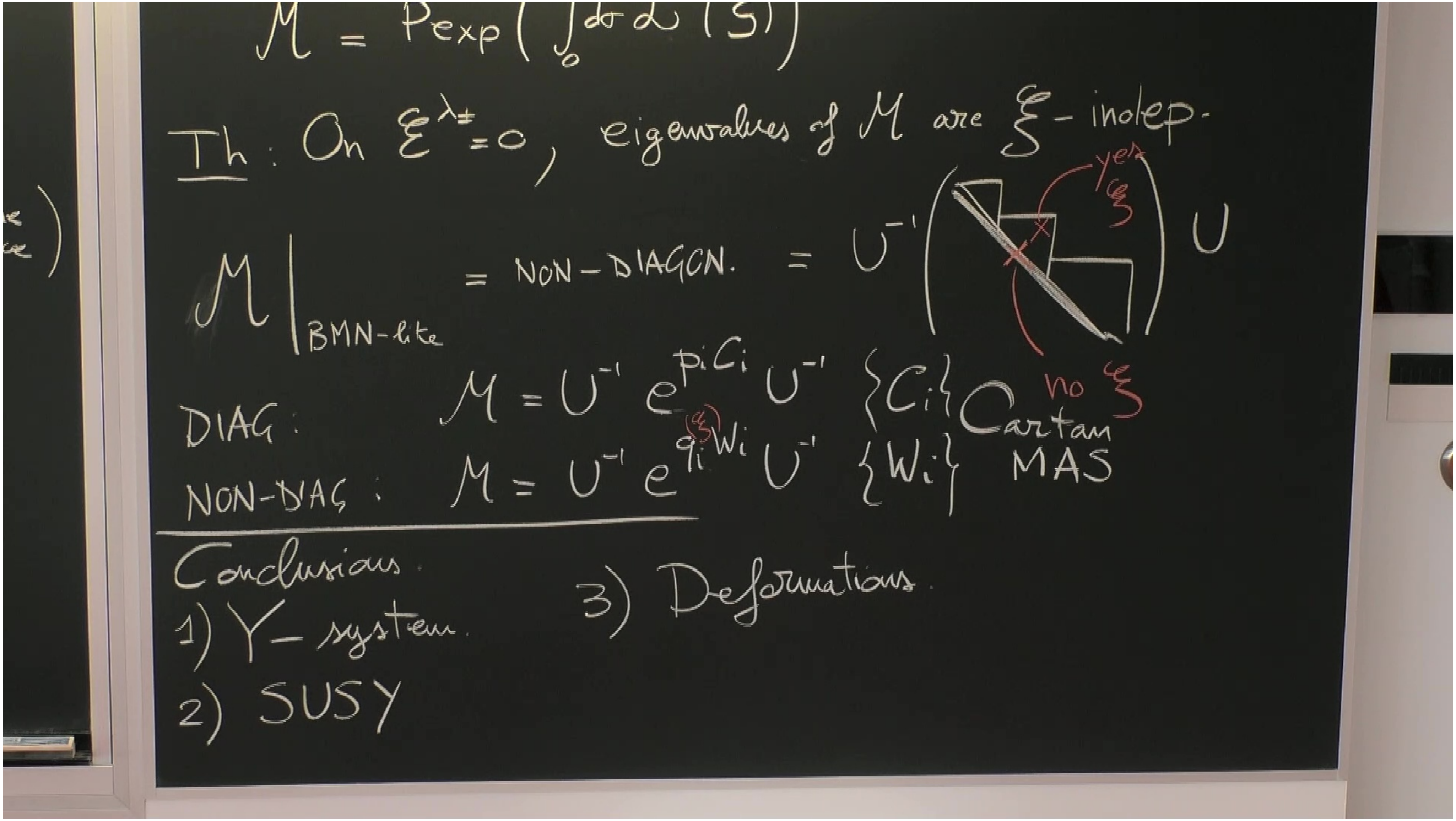
$B_{\mu\nu} = c^2$


$\mathcal{M} = P\exp\left(\int_0 d\sigma\, \mathcal{L}(\xi)\right)$
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$\mathcal{M}\big|_{\text{BMN-like}}$ = NON-DIAGON. = $U^{-1}(\ )U$
yes ξ
no ξ
DIAG: $\mathcal{M} = U^{-1} e^{p_i C_i} U^{-1}$ $\{C_i\} \subset$ Cartan
NON-DIAG: $\mathcal{M} = U^{-1} e^{q_i(\xi) W_i} U^{-1}$ $\{W_i\}$ MAS
Conclusions.
1) Y-system.
2) SUSY
3) Deformations.