Title: Constructing Conformal Bootstrap Equations from the Embedding Space OPE Formalism

Speakers: Valentina Prilepina

Series: Quantum Fields and Strings

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Abstract: In this talk, I will describe how to implement the conformal bootstrap program in the context of the embedding space OPE formalism introduced by Fortin and Skiba (DOI:10.1007/JHEP06(2020)028). To begin with, I will give some background on the formalism. In particular, I will map out how to build two-, three-, and four-point functions within this framework. I will then lay out how to construct tensorial generalizations of the well-known scalar four-point blocks for symmetric traceless exchange. As I will discuss, these generalized objects satisfy a number of contiguous relations. Together, these empower us to fully contract the four-point tensorial blocks, ultimately yielding finite spin-independent linear combinations of four-point scalar blocks potentially acted upon by first-order differential operators. I will next proceed to describe how to set up the conformal bootstrap equations directly in the embedding space. I will begin by mapping out a general strategy for counting the number of independent tensor structures, which leads to a simple path to generating the bootstrap equations. I will then examine how to implement this method to construct the two-point, three-point, and ultimately four-point conformal bootstrap equations. Lastly, I will illustrate this method in the context of a simple example.

Zoom link: https://pitp.zoom.us/j/98920533892?pwd=cDQvOExJWnBsUWNpZml5S1cxb0FJQT09

CB Eqns from Emb Space OPE

CB program → symm

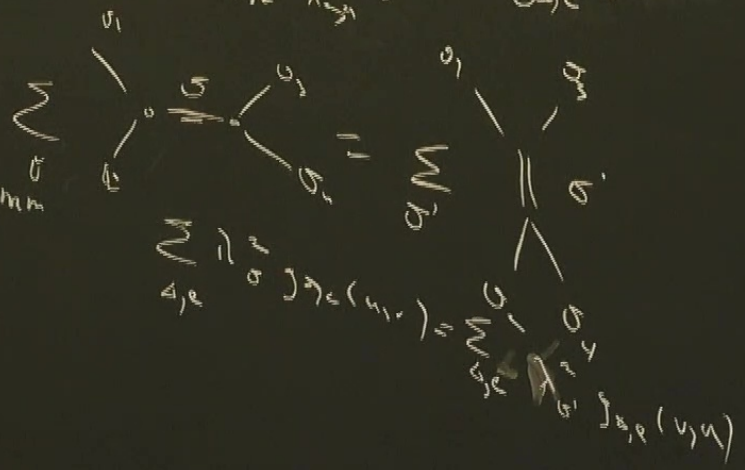
→ consistency conds

$$\langle \sigma(x_1)\sigma(x_2)\sigma(x_3)\sigma(x_4)\rangle = \sum_{\Delta,\ell} \lambda_{\sigma\sigma}^{\ 2}\, \frac{g_{\Delta,\ell}(u,v)}{x_{12}^{2\Delta_\sigma} x_{34}^{2\Delta_\sigma}}$$

CB Program

Compose crossing symm

$x \leftrightarrow x_3 \Rightarrow$ cross symm



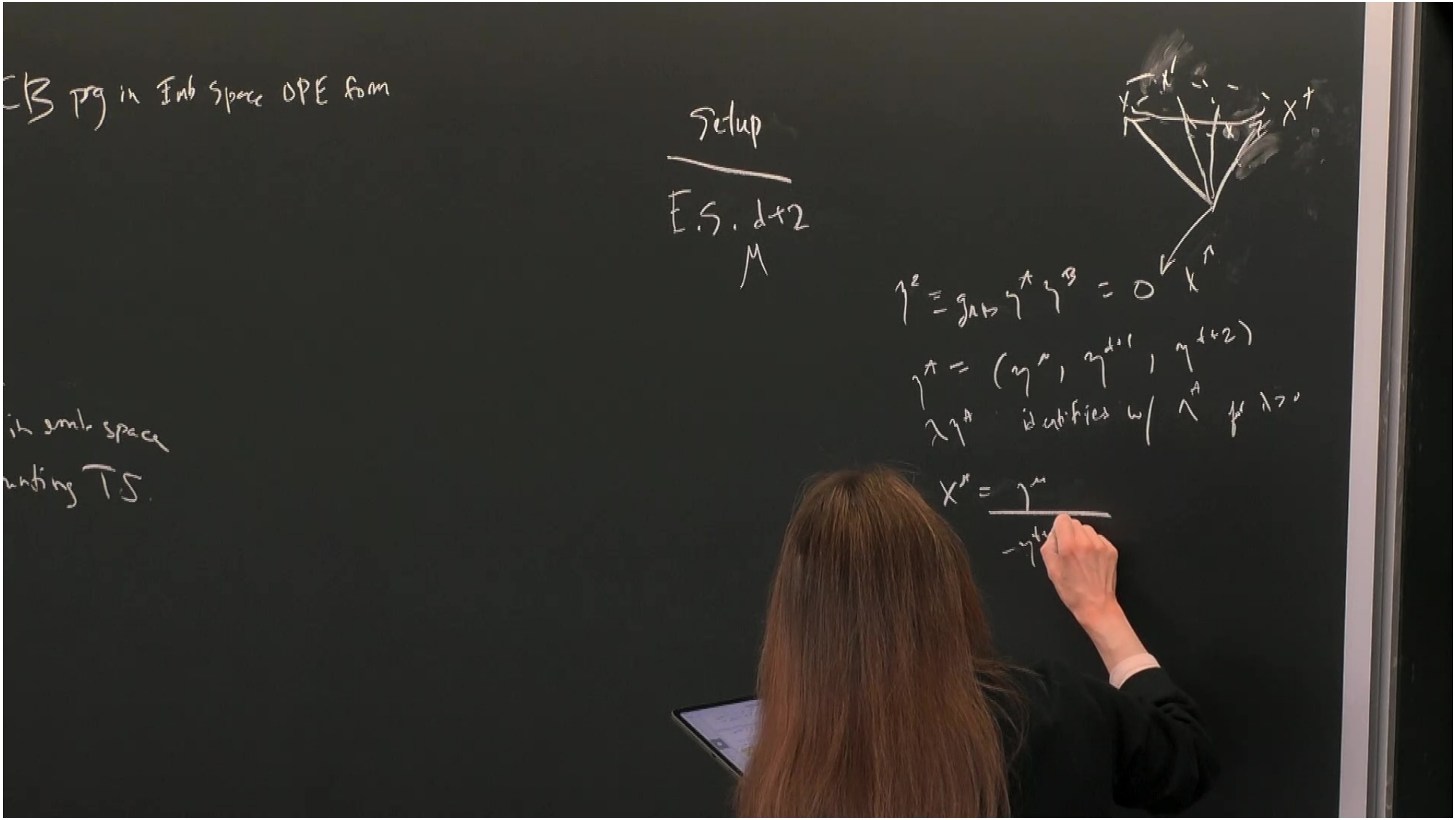
Use $\epsilon \circ \kappa \circ \sigma$

Goal of Talk:

* Describe how to implement CB prg in Fnb Space OPE form

Outline

* Background
* Tensorial gen of $G_{\mathbb{C}}(u,v)$ due STT exchange
* How to set up CB eqs in emb space
* Map out a strategy for counting TS
* Construct CB eqs
* Simple ex

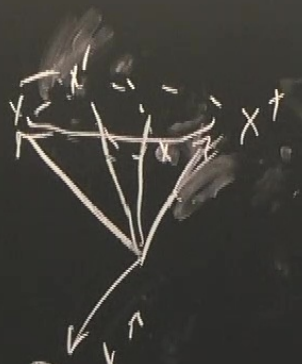

CB prg in Emb Space OPE form
Setup
E.S. d+2
M
in emb. space
unting TS.
x^+
$y^2 = g_{AB} y^A y^B = 0$
$y^A = (y^\mu, y^{d+1}, y^{d+2})$
λy^A identifies w/ y^A for $\lambda > 0$
$X^\mu = \frac{y^\mu}{-y^+}$

CB prg in Emb Space OPE form

Setup

E.S. $d+2$

M

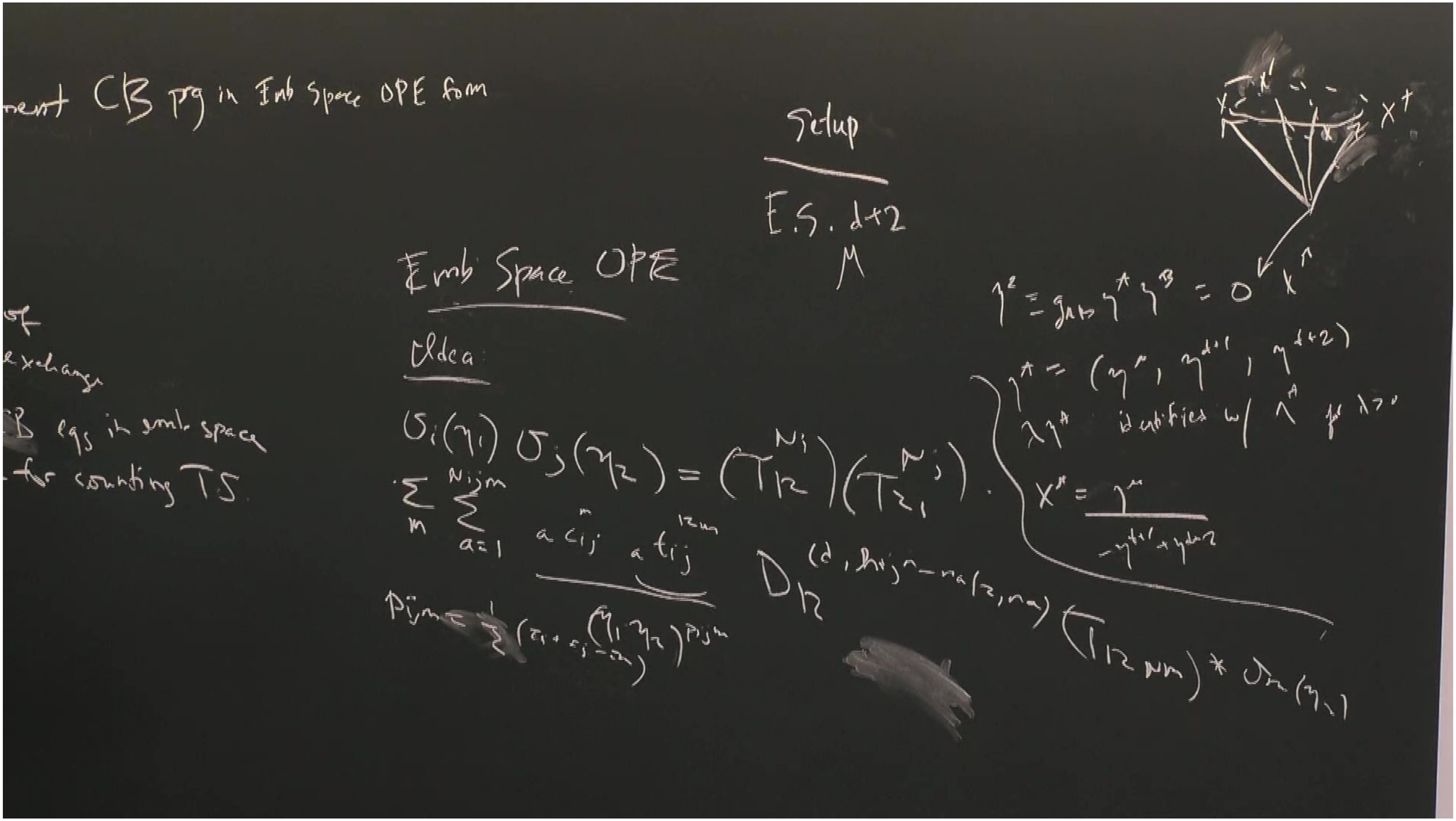


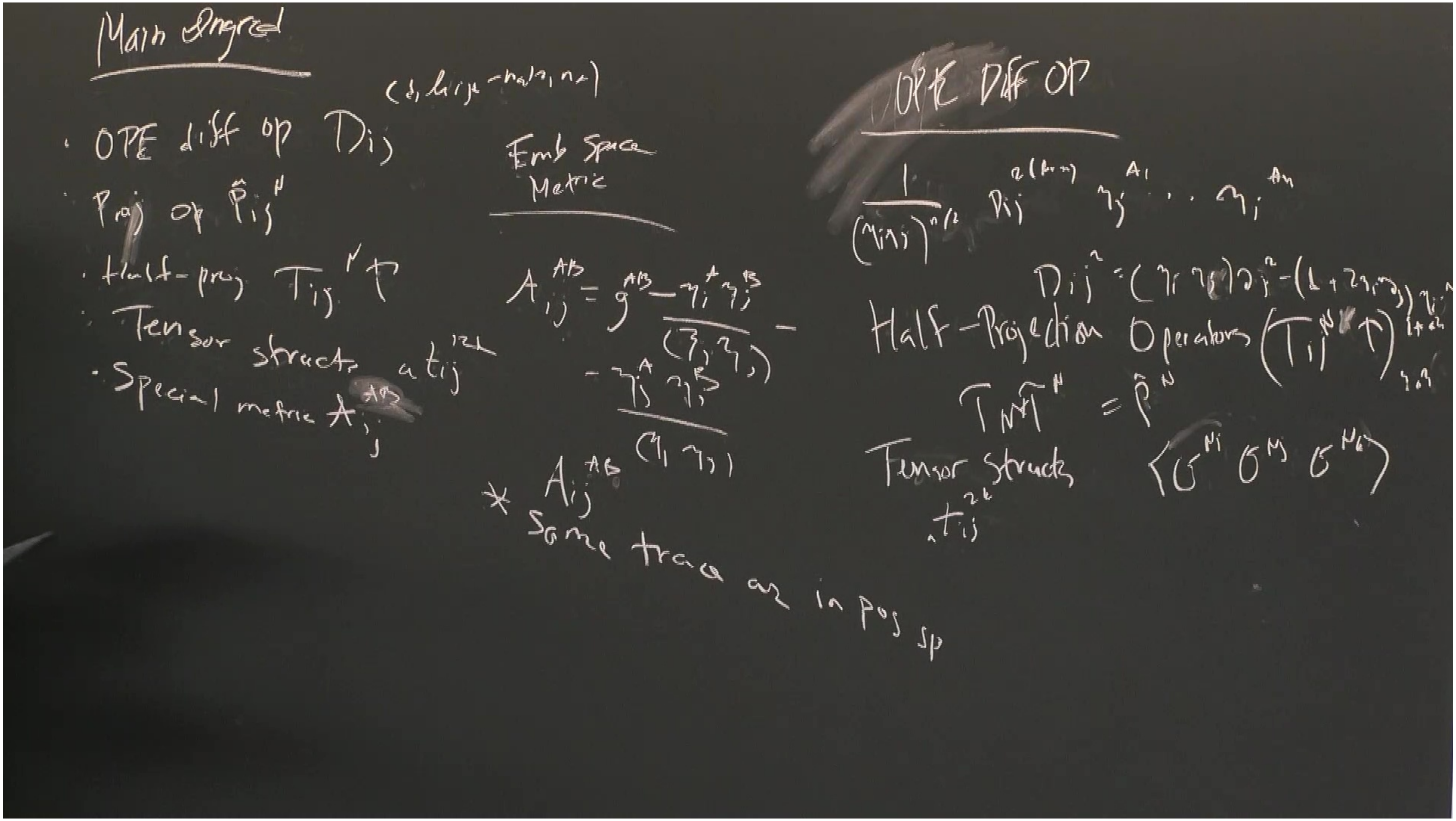
$\gamma^2 = g_{AB}\gamma^A\gamma^B = 0$ X^A

$\gamma^A = (\gamma^\mu, \gamma^{d+1}, \gamma^{d+2})$

$\lambda\gamma^A$ identifies w/ γ^A for $\lambda > 0$

$X^\mu = \frac{\gamma^\mu}{-\gamma^{d+1}+\gamma^{d+2}}$


ment CB pg in Emb Space OPE form
Setup
E.S. d+2
Emb Space OPE
Idea
eqs in emb. space
for counting TS
identified w/


Main Ingred
OPE diff op
Proj op
Half-proj
Tensor structs
Special metric
Emb Space
Metric
Same trace as in pos sp
OPE Diff Op
Half-Projection Operators
Tensor Structs

Main Ingred

$(\Delta_i, \ell_i, \ell_{ij} - n_{ij}, n_i)$

- OPE diff op $\mathcal{D}_{ij}$
- Proj op $\hat{\mathcal{P}}_{ij}^{N}$
- Half-proj $\mathcal{T}_{ij}^{N}\Gamma$
- Tensor structs t_{ij}^{12k}
- Special metric A_{ij}^{AB}

Building Blocks

$g^{\mu\nu}, \epsilon^{\mu_1 \cdots \mu_d}, \gamma^{\mu_1 \cdots \mu_n}$

$g^{\mu\nu} \to A_{12}^{AB}$

$\epsilon^{\mu_1 \cdots \mu_d} \to \epsilon_{12}^{A_1 \cdots A_d}$

$\gamma^{\mu_1 \cdots \mu_n} \to \Gamma_{12}^{A_1 \cdots A_n} \quad \forall\, n \in \{0, \ldots\}$

Emb Space Metric

$$A_{ij}^{AB} = g^{AB} - \frac{\eta_i^A \eta_j^B}{(\eta_i \cdot \eta_j)} - \frac{\eta_j^A \eta_i^B}{(\eta_i \cdot \eta_j)}$$

$*$ A_{ij}^{AB} Same trace as in pos sp

OPE Diff Op

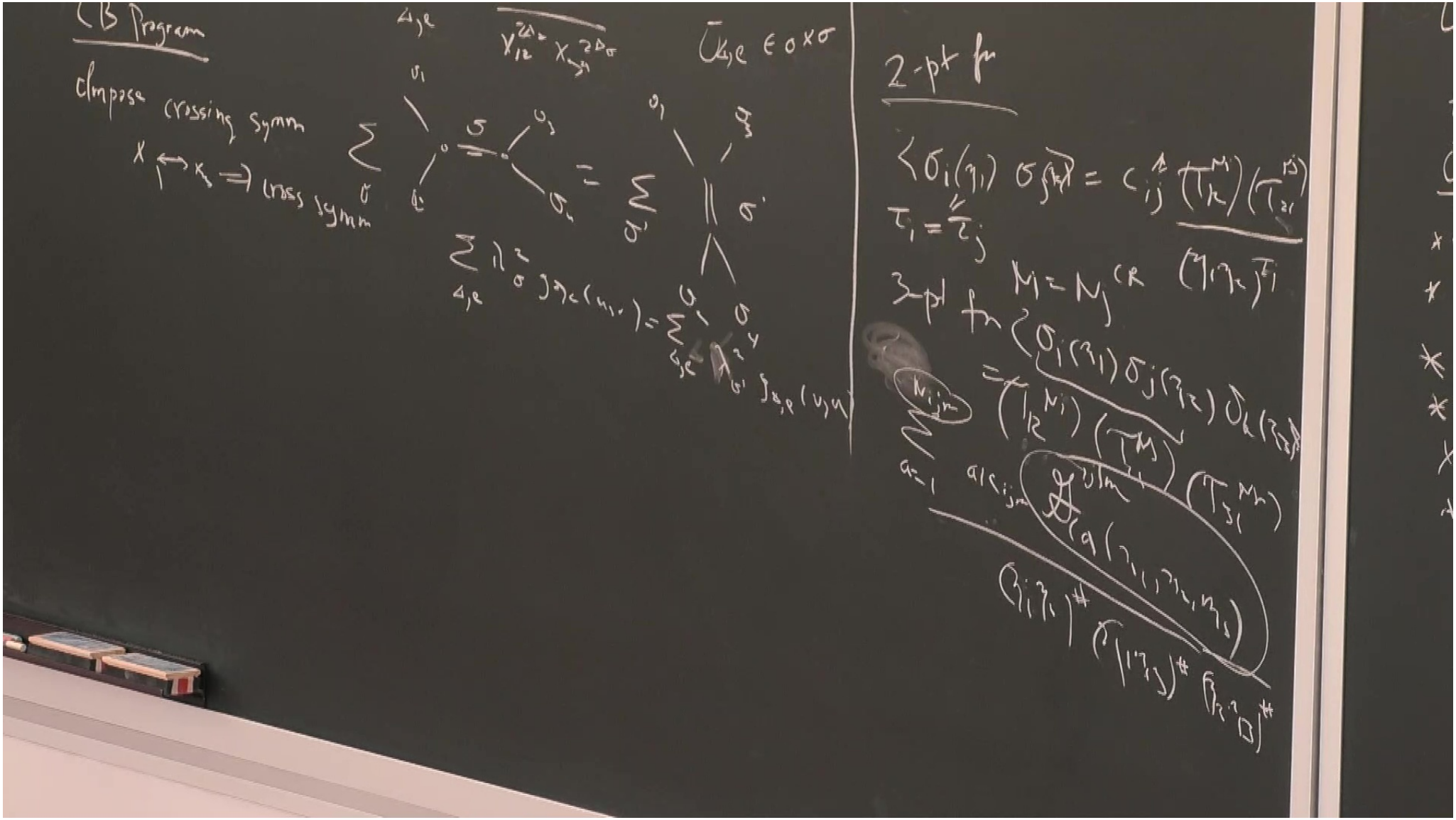
$$\frac{1}{(\eta_i \cdot \eta_j)^{n/2}} \mathcal{D}_{ij}^{2(\ldots)} \eta_j^{A_1} \cdots \eta_i^{A_n}$$

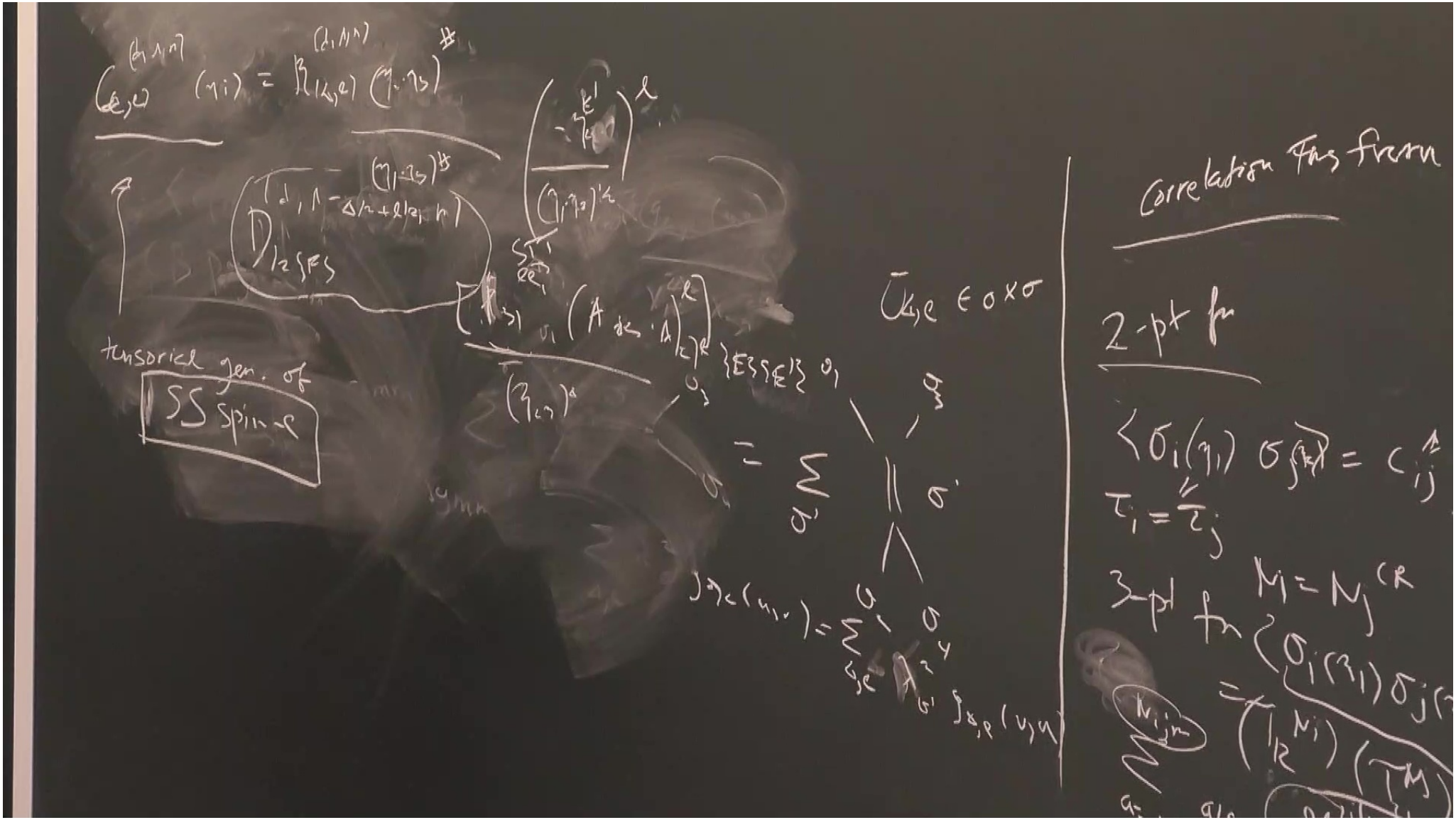
$$\mathcal{D}_{ij}^{A} = (\eta_i \cdot \eta_j)\partial_j^A - (d + 2\eta_i \cdot \partial_j)\eta_i^A$$

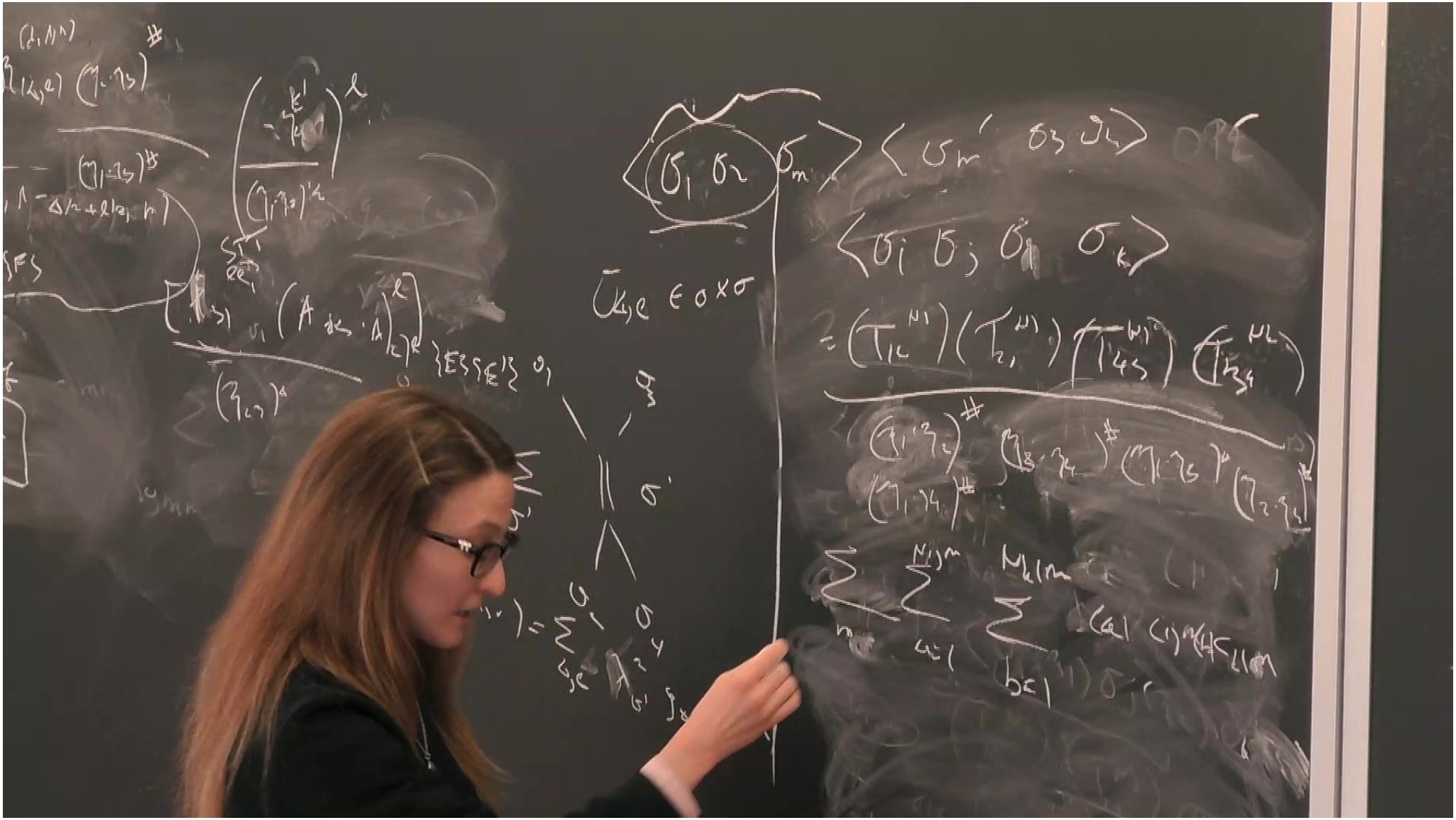
Half-Projection Operators $\left(\mathcal{T}_{ij}^{N}\Gamma\right)_{\eta_i}^{\ldots}$

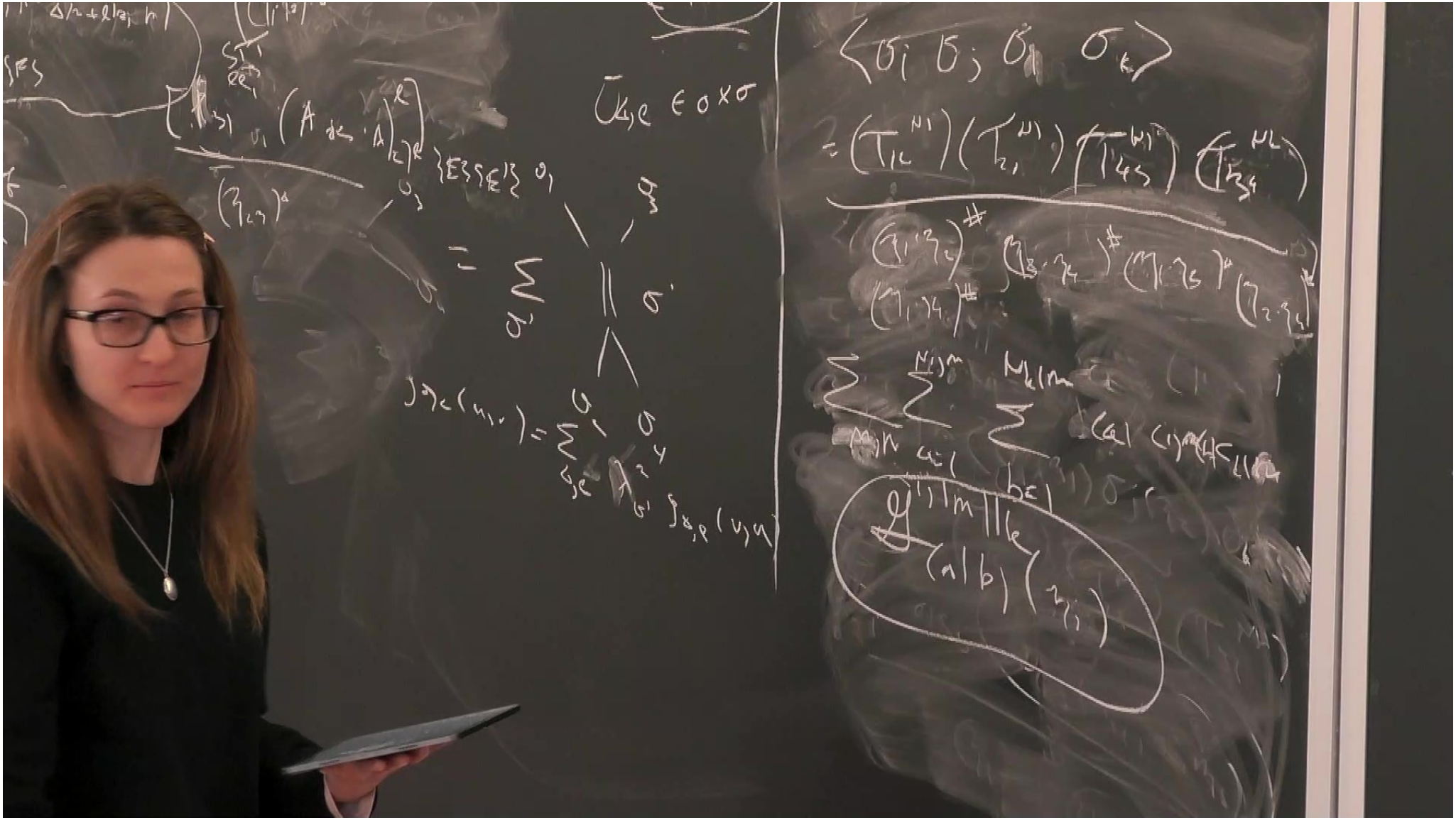
$\mathcal{T}_N \mathcal{T}^N = \hat{\mathcal{P}}^N$

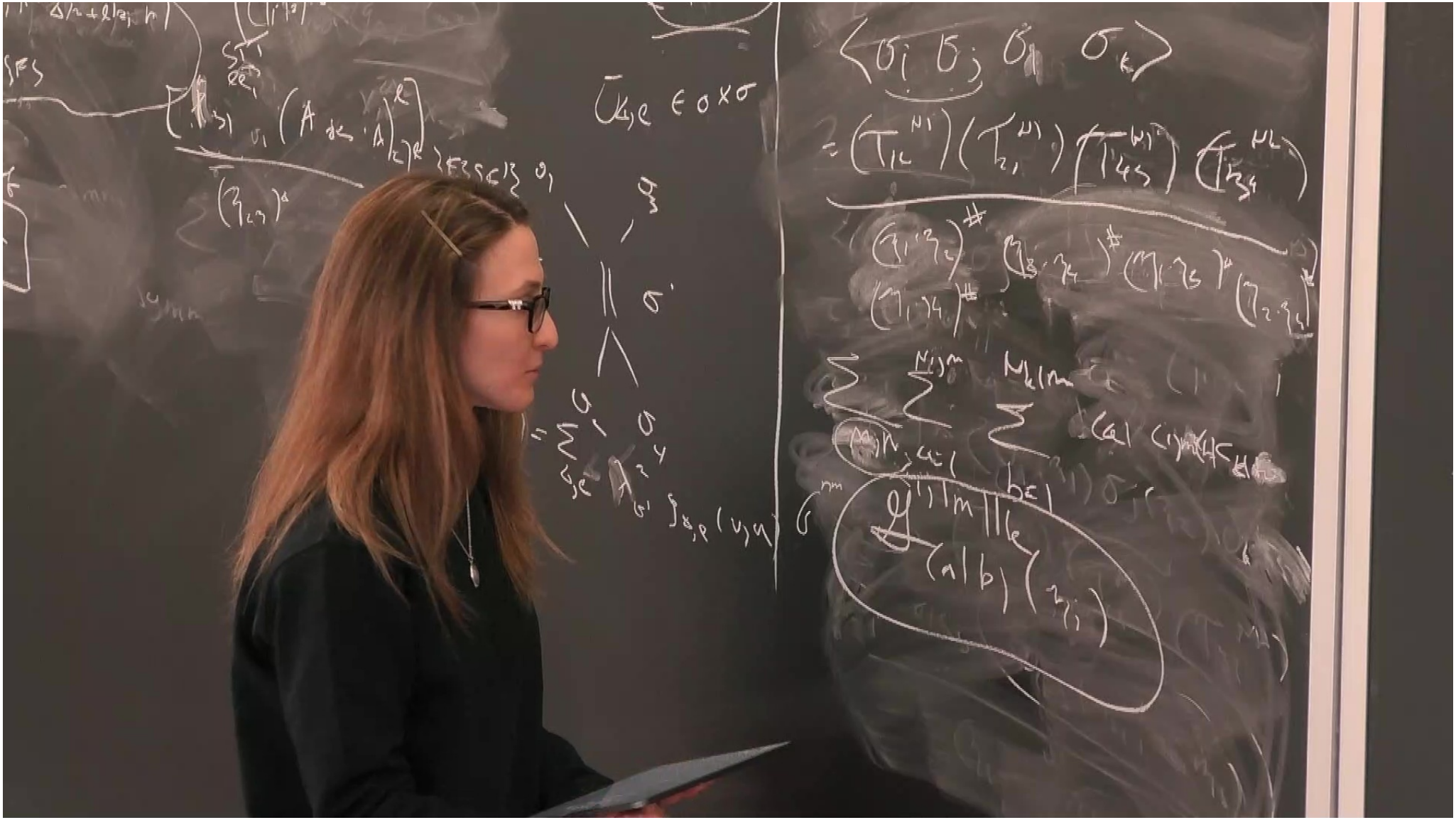
Tensor Structs t_{ij}^{2k} $\quad (\sigma^{\mu_1} \sigma^{\mu_2} \sigma^{\mu_3})$

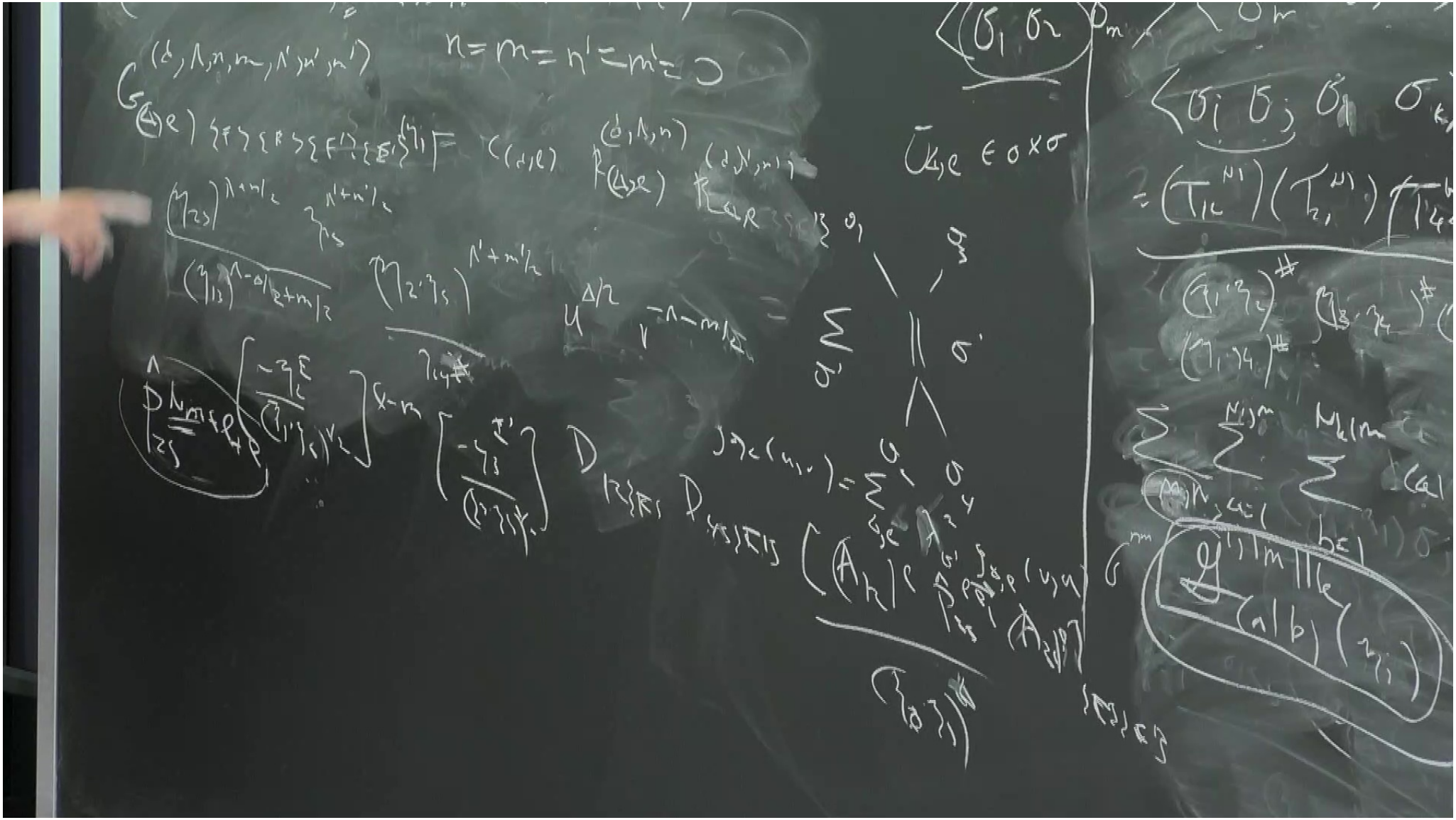

CB Program
compose
crossing Symm
cross symm
2-pt fn
3-pt fn

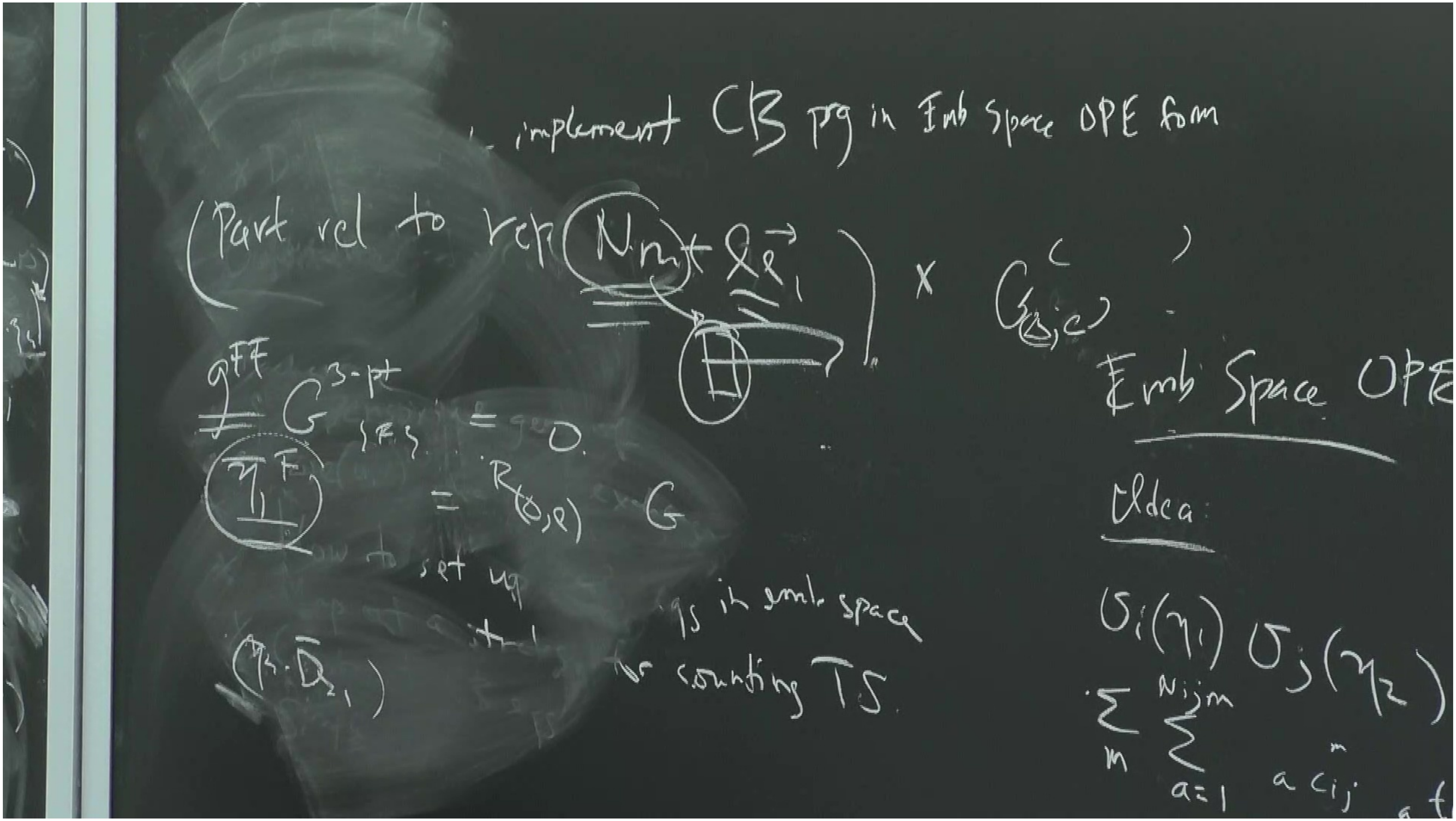

tensorial gen. of
SS spin-s
Correlation fns from fusion
2-pt fn
3-pt fn

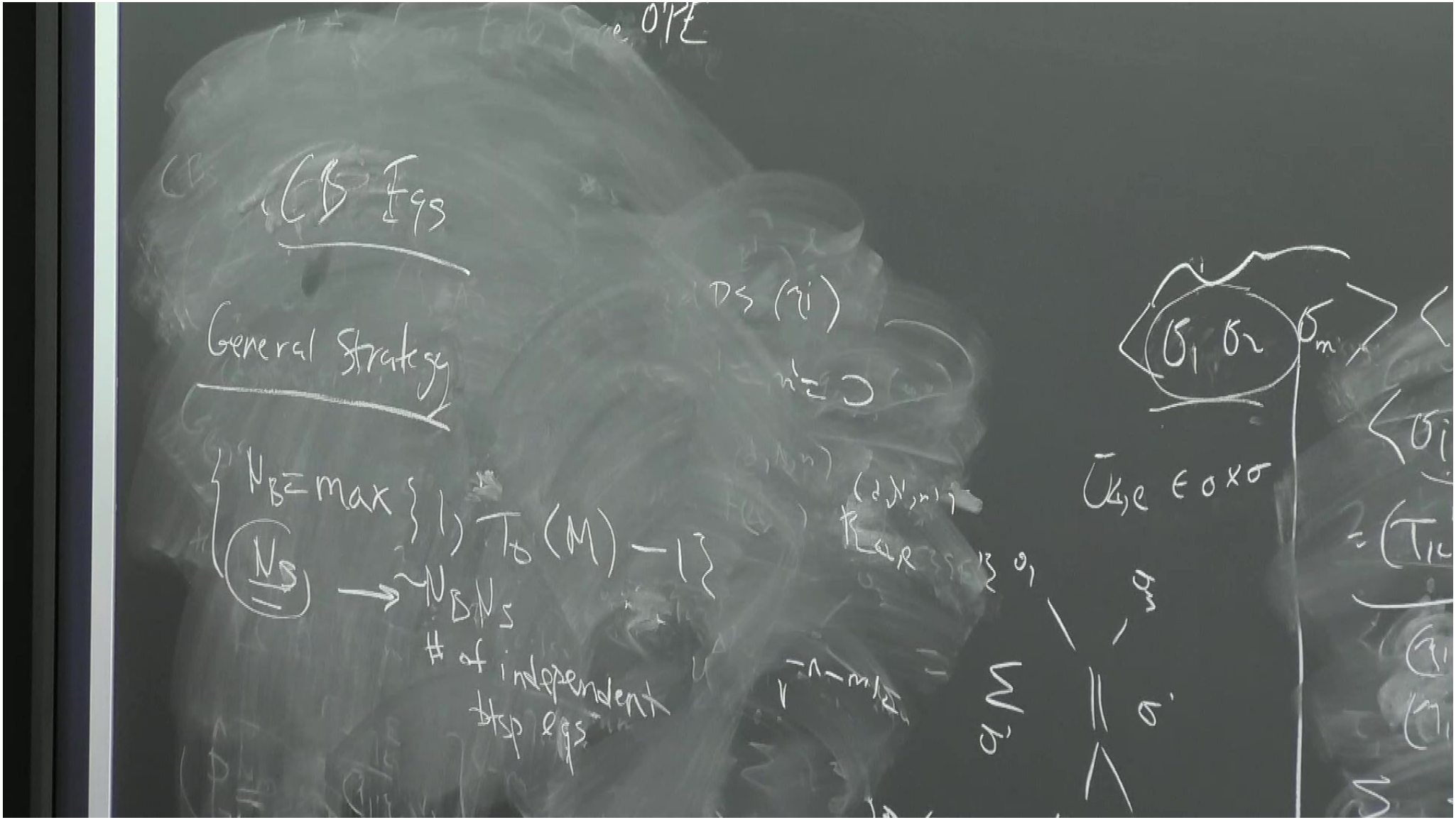


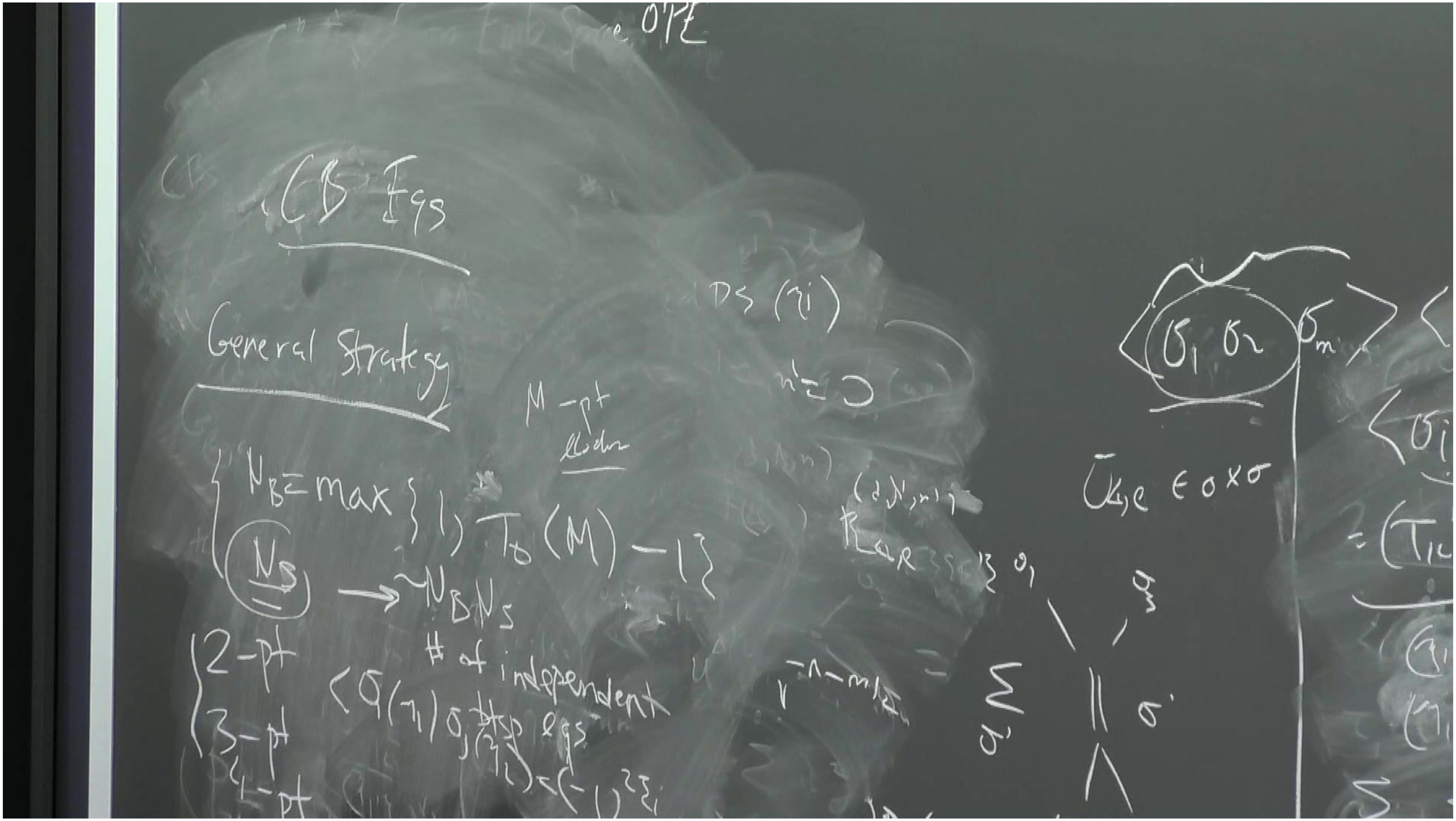


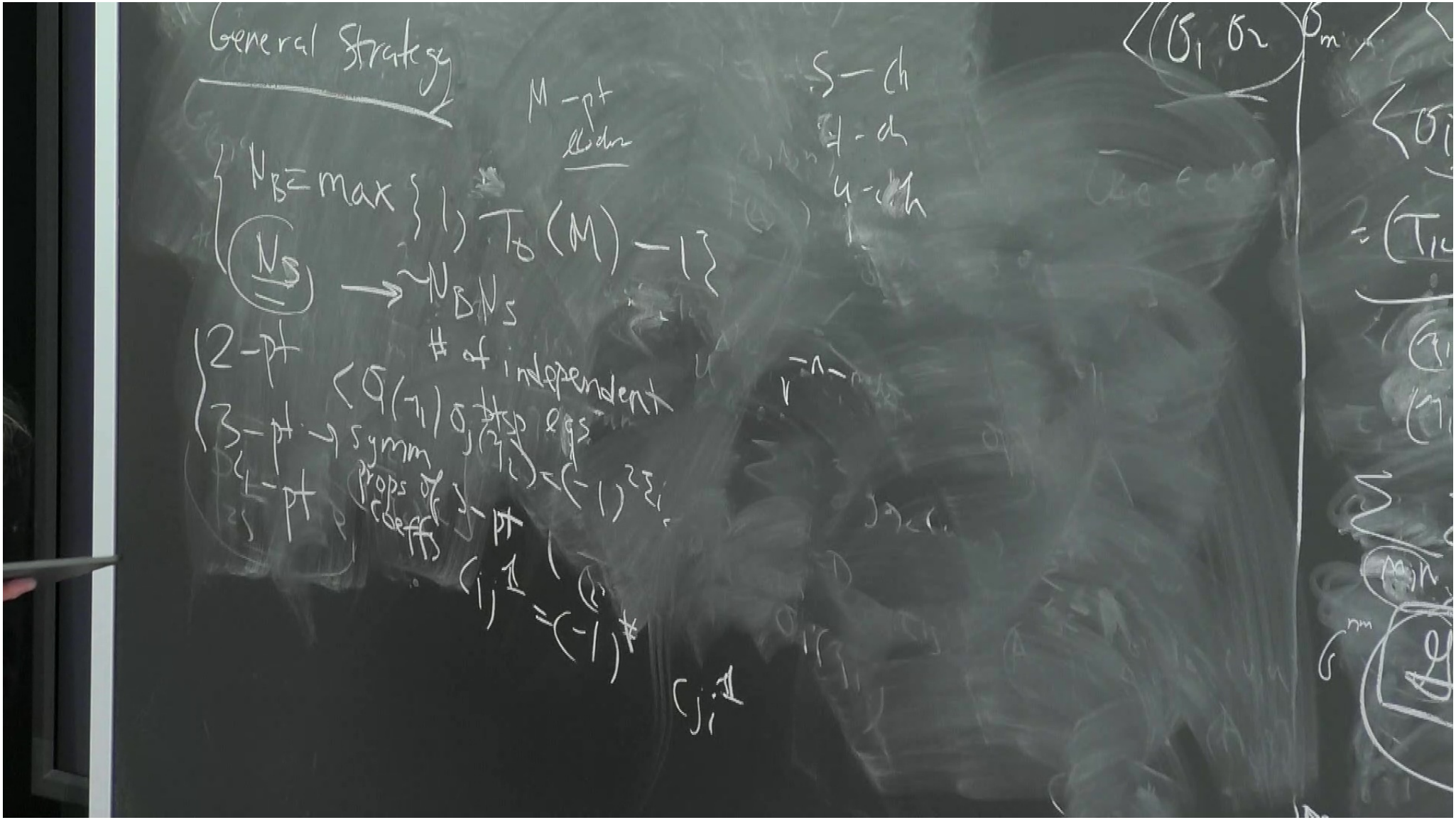


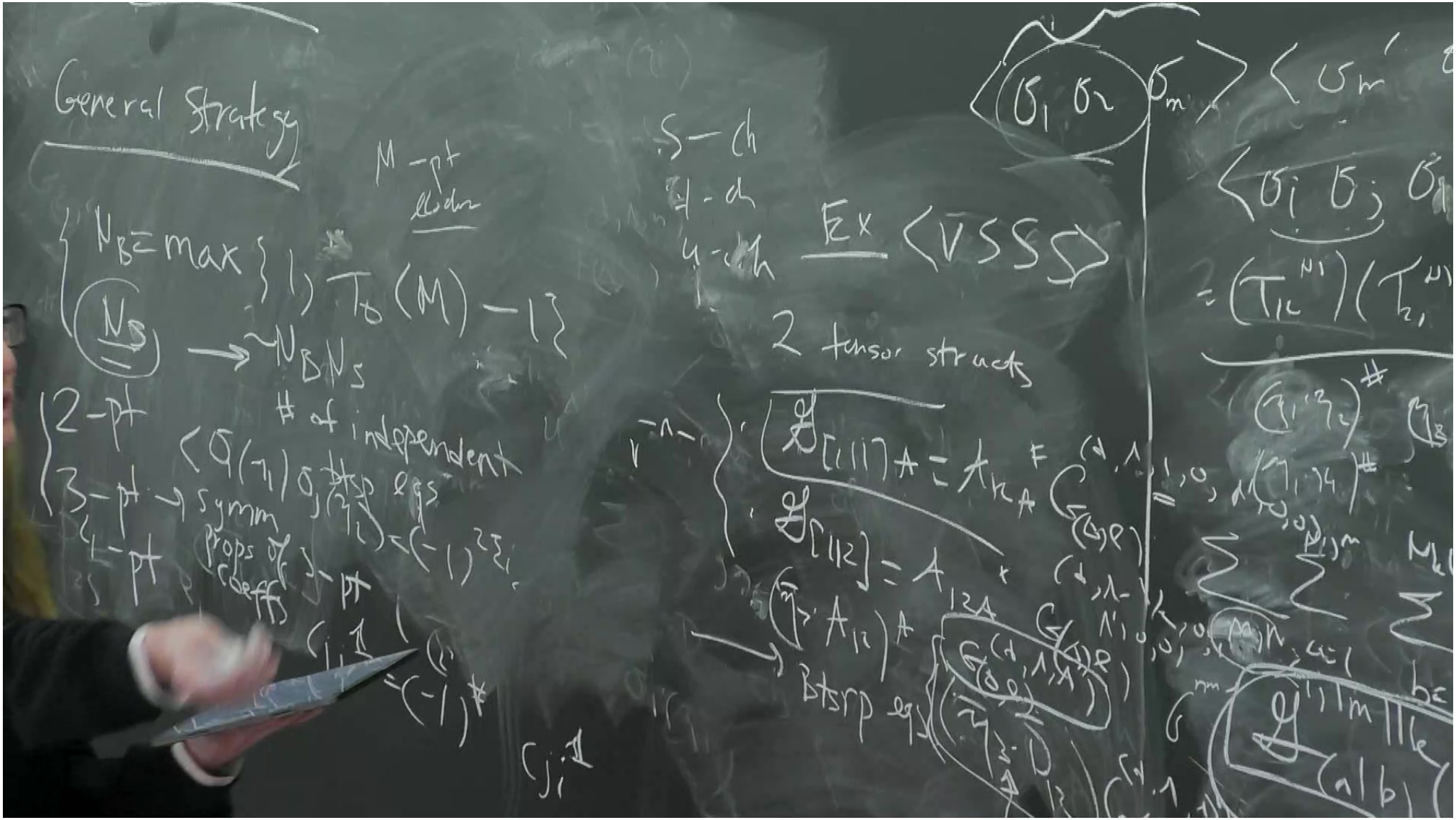



implement CB prg in Emb Space OPE form
(Part rel to rep
Emb Space OPE
Idea:
set up
in emb space
counting TS


OPE
CB Eqs
General Strategy
$N_B = \max\{1, T_0(M) - 1\}$
$N_S \to N_D N_S$
# of independent
btsp eqs
$D_S(\eta_i)$
$\eta_i = 0$
σ_1 σ_2 σ_m
$\bar{U}_{ij} \in O \times O$
σ_i
σ_j


CB Eqs
General Strategy
N_B = max{1, T_0(M) - 1}
N_S
~N_B N_S
# of independent
M-pt
2-pt
3-pt
4-pt


General Strategy
M-pt
eldim
$N_B = \max\{1, T_0(M) - 1\}$
N_S
$\sim N_B N_S$
# of independent
2-pt
3-pt
4-pt
symm
props of
coeffs
s-ch
t-ch
u-ch


General Strategy
M-pt
Ex ⟨VSS⟩
2 tensor structs
Btsrp eqs