Title: Pushing the frontiers of gravitational encounters and collisionless dynamics

Speakers: Uddipan Banik

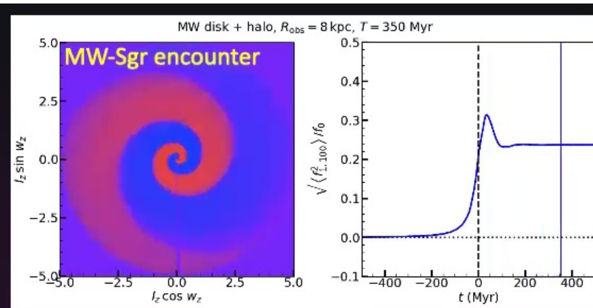
Series: Cosmology & Gravitation

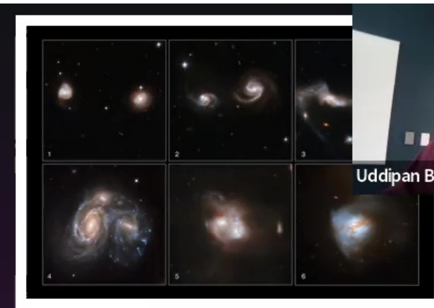
Date: November 22, 2022 - 11:00 AM

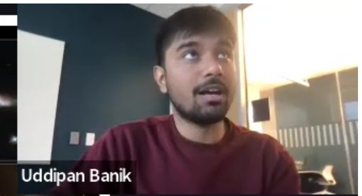
URL: https://pirsa.org/22110101

Abstract: The long range nature of gravity complicates the dynamics of self-gravitating many-body systems such as galaxies and dark matter (DM) halos. Relaxation/equilibration of perturbed galaxies and cold dark matter halos is typically a collective, collisionless process. Depending on the perturbation timescale, this process can be impulsive/fast, adiabatic/slow or resonant. First, I shall present a linear perturbative formalism to compute the response (in all three regimes) of disk galaxies to external perturbations such as satellite impacts. I shall elucidate how phase-mixing of the disk response gives rise to phase-space spirals akin to those observed by Gaia in the Milky Way disk, and how these features can be used to constrain the Milky Way's potential as well as its dynamical history. Next, I shall discuss the secular evolution of a massive perturber due to the back reaction of the near-resonant response of the host galaxy/halo. In this context I shall present two novel techniques to model the secular torque (dynamical friction) experienced by the perturber: 1. a self-consistent, time-dependent, perturbative treatment and 2. a non-perturbative orbit-based framework. These two approaches explain the origin of certain secular phenomena observed in N-body simulations of cored galaxies but unexplained in the standard Chandrasekhar and LBK theories of dynamical friction, namely core-stalling and dynamical buoyancy. I shall briefly discuss some astrophysical implications of these phenomena: potential choking of supermassive black hole mergers in cored galaxies, and the possibility of constraining the inner density profile (core vs cusp) of DM dominated dwarf galaxies and therefore the DM particle nature.

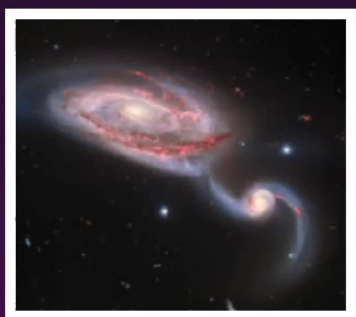
Zoom link: https://pitp.zoom.us/j/99089663538?pwd=aVVjV2ozMkZRTkE0ZW1Ib0dGUC9tdz09







Pushing the frontiers of gravitational encounters and collisionless dynamics



Uddipan Banik

Department of Astronomy,
Yale University

Collaborators: Frank C. van den Bosch, Martin Weinberg, Kathryn Johnston, Jason Hunt, Adrian Price-Whelan, Elise Darragh-Ford

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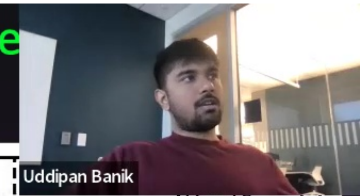
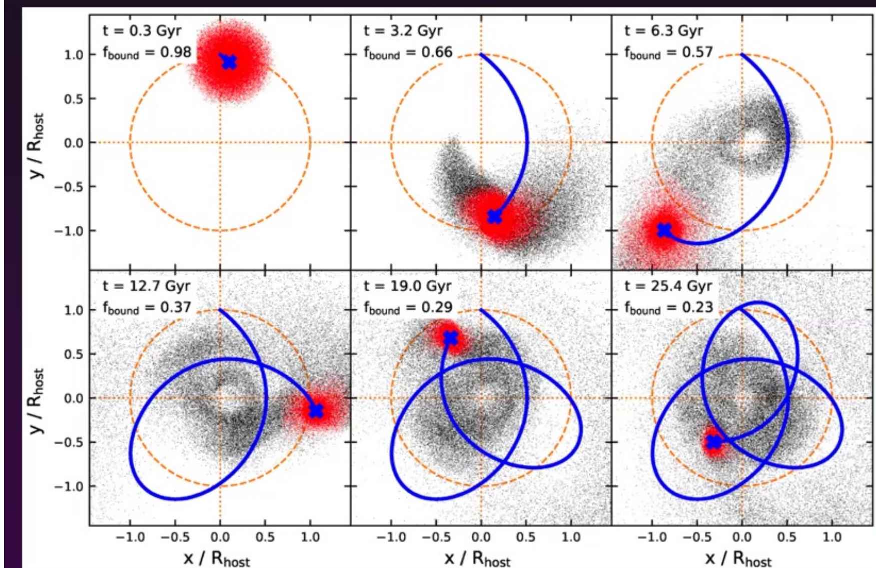
Cosmology Seminar
Perimeter Institute
November 22, 2022

Gravitational encounters

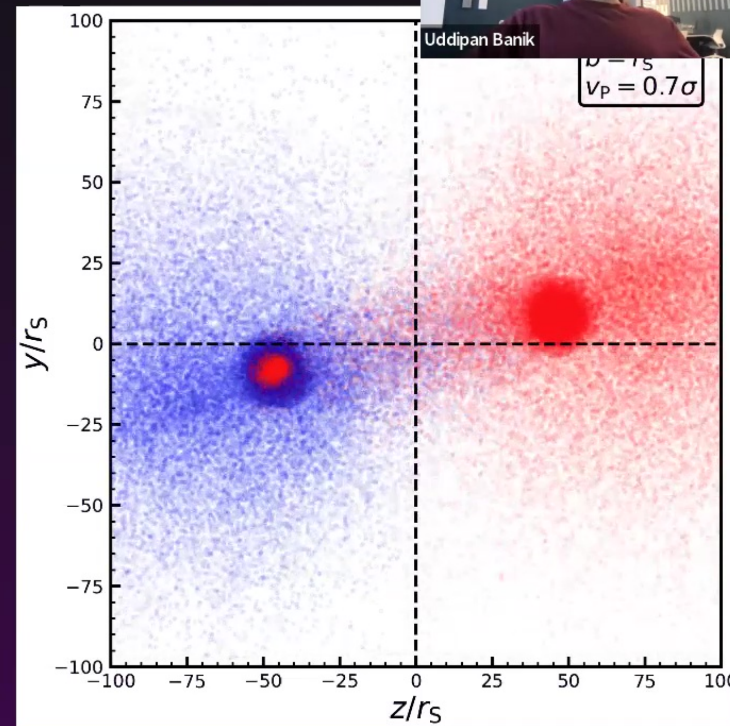
Galaxy-galaxy e
mergers



Inspiral and tidal mass loss of satellite galaxies



Miller et al. ’20



Banik & van den Bosch’21b (MNRAS)

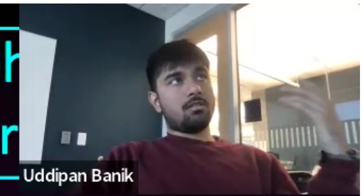
Galactic cannibalism

Perturbation of subject by perturber

Black hole mergers

Gravitational encounters

Orbits and ph
due to per



Ω_P = Encounter frequency Ω_* = intrinsic frequencies

Impulsive limit: $\Omega_P \gg \Omega_*$

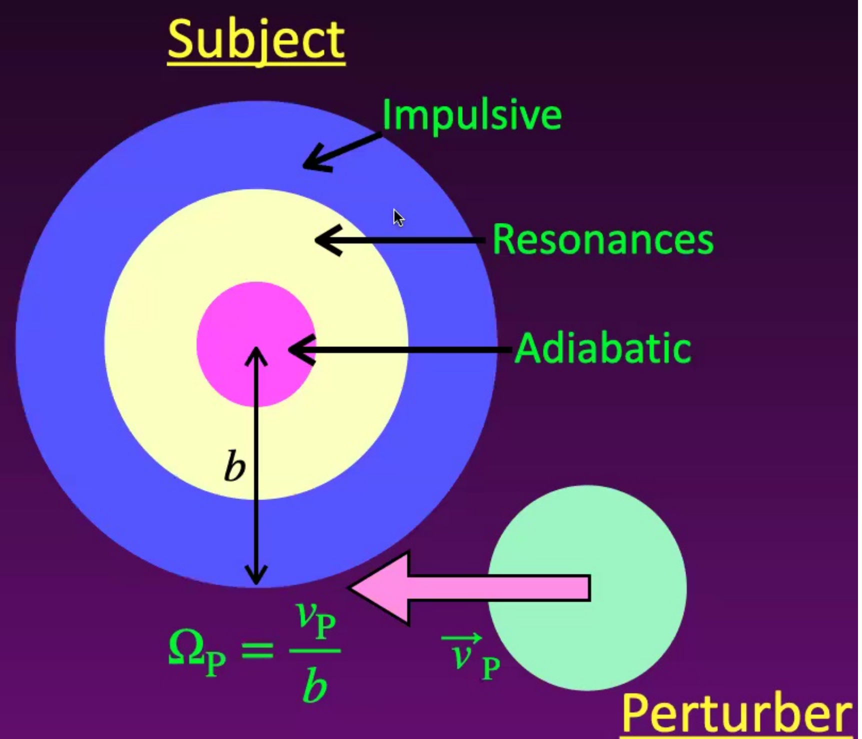
Tidal shocks Neglect internal motion

Adiabatic limit: $\Omega_P \ll \Omega_*$

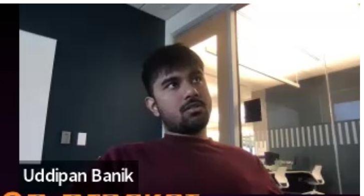
Adiabatic shielding Invariance of actions (orbits unaffected)

Intermediate regime: $\Omega_P \sim \Omega_*$

Secular evolution (dynamical friction) Resonances



Perturbation of collisionless systems: Eulerian approach



Collisionless Boltzmann Equation

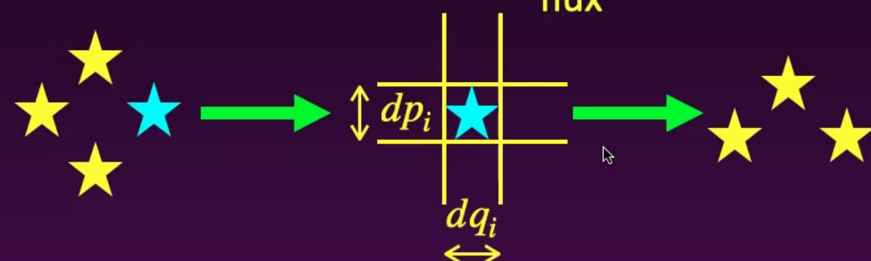
f = Distribution function

$$d^6N = f\, d^3q\, d^3p$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, H] = 0$$

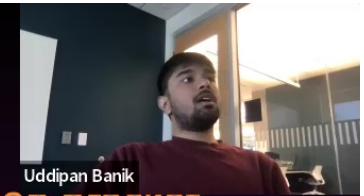
Poisson bracket

$$[f, H] = \sum_{i=1}^{3} \frac{\partial f}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial H}{\partial q_i}$$



Local time derivative
= − flux

Perturbation of collisionless systems: Eulerian approach



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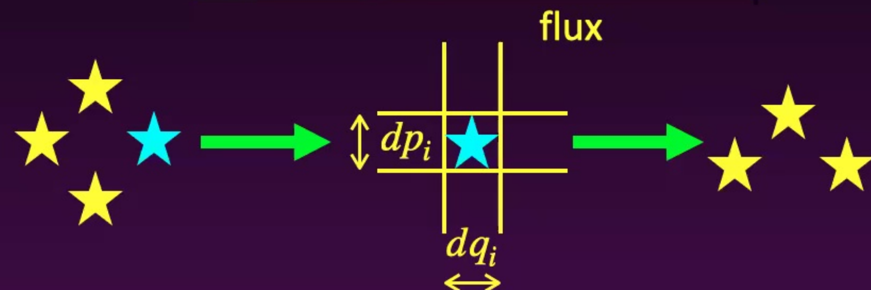


Local time derivative
$= -$ flux

Perturbing the system

Linear (weak) perturbation in potential

$$H = H_0 + \Phi_P(t)$$

Ignore self-gravity due to f_1
(Normal modes/point modes)

Perturbation in distribution function

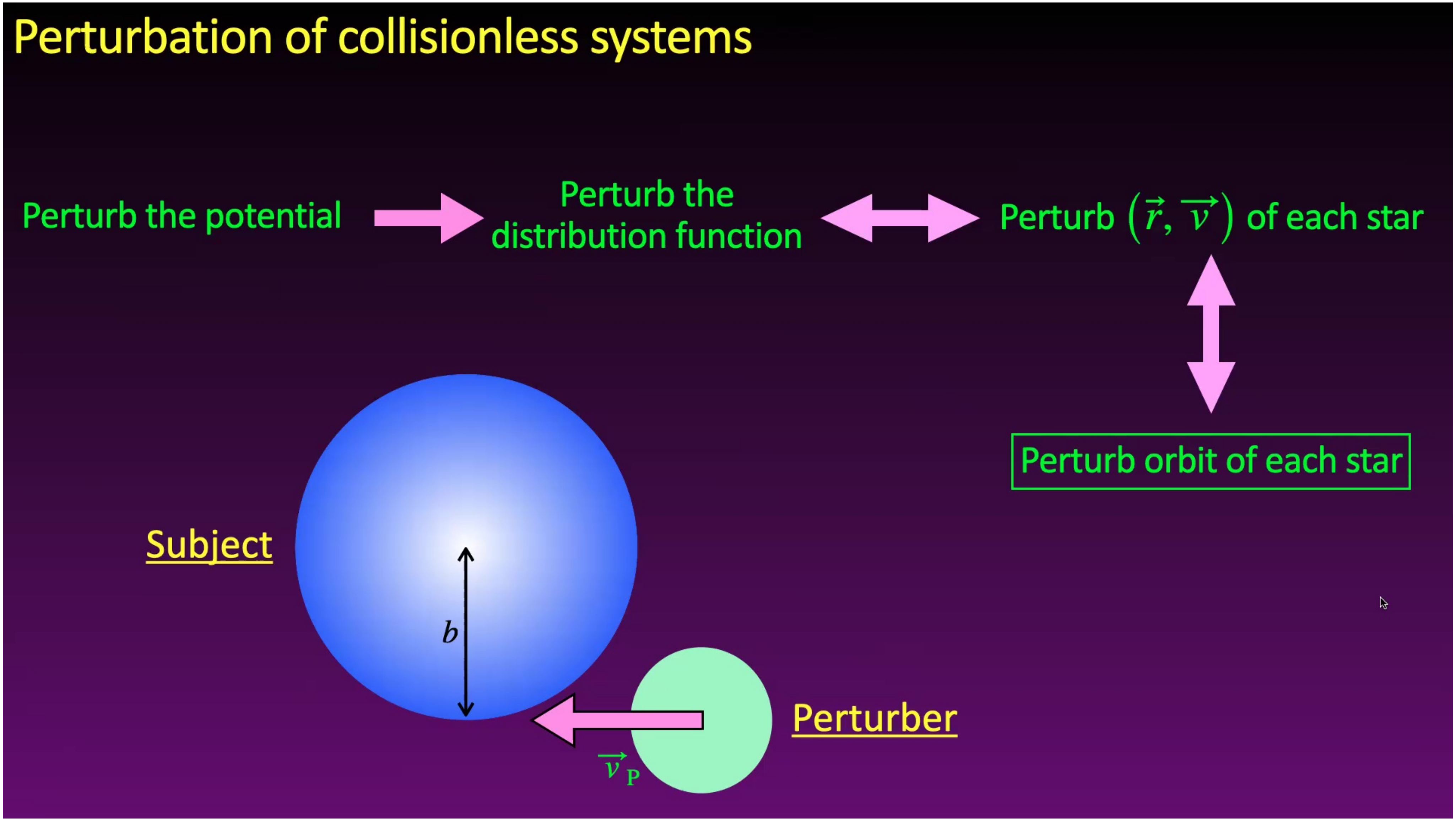
$$f = f_0 + f_1$$

$$[f_0, H_0] = 0$$

Linearized CBE

$$\frac{\partial f_1}{\partial t} + [f_1, H_0] + [f_0, \Phi_P] = 0$$

Direct response to perturber


Perturbation of collisionless systems
Perturb the potential
Perturb the distribution function
Perturb $(\vec{r}, \vec{v})$ of each star
Perturb orbit of each star
Subject
b
Perturber
$\vec{v}_P$

Orbital characteristics

Actions: Orbital sizes

$$I_1, I_2, I_3$$

$$I_i = \oint dr_i \, p_i$$

Conserved for symmetric, time-independent Hamiltonian/energy, H

$$H = H\left(I_1, I_2, I_3\right)$$

Angles: Orbital phases

$$w_1, w_2, w_3$$

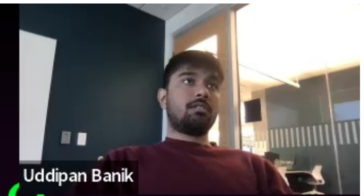
Change linearly with time

$$w_i = w_{i0} + \Omega_i t$$

$$\Omega_i = \frac{\partial H}{\partial I_i}$$

Each star is a combination of 3 oscillators!

Orbital characteristics

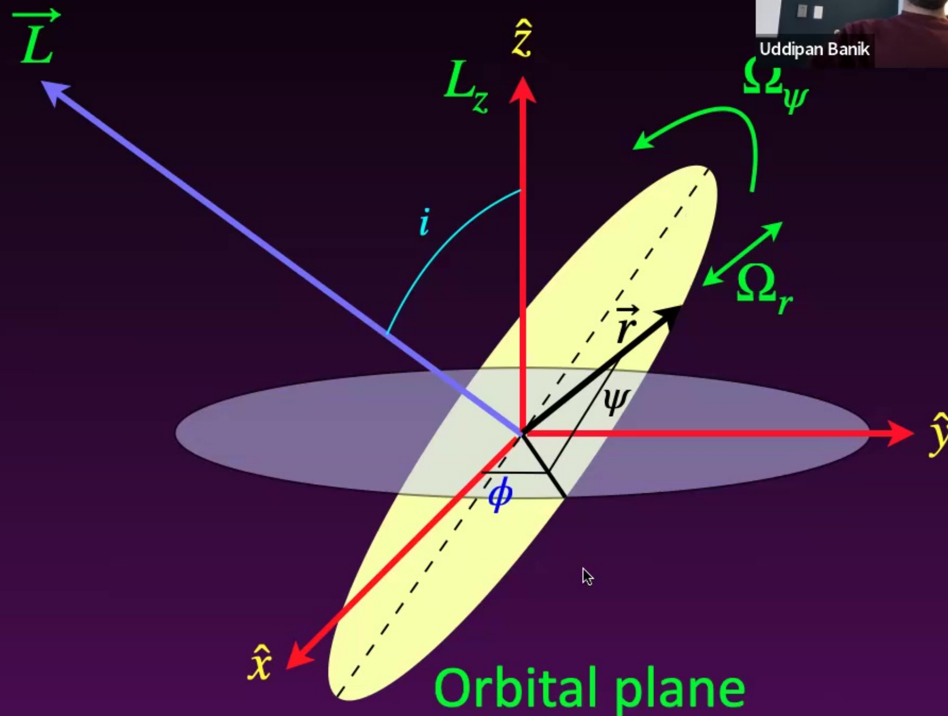


Spherical symmetry: $\Phi(r)$

Conserved: $H, \vec{L} \implies (a, e, i, \phi)$

Orbital plane fixed by

$$\phi\,, i = \cos^{-1}\left(\frac{L_z}{L}\right)$$



$$H = H\left(I_r, I_\psi = L\right)$$

$\implies \Omega_r = \dfrac{\partial H}{\partial I_r}, \Omega_\psi = \dfrac{\partial H}{\partial L}$ are the only 2 relevant frequencies.

Orbital characteristics

Spherical symmetry: $\Phi(r)$

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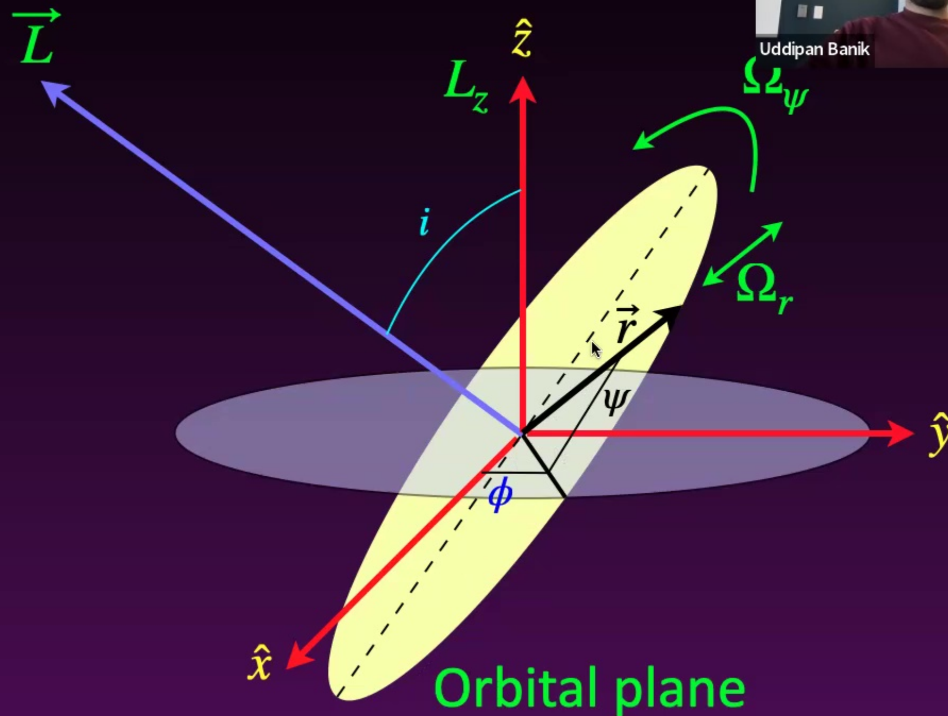
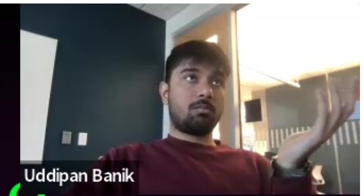


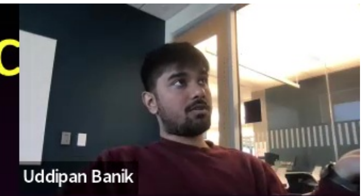


$$H = H\left(I_r, I_\psi = L\right)$$

Axisymmetric (disk): 3 frequencies

$\implies \Omega_r = \dfrac{\partial H}{\partial I_r}, \Omega_\psi = \dfrac{\partial H}{\partial L}$ are the only 2 relevant frequencies.

Perturbation of collisionless systems: Lagrangian approac



Perturbation of $(\vec{r}, \vec{v})$ or $(\vec{w}, \vec{I})$ of each star

Impulsive

$\Delta \vec{v} \neq 0$
$\Delta \vec{r} \approx 0$
Velocities change, positions don't.

$$\Delta \vec{v}(\vec{r}, t) \approx -\int_{-\infty}^{t} \vec{\nabla} \Phi_{\mathrm{P}}(\vec{r}, t')\, dt'$$

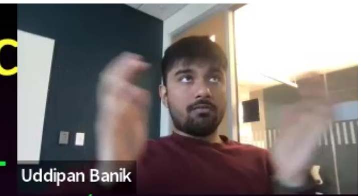
Adiabatic

$\Delta \vec{I} \approx 0$ Actions invariant away from resonances

$$I_i = \oint dr_i\, p_i$$

$$w_i = \Omega_i t$$

Perturbation of collisionless systems: Lagrangian approac



Perturbation of $(\vec{r}, \vec{v})$ or $(\vec{w}, \vec{I})$ of each star

$l_1\Omega_1 + l_2\Omega_2 + l_3\left(\Omega_3 -\right.$

Impulsive

$\Delta\vec{v} \neq 0$, $\Delta\vec{r} \approx 0$ — Velocities change, positions don't.

$$\Delta\vec{v}(\vec{r},t) \approx -\int_{-\infty}^{t} \vec{\nabla}\Phi_P(\vec{r},t')\,dt'$$

Adiabatic

$\Delta\vec{I} \approx 0$ — Actions invariant away from resonances

$$I_i = \oint dr_i\, p_i$$

$$w_i = \Omega_i t$$

Resonances

$\Delta I_s \neq 0$, $\Delta w_s \neq 0$, $\Delta I_{fi} = 0$ — Slow action-angle change, fast actions don't.

Slow action $I_s = \dfrac{I_3}{l_3}$

Fast actions $I_{fi} = I_i - \dfrac{l_i}{l_3} I_3 \quad \forall\, i = 1,2$

Slow angle $w_s = l_1 w_1 + l_2 w_2 + l_3\left(w_3 - \Omega_P t\right)$

Fast angles $w_{fi} = w_i \quad \forall i = 1,2$

Adiabatic invariance of actions is partially broken near the resonances.

Secular evolution!
Slow

Tremaine & Weinberg '84
Chiba & Schonrich '22
Hamilton et al. '22

Gravitational encounters

Orbits and ph
due to per

Ω_P = Encounter frequency Ω_* = intrinsic frequencies

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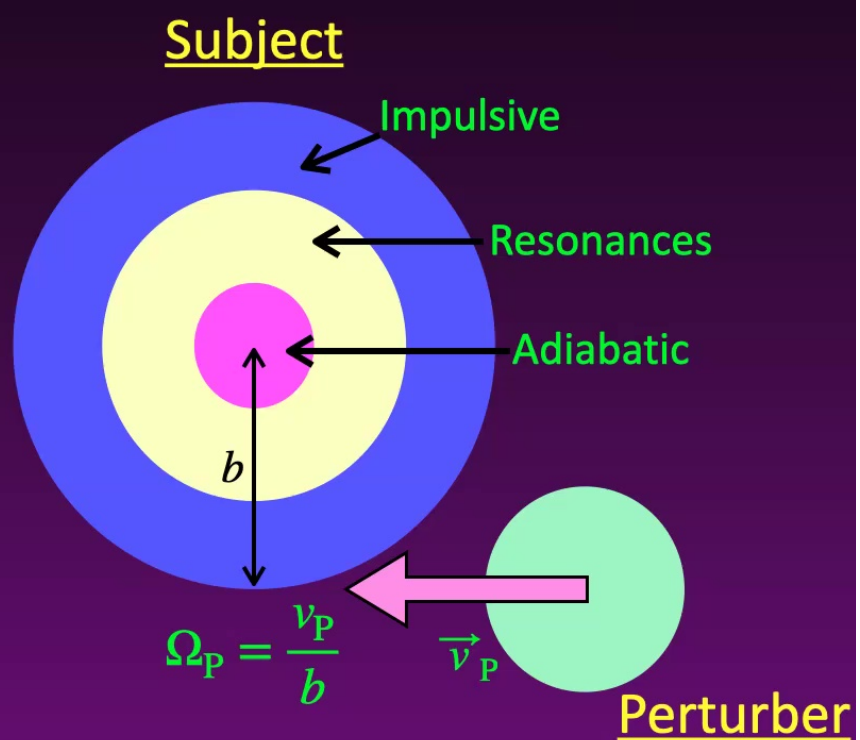
Secular evolution (dynamical friction) Resonances

Part I



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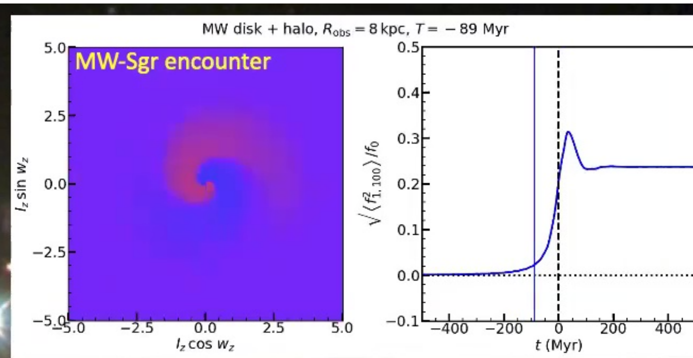

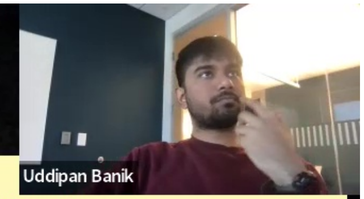

Papers

1. Banik et al. '22b (ApJ)

DOI: 10.3847/1538-4357/ac7ff9
arXiv: 2208.05038

2. Banik et al. '22a (submitted to ApJ)

Part I

Perturbative response of disk galaxies: phase-mixing & phase-spirals

Collaborators: Martin Weinberg, Frank C. van den Bosch, Kathryn Johnston, Jason Hunt, Adrian Price-Whelan, Elise Darragh-Ford

Image courtesy: NASA/ Space Telescope Science Institute

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Disk galaxies out of equilibrium

Milky Way is not in equilibrium!

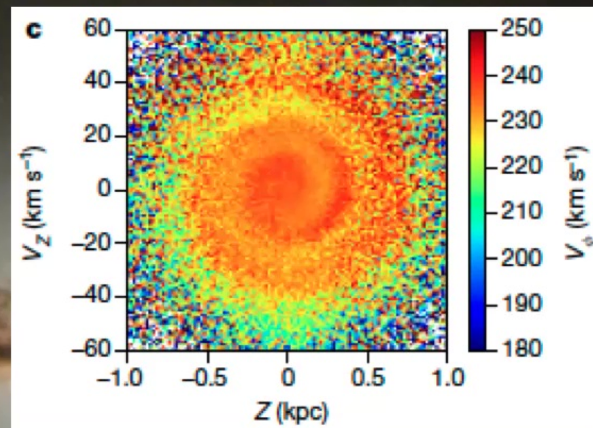
Phase-space spirals +
vertical bending waves, warp
observed by Gaia
(T. Antoja+18, Bennett & Bovy '18, '21)

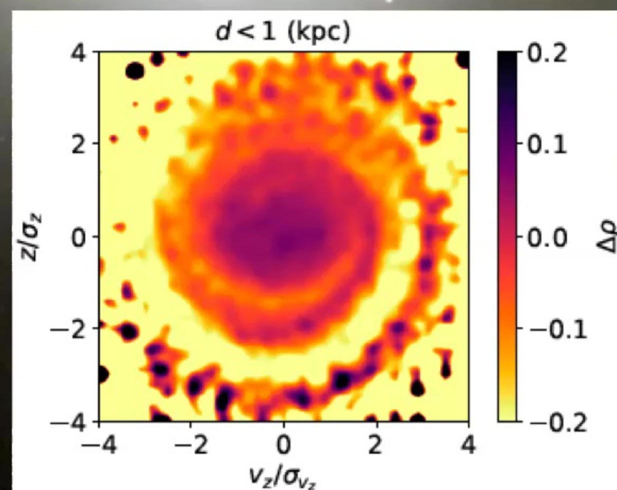
Perturbation + phase-mixing
(loss of coherence in oscillations)

Galactoseismology
Constrain dynamical history
of Milky Way

Constrain Milky Way potential,
dark matter distribution

Phase-space spiral
in solar neighborhood





Caution:
Spiral feature
NOT a physic

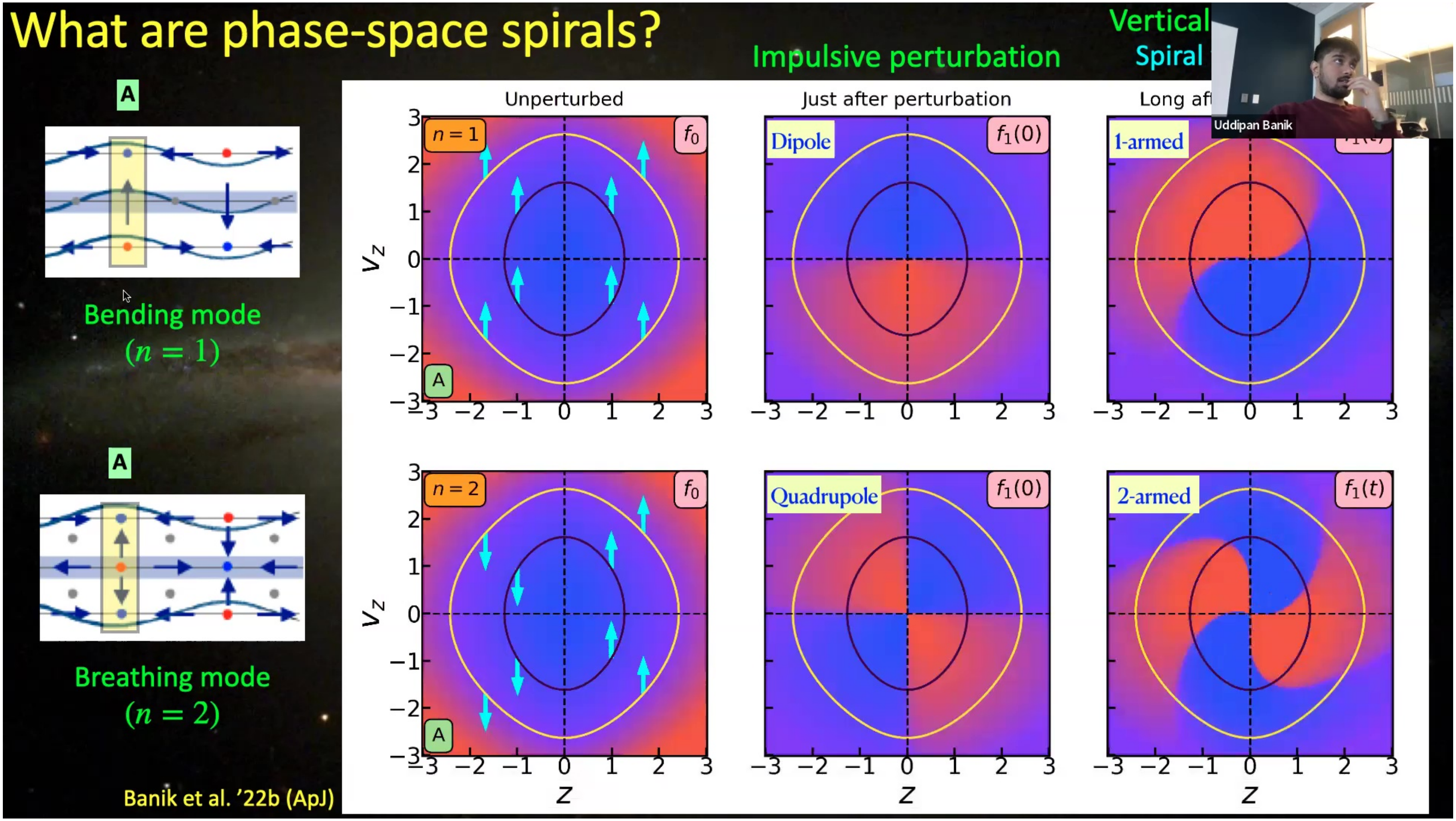
Observation

Gaia DR2
T. Antoja et al. 2018

Simulation

Perturbation ~ 500 Myr ago
Recent interaction with
Sagittarius dwarf galaxy?

MW-Sgr N-body simulation
J.A.S. Hunt et al. 2021


What are phase-space spirals?
Impulsive perturbation
Vertical
Spiral
Uddipan Banik
A
Bending mode
(n = 1)
A
Breathing mode
(n = 2)
Banik et al. '22b (ApJ)
Unperturbed
Just after perturbation
n = 1
f0
Dipole
f1(0)
1-armed
n = 2
Quadrupole
2-armed
f1(t)
vz
z

Linear perturbation theory for collisionless systems

$f =$ Distribution function

$$d^6N = f\, d^3q\, d^3p$$

Collisionless Boltzmann Equation

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, H] = 0$$

Poisson bracket

$$[f, H] = \sum_{i=1}^{3} \frac{\partial f}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial H}{\partial q_i}$$

Perturbing the galaxy

Linear (weak) perturbation in potential

$$H = H_0 + \Phi_P(t)$$

Ignore self-gravity due to f_1 (Normal modes/point modes)

→

Perturbation in distribution function

$$f = f_0 + f_1$$

$$[f_0, H_0] = 0$$

→

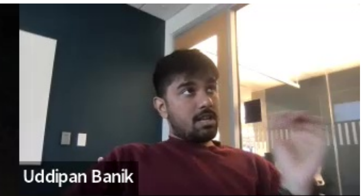
Linearized CBE

$$\frac{\partial f_1}{\partial t} + [f_1, H_0] + [f_0, \Phi_P] = 0$$

Direct response to perturber

$f_1 =$ Density contrast of phase-spiral

Linear perturbation theory for collisionless systems



Linearized CBE

$$\frac{\partial f_1}{\partial t} + [f_1, H_0] + [f_0, \Phi_{\mathrm{P}}] = 0$$

Linear perturbation theory for collisionless systems

Linearized CBE

$$\frac{\partial f_1}{\partial t} + [f_1, H_0] + [f_0, \Phi_{\mathrm{P}}] = 0$$

Action-angle variables $\vec{I}, \vec{w}$

Fourier series expansion

$$f_1(\vec{w}, \vec{I}) = \sum_{\vec{l}} e^{i\vec{l}\cdot\vec{w}} f_{1\vec{l}}(\vec{I})$$

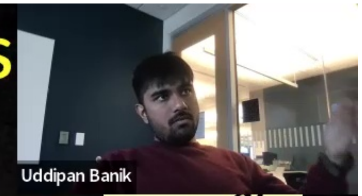
$$\Phi_{\mathrm{P}}(\vec{w}, \vec{I}) = \sum_{\vec{l}} e^{i\vec{l}\cdot\vec{w}} \Phi_{\vec{l}}(\vec{I})$$

$$\frac{\partial f_{1\vec{l}}}{\partial t} + i l_k \Omega_k f_{1\vec{l}} = i l_i \frac{\partial f_0}{\partial I_i} \Phi_{\vec{l}}\left(\vec{I}, t\right)$$

$$l_k\Omega_k = l_1\Omega_1 + l_2\Omega_2 + l_3\Omega_3$$

Forced oscillator

Linear perturbation theory for collisionless systems



Linearized CBE

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Action-angle variables $\vec{I}, \vec{w}$

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$$l_k \Omega_k = l_1 \Omega_1 + l_2 \Omega_2 + l_3 \Omega_3$$

Forced oscillator

Linear response

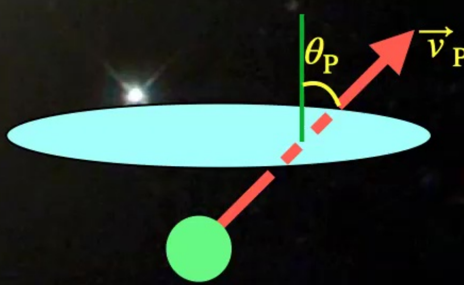
$$f_{1\vec{l}}\left(\vec{I}, t\right) = i l_i \frac{\partial f_0}{\partial I_i} \int_0^t d\tau \, e^{-i l_k \Omega_k \tau} \, \Phi_{\vec{l}}\left(\vec{I}, t - \tau\right)$$

Phase-spiral

Perturber

- Bar
- Spiral-arm
- Satellite galaxy

Satellite galaxy



Self-gravity of response

Linearized CBE

$$\frac{\partial f_1}{\partial t} + [f_1, H_0] + [f_0, \Phi_P] +$$

$$\nabla^2 \Phi_1 = 4\pi G \int d^3v\, f_1$$

Poisson eq

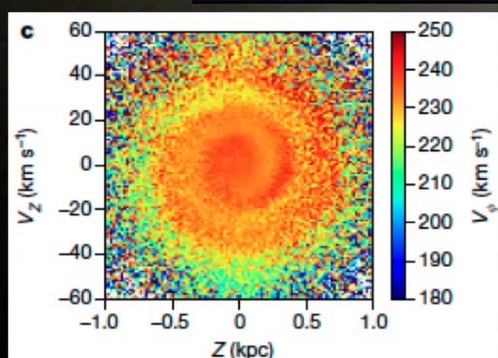
Linear response

Phase-spiral (transient, amplified)

$$f_{1\vec{l}}\left(\vec{I}, t\right) = i l_i \frac{\partial f_0}{\partial I_i} \int_0^t d\tau\, e^{-i l_k \Omega_k \tau}\, \Phi_{\vec{l}}\left(\vec{I}, t-\tau\right)$$

+ Normal/point modes

Phase-mixing



Landau damped coherent oscillations

Self-gravity of response

Linearized CBE

$$\frac{\partial f_1}{\partial t} + [f_1, H_0] + [f_0, \Phi_P] + [f_0, \Phi_1] = 0$$

$$\nabla^2 \Phi_1 = 4\pi G \int d^3 v\, f_1$$

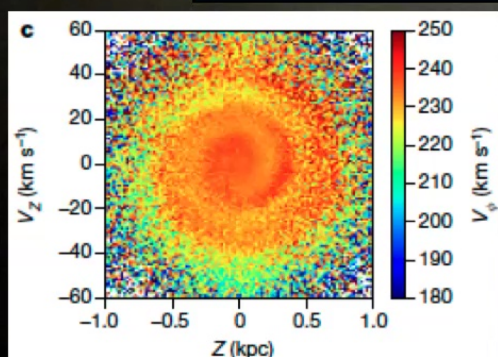
Poisson eq

Linear response

Phase-spiral (transient, amplified)

$$f_{1\vec{l}}\left(\vec{I}, t\right) = i l_i \frac{\partial f_0}{\partial I_i} \int_0^t d\tau\, e^{-i l_k \Omega_k \tau}\, \Phi_{\vec{l}}\left(\vec{I}, t - \tau\right)$$

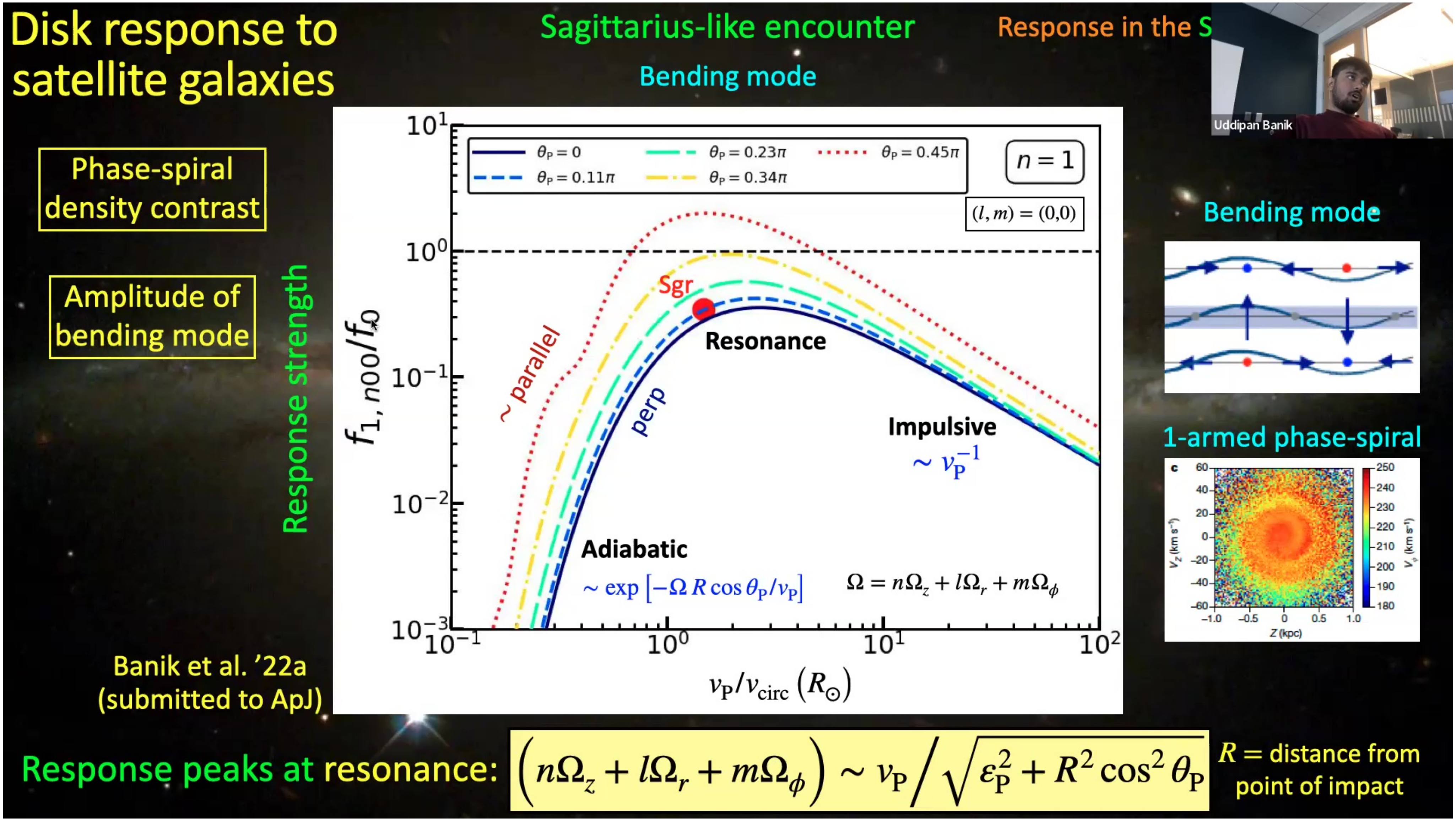
Phase-mixing



+ Normal/point modes

Landau damped coherent oscillations

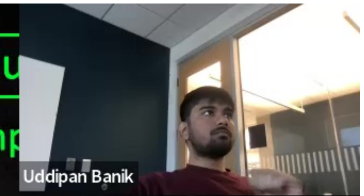
Ignore for phase-spiral!



Disk response to satellite galaxies
Sagittarius-like encounter
Bending mode
Response in the S
Uddipan Banik
Phase-spiral density contrast
Amplitude of bending mode
Response strength
$f_{1,n00}/f_0$
$\theta_P = 0$
$\theta_P = 0.11\pi$
$\theta_P = 0.23\pi$
$\theta_P = 0.34\pi$
$\theta_P = 0.45\pi$
$n = 1$
$(l, m) = (0,0)$
Sgr
Resonance
~ parallel
perp
Impulsive
$\sim v_P^{-1}$
Adiabatic
$\sim \exp[-\Omega R \cos\theta_P / v_P]$
$\Omega = n\Omega_z + l\Omega_r + m\Omega_\phi$
$v_P/v_{circ}(R_\odot)$
Bending mode
1-armed phase-spiral
V_z (km s^{-1})
V_ϕ (km s^{-1})
Z (kpc)
Banik et al. '22a (submitted to ApJ)
Response peaks at resonance: $\left(n\Omega_z + l\Omega_r + m\Omega_\phi\right) \sim v_P \Big/ \sqrt{\varepsilon_P^2 + R^2\cos^2\theta_P}$
R = distance from point of impact

Bending vs Breathing modes

Impact occurs at $\sim 20\,\mathrm{kpc}$. Sagittariu



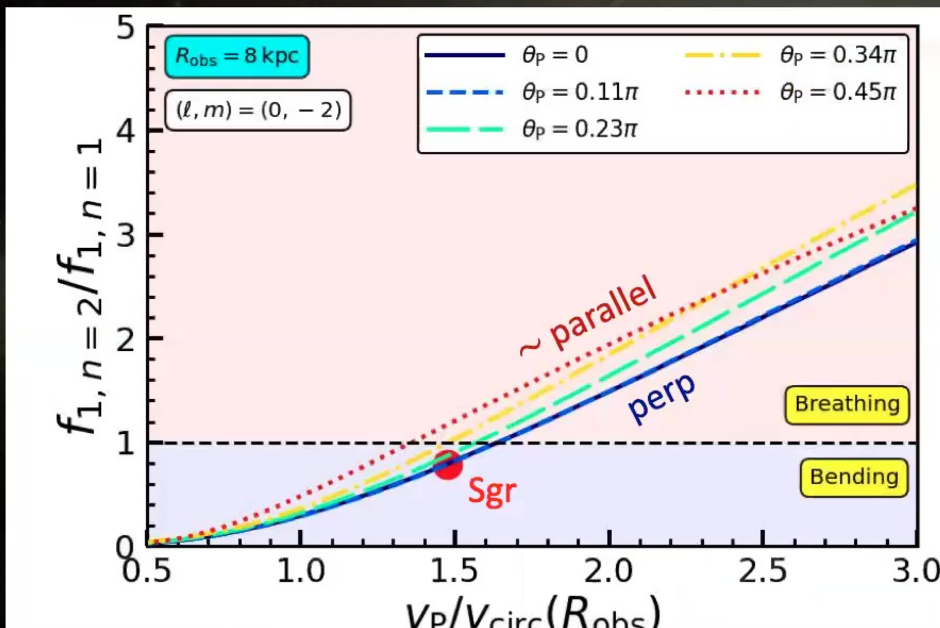
Far from point of impact

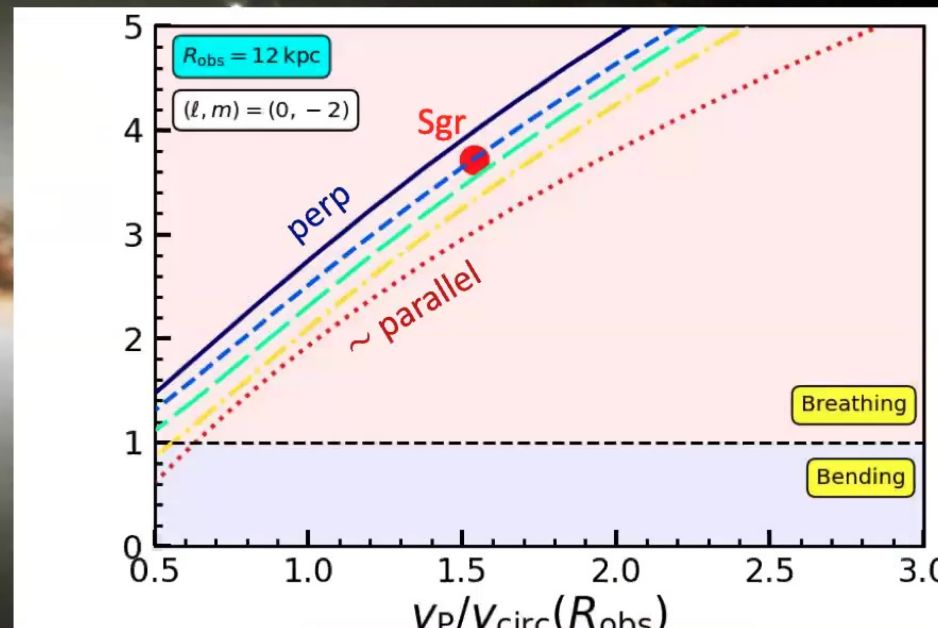
$R_{\mathrm{obs}} = 8\,\mathrm{kpc}$

Closer to point of imp

$R_{\mathrm{obs}} = 12\,\mathrm{kpc}$

Breathing /Bending





Banik et al. '22a (submitted to ApJ)

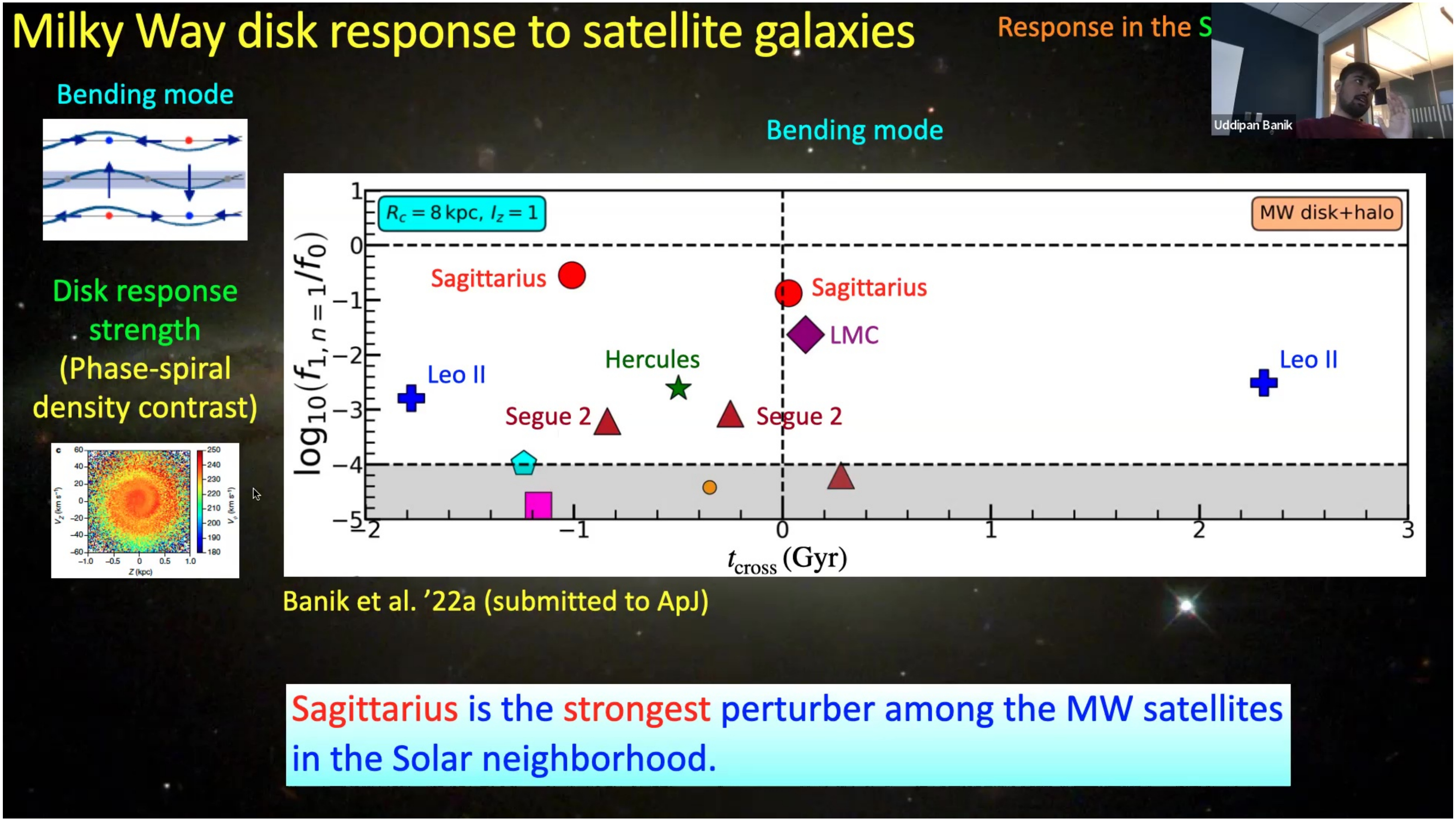
R = distance from point of impact

Breathing modes $\gtrsim$ bending modes for higher impact velocity and closer to the point of impact.

$$\frac{R\cos\theta_P}{v_P} \text{ vs } \frac{1}{\Omega_z}$$

Small v_P / Large R: Bending

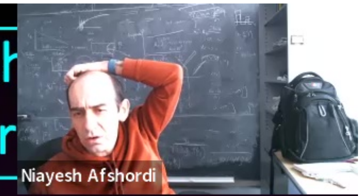
Large v_P / Small R: Breathing



Milky Way disk response to satellite galaxies
Response in the S
Uddipan Banik
Bending mode
Disk response strength (Phase-spiral density contrast)
Bending mode
$R_c = 8$ kpc, $l_z = 1$
MW disk+halo
Sagittarius
Sagittarius
LMC
Hercules
Leo II
Leo II
Segue 2
Segue 2
$\log_{10}(f_{1,n=1}/f_0)$
$t_{\rm cross}$ (Gyr)
Banik et al. '22a (submitted to ApJ)
Sagittarius is the strongest perturber among the MW satellites in the Solar neighborhood.

Gravitational encounters

Orbits and ph
due to per



Ω_P = Encounter frequency Ω_* = intrinsic frequencies

Impulsive limit: $\Omega_P \gg \Omega_*$

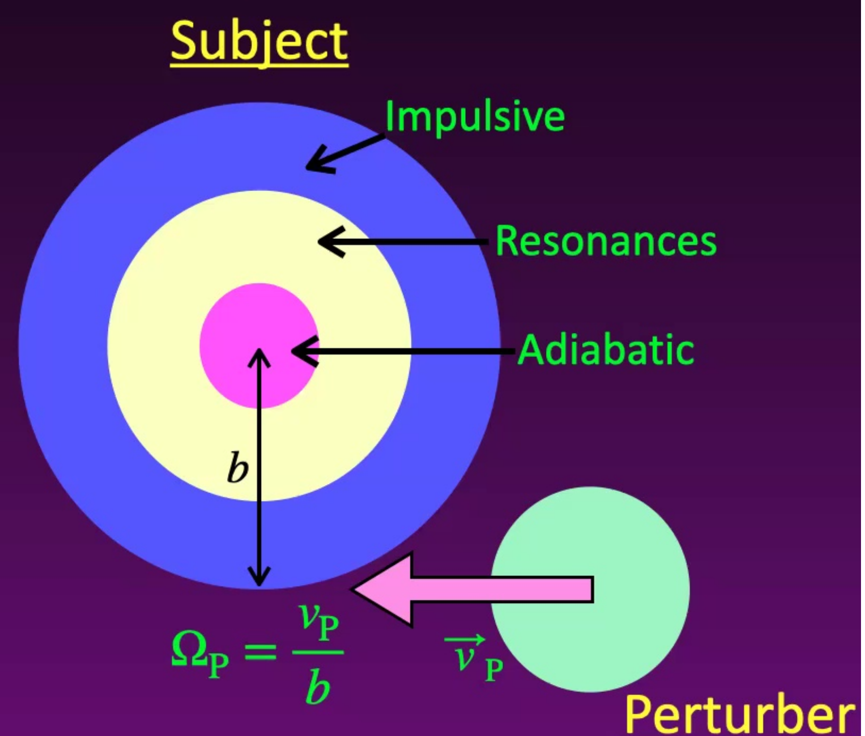
Tidal shocks — Neglect internal motion

Adiabatic limit: $\Omega_P \ll \Omega_*$

Adiabatic shielding — Invariance of actions (orbits unaffected)

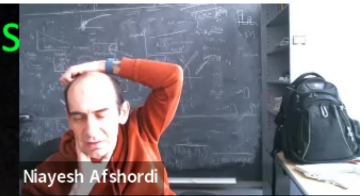
Intermediate regime: $\Omega_P \sim \Omega_*$

Secular evolution (dynamical friction) — Resonances — Part II

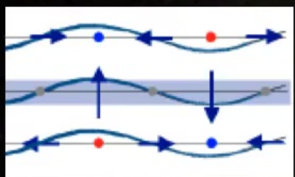


Milky Way disk response to satellite galaxies

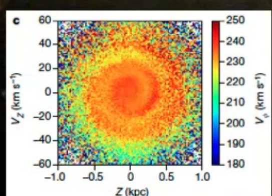
Response in the S



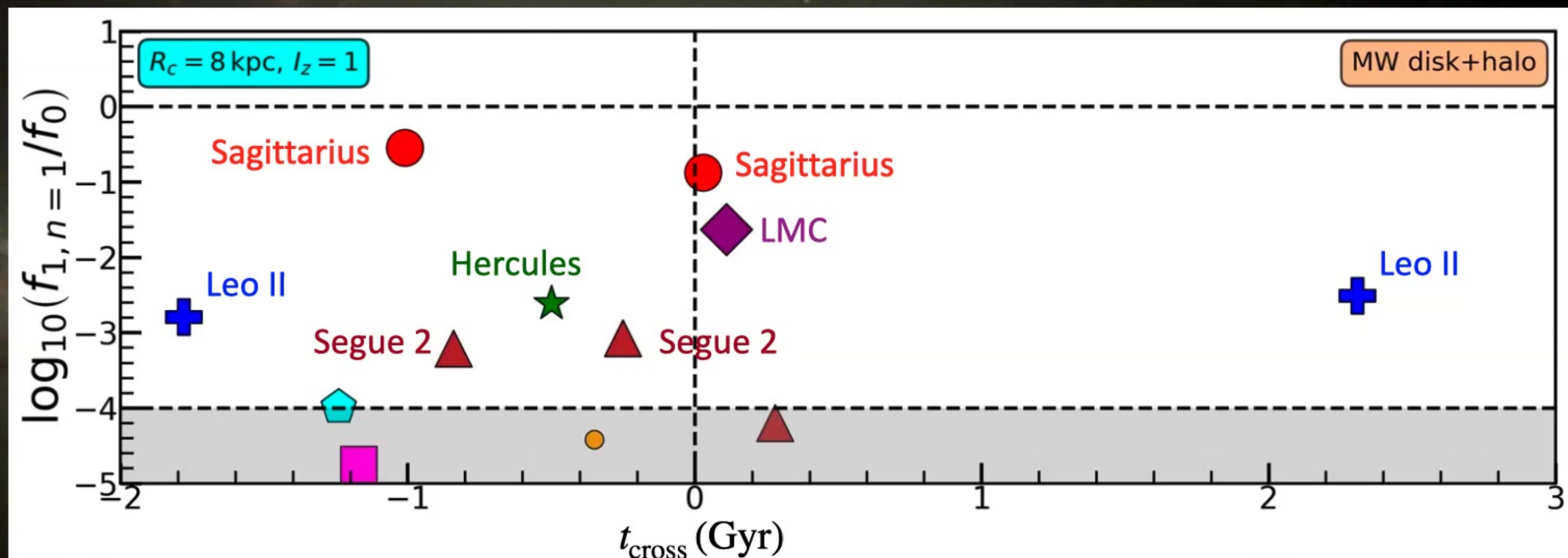
Bending mode



Disk response strength (Phase-spiral density contrast)

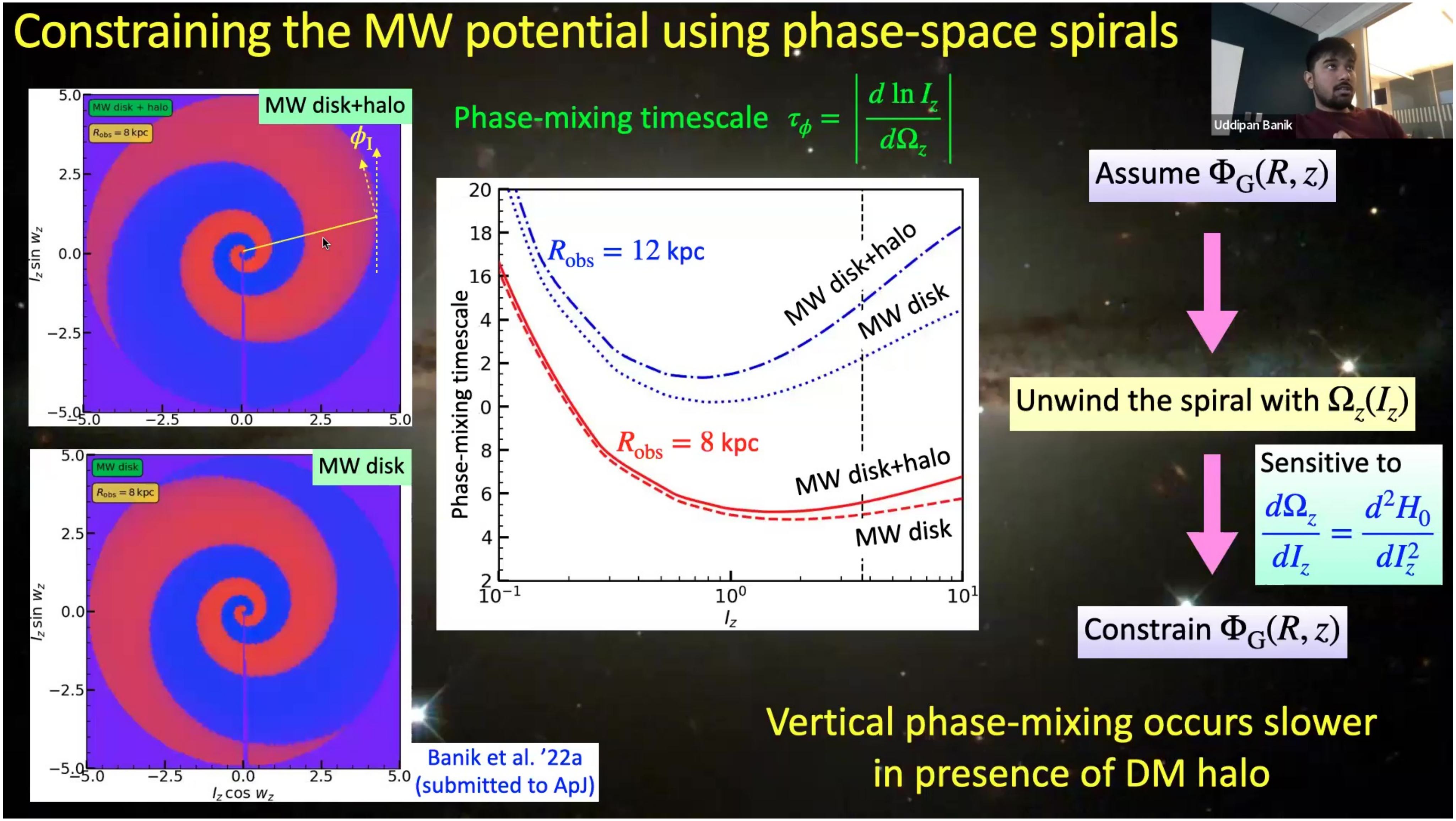


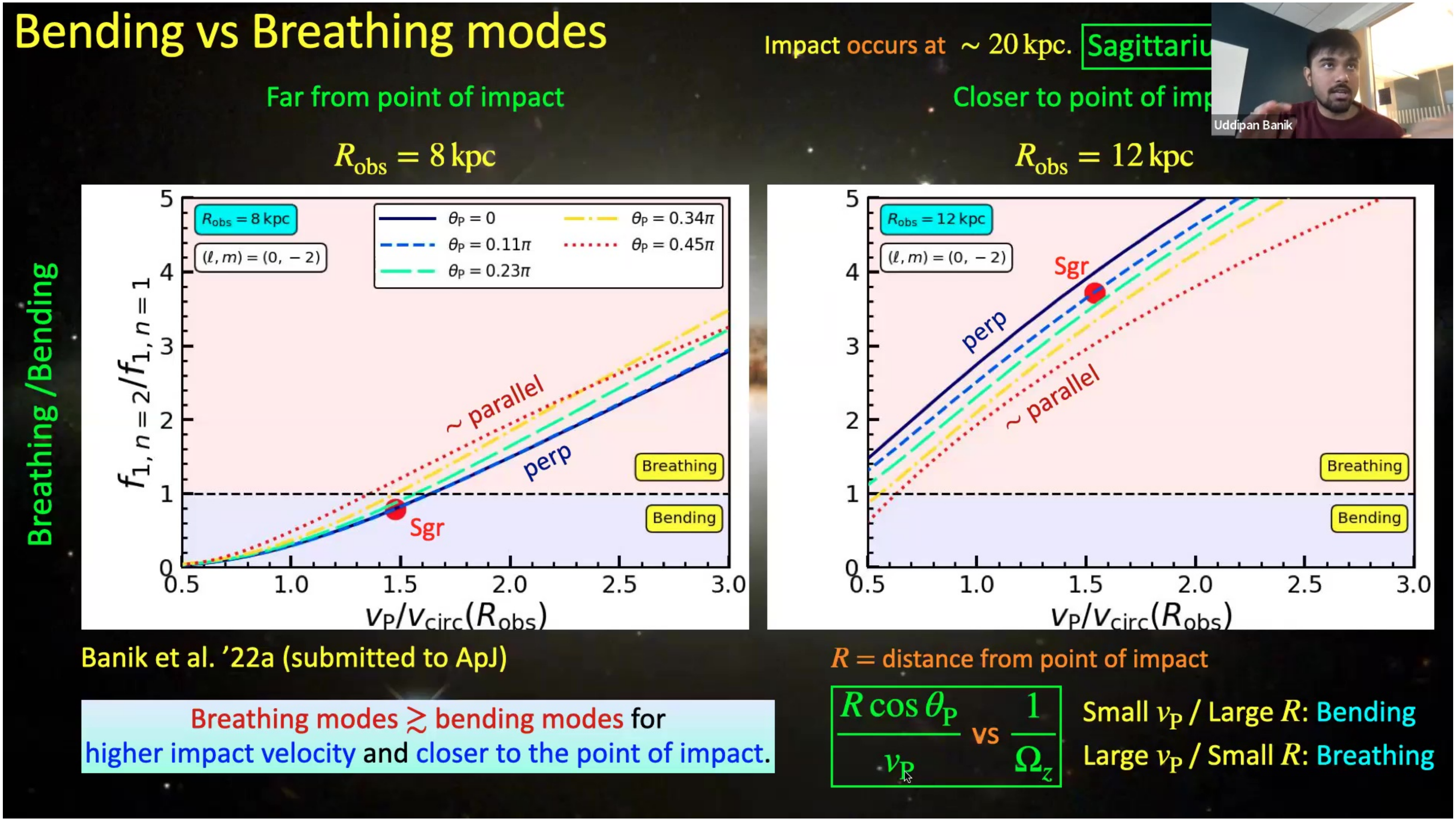
Bending mode



Banik et al. ’22a (submitted to ApJ)

Sagittarius is the strongest perturber among the MW satellites in the Solar neighborhood.


Constraining the MW potential using phase-space spirals
Phase-mixing timescale $\tau_\phi = \left| \frac{d \ln I_z}{d\Omega_z} \right|$
MW disk+halo
MW disk
$R_{obs} = 12$ kpc
$R_{obs} = 8$ kpc
Phase-mixing timescale
I_z
Assume $\Phi_G(R, z)$
Unwind the spiral with $\Omega_z(I_z)$
Sensitive to $\frac{d\Omega_z}{dI_z} = \frac{d^2 H_0}{dI_z^2}$
Constrain $\Phi_G(R, z)$
Vertical phase-mixing occurs slower in presence of DM halo
Banik et al. '22a (submitted to ApJ)
Uddipan Banik


Bending vs Breathing modes
Impact occurs at $\sim 20\,\mathrm{kpc}$. Sagittariu
Uddipan Banik
Far from point of impact
Closer to point of imp
$R_{\mathrm{obs}} = 8\,\mathrm{kpc}$
$R_{\mathrm{obs}} = 12\,\mathrm{kpc}$
Breathing /Bending
$f_{1,n=2}/f_{1,n=1}$
$R_{\mathrm{obs}} = 8\,\mathrm{kpc}$
$(\ell, m) = (0, -2)$
$\theta_{\mathrm{P}} = 0$
$\theta_{\mathrm{P}} = 0.11\pi$
$\theta_{\mathrm{P}} = 0.23\pi$
$\theta_{\mathrm{P}} = 0.34\pi$
$\theta_{\mathrm{P}} = 0.45\pi$
~ parallel
perp
Sgr
Breathing
Bending
$v_{\mathrm{P}}/v_{\mathrm{circ}}(R_{\mathrm{obs}})$
$R_{\mathrm{obs}} = 12\,\mathrm{kpc}$
$(\ell, m) = (0, -2)$
Sgr
perp
~ parallel
Breathing
Bending
$v_{\mathrm{P}}/v_{\mathrm{circ}}(R_{\mathrm{obs}})$
Banik et al. '22a (submitted to ApJ)
R = distance from point of impact
Breathing modes $\gtrsim$ bending modes for
higher impact velocity and closer to the point of impact.
$\frac{R\cos\theta_{\mathrm{P}}}{v_{\mathrm{P}}}$ vs $\frac{1}{\Omega_z}$
Small v_{P} / Large R: Bending
Large v_{P} / Small R: Breathing

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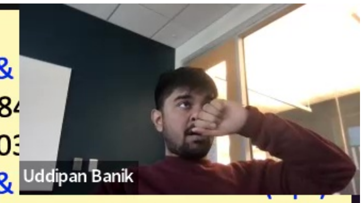
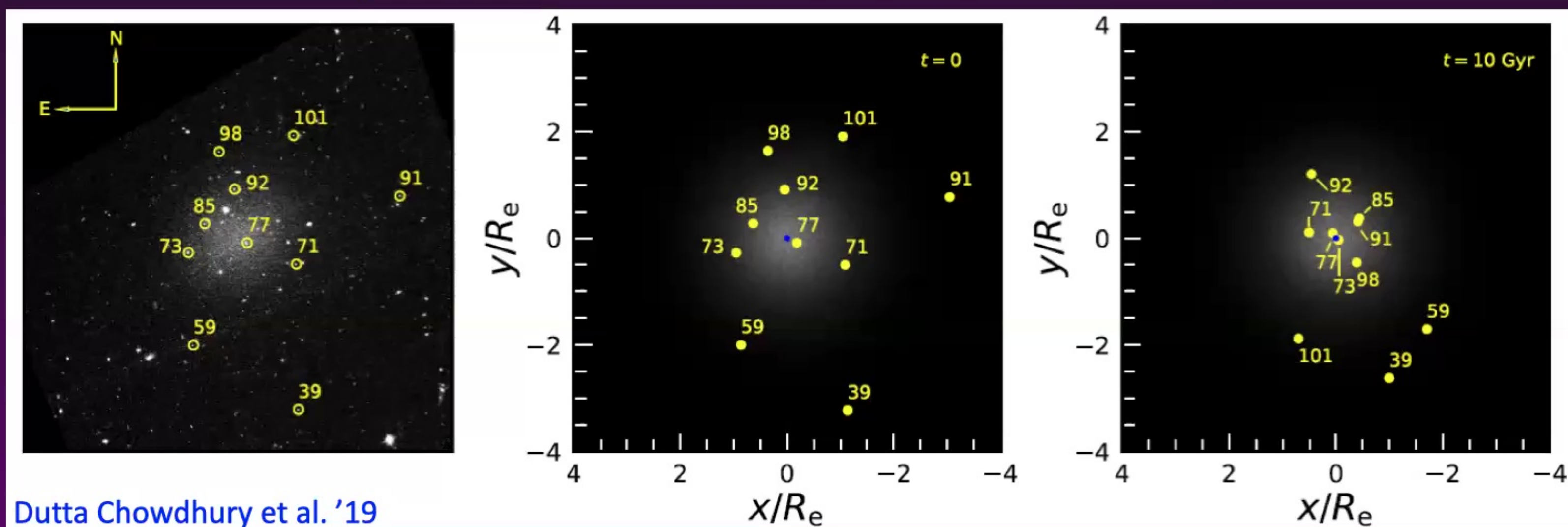
Part II

Papers
1. Banik &
DOI:10.384
arXiv: 210
2. Banik &
DOI: 10.3847/1538-4357/ac4242
arXiv: 2112.06944
3. Banik & van den Bosch in prep



Dynamical friction, buoyancy and core-stalling

Collaborator: Frank C. van den Bosch

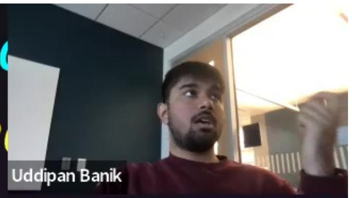


Dutta Chowdhury et al. '19

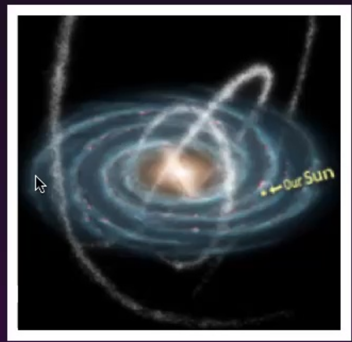
Visit my website: https://uddipanbanik.research.yale.edu

Dynamical friction

Loss of angular m
Key relaxation me
collisionless systems



Galaxy mergers

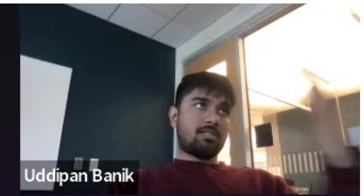


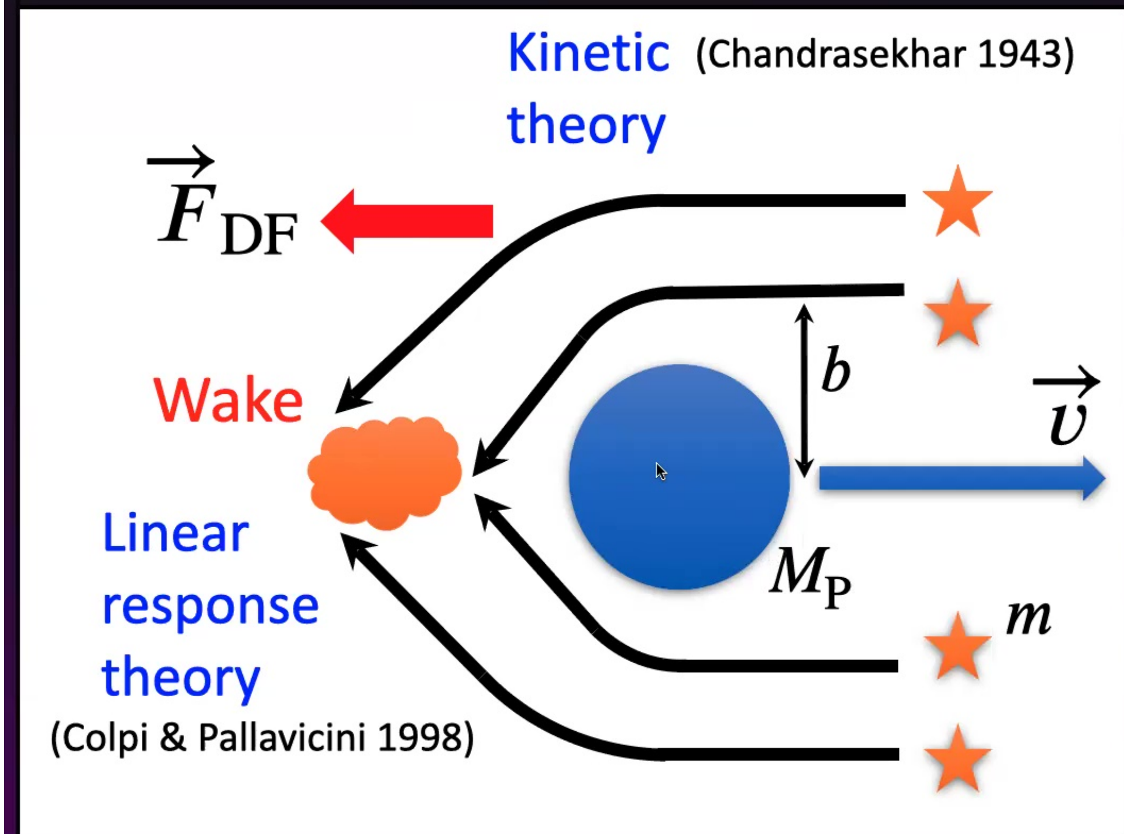
In-fall of galaxies in a cluster:
galactic cannibalism

Black hole mergers

In-fall of globular clusters
to the center of host galaxy:
bulges and nuclear star clusters

Dynamical friction- standard Chandrasekhar picture





$$\vec{F}_{\rm DF} = -4\pi G^2 M_{\rm P}^2 \ln\Lambda\, \rho(<v)\, \frac{\vec{v}}{v^3}$$

Approximations

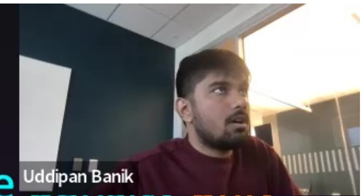
- Homogeneous background of stars
- Nearly straight orbits of stars (local)

Dynamical friction- standard resonance picture

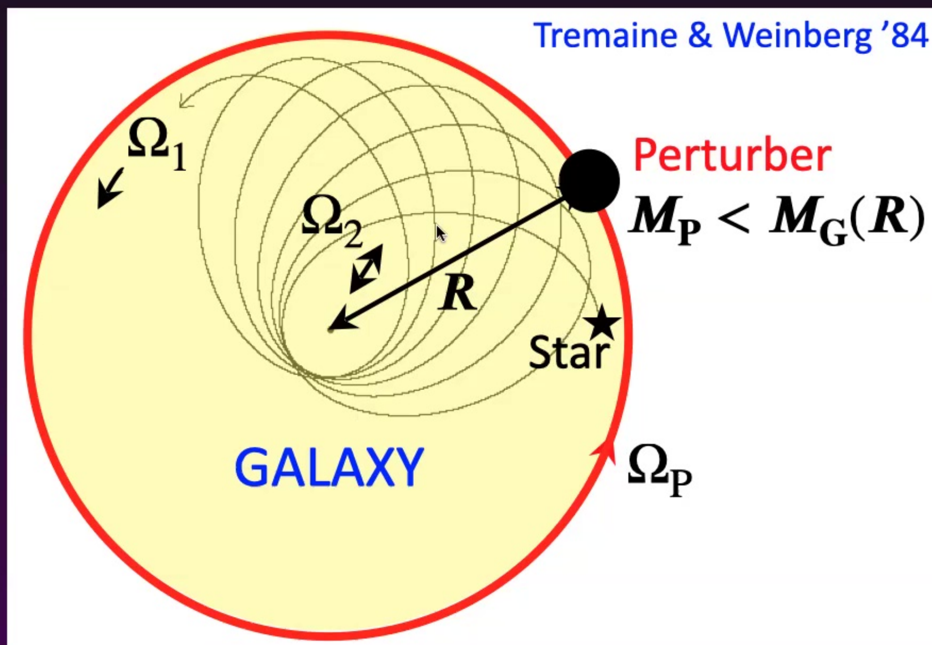
Rosette orbits of stars: conserved actions I, L, L_z and frequencies

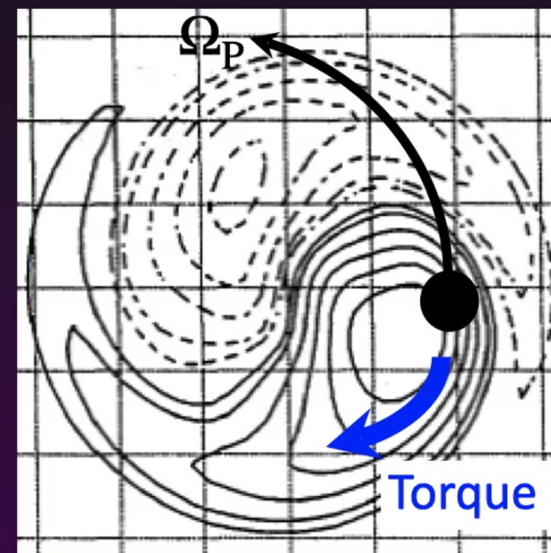
(oscillation amplitudes)

$$\Omega_1 = \frac{\partial H_0}{\partial L}, \Omega_2 = \frac{\partial H_0}{\partial I}$$



Response
from resonances





Weinberg '89

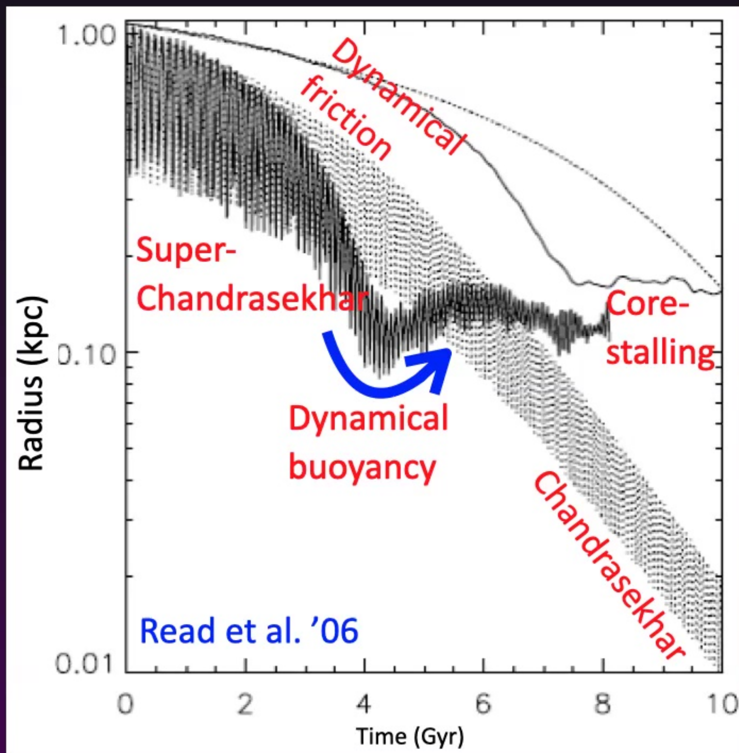
Only the resonant stars exert a non-zero torque: **LBK torque** Lynden-Bell & Kalnajs '72

$$l_1\Omega_1 + l_2\Omega_2 - l_3\Omega_P = 0$$

Weinberg '86, '89;
Kaur & Sridhar '18;
Kaur & Stone '21

Failure of standard theory

N-body simulations

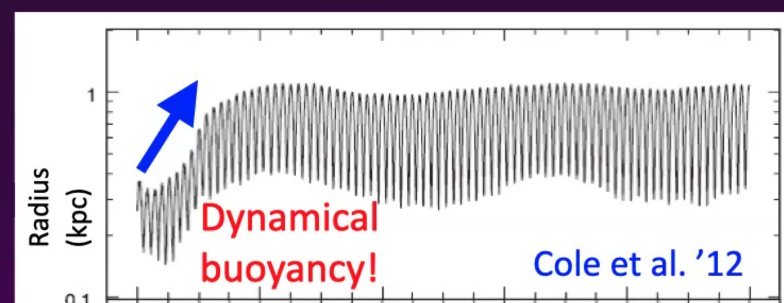


Chandrasekhar, resonance theories work in cuspy galaxy.

DO NOT work in cored galaxy.

Core-stalling

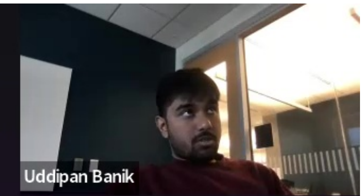
Dynamical buoyancy



Require general theory that can explain all aspects of perturbative dynamics.

Banik & van den Bosch ’21a (ApJ), Banik & van den Bosch ’22 (ApJ), Banik & van den Bosch in prep

Going beyond standard theory



Linear perturbation theory

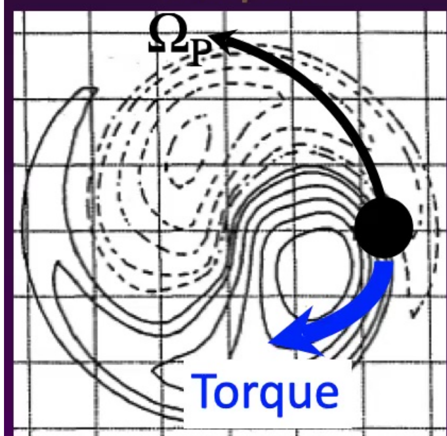
Perturber potential

Action-angle ↓ Fourier space of angles

Linear response

↓

Torque



Weinberg '89

Standard theory

Adiabatic approx

Secular approx → LBK Torque

Slow growth + Slow in-fall of perturber

$$l_k\Omega_k = l_1\Omega_1 + l_2\Omega_2 - l_3\Omega_P = 0$$

$$\tau_{LBK} = 16\pi^4\Omega_P \sum_{\vec{l}}^{\text{half}} l_3^2 \int d\vec{I}\, \delta\left(l_k\Omega_k\right) \frac{\partial f_0}{\partial E_0} \left|\Phi_{\vec{l}}\right|^2$$

Only resonances

< 0 : Always retarding

Only friction!

Going beyond standard theory

$l_1\Omega_1 + l_2$

~~Linear perturbation theory~~

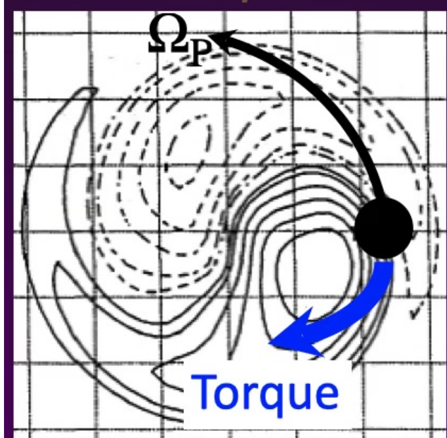
Perturber potential

Action-angle ↓ Fourier space of angles

Linear response

↓

Torque



Weinberg '89

Standard theory

Adiabatic approx

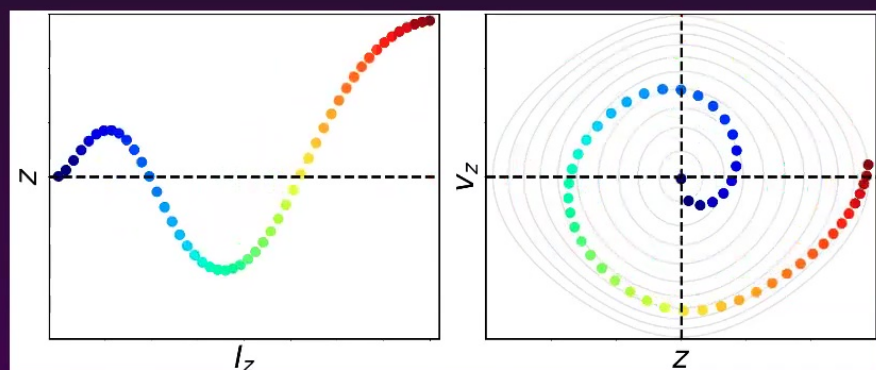
Secular approx

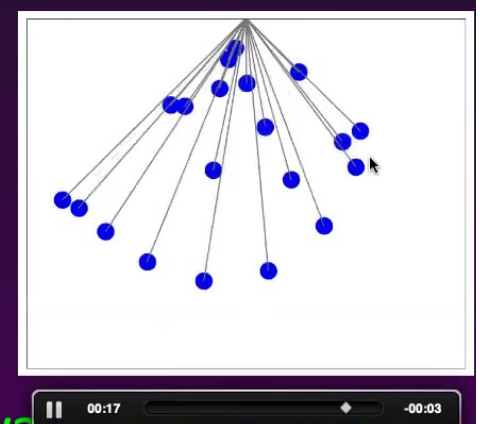
→ LBK Torque

- Only resonances
- Always friction

$$\sim \frac{\partial f_0}{\partial E_0} < 0$$

Slow growth + Slow in-fall of perturber





Near-resonant orbits phase-mix away: phase-spirals

$t \to \infty$: Only resonances

Going beyond standard theory

$l_1\Omega_1 + l_2$

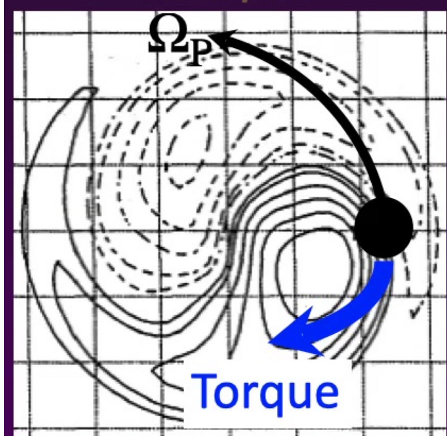
Linear perturbation theory

Perturber potential

Action-angle | Fourier space of angles

Linear response

Torque



Weinberg '89

Standard theory

Adiabatic approx

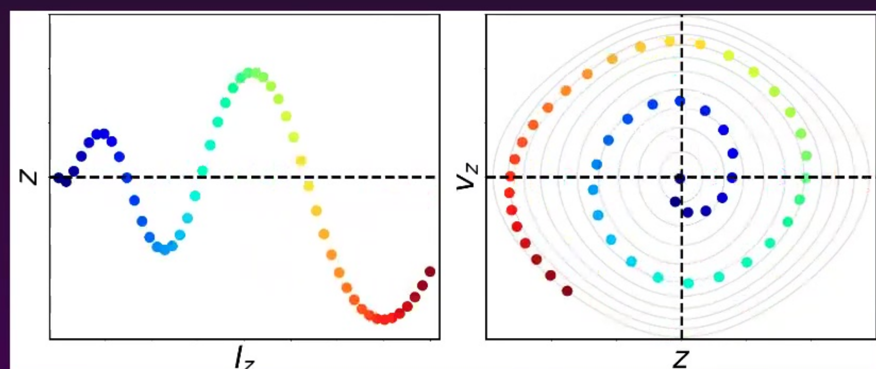
Secular approx

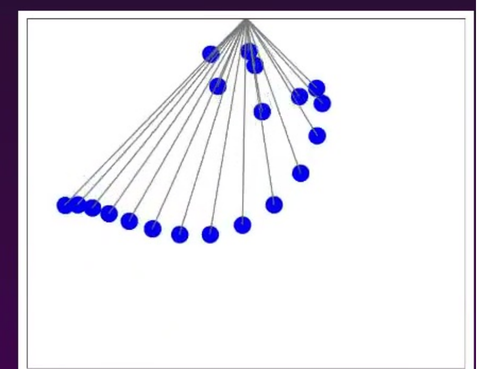
LBK Torque

- Only resonances
- Always friction

$$\sim \frac{\partial f_0}{\partial E_0} < 0$$

Slow growth + Slow in-fall of perturber

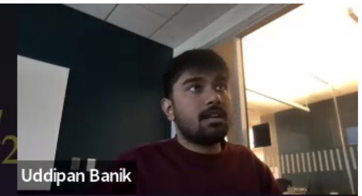




Near-resonant orbits phase-mix away: phase-spirals

$t \to \infty$: Only resonances

Going beyond standard theory



$l_1\Omega_1 + l_2$

Linear perturbation theory

Perturber potential

Action-angle ↓ Fourier space of angles

Linear response

↓

Torque

Standard theory

Adiabatic approx

Secular approx

→ LBK Torque

Slow growth + Slow in-fall of perturber

- Only resonances
- Always friction $\sim \dfrac{\partial f_0}{\partial E_0} < 0$

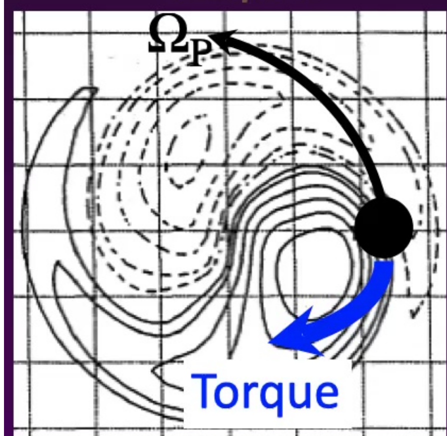
General theory → Self-consistent torque

Relax adiabatic and secular approximations

Self-consistent approach

Torque → Change in R → Torque

$\Longrightarrow$ Change in $\Phi'_{\mathrm{P}}, \Omega_{\mathrm{P}}$



Weinberg '89

Banik & van den Bosch '21a (ApJ)

Self-consistent Torque

$l_k \Omega_k = l_1 \Omega_1$

Time
through radius $R(t)$

Relax adiabatic and
secular approximations

LBK torque

$$\pi\, \delta(l_k \Omega_k) \left| \Phi_{\vec{l}} \right|^2$$

Self-consistent torque

$$\Phi^*_{\vec{l}}\left(\vec{I}, t\right) \int_0^t d\tau \, \cos \zeta(\tau)\, \Phi_{\vec{l}}\left(\vec{I}, t-\tau\right)$$

$$\zeta(\tau) = \left(l_1 \Omega_1 + l_2 \Omega_2\right) \tau - l_3 \int_0^\tau dt' \, \Omega_{\mathrm{P}}(t')$$

Torque

Change in R

$\Longrightarrow$ Change in $\Phi'_{\mathrm{P}}, \Omega_{\mathrm{P}}$

Self-consistent Torque

$l_k \Omega_k = l_1 \Omega_1$

Time
through radius $R(t)$

Relax adiabatic and
secular approximations

LBK torque

$$\pi\,\delta(l_k\Omega_k)\left|\Phi_{\vec{l}}\right|^2$$

Self-consistent torque

$$\Phi^*_{\vec{l}}\left(\vec{I},t\right)\int_0^t d\tau\,\cos\zeta(\tau)\,\Phi_{\vec{l}}\left(\vec{I},t-\tau\right)$$

$$\zeta(\tau) = \left(l_1\Omega_1 + l_2\Omega_2\right)\tau - l_3\int_0^\tau dt'\,\Omega_{\mathrm{P}}(t')$$

1. Temporal correlation

Torque depends on the *entire in-fall history* of the perturber.

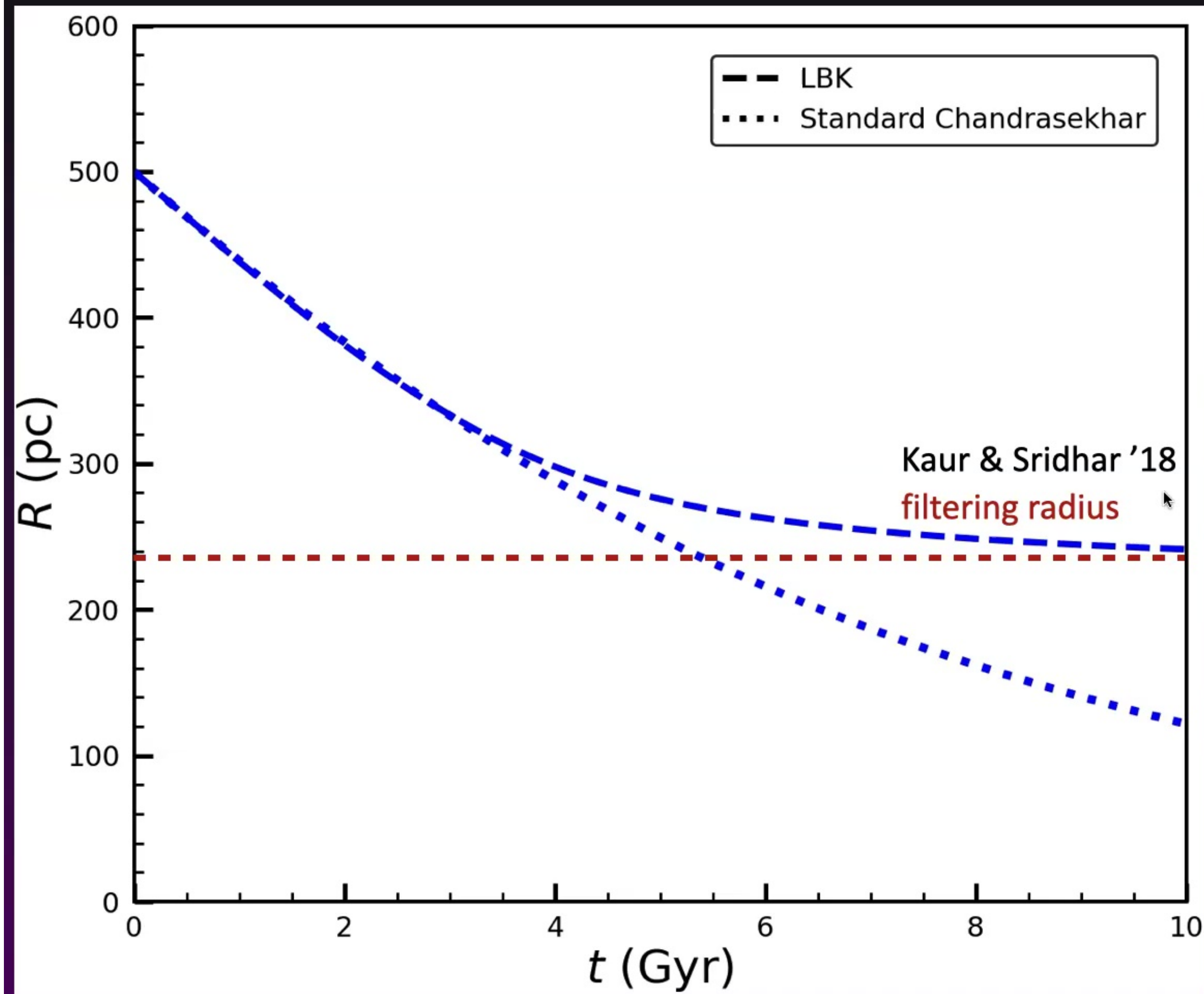
Memory Effect

2. Near-resonant orbits

Orbits fall *in and out of resonance* as the perturber moves.

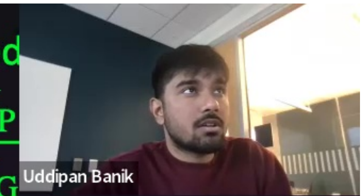
Sweeping through resonances

Orbital evolution



Cored

$\frac{M_P}{M_G}$



Core-stalling from LBK torque

Suppression of near-co-rotation resonances

$$R < R_*$$

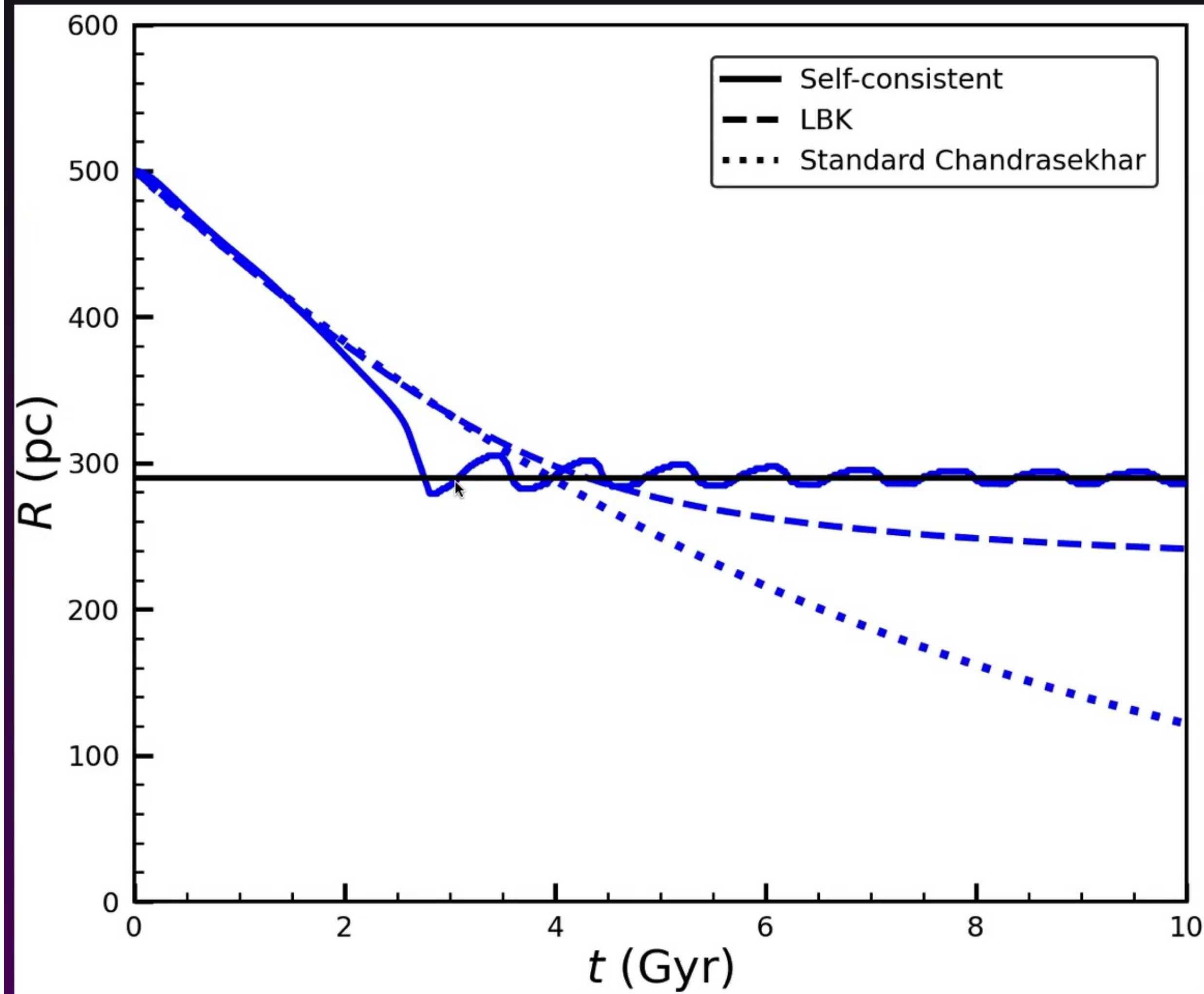
Circular frequency

$$\Omega_P = \sqrt{\frac{G\left[M_G(R) + M_P\right]}{R^3}} > \Omega_c \sim \sqrt{G\rho_c}$$

Central core frequency

Perturber orbits faster than core stars.

Orbital evolution



Cored

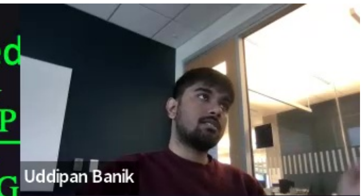
$\frac{M_P}{M_G}$

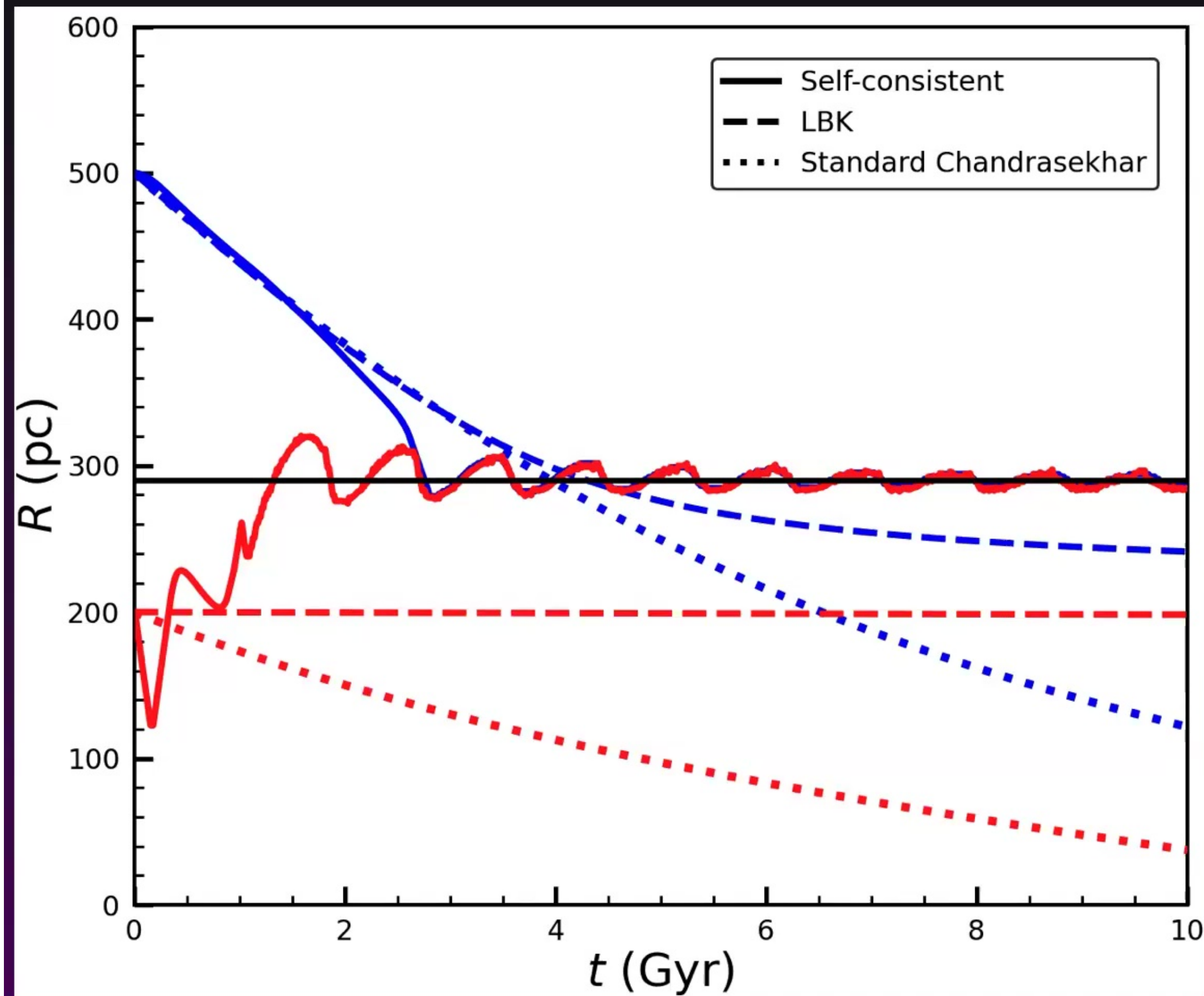
Uddipan Banik

Banik & van den Bosch '21a (ApJ)

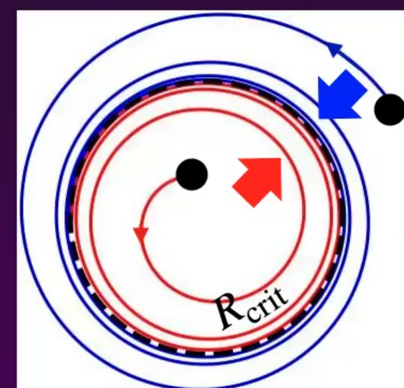
Orbital evolution

Cored $\frac{M_P}{M_G}$

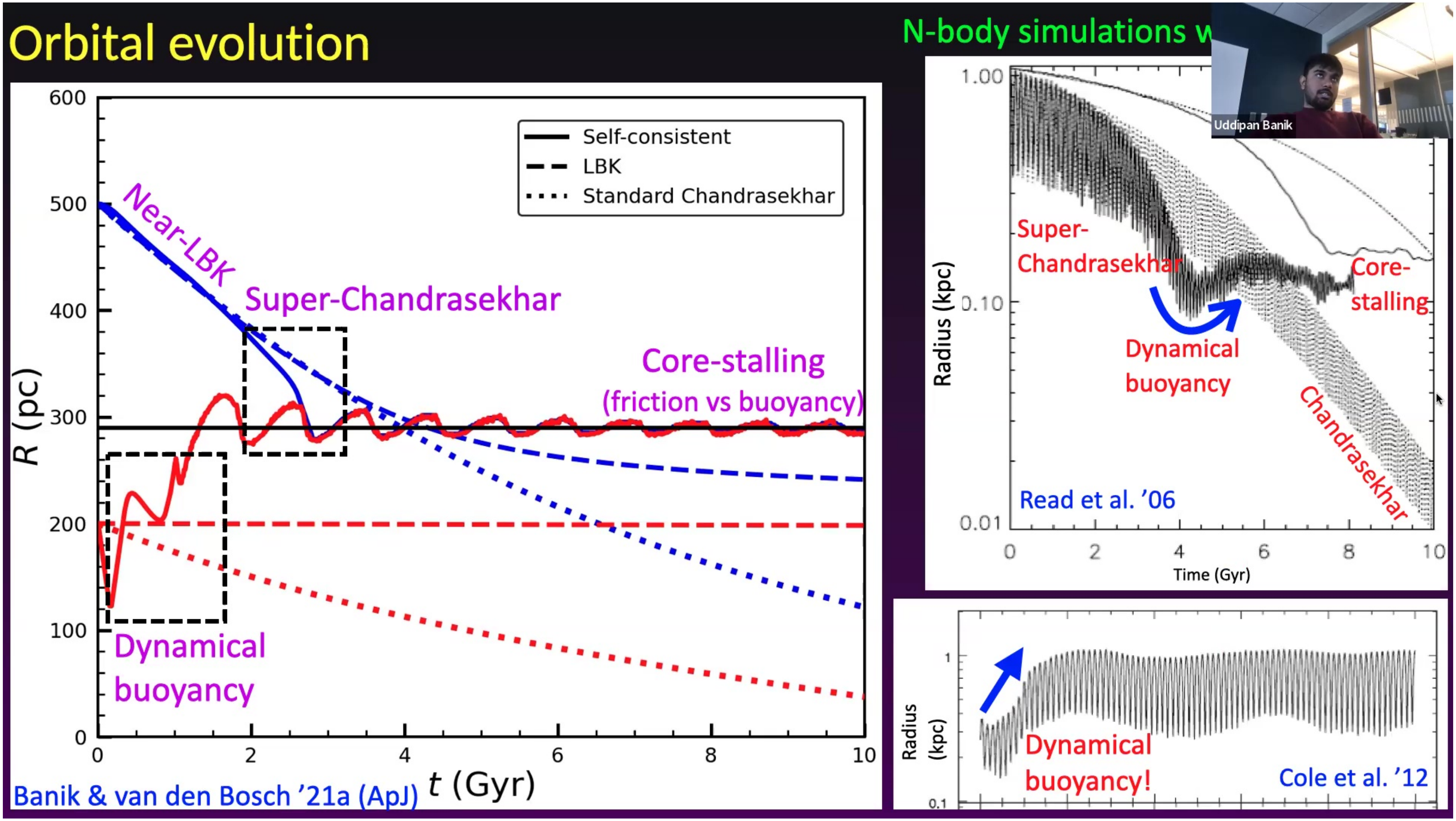




Core-stalling radius is a *stable limit cycle.*



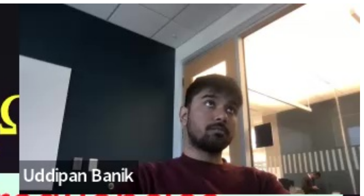
Banik & van den Bosch ’21a (ApJ)



Orbital evolution
N-body simulations w
Self-consistent
LBK
Standard Chandrasekhar
Near-LBK
Super-Chandrasekhar
Core-stalling
(friction vs buoyancy)
Dynamical
buoyancy
R (pc)
t (Gyr)
Banik & van den Bosch '21a (ApJ)
Super-
Chandrasekhar
Core-
stalling
Dynamical
buoyancy
Chandrasekhar
Read et al. '06
Radius (kpc)
Time (Gyr)
Dynamical
buoyancy!
Cole et al. '12
Radius
(kpc)
Uddipan Banik

Beyond perturbation theory

$l_1\Omega_1 + l_2\Omega$



1. Perturbation theory assumes $M_P < M_G(R)$.
   Core-stalling occurs when $M_P \sim M_G(R)$.

2. Resonances between unperturbed frequencies.
   But resonances themselves perturb the frequencies!

3. Linear perturbation theory is okay for 'fast in-fall' (somewhat far from stalling radius).

   Not okay for 'slow in-fall' (near stalling radius).
   Stars librate about resonances (adiabatic trapping).

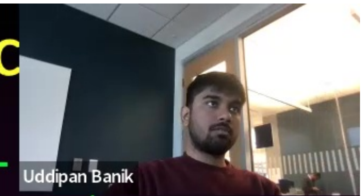
Non-linear perturbations develop.

Large changes in energy and angular momentum (slow action) of stars

Perturbation theory is questionable near the stalling radius. → Non-perturbative approach required!

Banik & van den Bosch '22 (ApJ)

Perturbation of collisionless systems: Lagrangian approac



Perturbation of $(\vec{r}, \vec{v})$ or $(\vec{w}, \vec{I})$ of each star

$l_1\Omega_1 + l_2\Omega_2 + l_3\left(\Omega_3 -\right.$

Impulsive

$\Delta\vec{v} \neq 0$, $\Delta\vec{r} \approx 0$: Velocities change, positions don't.

$$\Delta\vec{v}(\vec{r},t) \approx -\int_{-\infty}^{t} \vec{\nabla}\Phi_{\rm P}(\vec{r},t')\,dt'$$

Adiabatic

$\Delta\vec{I} \approx 0$: Actions invariant away from resonances

$$I_i = \oint dr_i\, p_i$$

$$w_i = \Omega_i t$$

Resonances

$\Delta I_s \neq 0$, $\Delta w_s \neq 0$: Slow action-angle change,

$\Delta I_{fi} = 0$: fast actions don't.

Slow action $I_s = \dfrac{I_3}{l_3}$

Fast actions $I_{fi} = I_i - \dfrac{l_i}{l_3} I_3 \quad \forall\, i = 1,2$

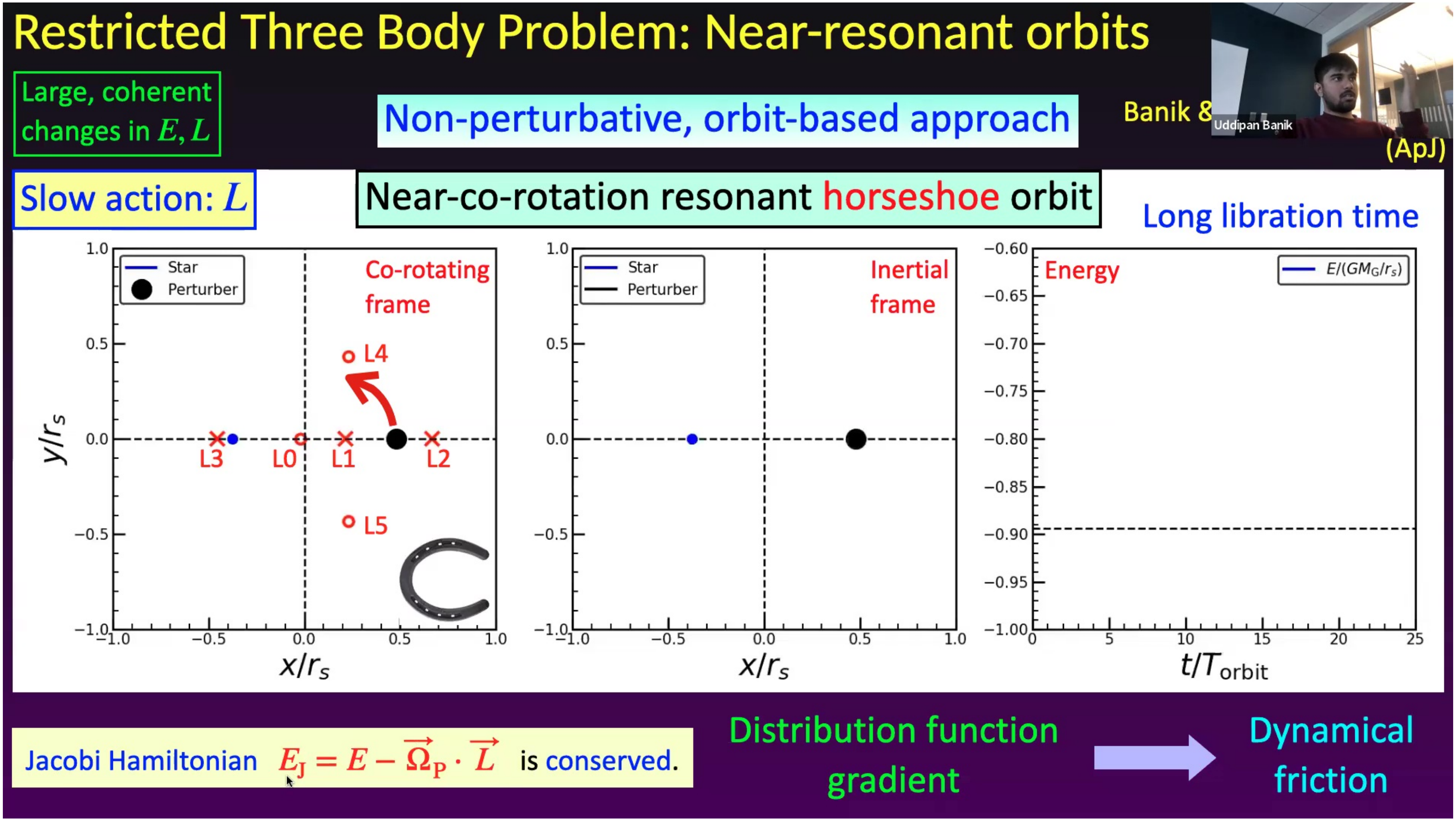
Slow angle $w_s = l_1 w_1 + l_2 w_2 + l_3\left(w_3 - \Omega_{\rm P} t\right)$

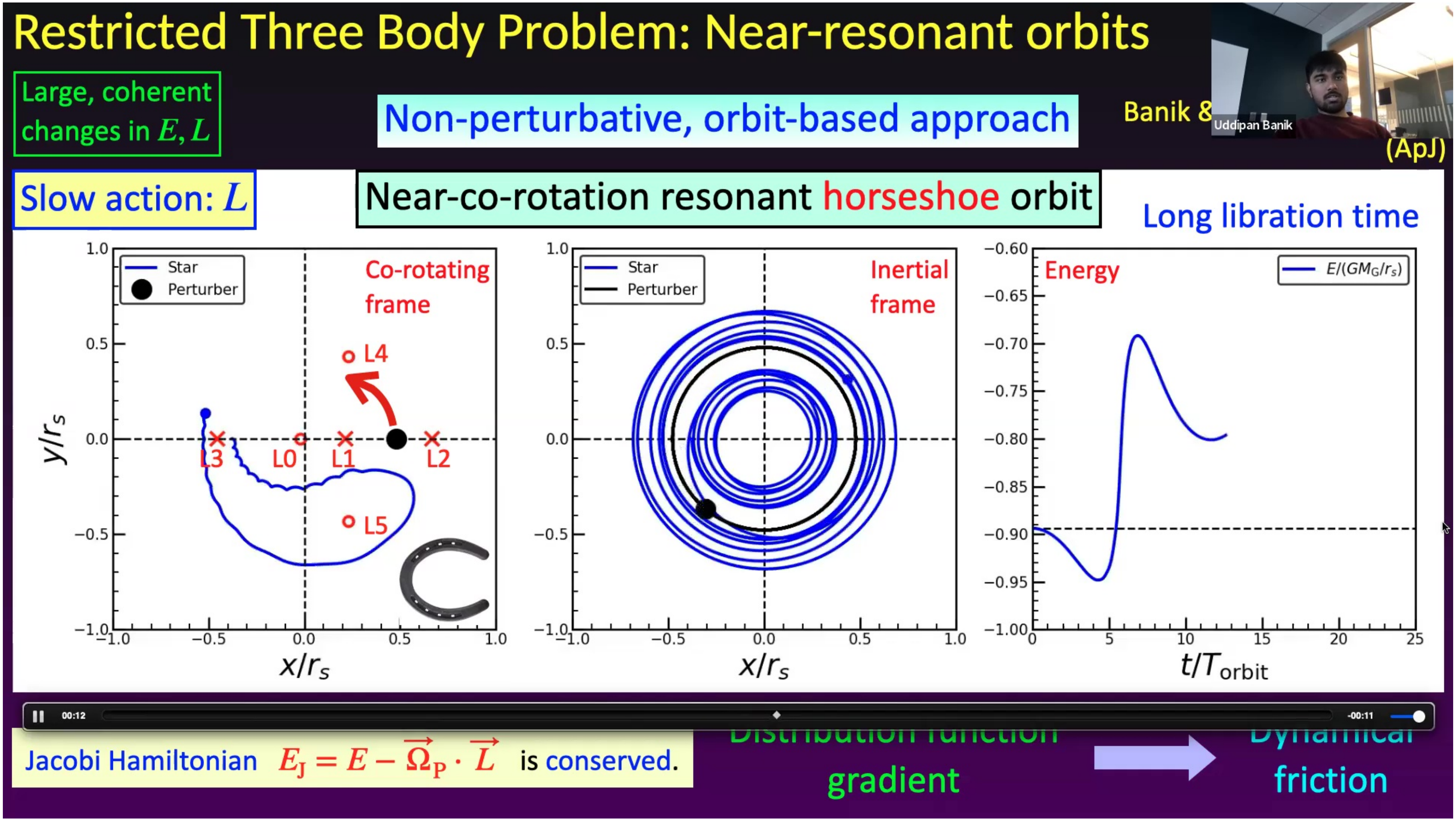
Fast angles $w_{fi} = w_i \quad \forall i = 1,2$

Adiabatic invariance of actions is partially broken near the resonances.

Secular evolution!
Slow

Tremaine & Weinberg ’84
Chiba & Schonrich ’22
Hamilton et al. ’22


Restricted Three Body Problem: Near-resonant orbits
Large, coherent changes in E, L
Non-perturbative, orbit-based approach
Banik &
Uddipan Banik
(ApJ)
Slow action: L
Near-co-rotation resonant horseshoe orbit
Long libration time
Star
Perturber
Co-rotating frame
L4
L3
L0
L1
L2
L5
y/r_s
x/r_s
Inertial frame
Energy
$E/(GM_G/r_s)$
$t/T_{\rm orbit}$
Jacobi Hamiltonian $E_J = E - \vec{\Omega}_P \cdot \vec{L}$ is conserved.
Distribution function gradient
Dynamical friction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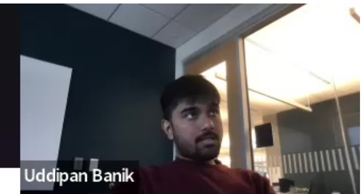
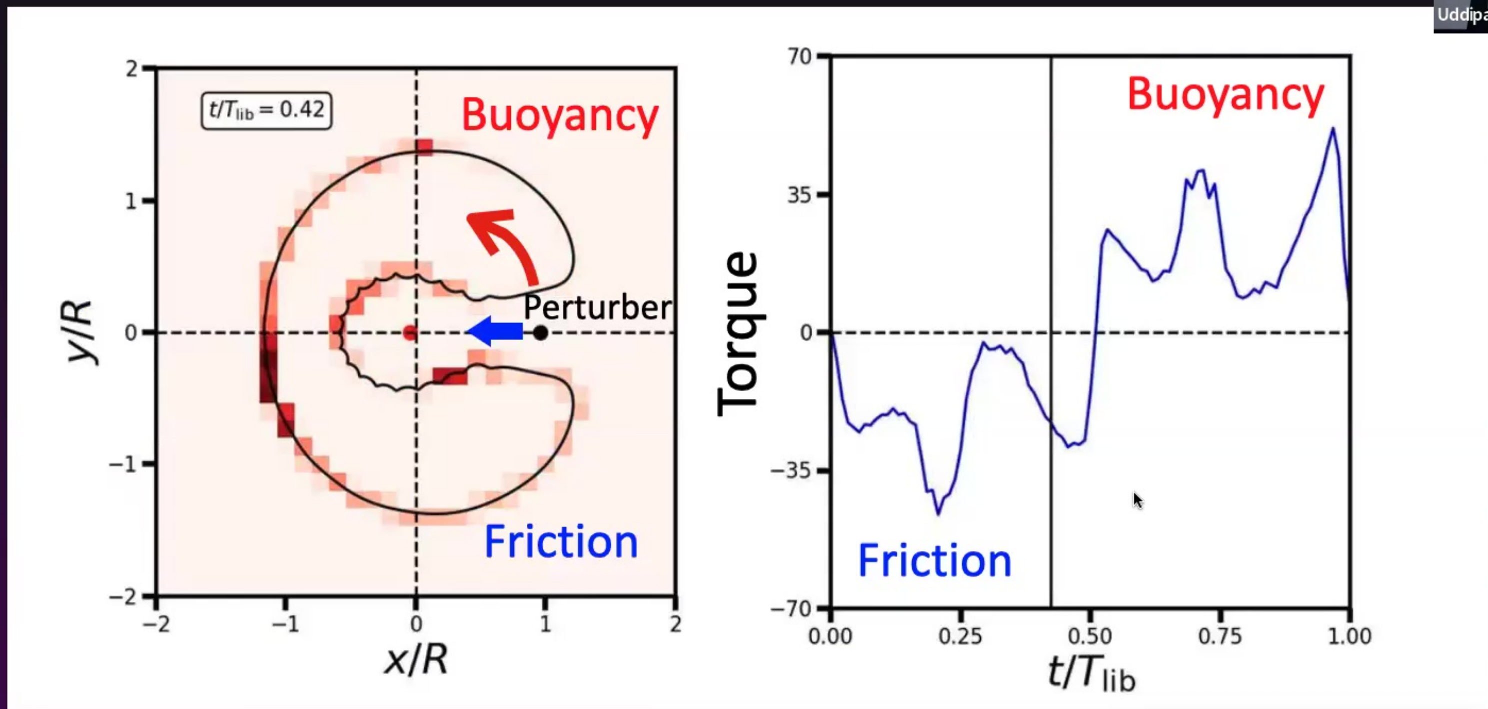
L4
L3
L0
L1
L2
L5
y/r_s
x/r_s
Inertial frame
Energy
$E/(GM_G/r_s)$
t/T_{orbit}
00:12
-00:11
Jacobi Hamiltonian $E_J = E - \vec{\Omega}_P \cdot \vec{L}$ is conserved.
gradient
friction

Dynamical friction from near-resonant orbits



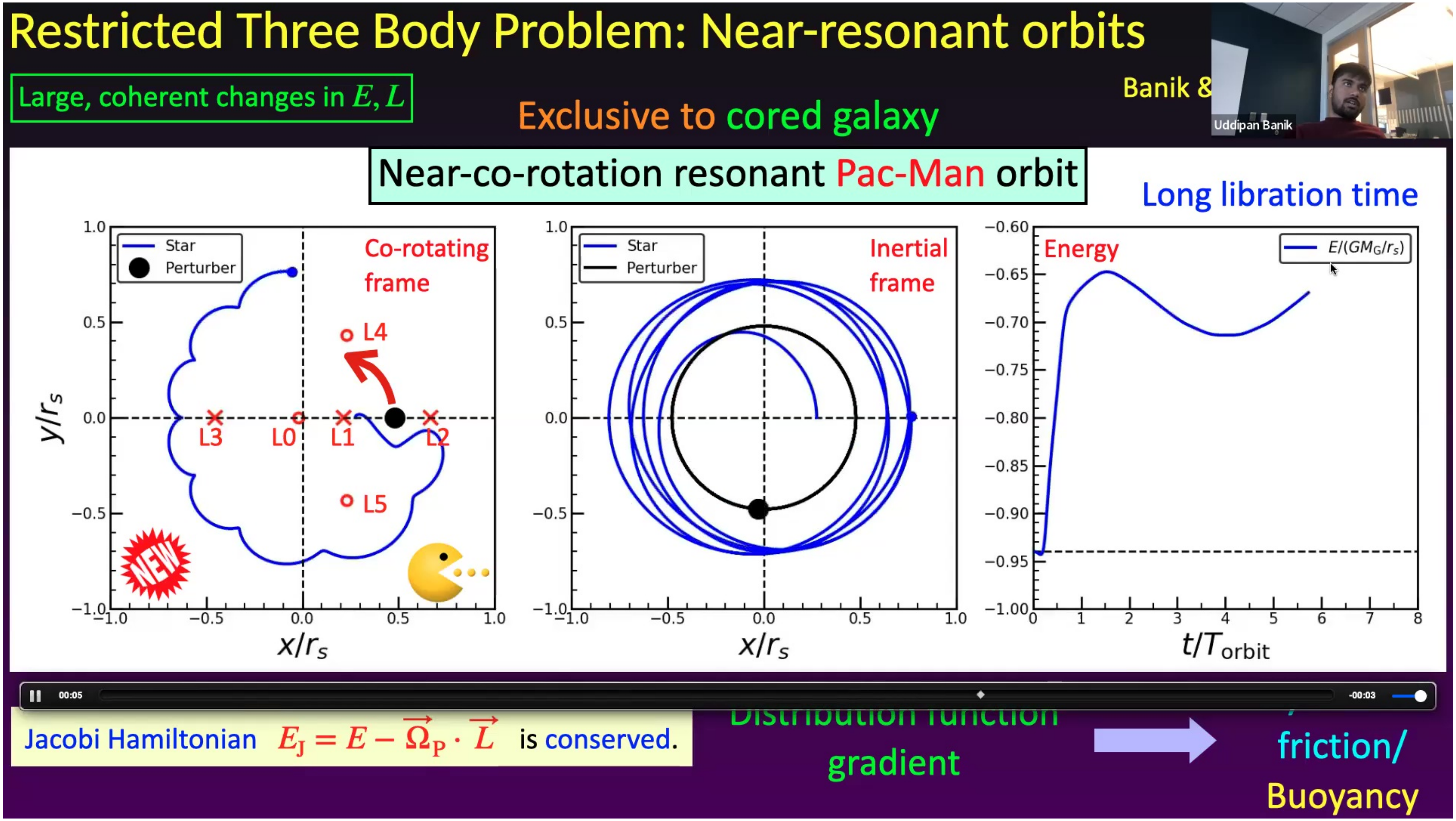
Outside
core



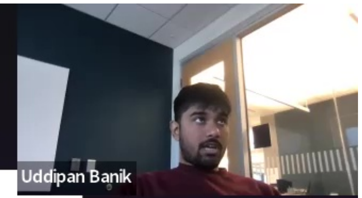
an den Bosch '22
(ApJ)

Distribution function gradient

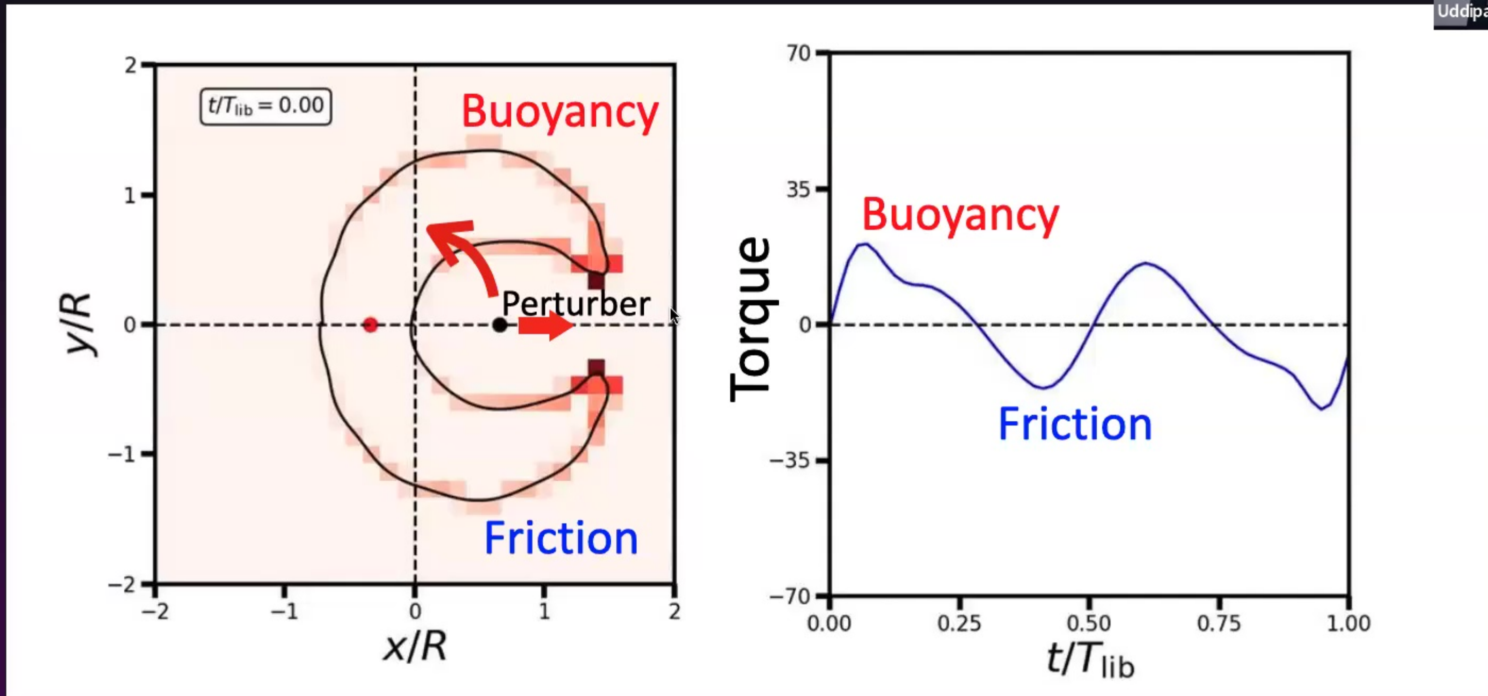
Initial retarding torque → Dynamical friction


Restricted Three Body Problem: Near-resonant orbits
Large, coherent changes in E, L
Banik &
Exclusive to cored galaxy
Uddipan Banik
Near-co-rotation resonant Pac-Man orbit
Long libration time
Star
Perturber
Co-rotating frame
L4
L3
L0
L1
L2
L5
NEW
y/r_s
x/r_s
Inertial frame
Energy
$E/(GM_G/r_s)$
t/T_{orbit}
00:05
-00:03
Jacobi Hamiltonian $E_J = E - \vec{\Omega}_P \cdot \vec{L}$ is conserved.
gradient
friction/
Buoyancy

Dynamical buoyancy from near-resonant orbits



Inside core

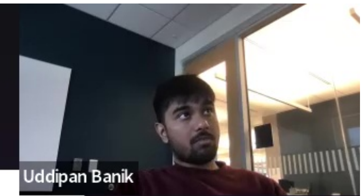


Banik & van den Bosch '22 (ApJ)

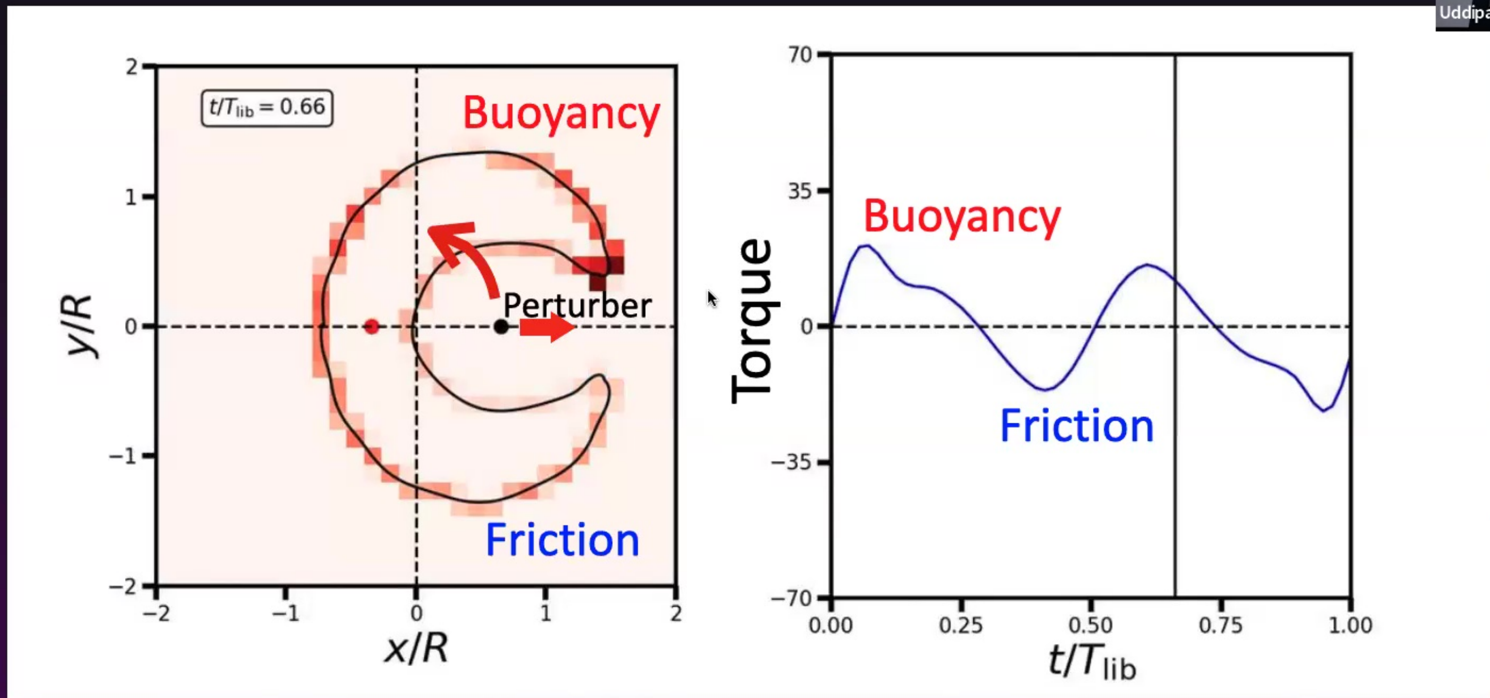
Distribution function gradient

Initial enhancing torque → Dynamical buoyancy

Dynamical buoyancy from near-resonant orbits



Inside core



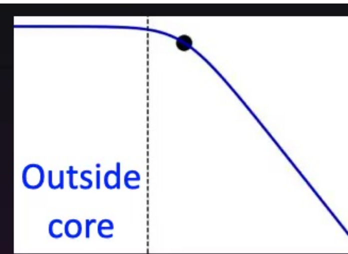
...an den Bosch '22 (ApJ)

Distribution function gradient

Initial enhancing torque → Dynamical buoyancy

Orbital Configuration

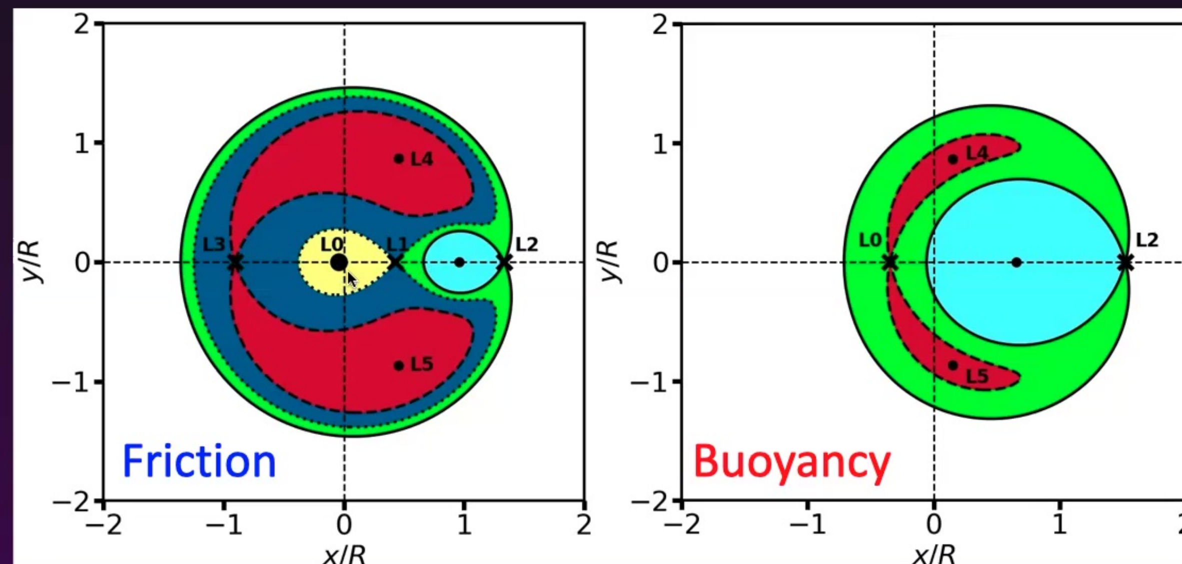
Plummer core



Orbit family	Importance for DF
Tadpoles	Y
Horseshoes	Y
Pac-Mans	Y
Center-phylic	N
Perturber-phylic	N

Tidal disruption of core

Petts et al. '16



Banik & van den Bosch '22 (ApJ)

Friction outside core, buoyancy inside core

Bifurcation: Stalling

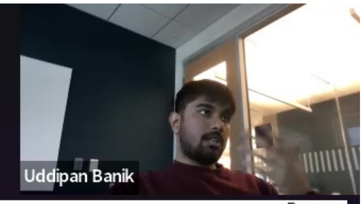
Circular frequency

$$\Omega_{\rm P} = \sqrt{\frac{G\left[M_{\rm G}(R) + M_{\rm P}\right]}{R^3}}$$

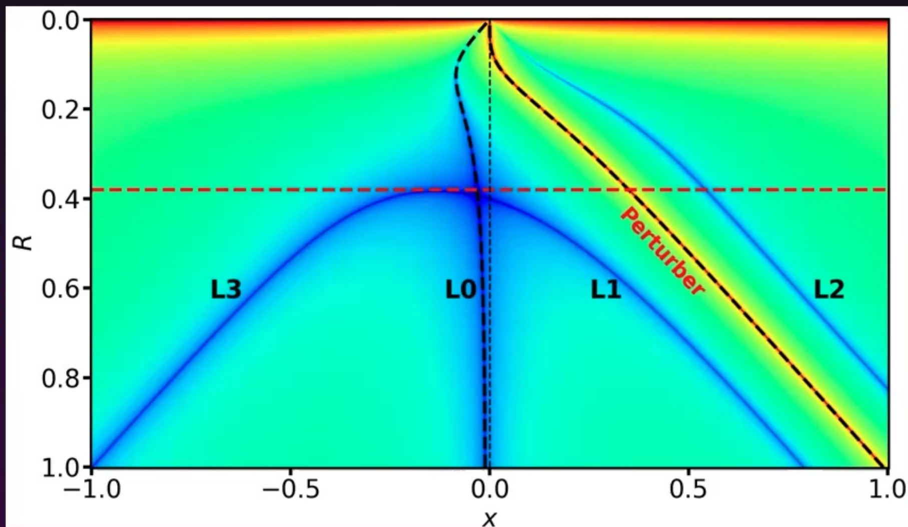
Equipotential contours

$$\Phi_{\rm eff} = \Phi_{\rm G} + \Phi_{\rm P} - \frac{1}{2}\left|\vec{\Omega}_{\rm P} \times \vec{r}\right|^2 = E_{\rm J}$$

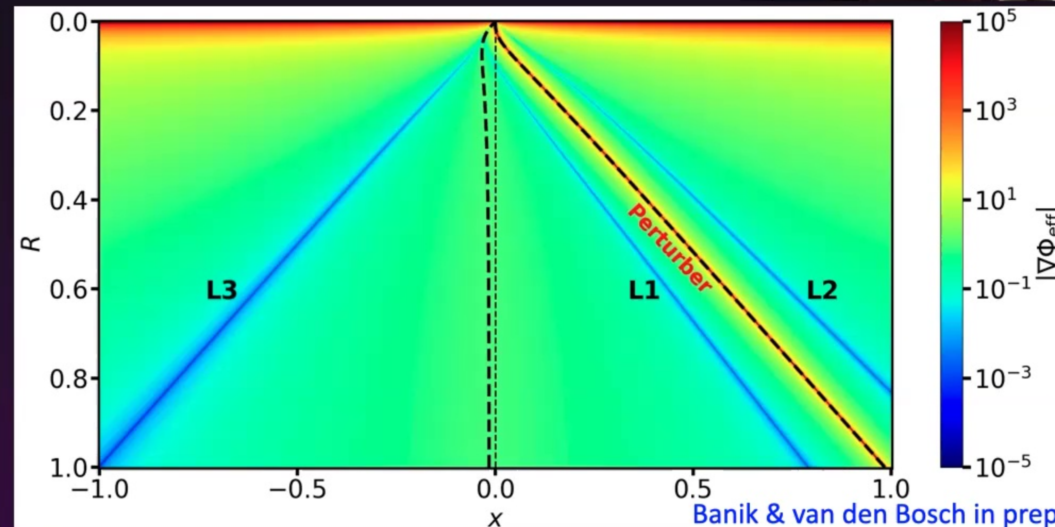
Bifurcation of Lagrange points: core-stalling



Plummer core



Hernquist cusp



Pitch-fork Bifurcation

Stalling: Yes

NO Bifurcation

Stalling: No

Effective potential

$$\Phi_{\rm eff} = \Phi_{\rm G} + \Phi_{\rm P} - \frac{1}{2}\left|\vec{\Omega}_{\rm P} \times \vec{r}\right|^2$$

Bifurcation radius: $\dfrac{\partial^2\Phi_{\rm eff}}{\partial x^2} = \dfrac{\partial\Phi_{\rm eff}}{\partial x} = 0$

Banik & van den Bosch in prep

- Horseshoe orbits disappear: suppression of lower order resonances (Kaur & Sridhar ’18)
- Libration time of near-resonant orbits $\rightarrow \infty$

Astrophysical Implications

Dynamical buoyancy & core-stalling

- *Supermassive black hole (SMBH) mergers* can be *choked* in *cored galaxies*.

  Off-center black holes (Banik & van den Bosch '19 (MNRAS); Mezcua & Dominguez Sanchez '20; Reines et al. '20)

- Implications for *SMBH merger rates* from *LISA* GW observations

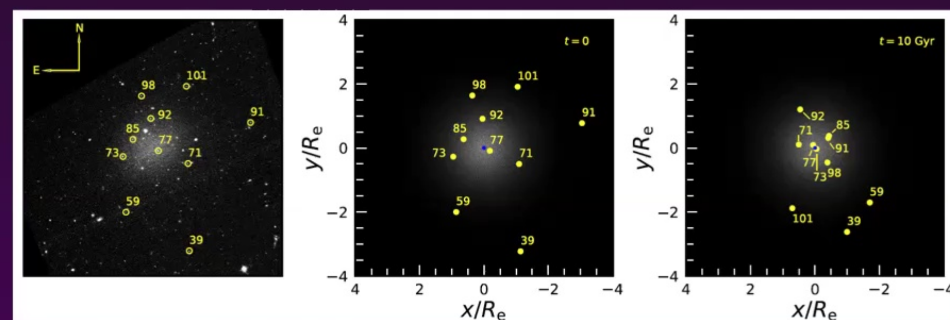
- *Stalled globular cluste*[rs in] dwarf galaxies like *Fornax*: *Dark matter core*?

  (Goerdt et al. 10, Cole et al. 12)

- *Stalled GCs* of *UDG 1052-DF2*?

  (Dutta Chowdhury et al. 19, 20)

L. Boco et al '20



Dutta Chowdhury et al. '19

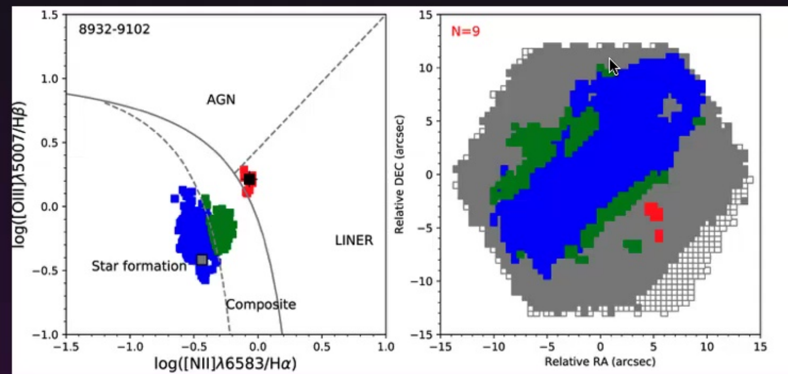
Constraining the nature of dark matter

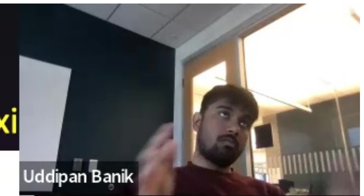
Off-center perturbers

- **Dark matter dominated galaxies** with *central nuclear star clusters (NSCs)* and/or *supermassive black holes (SMBHs) DISFAVOR A DARK MATTER CORE*.

- *Rule out cored dark matter*, unless *NSCs* and *SMBHs* are *OFFSET*.

Off-center AGNs in dwarf galaxi



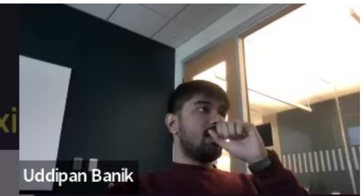


Mezcua & Dominguez Sanchez '20

Galactic nuclei

https://www.mpia.de/galactic-nuclei/projects/formation

Constraining the nature of dark matter



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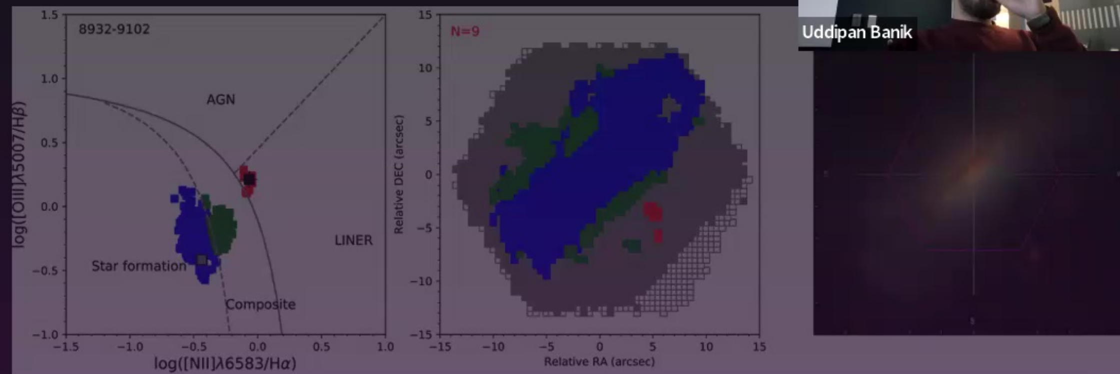

Mezcua & Dominguez Sanchez '20

Cold dark matter (CDM) vs Fuzzy dark matter (FDM)

CDM stalling radius

$$\frac{R_{cs}}{r_s} \sim \begin{cases} q^{\frac{1}{4}} \\ q^{\frac{1}{5}} \end{cases}$$

Mass ratio $q = \frac{M_P}{M_G}$

CDM: Stalling radius increases with mass-ratio

Opposite for FDM

Bar-Or et al. '19

Can infer if dark matter is cold or fuzzy!

Banik & van den Bosch in prep.

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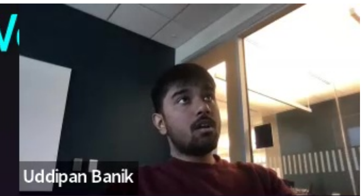
Constraining the nature of dark matter

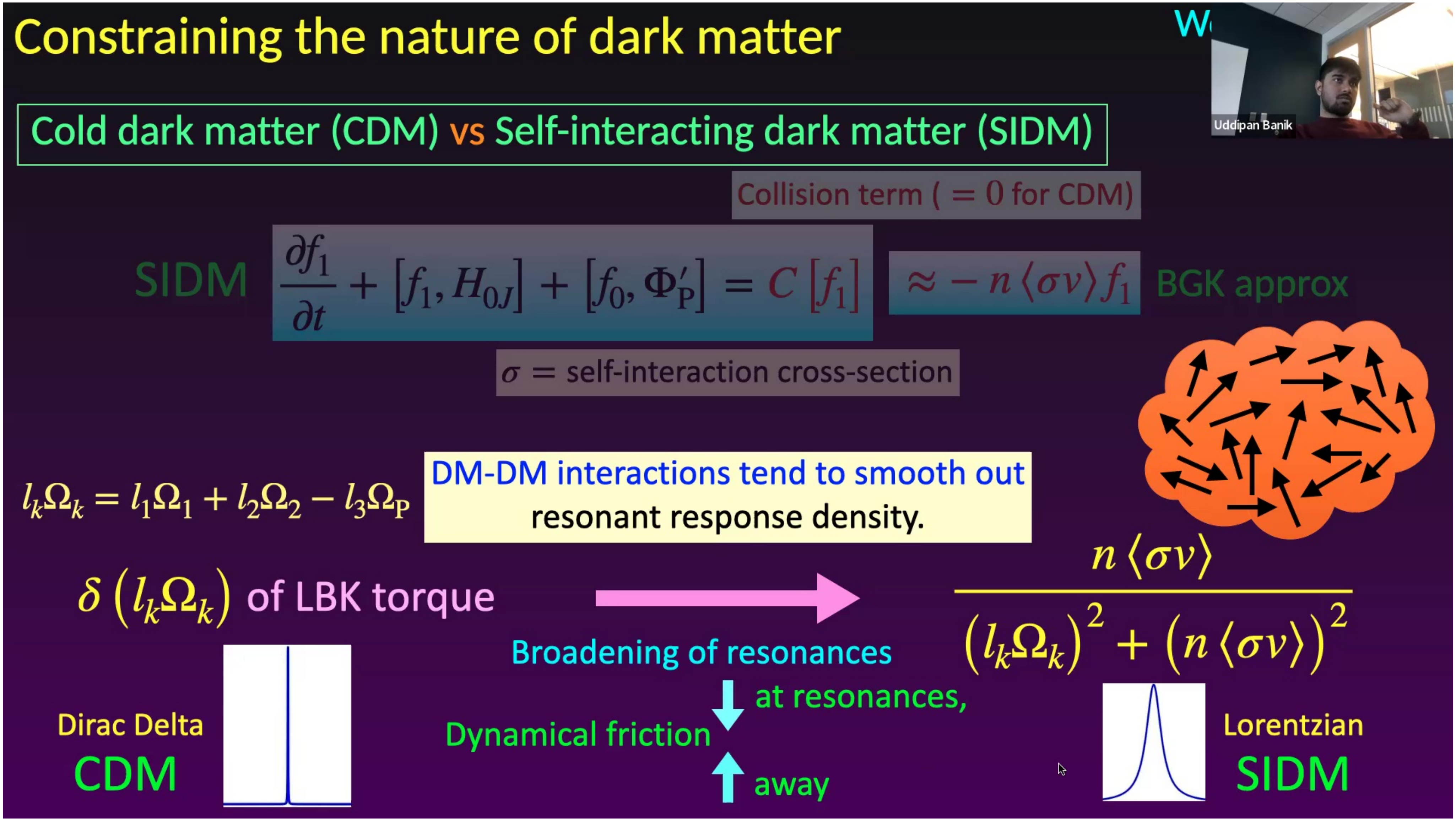


Cold dark matter (CDM) vs Self-interacting dark matter (SIDM)

Collision term ($= 0$ for CDM)

SIDM $$\frac{\partial f_1}{\partial t} + [f_1, H_{0J}] + [f_0, \Phi'_{\rm P}] = C[f_1] \approx -n\langle\sigma v\rangle f_1$$ BGK approx

σ = self-interaction cross-section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σ = self-interaction cross-section
$l_k\Omega_k = l_1\Omega_1 + l_2\Omega_2 - l_3\Omega_P$
DM-DM interactions tend to smooth out resonant response density.
$\delta(l_k\Omega_k)$ of LBK torque
$\frac{n\langle\sigma v\rangle}{(l_k\Omega_k)^2 + (n\langle\sigma v\rangle)^2}$
Broadening of resonances
Dirac Delta
CDM
at resonances,
Dynamical friction
away
Lorentzian
SIDM
Uddipan Banik

Summary

Visit my website: https://uddipanba

Banik & van den Bosch '21a, Banik & van den Bosch '21b, Banik & van den Bosch '22, Banik et al. '22b, Ba

Self-consistent, perturbative & *non-perturbative, orbit-based treatments of dynamical friction* in collisionless

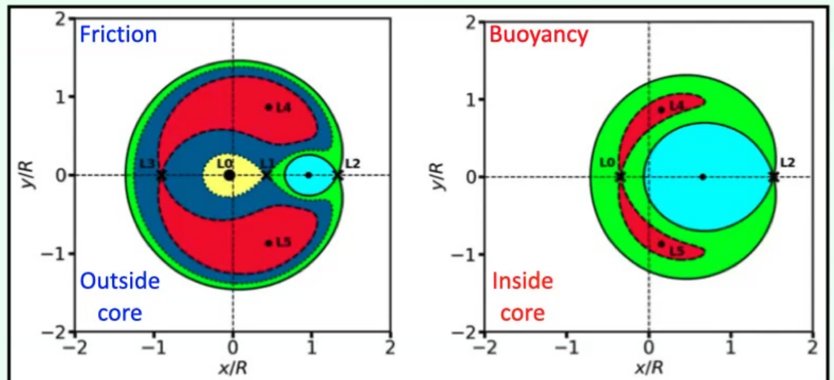
- Dynamical friction due to *near-resonant orbits*
  Near-co-rotation resonant orbits: Horseshoes, Pac-Mans, tadpoles

Core dynamics

- *Buoyancy* due to *non-linear* and *memory* effects,
  pushes out central objects (SMBHs, NSCs) in cored galaxies

- *Outside core*: *Friction* from mainly *horseshoes*
  Inside core: *Buoyancy* from *Pac-Mans*

Horseshoes, *Pac-Mans*, *tadpoles*



(a) Plummer sphere: outside of the core ($R = 0.5$) (b) Plummer sphere: inside of the core ($R = 0.2$)

Bifurcation of Lagrange points → *Core-stalling*

Perturbation theory for *disk response: phase-mixing & phase-spiral*

- Impulsive (large v_P) response is suppressed as v_P^{-1}, adiabatic (small v_P) response is exponentially suppressed.
- *Slower* (*faster*) perturbations cause *bending modes/1-armed phase-spiral* (*breathing modes/2-armed phase-spiral*).
- *Bending* modes are stronger for *smaller impact velocity* and *farther away from the point of impact with satellite galaxy*.
- *MW satellites* excited *bending* modes in the Solar neighborhood during *recent encounters*. Sagittarius dominates.

Scan QR code
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Constraining the nature of dark matter

Off-center AGNs in dwarf galaxi

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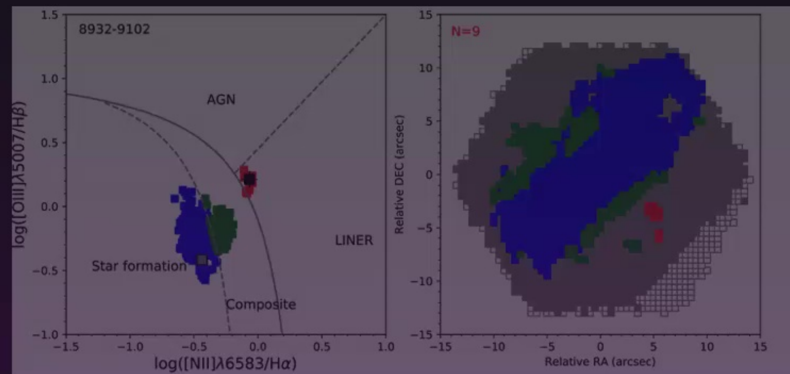

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